A new non-relativistic holography

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Talk at Eurostrings 2024

Based on:

- *Non-Relativistic Holography from AdS_5/CFT_4*, arXiv:2409.02267, to appear in Physical Review Letters

  Longer version: *Constructing non-relativistic AdS_5/CFT_4 holography*, arXiv:2403.02379, with J.M. Nieto

- *A perturbative approach to the non-relativistic string spectrum*, arXiv:2403.09563, with M. de Leeuw and J.M. Nieto

Motivations

- Non-relativistic holography is a natural example of
  (i) non-AdS, and
  (ii) non-Lorentzian holography
- timely with the current effort of exploring how general holography is (see e.g. flat space holography)
- is non-relativistic holography working as in the relativistic case? If not, in what does it differ?
- are the differences helping us understand how holography behaves universally?
- is non-relativistic holography a simplified setting where to test holography?

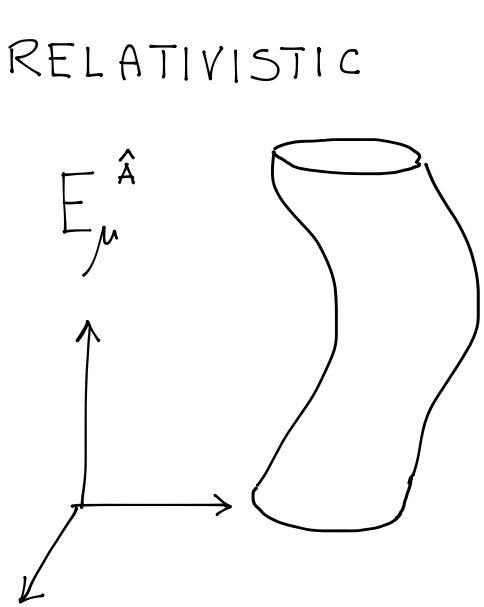
In this talk, we discuss:

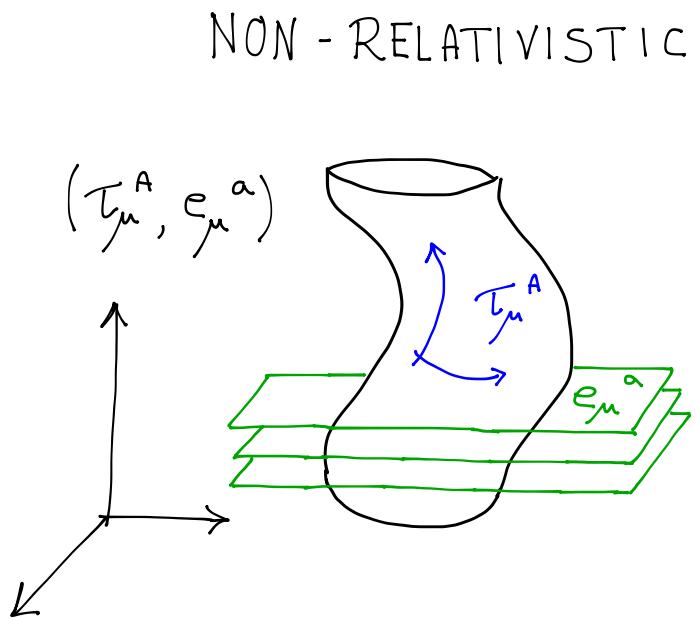
- the first example of non-relativistic AdS_5/CFT_4 holography (with relativistic w.s.), proposed in arXiv:2403.02379 [AF, Nieto]
- possible quantitative tests to support this claim

Non-relativistic limit in String Theory

- first proposed by Gomis-Ooguri in flat spacetime
  [Gomis, Ooguri (2000)][Danielsson, Guijosa, Kruczenski (2000)]
- generalised to $AdS_5 \times S^5$ by Gomis-Gomis-Kamimura
  [Gomis, Gomis, Kamimura (2005)]
- it requires a critical closed B-field
- Features: 1) NR target space, 2) relativistic world-sheet
- β-function vanishes [Gomis, Oh, Yan (2019)][Gallegos, Gursoy, Zinnato (2019)]

Target space is a String Newton-Cartan (SNC) geometry Review: [Oling, Yan (2022)]





Foliation "2+8" $\hat{A} = (A, a) \quad A = 0, 1 \quad a = 2, ..., 9$

Non-relativistic limit in Gauge Theory

Non-relativistic limit has been applied to several gauge theories. E.g.:

- Maxwell + scalar field $\rightarrow$ Galilean Electrodynamics (GED)
  [Santos, de Montigny, Khanna, Santana (2004)][Bergshoeff, Rosseel, Zojer (2016)]
- Super Yang Mills $\rightarrow$ Galilean version of SYM
  [Bagchi, Basu, Kakkar, Mehra (2016)]

and much more. (see the review arxiv:2311.00027 of S. Baiguera)

It is quite natural to ask:

Are non-relativistic string and gauge theories

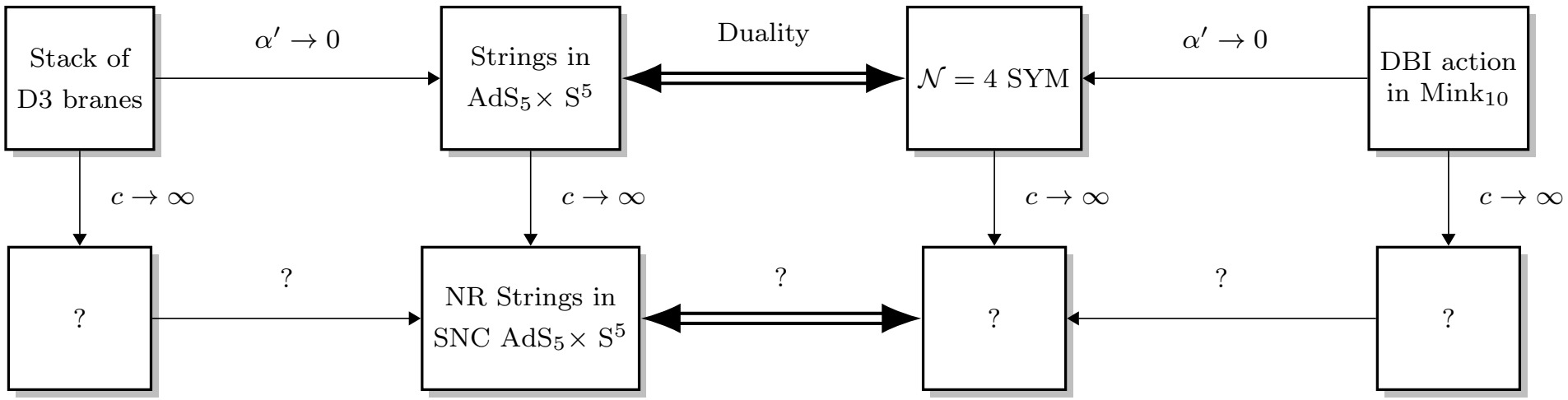
related by holography? If yes, how?

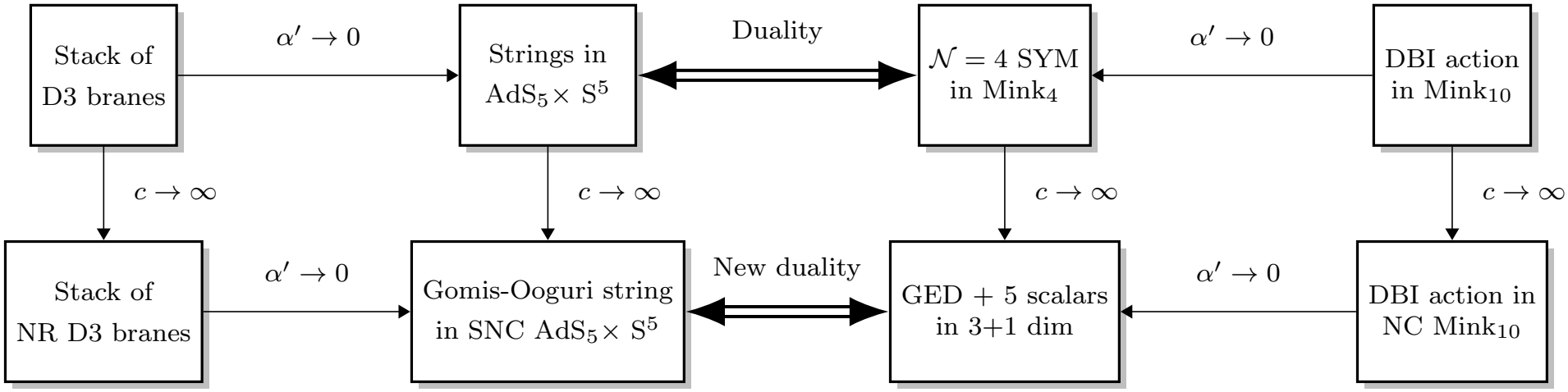
- conjectured duality between NR strings with Galilean world-sheet and Spin Matrix Theory (see talks by J. Hartong and S. Baiguera)
  [Harmark, Orselli (2006)][Harmark (2016)]
- in this talk: world-sheet is relativistic (i.e. Gomis-Ooguri action)

Our approach: constructing non-relativistic AdS_5/CFT_4 correspondence

Take Maldacena's setting of the AdS_5/CFT_4 correspondence.

- how does the non-relativistic limit enter in this construction?
- are the non-relativistic ($c \to \infty$) and near-horizon/decoupling ($\alpha' \to 0$) limits commuting?





[AF, Nieto (2024)]

Supported by a holographic realisation of symmetries:

boundary Killing vectors of SNC $AdS_5 \times S^5$ = On-shell symmetries of GED + 5 scalars in 3+1

[AF, Nieto (2024)] [Festuccia, Hansen, Hartong, Obers (2016)]

Penrose boundary realisation of this duality:

Penrose boundary of SNC $AdS_5 \times S^5$ = 3+1 NC $Mink_4$

Few detail of the derivation

String theory side:

- stack of D3-brane metric in NR limit retains a notion of horizon (useful to apply Maldacena's argument)
- "Gomis-Gomis-Kamimura" NR limit, where SNC $AdS_5 \times S^5$ is

$$\tau_\mu{}^A \ : \ AdS_2 \qquad\qquad e_\mu{}^a \ : \ \text{warped}_{AdS_2}\mathbb{R}^3 \times \mathbb{R}^5$$

Gauge theory side:

- what is the analogue of Gomis-Gomis-Kamimura limit on the DBI? It is a "stringy" NR limit acting on Minkowski's coordinates.
- Two possible ways to do it:
  1. $t \to ct,\ x^1 \to cx^1,\ x^i \to x^i$ and $c \to \infty \implies$ Galilean SYM in 3+1 d
  2. $t \to t,\ x^1 \to x^1,\ x^i \to \frac{1}{c}x^i$ and $c \to \infty \implies N^2$ copies of GED + 5 scalars
- option 2 requires an overall rescaling of gauge fields $A \to \frac{1}{c}A$ that abelianise the theory (resembles weak-coupling limit)
- option 1 does not realise some of the string symmetries, only option 2 does.

Generalisations

Regarding the non-relativistic limit:

- the NR limit is not "unique". There are many NR limits.
- in this work: Gomis-Gomis-Kamimura NR limit
- Recent generalisation to new NR limits by N. Lambert and J. Smith, arXiv:2405.06552
- Similar construction have also been applied to M2-branes in M-theory [Lambert, Smith (2024)]

Regarding other limits:

- repeat this construction for the Carroll limit? [AF, Nieto, In progress]
- flat space limit is more complicated, as the D3-brane metric loses the horizon

Quantitative test: $E = \Delta$

Famous test of AdS/CFT: $E = \Delta$
(string spectrum = scaling dimensions of gauge invariant operators)

- in AdS/CFT, integrability was crucial to solve this problem
- Is GGK theory integrable? We have a Lax pair [AF, van Tongeren (2022)] [AF, Nieto (2022)]
- its associated spectral curve is trivial [AF, Nieto, Ohlsson Sax (2022)]

Recently we solved the spectrum problem with heavy perturbative analysis, arXiv:2403.09563 [de Leeuw, AF, Nieto (2024)]

- famous BMN vacuum does not survive the NR limit [AF, Nieto (2023)]
- simplest NR vacuum compatible with light-cone gauge quantisation: folded version of the BMN string (infinitely extended) [AF, Nieto (2021)]
- After performing complicated field redefinitions, perturbatively:

  Spectrum: free massive and massless fields in AdS_2
  (we computed interactions up to six fields: all vanish!!)
  [de Leeuw, AF, Nieto (2024)]

Summary and future directions

Summary

- new approach to construct non-relativistic $\mathrm{AdS}_5/\mathrm{CFT}_4$ correspondence
- proposed a new duality:

$$\begin{array}{c} \text{Gomis-Ooguri string theory} \\ \text{SNC } \mathrm{AdS}_5 \times \mathrm{S}^5 \end{array} \quad \Longleftrightarrow \quad \begin{array}{c} N^2 \text{ copies of} \\ \text{GED + 5 scalars in 3+1 d} \end{array}$$

Future directions

- compute Δ for GED + 5 scalars, and check against E (i.e. spectrum of free fields in AdS_2)
- other quantitative tests: confinement/Hagedorn transition? Also with temperature/NR stringy black hole?
- generalisations to other limits: Carroll, new NR limits, more?

"Non-Lorentzian Geometries and their Applications"

Organisers: E. Bergshoeff, A. Fontanella

Workshop at Trinity College Dublin. Preliminary dates: April 29 - May 1, 2025

Topics:

1. non-relativistic strings, M-theory and holography
2. Carroll-related phenomena
3. Mathematics of non-Lorentzian geometries

Stay tuned!