Nambu-Goto equations from 3D gravity

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Based on work with Ayan Mukhopadhyay and Giuseppe Policastro (2404.02149)

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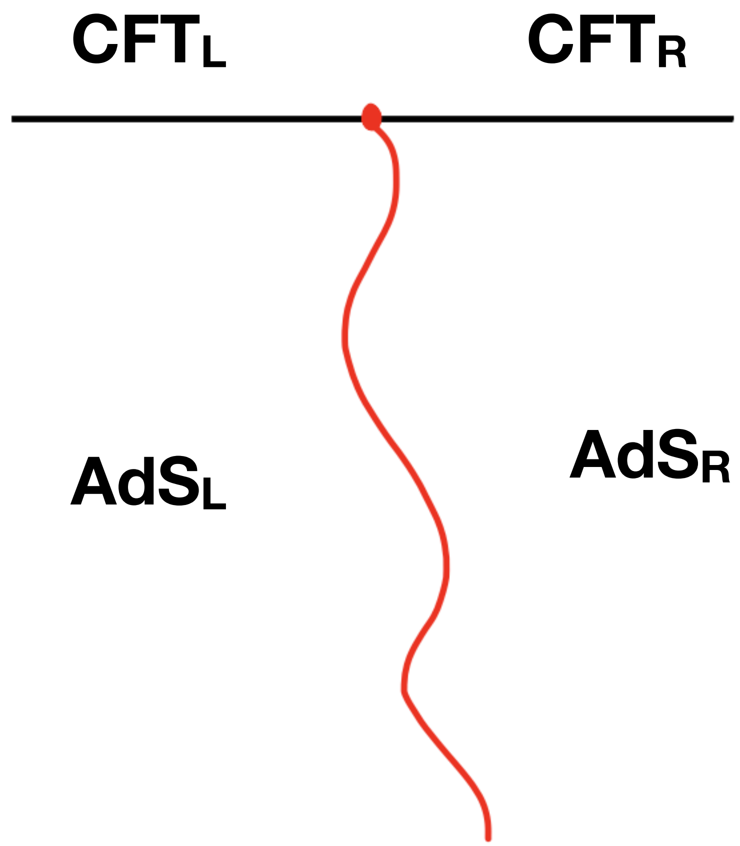
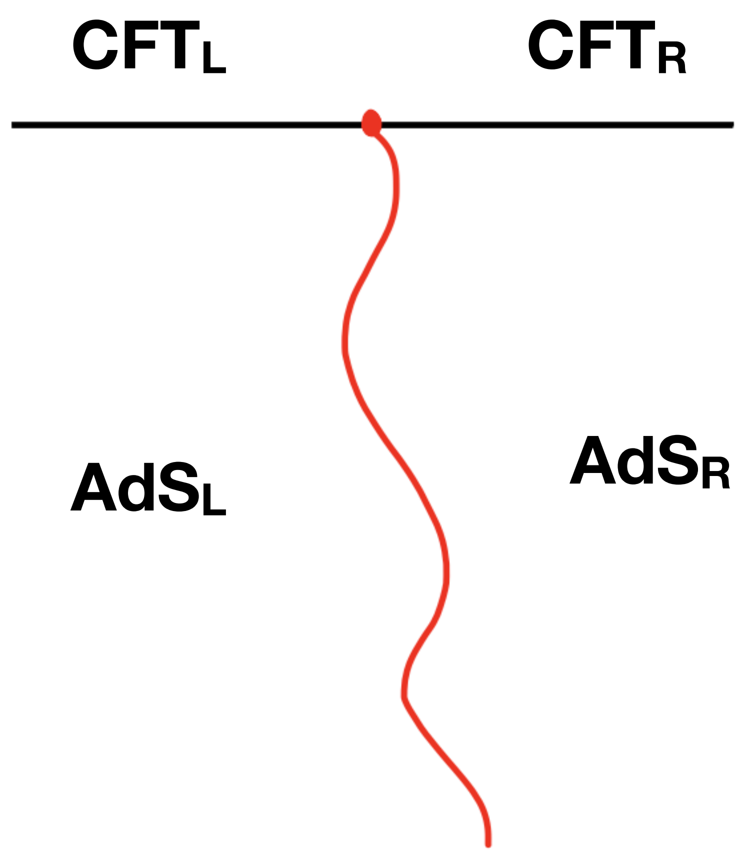
Introduction & Motivation



Introduction & Motivation

An ambitious question

Can we see the fundamental string in AdS from interfaces in the dual CFT?



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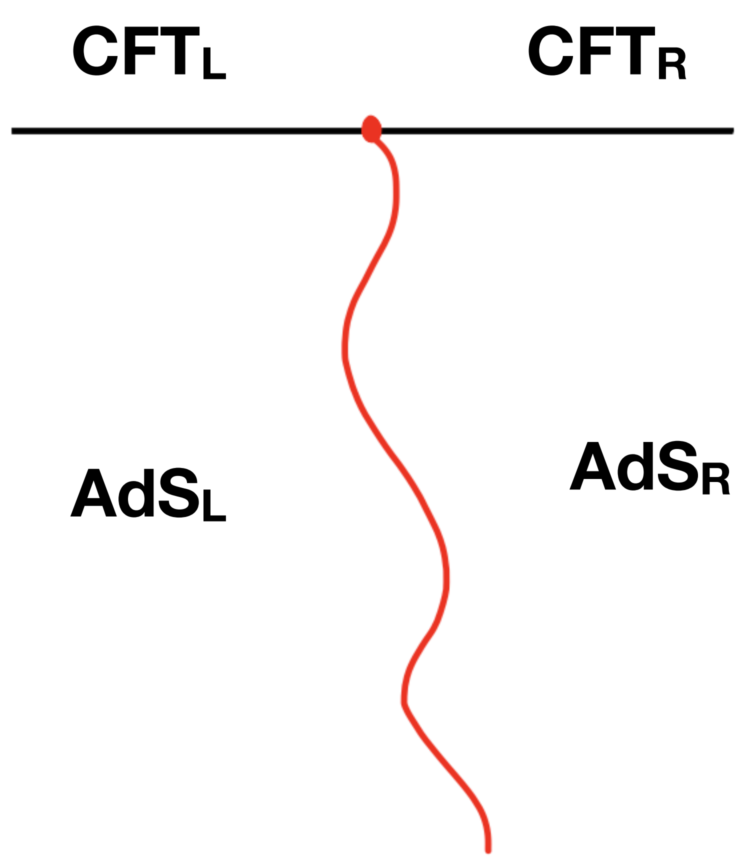
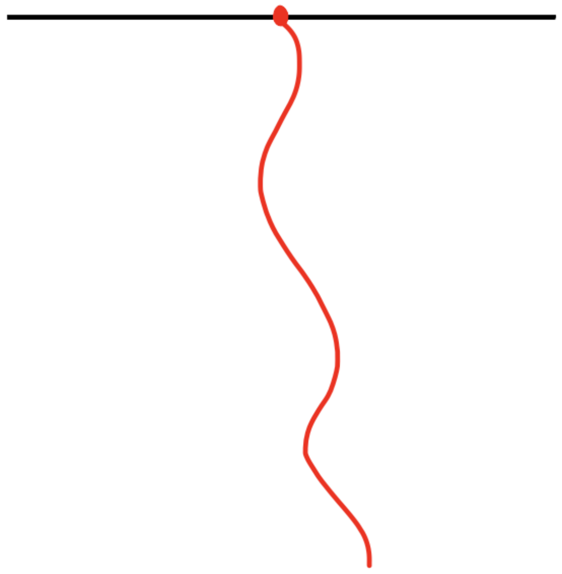


Can we see the fundamental string in AdS from interfaces in the dual CFT?

A slightly less ambitious question

Can Nambu-Goto equations emerge from gravity?

The Answer



Solution of gravitational junction (tensile string) equations in 3D = Solutions of Nambu-goto equation of a probe string + Corrections due to string tension + Corrections due rigid deformation of the worldsheet + Cross-terms

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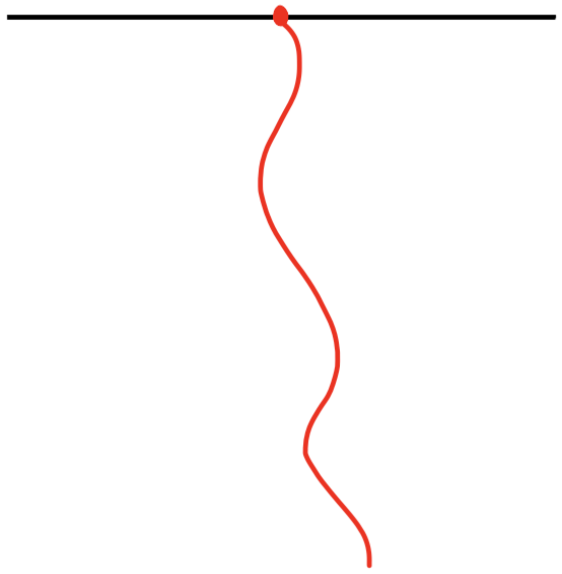

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Isometries of the worldsheet or isometries of the embedding space that shifts the worldsheet.

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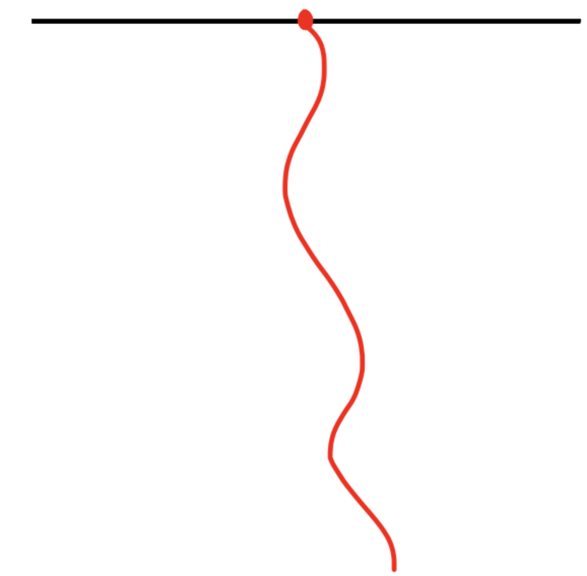

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Isometries of the worldsheet or isometries of the embedding space that shifts the worldsheet.

- Non-trivial as the string is not a probe

$$S = \frac{1}{16\pi G_N}\int_{\mathscr{M}} \mathrm{d}^D x\sqrt{-g}(R+2\Lambda) + T_0\int_{\Sigma}\mathrm{d}^{D-1}y\sqrt{-\gamma} + \text{GHY terms}$$

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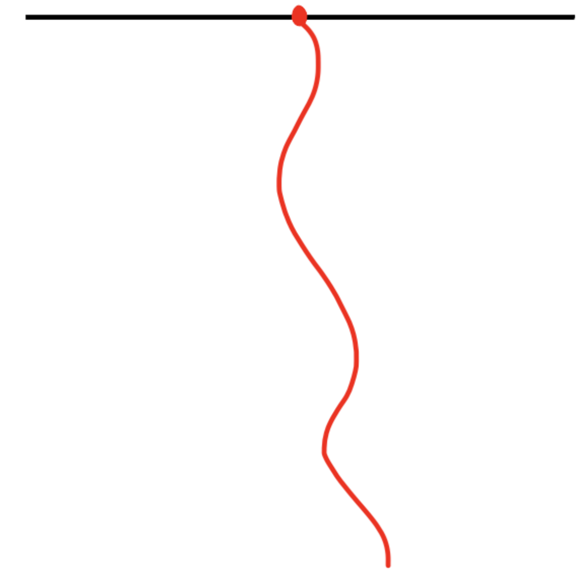

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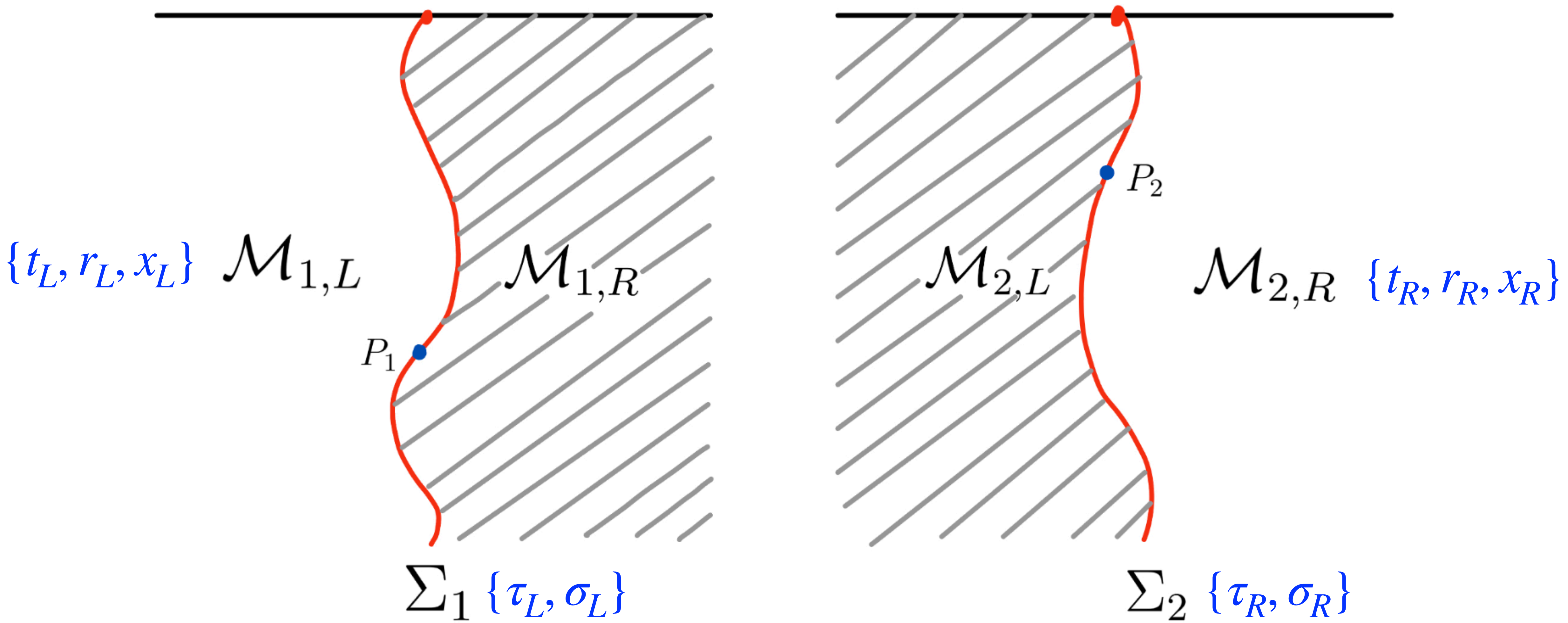
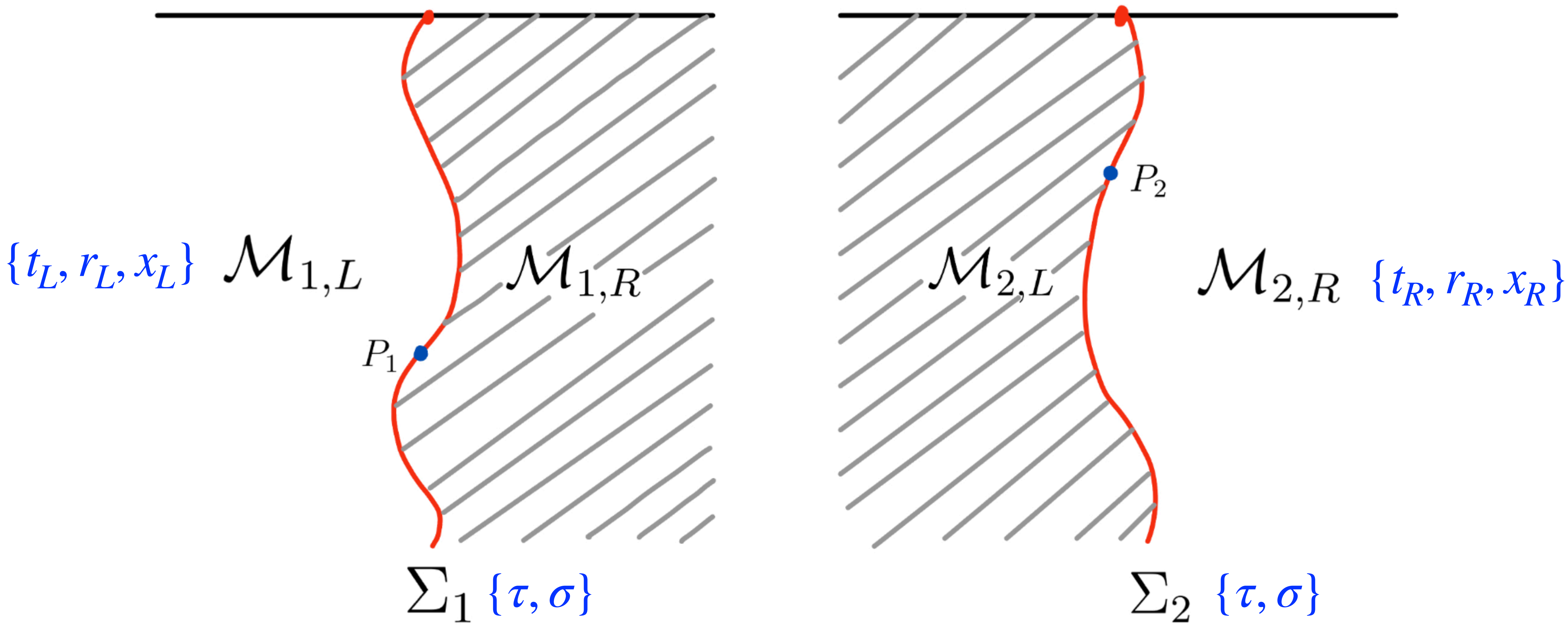
Nambu-Goto equation will follow from variation of the metric!!!

Our set up



$\{t_L, r_L, x_L\}$
$\mathcal{M}_{1,L}$
$\mathcal{M}_{1,R}$
P_1
Σ_1 $\{\tau_L, \sigma_L\}$
$\mathcal{M}_{2,L}$
$\mathcal{M}_{2,R}$
$\{t_R, r_R, x_R\}$
P_2
Σ_2 $\{\tau_R, \sigma_R\}$

Our set up



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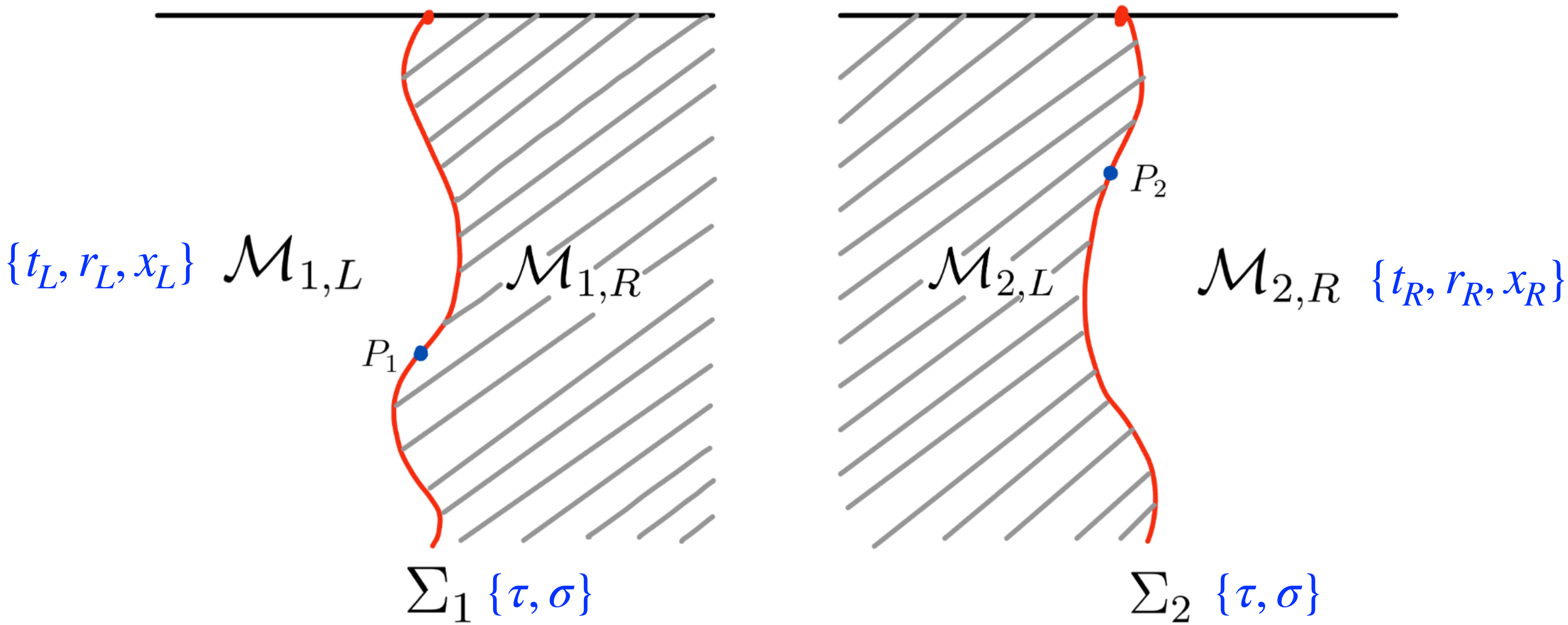

Gauge fixing $\tau = \frac{t_L + t_R}{2}$, $\sigma = \frac{r_L + r_R}{2}$

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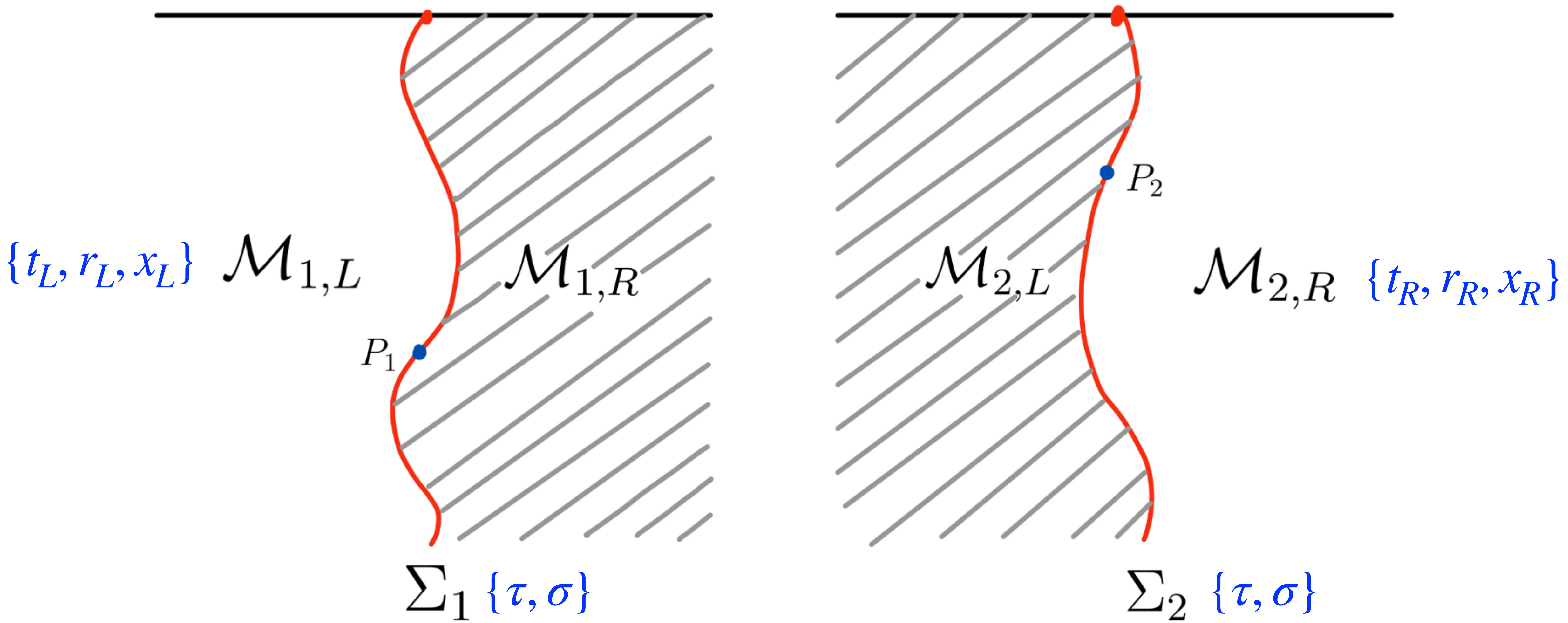

Gauge fixing $\quad \tau = \dfrac{t_L + t_R}{2}\ , \ \sigma = \dfrac{r_L + r_R}{2}$

Embedding

$$t_L(\tau,\sigma) = \tau - \tau_a(\tau,\sigma), \quad r_L(\tau,\sigma) = \sigma - \sigma_a(\tau,\sigma), \quad x_L(\tau,\sigma) = f_L(t_L(\tau,\sigma), r_L(\tau,\sigma))\,.$$

$$t_R(\tau,\sigma) = \tau + \tau_a(\tau,\sigma), \quad r_L(\tau,\sigma) = \sigma + \sigma_a(\tau,\sigma), \quad x_R(\tau,\sigma) = f_R(t_L(\tau,\sigma), r_L(\tau,\sigma))\,.$$

Variables and equations

- Dof

$$\tau_a(\tau,\sigma), \quad \sigma_a(\tau,\sigma), \quad x_s(\tau,\sigma) = \frac{f_L(\tau,\sigma) + f_R(\tau,\sigma)}{2}, \quad x_a(\tau,\sigma) = \frac{f_L(\tau,\sigma) - f_R(\tau,\sigma)}{2}.$$

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- Gluing conditions

$$[h_{ab}] = 0 \ , \qquad \text{(3 eqns)}$$

$$[K_{ab}] - h_{ab}[K] = 8\pi G T_0 \, h_{ab} \, . \qquad \text{(1 eqn)}$$

Conservation of Brown-York stress-tensor

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$$[K_{ab}] - h_{ab}[K] = 8\pi G T_0 \ h_{ab} \, .$$ (1 eqn) $x_a(\tau,\sigma)$

Conservation of Brown-York stress-tensor

Why D=3 is special

- Number of effective gravitational dof : $2D-(D-1)=D+1$

- Continuity of induced metric : $D(D-1)/2$ equations

- (Dis)continuity of extrinsic curvature: $D(D-1)/2$ equations

- Conservation of Brown-York: $(D-1)$ constraints.

- Number of independent equations: $D(D-1)-(D-1)=(D-1)^2$

Clearly $(D-1)^2 \geq (D+1)$ for $D \geq 3.$

Solving the conditions

- Dof $\tau_a(\tau,\sigma)$, $\sigma_a(\tau,\sigma)$, $x_s(\tau,\sigma) = \dfrac{f_L(\tau,\sigma) + f_R(\tau,\sigma)}{2}$, $x_a(\tau,\sigma) = \dfrac{f_L(\tau,\sigma) - f_R(\tau,\sigma)}{2}$

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- Perturbative ansatz

$$\tau_a(\tau,\sigma) = \sum_{n=1}^{\infty} \epsilon^n\ \tau_{a,n}(\tau,\sigma) \qquad f_L(\tau,\sigma) = x_0 + \sum_{n=1}^{\infty} \epsilon^n\ f_{L,n}(\tau,\sigma)$$

$$\sigma_a(\tau,\sigma) = \sum_{n=1}^{\infty} \epsilon^n\ \sigma_{a,n}(\tau,\sigma) \qquad f_R(\tau,\sigma) = x_0 + \sum_{n=1}^{\infty} \epsilon^n\ f_{R,n}(\tau,\sigma)$$

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Policastro et al.

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Policastro et al.

- We will choose a slightly less restrictive condition

$$\lim_{\sigma\to\infty} x_s = x_0 \qquad \lim_{\sigma\to\infty} x_{s,n} = 0 \ \text{ for } \ i = 1,2,.. \qquad \text{ingoing b.c at } \ \sigma = \sqrt{M}$$

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In a parallel exercise, we solve for Nambu-Goto equation for a probe string in a BTZ black hole of mass M, with precisely the same boundary condition.

Solving the conditions

Zeroth order

- Cond. h ⟶ Trivially satisfied
- Cond. K ⟶ Trivially satisfied

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First order

- Cond. h ⟶

$$\tau_{a,1} = \alpha_h + \frac{\sigma}{\sqrt{\sigma^2 - M}}\left(-\beta_h e^{\sqrt{M}\tau} + \gamma_h e^{-\sqrt{M}\tau}\right)$$

$$\sigma_{a,1} = \sqrt{M(\sigma^2 - M)}\left(\beta_h e^{\sqrt{M}\tau} + \gamma_h e^{-\sqrt{M}\tau}\right)$$

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$\alpha_h, \beta_h, \gamma_h$ are $SL(2,R)$ generators associated with the isometries of AdS_2 worldsheet.

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- Cond. K ⟶ $$x_{a,1} = -\frac{\lambda}{2\sigma} + \alpha_k + \frac{\sqrt{\sigma^2 - M}}{\sigma}\left(\beta_k e^{\sqrt{M}\tau} + \gamma_k e^{-\sqrt{M}\tau}\right), \qquad \lambda = 8\pi G T_0$$

Isometries of the BTZ that shifts the worldsheet without changing extrinsic curvature.

Asymptotics and boundary condition

- Near boundary

$$\tau_{a,1} = \alpha_h + \frac{\sigma}{\sqrt{\sigma^2 - M}}\left(-\beta_h e^{\sqrt{M}\tau} + \gamma_h e^{-\sqrt{M}\tau}\right) \sim \alpha_h + \left(-\beta_h e^{\sqrt{M}\tau} + \gamma_h e^{-\sqrt{M}\tau}\right)$$

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- In previous computations, these modes dropped out due to Dirichlet boundary condition.

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- Imposition of ingoing b.c at the horizon sets $\beta_h = 0,\ \beta_k = 0.$ $\qquad \{\alpha_h, \gamma_h, \alpha_k, \gamma_k\}$

Rigid deformation parameters

Asymptotics and boundary condition

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Rigid deformation parameters

$$\{\tau_{a,1}(\tau, \infty), \tau\}_{Sch} = -\frac{M}{2}$$ → Stress-tensor on the interface → Dynamics?!

Second order and higher $(n \geq 2)$

❋ Cond. h

$$\sigma'_{a,n} - \frac{\sigma}{\sigma^2 - M}\ \sigma_{a,n} - \frac{1}{2}(\sigma^2 - M)^{1/2}\left(\lambda\sqrt{\sigma^2 - M} + 2Me^{-\sqrt{M}\tau}\gamma_k\right) x'_{s,n-1} = \mathscr{A}_{n1}$$

$$\dot{\tau}_{a,n} + \frac{\sigma}{\sigma^2 - M}\ \sigma_{a,n} - \frac{\sqrt{M}\sigma}{\sqrt{\sigma^2 - M}}\gamma_k e^{-\sqrt{M}\tau}\ \dot{x}_{s,n-1} = \mathscr{A}_{n2}$$

$$\tau'_{a,n} - \frac{1}{(\sigma^2 - M)^2}\ \dot{\sigma}_{a,n} + \frac{1}{\sigma^2 - M}\left(\frac{\lambda}{2} + \frac{e^{-\sqrt{M}\tau}M}{\sqrt{\sigma^2 - M}}\gamma_k\right)\dot{x}_{s,n-1} - \frac{\sqrt{M}\sigma}{\sqrt{\sigma^2 - M}}e^{-\sqrt{M}\tau}\gamma_k\ x'_{s,n-1} = \mathscr{A}_{n3}$$

Second order and higher $(n \geq 2)$

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$$\frac{1}{\sigma^2 - M}\ddot{x}_{s,n-1} - (\sigma^2 - M)x''_{s,n-1} - \frac{2(2\sigma^2 - M)}{\sigma}x'_{s,n-1} = \widetilde{\mathscr{S}}^J_n$$

Second order and higher $(n \geq 2)$

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* Eom of a probe Nambu goto string in a BTZ background of mass M with the embedding

$$t = \tau, \quad r = \sigma, \quad x = x_0 + \sum_{n=1}^{\infty}\epsilon^n\,x_{NG,n}(\tau,\sigma)$$

Follows same eqtn (with different $\mathscr{S}^{NG}_n$)

✷ Upon imposing Dirichlet condition at the boundary and ingoing condition at the horizon

$$x_{s,1}(\tau,\sigma) = \sum_{n=0}^{\infty} A_n e^{-(2+n)\sqrt{M}\tau}\ \sigma^{-1}\ Q_1^{2+n}\left(\frac{\sigma}{\sqrt{M}}\right)$$

* Upon imposing Dirichlet condition at the boundary and ingoing condition at the horizon

$$x_{s,1}(\tau,\sigma) = \sum_{n=0}^{\infty} A_n e^{-(2+n)\sqrt{M}\tau}\ \sigma^{-1}\ Q_1^{2+n}\left(\frac{\sigma}{\sqrt{M}}\right)$$

* Superposition of quasi-normal modes with frequency $\omega_n = -\,i(2+n)\sqrt{M}$

⁕ Upon imposing Dirichlet condition at the boundary and ingoing condition at the horizon

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$$x_{s,1}(\tau,\sigma) = A_0 e^{-2\sqrt{M}\tau}\ \sigma^{-1}\ \frac{2M}{\sigma(M^2-\sigma)}$$

$$\tau_{a,2}(\tau,\sigma) = -A_0\lambda\ e^{-2\sqrt{M}\tau}\frac{M+\sigma^2}{2M^{3/2}(\sigma^2-M)} - 2A_0\gamma_k\ e^{-3\sqrt{M}\tau}\frac{\sqrt{M}}{(\sigma^2-M)^{\frac{3}{2}}}$$

$$\sigma_{a,2}(\tau,\sigma) = -A_0\lambda\ e^{-2\sqrt{M}\tau}\frac{M+\sigma^2}{M\sigma} - 2A_0\gamma_k\ e^{-3\sqrt{M}\tau}\frac{M}{\sigma\sqrt{\sigma^2-M}}$$

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Summary

Solution of gravitational junction (tensile string) equations in 3D = Solutions of Nambu-goto equation of a probe string + Corrections due to λ + Corrections due to $\{\alpha_h, \gamma_h, \alpha_k, \gamma_k\}$ + Cross-terms

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➡ Dual to $ICFT_2$ with dynamical interface (time-reparametrization modes).

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$$x_{s,3} = A_0^3 e^{-6\sqrt{M}\tau}\frac{4(7M-81\sigma^2)}{21\sigma^3(\sigma^2-M)^3} + A_0 M\lambda^2 e^{-2\sqrt{M}\tau}\frac{1}{4\sigma^3(\sigma^2-M)} + A_0 M\lambda\gamma_k e^{-3\sqrt{M}\tau}\frac{M-5\sigma^2}{\sigma^3(\sigma^2-M)^{\frac{3}{2}}} + A_0 M^2\gamma_k^2 e^{-4\sqrt{M}\tau}\frac{M-7\sigma^2}{\sigma^3(\sigma^2-M)^2}$$

Outlook

- Non-perturbative proof (Chesler-Yaffe with junction)

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- Are the rigid parameters physical? Transparency, entanglement…


Thank You