Soft factorization on AdS spacetimes

Karan Fernandes

National Taiwan Normal University

based on JHEP **08**, 105 (2021),JHEP **04**, 055 (2023)
in collaboration Nabamita Banerjee (IISER Bhopal, India)
and Arpita Mitra (POSTECH, South Korea)

Eurostrings & FPUK 2024, University of Southampton, UK

September 6, 2024

Outline

Single soft theorems on flat spacetime

- Factorization of scattering amplitudes involving soft particles

$$\lim_{k\to 0} \mathcal{M}_{n+1}(\varepsilon, k; \{p_a\}) = \text{Soft factor} \;\times\; \mathcal{M}_n(\{p_a\})$$

ε : polarization; k : soft momentum; p_a : hard momenta

$\mathcal{M}_n$: Amplitude with n external particles

- The soft factor admits a frequency expansion

$$S_{\text{em}} = \sum_i S_{\text{em}}^{(i)} \qquad S_{\text{em}}^{(0)} = \sum_{a=1}^{n} Q_{(a)}\eta_{(a)} \frac{\varepsilon . p_{(a)}}{p_{(a)} . k} \;\text{(leading)}$$

$\eta_{(a)} = +1(-1)$ for outgoing (incoming), $Q_{(a)}$ charges
and ε^{μ} the polarization

- Renewed interest in relation to asymptotic symmetries and holography on asymptotically flat spacetimes [see talk by Laura Donnay]

Soft theorem = Large gauge Ward identity

- The soft factor satisfies

$$\lim_{\omega\to 0}\langle \text{out}|a_+^{\text{out}}(\omega\hat{x})\mathcal{S}|\text{in}\rangle = S_{\text{em}}^{(0)}\langle\text{out}|\mathcal{S}|\text{in}\rangle = -\lim_{\omega\to 0}\langle\text{out}|\mathcal{S}a_-^{\text{in}\dagger}(\omega\hat{x})|\text{in}\rangle$$

  with $\mathcal{S}$ the S-matrix for the hard process, $\hat{x}$ and ω the soft photon orientation and frequency resp. $[k^\mu = \omega(1,\hat{x})]$

- Equivalent to *large gauge Ward identity* satisfied by soft modes, with gauge parameter $\epsilon(z_k\,,\bar{z}_k)$

$$\lim_{\omega\to 0}\frac{\sqrt{2}\omega}{1+z\bar{z}}\langle\text{out}|a_+^{\text{out}}(\omega\hat{x})\mathcal{S}|\text{in}\rangle = \sum_{k=1}^{n} Q_{(k)}\eta_{(k)}\epsilon(z_k\,,\bar{z}_k)\langle\text{out}|\mathcal{S}|\text{in}\rangle$$

$$\epsilon(z_k\,,\bar{z}_k) = \frac{1}{z-z_k} \quad (+\text{ve helicity})$$

- Also realized to subleading orders [see talks by Silvia Nagy, Sangmin Choi and Shreyansh Agrawal]

Motivation

- Broad Goal: Generalizations of soft factorization for scattering on non-asymptotically flat spacetimes?
- On spacetimes with a cosmological constant, we lack scattering amplitudes. They can be defined in appropriate flat spacetime limits [see talks by Luis Fernando Alday, Paul McFadden, Bin Zhu, and this session]
- If we can address soft factorization in AdS, we might better understand flat spacetime holography

Thus far for bulk scattering processes:

- Weinberg's soft theorems derivable from the flat limit $(L \to \infty)$ limit of a CFT_3 Ward identity [E. Hijano, D. Neuenfeld, JHEP **11** 009 (2020)]
- We can find AdS radius dependent (L^{-2n}) corrections of soft factors for flat spacetime amplitudes about the flat limit of AdS.
- These were derived using "classical soft theorems" [N. Banerjee, KF, A. Mitra, JHEP **08**, 105 (2021); N. Banerjee, A. Bhatacharjee, A. Mitra, JHEP **01**, 038(2020)] and from a CFT_3 Ward identity [N. Banerjee, KF, A. Mitra, JHEP **04**, 055 (2023)]

Outline

Classical soft theorems (CST)

- Classical soft theorems [A. Laddha, A. Sen, JHEP **10** 056 (2018); A. P. Saha, B. Sahoo, A. Sen, JHEP **06** 153 (2020)] :
  Classical limits of soft photon and soft graviton factors derivable from $\omega \to 0$ limit of radiative solutions from classical scattering.

- Applicabilty: $\Delta E_{\text{scatterer}} << 1$; $\lambda_{\text{radiation}} >> b$ (impact parameter)

- Derivable from energy flux in $\omega \to 0$

$$\lim_{\omega \to 0} \varepsilon^{\mu} \tilde{a}_{\mu}\,(\omega\,,\vec{x}) = -i\frac{e^{i\omega R}}{4\pi R} S_{\text{em}}$$

$$\lim_{\omega \to 0} \varepsilon^{\mu\nu} \tilde{e}_{\mu\nu}(\omega,\vec{x}) = -i\frac{e^{i\omega R}}{4\pi R} S_{gr}$$

where $\tilde{a}_{\mu}$ and $\tilde{e}_{\mu\nu}(\omega,\vec{x})$ are electromagnetic and gravitational perturbations in frequency space, ε^{μ} and $\varepsilon^{\mu\nu}$ are photon and graviton polarizations; R : distance to scatterer.

CST on AdS spacetimes about the flat limit

- *Hierarchy of length scales* for probe scattering

$$GM << r << L$$

Leads to $1/L^{2n}$ corrections of flat spacetime results.

- *Double scaling limit* : $\omega \to 0$ as $L \to \infty$, with $\gamma := \omega L >> 1$

Produces $1/\gamma^{2n}$ corrections of flat spacetime soft factors

- $1/L^n$ *Trajectory corrections* of flat asymptotic trajectories exist

Relevant for $\mathcal{O}(\gamma^{-4})$ corrected soft factors

- Derived leading (ω^{-1}) and subleading $(\ln \omega^{-1})$ soft photon and graviton factors, with their respective $1/\gamma^2$ corrections. Derived γ^{-2} corrected large gauge Ward identity in the leading soft photon case [N. Banerjee, KF, A. Mitra, JHEP **08**, 105 (2021)]

$1/\gamma^2$ corrected soft photon theorem on AdS$_4$

- Leading soft photon factor result $S^{(0)}_{\text{em}} = S^{\mathsf{f}(0)}_{\text{em}} + S^{\mathsf{L}(0)}_{\text{em}} + \mathcal{O}\left(\gamma^{-4}\right)$

$$S^{\mathsf{f}(0)}_{\text{em}} = \sum_{a=1}^{n} Q_{(a)}\eta_{(a)} \frac{\epsilon_\mu p^\mu_{(a)}}{p_{(a)}.k} \,, \qquad S^{L\,(0)}_{\text{em}} = \frac{\omega^2}{4\gamma^2} \sum_{a=1}^{n} Q_{(a)}\eta_{(a)} \frac{\epsilon_\mu p^\mu_{(a)}}{p_{(a)}.k} \frac{\vec{p}^{\,2}_{(a)}}{\left(p_{(a)}.k\right)^2}$$

- From leading (universal) term, we infer the single soft theorem

$$\begin{aligned}\lim_{\omega\to 0} \omega \left(\langle \text{out}| a_+(\omega\hat{x}) \mathcal{S} |\text{in}\rangle + \langle \text{out}| \tilde{a}_+(\omega\hat{x}) \mathcal{S} |\text{in}\rangle \right) \\ = \left(S^{\mathsf{f}\,(0)}_{\text{em}} + S^{L\,(0)}_{\text{em}} \right) \langle \text{out}|\mathcal{S}|\text{in}\rangle\end{aligned}$$

- Parametrizing for massless scattering, we have

$$\lim_{\omega\to 0} \frac{\sqrt{2}\omega\, n_L}{(1+z\bar{z})} \langle \text{out}|\tilde{a}_+ \mathcal{S}|\text{in}\rangle = \sum_{k=1}^{n} Q_{(k)}\eta_{(k)} \epsilon^{\text{cst}}_L(z_k\,,\bar{z}_k) \langle \text{out}|\mathcal{S}|\text{in}\rangle$$

$$\epsilon^{\text{cst}}_L(z_k\,,\bar{z}_k) = \frac{1}{16\gamma^2} \frac{\left(1+z_k\bar{z}_k\right)^2 \left(1+z\bar{z}\right)^2}{(\bar{z}-\bar{z}_k)^2 (z-z_k)^3} \qquad (+\text{ve helicity})$$

- Subtlety : Overall numerical constant n_L is *not fixed* by CST

Outline

Flat spacetime from Lorentzian AdS/CFT

- Analysis in global AdS_{d+1} with metric

$$ds^2 = \frac{L^2}{\cos^2\rho}\left(-d\tau^2 + d\rho^2 + \sin^2\rho\, d\Omega^2_{d-1}\right)$$

AdS boundary at $\rho = \frac{\pi}{2}$ with coordinates $x = \{\tau, z\,, \bar{z}\}$

- Define

$$\frac{r}{L} = \tan(\rho) \approx \rho\ , \qquad \frac{t}{L} = \tau$$

- AdS metric in the $L \to \infty$ limit goes to the flat spacetime metric

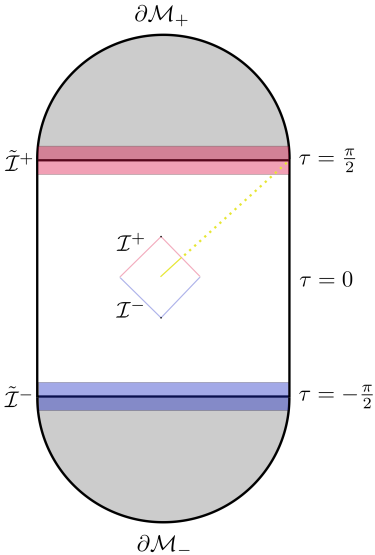
$$\lim_{L\to\infty} ds^2 = -dt^2 + dr^2 + r^2 d\Omega_2^2$$

Asymptotically flat patch centered about $\tau = 0$ in global AdS

- Bulk field modes from boundary operators by HKLL reconstruction [A. Hamilton, D. Kabat, G. Lifschytz, D. Lowe, PRD **74** 066009 (2006)]

Asymptotic mapping

$$\mathcal{I}^{\pm} :: \tilde{\mathcal{I}}^{\pm} \; ; \qquad i^{\pm} :: \partial\mathcal{M}_{\pm} \; ; \qquad i^{0} :: \tau \in \left\{ -\frac{\pi}{2}, \frac{\pi}{2} \right\}$$



Maxwell field in AdS from boundary sources

- Bulk field with magnetic boundary conditions (no Coulombic sources)

$$\hat{\mathcal{A}}_\mu(X) = \mathcal{A}_\mu(\rho, x) \xrightarrow[\rho\to\frac{\pi}{2}]{} (\cos\rho) j_\mu(x)$$

- Bulk field has HKLL reconstruction from boundary currents

$$\hat{\mathcal{A}}_\mu(X) = \int d^3x' \left[K^V_\mu(X;x')\epsilon_{\tau'}^{a'b'}\nabla_{a'} j^+_{b'} + K^S_\mu(X\,;x')\gamma^{a'b'}\nabla_{a'} j^+_{b'} + c.c.\right]$$

$\epsilon^{a'b'c'}$ and $\nabla_{a'}$: bndy Levi-Civita tensor and covariant derivative; $K^V_\mu(X;x')$ and $K^S_\mu(X;x')$: HKLL kernels from "vector" $(\Delta = 2)$ and "scalar" $(\Delta = 1)$ type solutions of Maxwell's equations.

- We can expand the kernels about $L \to \infty$ to find

$$\hat{\mathcal{A}}_\mu(X) = \hat{\mathcal{A}}^{\text{flat}}_\mu(X) + \hat{\mathcal{A}}^{L^{-2}}_\mu(X) + \mathcal{O}\left(L^{-4}\right)$$

Bulk fields about flat limit

- The HKLL kernels have a sum over discrete energies. These are large and scale like L to get continuous flat spacetime frequencies

$$2\kappa + l + \Delta = \omega_\kappa \to \omega L \qquad \Rightarrow \qquad \sum_\kappa \to \frac{1}{2}\int d\omega L$$

We find

$$\hat{\mathcal{A}}_z^{\text{out; flat}}(X) = \frac{1}{4\pi}\int d^3x' \int d\omega\, r\, j_l(r\omega)[[\cdots]]$$
$$\hat{\mathcal{A}}_z^{\text{out};L^{-2}}(X) = \frac{1}{16\pi}\int d^3x' \int d\omega\, \frac{r}{\gamma^2}\, j_l(r\omega) l(l+1)[[\cdots]]$$

with

$$[[\cdots]] = \left[\sum_{l,m} \frac{Y^*_{lm}(\Omega')}{-l(l+1)} \partial_z Y_{lm}(\Omega)\,(i)^{-l} e^{i\omega t} e^{-i\omega L\left(\tau' - \frac{\pi}{2}\right)} D^{\bar{z}'} j^+_{\bar{z}'} + c.c.\right]$$

[Likewise for ingoing modes, and other components]

Soft modes from bulk fields

- The bulk field solutions are about the flat limit, and we can define flat spacetime modes in the usual way

$$\hat{a}^{\text{out}}_{(\lambda)} = \lim_{t\to\infty} i \int d^3\vec{x}\, \varepsilon^{(\lambda)\mu} e^{-iq\cdot x} \overleftrightarrow{\partial_0} \hat{\mathcal{A}}^{\text{out}}_{\mu}(X)$$

- The $\hat{\mathcal{A}}^{\text{out; flat}}_{z}(X)$ and $\hat{\mathcal{A}}^{\text{out};L^{-2}}_{z}(X)$ pieces give

$$\hat{a}^{\text{out;flat}}_{+}(\omega\hat{x}) = \frac{1}{4}\frac{1+z\bar{z}}{\sqrt{2}\omega} \int d\tau'\, e^{i\omega L\left(\frac{\pi}{2}-\tau'\right)} \int d^2z' \epsilon(z') D^{\bar{z}'} j^{-}_{\bar{z}'}(x')$$

$$\hat{a}^{\text{out};L^{-2}}_{+}(\omega\hat{x}) = \frac{1}{4}\frac{1+z\bar{z}}{\sqrt{2}\omega} \int d\tau'\, e^{i\omega L\left(\frac{\pi}{2}-\tau'\right)} \int d^2z' \epsilon_L(z') D^{\bar{z}'} j^{-}_{\bar{z}'}(x')$$

with

$$\epsilon(z') = \frac{1}{(z-z')}\,, \quad \epsilon_L(x') = \frac{1}{8\pi\gamma^2} \int d\Omega_w \left[\frac{(1+z'\bar{z}')^2 (1+z_w\bar{z}_w)^2}{(\bar{z}'-\bar{z}_w)^2 (z-z_w)^3} \right]$$

- Soft limit: Phase dominated by $\tau = \frac{\pi}{2}$ around $\tilde{\mathcal{I}}^{+}$

$$\lim_{\omega\to 0} \lim_{L\to\infty} \int d\tau'\, e^{i\omega L\left(\frac{\pi}{2}-\tau'\right)} \int d^2z' \equiv \int_{\tilde{\mathcal{I}}^{+}} d^3x'$$

Recovering classical soft theorem gauge parameter

- Consider a "collinear expansion"

$$|z_w - z'| \approx |z - z'| + \delta \approx |z_w - z| \qquad \text{with } \delta << 1$$

This assumes $\{z_w \,, \bar{z}_w\}$ can be expanded about $\{z \,, \bar{z}\}$ or $\{z' \,, \bar{z}'\}$, with $|z - z'|$ as a minimal distance.

- Helps realize properties of the leading soft photon factor:

$$\text{diverges as } \omega^{-1} \text{ and collinearly diverges as } z \to z'$$

- We can now formally integrate $\epsilon_L(x')$ to find

$$\epsilon_L(x') = 8\epsilon_L^{\text{cst}}(x') + \text{`` corrections''}$$

with "corrections" involving derivatives of $\epsilon_L^{\text{cst}}(x')$

Ward identity to Soft theorem

- The integrated $U(1)$ Ward identity on the boundary

$$\int d^3x'\,\alpha(x')\partial'_\mu\langle 0|T\{j^\mu(x')X\}|0\rangle = \left(\sum_{i=1}^{n} Q_i\alpha(x'_i) - \sum_{j=1}^{m} Q_j\alpha(x'_j)\right)\langle 0|T\{X\}|0\rangle$$

  $T\{X\}$: n $(\tau > 0)$ and m $(\tau < 0)$ time ordered operators with charges Q_i $(i = 1, \cdots, n)$ and Q_j $(j = 1, \cdots, m)$

- $\langle 0|T\{X\}|0\rangle$ goes to the S-matrix on replacing operators with corresponding modes in the flat limit.

- $\alpha(x')|_{\tilde{\mathcal{I}}^\pm} = \epsilon(x')$ gives the Weinberg soft photon theorem [E. Hijano, D. Neuenfeld, JHEP **11** 009 (2020)]

- With $\alpha(x')|_{\tilde{\mathcal{I}}^\pm} = 8\epsilon_L^{\text{cst}}(x')$ we recover the γ^{-2} corrected soft photon theorem and fixes $n_L = \frac{1}{8}$ [N. Banerjee, KF, A. Mitra, JHEP **04**, 055 (2023)]

Outline

Conclusion and future directions

- Soft theorems about the flat spacetime limit of AdS involve corrections in powers of γ^{-2} where $\gamma = \omega L >> 1$
- Two independent derivations give a consistent result for γ^{-2} corrected soft photon factor
- Future: Derivation using Witten diagrams with a L^{-1} expansion
- Implications on CCFT sector within CFT about $\tau = \pm\frac{\pi}{2}$?
  [L. P. de Gioia, A-M Raclariu, 2303.10037 [hep-th]]
- Soft and flat limits of boundary correlators do not commute on AdS
  [C. Chowdhury, A. Lipstein, J. Mei, Y. Mo, 2407.16052 [hep-th]]
- Could a resummation of γ^{-2n} corrections relate the two limits?

Thank You