Amplitudes and celestial OPEs around curved backgrounds

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Motivation

Scattering around non-trivial backgrounds is important for many reasons!

- We know *almost nothing* compared to scattering on trivial backgrounds
- Pheno applications: non-linear regime of QED, heavy-ion collisions, gravitational waves and black holes *[Fedotov-Ilderton-Karbstein-King-Seipt, Iancu-Leonidov-McLerran, Sanchez, ...]*
- Strong-field scattering encodes *all-order* data for classical physical observables *[Adamo-Cristofoli-Ilderton-Klisch]*
- Tests for amplitude techniques beyond trivial background *[Adamo-Casali-Mason-Nekovar, GB-Mason, Adamo-Klisch]*
- Data for celestial holography *[Adamo-Bu-Zhu, Bittleston-Heuveline-Skinner, Costello-Paquette-Sharma '23, ...]*

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Today's plan

- How to define (and compute) amplitudes on curved backgrounds
- Gluon amplitudes around the self-dual dyon
- Graviton amplitudes around self-dual Taub-NUT
- Celestial chiral algebras for the Pedersen metric

We will focus on scattering around *strong self-dual backgrounds*

- background: a fixed, non-dynamical solution to the classical (non-linear) EOMs
- strong: we want to treat the background *non-perturbatively*
- self-dual: the background satisfies the appropriate SD equations

Perturbiner definition of amplitudes: *[Arefeva-Faddeev-Slavnov, Jevicki-Lee, Abbott-Grisaru-Schaefer]*

Scattering amplitudes ⇔ multi-linear pieces of the on-shell action

- Agrees with S-matrix elements, *when they exist!*

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- The perturbiner construction of amplitudes is *still* very cumbersome

$\Rightarrow$ Twistor theory is a very helpful framework!

- Twistor space $(\mathbb{P}\mathscr{T}, \bar{\nabla})$: 3-fold modelled on $\mathbb{CP}^3$ with homogeneous coordinates $Z^A = (\lambda_\alpha, \mu^{\dot{\alpha}})$ *[Penrose]*
  - ▷ spacetime point $x \Leftrightarrow$ holomorphic line $X \cong \mathbb{CP}^1 \hookrightarrow \mathbb{P}\mathscr{T}$
  - ▷ SD spacetime metric $\Leftrightarrow$ complex structure $\bar{\nabla}$ *[Penrose]*
  - ▷ SD background gauge field $\Leftrightarrow$ holomorphic vector bundle $(E, \bar{D}) \to \mathbb{P}\mathscr{T}$ *[Ward]*
  - ▷ scattering states $\Leftrightarrow$ cohomologies of $\bar{\nabla}$, $\bar{D}$ *[Penrose, Hitchin]*
  - ▷ MHV generating functionals $\Leftrightarrow$ non-local operators on $\mathbb{P}\mathscr{T}$ *[Mason, Mason-Boels-Skinner, Mason-Skinner]*

A crash course on twistors

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Warm up: gluon amplitudes around the self-dual dyon

- Background is an Abelian self-dual dyon (SDD)

$$A = \mathsf{c}\left(\frac{\mathrm{d}t}{r} + a\right) \qquad a = (1 - \cos\theta)\,\mathrm{d}\phi$$

- (Scalar) scattering states are

$$\Phi^{+} = \left(\frac{r}{1+\zeta\bar{\zeta}}\right)^{e} (\bar{\zeta}z + 1)^{2e}\mathrm{e}^{\mathrm{i}k\cdot x} \qquad \Phi^{-} = \left(\frac{r}{1+\zeta\bar{\zeta}}\right)^{-e} (\zeta - z)^{-2e}\mathrm{e}^{\mathrm{i}k\cdot x}$$

respectively for $e > 0$ and $e < 0$

  - $k_\mu = \kappa_\alpha\tilde{\kappa}_{\dot{\alpha}}$, $z = \kappa_1/\kappa_0$, $\zeta, \bar{\zeta} = (x^1 \pm \mathrm{i}x^2)/(r + x^3)$, $2e \in \mathbb{Z}$
  - $\Phi^{\pm}$ are *exact* (no partial waves, etc...)

- A similar story holds for all spins

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Twistor "recipe"

- Yang-Mills as an expansion around the SD sector *[Chalmers-Siegel]*

$$S = \int \mathrm{d}^4x \, \mathrm{tr}\,(B_{\alpha\beta}F^{\alpha\beta}) - \frac{g^2}{2}\int \mathrm{d}^4x \, \mathrm{tr}\,(B_{\alpha\beta}B^{\alpha\beta})$$

- The MHV generating functional is the twistor uplift of $\mathrm{tr}\,(B^2)$ *[Mason, Adamo-Mason-Sharma, Bu-Casali, GB-Mason]*

$$\int_{\mathbb{PT}\times_{\mathbb{M}}\mathbb{PT}} \mathrm{D}^3 Z_1 \wedge \mathrm{D}^3 Z_2 \wedge \mathrm{tr}\,(\mathsf{H}_1^{-1}\mathsf{b}_1\mathsf{H}_1\mathsf{H}_2^{-1}\mathsf{b}_2\mathsf{H}_2)$$

- Perturbative expansion is now a *linear* problem on twistor space

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- 2-point gluon amplitude is

$$\mathcal{A}_2 = 8e_2(2e_2)!\,\delta(\omega_1+\omega_2)\,\delta(e_1+e_2)\,\frac{(2\omega_1)^{2e_2-1}\langle 12\rangle^{2+2e_2}}{|\mathbf{k}_1+\mathbf{k}_2|^{2+4e_2}}$$

- Gluon MHV amplitude *at all multiplicity* is

$$\mathcal{A}_n = \frac{\langle 12\rangle^4}{\langle 12\rangle\ldots\langle n1\rangle}\,\delta\left(\sum_i \omega_i\right)\delta\left(\sum_i e_i\right) \times \int \mathrm{d}^3\mathbf{x}\,\mathrm{e}^{\mathrm{i}(\mathbf{k}_1+\ldots+\mathbf{k}_n)\cdot\mathbf{x}} \times \prod_{e_i>0}\left(\frac{r}{1+|\zeta|^2}\right)^{e_i}(\bar{\zeta}z_i+1)^{2e_i} \times \prod_{e_j<0}\left(\frac{r}{1+|\zeta|^2}\right)^{-e_j}(\zeta-z_i)^{-2e_j}$$

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Gluon celestial OPE around the SDD

- With the gluon MHV amplitude, we can compute the holomorphic splitting function for $z_{ij} = z_i - z_j \to 0$ *[Fan-Fotopoulos-Taylor, Pate-Raclariu-Strominger-Yuan]*

$$\mathcal{A}_n \to \frac{1}{\langle ij \rangle} \mathcal{S}(i,j) \mathcal{A}_{n-1}$$

- We find the same splitting function as around the trivial background

The gluon celestial OPE is undeformed around the self-dual dyon

- This is in agreement with works on twistorial monopoles/scattering *[Garner-Paquette]*

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Self-dual Taub-NUT

- Consider the SDTN metric

$$\mathrm{d}s^2 = V^{-1}(\mathrm{d}t - 2Ma)^2 + V\mathrm{d}\mathbf{x}^2 \qquad V \equiv 1 + \frac{2M}{r}$$

  - toy model for physical black holes

- The twistor space of SDTN is a line bundle over the total space of $\mathcal{O}(2) \to \mathbb{CP}^1$ *[Hitchin]*

Graviton MHV amplitude ⇔ tree-level correlator of a 2d CFT

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Graviton amplitudes around SDTN

- The 2-point graviton amplitude is the double-copy of the 2-point gluon amplitude

$$\mathcal{M}_2 = -2M(4M\omega_2)!\delta(\omega_1+\omega_2)\frac{(2\omega_1)^{4M\omega_2-2}\langle 12\rangle^{4+4M\omega_2}}{|\mathbf{k}_1+\mathbf{k}_2|^{2+8M\omega_2}}$$

- The MHV graviton amplitude can be compactly written as

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- Stay tuned for further properties of $\mathcal{M}_n$!

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- We can turn on a cosmological constant $\Lambda = -3/l^2$

- Important to study deformations of $Lw_{1+\infty}$ *[cf. Bin's talk]*

- The analogue of SDTN is the *Pedersen metric* *[Pedersen, Hitchin, Zoubas]*

$$ds^2 = V^{-1} f_l (\mathrm{d}t - 2Ma)^2 + V \left(\frac{\mathrm{d}r^2}{f_l} + r^2 \mathrm{d}\Omega_2^2 \right) \qquad f_l \equiv 1 + \frac{r(r+4M)}{l^2}$$

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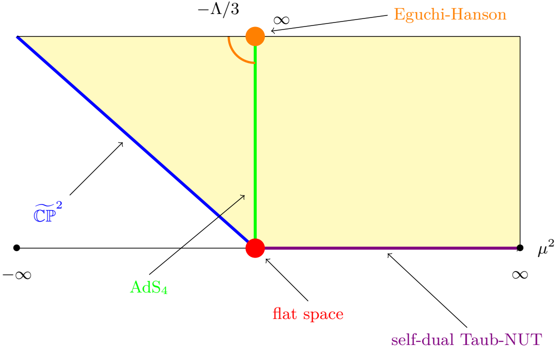
Most of the studied examples of curved spacetimes in celestial holography are particular limits of the Pedersen metric



- The celestial chiral algebra comes from the Poisson bracket of elements of $\mathcal{O}(2)$ *[Bittleston-Heuveline-Skinner, various talks here]*
- We conjecture a 2-parameter deformation of $Lw_{1+\infty}$, e.g.

$$\{w[p,q,2i,2j],w[r,s,2k,2l]\} = (ps-qr)w[p+r-1,q+s-1,2(i+k),2(j+l)] + 4\Lambda(il-jk)w[p+r,q+s,2(i+k-1)+1,2(j+l-1)+1] - \frac{\Lambda M}{4}((p-q)(k+l)-(r-s)(i+j))w[p+r+1,q+s+1,2(i+k-1),2(j+l-1)]$$

- In the relevant limits, it reduces to the known deformations of the celestial chiral algebra of AdS_4 *[Taylor-Zhu, Bittleston-GB-Heuveline-Kmec-Mason-Skinner, see also Bin's talk]*, Eguchi-Hanson *[Bittleston-Heuveline-Skinner]*, and is undeformed for SDTN

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Many more things to do

- Checks from graviton OPE on SDTN for the Pedersen CCA
- Ideally, we should be able to give all-multiplicity formulae at all MHV degrees for SDD and SDTN
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