DYNAMICS AND THE DISTANCE CONJECTURE

Filippo Revello, Utrecht University

Utrecht University

EUROSTRINGS 2024 MEETS FUNDAMENTAL PHYSICS UK

WiP with Thomas Grimm, Damian van de Heisteeg

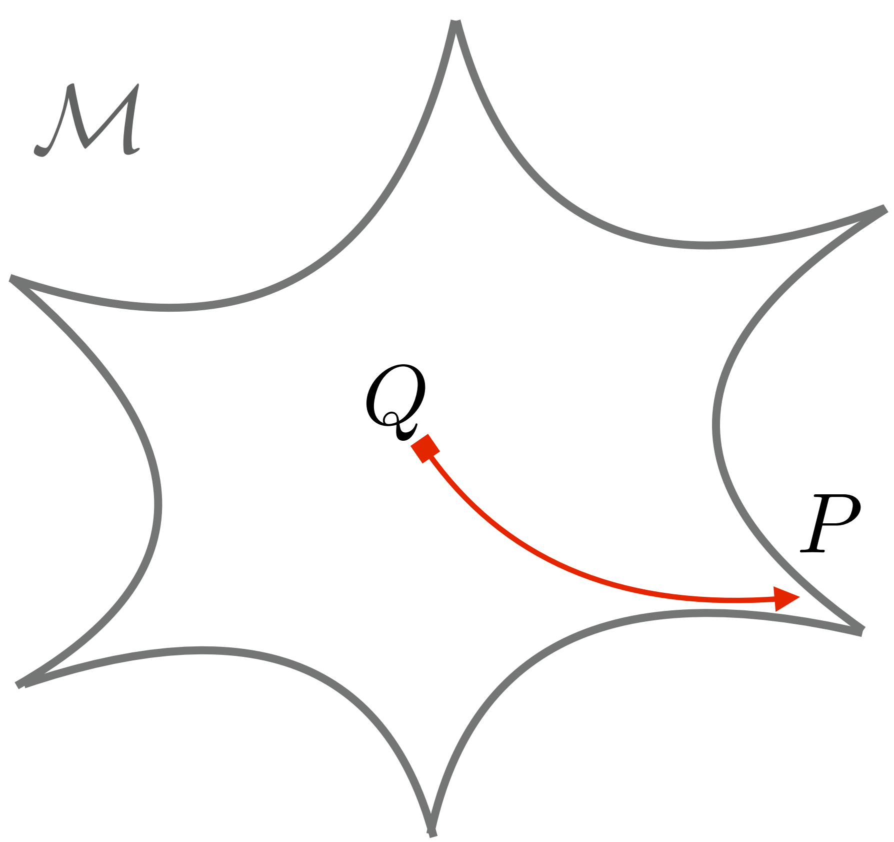
MOTIVATION – THE DISTANCE CONJECTURE

Swampland Distance Conjecture (SDC)

Infinite distance points in moduli space → *towers of light states*

[Ooguri, Vafa '06]
[Ooguri,Palti,Shiu,Vafa '19]

Invalidate EFT



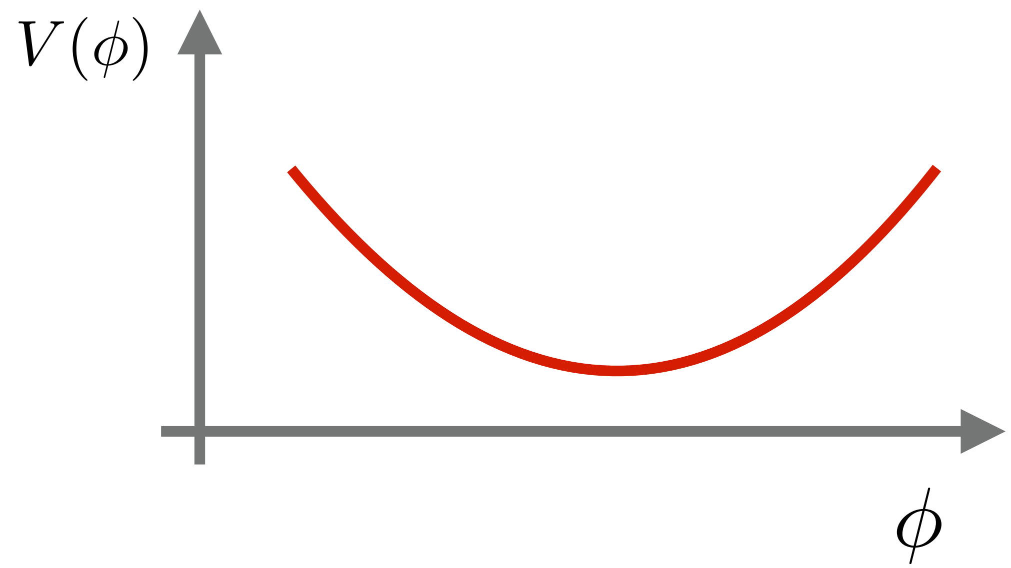
$$m(P) = m(Q)e^{-\lambda d(P,Q)}$$

geodesic distance

*Best established for **exact** moduli spaces*

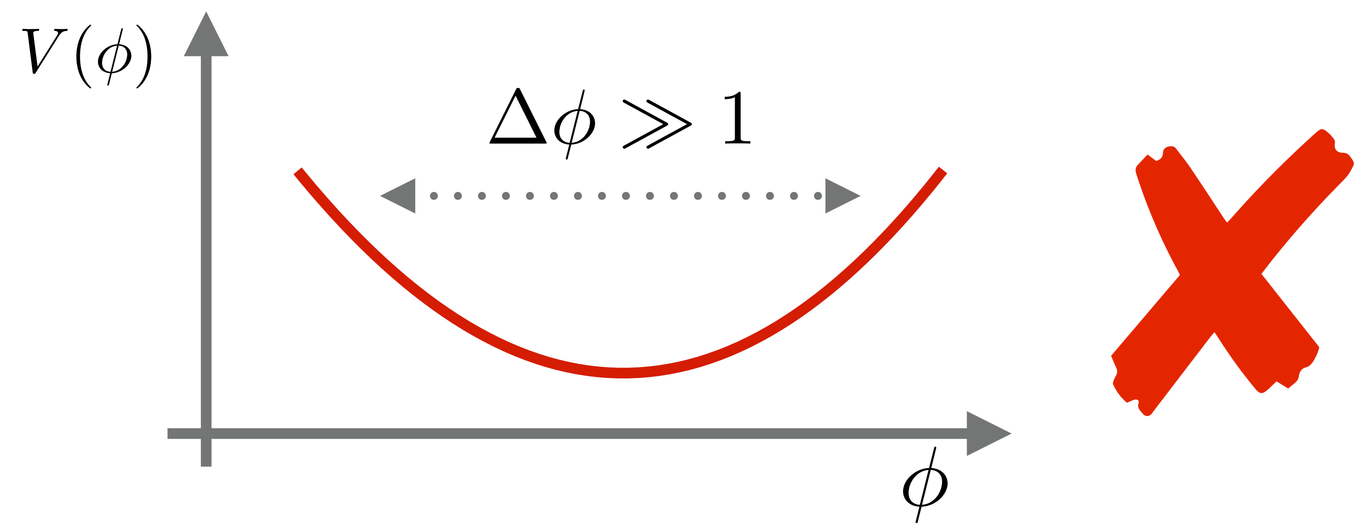
SDC WITH A POTENTIAL

In the real world, moduli must be stabilised with a potential



Typical application:
Rule out large field inflation

Compact directions (axions) important:
e.g. monodromy inflation



Still believed to hold, less evidence

[Klaewer,Palti '16]
[Calderon-Infante,Uranga,Valenzuela '20]+…

More recent:
generalised notion of distance, including V?

[Tonioni, Van Riet '24]
[Montero,Mohseini,Vafa,Valenzuela '24]

DYNAMICS

*Related issue: SDC applies to **adiabatic field variations***

$V = 0$	$V \neq 0$
adiabatic: $\dot{\phi} \to 0$	*(space)time dependence* $\dot{\phi} = 0$
geodesic trajectories	*non-geodesic trajectories*

$$\ddot{\phi}^i + \Gamma^i_{jk}\dot{\phi}^j\dot{\phi}^k + (d-1)H\dot{\phi}^i + \partial^i V = 0$$

What becomes of the SDC in a cosmological setting?

Some (sparse) comments appear in [Conlon,FR '22][Tonioni,Tran,Shiu' 23][Tonioni, Van Riet '24]

SETTING

Cosmology of asymptotic limits in type IIB/F-theory flux compactifications

[See also Calderon-Infante,Ruiz,Valenzuela '22,FR '23]

$$S = \frac{M_{P,d}^2}{2} \int d^d x \sqrt{-g} \left\{ \mathcal{R} + \frac{1}{2} G_{I\bar{J}} \partial_\mu \Phi^I \partial^\mu \bar{\Phi}^{\bar{J}} + V(\Phi, \bar{\Phi}) \right\}$$

Complex Structure moduli

EFTs classified with Asymptotic Hodge Theory

[Grana,Grimm,Herraez,Monnee,Plauschinn,Palti,Lanza,Li,Schlechter,Valenzuela,van de Heisteeg… '19-24]

$$V \sim \sum_{\ell \in \mathcal{E}} \left(\frac{s^1}{s^2}\right)^{\ell_1 - 4} \cdots \left(\frac{s^{\hat{n}-1}}{s^{\hat{n}}}\right)^{\ell_{\hat{n}-1} - 4} \left(s^{\hat{n}}\right)^{\ell_{\hat{n}} - 4} \|\rho_\ell\left(G_4, a_i\right)\|_\infty^2$$

Simple case: *single modulus* $\Phi = s + ia$

DYNAMICAL SYSTEM

Polynomial in w

$$V(s,a) = \frac{P_n(w)}{s^\lambda}$$

General case

$$V(s,a) = \frac{1}{s^\lambda}\sum_{i=0}^{n}\frac{P_i(w)}{s^i}$$

Autonomous system:

$$x = \frac{\dot{s}}{\alpha H s} \qquad y = \frac{\dot{a}}{\alpha H s} \qquad w = \frac{a}{s}$$

x^2, y^2 normalized kinetic terms

$$\begin{cases} \frac{dx}{dN} = -\alpha y^2 - \left(1 - x^2 - y^2\right)\left[(d-1)x - \frac{\alpha}{2}\left(\lambda + \frac{w\partial_w P_n(w)}{P_n(w)}\right)\right] \\ \frac{dy}{dN} = \alpha x y - \left(1 - x^2 - y^2\right)\left[(d-1)y + \frac{\alpha}{2}\frac{\partial_w P_n(w)}{P_n(w)}\right] \\ \frac{dw}{dN} = \alpha(y - wx) \end{cases}$$

[Copeland,Liddle,Wands '97][Russo,Townsend '06-'19]
[(Brinkmann),Cicoli, Dibitetto,Pedro '20-22],[Tonioni,Tran,Shiu' 23-24] , [FR'23]

SOME (GLOBAL!) RESULTS

Two options: → *Fixed point - easy*

→ $w \to \bar{w} \quad | \quad P_0(w) = 0$

New variable: $T = x + yw \sim \frac{1}{H^2} \frac{s\dot{s} + a\dot{a}}{s^2}$

$$V(s,a) = \frac{P_n(w)}{s^\lambda} \longrightarrow T \to \frac{\alpha\lambda}{2(d-1)}$$

$$V(s,a) = \frac{1}{s^\lambda} \sum_{i=0}^{n} \frac{P_i(w)}{s^i} \longrightarrow T \to \frac{\alpha\lambda}{2(d-1)} \times [0,1]$$

Using techniques from dynamical systems, e.g. Lyapunov functions

A POSSIBLE GENERALISATION*

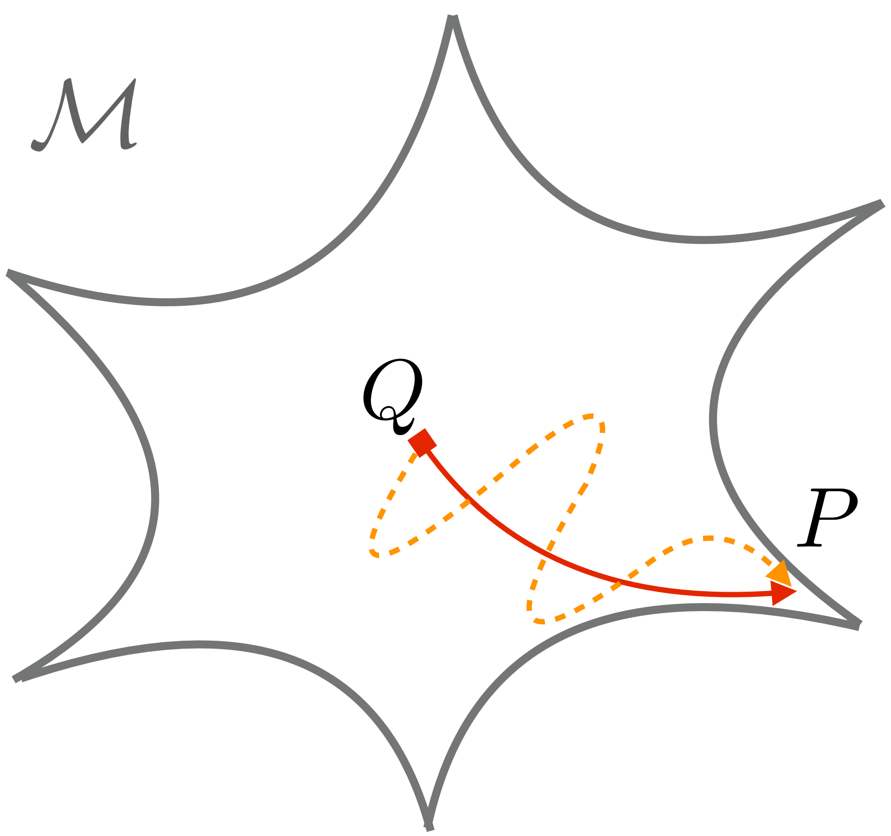
Question:

For trajectories approaching the boundary of moduli space, do towers of states become exponentially light in the dynamical distance ?

[Shiu,Landete '18]

From usual SDC, equivalent to relationship between length of trajectories and geodesics

We suspect yes



$$m(P) = m(Q)e^{-\lambda\Delta(P,Q)}$$

↓

$$\Delta = \int_{t_1}^{t_2} \mathrm{d}\tau \sqrt{G_{I\bar{J}}\dot{\Phi}^I\dot{\bar{\Phi}}^{\bar{J}}}$$

along trajectory

A COUNTER-EXAMPLE?

"Growing" trajectories → *claim easy to show*

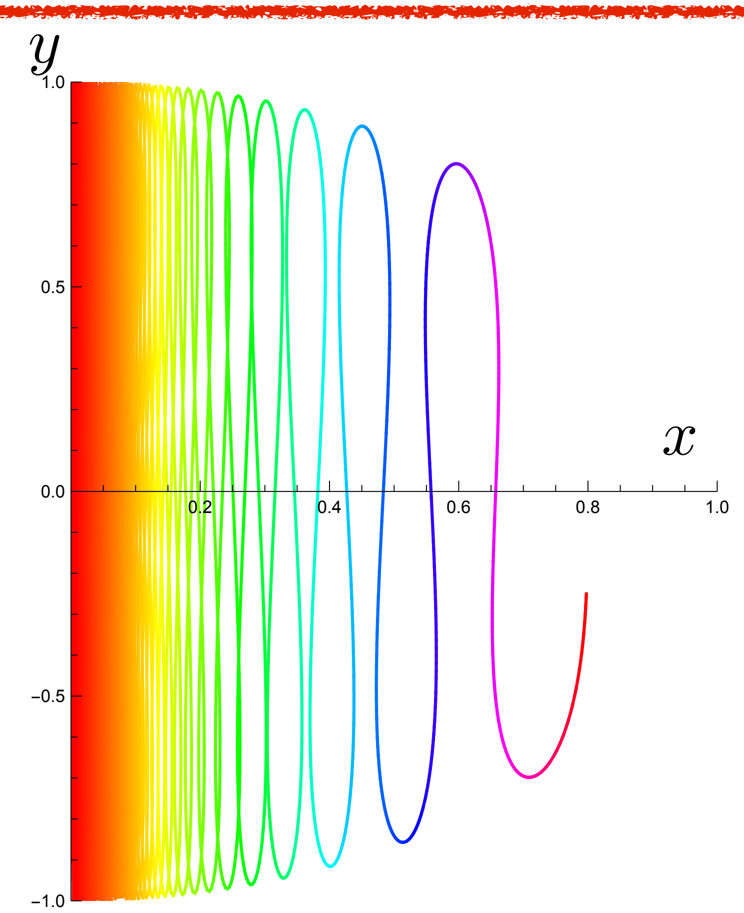
$$K = c \log s \qquad + \qquad V \sim \sum \frac{P_n(w)}{s^{\beta_n}}$$

Similar to axion backreaction, [Baume, Palti'16][Grimm,Li '20][Calderon-Infante,Uranga,Valenzuela '20]

"Oscillating" trajectories

$$V(a,s) = \# f^2 \frac{a^2}{s^2} \qquad (LCS)$$

$x \to 0,\ w \to 0$ *Fixed segment*



BUT corrections unavoidable & spoil behaviour

OUTLOOK

Cosmology of 1-modulus asymptotic limits

Analytical results from dynamical system approach

Dynamical version of SDC?

$$d(P,Q) \longleftrightarrow \Delta(P,Q)$$

Classification

Acc. expansion?

Long term: "Dynamical" Swampland

THANK YOU FOR YOUR ATTENTION!