Bra-ket Wormholes in Cosmology

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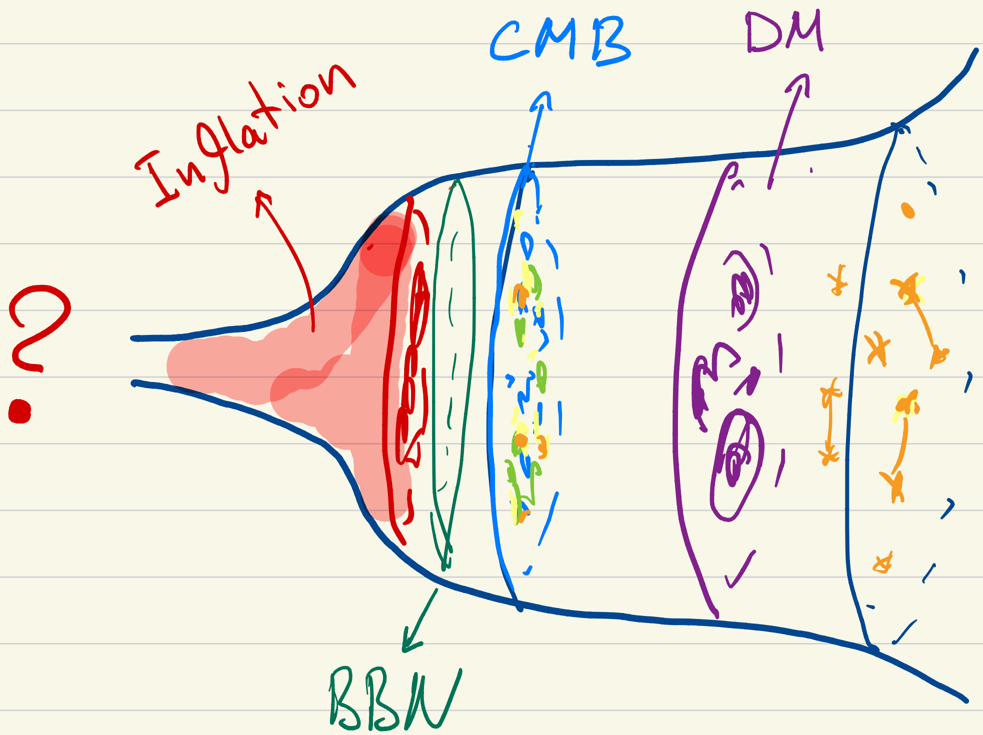
2007.16091 w Chen, Maldacena

2408.08351 w Fumagalli, Rames-King

Eurostrings 2024

- Status of the theory for initial state of the universe and holography in cosmology.
- No-boundary states, and bra-ket wormholes, in particular.
- Lorentzian bra-ket wormholes in (2D gravity) for Wigner distribution.
- Open questions and how we can make progress.

Expanding universe



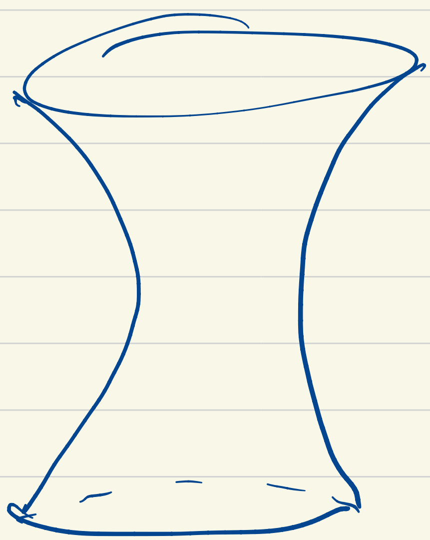
$$ds^2 = -dt^2 + e^{2H(t)\cdot t}\, d\vec{x}^{\,2}$$

- Inflation ~ period of quasi-dS expansion
- In fundamental cosmology we often try to understand this period at the microscopic level

- For most observations it is not very important what happened before inflation, but for a full theory we need also initial conditions

→ Inflation: singularity in the past

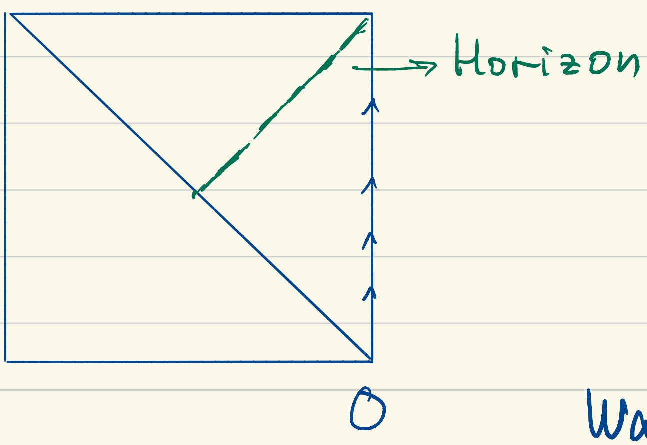
→ Pure dS: contraction in the past



$$ds^2 = -dt^2 + \cosh^2 Ht \, d\vec{x}^2$$

Holography

- de Sitter seems similar to (analytically continued) AdS space, and also to AdS-Schwarzschild, but so far we are lacking any concrete holographic models



dS Holography

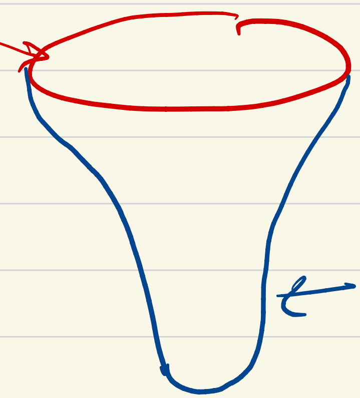
Global → Wave-function, Correlators

Local → ?

Gravitational path-integral and No-Boundary states

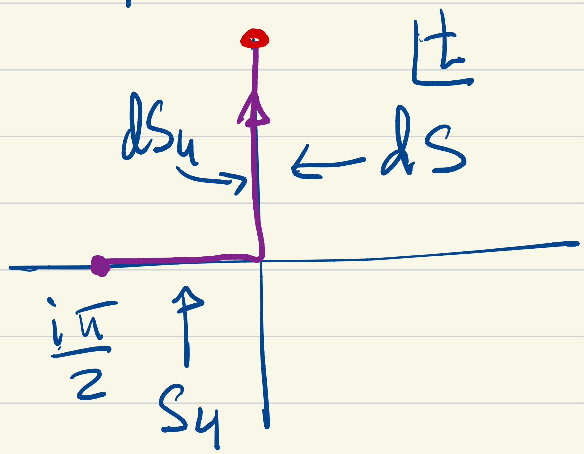
- From AdS/CFT we learned that a path-integral over smooth geometries (including non-trivial topologies). Gives a good approximation to (some observables) in CFT (which is a more fundamental description)
- A direct generalization of this idea to cosmology is the Hartle-Hawking no-boundary proposal.

• Original prescription is to calculate the wave function – one boundary geometry. State defined on a Cauchy slice:



← smooth complex metric

• in 4D pure dS: $ds^2 = -dt^2 + \cosh^2 t \, d^2 S_{S^3}$

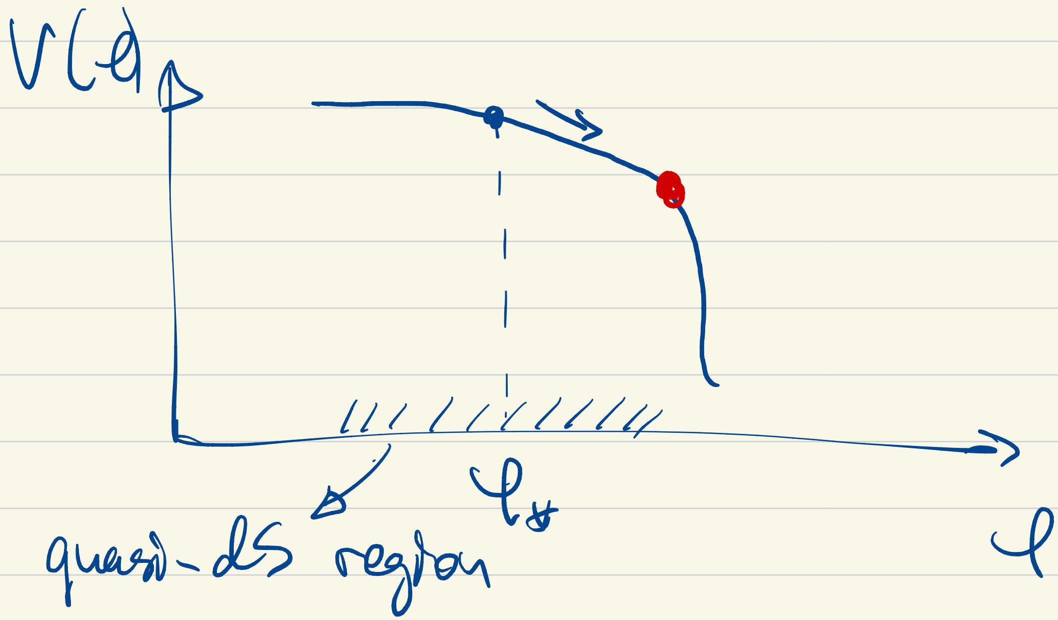


$$\Psi \sim e^{\frac{M_{pl}^2}{H^2} + i\varphi}$$

• in slow-roll inflation

$$\Psi \sim e^{\frac{M_{pl}^4}{V(\varphi_*)}}$$

$\varphi_* \rightarrow$ value of inflaton in the beginning of inflation



$$P(\varphi_*) \sim e^{\frac{1}{V(\varphi_*)}}$$

prefers very short inflation!

(inconsistent with observations)

- Let's discuss dS_2 JT gravity (related to KK reduction of near-extremal dS-Schwarzschild, which has $S_1 \times S_2$ spacial topology)

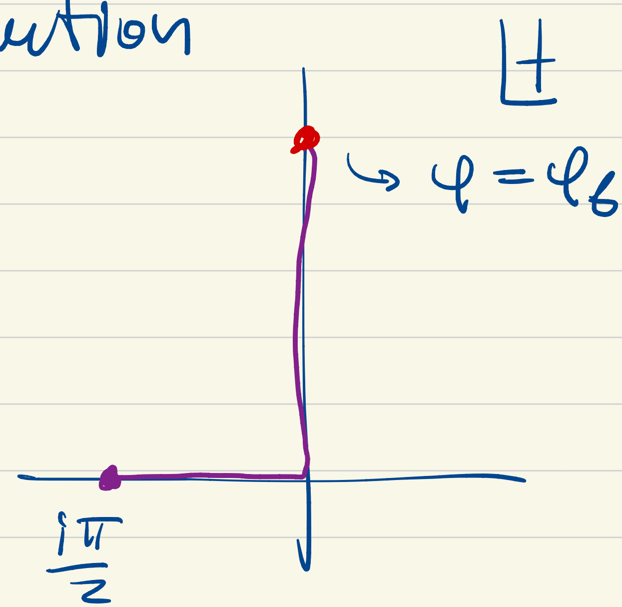
$$S^{dS} \simeq \varphi_0 \int d^2x \sqrt{g} R + \int d^2x \sqrt{-g}\, \varphi (R-2) + S_m$$

$\hookrightarrow \sim \frac{M_{pe}^2}{H^2}$ dilaton $(\sim R_{S_2})$

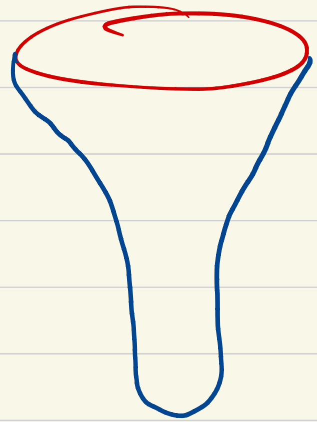
- There is also a HH solution

$$ds^2 = -dt^2 + \cosh^2 t\, dx^2$$

$$\varphi = \varphi_r \sinh t$$



$$\Psi(\varphi_b, \ell) \sim e^{\varphi_0}$$



- There is a different (theoretical) self-consistency problem!

- Consider adding CFT matter and computing entanglement entropy of this matter in a subregion.

- Since gravity is dynamical, we need to use the "island formula"

• Details in 2007.16091

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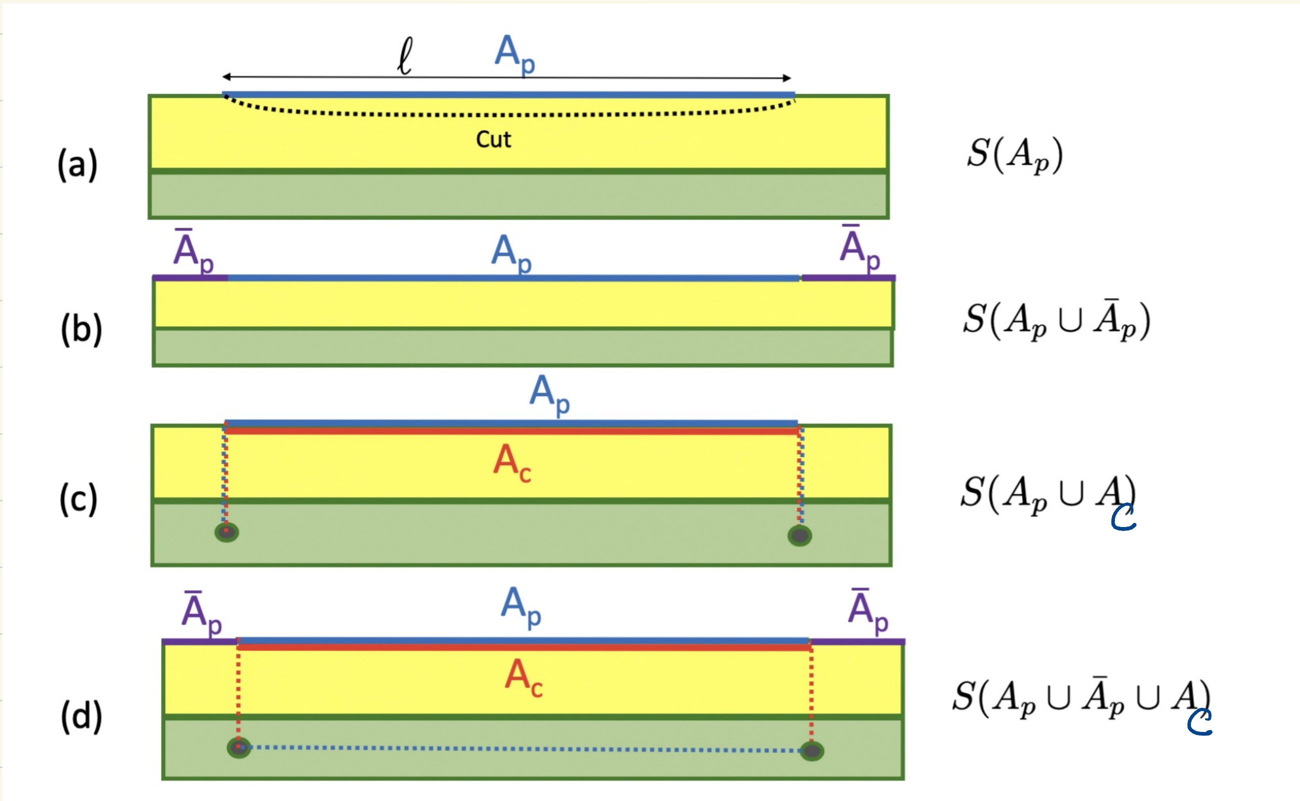


$$S(A_c \cup A_p) + S(A_p \cup \bar{A}_p) - S(A_c \cup A_p \cup \bar{A}_p) - S(A_p) = \frac{c_p}{3}\left[1 - 2\log\left(\frac{\ell c}{6\phi_r}\right)\right] < 0\,.$$

violation of strong subadditivity for large enough ℓ !

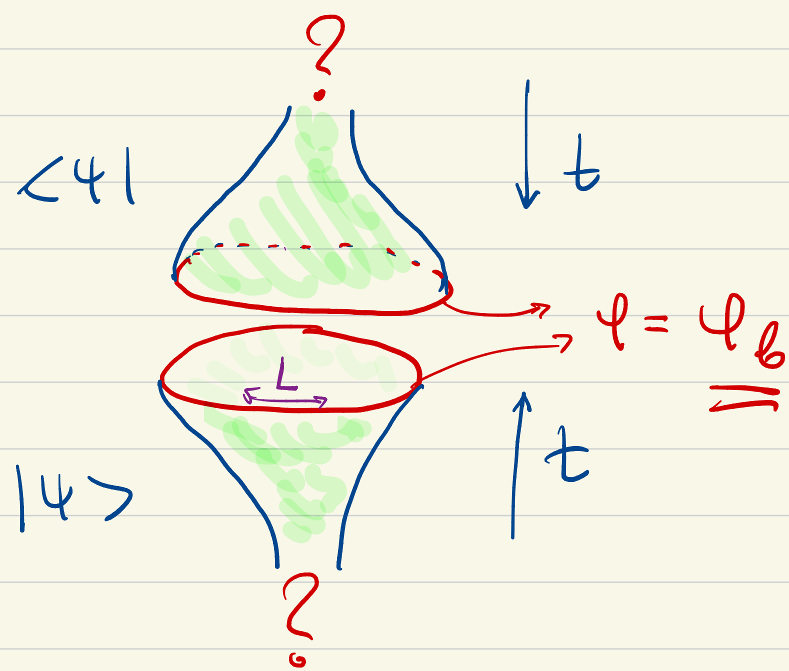
- Punchline:
  - → We do not have a working holographic model for cosmology, but we have a gravitational path integral from which we can infer would-be properties of a fundamental theory
  - → HH (no-boundary) state looks like the most natural to consider, but it has problems both in 4D and 2D, so it should not be the final answer

Bra-ket Wormholes

[We again consider JT gravity here]

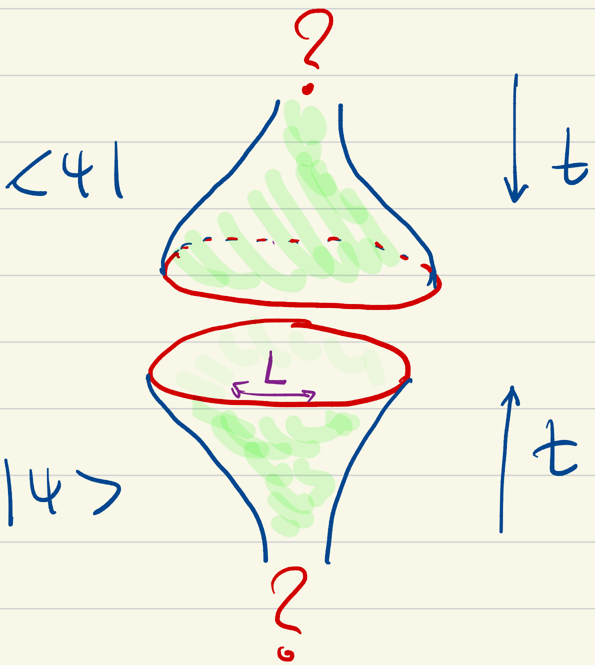
- In cosmology observables have the in-in nature, so we need two copies of space-time to compute them (bra and ket)

- Should we consider connected space-times?



- In particular, it is natural to study the following object:

(Wigner distribution)



$$P[L,P] = \int D\chi \, e^{2iP\chi} \, \Psi(L+\chi)\Psi^*(L-\chi)$$

Probability distribution of the size and expansion rate of the universe.

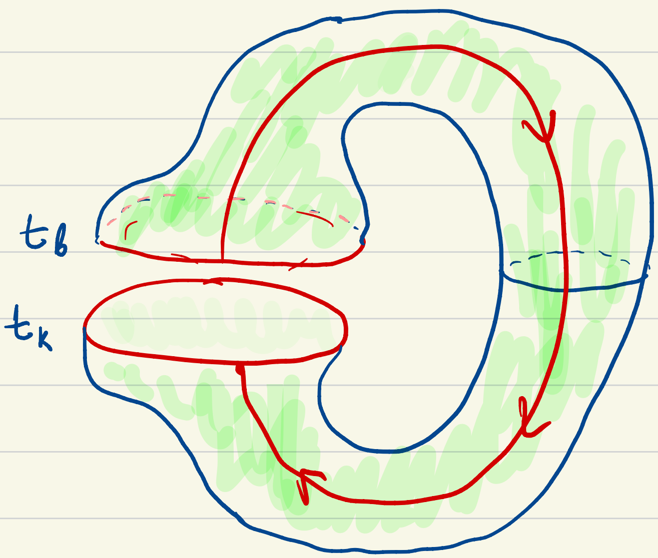
- A close analogy to this object in AdS/CFT is a spectral form-factor in microcanonical ensemble:

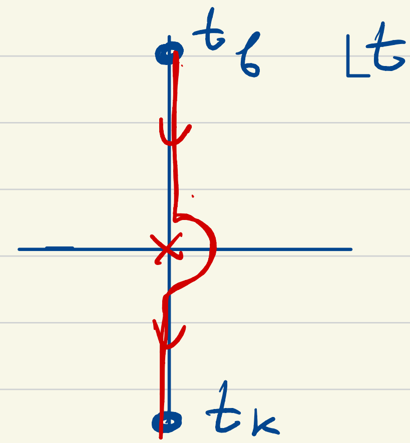
$$\bullet \, |Y_{E,\Delta E}(T)|^2 = \int d\beta_L \, d\beta_R \, e^{\beta_L E + \beta_L^2 \Delta E^2} Z(\beta_L + iT) \, e^{\beta_R E + \beta_R^2 \Delta E^2} Z(\beta_R - iT)$$

• With these boundary conditions there is a Bra-Ket WH solution:

$$ds^2 = -dt^2 + b^2 \sinh^2 t \, dx^2$$

$$\varphi = \varphi_r \cosh t$$





• Similar to the double-cone solution of

P. Saad, S. H. Shenker, and D. Stanford, "A semiclassical ramp in SYK and in gravity," arXiv:1806.06840 [hep-th].

producing ramp in the spectral FF.

- Until now we used the dilaton to fix time diffeos. We can also add an inflaton scalar field and use it as time.
- We then get a four-dimensional phase space (L, Φ, P, Q)

$$X \equiv \frac{1}{2}(X_6 + X_k)$$

- Classical equations of motion read

$$L = 6 \sinh t \qquad Q = 6 \cosh t$$

$$\Phi = \varphi_r \cosh t \qquad P = \varphi_r \sinh t$$

$$LP = \Phi Q$$

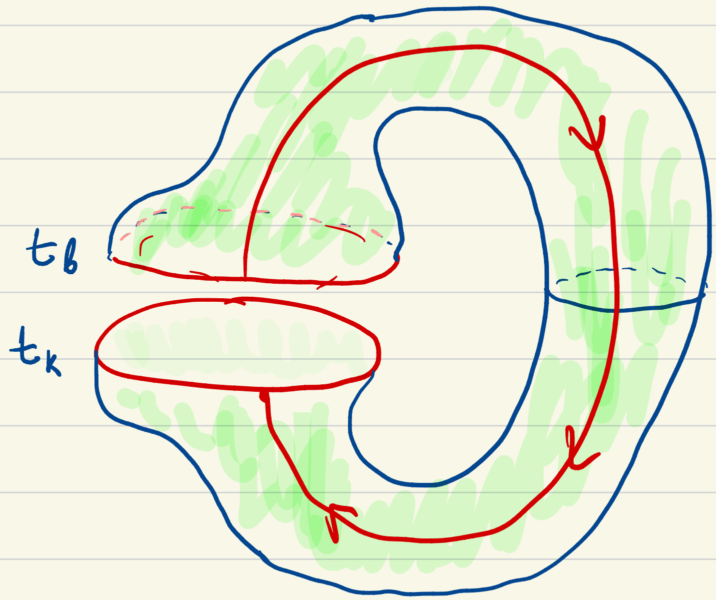
- Our distribution (at leading order) is localized on classical solutions:

$$P_W(L, P, \Phi, Q) = \delta(LP - \Phi Q)$$

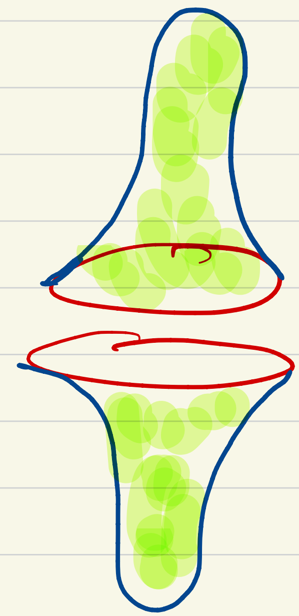
details in 2408.08351

- We also computed the one-loop determinant from the Schwarzian mode (at late times its a constant)

- Solution exists, but how is it compared to the disconnected contribution?



bra - ket



HH

$$\sim e^{2 S_0}$$

$$\frac{P_{b.k}}{P_{HH}} = e^{-2 S_0} \frac{L^4}{\varphi^2} \rightarrow \text{bra-ket dominates for large } L$$

- In particular, the SSA paradox is resolved by the wormhole contribution

Outlook

- We constructed bra-ket wormhole solutions that dominate over HH for large universes and give locally well-defined probability measures on phase space

- However, they are not normalizable:

  $L \to \infty$ divergence $\rightsquigarrow$ higher topologies?

  $t_c \to 0$ divergence $\rightsquigarrow$ Not universal?

- We believe that our solution can be lifted to 4D with $S_1 \times S_2$ spacial topology.
  - → Other topologies?
  - → Universal "ramp-like" enhancement for S_1?
  - → what does it correspond to in a "CFT"
  - → Which topology dominates
- Gravitational path integral allows us to calculate many of these features, and knowing enough of them could lead us to the microscopic theory