Cosmological Correlators from Flat Space Feynman Graphs

Sadra Jazayeri

courtesy: quanta magazine/Planck 2018

work in progress with Sebastián Céspedes

earlier related papers (2022,2023) with

S Renaux-Petel (IAP), D Werth(IAP)

See also Raffaele's talk
on (Marotto, Skenderis, Verma 2024)

Imperial College London

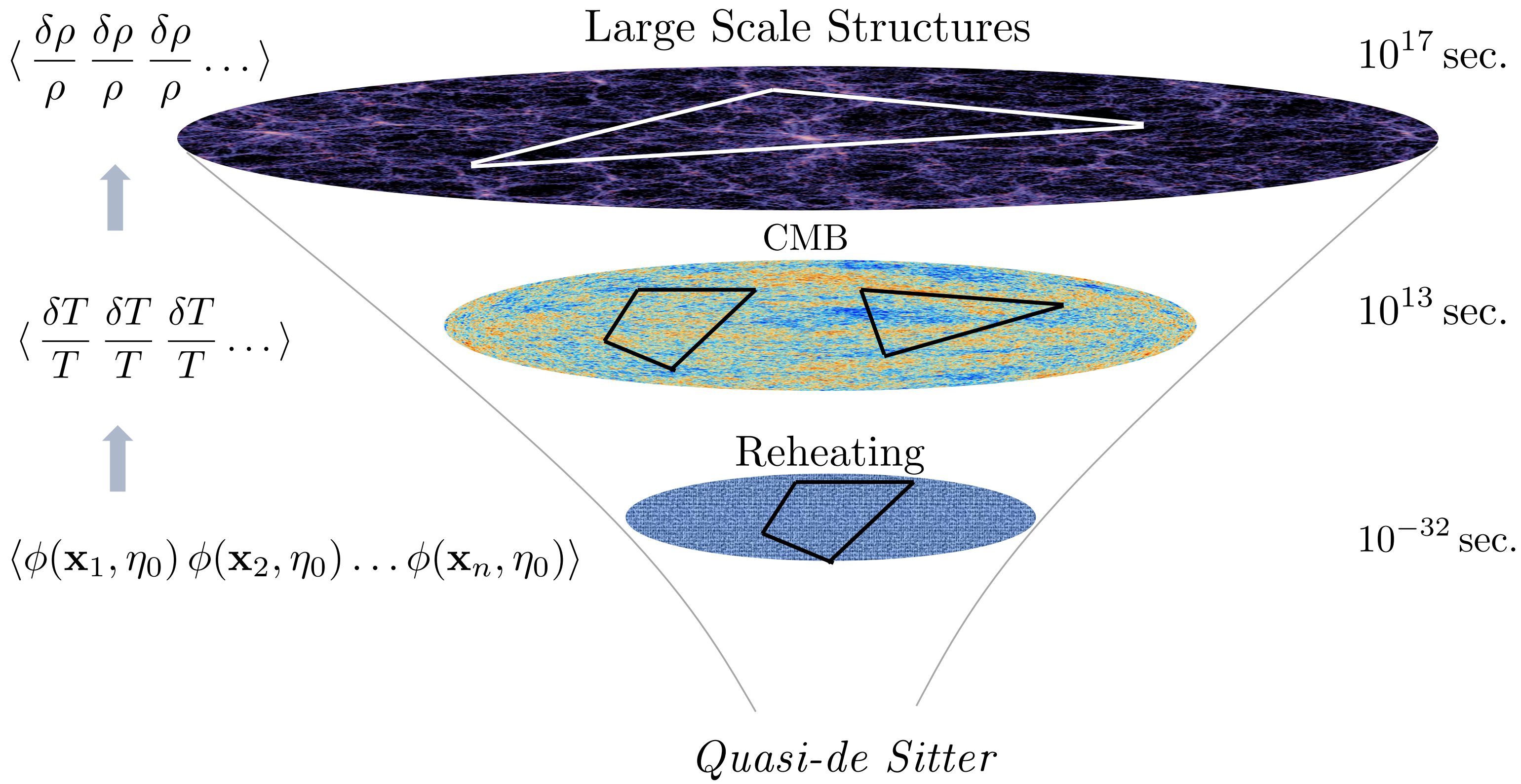
S F

Massive Flat Space Limit of Correlators

Application: Signatures of Massive Fields

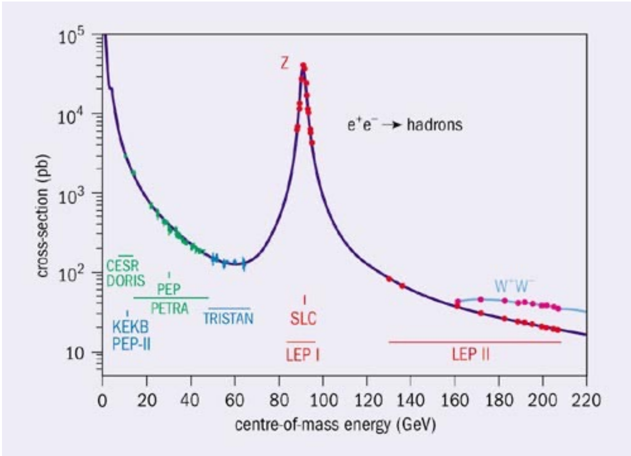
Speculations

- Cosmology gives us the unique opportunity to learn about fundamental physics by gazing at sky.

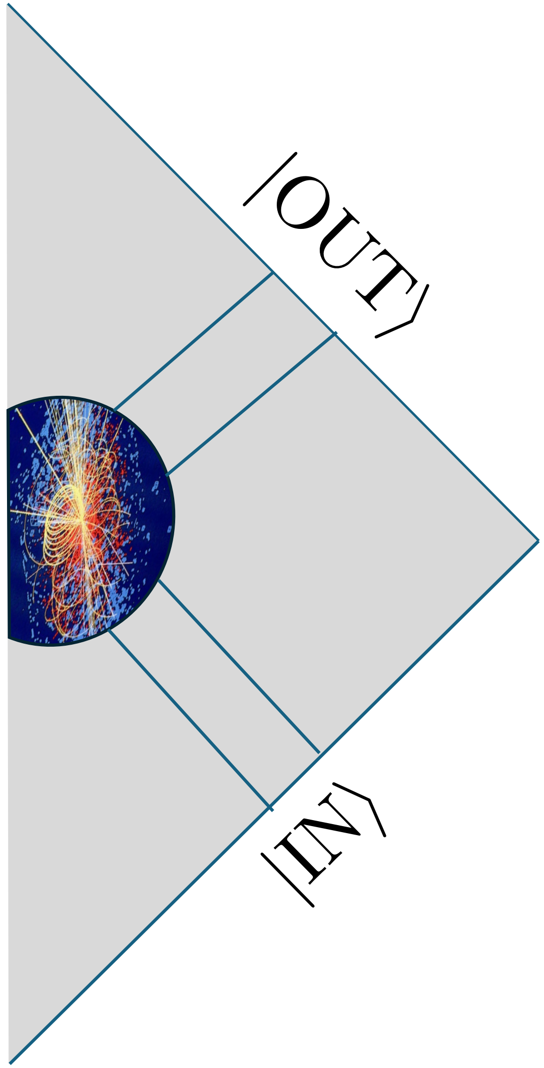


- New physics in flat space

$$ds^2 = -d\eta^2 + d\boldsymbol{x}^2$$

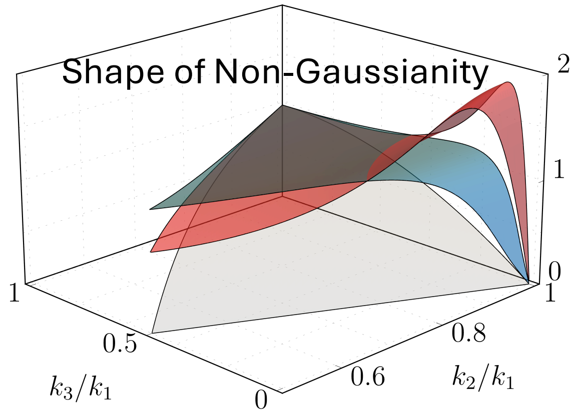


$$E_{\rm CM} \sim 10\,\text{Tev}$$



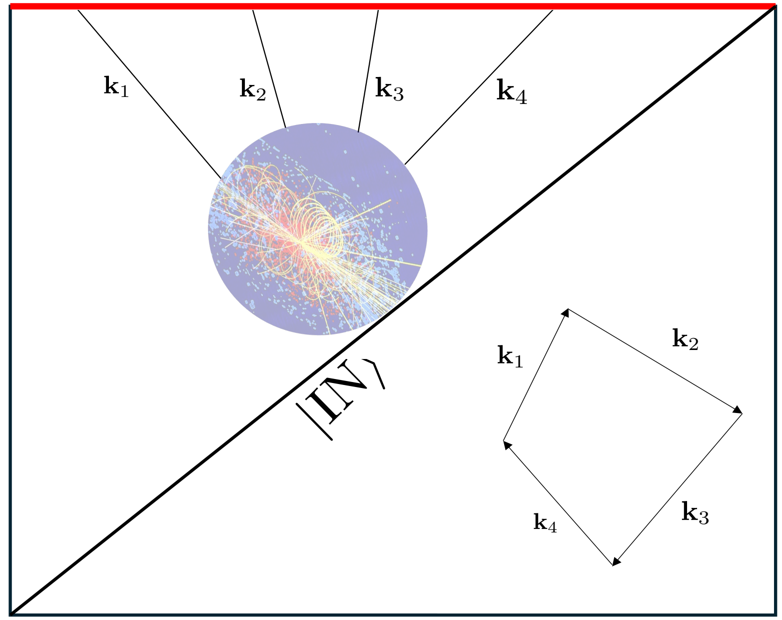
- New physics in cosmology

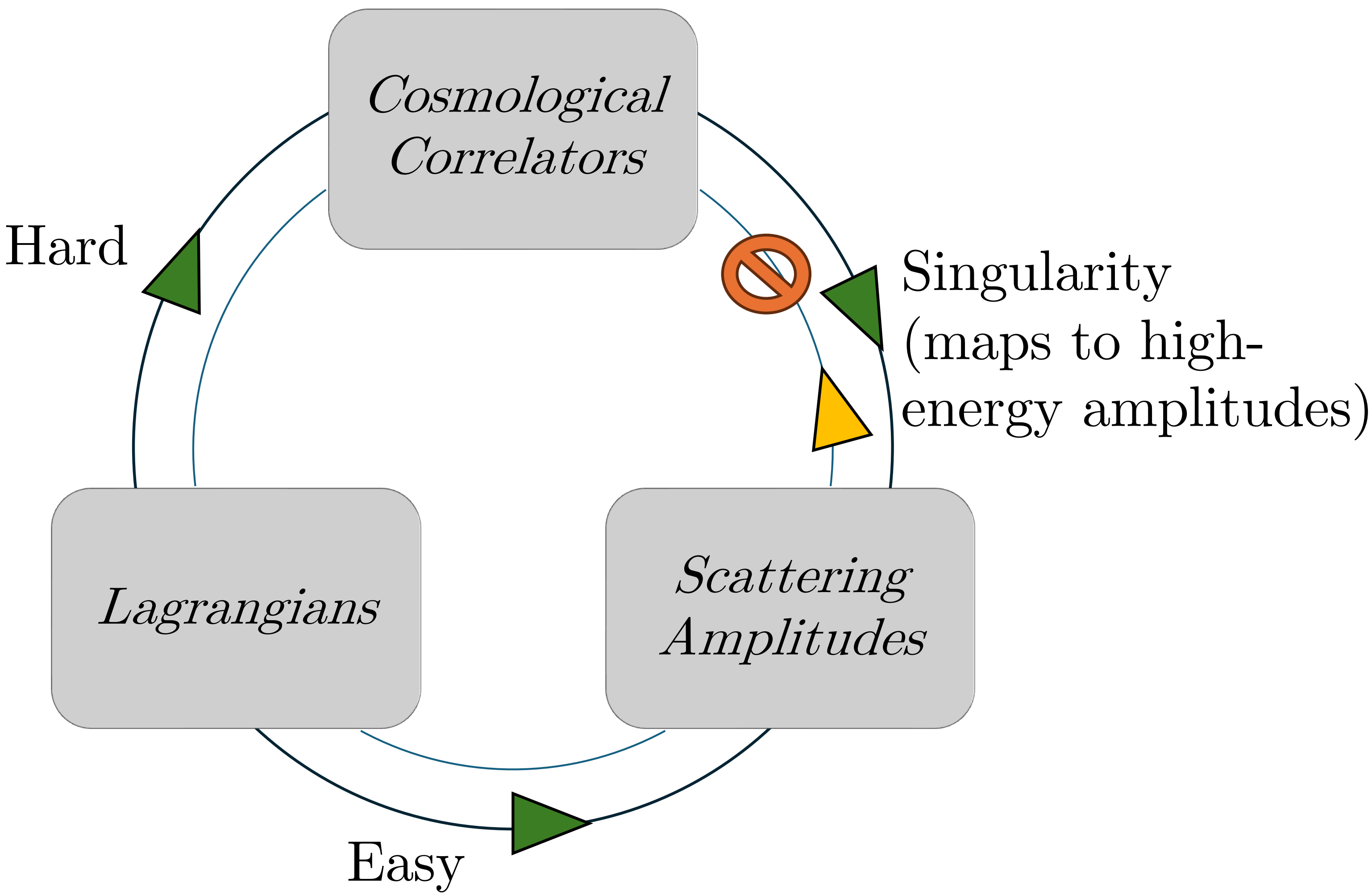
$$ds^2 = \frac{1}{\eta^2 H_I^2}(-d\eta^2 + d\boldsymbol{x}^2)$$

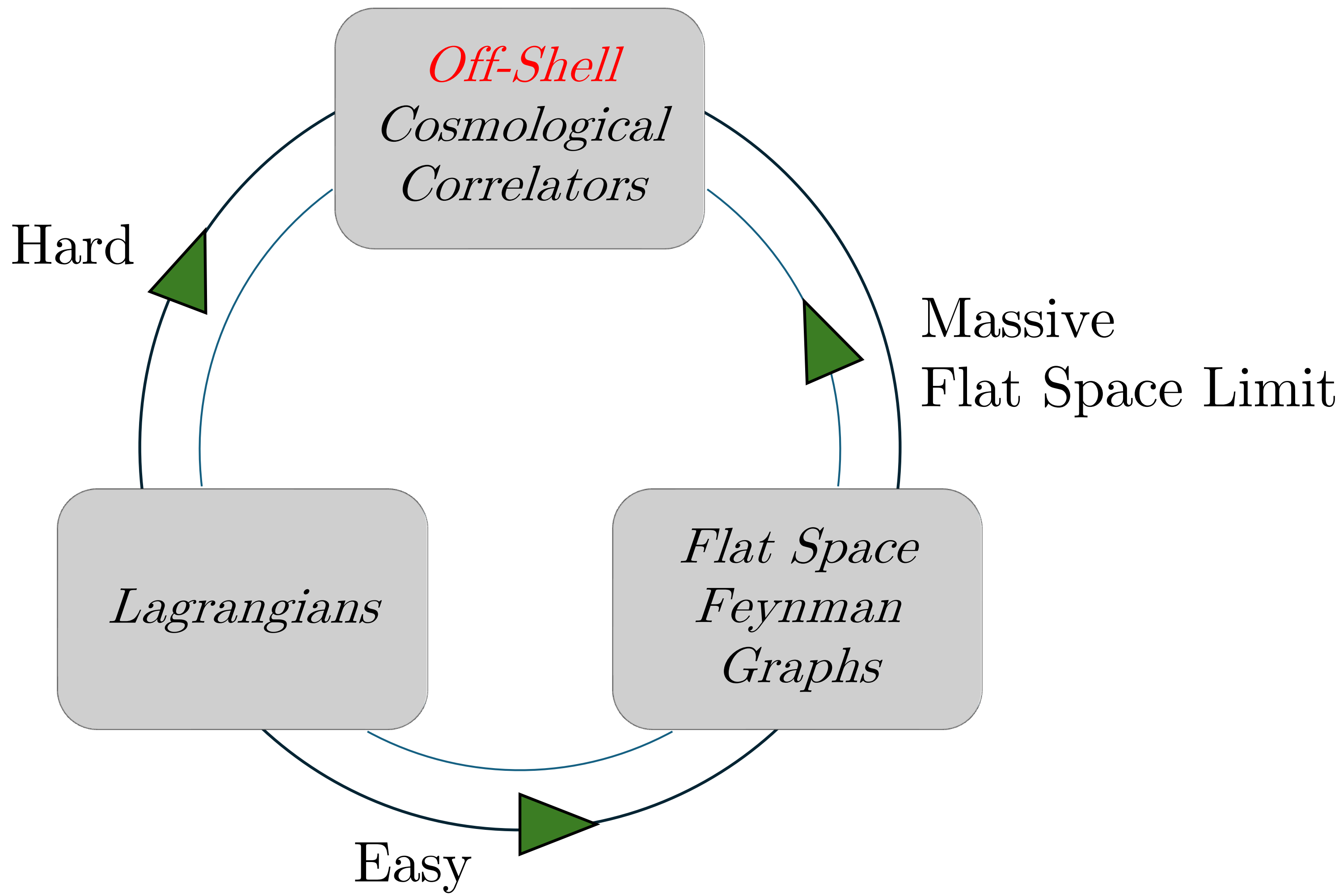


$$E_{\rm CM} \sim H_I \sim 10^{11}\text{Tev}$$

$$\langle \text{IN}|\phi(\mathbf{k}_1)\phi(\mathbf{k}_2)\phi(\mathbf{k}_3)\phi(\mathbf{k}_4)|\text{IN}\rangle$$



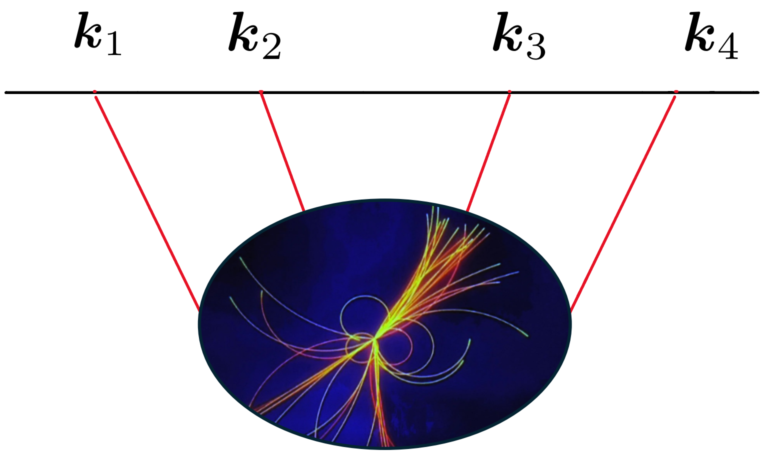

Cosmological Correlators
Hard
Singularity (maps to high-energy amplitudes)
Lagrangians
Scattering Amplitudes
Easy



Off-Shell
Cosmological
Correlators
Hard
Massive
Flat Space Limit
Lagrangians
Flat Space
Feynman
Graphs
Easy

Massive Flat Space Limit of Correlators

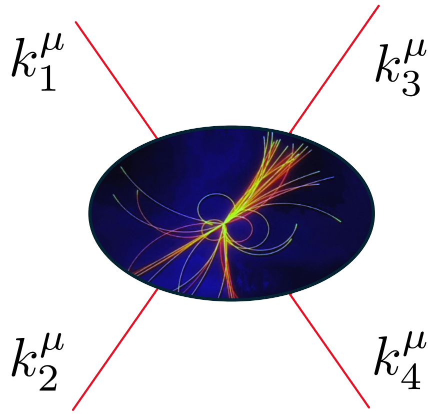
- Scattering amplitudes are encoded in the total energy singularity of de Sitter Witten graphs



$$\boldsymbol{k}_a = k_a\,\hat{k}_a$$

$$\Big\langle \phi(\boldsymbol{k}_1)\phi(\boldsymbol{k}_2)\phi(\boldsymbol{k}_2)\phi(\boldsymbol{k}_4) \Big\rangle$$

$$\supset$$



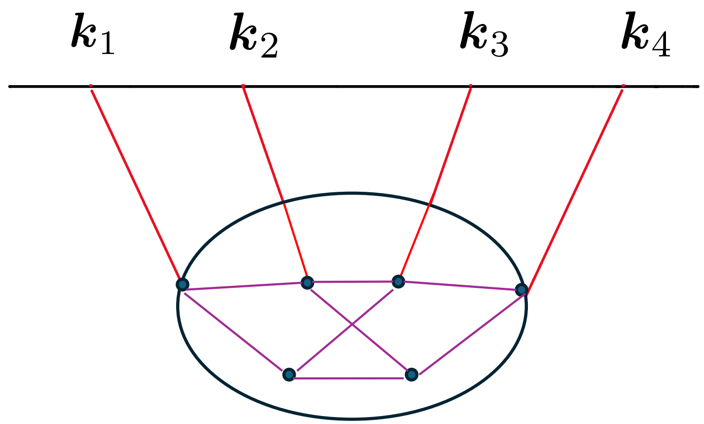
$$k_a^\mu = (k_a, \boldsymbol{k}_a)$$

$$\mathcal{A}(k_1^\mu, k_2^\mu, k_3^\mu, k_4^\mu)$$

- Scattering amplitudes are encoded in the total energy singularity of de Sitter Witten graphs

$$k_T = \sum_{a=1}^{n} k_a \to 0$$

$$\Big\langle \phi(\boldsymbol{k}_1)\phi(\boldsymbol{k}_2)\phi(\boldsymbol{k}_2)\phi(\boldsymbol{k}_4) \Big\rangle \;\propto \frac{1}{k_T^{\delta}} \;\; \mathcal{A}(k_1^{\mu}, k_2^{\mu}, k_3^{\mu}, k_4^{\mu})$$

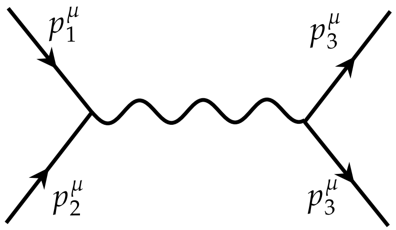


High-Energy Limit of the Amplitude (i.e. zero mass)

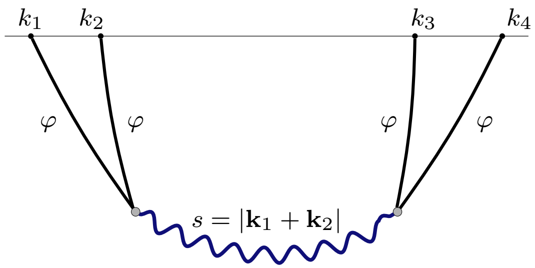
The Degree of Divergence

$$\delta = 1 + \sum_{i=1}^{V} (\Delta_V - 4)$$

- Is there a direct relation between *correlators* and *massive* amplitudes?



$$\mathcal{A} = \frac{1}{-(k_1+k_2)^{+}(\boldsymbol{k}_1+\boldsymbol{k}_2)^2 + m^2} + (t-u) \text{ channels}.$$

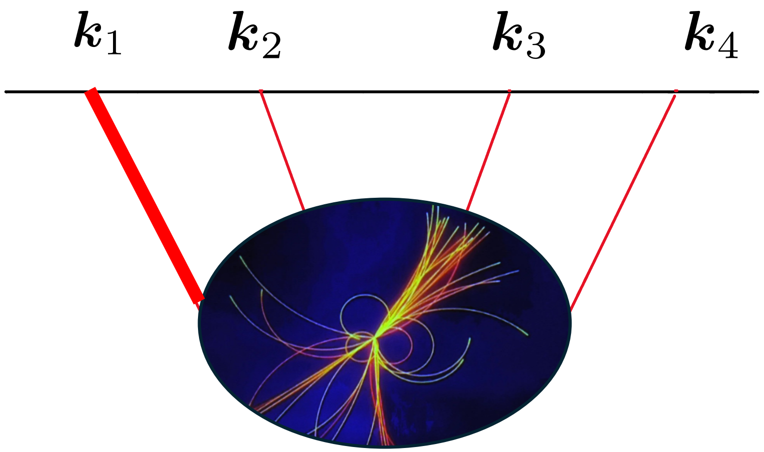


$$F_4(k_1, k_2, k_3, k_4, |\boldsymbol{k}_1 + \boldsymbol{k}_2|; \frac{m}{H}) + (t-u) \text{ channels}.$$

- Is there a direct relation between *correlators* and *massive* amplitudes?

off-shell

off-shell graphs in flat space



$$\phi_{\pm}(k_1 = |\boldsymbol{k}_1|, \eta) \to \phi_{\pm}(\omega_1, \eta)$$

$$\omega_1^2 \neq \boldsymbol{k}_1.\boldsymbol{k}_1$$

$$F_4(k_1, k_2, k_3, k_4, \underbrace{\{\boldsymbol{k_a}\}}_{\text{Internal Momenta/Energies}}) \to F_4(\omega_1, \underbrace{\omega_2, \omega_3, \omega_4, \{\boldsymbol{k_a}\}}_{\text{External Energies}})$$

- *Definition:* **The massive flat-space limit** (MFS) of an off-shell Witten graph with heavy internal lines is

$$\frac{M}{H} \to \infty\,, \qquad \omega_a \to 0\,, \qquad \text{whilst} \qquad \frac{M}{H} \times \omega_a = \text{finite}\,.$$

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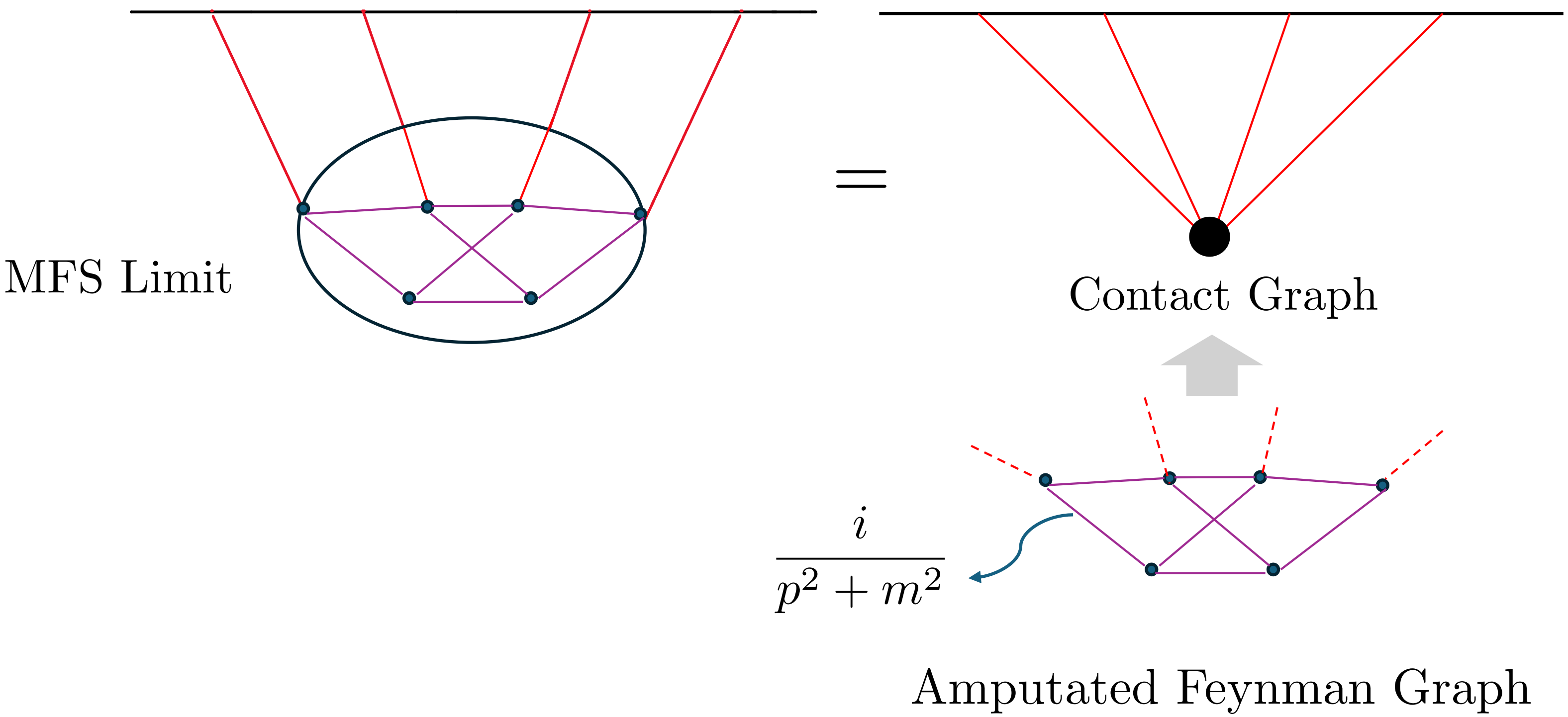
- *Motivation: Correlators of the scalar fluctuations during inflation*

$$\omega_i = c_s|\boldsymbol{k}_i| \qquad c_s \to 0\,, \qquad \frac{M}{H} \to \infty\,, \qquad c_s \times \frac{M}{H} = \text{fixed}\,.$$

- *Theorem:* The MFS limit of a Witten graph is entirely fixed by the same graph in flat space with **amputated external legs.**



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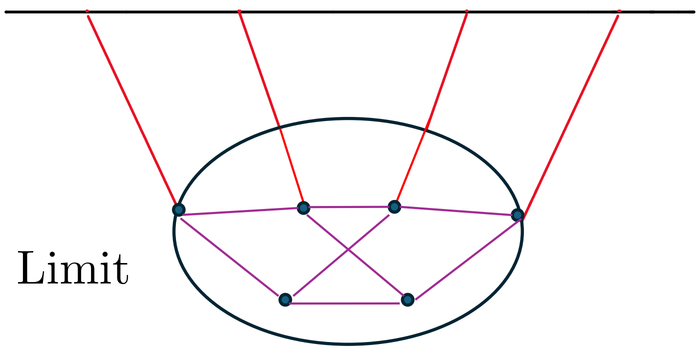
MFS Limit



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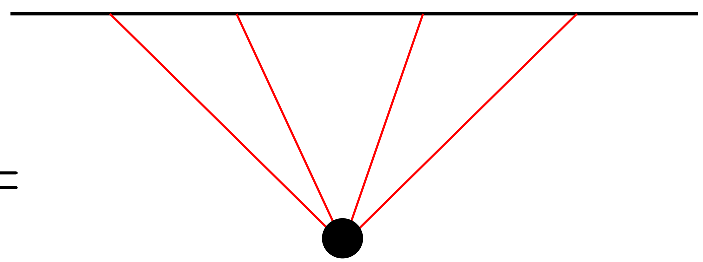


Contact Graph

$$F_n^{\mathrm{MFS}}\Big(\frac{M}{H}\omega_i, \boldsymbol{k}_i; M\Big) = \mathrm{Im}\int_{-\infty}^{0} \frac{d\eta}{\eta^4} G_n[\underbrace{p_1^\mu(\eta)}, \dots, p_n^\mu(\eta); M]\,\phi_+(\omega_1,\eta)\dots\phi_+(\omega_n,\eta)\,.$$

$$p_i^\mu(\eta) = \Big(0, \frac{\boldsymbol{k}_i}{a}\Big) \qquad \text{with } a = -\frac{1}{\eta H}$$

- *Sketch of The Proof:*

Building Block I.

$$\frac{1}{M} \ll \frac{1}{H_I} \qquad \exp(iS_{\text{eff}}(\phi)) = \underbrace{\int}\mathcal{D}\sigma\ \exp(iS_{\text{eff}}(\phi,\sigma))$$

Do the path integral as if you were
In flat space (instead of Schwinger-Keldysh)

Building Block II.

$$F_n = \int d\eta_1 \ldots d\eta_V\ \underbrace{e^{i\omega_1\eta_1}\ldots e^{i\omega_V\eta_V}} \times \ldots$$

Zero energy limit picks up the UV limit $\eta_i \to -\infty$
Such that the kinetic energy $|\boldsymbol{k}_i\eta_j|$
is comparable to the mass

- Particle production vanishes in this limit.
- Only "++" propagators contribute to the correlators.
- Explicitly checked for tree-Level and one-Loop bubble diagrams.

Signatures Of Massive Fields

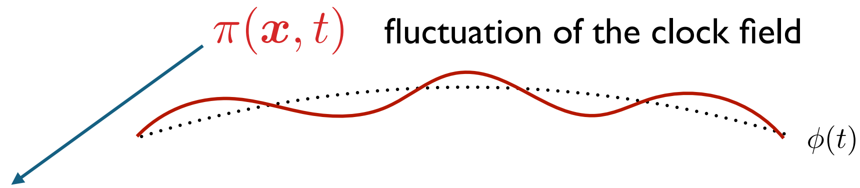
- During inflation time translation symmetry is spontaneously broken. The long wavelength fluctuations of the system can be described with the associated Goldstone boson (aka phonon)

$$U(1) \times \mathrm{ISO}(3,1) \to U_{\mathrm{diag.}}(1) \times \mathrm{ISO}(3)$$

$$\langle \hat{\phi} \rangle = \dot{\bar{\phi}}\, t \qquad\qquad \phi = \dot{\bar{\phi}}\,(t + \pi(t, \mathbf{x}))$$

$$S = \int d^3x\, dt\, a^3\, \frac{M_P^2 |\dot{H}|}{c_s^2} \left[\dot{\pi}^2 - c_s^2 (\tilde{\partial}_i \pi)^2 + (1 - c_s^2) \dot{\pi} (\tilde{\partial}_i \pi)^2 + \dots \right]$$

$$\tilde{\partial}_i = \frac{1}{a} \partial_i$$



scalar fluctuations during inflation

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$$f_{NL} \sim \frac{1}{c_s^2}$$

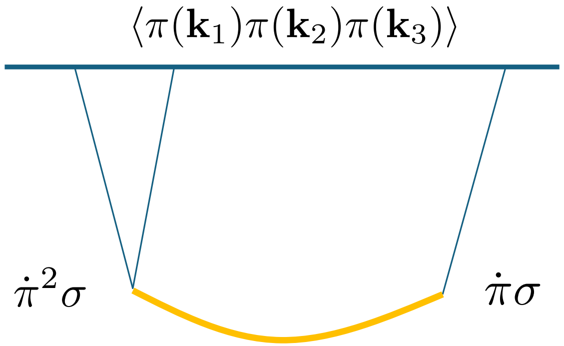
$$c_s \geq 0.021$$

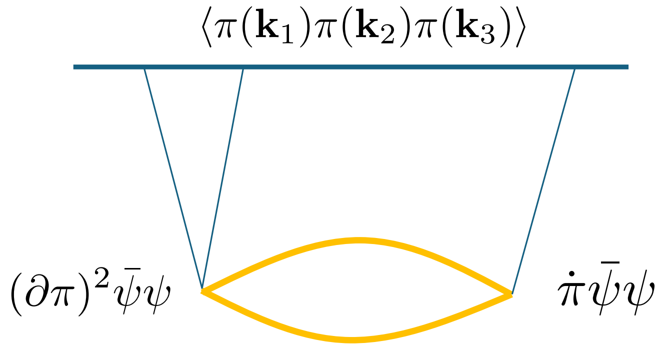
Planck 2018

$$\pi_c(k, \eta) = \frac{H}{\sqrt{2 c_s^3 k^3}} (1 - i c_s k \eta) \exp(i c_s k \eta)$$

- We can couple additional matter fields to the phonons

$$\mathcal{L}_{\text{eff}} = \left[-2\dot{\pi} + (\partial\pi)^2\right]^n \mathcal{O}_{\text{heavy}}$$



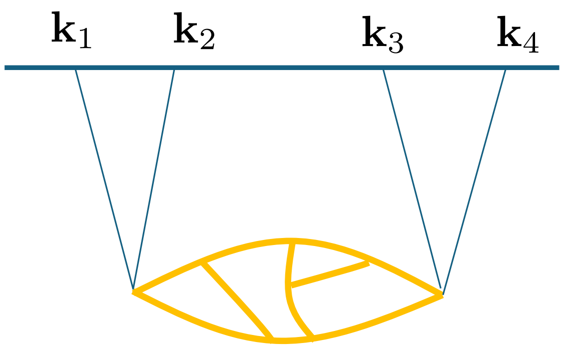


Cosmological Bootstrap

?

SJ, S Renaux-Petel 2022
G Pimentel, D Wang 2022

- Bubble-Diagrams Contribution to Four-Point



Feynman Loop Integrals

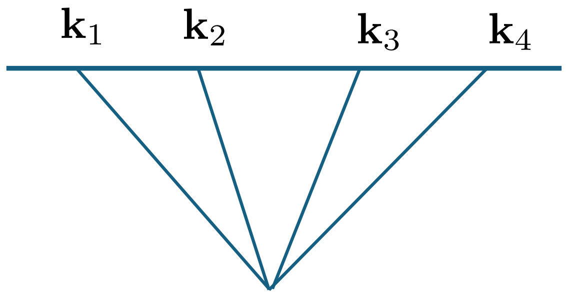
$\Pi(p^2)$

e.g. $\quad \mathcal{O}(\pi)\sigma \qquad \Pi(p^2) = \dfrac{i}{p^2 + m^2}$

$$\mathcal{O}(\pi)\sigma^2 \qquad \Pi(p) \equiv \int_0^1 dx \, \log\left[\frac{m^2 + x(1-x)p^2}{m^2 + 4x(1-x)\frac{\mu^2}{3}}\right]$$

- The general structure of the four-point function (up to weight-shifting operators)

$$F(k_1, k_2, k_3, k_4, s) = \text{Im} \int_{-\infty}^{0} d\eta \quad \Pi(s^2\eta^2)\, e^{ic_s k_T \eta}$$



$$s = |\mathbf{k}_1 + \mathbf{k}_2|$$

$$k_T = k_1 + k_2 + k_3 + k_4$$

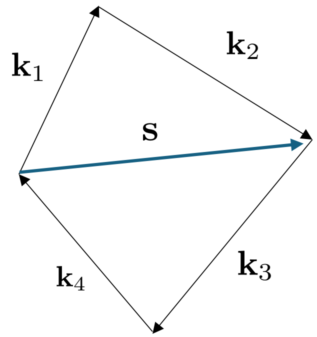
Melling transformation simplifies the computation a lot

$$\Pi(p^2) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} dz\, \tilde{\Pi}(z)\, (-p^2)^{-z/2} \,.$$

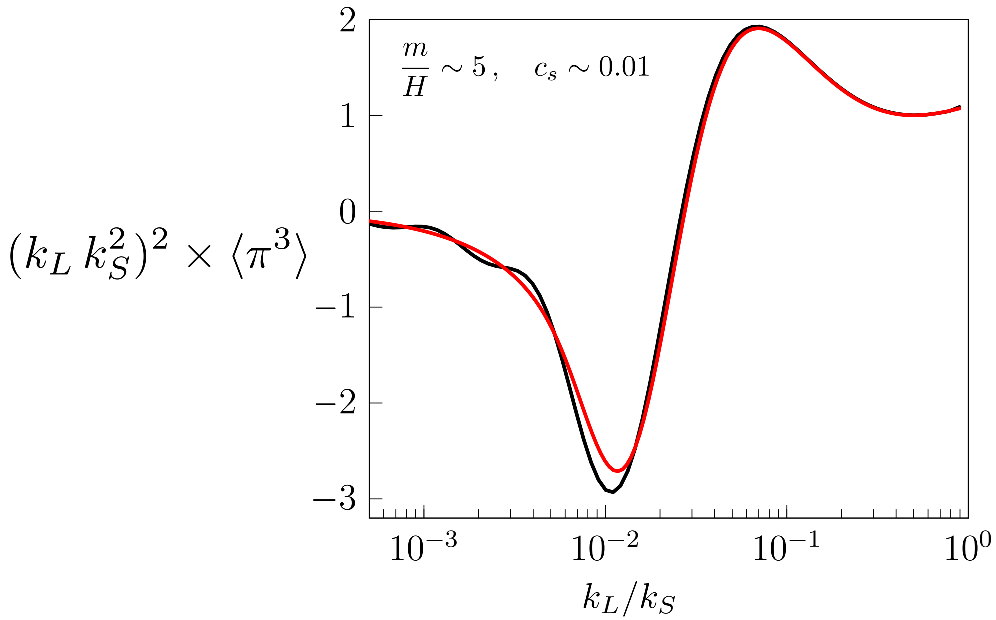
$$\int d(s\eta)\, (-s\eta)^{-z}\, e^{ic_s k_T \eta} = \left(\frac{ic_s k_T}{s}\right)^{z-1} \Gamma(1-z)$$

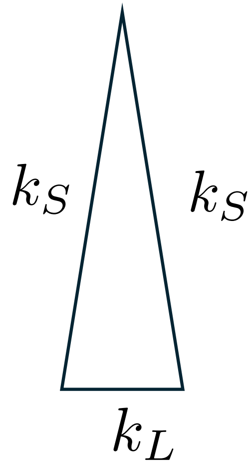
- Tree-level signal $\qquad \alpha = \frac{c_s m}{H} \lesssim 1$

$$F = -\frac{1}{s}\sum_{n=1}^{\infty}\frac{1}{(2n-1)!}\left(\frac{\alpha k_T}{s}\right)^{2n-1}\left[\log\left(\frac{\alpha k_T}{s}\right) - \psi^{(0)}(2n)\right]$$



$k_4 \to 0$ + Weight-Shifting $\hat{\mathcal{W}}$





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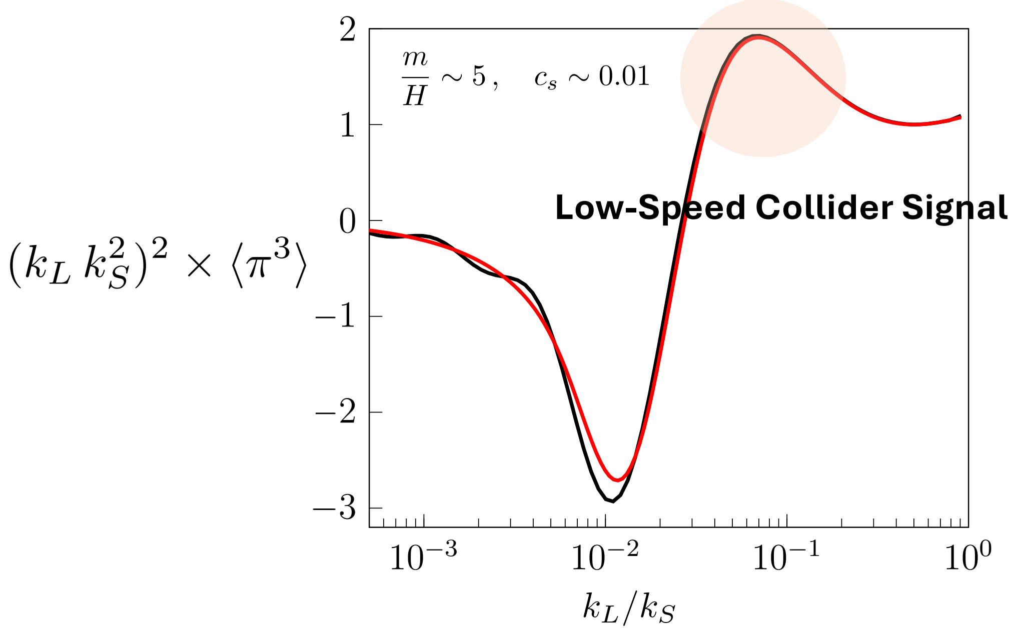


BOSS Galaxy data: Cabass et al 2024

CMB data: Sohn et al 2024

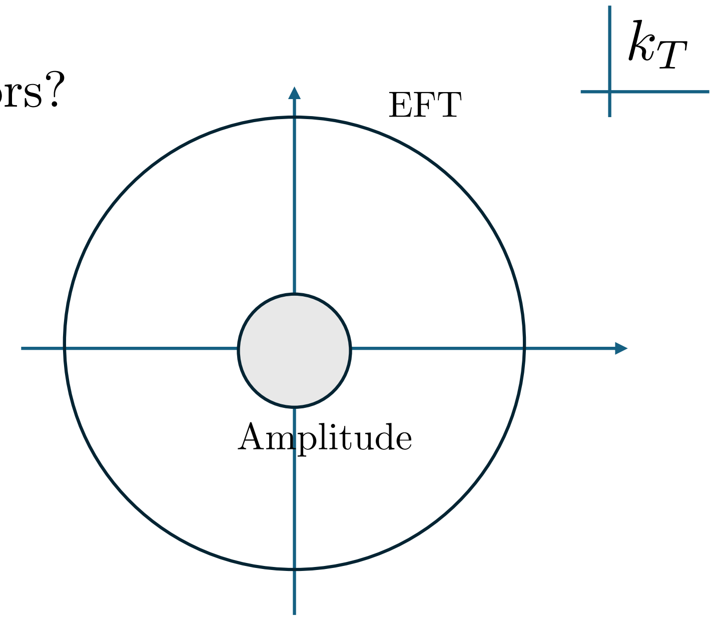
Speculations

- What happens in the massive flat space of *on-shell correlators*? What is its relation to scattering amplitudes? (particle production might become important unlike the off-shell case)

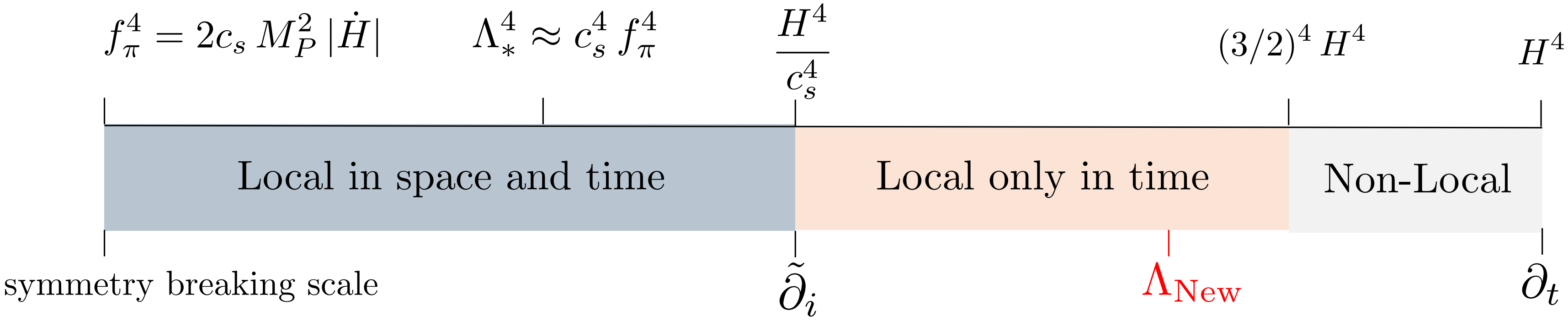
$$k_T \to 0\,, \quad m \to \infty\,, \qquad m \times k_T = \text{fixed}.$$

- Can we learn about positivity bounds on correlators?

Correlators EFT: $$F = \sum_m c_m \frac{\text{Poly}(\boldsymbol{k}_i)}{k_T^m}\,.$$

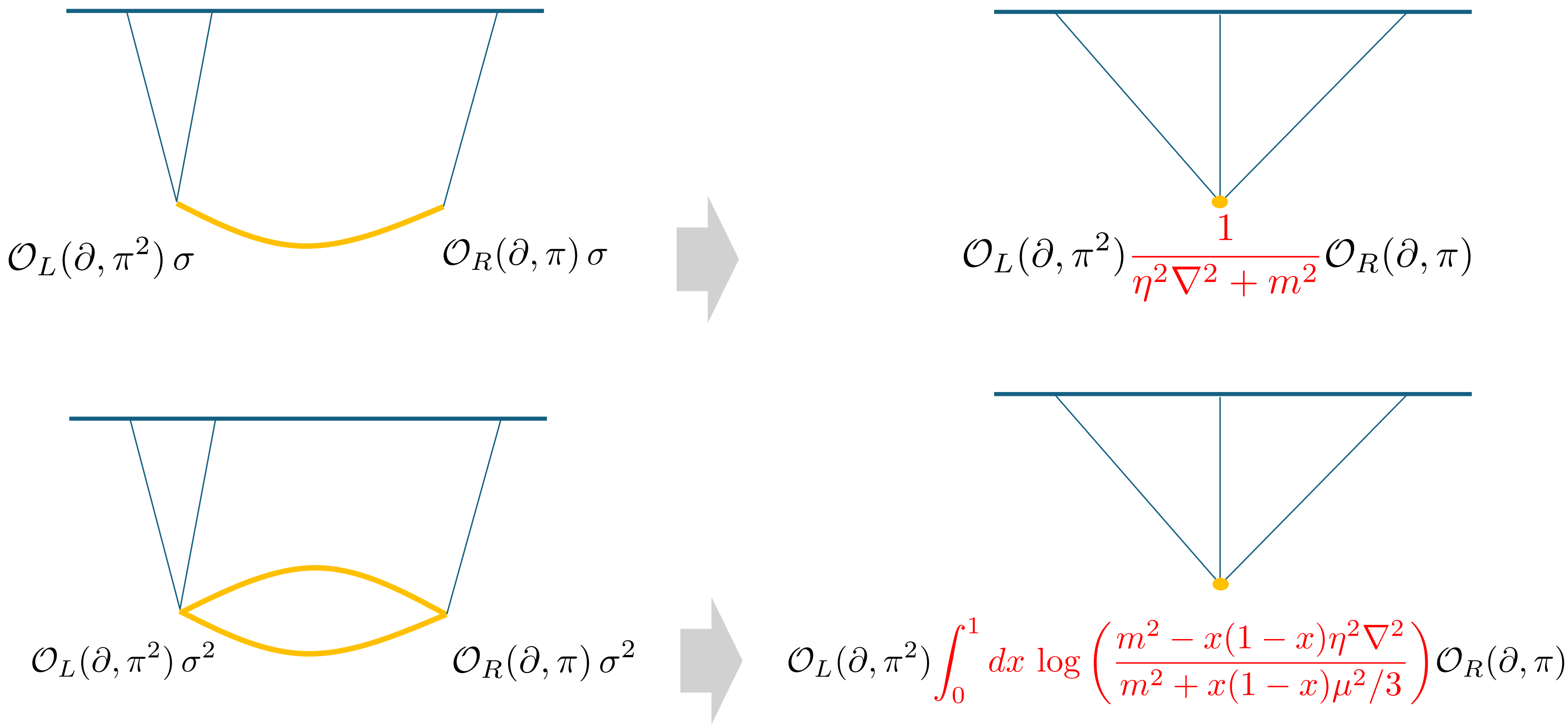


Backup



$$e^{iS_{\rm eff}[\pi]} = \int D\sigma\, e^{iS[\pi,\sigma]}$$

$$\mathcal{L}_{\rm EFT}\Big(\frac{1}{\Lambda}\partial_t, \frac{1}{\Lambda}\partial_i, \frac{\pi}{f_\pi}\Big) = \sum_{n,m} \underbrace{f_{mn}\Big(\frac{1}{\Lambda}\partial_i\Big)}_{\text{Non-Local in space}}\ \underbrace{\left[\frac{1}{\Lambda}\partial_t\right]^n}_{\text{Local in time}}\ \left[\frac{\pi_c}{f_\pi}\right]^m$$



$\mathcal{O}_L(\partial,\pi^2)\,\sigma$
$\mathcal{O}_R(\partial,\pi)\,\sigma$
$\mathcal{O}_L(\partial,\pi^2)\,\frac{1}{\eta^2\nabla^2+m^2}\,\mathcal{O}_R(\partial,\pi)$
$\mathcal{O}_L(\partial,\pi^2)\,\sigma^2$
$\mathcal{O}_R(\partial,\pi)\,\sigma^2$
$\mathcal{O}_L(\partial,\pi^2)\int_0^1 dx\,\log\left(\frac{m^2-x(1-x)\eta^2\nabla^2}{m^2+x(1-x)\mu^2/3}\right)\mathcal{O}_R(\partial,\pi)$