Emergent Geometry in String Theory

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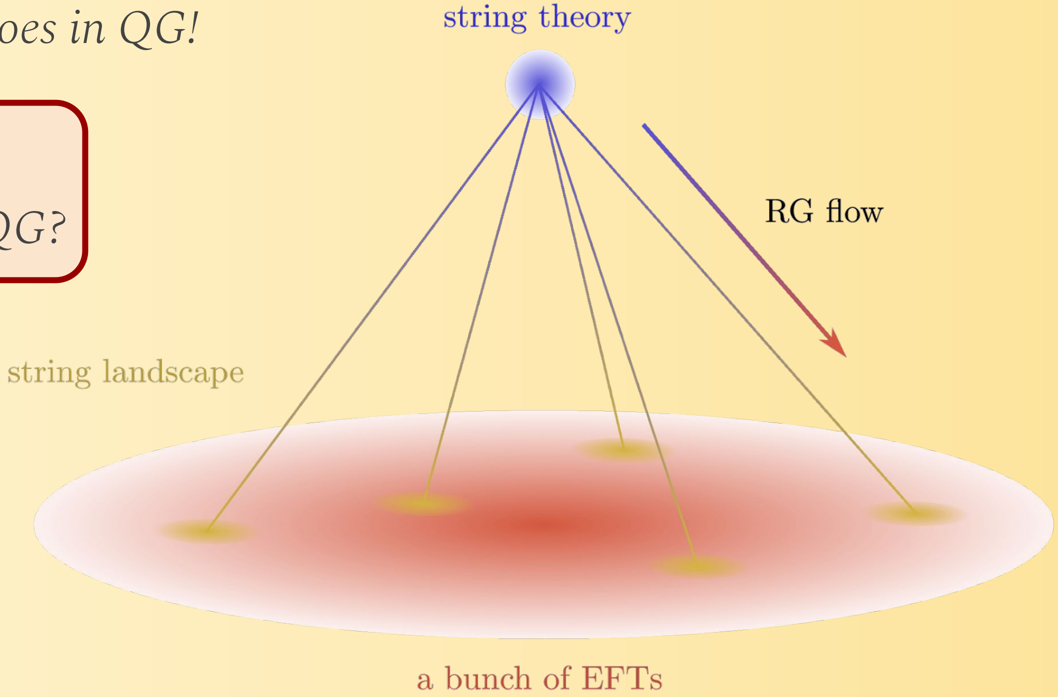
Obligatory slide on the swampland program

swampy lesson

➢ *almost **nothing** goes in QG!*

swampy question

➢ *what **does** go in QG?*



New physics in quantum gravity

$$\mathcal{L}_{\text{eff}} = M_{\text{Pl}}^{d-2}\left(R + \frac{\mathcal{R}^2}{\Lambda_{\text{UV}}^2} + \dots\right) + \sum_k m_{\text{gap}}^{d-2k}\mathcal{R}^{2k}$$

genuine QG effects

field theory d.o.f. to "integrate in"

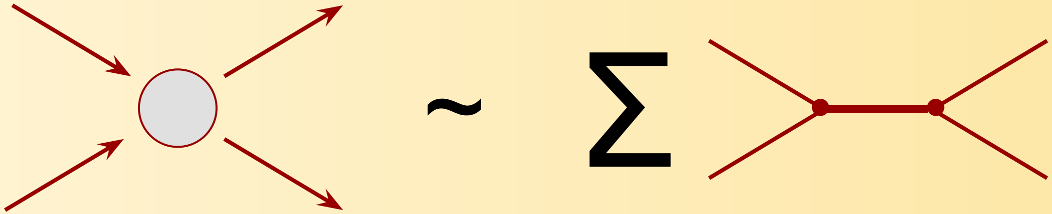
(Castellano, Herráez, Ibáñez, 2023)

- *if new physics is **Planckian*** ⟶ *we're screwed (?)*
- *if* $\Lambda_{\text{UV}} \ll M_{\text{Pl}}$ ⟶ *hope for experiments?*
  - *gravitons are **weakly coupled**: extra dims, or HS towers* *(CEMZ, 2014)*
  - *argued to be **infinite-distance** limits* *(Stout, 2021-2022)*

Living in the (S-)matrix

- *a weakly coupled completion of GR must be* ***consistent @ tree level***

  (Cheung, Remmen, 2017)



- *can "bootstrap" suitable meromorphic (tree-level) amplitudes*
  - *causality (Shapiro delay):* ***higher-spin tower*** *(Camanho, Edelstein, Maldacena, Zhiboedov, 2014)*
  - *graviton amplitudes are* ***restricted*** *to string-like structure & spectra of poles*

(Caron-Huot, Komargodski, Sever, Zhiboedov, 2016) (Geiser, Lindwasser, 2022) (Caron-Huot, Li, Parra-Martinez, Simmons-Duffin, 2022) (Cheung, Remmen, 2022-2024) (Häring, Zhiboedov, 2023) (Arkani-Hamed, Cheung, Figueiredo, Remmen, 2023) (Eckner, Figueroa, Tourkine, 2024) (Berman, Elvang, 2024) (Bhardwaj, Spradlin, Volovich, Weng, 2024) (Albert, Knop, Rastelli, 2024) (Cheung, Remmen, 2024)

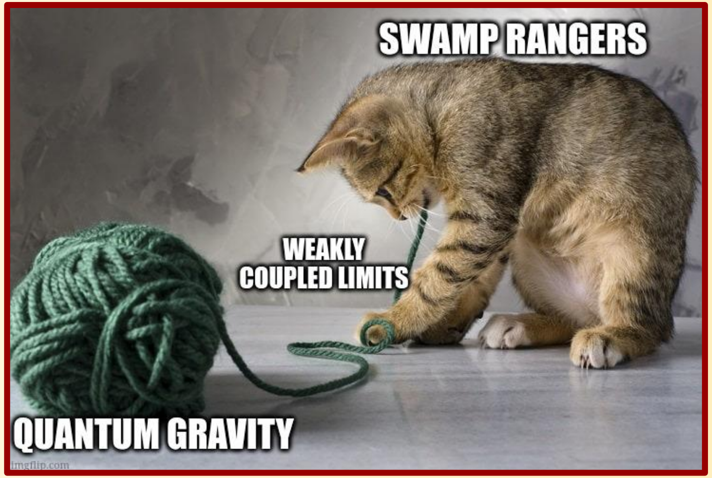
Minimal black holes & thermodynamics

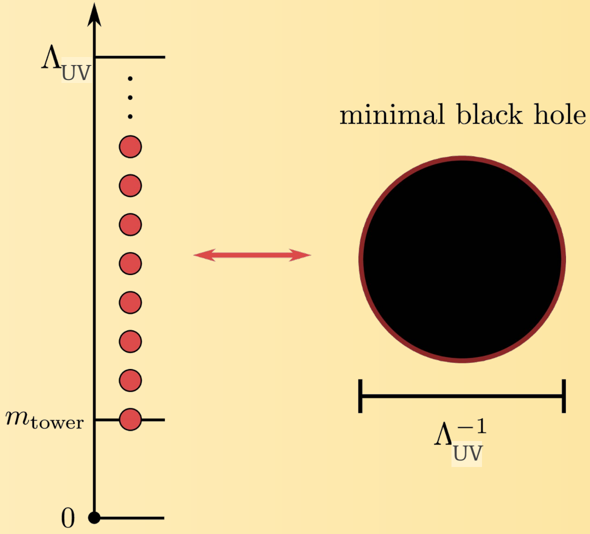
❖ *when UV cutoff is small, BHs know about it:* ***minimal size*** $1/\Lambda_{\rm UV}$

(Veneziano, 2001) (Dvali, 2007) (Dvali, Gomez, Lüst, Isermann, Stieberger, 2009-2014) (Cribiori, Lüst, Staudt, 2022)

➢ *"tower-BH correspondence": **thermodynamic constraints** for minimal BHs*

(IB, (Cribiori), Lüst, Montella, 2023-2024) (Bedroya, Mishra, Wiesner, 2024) (Herráez, Lüst, Masias, Scalisi, 2024)





Game plan

- *weakly coupled string theory (worldsheet)*
- *when is the* ***UV cutoff*** *small? what's the* ***new physics?***
  - *small string coupling* $g_s \ll 1$*: light strings (duh)* 🙄
  - ***other options?*** *central charge requires* ***other d.o.f.*** 🤔

moduli of w.s. CFT

plan: *identify UV cutoff, check* ***what makes it small***

I know what you're thinking...

- *obvious option:* ***extra d.o.f.*** *=* ***extra dimensions***
  - *includes dualities & strong coupling subtleties*

(Lee, Lerche, Weigand, 2019) (Kläwer, Lee, Weigand, Wiesner, 2020) (Álvarez-García, Kläwer, Weigand, 2021)
(Demulder, Lüst, Raml, 2023) (IB, (Cribiori), Lüst, Montella, 2023-2024) (Bedroya, Mishra, Wiesner, 2024)
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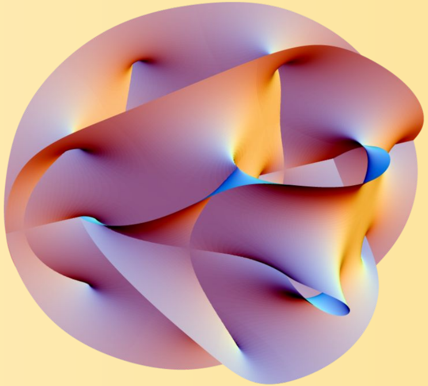

- ***but:*** *"non-geometric" stuff?*
  - *asymmetric orbifolds, rigid mirrors, holography, matrix models, fluxes, ...*

(Kawai, Lewellen, Tye, 1986) (Narain, Sarmadi, Vafa, 1986-1991) (Lerche, Lüst, Schellekens, 1987) (Antoniadis, Bachas, Kounnas, 1987)
(Dixon, Kaplunovsky, Vafa, 1987) (Gepner, 1988) (Seiberg, 1988) (Antoniadis, Bachas, 1988) (Green, Hubsch, 1988) (Kazama, Suzuki, 1989)
(Vafa, Warner, 1989) ((Greene), Vafa, Warner, 1989) (Candelas, de la Ossa, 1990) (Witten, 1993) (Aspinwall, Greene, Morrison, 1993)
(Kachru, Vafa, 1995) (Angelantonj, Bianchi, Pradisi, Sagnotti, Stanev, 1996) (Blumenhagen, Wisskirchen, 1998) (Bianchi, Morales, Pradisi, 2000)
(Blumenhagen, Gorlich, Kors, Lüst, 2000) (Angelantonj, Blumenhagen, Gaberdiel, 2000) (Dabholkar, Hull, 2003) (Hull, 2004) (Shelton, Taylor, Wecht, 2005)
(Israël, Thiéry, 2013) (Hull, Israël, Sarti, 2017) (Demulder, Raml, 2022) (Gkountoumis, Hull, Stemerdink, Vandoren, 2023) (Baykara, (Hamada), Tarazi, Vafa, 2023-2024)

A: when cutoff is small, extra d.o.f. are geometry

Scattering gravitons to get the UV cutoff

- *match EFT w/ 2->2 graviton scattering @ 1-loop*
  - **R^4 coefficient** *from graviton scattering* $\alpha = \Lambda_{\mathrm{UV}}^{-6}$

$$\Lambda_{\mathrm{UV}} \ll M_{\mathrm{Pl}}$$

(Green, Schwarz, Brink, 1982) (Kiritsis, Pioline, 1997) (Green, Gutperle, Vanhove, 1997) (Obers, Pioline, 1999)

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(Blumenhagen, Cribiori, Gligovic, Paraskevopoulou, 2024) (Herráez, Castellano, Ibáñez, 2023) (Bedroya, van de Heisteeg, Vafa, Wiesner, Wu, 2023)

$$\alpha = \left(\frac{M_{\mathrm{Pl}}}{M_s}\right)^{8-d}\left(\underbrace{2\zeta(3)}_{\text{tree- level}}\left(\frac{M_{\mathrm{Pl}}}{M_s}\right)^{d-2} + 2\pi\int_{\mathcal{F}} d\mu\, \underbrace{\mathcal{Z}_{T^2}^{\mathrm{reg}}}_{\text{1-loop}}\right)$$

"reduced" partition function

(Afkhami-Jeddi, Cohn, Hartman, Tajdini, 2021)

depends on CFT moduli {t}

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"emergent string" limit

$$\alpha \overset{g_s \ll 1}{\sim} \left(\frac{M_{\rm Pl}}{M_s}\right)^6$$

p-dims decompactify

$$\int_{\mathcal{F}} d\mu\, \mathcal{Z}^{\rm reg}_{T^2} \sim \left(\frac{m_{\rm gap}}{M_s}\right)^{-p}$$

If it quacks like geometry...

- *using modular invariance,* ***can prove*** *(Aoufia, IB, Leone, 2024)*
  - *Wilson coefficient* ***diverges*** $\Leftrightarrow$ $\exists$ ***light tower***
  - *scaling w/ spectral gap is* ***geometric*** *("emergent geometry")*

∞ distance
(Stout, 2021-2022)
(Ooguri, Wang, 2024)

$$\int_{\mathcal{F}} d\mu\, \widetilde{Z}(t) \overset{t\gg1}{\sim} \Delta_{\text{gap}}(t)^{-\frac{c_{\text{int}}}{2}}$$

bonus: *limiting CFT contains* $\mathbb{R}^N$ *sigma model + KK tower* ✔ *(Ooguri, Wang, 2024)*

Some gory details — existence of light tower

- *assume* N *states go below some threshold weight* Δ_{th}
  - *bound modular integral with strip integral* [τ = x+iy]

$$|\widetilde{\mathcal{Z}_{T^2}}| \leq y^{\frac{c}{2}} \sum_{j,\Delta>0} e^{-2\pi\Delta y} + |E_{\frac{c}{2}} - y^{\frac{c}{2}}|$$

  - *split sum into* $Z_{\text{below}} + Z_{\text{above}}$ *with* $Z_{\text{below}} \leq N$ ⟶ *modular invariance*

$$Z_{\text{above}} \overset{y>1}{\leq} \frac{e^{2\pi\left(\frac{1}{y}-y\right)\Delta_{\text{th}}}}{1-e^{2\pi\left(\frac{1}{y}-y\right)\Delta_{\text{th}}}} Z_{\text{below}} \qquad Z_{\text{above}} \overset{y<1}{\leq} \frac{1}{1-e^{2\pi\left(y-\frac{1}{y}\right)\Delta_{\text{th}}}} Z_{\text{below}}$$

⟶ ***finite N ⇒ finite Wilson coeff.***

More gory details — modular differential equation

- *assume weights are* ***light*** $\Delta = \Delta_0 f(t) \sim \Delta_0/t$ *or* ***heavy*** $\Delta >> 1$
  - ***asymptotic*** *differential equation (akin to Narain lattice sum)*

$$\left(-t^2\partial_t^2 - (2-c)t\partial_t\right)\mathcal{Z}_{T^2} \overset{t\gg1}{\sim} \left(\Delta_\tau - \frac{c}{2}\left(1-\frac{c}{2}\right)\right)\mathcal{Z}_{T^2}$$

- *regulate & integrate over fundamental domain*

(Rankin, 1939) (Selberg, 1940) (Zagier, 1982) (Angelantonj, Florakis, Pioline, 2011) (Angelantonj, Cardella, Elitzur, Rabinovici, 2011)

  - ***geometric scaling*** *of integral*

$$I(t) \overset{t\gg1}{\sim} t^{\frac{c}{2}} \sim \Delta_{\text{gap}}^{-\frac{c}{2}}$$

Last one, I promise — factorized CFT limits

- *relax spectrum to factorization* Z(t) = A(t)B *("partial decompactification")*
  - ➢ ***harmonic decomposition*** *w.r.t. fundamental domain*

    (Benjamin, Collier, Fitzpatrick, Maloney, Perlmutter, 2021)

$$\widetilde{A} = \frac{3}{\pi} I_A(t) + \sum_{n>0} a_n(t)\, \nu_n + \int_{\mathrm{Re}(s)=\frac{1}{2}} ds\, \alpha_s\, E_s$$

preceding result — *Maass cusp forms* — *real analytic Eisenstein series*

$$I_{AB}(t) \overset{t \gg 1}{\sim} a\, t^{\frac{c_A}{2}} + b\, t^{\frac{c_A + c_B - 2}{2}}$$

QG geometric scaling — *field theory gap contribution*

bonus: *log threshold terms [when expected]* ✔

Outlook

❖ *what kind of **new physics** from string theory?*

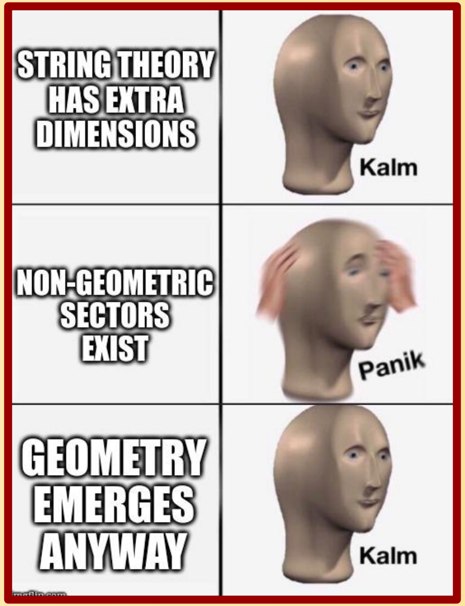
➢ *if **Planckian**: who knows*

➢ *if **sub-Planckian**: extra dimensions (or light strings)*

❖ ***outlook:** combine w/ **observations** for profit & for fun*

➢ *explore more **UV/IR mixing relations***

thank you!



Geometric decompactifications — EFT estimates

- *compactify on* n-*dim. manifold* X
  - *heat kernel* $K_X(t)$ *determines one-loop contribution*

$$S_{\text{1-loop}} \sim -\frac{1}{2(4\pi)^{\frac{d}{2}}} \int_{\Lambda_{\text{st}}^{-2}}^{\infty} \frac{dt}{t^{1+\frac{d}{2}}} K_X(t) \sum_{k\geq 0} a_{2k}(\mathcal{R})\, t^k$$

curvature ops.

  - *"relevant" vs. "irrelevant" ops.*

(IB, Lüst, Montella, 2023) (Aoufia, IB, Leone, 2024)

$$m_{\text{gap}}^{d-2k} \int_{\frac{m_{\text{gap}}^2}{\Lambda_{\text{st}}^2}} \frac{ds}{s}\, s^{k-\frac{d+n}{2}} \sim \begin{cases} \Lambda_{\text{st}}^{2-2k} & 2k<d+n \\ m_{\text{gap}}^{d-2k} & 2k>d+n \end{cases}$$

2k<d+n: $\Lambda_{\text{st}}^{2-2k}$ — Planck scale appears (QG effect)

2k>d+n: m_{gap}^{d-2k} — no Planck scale (field theory effect)

(Castellano, Herráez, Ibáñez, 2023)
(Bedroya, Mishra, Wiesner, 2024)