

Soft factorization on AdS spacetimes

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based on JHEP **08**, 105 (2021), JHEP **04**, 055 (2023)
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Outline

Overview and Motivation

Derivation from Classical soft theorems

Derivation from CFT_3 Ward identity

Conclusion and future directions

Single soft theorems on flat spacetime

- ▶ Factorization of scattering amplitudes involving soft particles

$$\lim_{k \rightarrow 0} \mathcal{M}_{n+1}(\varepsilon, k; \{p_a\}) = \text{Soft factor} \times \mathcal{M}_n(\{p_a\})$$

ε : polarization; k : soft momentum; p_a : hard momenta

\mathcal{M}_n : Amplitude with n external particles

- ▶ The soft factor admits a frequency expansion

$$S_{\text{em}} = \sum_i S_{\text{em}}^{(i)} \quad S_{\text{em}}^{(0)} = \sum_{a=1}^n Q_{(a)} \eta_{(a)} \frac{\varepsilon \cdot p_{(a)}}{p_{(a)} \cdot k} \quad (\text{leading})$$

$\eta_{(a)} = +1(-1)$ for outgoing (incoming), $Q_{(a)}$ charges
and ε^μ the polarization

- ▶ Renewed interest in relation to asymptotic symmetries and holography on asymptotically flat spacetimes [see talk by Laura Donnay]

Soft theorem = Large gauge Ward identity

- ▶ The soft factor satisfies

$$\lim_{\omega \rightarrow 0} \langle \text{out} | a_+^{\text{out}}(\omega \hat{x}) \mathcal{S} | \text{in} \rangle = S_{\text{em}}^{(0)} \langle \text{out} | \mathcal{S} | \text{in} \rangle = - \lim_{\omega \rightarrow 0} \langle \text{out} | \mathcal{S} a_-^{\text{in}\dagger}(\omega \hat{x}) | \text{in} \rangle$$

with \mathcal{S} the S -matrix for the hard process, \hat{x} and ω the soft photon orientation and frequency resp. [$k^\mu = \omega(1, \hat{x})$]

- ▶ Equivalent to *large gauge Ward identity* satisfied by soft modes, with gauge parameter $\epsilon(z_k, \bar{z}_k)$

$$\lim_{\omega \rightarrow 0} \frac{\sqrt{2}\omega}{1 + z\bar{z}} \langle \text{out} | a_+^{\text{out}}(\omega \hat{x}) \mathcal{S} | \text{in} \rangle = \sum_{k=1}^n Q_{(k)} \eta_{(k)} \epsilon(z_k, \bar{z}_k) \langle \text{out} | \mathcal{S} | \text{in} \rangle$$

$$\epsilon(z_k, \bar{z}_k) = \frac{1}{z - z_k} \quad (+\text{ve helicity})$$

- ▶ Also realized to subleading orders [see talks by Silvia Nagy, Sangmin Choi and Shreyansh Agrawal]

Motivation

- ▶ Broad Goal: Generalizations of soft factorization for scattering on non-asymptotically flat spacetimes?
- ▶ On spacetimes with a cosmological constant, we lack scattering amplitudes. They can be defined in appropriate flat spacetime limits [see talks by Luis Fernando Alday, Paul McFadden, Bin Zhu, and this session]
- ▶ If we can address soft factorization in AdS, we might better understand flat spacetime holography

Thus far for bulk scattering processes:

- ▶ Weinberg's soft theorems derivable from the flat limit ($L \rightarrow \infty$) limit of a CFT_3 Ward identity [E. Hijano, D. Neuenfeld, JHEP **11** 009 (2020)]
- ▶ We can find AdS radius dependent (L^{-2n}) corrections of soft factors for flat spacetime amplitudes about the flat limit of AdS.
- ▶ These were derived using “classical soft theorems” [N. Banerjee, KF, A. Mitra, JHEP **08**, 105 (2021); N. Banerjee, A. Bhattacharjee, A. Mitra, JHEP **01**, 038(2020)] and from a CFT_3 Ward identity [N. Banerjee, KF, A. Mitra, JHEP **04**, 055 (2023)]

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Classical soft theorems (CST)

- ▶ Classical soft theorems [A. Laddha, A. Sen, JHEP **10** 056 (2018); A. P. Saha, B. Sahoo, A. Sen, JHEP **06** 153 (2020)] :
Universal contributions to soft photon and soft graviton factors from $\omega \rightarrow 0$ limit of classical radiative solutions.
- ▶ Applicability: $\Delta E_{\text{scatterer}} \ll 1$; $\lambda_{\text{radiation}} \gg b$ (impact parameter)
- ▶ Derivable from energy flux in $\omega \rightarrow 0$

$$\lim_{\omega \rightarrow 0} \varepsilon^\mu \tilde{a}_\mu(\omega, \vec{x}) = -i \frac{e^{i\omega R}}{4\pi R} S_{\text{em}}$$

$$\lim_{\omega \rightarrow 0} \varepsilon^{\mu\nu} \tilde{e}_{\mu\nu}(\omega, \vec{x}) = -i \frac{e^{i\omega R}}{4\pi R} S_{\text{gr}}$$

where \tilde{a}_μ and $\tilde{e}_{\mu\nu}(\omega, \vec{x})$ are electromagnetic and gravitational perturbations in frequency space, ε^μ and $\varepsilon^{\mu\nu}$ are photon and graviton polarizations; R : distance to scatterer.

CST on AdS spacetimes about the flat limit

- ▶ *Hierarchy of length scales* for probe scattering

$$GM \ll r \ll L$$

Leads to $1/L^{2n}$ corrections of flat spacetime results.

- ▶ *Double scaling limit* : $\omega \rightarrow 0$ as $L \rightarrow \infty$, with $\gamma := \omega L \gg 1$

Produces $1/\gamma^{2n}$ corrections of flat spacetime soft factors

- ▶ $1/L^n$ *Trajectory corrections* of flat asymptotic trajectories exist

Relevant for $\mathcal{O}(\gamma^{-4})$ corrected soft factors

- ▶ Derived leading (ω^{-1}) and subleading ($\ln \omega^{-1}$) soft photon and graviton factors, with their respective $1/\gamma^2$ corrections.
Derived γ^{-2} corrected large gauge Ward identity in the leading soft photon case [N. Banerjee, KF, A. Mitra, JHEP 08, 105 (2021)]

$1/\gamma^2$ corrected soft photon theorem on AdS_4

- ▶ Leading soft photon factor result $S_{\text{em}}^{(0)} = S_{\text{em}}^{\text{f}(0)} + S_{\text{em}}^{\text{L}(0)} + \mathcal{O}(\gamma^{-4})$

$$S_{\text{em}}^{\text{f}(0)} = \sum_{a=1}^n Q_{(a)} \eta_{(a)} \frac{\epsilon_{\mu} p_{(a)}^{\mu}}{p_{(a)} \cdot k}, \quad S_{\text{em}}^{\text{L}(0)} = \frac{\omega^2}{4\gamma^2} \sum_{a=1}^n Q_{(a)} \eta_{(a)} \frac{\epsilon_{\mu} p_{(a)}^{\mu}}{p_{(a)} \cdot k} \frac{\bar{p}_{(a)}^2}{(p_{(a)} \cdot k)^2}$$

- ▶ From leading term, we infer the single soft theorem

$$\begin{aligned} \lim_{\omega \rightarrow 0} \omega (\langle \text{out} | a_+(\omega \hat{x}) \mathcal{S} | \text{in} \rangle + \langle \text{out} | \tilde{a}_+(\omega \hat{x}) \mathcal{S} | \text{in} \rangle) \\ = (S_{\text{em}}^{\text{f}(0)} + S_{\text{em}}^{\text{L}(0)}) \langle \text{out} | \mathcal{S} | \text{in} \rangle \end{aligned}$$

- ▶ Parametrizing for massless scattering, we have

$$\lim_{\omega \rightarrow 0} \frac{\sqrt{2}\omega n_L}{(1 + z\bar{z})} \langle \text{out} | \tilde{a}_+ \mathcal{S} | \text{in} \rangle = \sum_{k=1}^n Q_{(k)} \eta_{(k)} \epsilon_L^{\text{cst}}(z_k, \bar{z}_k) \langle \text{out} | \mathcal{S} | \text{in} \rangle$$

$$\epsilon_L^{\text{cst}}(z_k, \bar{z}_k) = \frac{1}{16\gamma^2} \frac{(1 + z_k \bar{z}_k)^2 (1 + z\bar{z})^2}{(\bar{z} - \bar{z}_k)^2 (z - z_k)^3} \quad (\text{+ve helicity})$$

- ▶ Subtlety : Overall numerical constant n_L is *not fixed* by CST

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Flat spacetime from Lorentzian AdS/CFT

- ▶ Analysis in global AdS_{d+1} with metric

$$ds^2 = \frac{L^2}{\cos^2 \rho} \left(-d\tau^2 + d\rho^2 + \sin^2 \rho d\Omega_{d-1}^2 \right)$$

AdS boundary at $\rho = \frac{\pi}{2}$ with coordinates $x = \{\tau, z, \bar{z}\}$

- ▶ Define

$$\frac{r}{L} = \tan(\rho) \approx \rho, \quad \frac{t}{L} = \tau$$

- ▶ AdS metric in the $L \rightarrow \infty$ limit goes to the flat spacetime metric

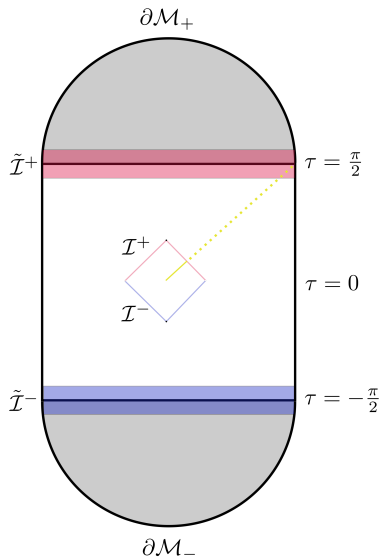
$$\lim_{L \rightarrow \infty} ds^2 = -dt^2 + dr^2 + r^2 d\Omega_d^2$$

Asymptotically flat patch centered about $\tau = 0$ in global AdS

- ▶ Bulk field modes from boundary operators by HKLL reconstruction
[A. Hamilton, D. Kabat, G. Lifschytz, D. Lowe, PRD **74** 066009 (2006)]

Asymptotic mapping

$$\mathcal{I}^\pm \:: \tilde{\mathcal{I}}^\pm; \quad i^\pm \:: \partial\mathcal{M}_\pm; \quad i^0 \:: \tau \in \left\{ -\frac{\pi}{2}, \frac{\pi}{2} \right\}$$



Maxwell field in AdS from boundary sources

- ▶ Bulk field with magnetic boundary conditions (no Coulombic sources)

$$\hat{\mathcal{A}}_\mu(X) = \mathcal{A}_\mu(\rho, x) \xrightarrow{\rho \rightarrow \frac{\pi}{2}} (\cos \rho) j_\mu(x)$$

- ▶ Bulk field has HKLL reconstruction from boundary currents

$$\hat{\mathcal{A}}_\mu(X) = \int d^3x' \left[K_\mu^V(X; x') \epsilon_{\tau'}^{a'b'} \nabla_{a'} j_{b'}^+ + K_\mu^S(X; x') \gamma^{a'b'} \nabla_{a'} j_{b'}^+ + c.c. \right]$$

$\epsilon^{a'b'c'}$ and $\nabla_{a'}$: bndy Levi-Civita tensor and covariant derivative;
 $K_\mu^V(X; x')$ and $K_\mu^S(X; x')$: HKLL kernels from "vector" ($\Delta = 2$) and "scalar" ($\Delta = 1$) type solutions of Maxwell's equations.

- ▶ We can expand the kernels about $L \rightarrow \infty$ to find

$$\hat{\mathcal{A}}_\mu(X) = \hat{\mathcal{A}}_\mu^{\text{flat}}(X) + \hat{\mathcal{A}}_\mu^{L^{-2}}(X) + \mathcal{O}(L^{-4})$$

Bulk fields about flat limit

- ▶ The HKLL kernels have a sum over discrete energies. These are large and scale like L to get continuous flat spacetime frequencies

$$2\kappa + l + \Delta = \omega_\kappa \rightarrow \omega L \quad \Rightarrow \quad \sum_\kappa \rightarrow \frac{1}{2} \int d\omega L$$

We find

$$\hat{\mathcal{A}}_z^{\text{out}; \text{flat}}(X) = \frac{1}{4\pi} \int d^3x' \int d\omega r j_l(r\omega) [[\dots]]$$
$$\hat{\mathcal{A}}_z^{\text{out}; L^{-2}}(X) = \frac{1}{16\pi} \int d^3x' \int d\omega \frac{r}{\gamma^2} j_l(r\omega) l(l+1) [[\dots]]$$

with

$$[[\dots]] = \left[\sum_{l,m} \frac{Y_{lm}^*(\Omega')}{-l(l+1)} \partial_z Y_{lm}(\Omega) (i)^{-l} e^{i\omega t} e^{-i\omega L(\tau' - \frac{\pi}{2})} D^{\bar{z}'} j_{\bar{z}'}^+ + c.c. \right]$$

[Likewise for ingoing modes, and other components]

Soft modes from bulk fields

- ▶ The bulk field solutions are about the flat limit, and we can define flat spacetime modes in the usual way

$$\hat{a}_{(\lambda)}^{\text{out}} = \lim_{t \rightarrow \infty} i \int d^3 \vec{x} \varepsilon^{(\lambda)\mu} e^{-iq \cdot x} \overleftrightarrow{\partial}_0 \hat{\mathcal{A}}_{\mu}^{\text{out}}(X)$$

- ▶ The $\hat{\mathcal{A}}_z^{\text{out}; \text{flat}}(X)$ and $\hat{\mathcal{A}}_z^{\text{out}; L^{-2}}(X)$ pieces give

$$\hat{a}_+^{\text{out}; \text{flat}}(\omega \hat{x}) = \frac{1}{4} \frac{1 + z \bar{z}}{\sqrt{2\omega}} \int d\tau' e^{i\omega L(\frac{\pi}{2} - \tau')} \int d^2 z' \epsilon(z') D^{\bar{z}'} j_{\bar{z}'}^-(x')$$

$$\hat{a}_+^{\text{out}; L^{-2}}(\omega \hat{x}) = \frac{1}{4} \frac{1 + z \bar{z}}{\sqrt{2\omega}} \int d\tau' e^{i\omega L(\frac{\pi}{2} - \tau')} \int d^2 z' \epsilon_L(z') D^{\bar{z}'} j_{\bar{z}'}^-(x')$$

with

$$\epsilon(z') = \frac{1}{(z - z')}, \quad \epsilon_L(x') = \frac{1}{8\pi\gamma^2} \int d\Omega_w \left[\frac{(1 + z' \bar{z}')^2 (1 + z_w \bar{z}_w)^2}{(\bar{z}' - \bar{z}_w)^2 (z - z_w)^3} \right]$$

- ▶ Soft limit: Phase dominated by $\tau = \frac{\pi}{2}$ around $\tilde{\mathcal{I}}^+$

$$\lim_{\omega \rightarrow 0} \lim_{L \rightarrow \infty} \int d\tau' e^{i\omega L(\frac{\pi}{2} - \tau')} \int d^2 z' \equiv \int_{\tilde{\mathcal{I}}^+} d^3 x'$$

Recovering classical soft theorem gauge parameter

- ▶ Consider a “collinear expansion”

$$|z_w - z'| \approx |z - z'| + \delta \approx |z_w - z| \quad \text{with } \delta \ll 1$$

This assumes $\{z_w, \bar{z}_w\}$ can be expanded about $\{z, \bar{z}\}$ or $\{z', \bar{z}'\}$, with $|z - z'|$ as a minimal distance.

- ▶ Helps realize properties of the leading soft photon factor:

diverges as ω^{-1} and collinearly diverges as $z \rightarrow z'$

- ▶ We can now formally integrate $\epsilon_L(x')$ to find

$$\epsilon_L(x') = 8\epsilon_L^{\text{cst}}(x') + \text{“ corrections”}$$

with “corrections” involving derivatives of $\epsilon_L^{\text{cst}}(x')$

Ward identity to Soft theorem

- ▶ The integrated $U(1)$ Ward identity on the boundary

$$\int d^3x' \alpha(x') \partial'_\mu \langle 0|T\{j^\mu(x')\mathcal{X}\}|0\rangle = \left(\sum_{i=1}^n Q_i \alpha(x'_i) - \sum_{j=1}^m Q_j \alpha(x'_j) \right) \langle 0|T\{\mathcal{X}\}|0\rangle$$

$T\{\mathcal{X}\}$: n ($\tau > 0$) and m ($\tau < 0$) time ordered operators with charges Q_i ($i = 1, \dots, n$) and Q_j ($j = 1, \dots, m$)

- ▶ $\langle 0|T\{\mathcal{X}\}|0\rangle$ goes to the S -matrix on replacing operators with corresponding modes in the flat limit.
- ▶ $\alpha(x')|_{\tilde{\mathcal{I}}^\pm} = \epsilon(x')$ gives the Weinberg soft photon theorem
[E. Hijano, D. Neuenfeld, JHEP 11 009 (2020)]
- ▶ With $\alpha(x')|_{\tilde{\mathcal{I}}^\pm} = 8\epsilon_L^{\text{cst}}(x')$ we recover the γ^{-2} corrected soft photon theorem and fixes $n_L = \frac{1}{8}$ [N. Banerjee, KF, A. Mitra, JHEP 04, 055 (2023)]

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- ▶ Soft theorems about the flat spacetime limit of AdS involve corrections in powers of γ^{-2} where $\gamma = \omega L \gg 1$
- ▶ Two independent derivations give a consistent result for γ^{-2} corrected soft photon factor
- ▶ Future: Derivation using Witten diagrams with a L^{-1} expansion
- ▶ Implications on CCFT sector within CFT about $\tau = \pm \frac{\pi}{2}$?
[L. P. de Gioia, A-M Raclariu, 2303.10037 [hep-th]]
- ▶ Soft and flat limits of boundary correlators do not commute on AdS
[C. Chowdhury, A. Lipstein, J. Mei, Y. Mo, 2407.16052 [hep-th]]
- ▶ Could a resummation of γ^{-2n} corrections relate the two limits?

Thank You