# Spin Matrix Theory as Conformal Field Theory

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Based on work to appear with T. Harmark, Y. Lei and Z. Yan
and on [SB, Harmark, Lei, 2211.16519]

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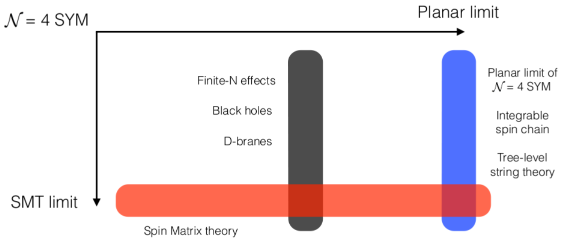
# Outline

1. Motivations: Spin Matrix Theory
2. Conformal map between SU(1,2) Non-Lorentzian geometries
3. Conclusions and perspectives

# Spin Matrix Theory (SMT)

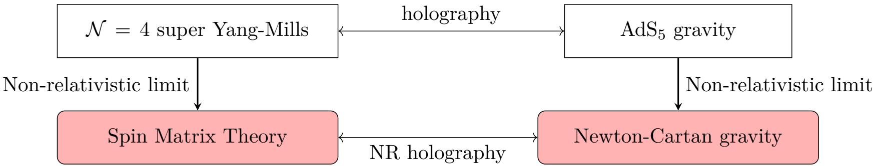
Controlled finite $N$ effects: Spin Matrix Theory limits [Harmark, Orselli, 2014]

- Decoupling limits of $\mathcal{N}=4$ SYM on $\mathbb{R}\times S^3$ $\Rightarrow$ the theory reduces to a subsector with only one-loop contributions of the dilatation operator [Harmark, Orselli, 2006][Harmark, Kristjansson, Orselli, 2006-07]



# Non-relativistic nature of SMT

- Emergent $\mathrm{U}(1)$ global symmetry $\Rightarrow$ mass conservation
- Antiparticles decouple
- Low-energy excitations (magnons) have non-relativistic dispersion relations
- Bulk duals are non-relativistic string theories (see review [Oling, Yan, 2022])



# Decoupling limits

Zoom-in close to a unitarity (BPS) bound

$$E \geq J \equiv \sum_{i=1,2} a_i S_i + \sum_{j=1}^{3} b_j Q_j \tag{1}$$

($S_i$ rotations on $S^3$, $Q_i$ Cartan generators of $\mathrm{SU}(4)$ R-symmetry)

SMT limit ($\lambda = g^2 N$)

$$\boxed{\lambda \to 0\,, \qquad H_{\text{int}} = \frac{H-J}{\lambda} \text{ finite}\,, \qquad N \text{ fixed}} \tag{2}$$

PSU$(2,2|4)$ symmetry group of $\mathcal{N}=4$ SYM gets reduced to a spin subgroup

**PSU(1,2|3) sector contains all the others!**

# What to do next?

**PSU(1,2|3) SMT Hamiltonian computed in** [SB, Lei, Harmark, 2022]

- Find a local formulation of SMTs with SU(1,2) bosonic symmetry group
- Determine a geometry with SU(1,2) conformal isometry
- Build a corresponding state-operator correspondence

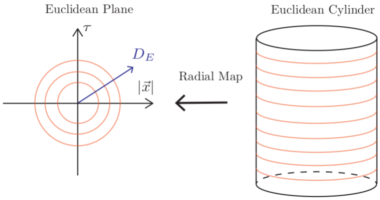
# Conformal map between SU(1,2) Non-Lorentzian geometries

# Relativistic state-operator correspondence (Euclidean)

$$\text{Local operators on } \mathbb{R}^D \quad \Leftrightarrow \quad \text{States on } S^{D-1} \tag{3}$$

Geometric mapping between Euclidean plane and cylinder

$$ds^2 = d\rho^2 + \rho^2 d\Omega^2_{D-1} \quad \underset{\tau = \log \rho}{\longleftarrow} \quad ds^2 = e^{2\tau}\left(d\tau^2 + d\Omega^2_{D-1}\right) \tag{4}$$




Map between generators

$$D^E = \rho \partial_\rho \quad \underset{\tau = \log \rho}{\longleftarrow} \quad H^E = \partial_\tau \, . \tag{5}$$

# Relativistic state-operator correspondence (Lorentzian)

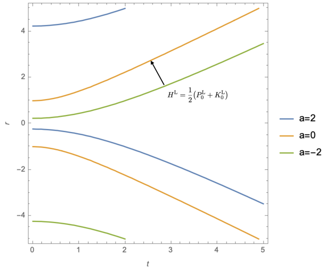
Hyperbolic map

$$t = \frac{1}{2}\left[\tan\left(\frac{\tau+\chi}{2}\right) + \tan\left(\frac{\tau-\chi}{2}\right)\right], \quad r = \frac{1}{2}\left[\tan\left(\frac{\tau+\chi}{2}\right) - \tan\left(\frac{\tau-\chi}{2}\right)\right] \tag{6}$$

relates Lorentzian plane and cylinder

$$ds^2_{\text{state}} = -dt^2 + dr^2 + r^2 d\Omega_2^2 \tag{7}$$

$$ds^2_{\text{ope}} = (\cos\tau + \cos\chi)^{-2}\left[-d\tau^2 + d\chi^2 + \sin^2\chi d\Omega_2^2\right] \tag{8}$$




Map of generators [Minwalla, 1997] [Chagnet, Chapman, de Boer, Zukowski, 2021]

$$H^L_\tau \to \frac{1}{2}(P_0^L + K_0^L) \tag{9}$$

# Geometries with SU(1,2) conformal isometry

State picture

- Start from $\mathbb{R} \times S^3$

$$ds^2 = -dt^2 + \tfrac{1}{4} R^2 \left[ d\theta^2 + \sin^2\theta \, d\varphi^2 + \left( d\eta + \cos\theta \, d\varphi \right)^2 \right] \qquad (10)$$

  with energy $E = iR\partial_t$ and total angular momentum $S = -2\, i\, \partial_\eta$

- Introduce a null isometry along $u$
- Fix the null momentum $P_u = E - S$ to preserve SU(1,2) isometry [Harmark, Kristjansson, Orselli, 2007] [Harmark, Hartong, Obers, Oling, 2020]
- Require energy is associated with the new time coordinate $E = i\partial_{x^0}$

$$ds^2 = 2\tau(du - m) + h_{\mu\nu} dx^\mu dx^\nu \qquad (11)$$

$$\tau = R^2 dx^0 - \frac{R^2}{2} \left( \cos\theta - 1 \right) \, d\varphi \,, \qquad m = \frac{1}{2} dx^0 + \frac{1}{4} \left( \cos\theta + 1 \right) \, d\varphi$$
$$h_{\mu\nu} dx^\mu dx^\nu = \frac{R^2}{4} (d\theta^2 + \sin^2\theta d\varphi^2) \qquad (12)$$

Operator picture

- Take a $R \to \infty$ limit such that $S^2 \to \mathbb{R}^2$

$$\theta = \frac{2\hat{r}}{R}\,, \qquad \varphi = \hat{\varphi}\,, \qquad x^0 = \frac{\hat{\varphi}}{4} + \frac{\hat{t}}{2R^2}\,, \qquad u = \hat{u} \tag{13}$$

- Leads to a Newton-Cartan geometry with $\Omega$-deformation [Lambert, Mouland, Orchard, 2022]

$$d\hat{s}^2 = 2\hat{\tau}(d\hat{u} - \hat{m}) + \hat{h}_{\mu\nu} d\hat{x}^\mu d\hat{x}^\nu \tag{14}$$

$$\begin{aligned} \hat{\tau} = d\hat{t} + \hat{r}^2 d\hat{\varphi}\,, \qquad \hat{m} = \frac{1}{2} d\hat{\varphi} \\ \hat{h}_{\mu\nu} d\hat{x}^\mu d\hat{x}^\nu = d\hat{r}^2 + \hat{r}^2 d\hat{\varphi}^2 \end{aligned} \tag{15}$$

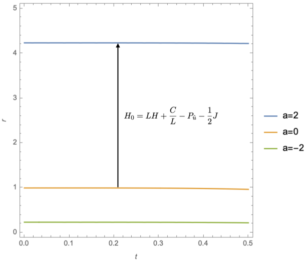
- Geometry has SU(1,2) conformal isometry

# State-operator map

**There exists an exact conformal map between the two geometries!**

$$
\begin{aligned}
x^0 &= -\mathrm{arccot}\left(\frac{\mathbf{t}^2+\mathbf{r}^4-1}{\mathbf{t}}\right), \qquad \varphi = \hat{\varphi} - \arctan\left(\frac{\mathbf{r}^2-1}{\mathbf{t}}\right), \\
\theta &= \arccos\left[1-\frac{8\,\mathbf{r}^2}{\mathbf{t}^2+\left(\mathbf{r}^2+1\right)^2}\right], \qquad u = \hat{u} + \frac{1}{2}\arctan\left(\frac{\mathbf{r}^2-1}{\mathbf{t}}\right)
\end{aligned} \tag{16}
$$

where $\mathbf{t} = (2R^2)^{-1}\,\hat{t}$ and $\mathbf{r} = (2R)^{-1}\,\hat{r}$.




Map of generators

$$
\boxed{H_0 = \partial_{x^0} \to L\,H + \frac{C}{L} - P_{\hat{u}} - \frac{1}{2}J} \tag{17}
$$

$H$ Hamiltonian, $C$ special conformal generator,
$J$ angular momentum,
$P_{\hat{u}}$ U(1) central generator

# SU(1,2) state-operator correspondence

- Schrödinger case [Nishida, Son, 2007]

$$|\psi_{\mathcal{O}}\rangle = e^{-H}\mathcal{O}^\dagger|0\rangle \tag{18}$$

  Primaries correspond to eigenstates of a system in a harmonic potential

$$H_{\text{osc.}}|\psi_{\mathcal{O}}\rangle = \Delta_{\mathcal{O}}|\psi_{\mathcal{O}}\rangle\,, \quad H_{\text{osc.}} = H + C \tag{19}$$

- Map between eigenstates also applies to systems with SU$(1,n)$ invariance [Lambert, Mouland, Orchard, 2022]

**Geometric map between geometries with SU(1,2) conformal invariance naturally leads to the proposal**

$$D^E \equiv -i\left[LH + \frac{C}{L} - P_{\hat{u}} - \frac{1}{2}J\right]\,, \qquad H^E \equiv -iH_0\,. \tag{20}$$

# Conclusions and perspectives

Summary of results:

- Non-relativistic theories from near-BPS limits of $\mathcal{N} = 4$ SYM
- Geometric formulation of SU(1,2) state-operator correspondence

Future developments:

- Find non-relativistic QFT with PSU(1,2|3) invariance from null reduction
- Bulk side and dual D3-branes perspective [Yan, Gomis, 2023][Blair, Lahnsteiner, Obers, Yan, 2023]

# Thank you!