Categorical Landau Paradigm

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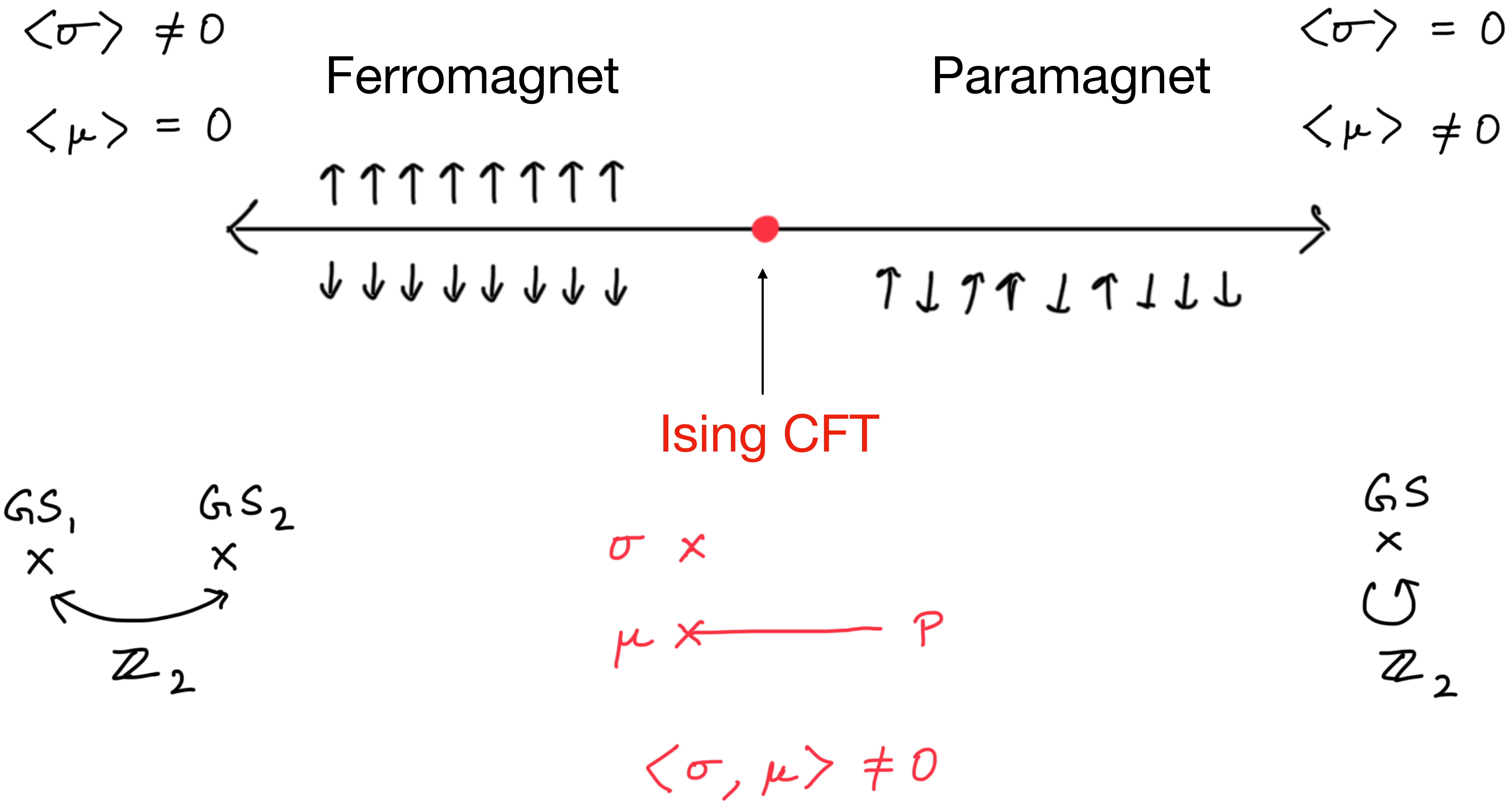
Works: 2310.03786, 2310.03784, 2312.17322, 2403.00905, 2405.05302, 2405.05964, 2408.05266

Gapped Gapless Lattice

(1+1)d Ising Model



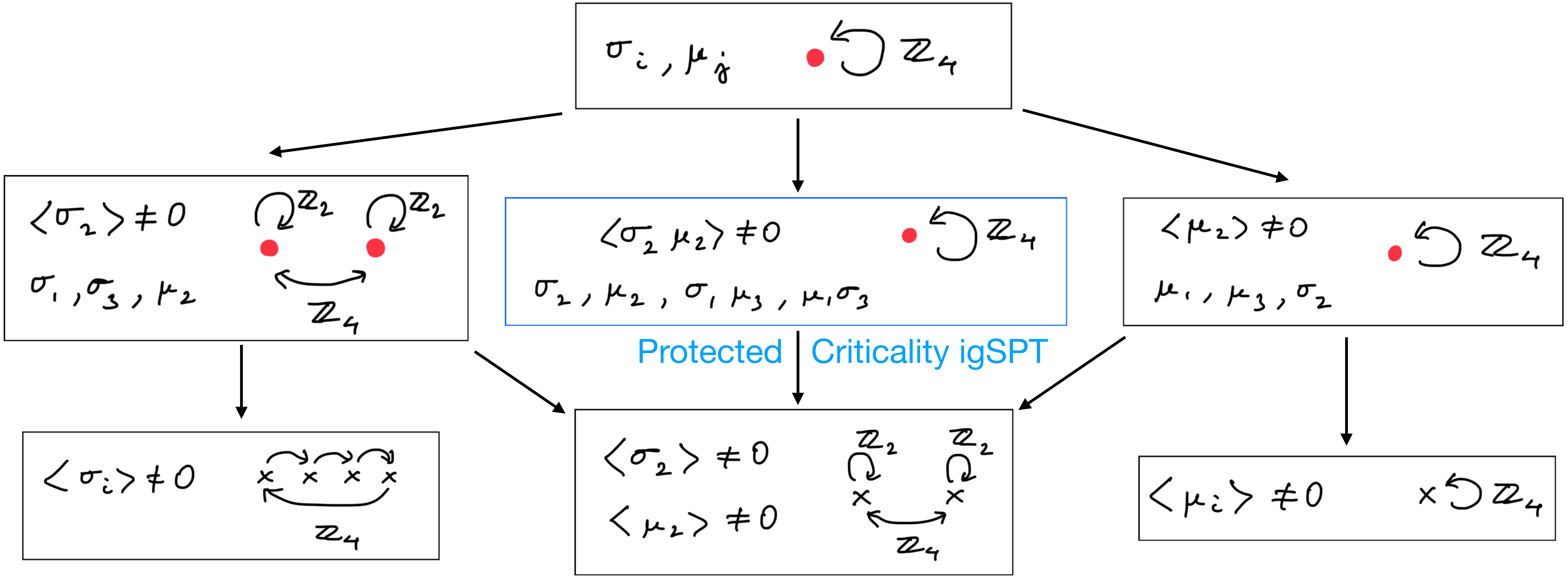
$\mathbb{Z}_4$ Symmetry

$i = 1, 2, 3$

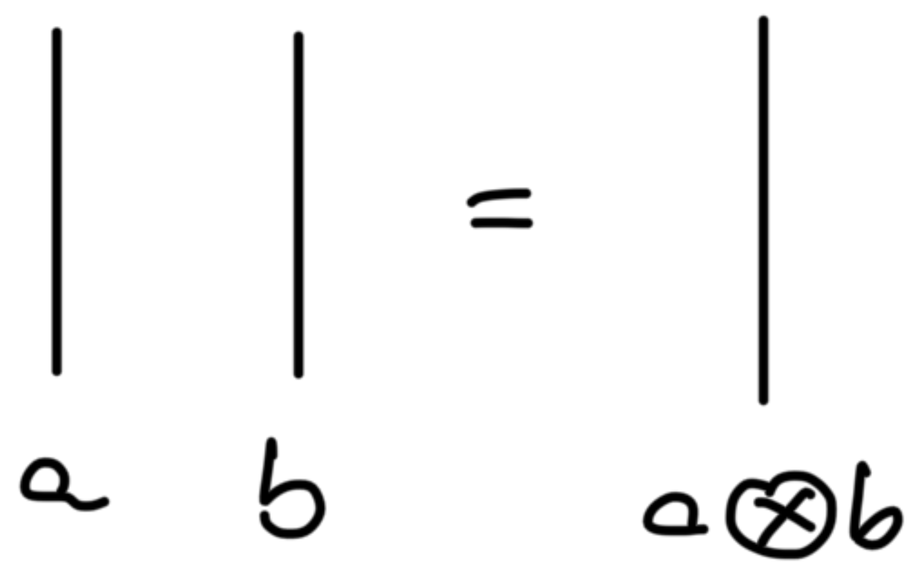
$\sigma_i \times$

$\mu_i \times \longleftarrow p^i$

$\langle \sigma_i, \mu_j \rangle = i + j \mod 4$



Non-Invertible Symmetries in (1+1)d



$a, b \in \mathcal{S} =$ fusion category

e.g. $\mathcal{S} =$ Rep(G), in particular Rep(S_3)

Permutation group of 3 objects

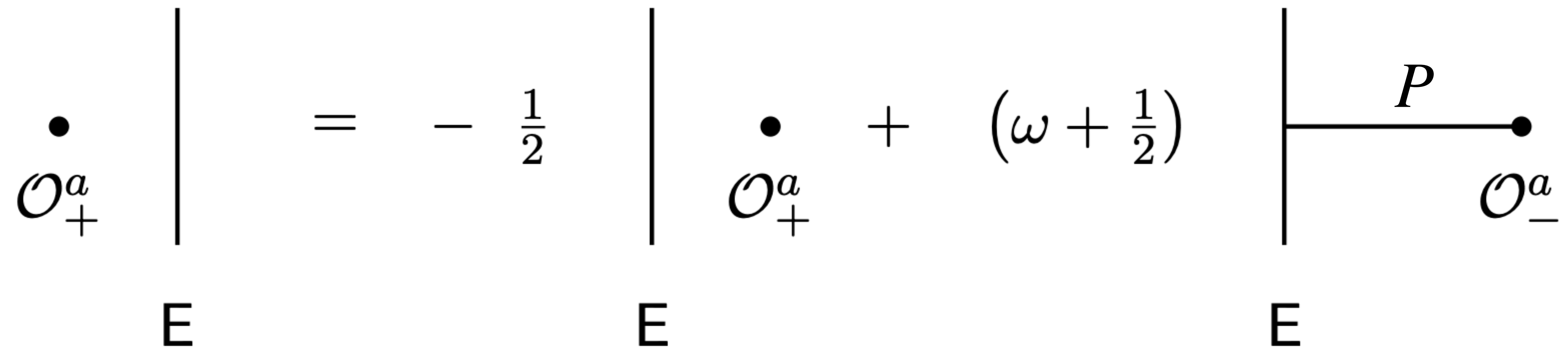
$\mathbb{Z}_2$ subsymmetry $\longrightarrow$ $P \otimes P = 1$

$P \otimes E = E$

Non-invertible element $\longrightarrow$ $E \otimes P = E$

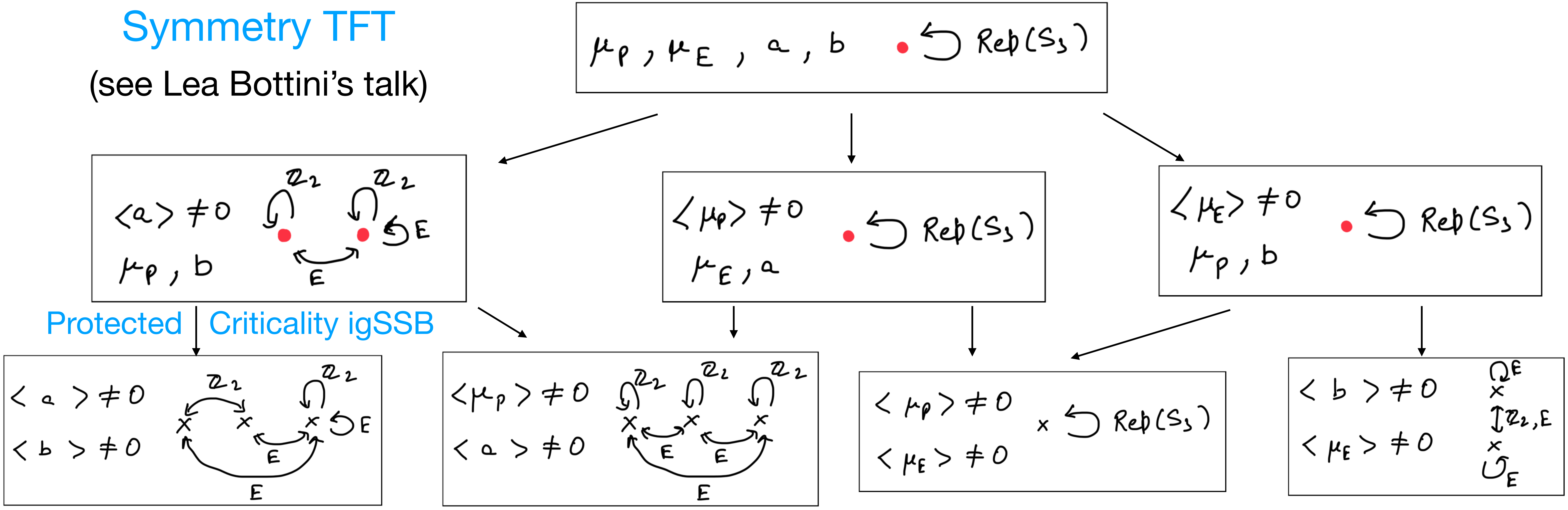
$E \otimes E = 1 + P + E$

Rep(S_3) Landau Paradigm



Symmetry TFT

(see Lea Bottini's talk)



$\cdots \quad \mathbb{C}^3 \quad \mathbb{C}^2 \quad \mathbb{C}^3 \quad \mathbb{C}^2 \quad \mathbb{C}^3 \quad \cdots$

$\times \quad \times \quad \times \quad \times \quad \times$

$j-1 \quad j-\frac{1}{2} \quad j \quad j+\frac{1}{2} \quad j+1$

$$H = -\sum_j \sum_{I=0}^{5} \left[\lambda_I P^{(I)}_{j-\frac{1}{2}} + t_I X^{(I)}_j\right]$$

$$E = \frac{1}{2}\left(1 + \prod_j \sigma^z_{j+\frac{1}{2}}\right)(T_1 + T_2) \qquad P = \prod_j \sigma^z_{j+\frac{1}{2}}$$

$$T_s = \frac{1}{2}\prod_j \sum_{n=1,2}\left[\left(1 + (-1)^{n+1}\prod_{i=0}^{j-1}\sigma^z_{i+\frac{1}{2}}\right)X_j^{ns}\right]$$

$$P^{(2I+s)}_{j+\frac{1}{2}} = \frac{1}{6}\left[1 + (-1)^s \sigma^z_{j+\frac{1}{2}}\right]\left[\sum_{n=0}^{2}\omega^{-In} Z_j^n Z_{j+1}^{2s-n}\right]$$

$$X_j^{(2I+s)} = (X_j)^I \left(\sigma^x_{j-\frac{1}{2}}\Gamma_j \sigma^x_{j+\frac{1}{2}}\right)^s,$$

Four Gapped Phases

$$H_1 = -\sum_j \left[\frac{1+\sigma^z_{j+\frac{1}{2}}}{2} + \frac{1 + X_j + X_j^2}{3}\right]$$

1 Ground State ✅

$$H_2 = -\sum_j \frac{1}{6}\left(1 + X_j + X_j^2\right)\left(1 + \sigma^x_{j-\frac{1}{2}}\Gamma_j\sigma^x_{j+\frac{1}{2}}\right)$$

2 Degenerate Ground States ✅

$$H_3 = -\sum_j \frac{1}{6}\left[1 + \sigma^z_{j+\frac{1}{2}}\right]\left[\sum_{n=0}^{2} Z_j^n Z_{j+1}^{-n}\right]$$

3 Degenerate Ground States ?

= 3 x $\mathbb{Z}_2$ Trivial

$$H_4 = -\frac{1}{2}\sum_j \left(1 + \sigma^x_{j-\frac{1}{2}}\Gamma_j\sigma^x_{j+\frac{1}{2}}\right) - \frac{1}{6}\sum_j \sum_{\alpha=\pm 1}\left[1 + \alpha\sigma^z_{j+\frac{1}{2}}\right]\left[\sum_{n=0}^{2} Z_j^n Z_{j+1}^{-\alpha n}\right]$$

3 Degenerate Ground States ?

= $\mathbb{Z}_2$ Trivial + $\mathbb{Z}_2$ Broken

Conclusions

1. Generalizing Landau paradigm to categorical or non-invertible symmetries leads to a plethora of new phases.
2. In (1+1)d, these phases are characterized in terms of degeneracies unexpected from spontaneous breaking of traditional group-like symmetries.
3. This includes both gapped and gapless phases, and transitions between them.
4. For gapped phases, we have degenerate gapped vacua appearing in the IR.
5. For gapless phases, we have degenerate CFTs appearing in the IR.
6. Using Symmetry TFTs we can classify all possible gapped and gapless phases for any given categorical symmetry.

Future Directions

1. On-going work on experimental realisation of phases and transitions with non-invertible symmetries on quantum simulators.
2. Phases in higher-dimensions:
   Symmetries are higher-categories. [LB, Bottini, Schafer-Nameki, Tiwari 2022]
   Infinite number of gapped phases for every symmetry in (2+1)d.
   [LB, Pajer, Schafer-Nameki, Tiwari, Warman 2024]
   See also [Aguilera Damia, Argurio, Benini, Benvenuti, Copetti, Tizzano 2023], [Antinucci, Copetti, Schafer-Nameki 2024] for phases in (3+1)d.
3. Include symmetries of systems involving fermions and time-reversal in various d.
   For fermionic non-invertible symmetries in (1+1)d, see [LB, Inamura, Tiwari 2024]
4. Phases of systems out-of-equilibrium (with categorical symmetries) can also be studied using SymTFT. [LB, Moudgalya, Tiwari (to appear soon)]