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Spinning Massive Amplitudes from The Flat limit of CFT correlators

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Based on:

INFN

- Kostas Skenderis, Mritunjay Verma and R. M., Momentum space CFT correlators of non-conserved spinning operators, JHEP 03 (2023) 196.
- Kostas Skenderis, Mritunjay Verma and R. M., Flat space spinning massive amplitudes from momentum space CFT, JHEP 08 (2024) 226.

Plan of the Talk

Motivations

- □ Momentum space CFT correlators of non-conserved spinning operators
- Bulk 3-point functions.
- □ Flat limit of the three points.
- Conclusions

- Holographic Principle: In quantum gravity, the physics of the interior of a region can be entirely described by the degrees of freedom on the boundary of that region. ('t Hoft 9310022, Susskind 9409089.)
- AdS/CFT duality: In its most general formulation, is an equivalence between a (d+1)dimensional gravitational theory on and asymptotically AdS space time and a d-dimensional conformal field theory (CFT) located on the boundary at spatial infinity.
- Most studied case:

String Theory on
$$AdS_5 \times S_5 \cong (\mathcal{N} = 4, \ d = 4)$$
 $SU(N)$ $SYM - theory$
 $g_{S, \ L}$
 $g_{YM}^2 = 4\pi g_{S, \ \lambda} = g_{YM}^2 N$

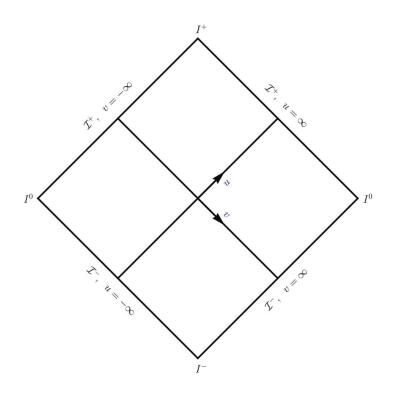
String Coupling AdS-Radius Yang-Mills Coupling 't Hooft Coupling
$$\frac{L^2}{\alpha'} = g_{YM} N^{\frac{1}{2}}$$

(Maldacena/9711200; Gubser, Klebanov and Polyakov/9802109; Witten/9802150.)

SPINNING MASSIVE AMPLITUDES FROM THE FLAT LIMIT OF CFT CORRELATORS

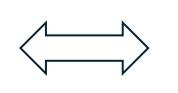
- In the very large N limit, the curvature of the dual geometry vanishes, and string theory in flat space should be recovered. (Polchinski/991076, Susskind/9901079).
- As the AdS radius approaches infinity, flat space geometry should be recovered. In principle, this could provide insights into quantum gravity in flat space by exploring the flat limit of the AdS/CFT correspondence.
- The flat space limit establishes a connection between the coefficients of the effective action in AdS and those in flat space. This relationship could potentially determine the electromagnetic form factors in AdS based on their established values in flat space.

- The flat-space limit also provides a link to flat-space holography: the duality between (d+1)-dimensional gravity in asymptotically flat spacetime and lower-dimensional (d or d-1) field theory located at infinity. (Alday, Nocchy, Ruzziconi and Srikant/2406.19343.)
- For massive particles, there are proposal suggesting that the dual theory may be located at either timelike infinity or spacelike infinity.
- To further explore this connection for massive particles, a detailed understanding of the flat limit of massive particles is essential.



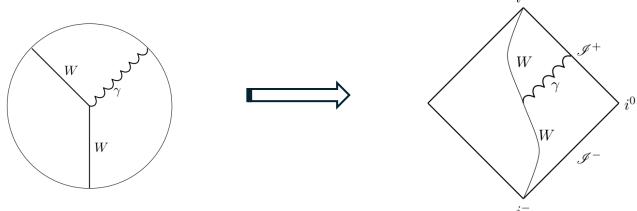
• We describe the flat limit as a simple yet physically interesting setup.

3-point CFT correlator involving a conserved current and two spin-1 operators in momentum space.



3-point function of an abelian gauge field with a massive spin-1 complex Proca field.

Our goal is to derive the scattering process of a photon with a massive particle from flat limit CFT correlators.



• In momentum space, both sides of the duality need to be determined.



• The CFT 3-point involving a conserved current $\mathcal{J}_{\mu}(p)$ and two non conserved spin-1 fields of conformal dimension Δ , $\mathcal{O}_{\Delta}(p)$ can be expressed as:

$$\mathcal{A}_{3}^{\mu_{1}\mu_{2}\mu_{3}} = (2\pi)^{d} \delta^{d}(\mathbf{p}_{1} + \mathbf{p}_{2} + \mathbf{p}_{3}) \left\langle \left\langle \mathcal{O}_{\Delta}^{\mu_{1}}(\mathbf{p}_{1}) \mathcal{J}^{\mu_{2}}(\mathbf{p}_{2}) \mathcal{O}_{\Delta}^{\mu_{3}}(\mathbf{p}_{3}) \right\rangle \right\rangle$$

The current is decomposed in its longitudinal and transverce components

$$j^{\mu}(\mathbf{p}_2) = \pi^{\mu}{}_{\nu}(\mathbf{p}_2)\mathcal{J}^{\nu}(\mathbf{p}_2), \qquad \pi^{\mu\nu}(\mathbf{p}_2) = \delta^{\mu\nu} - \frac{p_2^{\mu}p_2^{\nu}}{p_2^2}, \qquad p_2^{\mu}\pi_{\mu\nu}(\mathbf{p}_2) = 0$$

• The form factors of the CFT- correlator are obtained by solving the momentum space CFT-Ward identities.

$$\mathcal{D}\mathcal{A}_{3}^{\mu_{1}\mu_{2}\mu_{3}} = \mathcal{K}_{\mu}\mathcal{A}_{3}^{\mu_{1}\mu_{2}\mu_{3}} = 0$$
Generator of the Dilatations

Generator of special conformal transformations

• Focusing on the transverse part:

$$\langle \langle \mathcal{O}_1^{\mu_1}(\mathbf{p}_1) \, j^{\mu_2}(\mathbf{p}_2) \mathcal{O}_3^{\mu_3}(\mathbf{p}_3) \rangle \rangle = (\pi \cdot p_1)^{\mu_2} A^{\mu_1 \mu_3} + \pi^{\mu_2 \mu_1} B^{\mu_3} + \pi^{\mu_2 \mu_3} C^{\mu_1}$$
$$\mu = 1, \dots d$$

 \succ The form factors A_i , i = 1, ..., 5, B_i and C_i , i = 1, 2 depend on the magnitude of the momenta.

$$A^{\mu_1\mu_3} = A_1 \,\delta^{\mu_1\mu_3} + A_2 \,p_1^{\mu_1} \,(p_1 + p_2)^{\mu_3} + A_3 \,p_2^{\mu_1} \,(p_1 + p_2)^{\mu_3} + A_4 \,p_1^{\mu_1} p_2^{\mu_3} + A_5 \,p_2^{\mu_1} p_2^{\mu_3};$$

$$B^{\mu_3} = B_1 (p_1 + p_2)^{\mu_3} + B_2 p_2^{\mu_3};$$

 $C^{\mu_1} = C_1 p_1^{\mu_1} + C_2 p_2^{\mu_1}.$

• Focusing on the transverse part

➤ The form factors A_i , i = 1, ..., 5, B_i and C_i , i = 1, 2 depend on the magnitude of the momenta and are antisymmetric in the exchange of (μ_1, p_1) and (μ_3, p_3) $\square A_3 = A_4$, $B_1 = C_1$, $B_2 = -C_2$.

$$\begin{aligned} A_1 &= -a_5 J_{2\{0,1,0\}} + a_1 J_{1\{0,0,0\}}; A_2 = -a_5 J_{3\{-1,2,-1\}} + a_2 J_{1\{-1,0,-1\}} + 2a_4 J_{2\{-1,1,-1\}}; \\ A_3 &= a_5 J_{3\{0,1,-1\}} - a_4 J_{2\{0,0,-1\}}; A_5 = a_5 J_{3\{0,0,0\}} ; B_2 = -a_5 J_{2\{0,0,1\}} + b_2 J_{1\{0,0,0\}}; \\ B_1 &= -a_5 J_{2\{0,1,0\}} + b_1 J_{1\{0,1,-1\}} + (b_1 - b_2) J_{1\{1,0,-1\}} + (b_1 - b_2 + a_4) J_{1\{0,0,0\}}; \end{aligned}$$

$$\begin{array}{ll} \succ \text{ Triple K Integrals} \\ J_{N\{k_1,k_2,k_3\}}(p_1,p_2,p_3) & \equiv \int_0^\infty dx \, x^{\frac{d}{2}+N-1} \prod_{i=1}^3 p_i^{\Delta_i - \frac{d}{2}+k_i} \, K_{\Delta_i - \frac{d}{2}+k_i}(xp_i) \, . \end{array}$$

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• Only three parameters are independents

$$a_1 = (d-2)\Delta a_5 - (\Delta - 1)a_4 + b_2 \quad ; \quad a_2 = 2(d-2)\Delta a_5 - (2\Delta + d - 4)a_4 + \frac{(2\Delta - d)}{(\Delta - 1)}b_2$$
$$b_1 = \frac{(2\Delta - d)}{2(\Delta - 1)}b_2$$

The most general Euclidean bulk action linear in the gauge field A_M, (M=1,...d+1) and quadratic in the Proca fields, in d+1 AdS-spacetime and up to 3 derivative terms is:

$$S = \int d^{d+1}x \sqrt{G} \left[-\frac{1}{16\pi G_N} (R - 2\Lambda) + \frac{1}{4} F^{MN} F_{MN} + \frac{1}{2} W^*_{MN} W^{MN} + m^2 W^*_M W^M - ig \alpha F^{MN} W^*_M W_N + ig \beta F^{MN} \left(\nabla_M W^*_P \nabla^P W_N - \nabla_M W_P \nabla^P W^*_N \right) \right],$$

 $F_{MN} = \partial_M A_N - \partial_N A_M$

 Λ -Cosmological constant

 $W_{MN} = (\nabla_M + igA_M)W_N - (\nabla_N + igA_N)W_M$

✓ We Fourier transform the boundary coordinates $(x, x^{\mu}) \rightarrow (z, k^{\mu})$, $(\mu = 1, ..., d)$ and choose the axial gauge $A_0(z, k)=0$.

✓ For the three-points we need the perturbative solution of the gauge field up to the first order in the gauge coupling.

$$A_{\mu}(z, k) = \mathbb{K}_{\mu}{}^{\nu}(z, k)A_{(0)\nu}(k) + \int dw \sqrt{G} \ \mathcal{G}_{\mu\nu}(z, w; k) J^{\nu}(w, k) ,$$

Bulk-to boundary propagator
Boundary value of gauge field
Bulk to bulk propagator

✓ For the 3-point function we only need the free classical solution for the massive fields since it is determined through the back reaction of the massive fields to A_{μ} .

$$W_M(z, k) = \mathcal{K}_M^{\ \mu}(z, k) w_\mu(k) \quad ; \quad W_M^*(z, k) = \bar{\mathcal{K}}_M^{\ \mu}(z, k) w_\mu^*(k)$$

Bulk to boundary propagator

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FUNDAMENTAL PHYSICS

✓ The Proca fields are dual to the boundary operators \mathcal{O}_{Δ} , their mass is related to the conformal dimension Δ by :

$$L^2 m^2 = (\Delta - 1)(\Delta + 1 - d)$$

- ✓ The flat limit that we shall consider consists in taking $L, \Delta \rightarrow \infty$ kepping finite the mass of the particle.
- ✓ According to the AdS/CFT correspondence the on-shell bulk partition function is the generating functional of the dual CFT-correlation functions.
- ✓ The generator of the connected QFT correlators is the classical on-shell action.

• Holographic Renormalization: To regularize the on-shell action we restrict the radial coordinate $\varrho = \frac{z^2}{L} > \epsilon$ and add counter terms to cancel infinities. The full renormalized action is:

$$S_{\text{ren}} = \lim_{\epsilon \to 0} \left(S_{\text{reg}} + S_{\text{ct}} \right)$$

✓ From the renormalized action one gets the exact renormalized 1-point function

$$\langle \mathcal{J}^{\mu}(k) \rangle = \lim_{\epsilon \to 0} \frac{1}{\epsilon^{\frac{d}{2}} \sqrt{\gamma}} \frac{\delta S_{\text{ren}}}{\delta A_{\mu}(k,\epsilon)} = \delta^{\mu\tau} \int d^{d}y \, dw \sqrt{G} \, \mathbb{K}_{\tau}^{\ \nu}(w;x,y) J_{\nu}(y,w)$$

✓ The CFT 3-point function of a conserved current and two spin-1 operators is obtained by differentiating with respect to the sources

$$\left\langle \mathcal{O}^{*\nu}(x_1) \mathcal{J}^{\mu}(x_2) \mathcal{O}^{\sigma}(x_3) \right\rangle = \frac{\delta^2 \left\langle \mathcal{J}^{\mu}(x_2) \right\rangle}{\delta \mathcal{W}_{\nu}^{(0)}(x_1) \delta \mathcal{W}_{\sigma}^{*(0)}(x_3)} = \delta^{\mu\tau} \int d^d y \, dw \sqrt{G} \, \mathbb{K}_{\tau}^{\lambda}(w; x_2, y) \, \frac{\delta^2 J_{\lambda}(y)}{\delta \mathcal{W}_{\nu}^{(0)}(x_1) \delta \mathcal{W}_{\sigma}^{*(0)}(x_3)}$$

BULK Theory 3-Point Function

The CF The CFT 3-point function obtained from the bulk dual theory exactly matches the boundary CFT result after identifying the bulk parameters with the three independent CFT coefficients:

$$g = -\frac{(\Delta - 1)\left(-a_4 + (d - 2)a_5\right) + b_2}{2C_0} \quad ; \quad \alpha = -\frac{(\Delta - 1)(-a_4 + da_5) + 2a_5 - b_2}{(\Delta - 1)\left(-a_4 + (d - 2)a_5\right) + b_2} \quad ; \quad \beta = -\frac{a_5L^2}{(\Delta - 1)\left(-a_4 + (d - 2)a_5\right) + b_2}$$

$$C_0 = -\frac{2^{2-\frac{d}{2}}}{\Gamma\left(\frac{d}{2}-1\right)} \left[\frac{2^{\frac{d}{2}+1-\Delta}}{\Gamma\left(\Delta-\frac{d}{2}\right)}\right]^2 L^{2\Delta-d-1}$$

- ✓ The number of the bulk parameter coincides with those of the CFT and we spann the full 3dimensional CFT space.
- Each coupling in the bulk, whether minimal, gyromagnetic, or quadrupole, is consistent with the CFT 3-point function on its own, because the bulk action remains AdS-invariant for any value of the coupling

Flat Space Limit of AdS Metric and Isometries

• AdS- metric in the Poincaré coordinates

$$ds^{2} = \frac{L^{2}}{z^{2}} \left(dz^{2} + \delta_{\mu\nu} dx^{\mu} dx^{\nu} \right) \qquad ; \qquad x^{a} = (z, x^{\mu})$$

✓ In the limit L → ∞ the Riemann, Ricci and scalar curvatutes vanishes getting the flat geometry.
 ✓ To analyse the flat limit, it is convenient to parametrize the radial coordinate as:

$$\frac{z}{L} = e^{\frac{\tau}{L}}$$
; $\tau \in (-\infty, \infty)$ Gubser,Klebanov Polyakov/9802109
Yue-Zhou Li/2106.04606

✓ In the large L limit, τ becomes the $(d + 1)^{th}$ flat (euclidean) time direction and the AdS-metric reduces to Minkowsky one:

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Minka

• The isometry algebra of Euclidead AdS_{d+1} is $\mathfrak{so}(d+1,1)$

$$[M_{AB}, M_{CD}] = \eta_{BC} M_{AD} - \eta_{AC} M_{BD} + \eta_{AD} M_{BC} - \eta_{BD} M_{AC}$$

 $\eta_{AB} = (+, \dots, +, -) \quad , \quad A, B, C, D = 1, 2, \cdots, d + 1, d + 2 \equiv \{a, d + 2\} \equiv \{\mu, d + 1, d + 2\}$

✓ Inönü-Wigner contraction: Upon splitting the $(d + 2)^{th}$ component of the algebra and writing $M_{a,d+2} = L P_a$

In the flat limit, the (Euclidean) AdS isometry algebra reduces to the algebra of the Euclidean group.
 Upon Wick rotation this becomes the Poincaré group.

- We shall consider the flat limit $L, \Delta \to \infty$ while keeping τ ($z = L e^{\frac{t}{L}}$) and the bulk parameters, m, g, α and β finite.
- ✓ The flat space limit of the bulk-to-boundary propagator determines the external leg factors of the corresponding fields.
- The flat limit of the bulk to boundary propagator of thre Proca-field requires uniform expansion of the modified Bessel function:

$$K_{\nu}(\nu z)\Big|_{\nu \to \infty} \simeq \left(\frac{\pi}{2\nu}\right)^{\frac{1}{2}} \frac{e^{-\nu \xi(z)}}{(1+z^2)^{\frac{1}{4}}} \quad ; \qquad \xi(z) = (1+z^2)^{\frac{1}{2}} + \ln\left(\frac{z}{1+(1+z^2)^{\frac{1}{2}}}\right)$$
$$\implies \qquad K_{\Delta - \frac{d}{2} + \ell}(z \, k) = \left(\frac{\pi}{2L}\right)^{\frac{1}{2}} (k^2 + m^2)^{-\frac{1}{4}} \left(\frac{k}{m + \sqrt{k^2 + m^2}}\right)^{-mL-\ell} e^{-\sqrt{k^2 + m^2}(L+\tau)} + \mathcal{O}\left(\frac{1}{L}\right)$$

✓ In the limit, the Proca field turns out to be:

$$W_{a}(k) \simeq \mathcal{W}_{a}(k) e^{-\sqrt{k^{2}+m^{2}\tau}} + O\left(\frac{1}{L^{\frac{d-5}{2}}}\right), \quad \mathcal{W}_{a}(k) = \frac{1}{\sqrt{Z_{W}}} \left(i\frac{k_{\mu}w^{\mu}}{m}, \,\tilde{\pi}_{\mu\nu}w^{\nu}\right),$$
$$\tilde{\pi}_{\mu\nu} = \delta_{\mu\nu} + \frac{k_{\mu}k_{\nu}}{m(m+\sqrt{k^{2}+m^{2}})} \qquad \qquad \frac{1}{\sqrt{Z_{W}}} \equiv \frac{2^{1-mL}\pi^{\frac{1}{2}}L^{\frac{d-3}{2}}e^{-L\sqrt{k^{2}+m^{2}}}}{\Gamma(mL)} \frac{(m+\sqrt{m^{2}+k^{2}})^{mL}}{\sqrt{2\sqrt{k^{2}+m^{2}}}}$$

✓ The uplifted (d+1)-dimensional Euclidean momenta:

$$q^a = (\pm i\sqrt{k^2 + m^2}, k^{\mu}) , q^2 = -m^2$$

✓ The gauge field in the flat limit becomes:

$$A_a(\tau,k) = \mathcal{A}_a e^{-k\tau}, \qquad \mathcal{A}_a \equiv \left(0, \, \pi_{\mu\nu} \frac{1}{\sqrt{Z_A}} A^{\mu}_{(0)}(k)\right)$$

$$\frac{1}{\sqrt{Z_A}} = \pi^{\frac{1}{2}} k^{\frac{d-3}{2}} L^{\frac{d-3}{2}} e^{-kL}$$

 \checkmark The (d+1)-dimensional null momenta is defined as:

$$q^{a} = (q^{0}, q^{\mu}) = (\pm ik, k^{\mu}), \qquad q^{2} \equiv \delta_{ab} q^{a} q^{b} = 0$$

• The 3-point CFT d-dimensional correlator in momentum space are expressed in terms of triple-K integrals. Their flat limit reads:

✓ At the leading order in the AdS-radius L the correlator becomes:

$$A_3^{\mu_1\mu_2\mu_3}\Big|_{L\to\infty} = 2\pi i \,\delta(q_1^{(0)} + q_2^{(0)} + q_3^{(0)}) \,\frac{g}{\sqrt{Z_{W_1} \, Z_A \, Z_{W_3}}} \,\mathcal{C}^{\mu_1\mu_2\mu_3}$$

Flat Limit of the Three-point

✓ with

$$\begin{split} \mathcal{C}^{\mu_{1}\mu_{2}\mu_{3}} &= -(1+\alpha)\pi_{\ \mu}^{\mu_{2}}\left[\left(\eta^{\mu_{1}\mu} + \frac{p_{1}^{\mu_{1}} p_{1}^{\mu}}{m(E_{1}+m)}\right)\left(\frac{(p_{1}+p_{2})^{\mu_{3}} p_{2}}{E_{3}+m} + p_{2}^{\mu_{3}}\right)\right. \\ &+ \left(\eta^{\mu\mu_{3}} + \frac{p_{3}^{\mu_{3}} p_{3}^{\mu}}{m(E_{3}+m)}\right)\left(\frac{p_{1}^{\mu_{1}} p_{2}}{E_{1}+m} - p_{2}^{\mu_{1}}\right)\right] - 2p_{1\,\mu}\pi^{\mu\mu_{2}}\left[\eta^{\mu_{1}\mu_{3}} - \frac{p_{1}^{\mu_{1}} p_{2}^{\mu_{3}}}{m(E_{1}+m)}\right. \\ &+ \left.\frac{2p_{1}^{\mu_{1}} (p_{1}+p_{2})^{\mu_{3}}}{(E_{1}+m) (E_{3}+m)} - \left.\frac{2E_{2} p_{1}^{\mu_{1}} (p_{1}+p_{2})^{\mu_{3}}}{m(E_{1}+m) (E_{3}+m)} + \left.\frac{p_{2}^{\mu_{1}} (p_{1}+p_{2})^{\mu_{3}}}{m(E_{3}+m)}\right]\right. \\ &+ 2\beta p_{1\,\mu}\pi^{\mu\mu_{2}}\left[\frac{p_{1}^{\mu_{1}} E_{2}}{(E_{1}+m)} \frac{(p_{1}+p_{2})^{\mu_{3}} E_{2}}{(E_{3}+m)} - \frac{p_{2}^{\mu_{1}} (p_{2}+p_{1})^{\mu_{3}} E_{2}}{(E_{3}+m)} + \frac{p_{1}^{\mu_{1}} p_{2}^{\mu_{3}} p_{2}}{E_{1}+m} - p_{2}^{\mu_{1}} p_{2}^{\mu_{3}}\right] \end{split}$$

✓ By using the polarizations suggested by the flat limit of the Btb-propagators

$$\varepsilon_a^W = \left(\frac{(p \cdot \varepsilon)}{m}, \, \varepsilon_\mu + \frac{(p \cdot \varepsilon)}{m(E+m)} \, p_\mu\right) \quad ; \quad \varepsilon_a^A = (0, \, \pi_{\mu\nu} \epsilon^\nu) \,,$$

✓ In the flat limit, the d-dimensional correlator dressed by the (d+1)-dimensional polarization vectors:

$$C = \left[2(\varepsilon_1 \cdot \varepsilon_3)(\varepsilon_2 \cdot k_1) + (1+\alpha)(\varepsilon_2 \cdot \varepsilon_1)(\varepsilon_3 \cdot k_2) - (1+\alpha)(\varepsilon_2 \cdot \varepsilon_3)(\varepsilon_1 \cdot k_2) + 2\beta(\varepsilon_2 \cdot k_1)(\varepsilon_3 \cdot k_2)(\varepsilon_1 \cdot k_2) \right]$$

This is exactly the (d+1) dimensional three- point amplitude, $\mathcal{M}_{3}^{\mu_{1}\mu_{2}\mu_{3}}$, stripped by the delta over the momenta, of a photon in interaction with two complex Proca fields.

$$\lim_{L \to \infty} \frac{i}{2\pi} \sqrt{Z_{W_1} Z_A Z_{W_3}} A_3^{\mu_1 \mu_2 \mu_3} = \delta(q_0^{(1)} + q_0^{(2)} + q_0^{(3)}) \mathcal{M}_3^{\mu_1 \mu_2 \mu_3}$$

Conclusions

- We have solved the CFT Ward identities defining the 3-point function involving a conserved current and two non-conserved operators.
- ✓ Using the holographic renormalization approach, we have computed the dual "Witten diagram".
- ✓ We have shown that the AdS calculation is in agreement with the CFT result, providing a new test of the AdS/CFT correspondence.
- ✓ The matching is valid for each bulk coupling separately.
- We have computed the flat limit of the d-dimensional CFT correlator, keeping the mass and gauge coupling of the dual theory fixed, reproducing the (d+1)-dimensional flat amplitude with two complex Proca fields interacting with a U(1) gauge field.
- ✓ In the flat limit, the Euclidean time coordinate and the corresponding delta function on the energies are emerging quantities.
- ✓ From the CFT perspective, one zooms into the IR region while sending the conformal dimension of the operator dual to the massive fields to infinity.
- ✓ The flat limit transforms the AdS isometries into Poincaré isometries.

- We expect that our analysis will extend to higher-point functions.
- As we have seen, holography allows for the construction of flat space couplings from CFT correlators, which are completely fixed by the CFT Ward identities. This approach appears promising for constructing massive higher-spin couplings in flat space.
- Gain insights into flat space holography through the flat limit of the AdS/CFT duality

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Thank you!