Holographic Correlators for all Λ

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based on work in collaboration with Charlotte Sleight!

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erc
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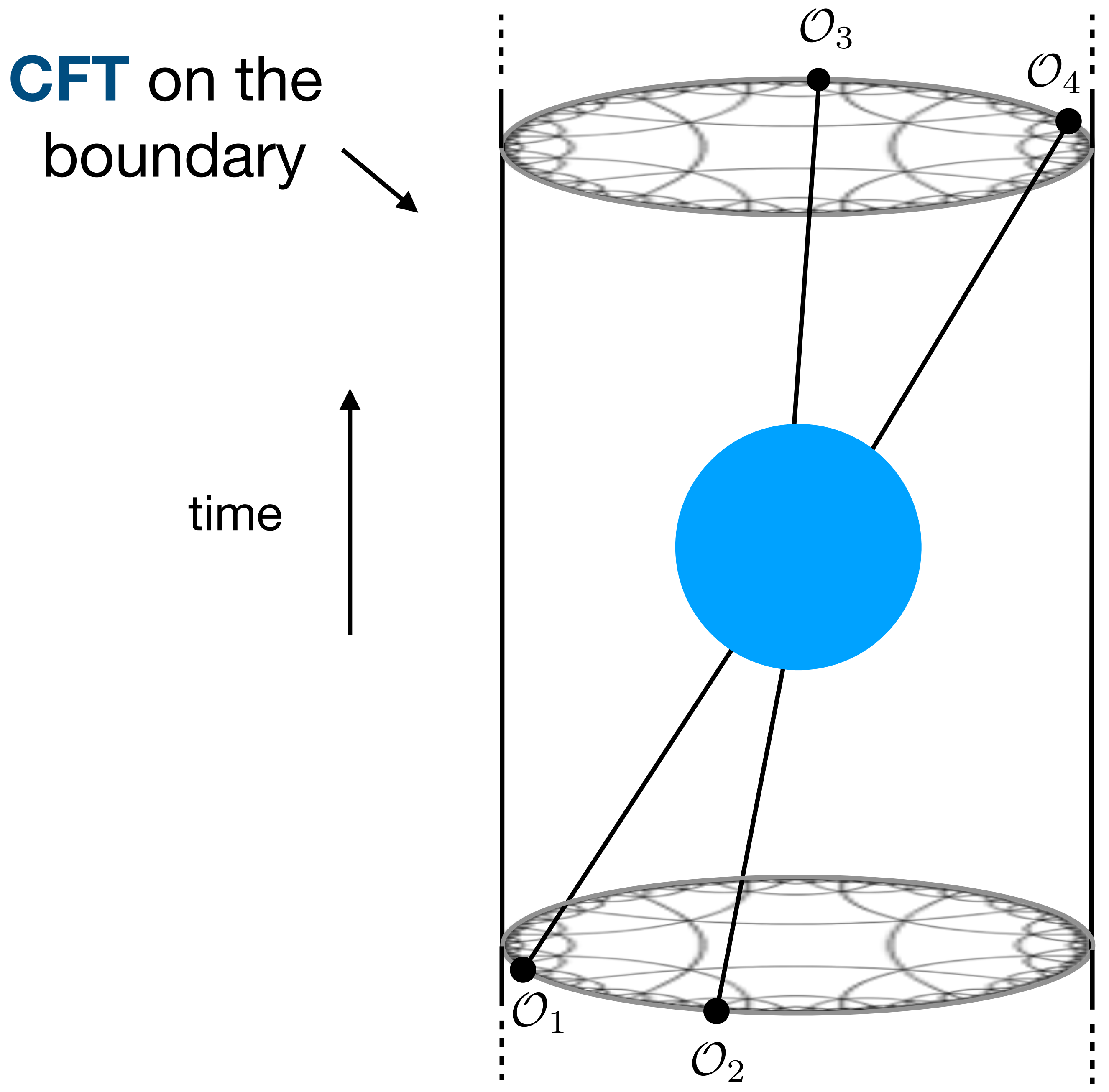
AdS-CFT

Quantum Gravity
in **a**nti-**d**e **S**itter space

=

(non-gravitational)
Conformal **F**ield **T**heory
in Minkowski space

Observables ?! ⟷ **Correlation functions**



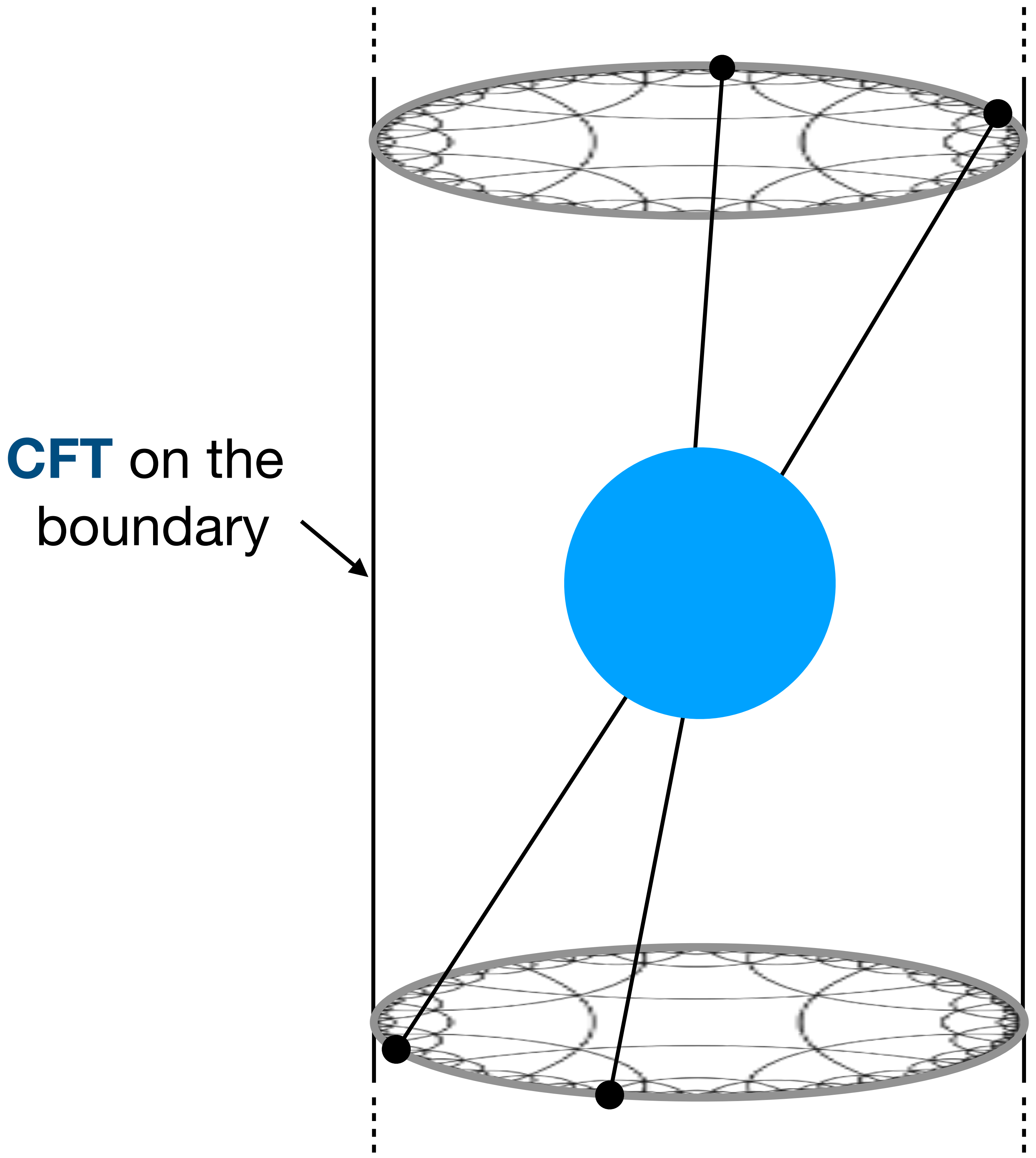
Constrained non-perturbatively by
the Conformal Bootstrap:

- Conformal symmetry
- Unitarity
- Associative operator algebra (crossing symmetry)

$$(\mathcal{O}_1\mathcal{O}_2)\,\mathcal{O}_3 = \mathcal{O}_1\,(\mathcal{O}_2\mathcal{O}_3)$$

AdS-CFT

In Anti de Sitter space we can write down the fundamental axioms of Gravity!

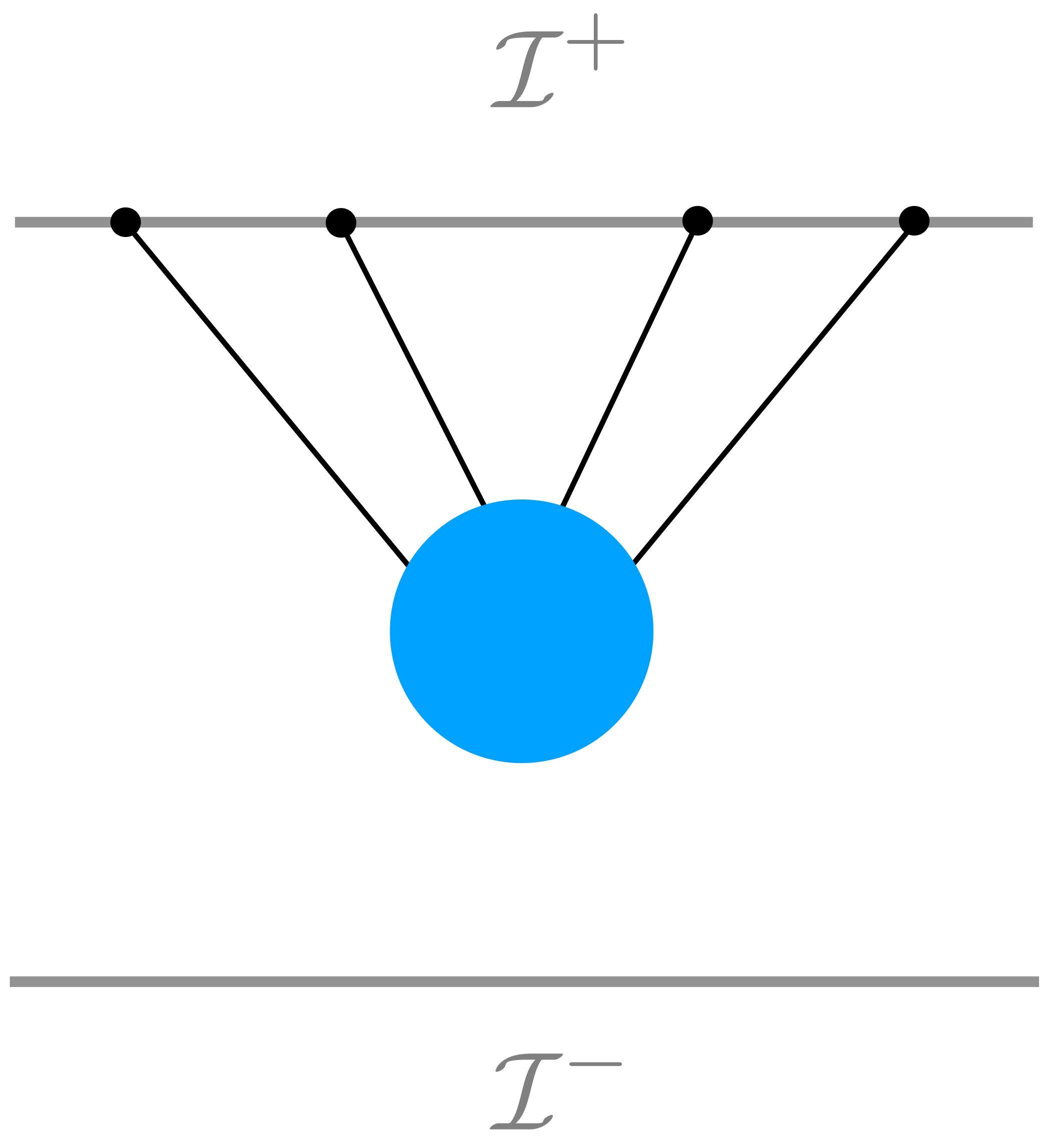


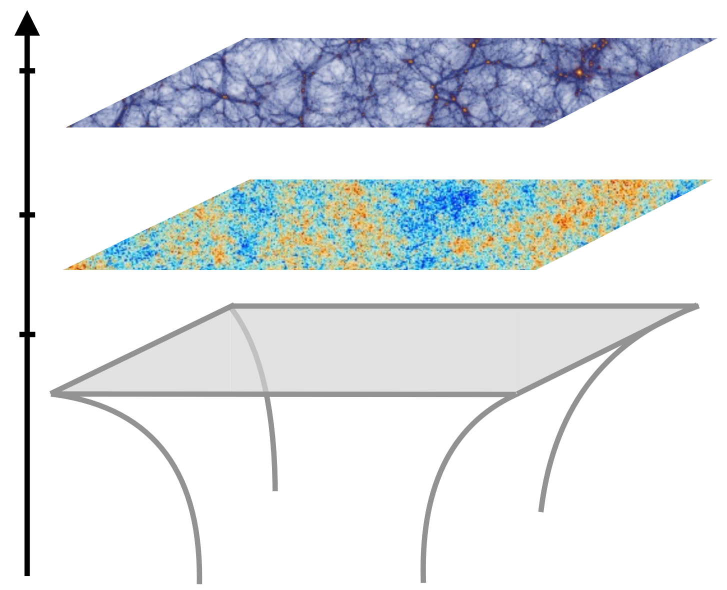
Can we extend this understanding to our own universe?

Holography for all Λs?

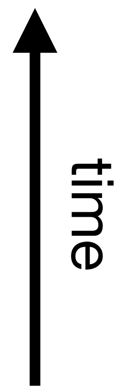
The maximally symmetric cousins of AdS

$\Lambda > 0$ de Sitter

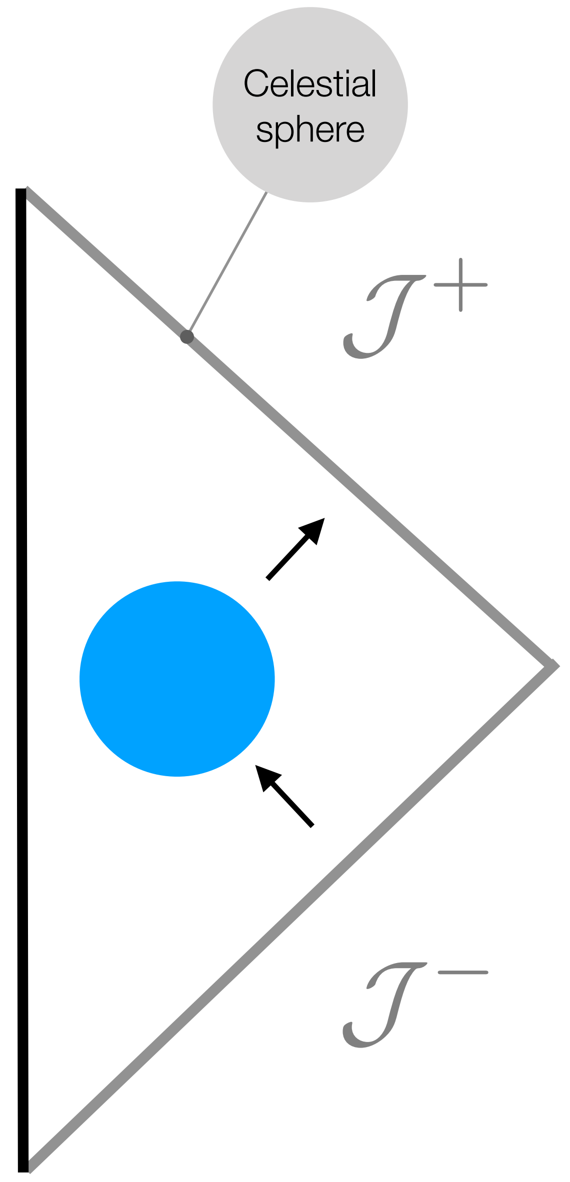




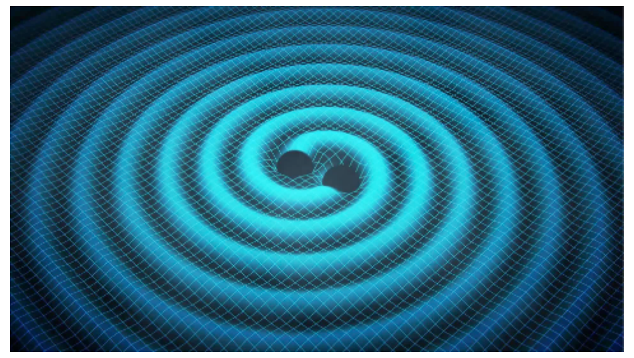
- Cosmological scales
- Primordial inflation



$\Lambda = 0$ Minkowski



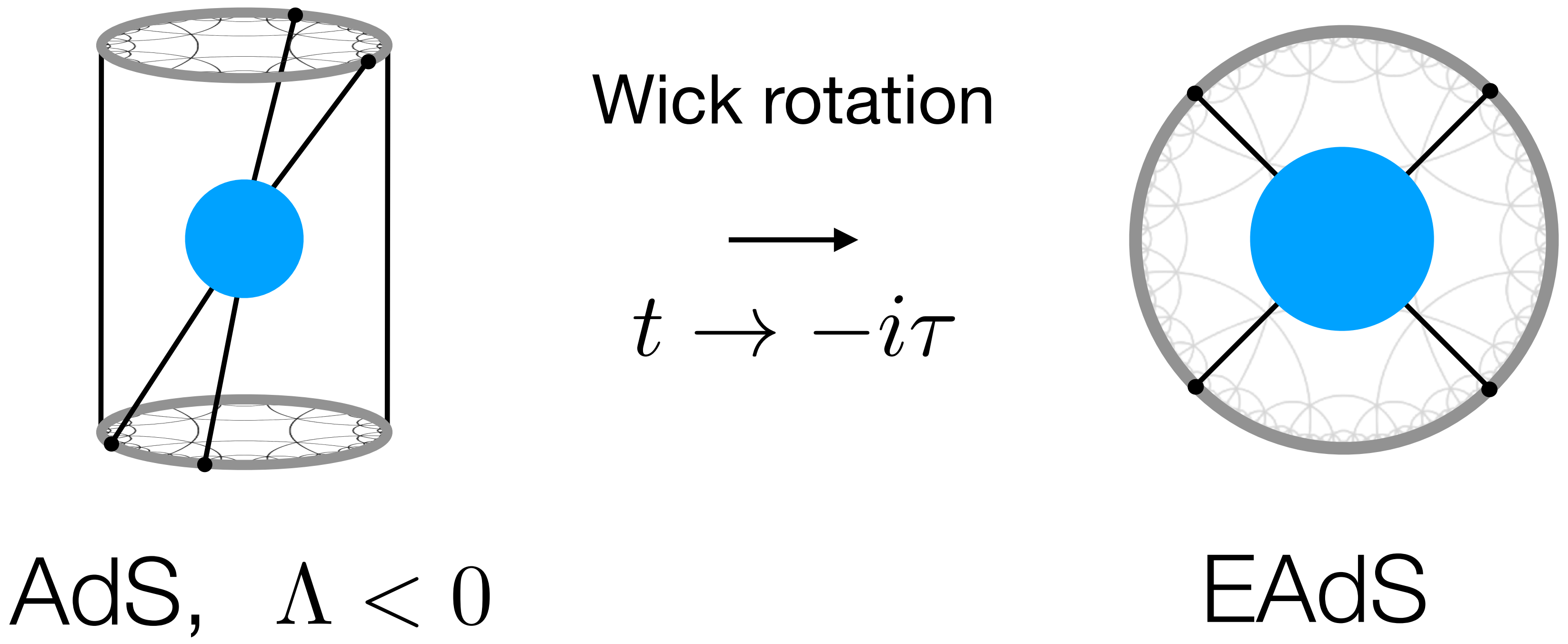
- intermediate scales

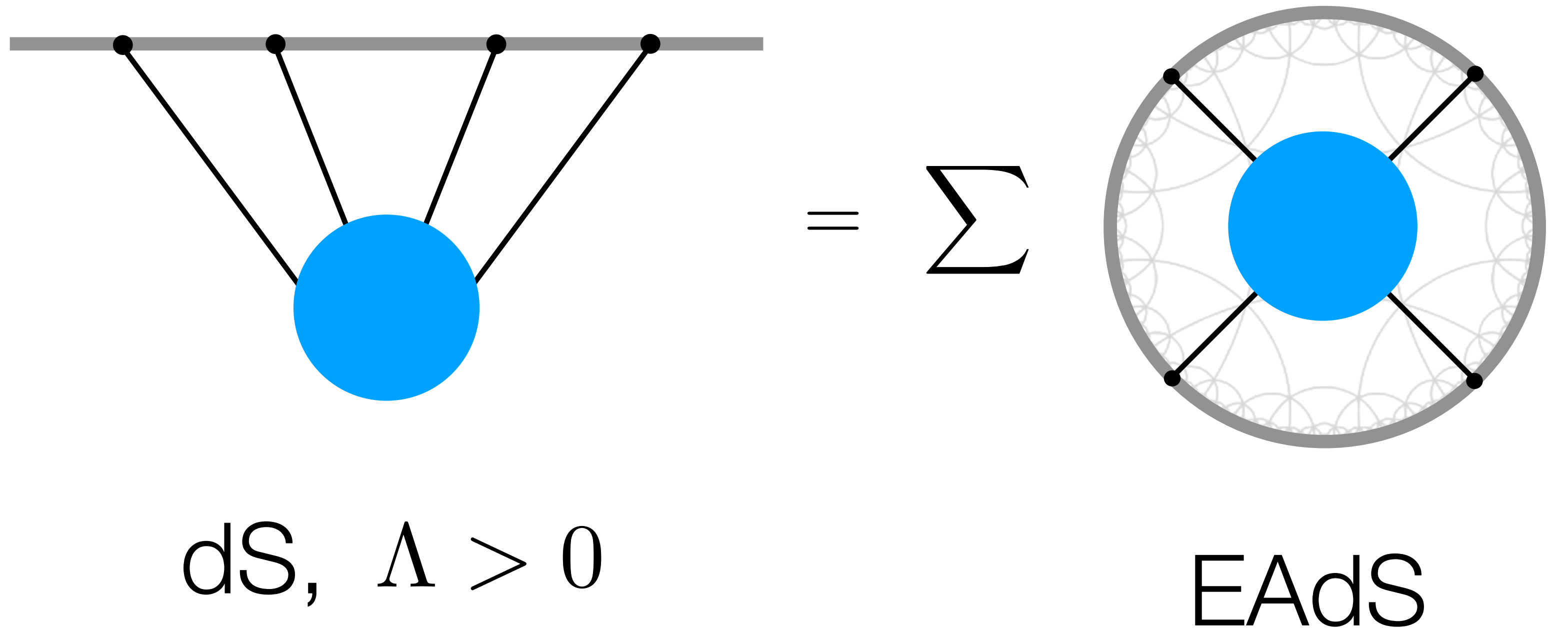


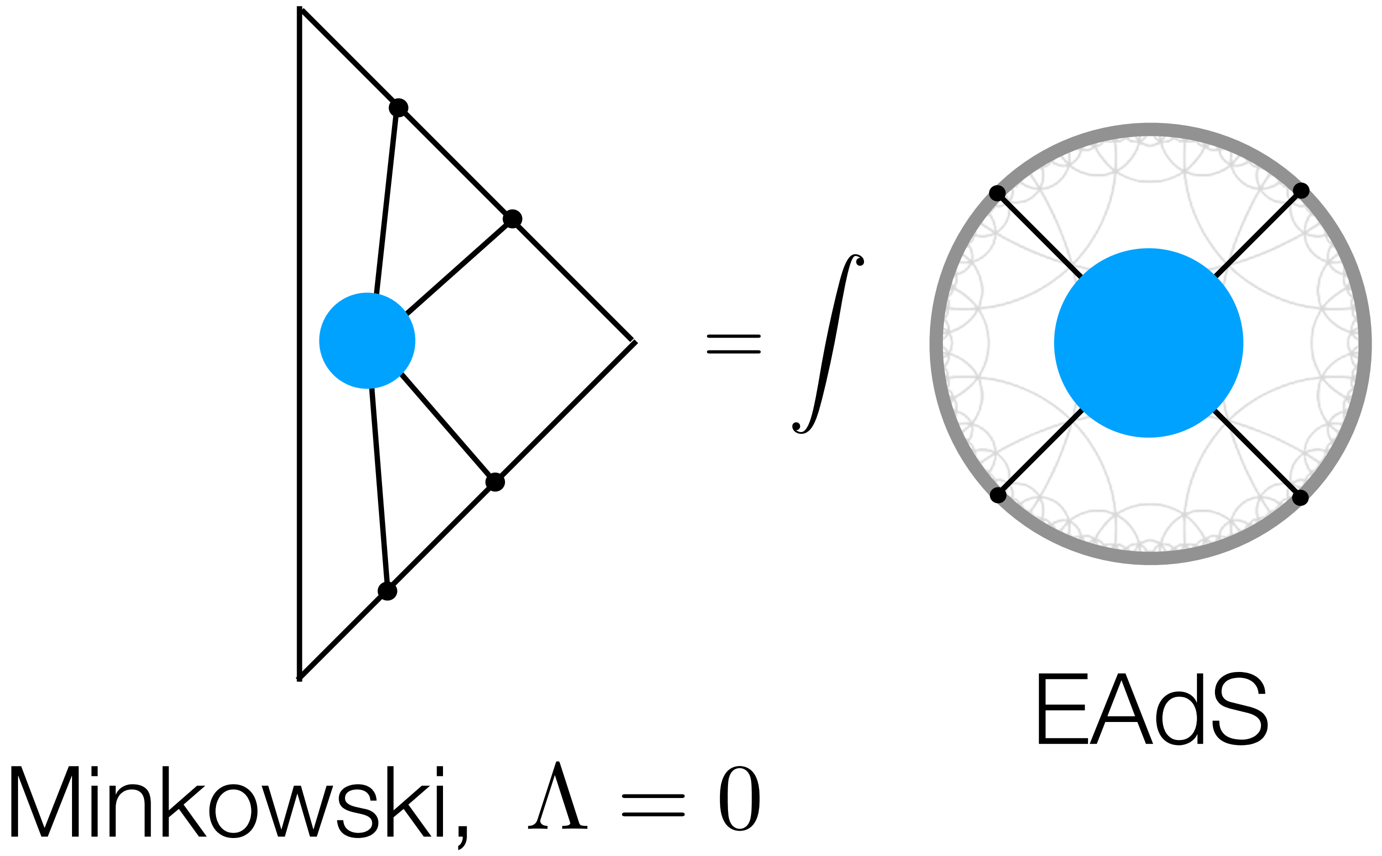
Holography for all Λs?

[Sleight, Taronna]

The strategy: connect dS and Minkowski boundary observables to those in AdS-CFT







[C.S. & Taronna ’19, ’20, ’21]

[C.S. & Taronna ’23]
[Iacobacci, C.S. & Taronna ’24]

dS and Celestial correlators therefore have a similar analytic structure to their EAdS counterparts!
On a practical level, can use such identities to import techniques and understanding from AdS.

Outline

I. $\Lambda < 0$

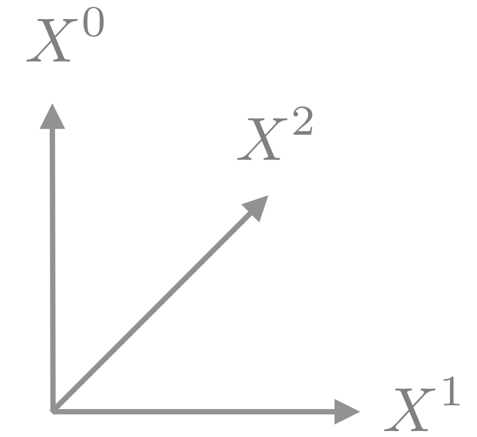
II. $\Lambda > 0$

$\Big($ III. $\Lambda = 0\Big)$

IV. Some applications.

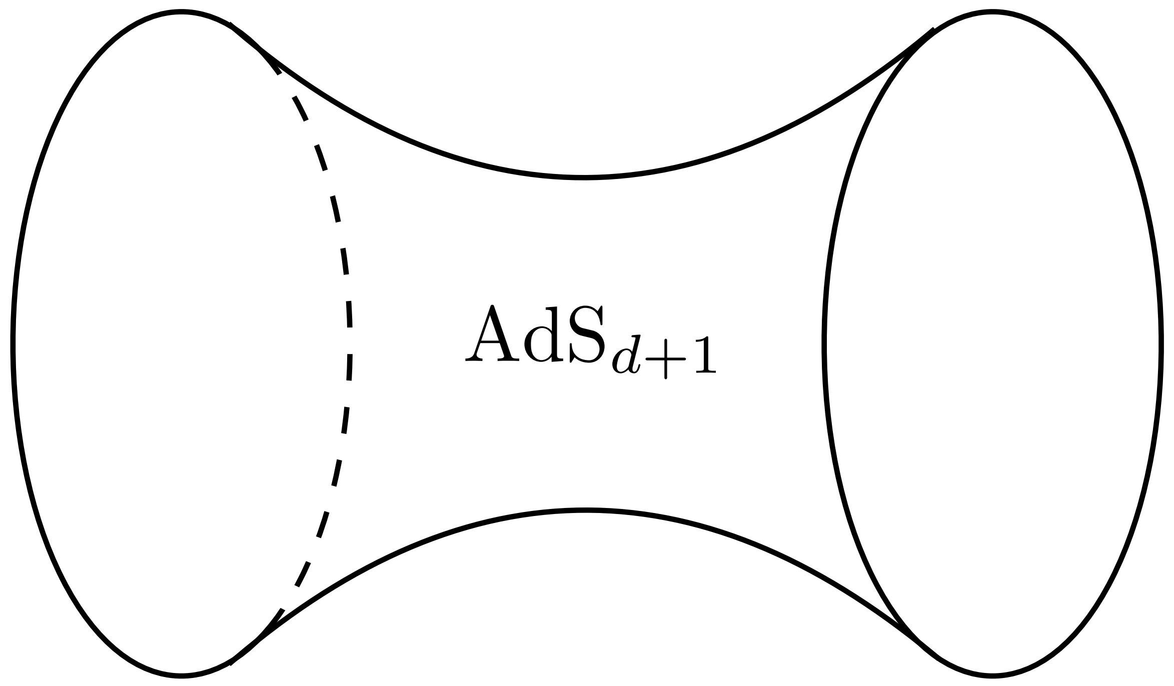
$$\Lambda < 0$$

Anti-de Sitter space-time



$\mathrm{AdS}_{d+1} \subset \mathbb{R}^{d,2}$:

$$-\left(X^0\right)^2 - \left(X^{d+1}\right)^2 + \sum_{i=1}^{d} \left(X^i\right)^2 = -R_{\mathrm{AdS}}^2$$

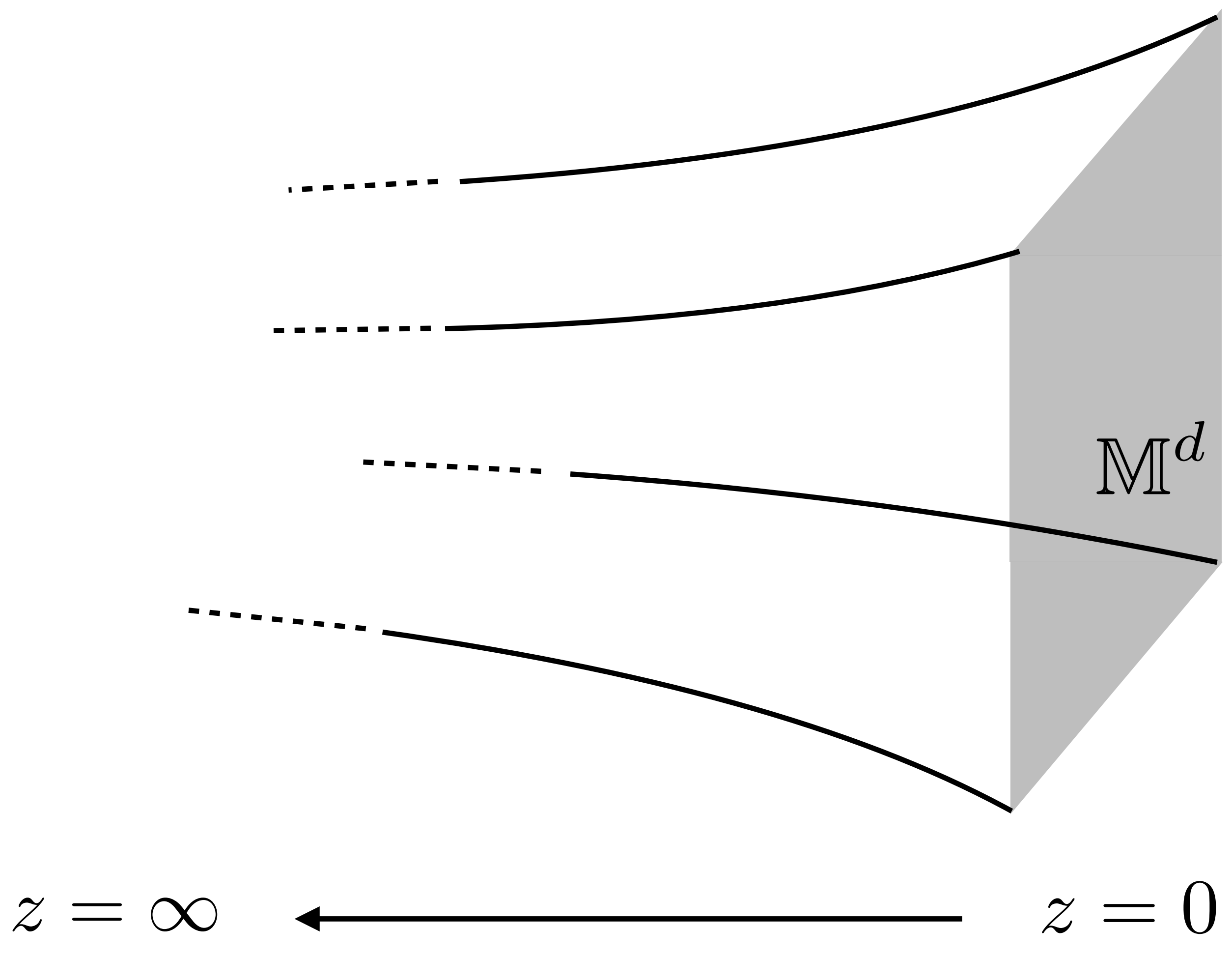


It is manifest that

Isometry group: $SO\left(d,2\right)$ = conformal group in $\mathbb{M}^d$

Poincaré coordinates:

$$\mathrm{d}s^2 = R_{\mathrm{AdS}}^2 \frac{\mathrm{d}z^2 + \eta_{\mu\nu} dx^\mu dx^\nu}{z^2}$$

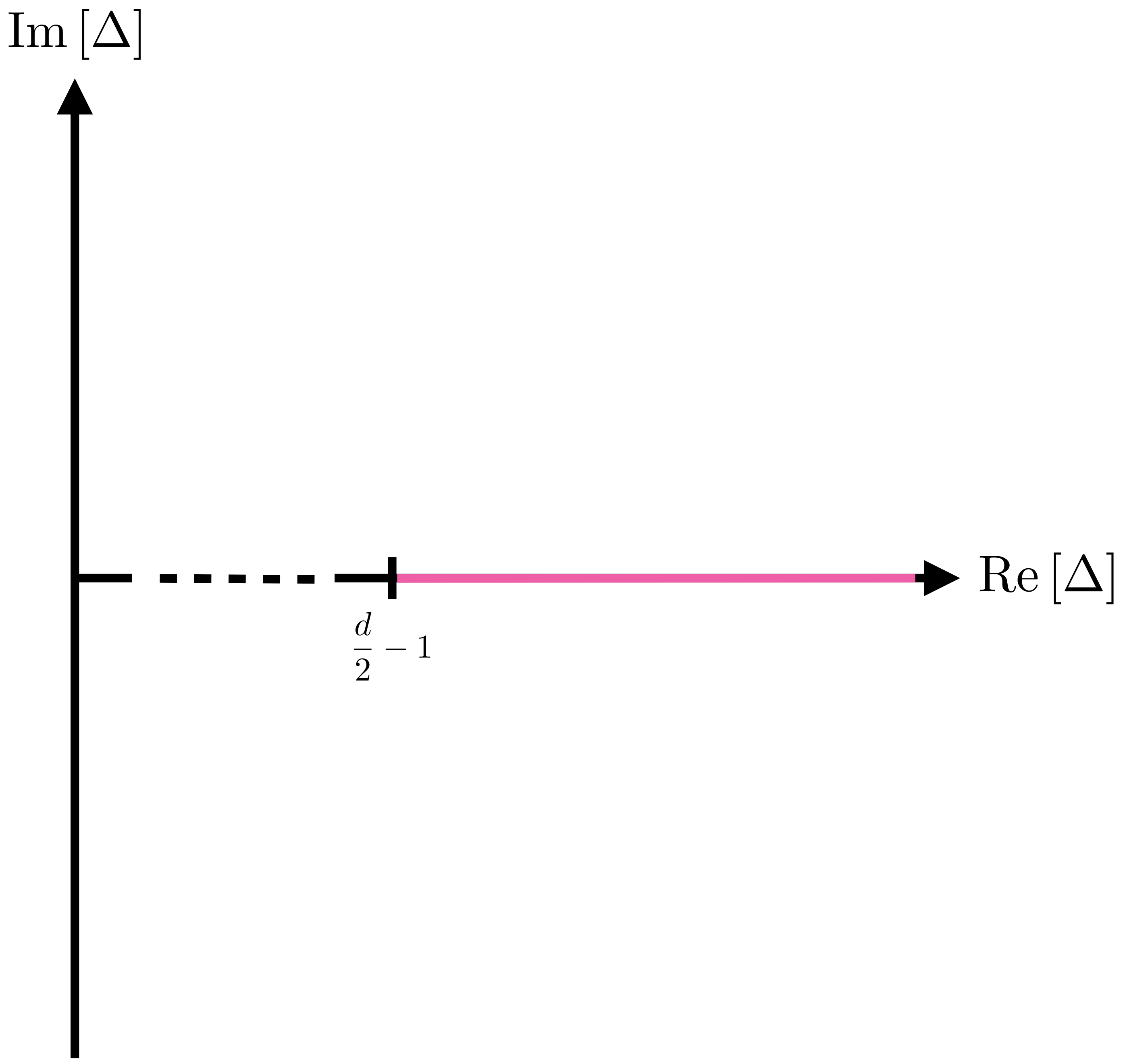


Particles in AdS

Particles in AdS$_{d+1}$ $\longleftrightarrow$ unitary irreducible representations of $SO(d,2)$

Labelled by a scaling dimension Δ and spin J. Unitarity constrains Δ :

E.g. Spin J=0 representations



Notes:

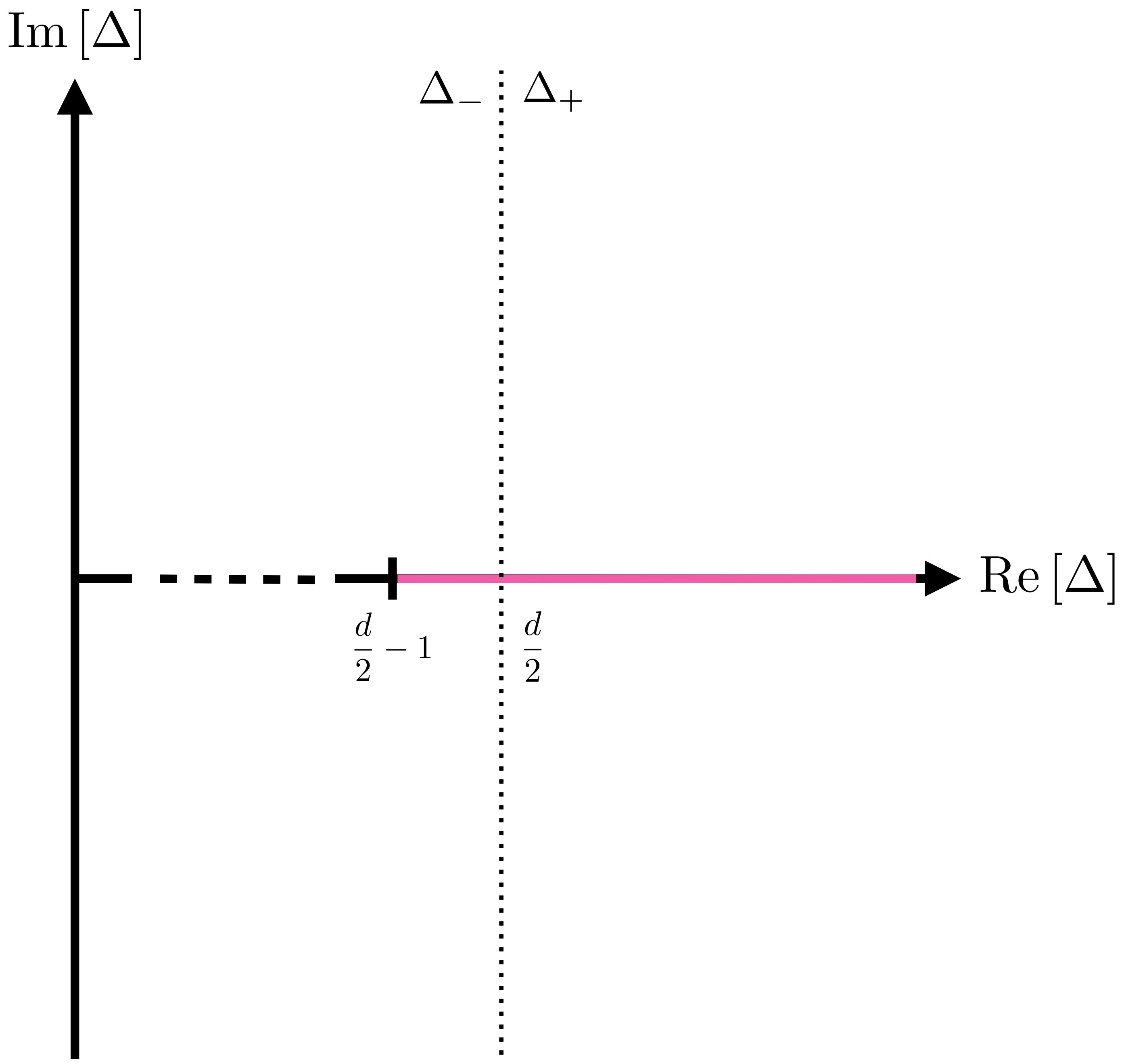
- $\Delta \in \mathbb{R}$
- Bounded from below $\Delta \geq \dfrac{d}{2} - 1$

Particles in AdS

Particles in AdS$_{d+1}$ ⟷ unitary irreducible representations of $SO\left(d,2\right)$

Labelled by a scaling dimension Δ and spin J. Can be realised by fields in AdS$_{d+1}$:

E.g. Spin J=0 representations



$$\left(\nabla^2 - m^2\right)\varphi = 0 \quad \leftrightarrow \quad \overbrace{\left(\mathcal{C}_2 - \langle\mathcal{C}_2\rangle\right)\varphi = 0}^{\substack{\text{Quadric Casimir equation} \\ \langle\mathcal{C}_2\rangle = \Delta\left(\Delta - d\right)}}$$

$$m^2 R^2_{\mathrm{AdS}} = \Delta\left(\Delta - d\right)$$

Boundary behaviour ($\Delta_- = d - \Delta_+$):

$$\lim_{z\to 0}\varphi\left(z,x\right) = \underbrace{O_{\Delta_+}\left(x\right)}_{\substack{\text{Dirichlet} \\ \text{boundary condition}}} z^{\Delta_+} + \underbrace{O_{\Delta_-}\left(x\right)}_{\substack{\text{Neuman} \\ \text{boundary condition}}} z^{\Delta_-}$$

N.B. Δ_- may be ruled out by unitarity

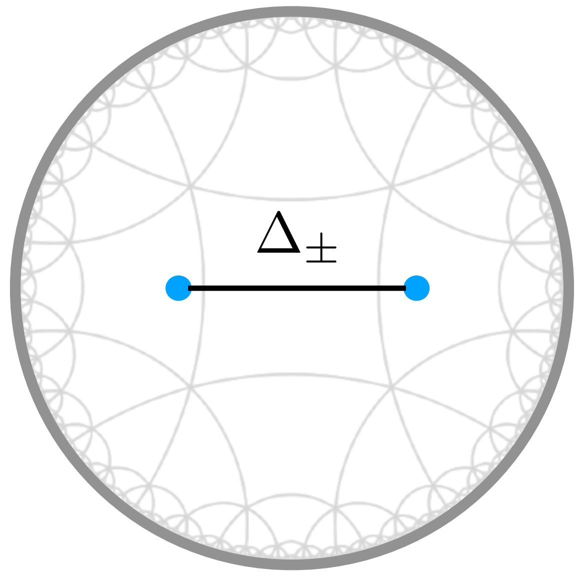
$O_{\Delta_\pm}\left(x\right)$ transform as primary fields with scaling dimension $\Delta_\pm$ in Minkowski CFT$_d$

AdS boundary correlators

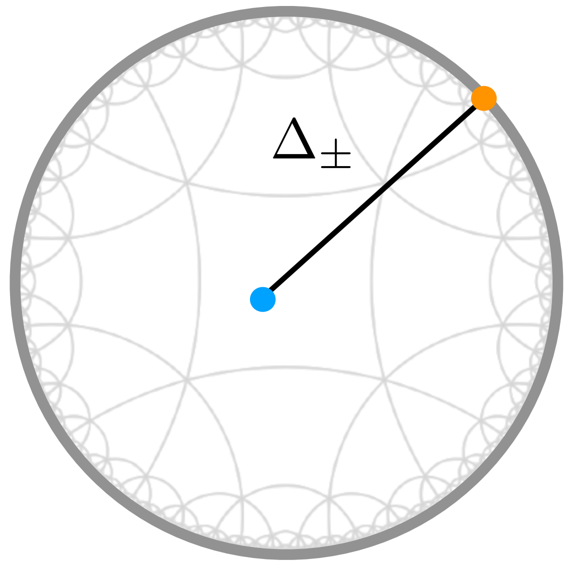
$$\lim_{z\to 0} z^{-(\Delta_1+\ldots+\Delta_n)}\langle\varphi_1\left(x_1,z\right)\ldots\varphi_n\left(x_n,z\right)\rangle \overset{!}{=} \langle\mathcal{O}_{\Delta_1}\left(x_1\right)\ldots\mathcal{O}_{\Delta_n}\left(x_n\right)\rangle$$

Feynman rules:

Bulk-to-bulk propagator, $\Delta_\pm$ boundary condition:

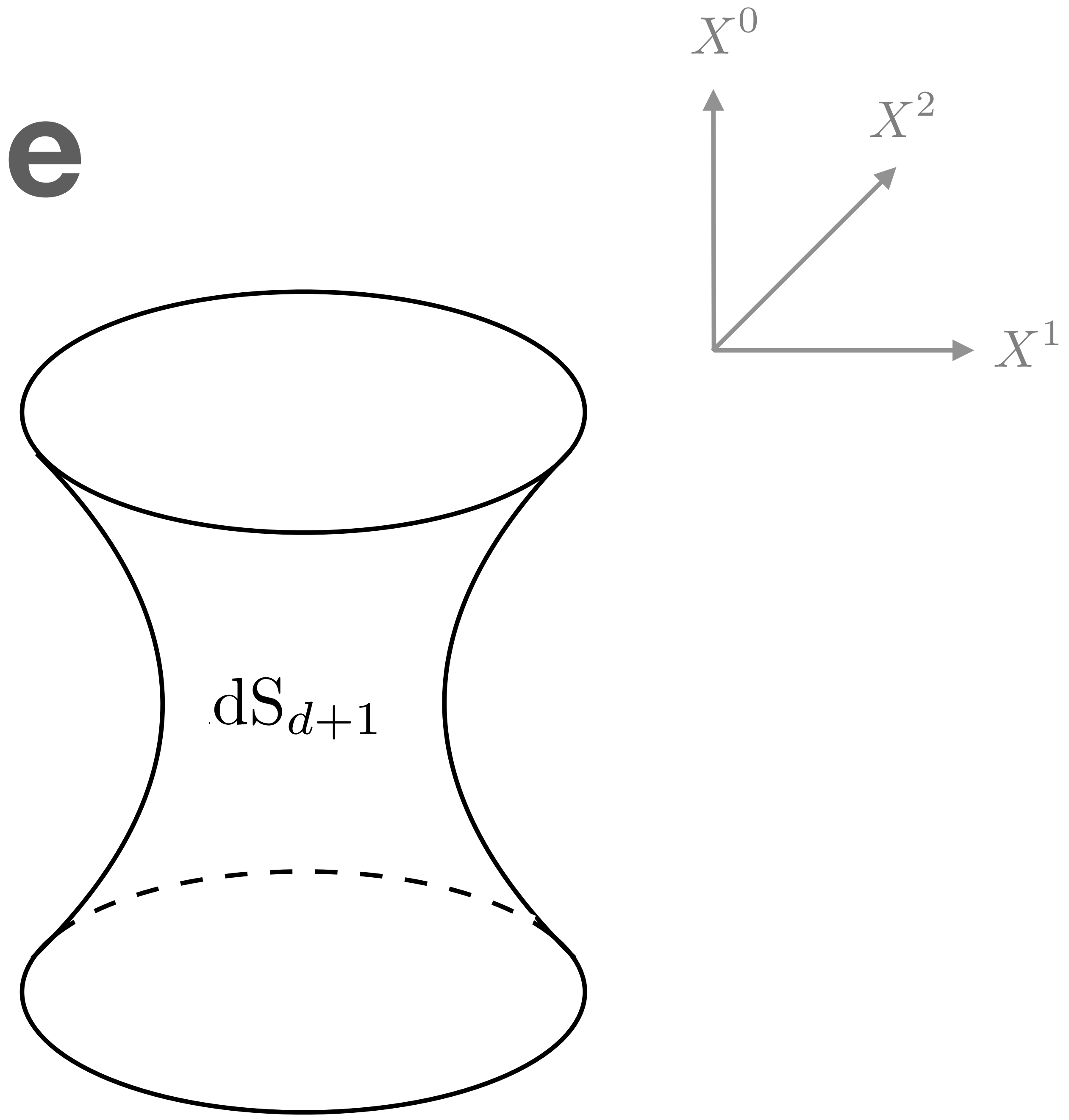


Bulk-to-boundary propagator, $\Delta_\pm$ boundary condition:



$$\Lambda > 0$$

de Sitter space-time



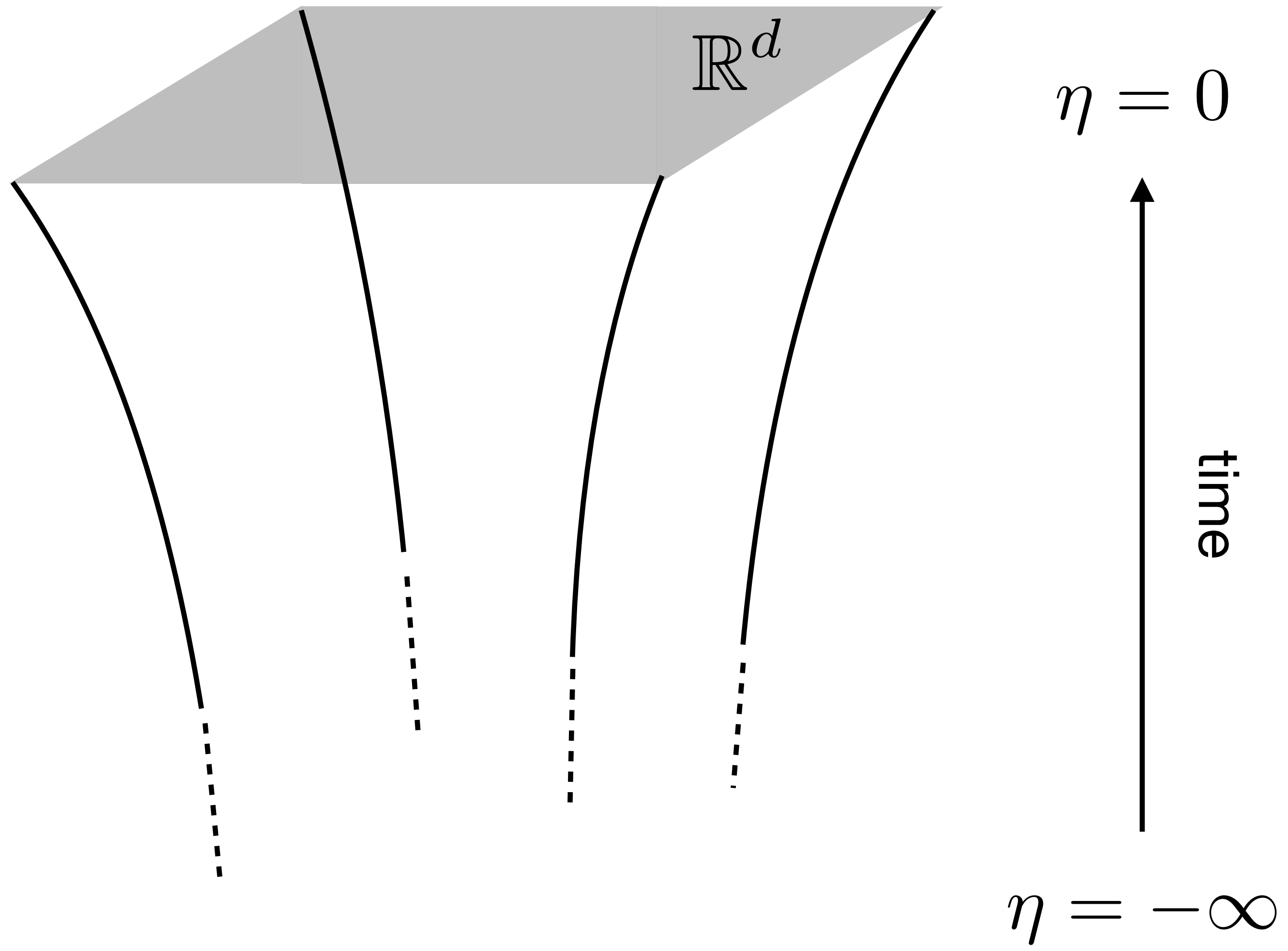
$\mathrm{dS}_{d+1} \subset \mathbb{M}^{d+2}$:

$$-\left(X^0\right)^2 + \sum_{i=1}^{d+1} \left(X^i\right)^2 = R_{\mathrm{dS}}^2$$

Isometry group: $SO\left(d+1,1\right)$ = conformal group in $\mathbb{R}^d$

Poincaré coordinates:

$$\mathrm{d}s^2 = R_{\mathrm{dS}}^2 \frac{-\mathrm{d}\eta^2 + \mathrm{d}\mathbf{x}^2}{\eta^2}$$

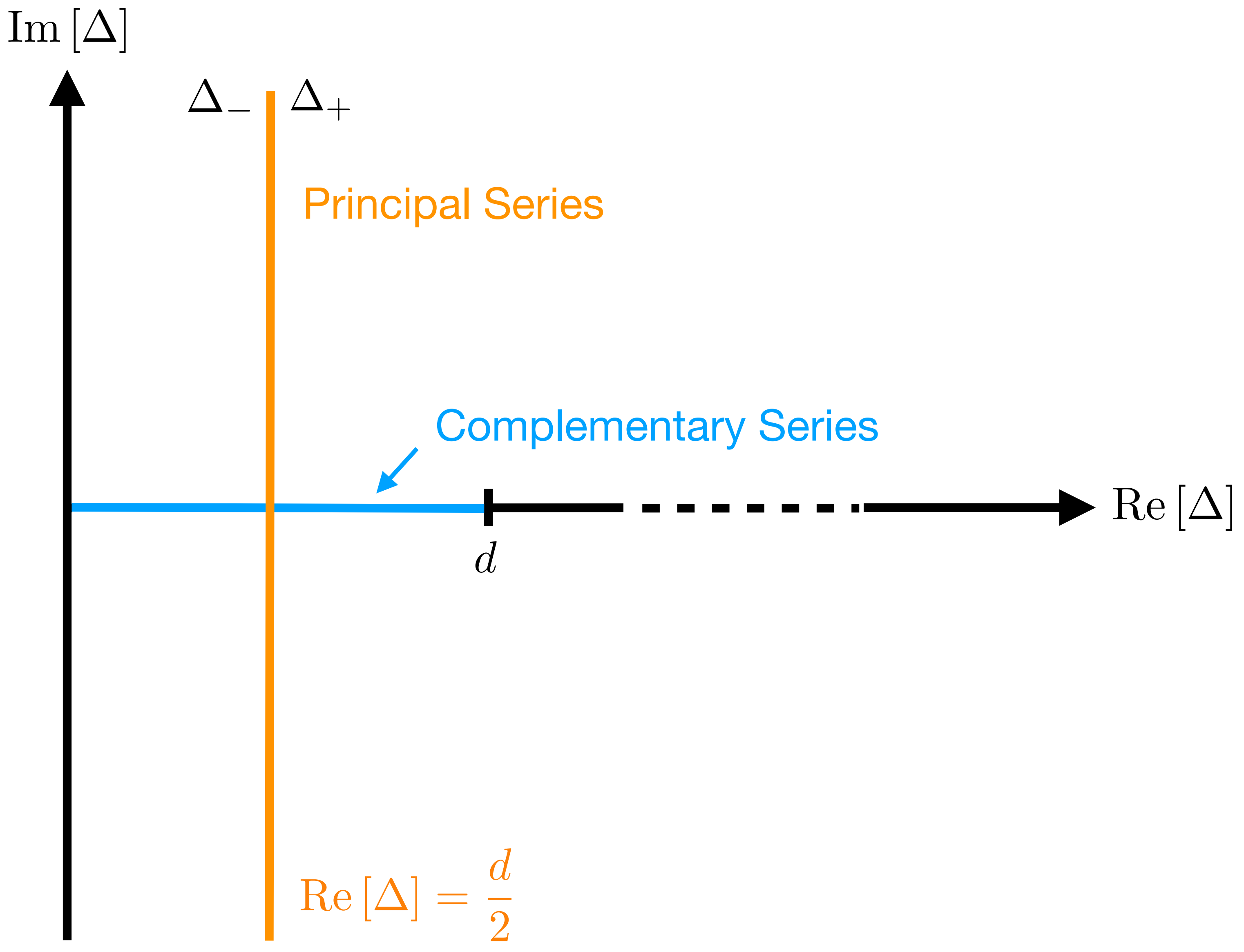


Particles in dS

Particles in dS_{d+1} $\longleftrightarrow$ unitary irreducible representations of $SO\left(d+1,1\right)$

Labelled by a scaling dimension Δ and spin J. Unitarity constrains Δ :

E.g. Spin J=0 representations



Notes:

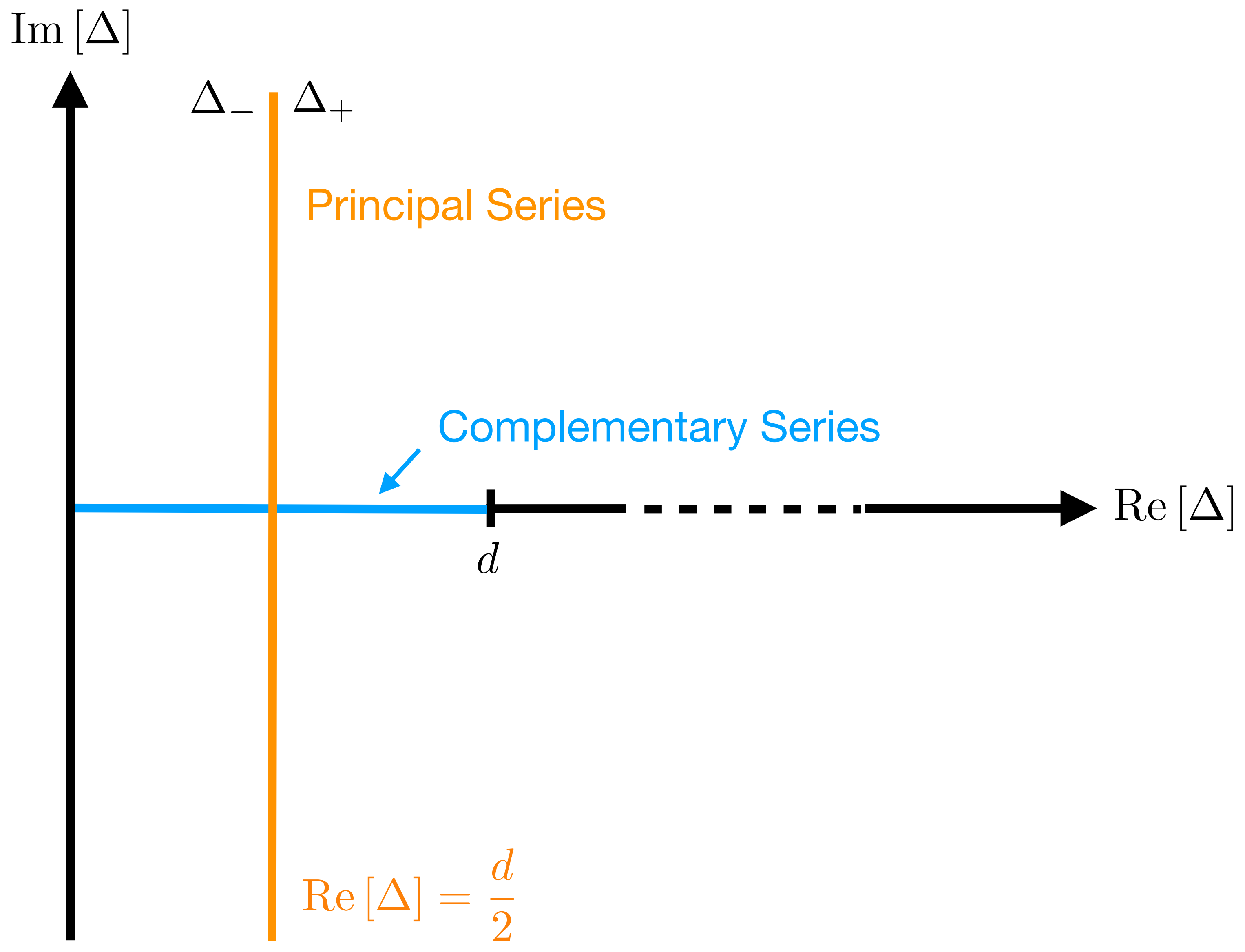
- Both Δ_+ and Δ_- are unitary
- Δ can be complex (Principal Series)

Particles in dS

Particles in dS_{d+1} ⟷ unitary irreducible representations of $SO(d+1,1)$

Labelled by a scaling dimension Δ and spin J. Can be realised by fields in dS_{d+1}.

E.g. Spin J=0 representations



Quadric Casimir equation

$$\left(\nabla^2 - m^2\right)\varphi = 0 \quad \leftrightarrow \quad \overbrace{\left(\mathcal{C}_2 - \langle\mathcal{C}_2\rangle\right)\varphi = 0}^{\langle\mathcal{C}_2\rangle = \Delta(d-\Delta)}$$

$$m^2 R_{\mathrm{dS}}^2 = \Delta(d-\Delta)$$

Boundary behaviour:

$$\lim_{\eta\to 0}\varphi(\eta,x) = O_{\Delta_+}(\mathbf{x})\,\eta^{\Delta_+} + O_{\Delta_-}(\mathbf{x})\,\eta^{\Delta_-}$$

Determined by the initial state

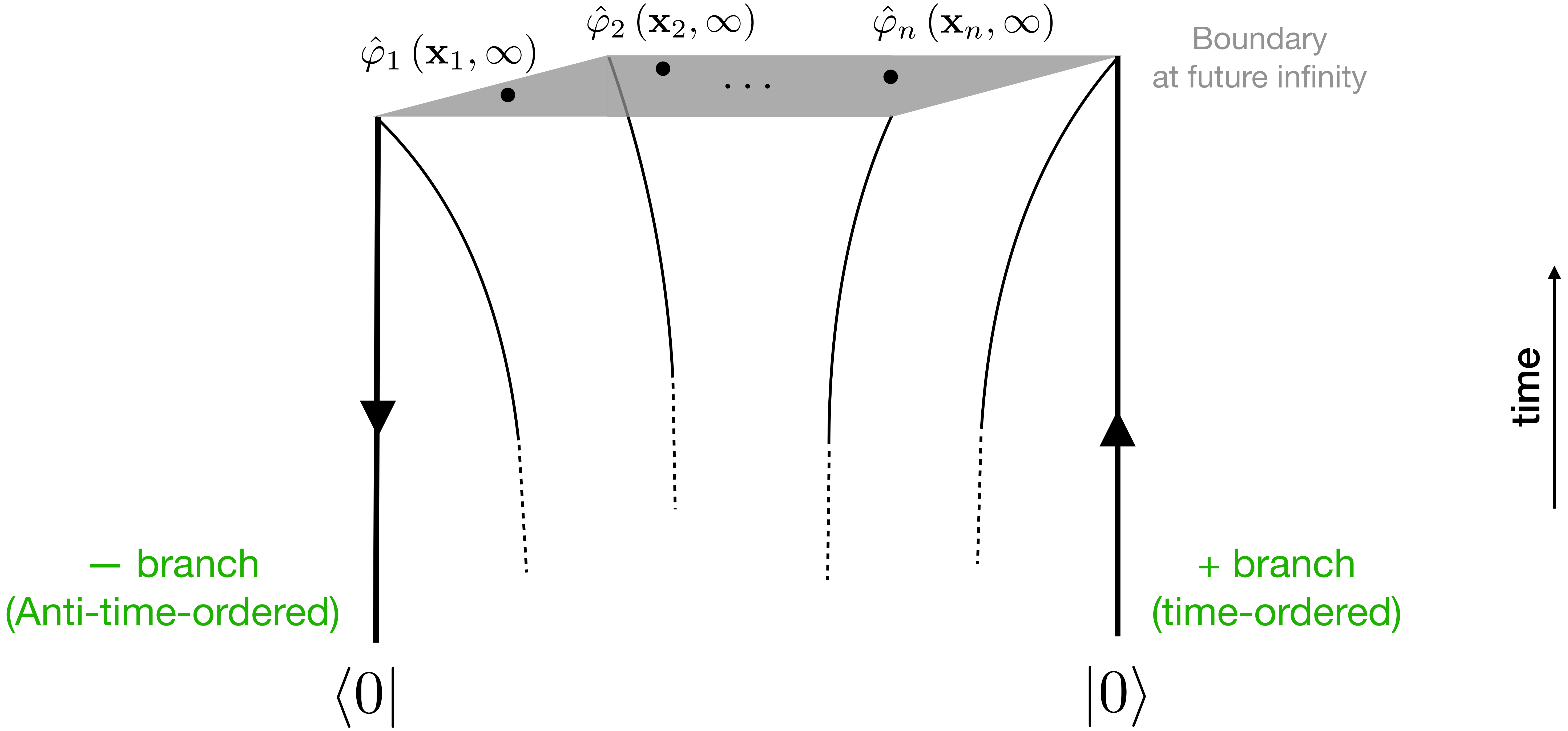
$O_{\Delta_\pm}(\mathbf{x})$ transform as primary fields with scaling dimension $\Delta_\pm$ in Euclidean CFT_d

dS Boundary Correlators

in-in formalism

[Maldacena ’02, Weinberg '05]

$$\lim_{\tau \to \infty} \langle 0 | \hat{\varphi}_1 \left(\mathbf{x}_1, \tau \right) \ldots \hat{\varphi}_n \left(\mathbf{x}_n, \tau \right) | 0 \rangle$$



Take $|0\rangle$ to be the de Sitter vacuum which reduces to the Minkowski vacuum at early times.

(Bunch Davies vacuum)

dS Boundary Correlators

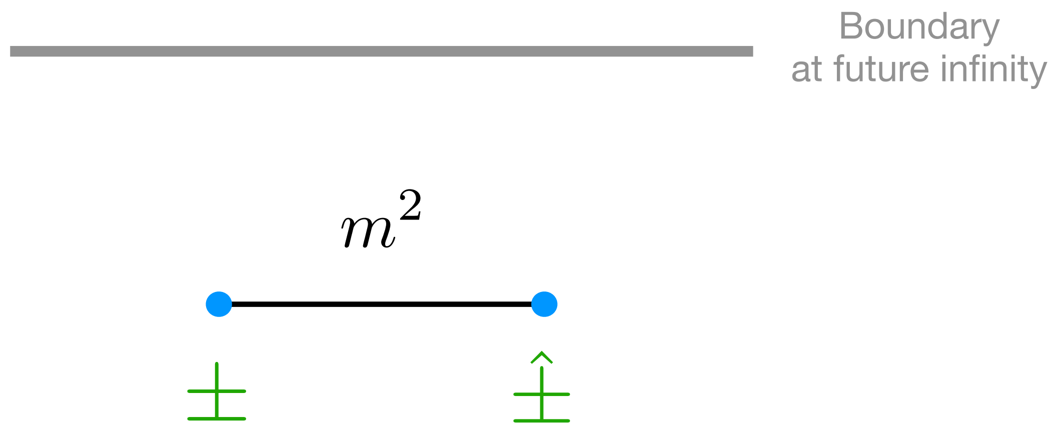
in-in formalism

[Maldacena ’02, Weinberg '05]

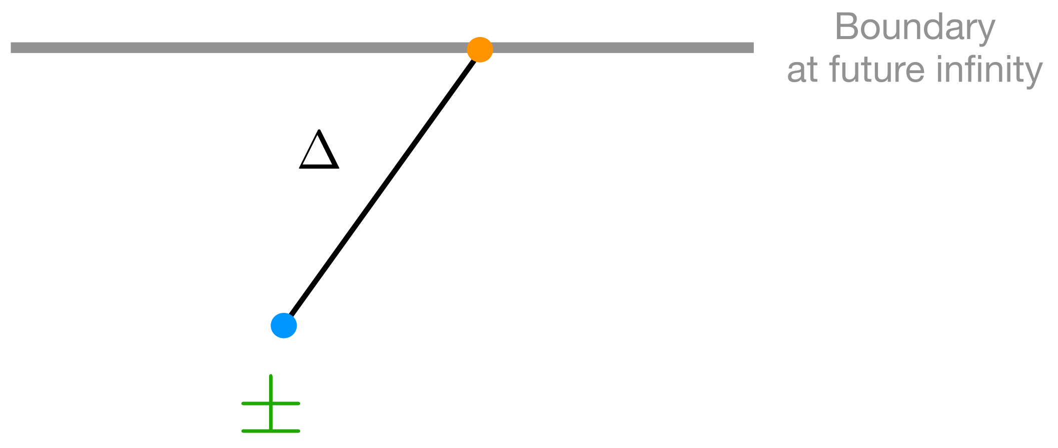
$$\lim_{\tau\to\infty} \langle 0|\hat{\varphi}_1\left(\mathbf{x}_1,\tau\right)\ldots\hat{\varphi}_n\left(\mathbf{x}_n,\tau\right)|0\rangle$$

Feynman rules:

$\pm$ bulk-to- $\hat{\pm}$ bulk propagator:



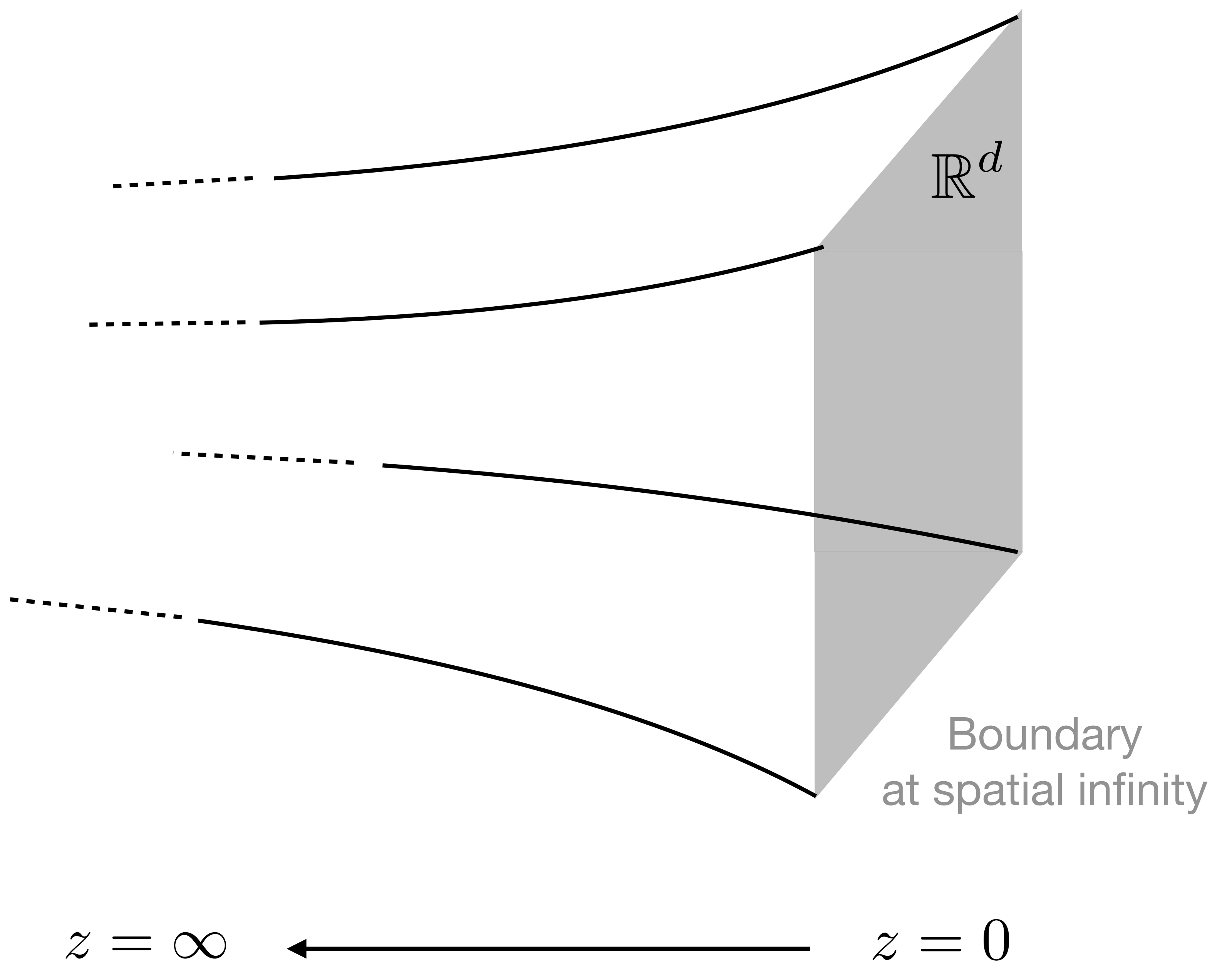
$\pm$ bulk-to-boundary propagator:



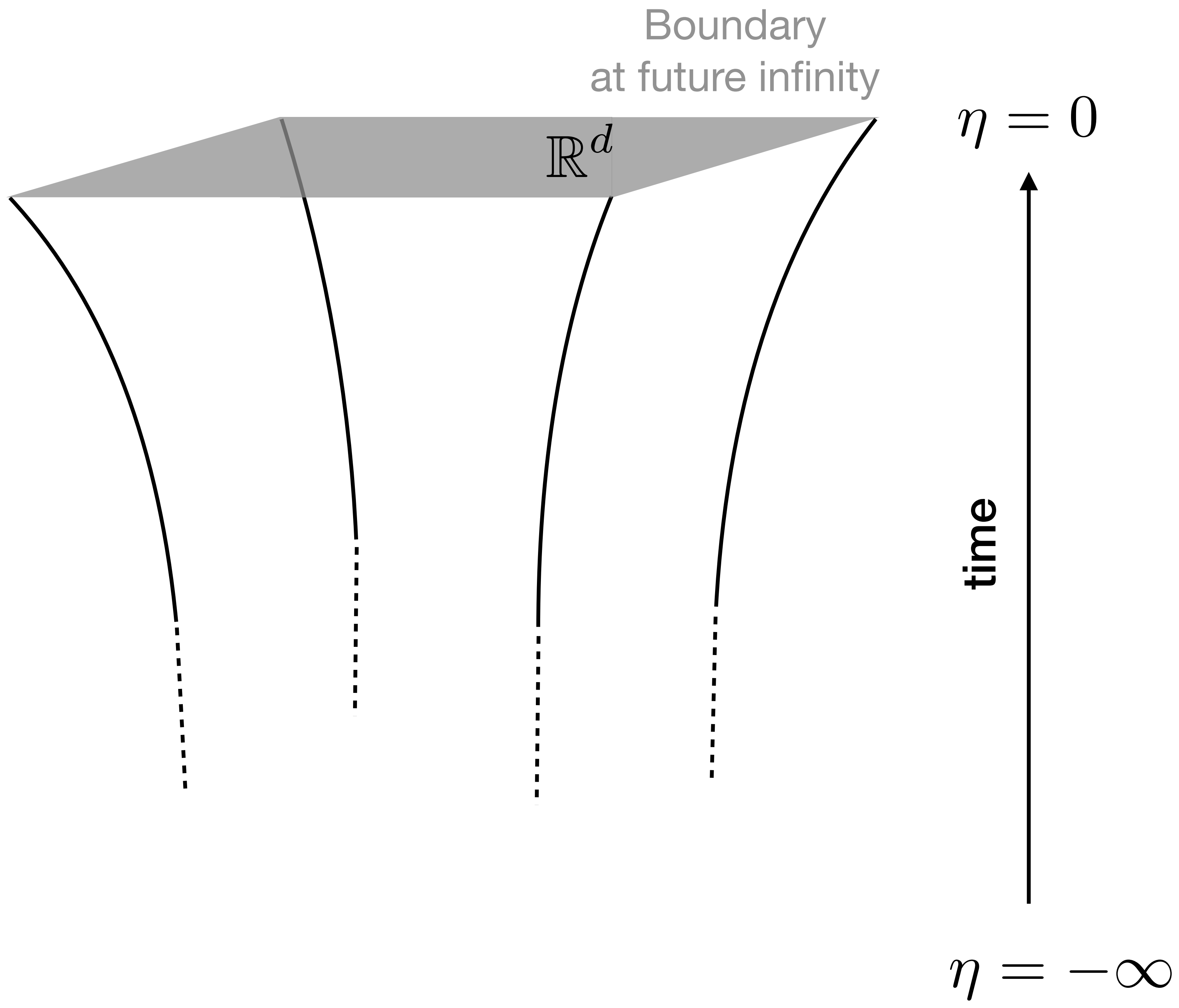
Sum contributions from each branch (±) of the time (in-in) contour!

From dS to Euclidean AdS

Euclidean AdS



dS



$$\mathrm{d}s^2 = R_{\mathrm{AdS}}^2 \frac{\mathrm{d}z^2 + \mathrm{d}\mathbf{x}^2}{z^2} \quad \longleftrightarrow \quad \mathrm{d}s^2 = R_{\mathrm{dS}}^2 \frac{-\mathrm{d}\eta^2 + \mathrm{d}\mathbf{x}^2}{\eta^2}$$

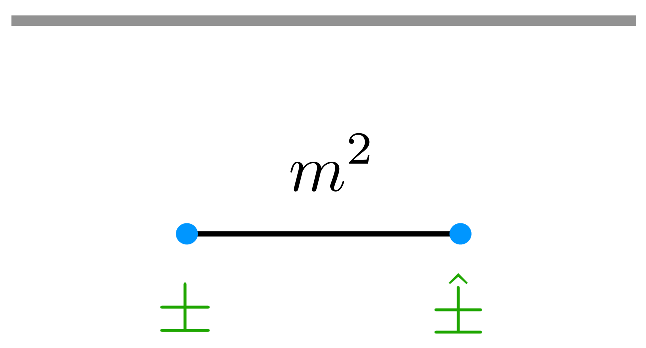
EAdS and dS are identified under:

$$R_{\mathrm{AdS}} = \pm i R_{\mathrm{dS}} \qquad z = \pm i \left(-\eta\right)$$

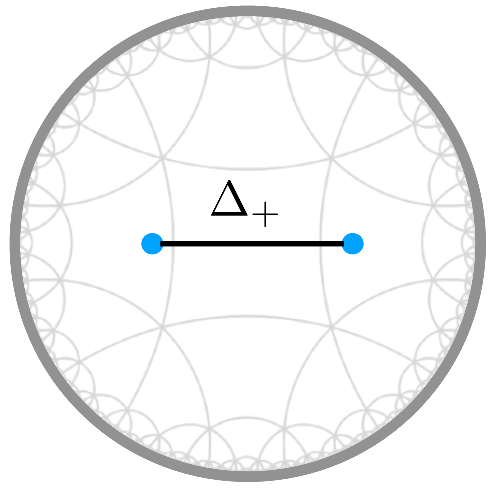
From dS to Euclidean AdS

$\pm$ bulk-to- $\hat{\pm}$ bulk propagator:

[C.S. and M. Taronna ’19, ’20, ’21]

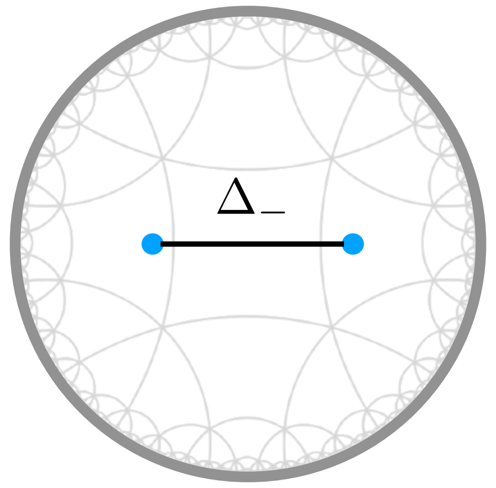


$= e^{\mp\frac{i\pi}{2}\Delta_+} e^{\hat{\mp}\frac{i\pi}{2}\Delta_+}$



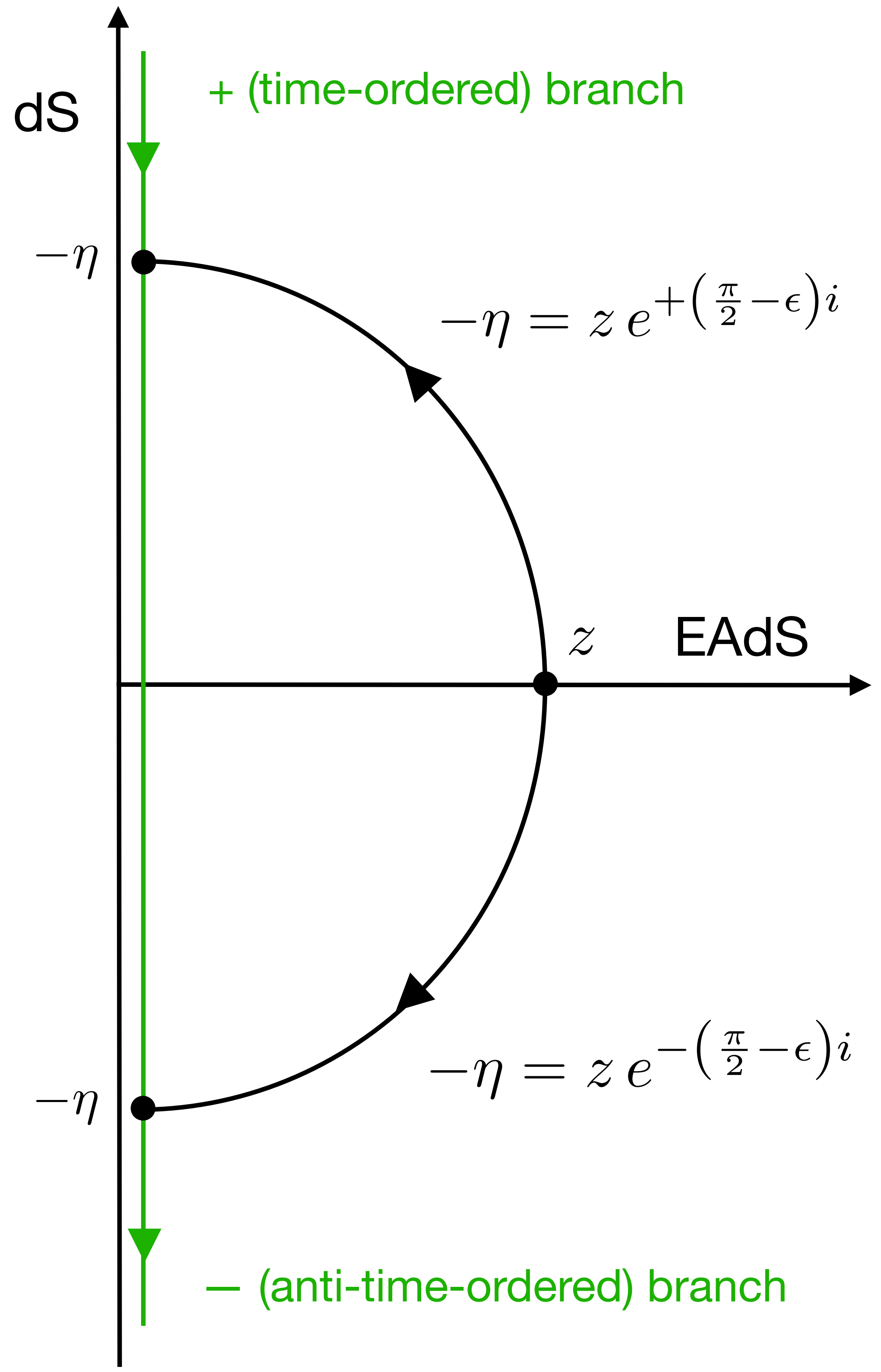
Dirichlet
boundary condition

$+ e^{\mp\frac{i\pi}{2}\Delta_-} e^{\hat{\mp}\frac{i\pi}{2}\Delta_-}$

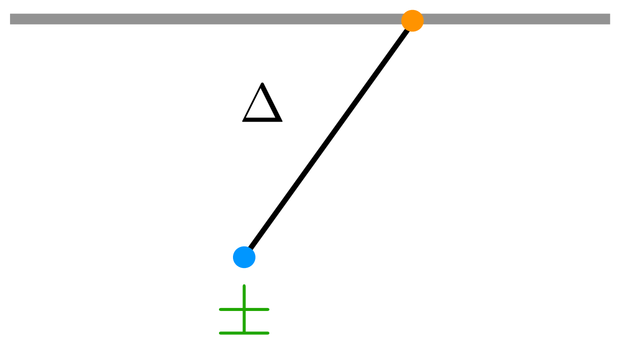


Neumann
boundary condition

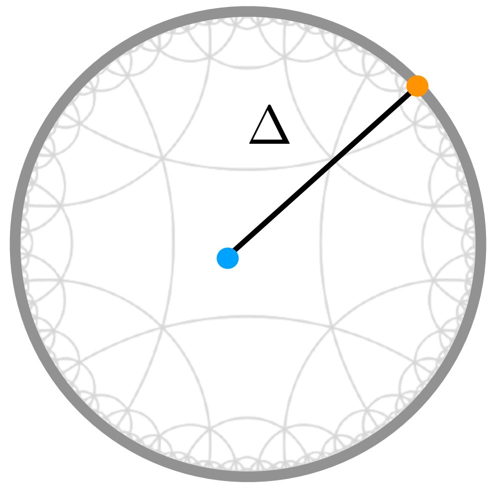
$m^2 R_{\text{dS}}^2 = \Delta_+\Delta_-$



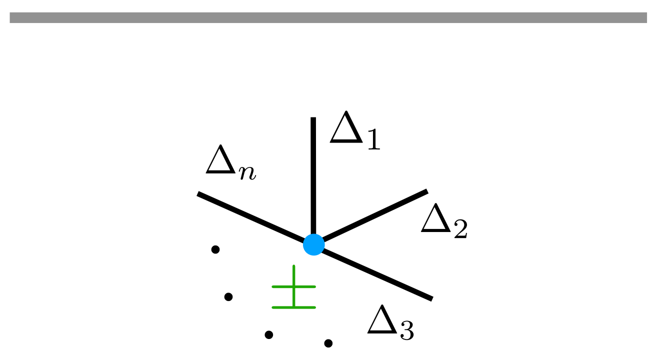
$\pm$ bulk-to-boundary propagator:



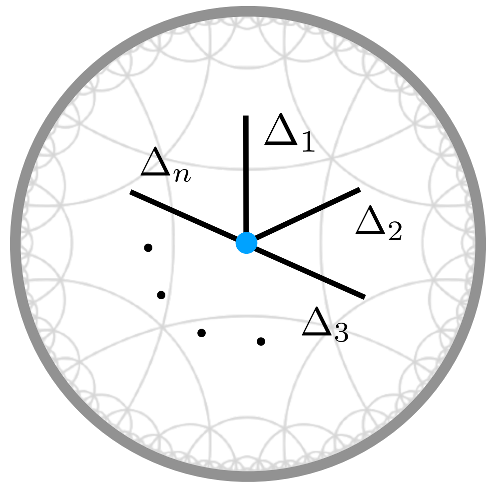
$= e^{\mp\frac{i\pi}{2}\Delta}$



$\pm$ bulk integrals:



$= e^{\pm(d-1)\frac{\pi i}{2}}$



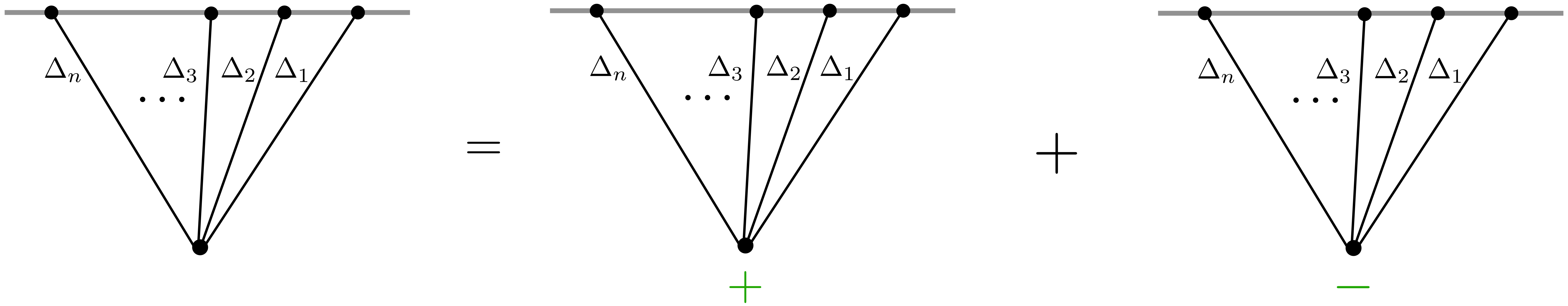
From dS to Euclidean AdS

Examples.

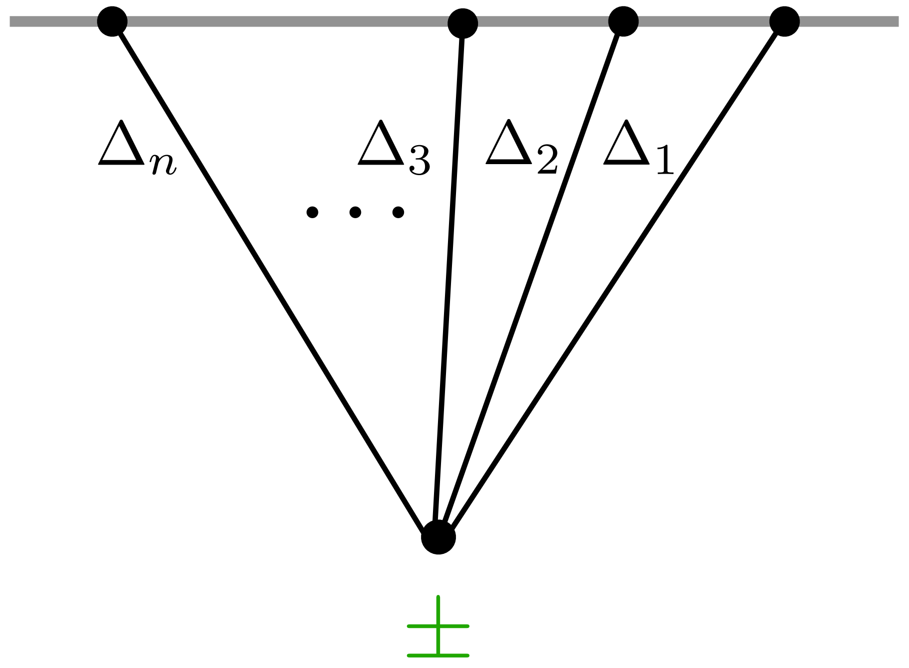
[C.S. and M. Taronna ’19]

Non-derivative vertex of scalars fields $\mathcal{V}(X) = g\phi_1(X)\ldots\phi_n(X)$

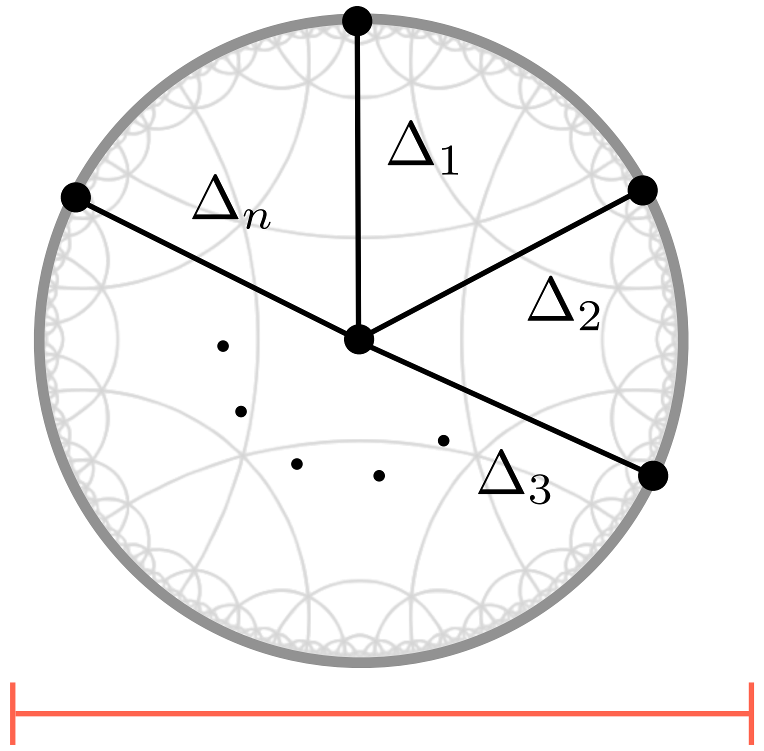
Contact diagram:



Where



$$= e^{\pm\frac{i\pi}{2}(d-1)}\prod_{j=1}^{n} e^{\mp\frac{i\pi}{2}\Delta_j}$$



Same contact diagram in EAdS

From dS to Euclidean AdS

Examples.

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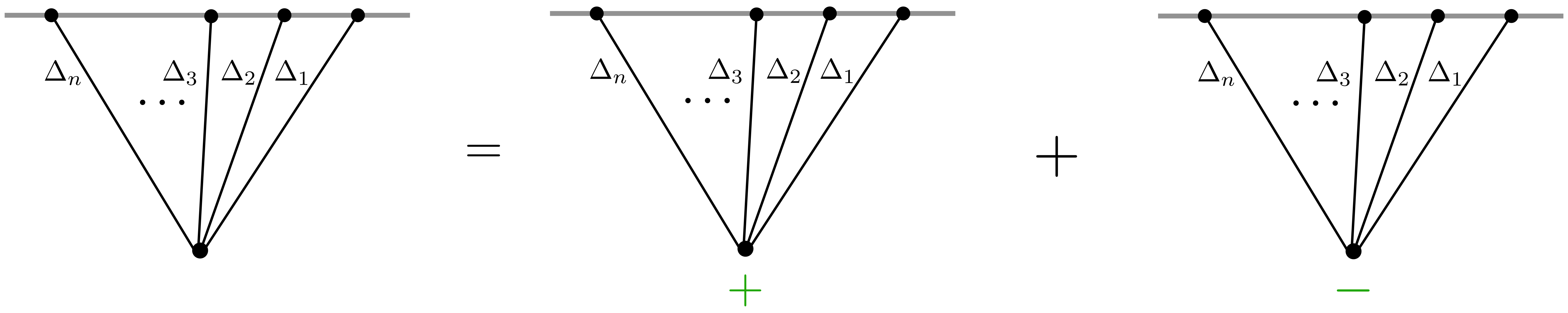
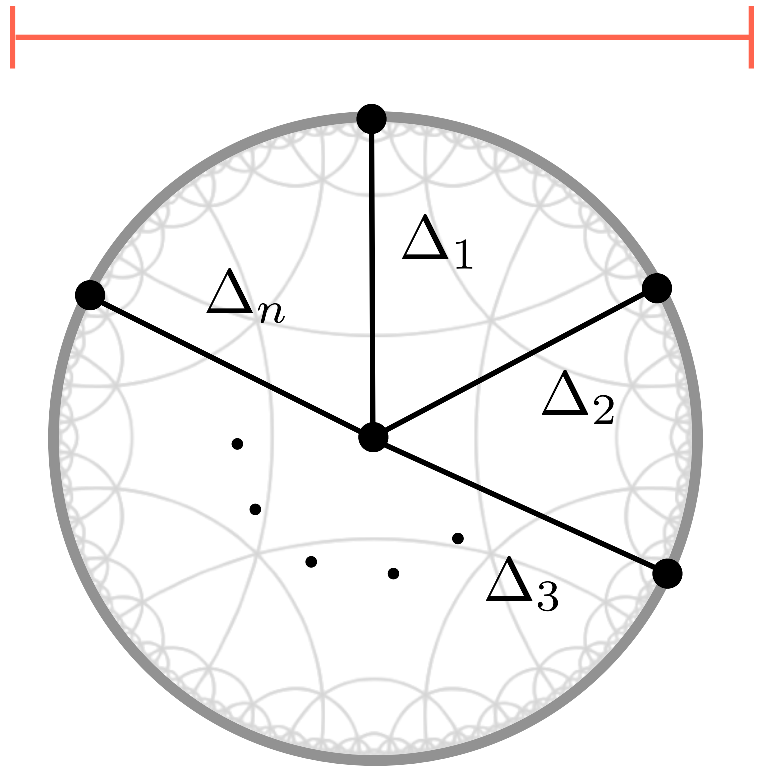
Contact diagram:



$$=\sin\left(-\frac{d}{2}+\frac{1}{2}\sum_{i=1}^{n}\Delta_i\right)\pi$$

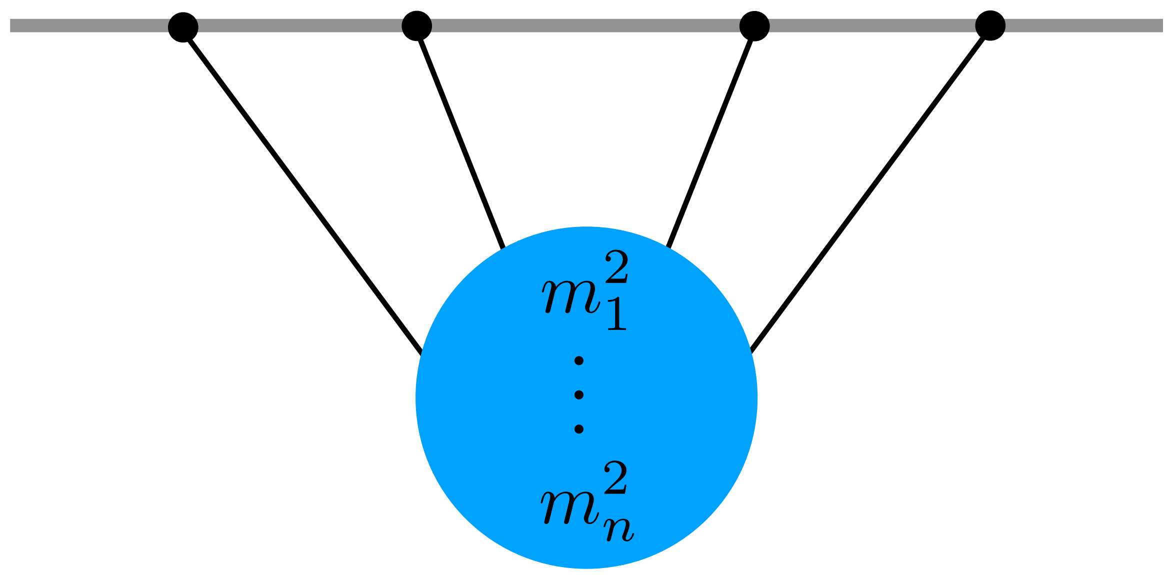
Same contact diagram in EAdS



From dS to EAdS, and back

dS boundary correlators are perturbatively recast as Witten diagrams in EAdS:

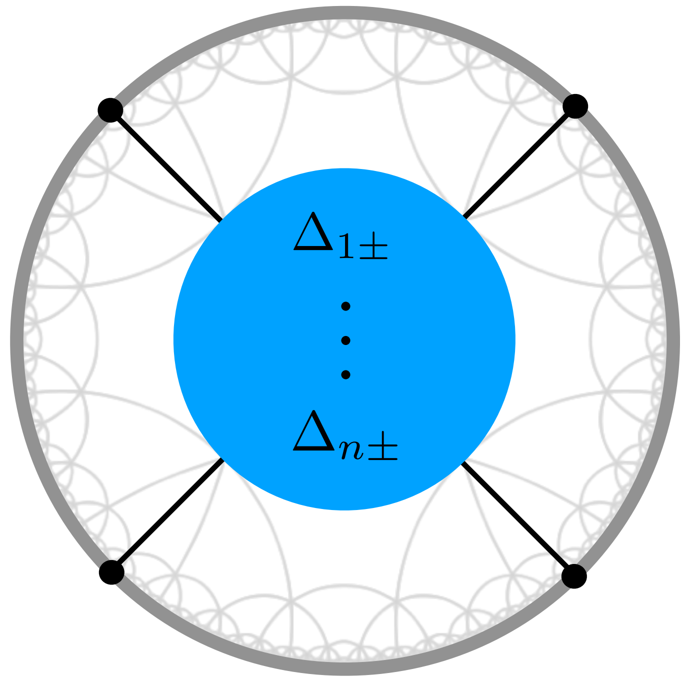
e.g. four-points



Process with M vertices

Combines contributions from each branch of the in-in contour

$$= \underbrace{\sum_{\Delta_{1\pm}\ldots\,\Delta_{n\pm}}}_{\text{Sum over boundary conditions for exchanged particles}} \underbrace{C_{\Delta_{1\pm}\ldots\,\Delta_{n\pm}}}_{\text{factorises into: } C_1^{\text{contact}} \times \ldots \times C_M^{\text{contact}}}$$



Notes:

- Contributions from both $\Delta_\pm$ modes, which is not always possible in AdS
- $\Delta_{i\,\pm}$ $\in$ Unitary Irreducible Representation of **dS** isometry

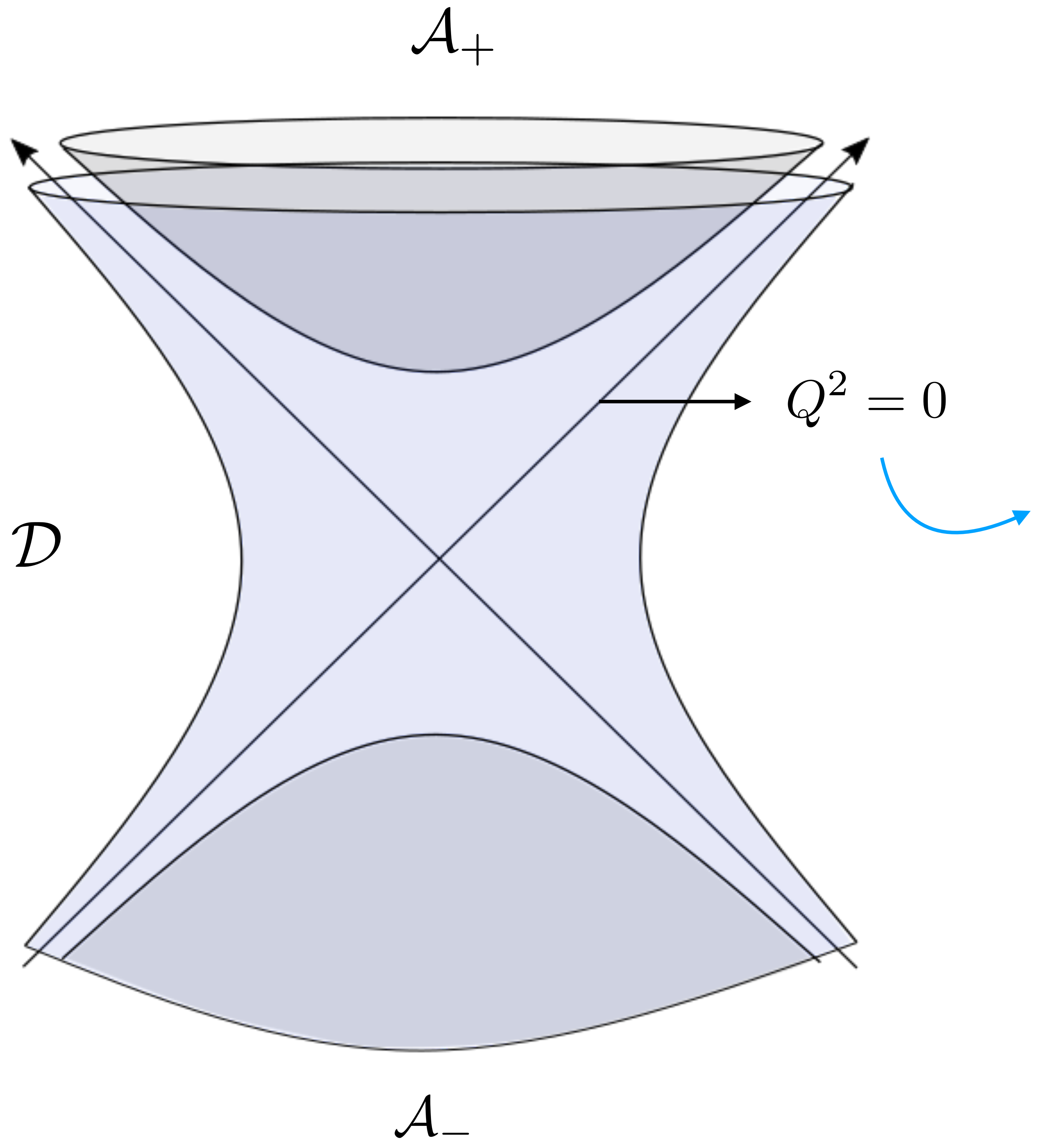
Can use to import techniques and results from AdS to dS!

$$\Lambda = 0$$

Hyperbolic slicing of Minkowski space

[de Boer and Solodukhin '03]

(d+2)-dimensional Minkowski space $\mathbb{M}^{d+2}$, coordinates $X^A, \quad A = 0, \ldots d+1$



$\mathcal{A}_\pm : \quad X^2 = -t^2$ (EAdS$_{d+1}$, radius t)

$\mathcal{D} : \quad X^2 = R^2$ (dS$_{d+1}$, radius R)

Conformal boundary:

$$Q^2 = 0, \quad Q \equiv \lambda Q, \quad \lambda \in \mathbb{R}^+$$

Introduce projective coordinates:

$$\xi_i = Q^i / Q^0, \quad i = 1, \ldots, d+1$$

$$\xi_1^2 + \ldots + \xi_{d+1}^2 = 1$$

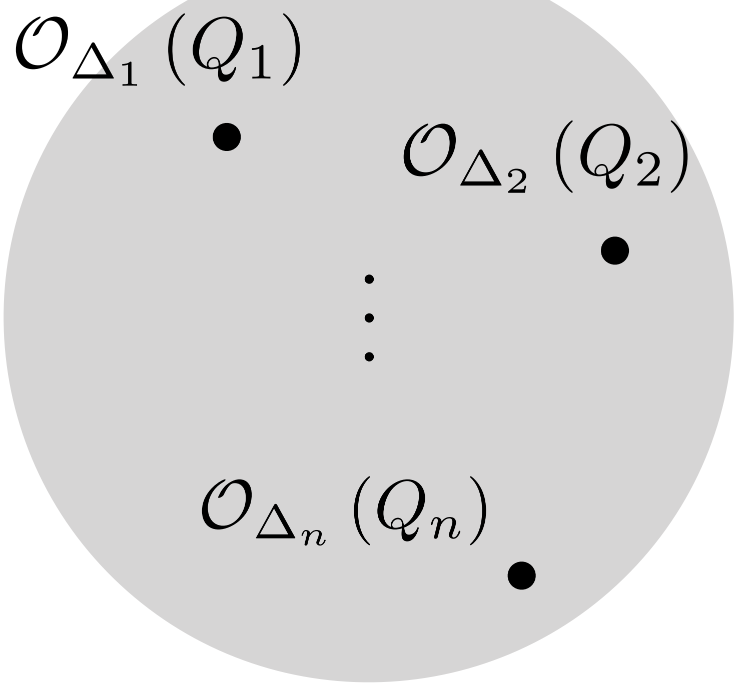
d-dimensional Celestial sphere

$SO\left(d+1, 1\right)$ acts on the celestial sphere as the Euclidean conformal group!

Minkowski boundary correlators

[C.S. and M. Taronna '23]

Radial Mellin transform of Minkowski correlators implements a radial reduction onto the hyperbolic slicing:



$$= \prod_i \lim_{\hat{X}_i \to Q_i} \underbrace{\int_0^\infty \frac{dt_i}{t_i}\, t_i^{\Delta_i}}_{\text{Mellin transform}} \left\langle \phi_1(\underset{\uparrow\,\text{radial coordinate}}{t_1} \overset{\text{Hyperbolic coordinate}\,\downarrow}{\hat{X}_1}) \ldots \phi_n(t_n \hat{X}_n) \right\rangle$$

Celestial correlators then arise in the boundary limit $\hat{X}_i \to Q_i$!

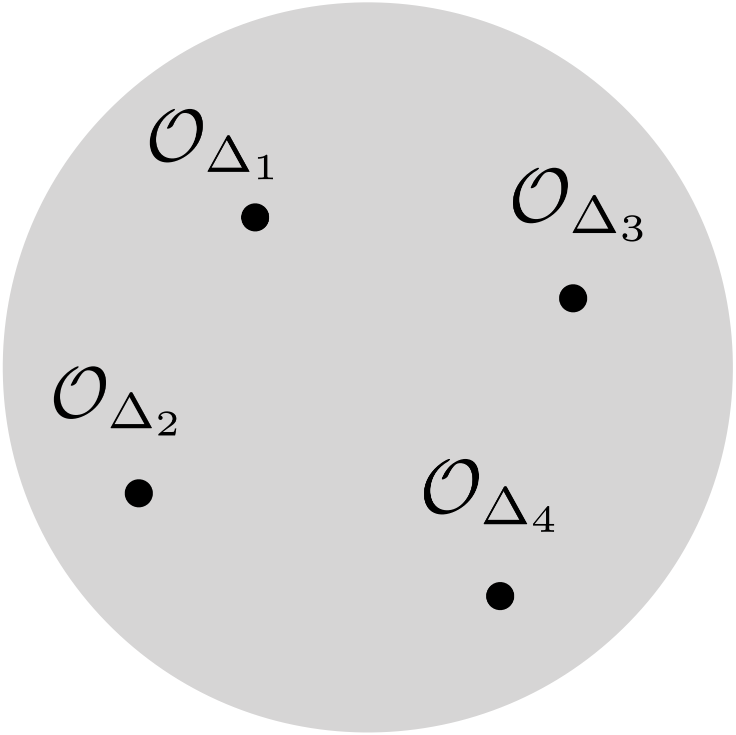
Radial Mellin Transform for both massless and massive particles in Minkowski

S-matrix is what we measure in experiments but AdS/CFT puts bulk correlators at the center

From the Celestial Sphere to EAdS

[C.S. and M. Taronna ’23; L. Iacobacci, C.S. and M. Taronna ’24]

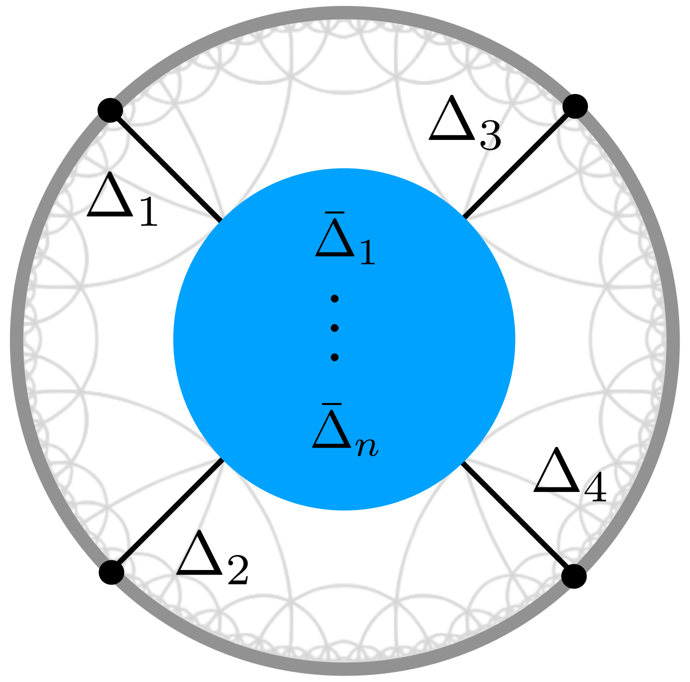
In general, for exchanges of particles of mass $m_i, \quad i = 1, \ldots, n$



Process with M vertices

Unitary Principal Series representations of SO(d+1,1)

$$= \overbrace{\int_{\frac{d}{2}-i\infty}^{\frac{d}{2}+i\infty}} \frac{d\bar{\Delta}_1}{2\pi i} \cdots \frac{d\bar{\Delta}_n}{2\pi i} \underbrace{C_{\bar{\Delta}_1\ldots\bar{\Delta}_n}\left(m_1, \ldots, m_n\right)}$$



Minkowski exchanges are a *continuum* of EAdS exchanges

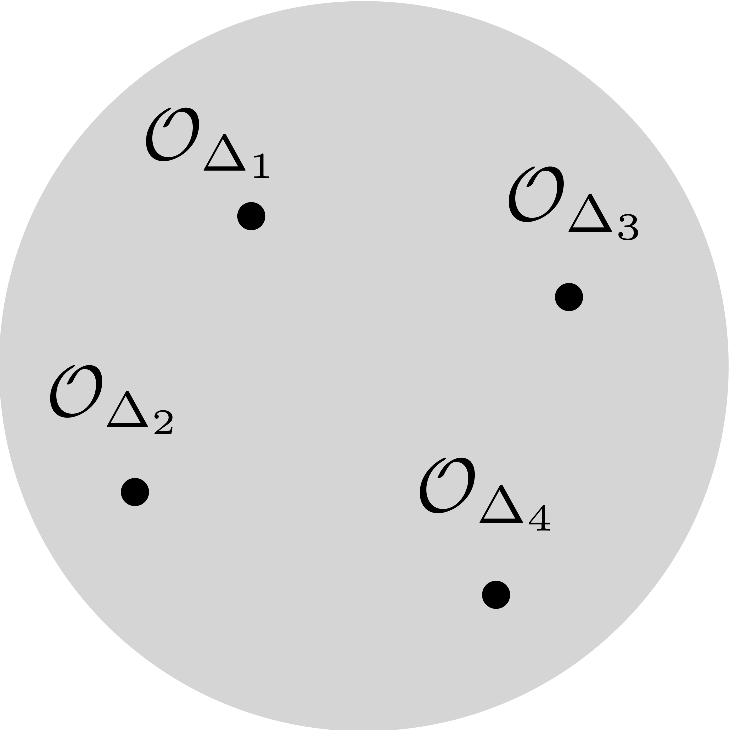
factorises into:

$$C_1^{\text{contact}} \times \ \ldots \ \times C_M^{\text{contact}}$$

From the Celestial Sphere to EAdS

[C.S. and M. Taronna ’23; L. Iacobacci, C.S. and M. Taronna ’24]

In general, for exchanges of particles of mass $m_i, \quad i = 1, \ldots, n$

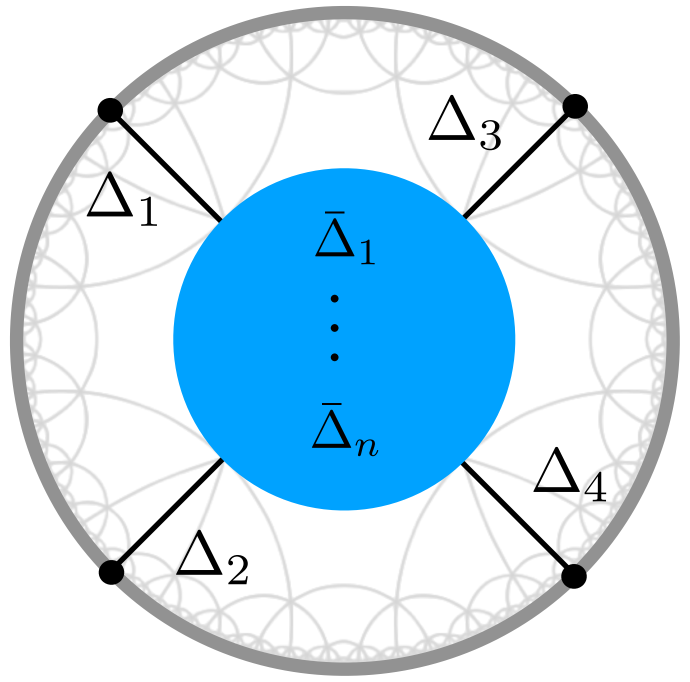


Process with M vertices

Unitary Principal Series representations of SO(d+1,1)

$$= \int_{\frac{d}{2}-i\infty}^{\frac{d}{2}+i\infty} \frac{d\bar{\Delta}_1}{2\pi i} \cdots \frac{d\bar{\Delta}_n}{2\pi i} \, C_{\bar{\Delta}_1 \ldots \bar{\Delta}_n}\left(m_1, \ldots, m_n\right)$$

Minkowski exchanges are a *continuum* of EAdS exchanges

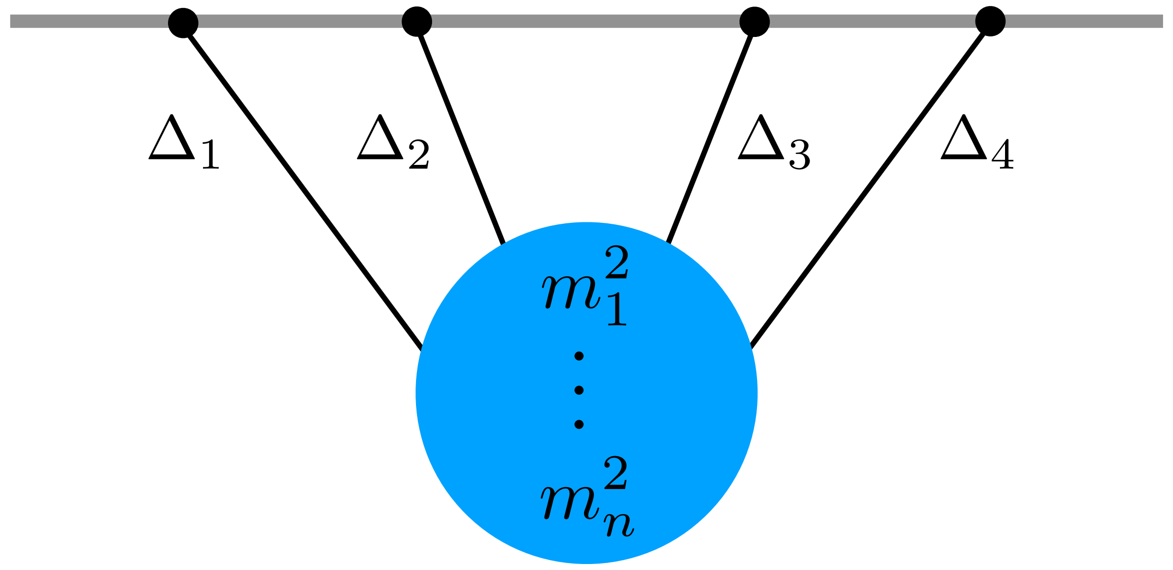


factorises into:

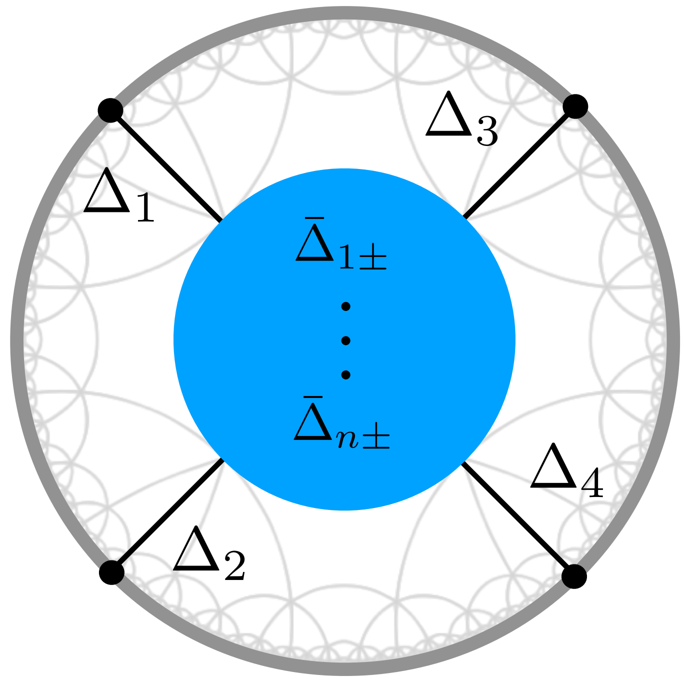
$$C_1^{\text{contact}} \times \ldots \times C_M^{\text{contact}}$$

Makes manifest conformal symmetry

Compare with de Sitter:



$$= \sum_{\bar{\Delta}_{1\pm} \ldots \bar{\Delta}_{n\pm}} C_{\bar{\Delta}_{1\pm} \ldots \bar{\Delta}_{n\pm}}$$



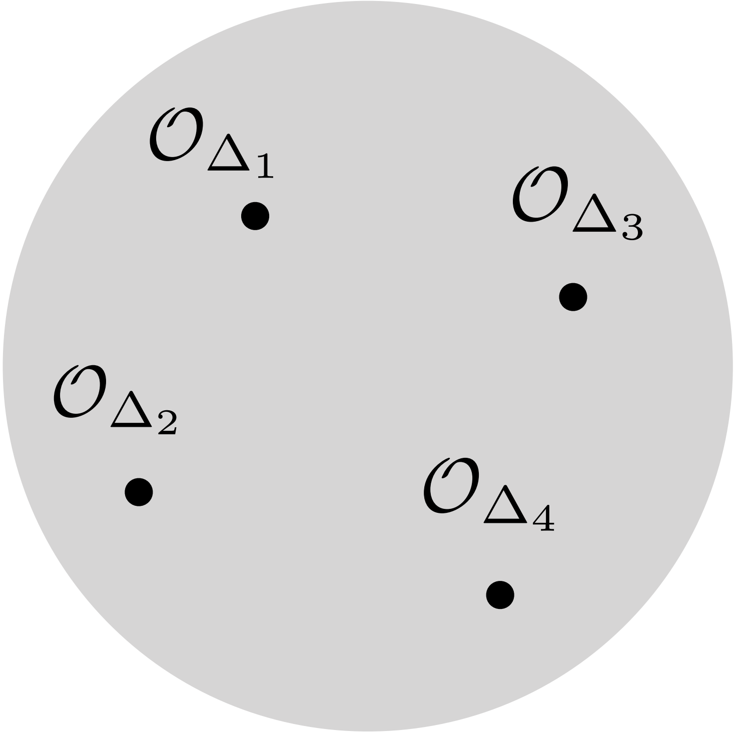
Process with M vertices

dS exchanges are a *discrete sum* of EAdS exchanges

From the Celestial Sphere to EAdS

[C.S. and M. Taronna ’23; L. Iacobacci, C.S. and M. Taronna ’24]

In general, for exchanges of particles of mass $m_i, \quad i = 1, \ldots, n$



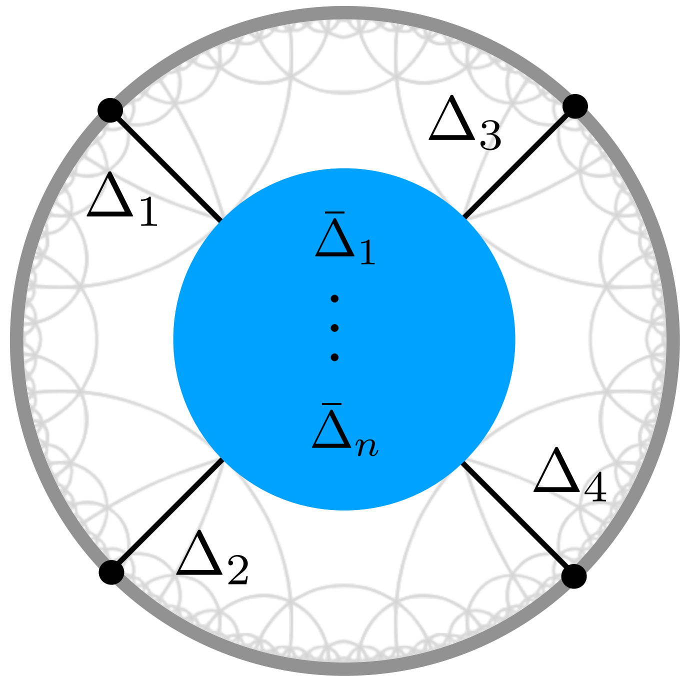
Process with M vertices

$$= \int_{\frac{d}{2}-i\infty}^{\frac{d}{2}+i\infty} \frac{d\bar{\Delta}_1}{2\pi i} \cdots \frac{d\bar{\Delta}_n}{2\pi i} \underbrace{C_{\bar{\Delta}_1 \ldots \bar{\Delta}_n}(m_1, \ldots, m_n)}$$

Minkowski exchanges are a *continuum* of EAdS exchanges

factorises into:

$C_1^{\text{contact}} \times \ldots \times C_M^{\text{contact}}$



Makes manifest conformal symmetry

Comments:

- Relation to definition [Pasterski, Shao, Strominger ’17] of celestial correlators as scattering amplitudes in a conformal basis?

[Pasterski, Shao, Strominger ’17] = LSZ ([Sleight, Taronna ’23]) ?

- Celestial correlators defined as an extrapolation of bulk Minkowski correlators give a definition of celestial correlators for theories without an S-matrix.

What lessons can we draw from Minkowski CFT?

Some applications.

Perturbative OPE data

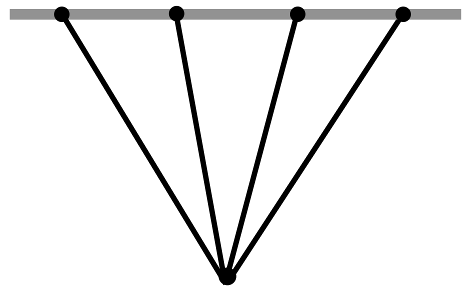
Perturbative OPE data on the boundary of dS and Minkowski space from EAdS

E.g. Composite operators on the boundary [C.S. and M. Taronna ’20]

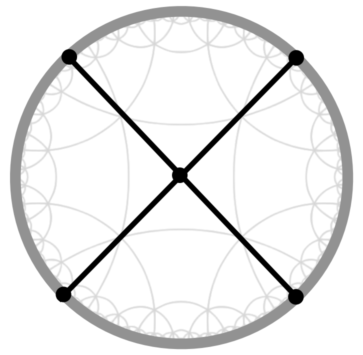
$$[\mathcal{O}\mathcal{O}]_{n,\ell} \sim \mathcal{O}\left(\partial^2\right)^n \partial_{i_1}\dots\partial_{i_\ell}\mathcal{O}+\dots$$

scaling dimension: $\Delta_{n,\ell} = \underbrace{2\Delta + 2n + \ell}_{\text{Free theory}} + \underbrace{\gamma_{n,\ell}}_{\text{anomalous dimension}}$

- $\gamma_{n,\ell}$ induced by bulk ϕ^4 contact diagram in dS:

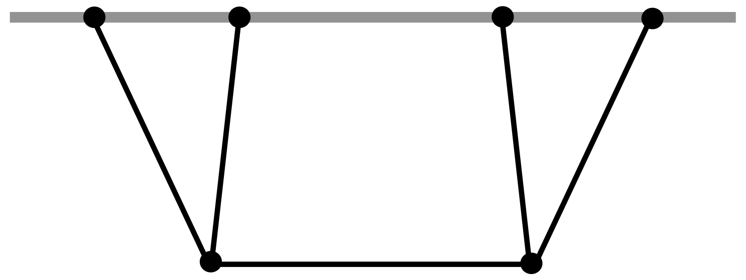


$= \sin\left(-\frac{d}{2}+2\Delta\right)\pi$

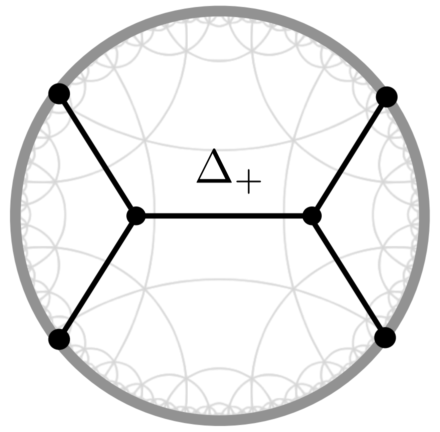


$$\longrightarrow \quad \gamma^{\phi^4}_{n,\ell} = \sin\left(-\frac{d}{2}+2\Delta\right)\pi \times {}^{(\text{EAdS})}\gamma^{\phi^4}_{n,\ell}$$

- $\gamma_{n,\ell}$ induced by an exchange diagram in dS:



$= \sin\left(\frac{-d+2\Delta+\Delta_+}{2}\right)\pi \, \sin\left(\frac{-d+2\Delta+\Delta_+}{2}\right)\pi$



$+ \quad (\Delta_+ \to \Delta_-)$

$$\longrightarrow \quad \gamma^{\phi^3\,\text{exch}}_{n,\ell} = \sin\left(\frac{-d+2\Delta+\Delta_+}{2}\right)\pi \, \sin\left(\frac{-d+2\Delta+\Delta_+}{2}\right)\pi \times {}^{(\text{EAdS})}\gamma^{\phi^3\,\text{exch}\,\Delta_+}_{n,\ell} + (\Delta_+ \to \Delta_-)$$

Perturbative OPE data

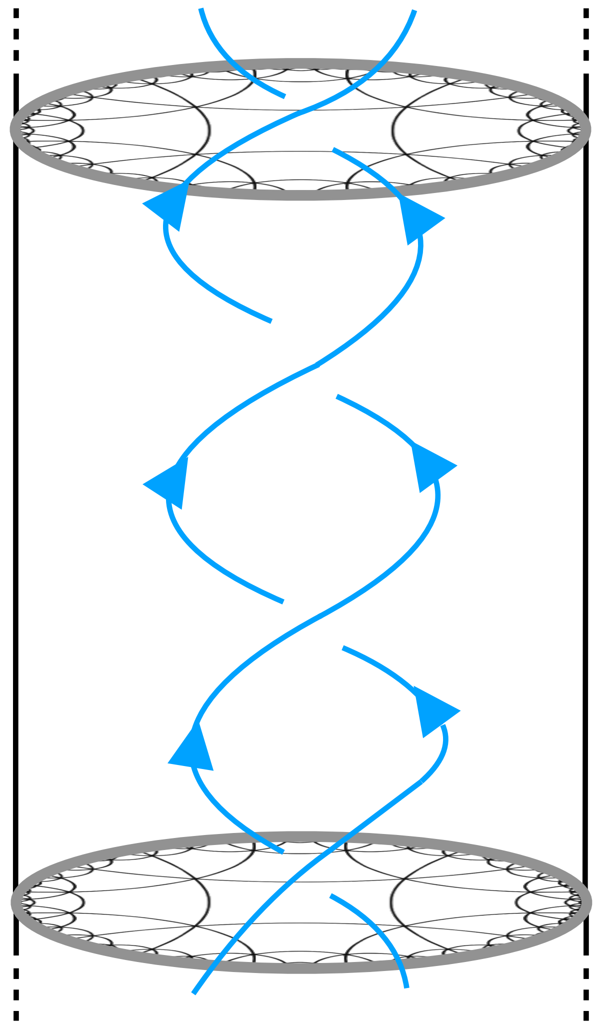
Perturbative OPE data on the boundary of dS and Minkowski space from EAdS

E.g. Composite operators on the boundary

$$[\mathcal{O}\mathcal{O}]_{n,\ell} \sim \mathcal{O}\left(\partial^2\right)^n \partial_{i_1} \ldots \partial_{i_\ell} \mathcal{O} + \ldots$$

scaling dimension: $$\Delta_{n,\ell} = \underbrace{2\Delta + 2n + \ell}_{\text{Free theory}} + \underbrace{\gamma_{n,\ell}}_{\text{anomalous dimension}}$$

AdS

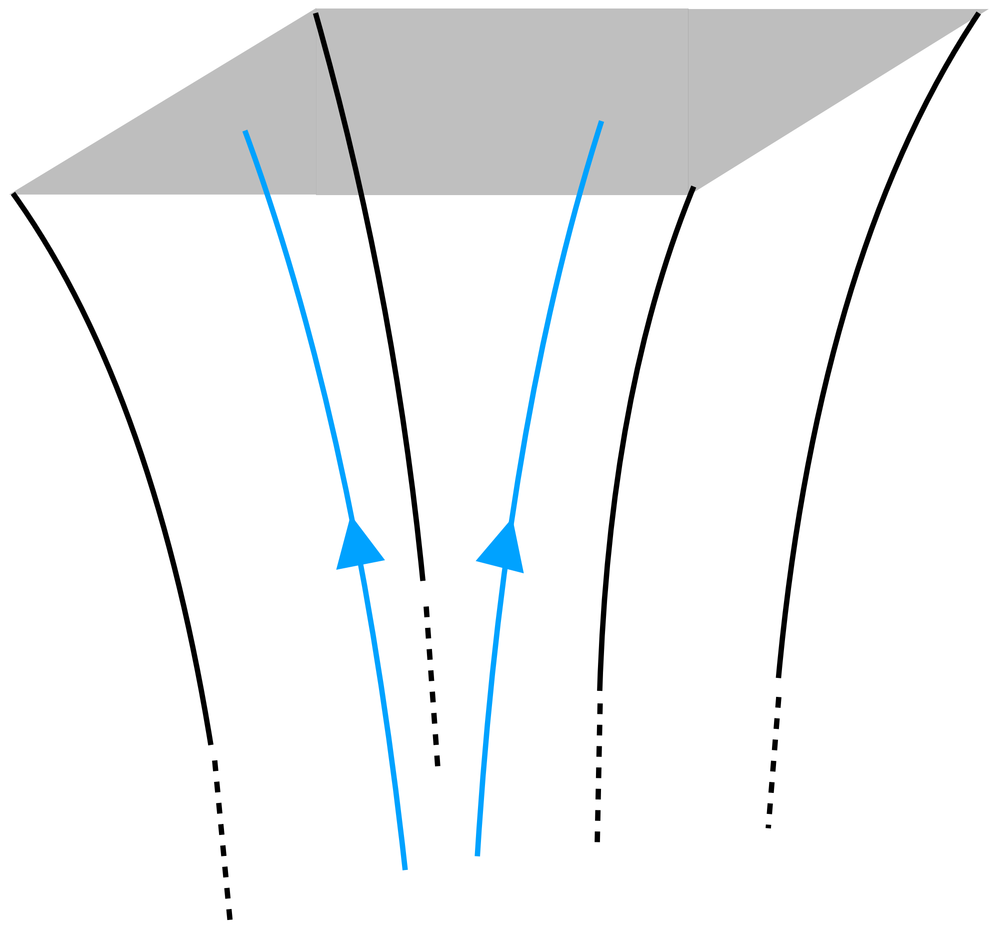


$\Delta_{n,\ell}$ is unitary

⟶ stable particle (bound state)

vs.

dS



$\Delta_{n,\ell}$ is (generally) non-unitary

⟶ resonance

[Bros et al ’06-’09, Marolf and Morrison ’10, Krotov and Polyakov ’10, Arkani-Hamed and Maldacena ’15]

Conformal Partial Wave Expansion

[Sleight, Taronna ’20] [Hogervorst, Penedones, Vaziri ’21] [di Pietro, Komatsu, Gorbenko '21]

Perturbative dS and celestial correlators have a similar analytic structure to those in AdS.

⟶ Like in AdS they admit a conformal partial wave expansion

$$\langle \mathcal{O}(\mathbf{x}_1)\,\mathcal{O}(\mathbf{x}_2)\,\mathcal{O}(\mathbf{x}_3)\,\mathcal{O}(\mathbf{x}_4)\rangle = \sum_{J=0}^{\infty}\int_{\frac{d}{2}-i\infty}^{\frac{d}{2}+i\infty}\frac{d\Delta}{2\pi i}\,\overset{\substack{\text{Spectral density, meromorphic in }\Delta\\ \downarrow}}{\rho_J}(\Delta)\;\underbrace{\mathcal{F}_{\Delta,J}(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3,\mathbf{x}_4)}_{\text{Conformal Partial Wave}}$$

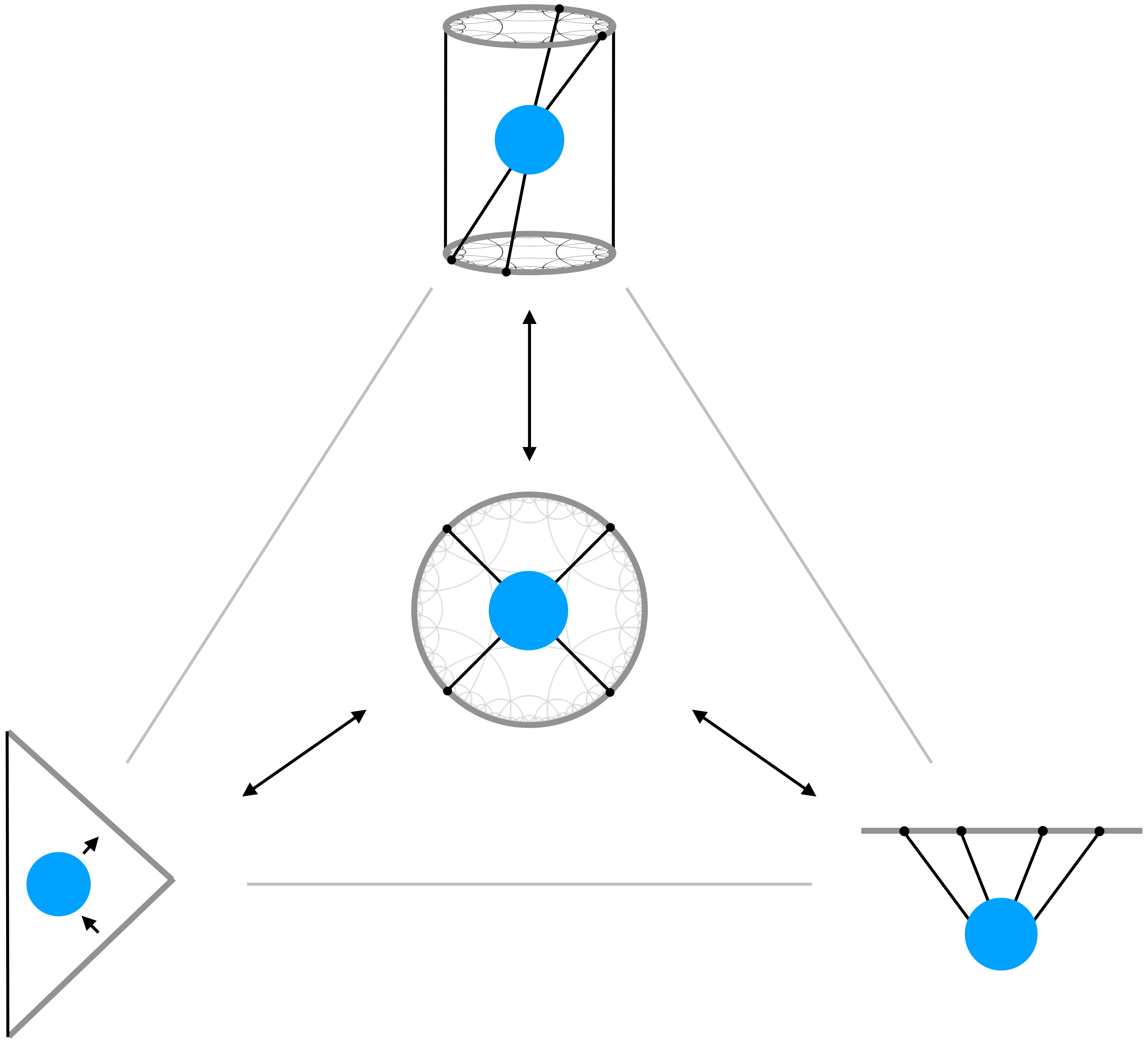
The spectral function has to be positive as prescribed by SO(d+1,1) Unitarity (This is EAdS, not AdS, unitarity)

Unitarity: $\rho_J(\Delta) \geq 0$ + crossing ⟶ Bootstrap for Euclidean CFTs?

Cf. Lorentzian CFT:

$$\langle \mathcal{O}(\mathbf{x}_1)\,\mathcal{O}(\mathbf{x}_2)\,\mathcal{O}(\mathbf{x}_3)\,\mathcal{O}(\mathbf{x}_4)\rangle = \sum_{\Delta,J}^{\infty} C^2_{\Delta,J}\,\underbrace{G_{\Delta,J}(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3,\mathbf{x}_4)}_{\text{Conformal Block}}$$

Unitarity: $C^2_{\Delta,J} \geq 0$ + crossing ⟶ Conformal Bootstrap



Thank you.