Non-planar integrated correlator in $\mathcal{N} = 4$ SYM

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Shun-Qing Zhang (MPP Munich)

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Motivation

- 4-point function

dynamic part

$$\langle \mathcal{O}_2\mathcal{O}_2\mathcal{O}_2\mathcal{O}_2\rangle = \text{free} + \frac{I_4(x_i, Y_i)}{x_{12}^4 x_{34}^4} T_N(U, V; g_{\rm YM})$$

Pert. Integrands known:
L=10 (planar), L=4 (non-planar)

- *Integrate* over *U,V* [Binder, Chester, Pufu, Wang].

$$\mathcal{C}_{SU(N)} = \int dM_{U,V}\, T_N(U, V; g_{\rm YM})$$

Exact, talk by Dorigoni

At small $g_{\rm YM}$ [Dorigoni, Green, Wen]

non – planar

- $$\mathcal{C}^{\rm pert}_{SU(N)} = 4c\left[\frac{3\zeta(3)a}{2} - \frac{75\zeta(5)a^2}{8} + \frac{735\,\zeta(7)a^3}{16} - \frac{6615\,\zeta(9)(1+\frac{2}{7N^2})a^4}{32} + \mathcal{O}(a^5)\right]$$

$$\left(c = \frac{N^2-1}{4}, \quad a = \frac{\lambda}{4\pi^2}\right)$$

$$T_N(U, V; g_{\rm YM})$$

$$T_N(U, V; g_{\rm YM}) = 2\,c\,\frac{U}{V}\sum_{L=1}^{\infty} a^L\, x_{13}^2 x_{24}^2\, F^{(L)}(x_i)$$

$$F^{(L)}(x_1, \ldots, x_4) = \frac{\prod_{1\le i<j\le 4} x_{ij}^2}{L!(-4\pi^2)^L}\int \boxed{d^4x_5 \cdots d^4x_{4+L}}\, f^{(L)}(x_1, \ldots, x_4, \boxed{x_5, \ldots, x_{4+L}})$$

- $f^{(L)}(x_1, \cdots, x_{4+L})$: $\boldsymbol{S_{4+L}}$ permutation sym. [Eden, Heslop, Korchemsky, Sokatchev]
- To get $\mathscr{C}^{\rm pert.}$:

-From $F^{(L)}$: difficult, even though $F^{(3)}$ is known [Drummond, Duhr, Eden, Heslop, Pennington]

-From $f^{(L)}$: Done for P/NP at $L = 4$

$$\mathscr{P}_{f_\alpha^{(L)}} := \int_{(0,1,\infty)} \frac{d^4x_1 \cdots d^4x_{4+L}}{(\pi^2)^{L+1}}\, f_\alpha^{(L)}(x_1, \ldots, x_{4+L})$$

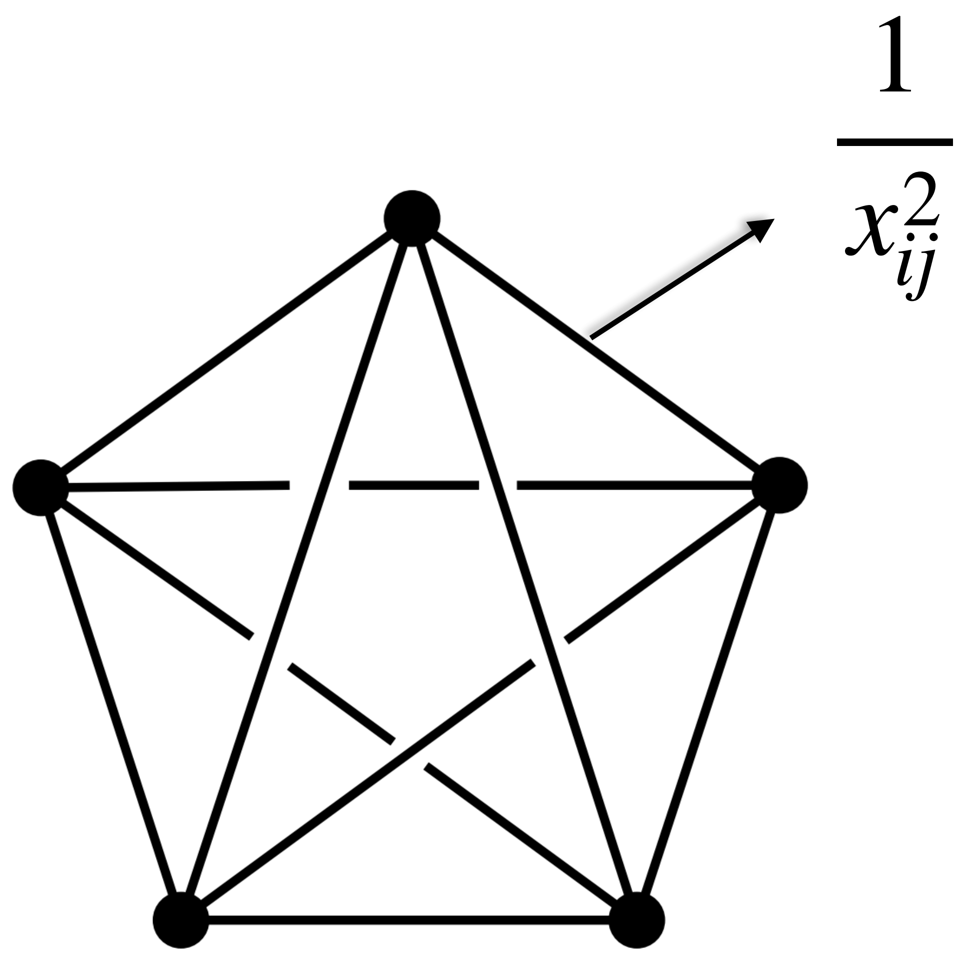
$\mathscr{C}^{\rm pert}_{SU(N)}$: periods of *f*-graphs [Wen, SQZ]

[Bourjaily, Eden, Heslop, Korchemsky, Sokatchev, Tran]

HyperlogProcedures [Schnetz] evaluates $\mathscr{P}_{f^{(L)}}$

$L = 1:$

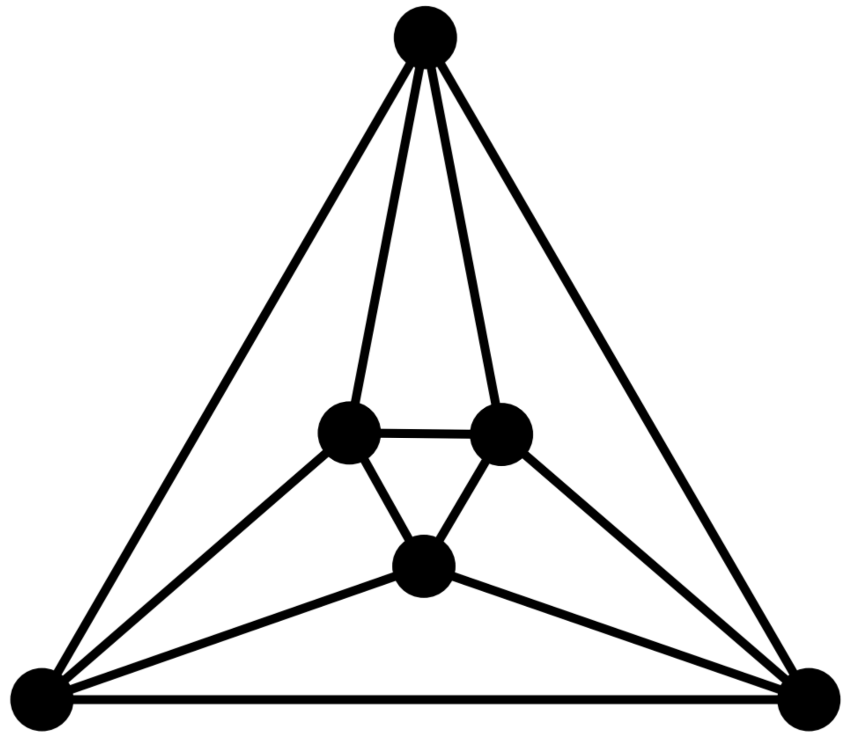
$$\frac{-1}{1!(-4)^1} \times \mathscr{P}_{f^{(1)}} = \boxed{\frac{3\zeta(3)}{2}}, \quad f^{(1)}(x_i) = \frac{1}{\prod_{1\le i<j\le 5} x_{ij}^2}$$



$6\zeta(3)$

$L = 2:$

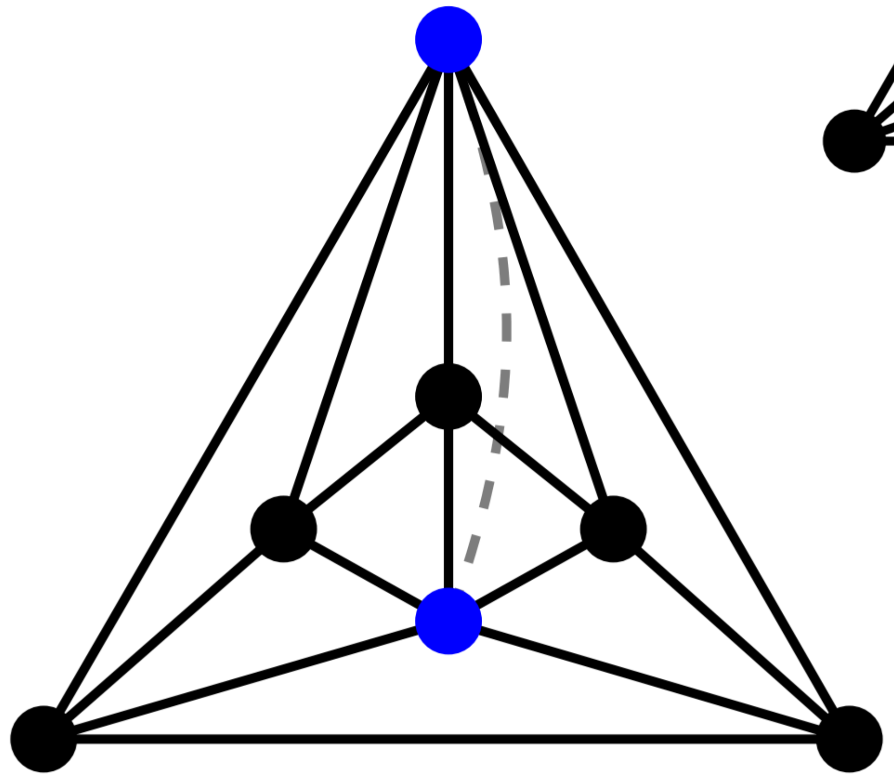
$$\frac{-1}{2!\,(-4)^2} \times \mathscr{P}_{f^{(2)}} = \boxed{\frac{-75\zeta(5)}{8}}, \quad f^{(2)}(x_i) = \frac{x_{12}^2 x_{34}^2 x_{56}^2}{48\prod_{1\le i<j\le 6} x_{ij}^2} + S_6$$



$\frac{6!}{48} \times 20\zeta(5)$

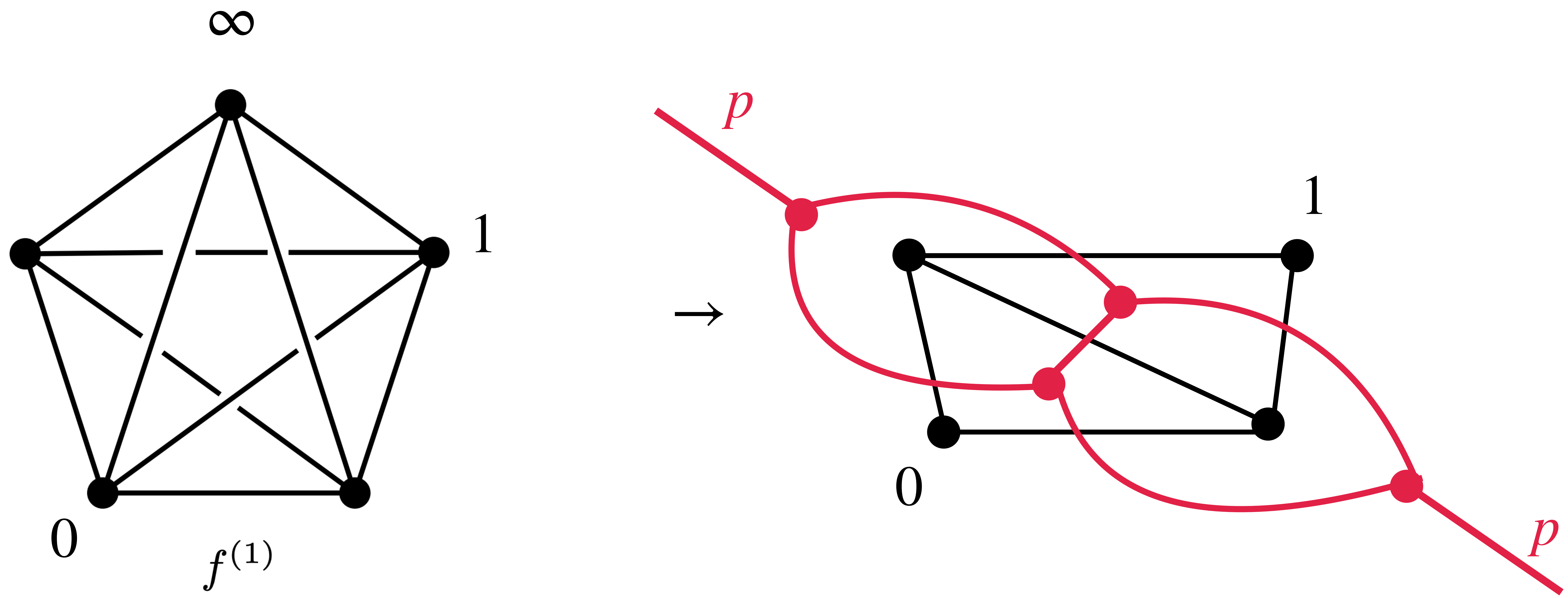
$L = 3:$

$$\frac{-1}{3!\,(-4)^3} \times \mathscr{P}_{f^{(3)}} = \boxed{\frac{735\zeta(7)}{16}}, \quad f^{(3)}(x_i) = \frac{x_{12}^4 x_{34}^2 x_{45}^2 x_{56}^2 x_{67}^2 x_{37}^2}{20\prod_{1\le i<j\le 7} x_{ij}^2} + S_7$$



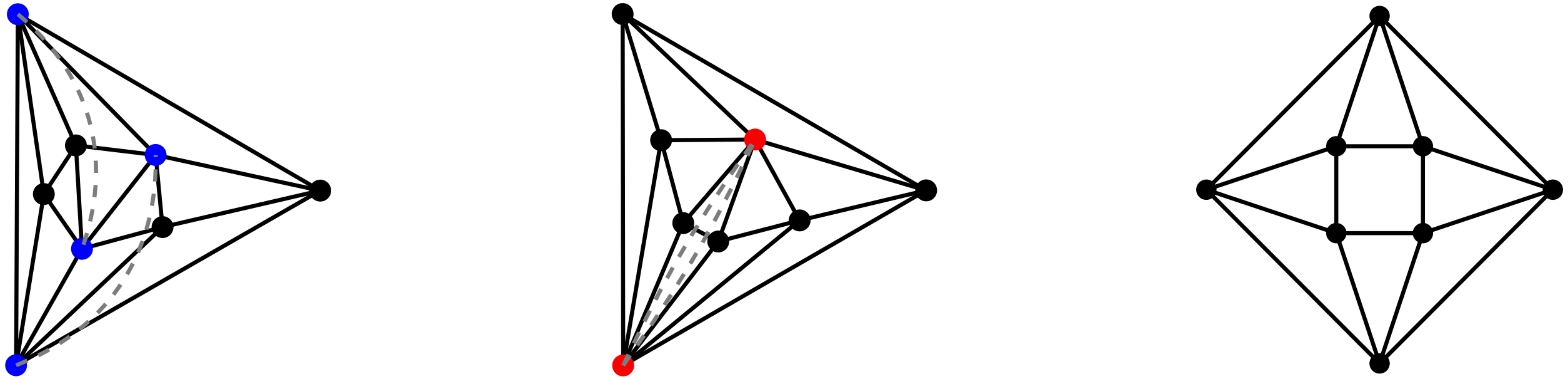
$\frac{7!}{20} \times 70\zeta(7)$

Planar duality ($x \leftrightarrow p$)



$\mathscr{C}^{\text{pert}}_{SU(N)}$: periods of *f*-graphs

L=4 (planar): 3 topologies



$f_1^{(4)}$ $f_2^{(4)}$ $f_3^{(4)}$

$$\frac{-1}{4!\,(-4)^4} \times \left(\mathscr{P}_{f_1^{(4)}} + \mathscr{P}_{f_2^{(4)}} - \mathscr{P}_{f_3^{(4)}} \right) = \frac{-6615\zeta(9)}{32}\,.$$

$\mathscr{P}_{f_1^{(4)}}$: $\frac{8!}{8} \times 252\zeta_9$

$\mathscr{P}_{f_2^{(4)}}$: $\frac{8!}{24} \times 252\zeta_9$

$\mathscr{P}_{f_3^{(4)}}$: $\frac{8!}{16} \times 168\zeta_9$

Non-planar sector at $L = 4$

- **Non-planar data** [Fleury, Pereira] and **Gram det.**

$$c^{(4)}_{1;\alpha} = 2 \times \{\,12,10,-14,8,-4,6,0,-1,-4,0,4,-2,-1,0^5,4,-2,4,0,-2,0^2,-2,0^6\}\,.$$

$$\frac{-1}{4!\,(-4)^4} \times \frac{1}{N^2} \times \sum_{\alpha=1}^{32} \boxed{c^{(4)}_{1;\alpha}}\, \mathscr{P}_{f^{(4)}_\alpha} = -\frac{2}{7N^2} \times \frac{6615\zeta(9)}{32}\,.$$

- **MZV's cancel**

$$\mathscr{P}_{f^{(4)}_4} = \frac{8!}{16} \times \left(\frac{432}{5}\zeta(5,3) + 252\zeta(5)\zeta(3) - \frac{58\pi^8}{2625} \right),$$

$$\mathscr{P}_{f^{(4)}_{12}} = \frac{8!}{4} \times \left(\frac{432}{5}\zeta(5,3) - 36\zeta(3)^2 + 360\zeta(5)\zeta(3) + \frac{189\zeta(7)}{2} - \frac{131\zeta(9)}{2} - \frac{58\pi^8}{2625} \right).$$

2nd Correlator: $\mathcal{H}^{\text{pert}}_{G_N}$

[Chester, Pufu], [Alday, Chester, Dorigoni, Green, Wen]

$$1\text{st}: \quad \frac{1}{4}\Delta_\tau \partial_m^2 \log Z(m;\tau)\Big|_{m\to 0}:$$

$$\mathcal{C}^{\text{pert}}_{G_N} = -\frac{8}{\pi}\int_0^\infty dr \int_0^\pi d\theta \frac{r^3\sin^2(\theta)}{U^2} T_N(U,V)$$

$$2\text{nd}: -48\zeta(3)c + \partial_m^4 \log Z(m;\tau)\Big|_{m\to 0}:$$

$$\mathcal{H}^{\text{pert}}_{G_N} = -\frac{32}{\pi}\int_0^\infty dr \int_0^\pi d\theta \frac{r^3\sin^2(\theta)}{U^2}(1+U+V)\boxed{\bar{D}_{1111}(U,V)}\, T_N(U,V)$$

$$g^{(5+L)}_{(1,2,3,4)} = \frac{1}{x^2_{1,5+L}\, x^2_{2,5+L}\, x^2_{3,5+L}\, x^2_{4,5+L}}$$

$$(\, U = 1 + r^2 - 2r\cos(\theta), \quad V = r^2\,)$$

$$S_{4+L} \to S_4 \times S_L$$

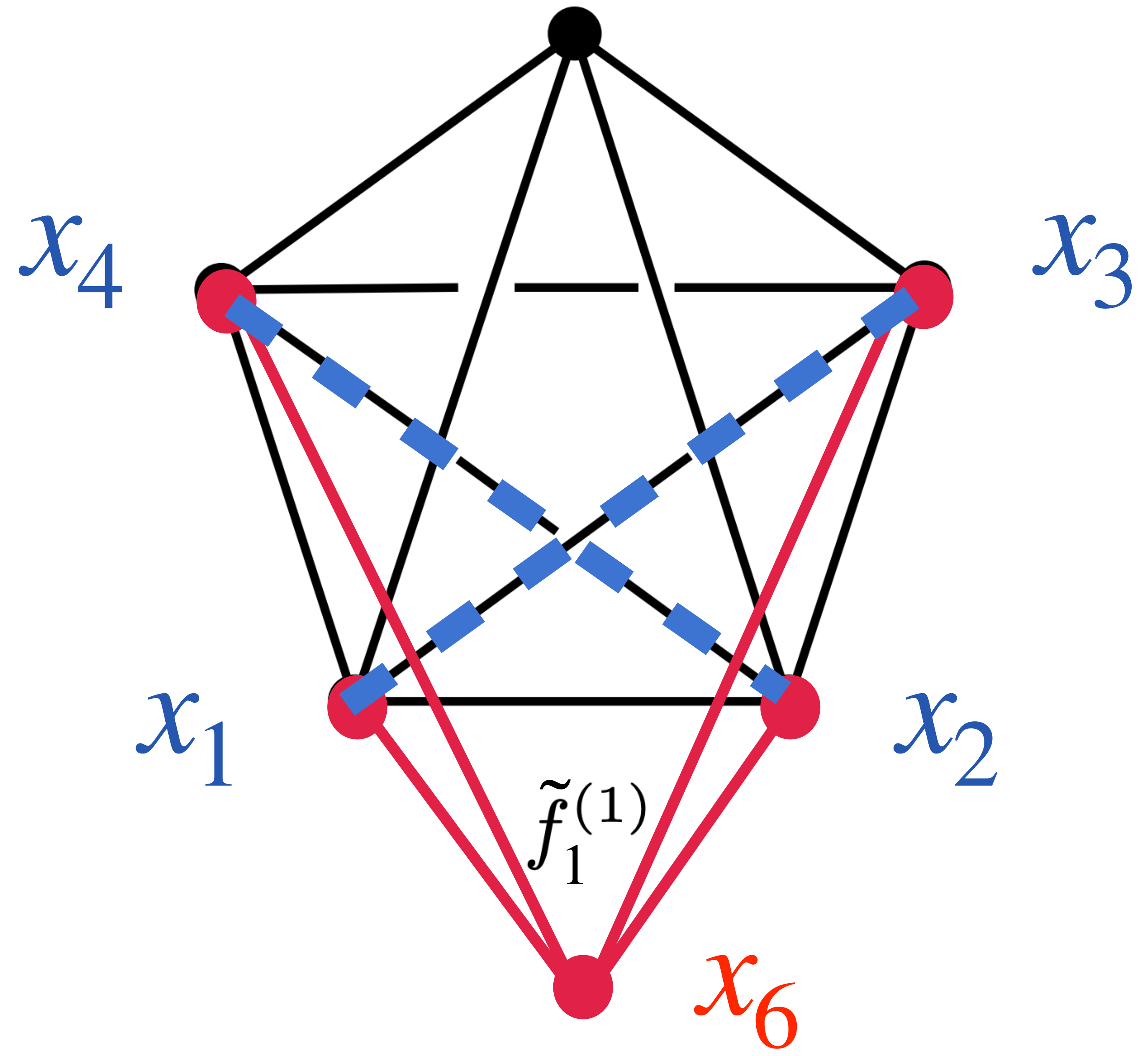
$\mathscr{H}^{\text{pert}}_{G_N}$: periods of $\tilde{f}$

$$\mathscr{H}^{\text{pert}}_{G_N} = 4c\left[-60\zeta_5\, a_{G_N} + \frac{3\,(36\zeta_3^2 + 175\zeta_7)\, a^2_{G_N}}{2} - \frac{45\,(20\zeta_3\zeta_5 + 49\zeta_9)\, a^3_{G_N}}{2} + \mathcal{O}(a^4_{G_N})\right].$$

$L = 1$:

$$\tilde{f}^{(1)} = \frac{x_{13}^2 x_{24}^2\,\boxed{(1 + U + V)}}{\prod_{1\le i<j\le 4} x_{ij}^2}\, g^{(5)}_{(1,2,3,4)} \times g^{(6)}_{(1,2,3,4)}.$$

($U \to x_{12}^2 x_{34}^2$, $V \to x_{14}^2 x_{23}^2$)

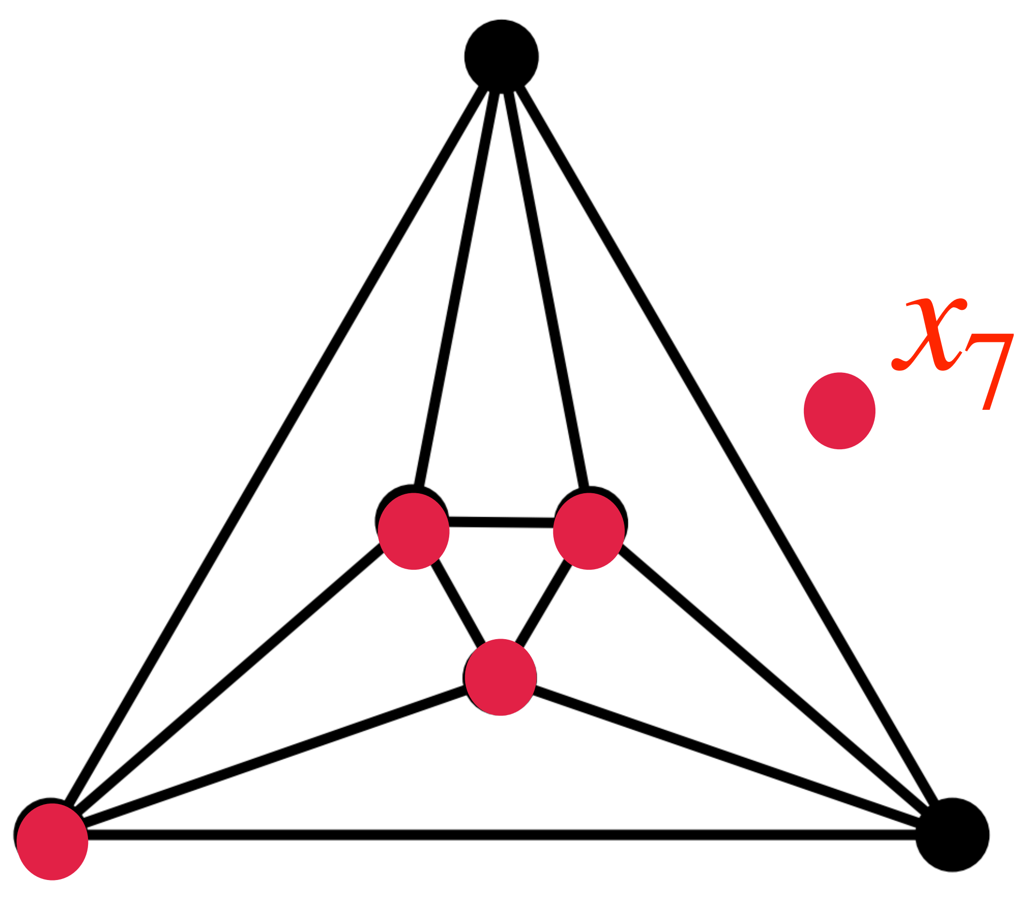


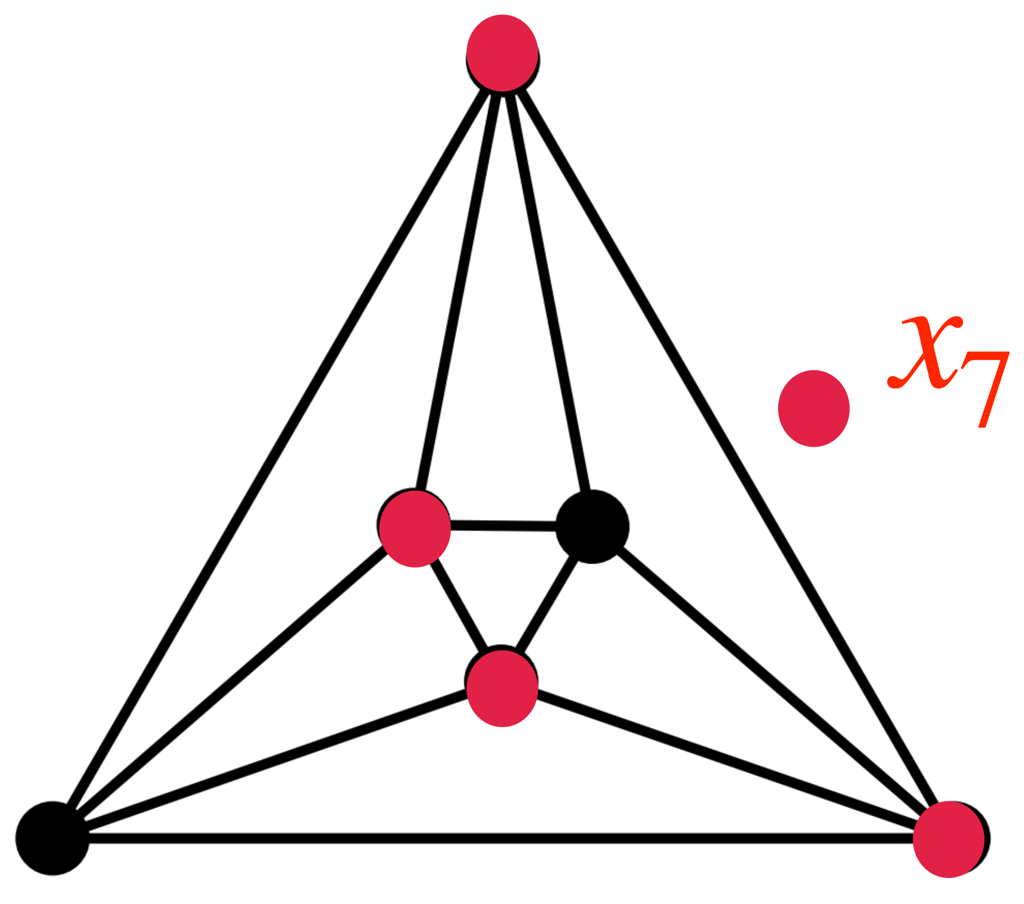
$$\mathscr{P}_{\tilde{f}^{(1)}} = \sum_{\blacksquare=1,U,V} \mathscr{P}_{\tilde{f}^{(1)}_{\blacksquare}} = 20\zeta(5) + 20\zeta(5) + 20\zeta(5) \longrightarrow 4 \times \frac{1}{(-4)} \times \mathscr{P}_{\tilde{f}^{(1)}} = \boxed{-60\zeta(5)}$$

$L = 2:$

$$\mathscr{P}_{\tilde{f}^{(2,1)}} = 12 \times \left(\frac{441}{8}\zeta(7) + 70\zeta(7) + \frac{441}{8}\zeta(7) \right)$$

$$\mathscr{P}_{\tilde{f}^{(2,2)}} = 3 \times \left[36\zeta(3)^2 + \left(72\zeta(3)^2 - 21\zeta(7)\right) + 36\zeta(3)^2 \right]$$





$$\frac{4}{2!(-4)^2} \times \left(\mathscr{P}_{\tilde{f}^{(2,1)}} + \mathscr{P}_{\tilde{f}^{(2,2)}} \right) = \boxed{\frac{3}{2} \times \left(36\zeta(3)^2 + 175\zeta(7)\right)}$$

$L = 3:$ $\quad \sum_{i=1}^{5} \mathscr{P}_{\tilde{f}^{(3,i)}}$ $\quad g \times \left(T, E, L, g \times h, H\right)$

Done similarly, $\zeta(3)^3$ cancelled.

Conclusion & Outlook

- **1st & 2nd int. correlator are periods of f, and $\tilde{f}$-graphs**

- **2nd int. correlator at $a_{G_N}^4$:**

$$\frac{45a_{G_N}^4}{16} \times \left(340\zeta(5)^2 + 588\zeta(3)\zeta(7) + 1617\zeta(11) + P_{G_N,1}\left(840\zeta(5)^2 + 1617\zeta(11)\right)\right)$$

- $\langle \mathcal{O}_2\mathcal{O}_2\mathcal{O}_p\mathcal{O}_p\rangle$ [Paul, Perlmutter,Raj], and $\langle \mathcal{O}_{p_1}\mathcal{O}_{p_2}\mathcal{O}_{p_3}\mathcal{O}_{p_4}\rangle$ [Brown, Heslop, Wen, Xie] using **10D symmetry** [Caron-Huot, Coronado]

- $\mathcal{N} = 2$ **SYM** [Billo, Frau, Lerda, Pini, Vallarino], **Wilson line** [Pufu, Rodriguez, Wang; Billo, Frau, Lerda], **determinant operators** $\mathcal{D}$, $\langle \mathcal{D}\mathcal{D}\mathcal{O}_2\mathcal{O}_2\rangle$ [Jiang, Wu, Zhang; Brown, Galvagno, Wen].