Categorical Symmetry Resolution of Entanglement in Rational CFT

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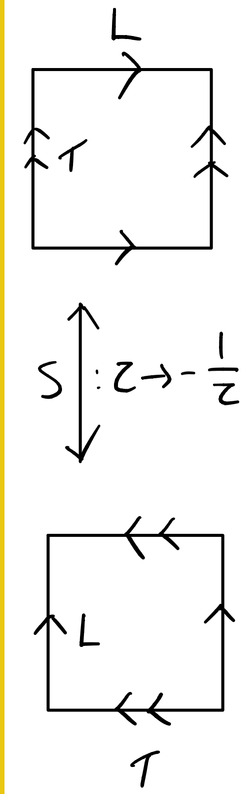
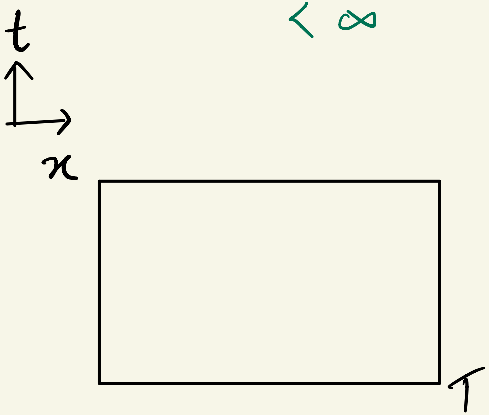
[EuroStrings '24]



ENTANGLEMENT ENTROPY:

* GIVEN A $\underset{\wedge}{\text{(DIAG)}}$ 2D RCFT w/ CHIRAL ALG. $\mathcal{A}$
  & $\underbrace{\text{IRREPS}}_{<\infty}$ $\longleftrightarrow$ PRIMARIES $\{|\varphi_a\rangle\}_{a\in\mathcal{A}}$:

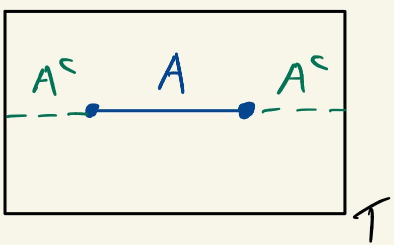
$$\mathcal{H}_1 = \bigoplus_a V_a \otimes \overline{V_a}$$



$$= Z[q,\bar{q}] = \mathrm{Tr}\left(q^{L_0 - c/24}\,\bar{q}^{\bar{L}_0 - \bar{c}/24}\right)$$

$$= \sum_i M_{ii}\, \chi_i(q)\, \overline{\chi_i(\bar{q})}$$

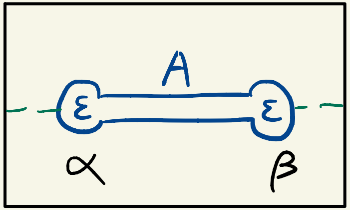
* EE:



FACTORIZAT^N: $\mathcal{H}_1 = \mathcal{H}_A \otimes \mathcal{H}_{A^c}$

EE: $S_A = \frac{c}{3}\ln\left(\frac{\ell}{\varepsilon}\right) + O(1)$

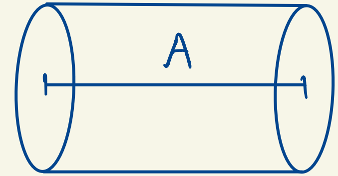
BOUNDARY CFT APPROACH:

* 

T

$\circlearrowleft 2\pi$

$$\xrightarrow[\underbrace{\text{\& CONF. MAP}}_{\tau = \frac{i\pi}{W}\ ;\ q = e^{-2\pi^2/W}}]{\overbrace{\text{REPLICATE}}^{q \to q^n}}$$

$\langle\tilde{\alpha}|$ 

$|\tilde{\beta}\rangle$

$\leftarrow W \sim 2 \ln \frac{l}{\varepsilon} \rightarrow$

$\circlearrowleft 2\pi n$

w/ $|\tilde{\alpha}\rangle$; $|\tilde{\beta}\rangle$: $\underbrace{\text{CONF.-INV. BDRY STATES}}_{\text{PRESERVE CONF. INV.}}$ TO

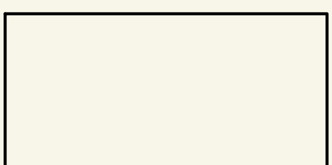
$\rightarrow$ [CARDY STATES]

$\updownarrow$ 1-1

[PRIM. IN DIAG. CFT]

*FACTORIZAT^{N}:

$$\mathcal{H}_1 = \mathcal{H}_{A,\alpha\beta} \otimes \mathcal{H}_{A^c,\beta\alpha}$$

α  β $= Z_{\alpha\beta}(q) = \mathrm{Tr}_{\alpha\beta}\left(q^{L_0 - c/24}\right)$

A

[w/ $q = e^{-2\pi^2/W}$]

BCFT → EE:

$$* \; S_A[q] = \lim_{n\to 1} \frac{1}{1-n} \ln \underbrace{\left(\frac{Z_{\alpha\beta}[q^n]}{(Z_{\alpha\beta}[q])^n} \right)}_{\equiv \, \mathrm{Tr}(\rho_A^n)}$$ [REPLICA TRICK]

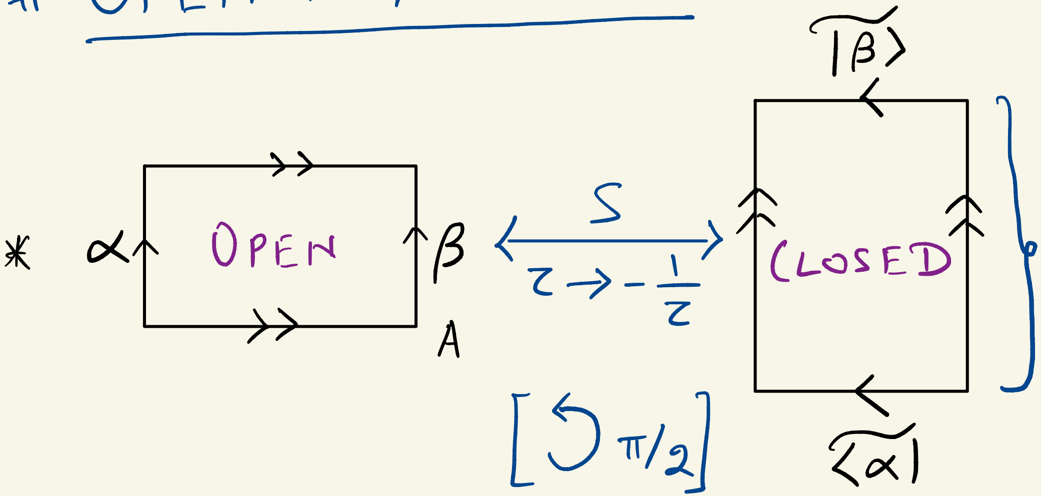
$$\text{w/} \quad Z[q^n] \equiv Z_{\alpha\beta}[q^n] = \mathrm{Tr}_{\alpha\beta}\, q^{n(L_0 - c/24)} = \sum_i n^i_{\alpha\beta}\, \chi_i(q^n)$$

$$\overset{S}{\equiv} \widetilde{\langle\alpha|}\, \tilde{q}^{\frac{1}{n}(L_0 - c/24)}\, \widetilde{|\beta\rangle}$$

$$\underset{\ell \gg \varepsilon}{\underbrace{\tilde{q}\to 0}} \; \widetilde{\langle\alpha|}\, 1 \times 1 \,\widetilde{|\beta\rangle}\, e^{g^*_\alpha + g_\beta}\, \tilde{q}^{\frac{1}{n}\left(-\frac{c}{24}\right)}$$

$$\left[S: \tau \to -\frac{1}{\tau} \; \left(q \to \tilde{q} = e^{-2W} \right) \right]$$

Open ⟷ Closed:



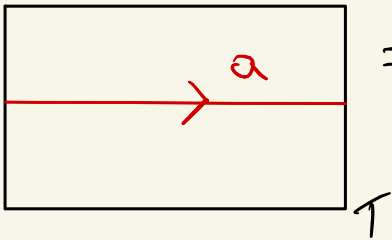
TRACE ⟷ PROP. AMP.

$$\Rightarrow S_A = \frac{c}{3}\ln\frac{\ell}{\epsilon} + \underbrace{(g_\alpha^* + g_\beta)} + \underbrace{O(\epsilon/\ell)}$$

$g_\alpha \equiv \ln\langle 1|\widetilde{\alpha}\rangle$
[AFFLECK-LUDWIG BDRY ENTROPY]

TOPOLOGICAL DEFECT LINES:

* IN DIAG RCFT : $|\varphi_a\rangle \longleftrightarrow a$ & ACT^N:



$$= Z^a(q,\bar q) = \mathrm{Tr}\left(a\, q^{L_0-\frac{c}{24}}\, \bar q^{\bar L_0-\frac{\bar c}{24}}\right)$$

$a: V_b\otimes \bar V_b \to V_b\otimes \bar V_b$; $a|\varphi_b\rangle = \frac{S_{ab}}{S_{1b}}|\varphi_b\rangle$

$\Downarrow$

$$\left\{a|1\rangle = \frac{S_{a1}}{S_{11}}|1\rangle = d_a|1\rangle\right\}$$

* $[a, L_n] = 0 = [a, \bar L_n]$

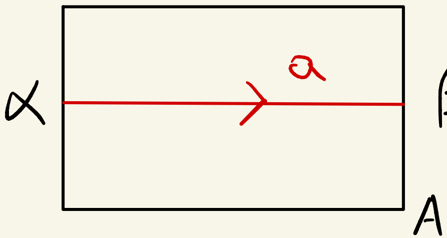
$\Rightarrow$ a GENERATES 0-FORM GLOBAL SYMM. IN CFTs

* FUSION: SAME AS PRIM: $a\times b = \sum_c \underbrace{N_{ab}^{c}}_{\geq 0}\, c$

i) INV.: $a\bar a = 1 \Rightarrow a^m = 1$; ii) NON-INV: $a^2 = 1 + \dots$

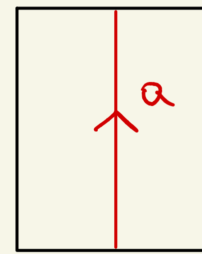
#Symm. Resolved EE:

① G-Symm. Conf. Bdry:



$$\beta = \overbrace{Z^a(q)}^{\text{Charged Moments}} = \mathrm{Tr}\left(a\, q^{L_0 - c/24}\right)$$

$$\overset{S}{=} \langle\tilde{\alpha}|\, \tilde{q}^{L_0 - c/24}\, |\tilde{\beta}\rangle = {}_a\langle\tilde{\beta}| \ \ldots \ |\tilde{\alpha}\rangle_a$$



$$= Z_a(\tilde{q})$$

w/ a Inv. & $a|\tilde{\alpha}\rangle = \underbrace{|\tilde{\alpha}\rangle}_{\text{G-Symm. Cardy St.}} \quad \forall\, a \in G$

② PROJECTORS:

• LET 'r' BE AN IRREP. OF G:

$$P_r = \frac{d_r}{|G|}\sum_{g\in G}\chi_r^*(g) \xrightarrow{\quad g \quad} , \ \forall\ g \in G$$

[CHAR-ORTHO. THM]

• DEFINE PROJ. PART^N F^N:

$$Z_{\alpha\beta}[q^n, r] = \mathrm{Tr}\left(P_r\, \rho_A^n\right) = \frac{d_r}{|G|}\sum_{g\in G}\chi_r^*(g)\,\frac{Z_{\alpha\beta}[q^n, g]}{\left(Z_{\alpha\beta}[q, g]\right)^n}$$

• SREE:

$$S_A[q,r] = \lim_{n\to 1}\frac{1}{1-n}\ln\frac{Z_{\alpha\beta}[q^n, r]}{\left(Z_{\alpha\beta}[q, r]\right)^n} = \underbrace{\frac{c}{3}\ln\frac{l}{\epsilon}}_{\left\{\begin{array}{l}\text{EQUIPARTITION} \\ \text{OF EE} \sim O(l/\epsilon)\end{array}\right\}} + \frac{d_r^2}{|G|} + \left(g_\alpha^* + g_\beta\right) + O(\epsilon/l)$$

CATEGORICAL SYMM. RESOLVED EE:

* LET $\mathcal{C}$ BE THE CAT. OF TDLs:

① $\mathcal{C}$-SYMM. CONF. BDRY:

A) $\mathcal{C}$-STRONGLY SYMM.: $a|\widetilde{\alpha}\rangle = d_a|\widetilde{\alpha}\rangle \quad \forall \; a \in \mathcal{C}$

$\Rightarrow \mathcal{C}$ IS NON-ANOMALOUS

B) $\mathcal{C}$-WEAKLY SYMM.: $a|\widetilde{\alpha}\rangle = |\widetilde{\alpha}\rangle + \ldots \quad \forall \; a \in \mathcal{C}$

$\Rightarrow \mathcal{C}$ CAN BE GAUGED

② PROJECTORS: $P_1^{(a,a)} : \mathcal{H}_1 \longrightarrow V_a \otimes \overline{V}_a$

& $$P_1^{(a,a)} = \frac{d_a}{\underbrace{|\mathcal{C}|}_{\sum d_b^2 =}} \sum_{b \in \mathcal{C}} \underbrace{\chi_a^*(b)}_{= \frac{S^*_{ba}}{S_{11}}} \xrightarrow{\quad b \quad}$$

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- **CAT-SREE:**

$$S_A[q,(a,a)] = \underbrace{\frac{c}{3}\ln\frac{\ell}{\varepsilon}}_{\text{[EQUIPARTIT}^{\text{N}}\text{]}} + \frac{d_a^2}{|C|} + (g_\alpha^* + g_\beta) + \mathcal{O}(\varepsilon/\ell)$$

- **EXAMPLE: TRI-CRIT ISING:** $\mathcal{M}(5,4)$; $c = \frac{7}{10}$

TDLs: $\{\underbrace{1,\eta}_{\text{INV.}}, \underbrace{N, W, \eta W, WN}_{\text{NON-INV.}}\}$

i) GRP: $\{1,\eta\} \equiv \mathbb{Z}_2$ w/ FUSION: $\eta^2 = 1 \xrightarrow{\text{ROOTS}} \{\pm 1\}$

[G-SYMM. BDRY: $\eta N = N$
$\Rightarrow \eta|\widetilde{N}\rangle = |\widetilde{N}\rangle$]

$P_r \in \{P_+, P_-\}$:

$$P_+ = P_1^{(1,1)} + P_1^{(\eta,\eta)} + P_1^{(W,W)} + P_1^{(\eta W,\eta W)}; \quad P_- = P_1^{(N,N)} + P_1^{(WN,WN)}$$

$$\Rightarrow \mathcal{H}_1 = \underbrace{\mathcal{H}_1 \oplus \mathcal{H}_\eta \oplus \mathcal{H}_W \oplus \mathcal{H}_{\eta W}}_{\mathcal{H}_+} \oplus \underbrace{\mathcal{H}_N \oplus \mathcal{H}_{WN}}_{\mathcal{H}_-}$$

$$S_A[q,\pm] = \underbrace{\frac{c}{3}\ln\left(\frac{\ell}{\varepsilon}\right) + (g_N^* + g_N)}_{\left[\text{EQUIPART}^N \sim O(1)\right]} + O(\varepsilon/\ell) \left\{\because d_\pm = 1\right\}$$

ii) CAT. : $\mathcal{C}_{FIB} \equiv \{1, W\}$ w/ FUSION:

$\left\{\frac{1\pm\sqrt{5}}{2}\right\} \xleftarrow{\text{ROOTS}} W^2 = 1 + W \Rightarrow$ [$\mathcal{C}$-WEAKLY SYMM.: $W|\widetilde{W}\rangle = |\widetilde{1}\rangle + |\widetilde{W}\rangle$]

$\hookrightarrow \{\phi, -\phi^{-1}\}$

$$P_\phi = P_1^{(1,1)} + P_1^{(\eta,\eta)} + P_1^{(N,N)}; \quad P_{(-\phi^{-1})} = P_1^{(W,W)} + P_1^{(WN,WN)} + P_1^{(\eta W,\eta W)}$$

$$\Rightarrow \mathcal{H}_1 = \underbrace{\mathcal{H}_1 \oplus \mathcal{H}_\eta \oplus \mathcal{H}_N}_{\mathcal{H}_\phi} \oplus \underbrace{\mathcal{H}_w \oplus \mathcal{H}_{\eta w} \oplus \mathcal{H}_{wN}}_{\mathcal{H}_{(-\phi^{-1})}}$$

$$S_A[q,\phi] = \frac{c}{3}\ln\frac{\ell}{\varepsilon} + (g_w^* + g_w) + \mathcal{O}(\varepsilon/\ell)$$

$$S_A[q,(-\phi^{-1})] = \frac{c}{3}\ln\frac{\ell}{\varepsilon} + (g_w^* + g_w) + \underbrace{\frac{\phi}{1+\phi^2}}_{= d_w/|e|} + \mathcal{O}(\varepsilon/\ell)$$

THUS: "INV. SYMM: EE EQUIPART^N @ $\mathcal{O}(1)$

NON-INV. SYMM: EE EQUIPART^N @ $\mathcal{O}(\ell/\varepsilon)$"

THANK YOU!!!