Bootstrapping string and M-theory

Shai M. Chester

Imperial College London

Based on 2312.12576 and TBA with S. Pufu and R. Dempsey

Non-perturbative string theory

- Only non-perturbative def of string theory from holography. e.g.:
- IIB string theory on $AdS_5 \times S^5 \Leftrightarrow$ 4d $\mathcal{N} = 4$ $SU(N)$ SYM.
  - String length ℓ_s and complex string coupling $\tau_s = \chi + i/g_s$ related to SYM $\tau \equiv \frac{4\pi i}{g_{\text{YM}}^2} + \frac{\theta}{2\pi}$ as $\tau_s = \tau$ and $L_{\text{AdS}}^4/\ell_s^4 = \lambda \equiv g_{\text{YM}}^2 N$.
- M-theory on $AdS_4 \times S^7/\mathbb{Z}_k \Leftrightarrow$ 3d $U(N)_k \times U(N)_{-k}$ ABJM.
  - Planck length ℓ_{11} related as $L_{\text{AdS}}^9/\ell_{11}^9 = kN^{3/2}$.
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  - For SYM can do weak coupling, but gravity at strong coupling.
- Today we combine three non-perturbative constraints to numerically study holographic CFTs (and thus quantum gravity):
  1. Unitarity: OPE coefficients real and Δ bounded.
  2. Crossing: combine with unitarity for numerical bootstrap algorithm.
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4d $\mathcal{N} = 4$ SYM stress tensor correlator

- Expand stress tensor superprimary correlator in blocks:

$$\langle S(x_1)S(x_2)S(x_3)S(x_4)\rangle = G_{prot}(c, x_i) + \Theta_{\text{kin}}(x_i) \sum_{\Delta,\ell} \lambda^2_{\Delta,\ell} g_{\Delta,\ell}(x_i)$$

  - Susy ward identities solved by this expansion, where all dynamical info in λ, Δ of long multiplets in singlet of $SU(4)_R$.
- Theory specified by $c \sim N^2$ and τ.
  - $1/c$ enters linearly into $G_{prot}(c, x_i)$.
  - Both τ and c effect long multiplet CFT data nontrivially.

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Localization constraints

- Derivatives of free energy $F(m)$ deformed by hyper mass relate to S^4 integrals of correlator [Binder, SMC, Pufu, Wang '19; SMC, Pufu '20] :

$$\frac{1}{8c}\frac{\partial_m^2\partial_\tau\partial_{\bar\tau}F}{\partial_\tau\partial_{\bar\tau}F}\Big|_{m=0} = I_2\Big[\sum_{\Delta,\ell}\lambda^2_{\Delta,\ell}g_{\Delta,\ell}(x_i) - f_{\text{prot}}(c,x_i)\Big],$$

$$-48\zeta(3)c^{-1} - c^{-2}\partial_m^4 F\big|_{m=0} = I_4\Big[\sum_{\Delta,\ell}\lambda^2_{\Delta,\ell}g_{\Delta,\ell}(x_i) - f_{\text{prot}}(c,x_i)\Big].$$

- LHS can be computed as function of N and τ in terms of N dimensional matrix model from localization [Pestun '08] .
  - LHS computed in closed form for any N and τ to high accuracy [Alday, SMC, Dorigoni, Green, Wen '23] .
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Perturbative results

- Small g_{YM} expansion at finite N, e.g. Konishi [Velizhanin '09]

$$\Delta = 2 + \frac{3\lambda}{4\pi^2} - \frac{3\lambda^2}{16\pi^4} + \frac{21\lambda^3}{256\pi^6} + \frac{\lambda^4\left(-1440\left(\frac{12}{N^2}+1\right)\zeta(5) + 576\zeta(3) - 2496\right)}{65536\pi^8} + O(\lambda^5)$$

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$$\Delta = 4 - \frac{4}{c} + \frac{135}{7\sqrt{2}\pi^{3/2}c^{7/4}}E(\tfrac{3}{2},\tau) + \frac{1199}{42c^2} - \frac{3825}{32\sqrt{2}\pi^{5/2}c^{9/4}}E(\tfrac{5}{2},\tau) + \ldots$$

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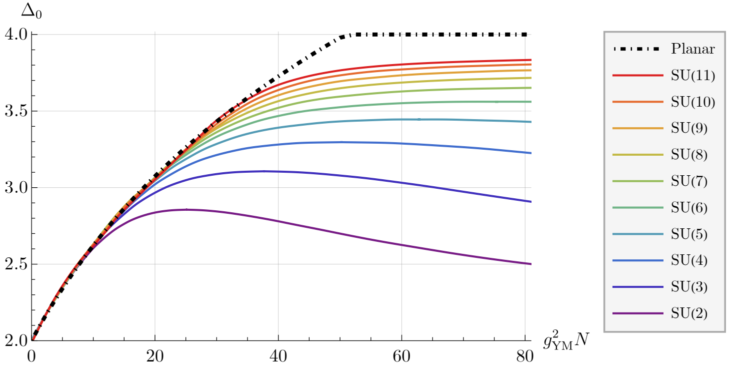
Bootstrap bounds on lowest Δ for various N



- Bounds are converging to Planar integrability spectrum (similar to Pade resummed 4-loop weak coupling in this regime).
- Planar integrability for double trace is trivially $4 = 2 + 2$.

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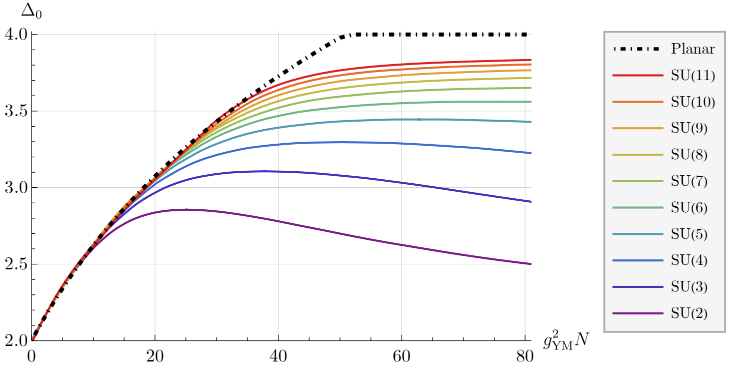

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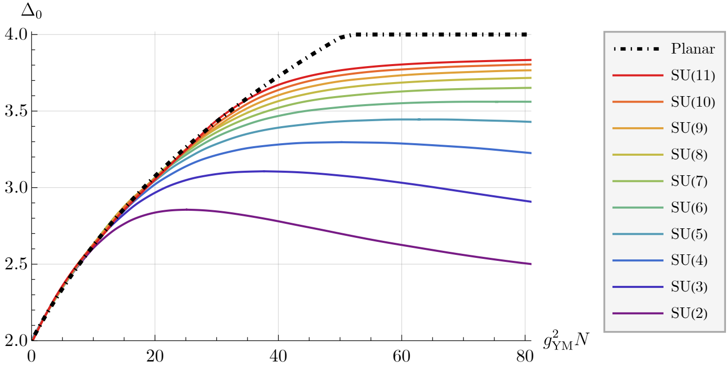

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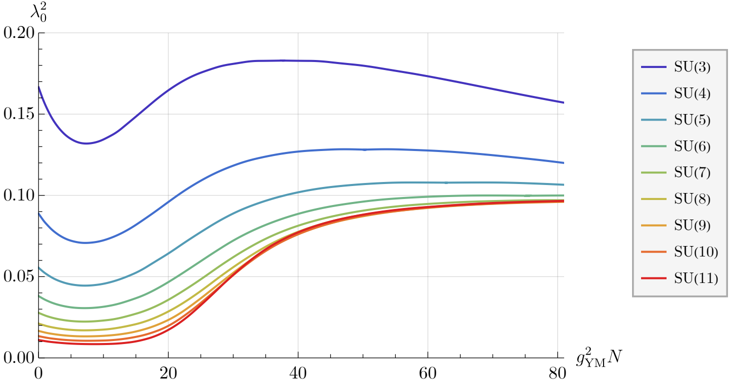
Bounds on lowest λ^2 for various N



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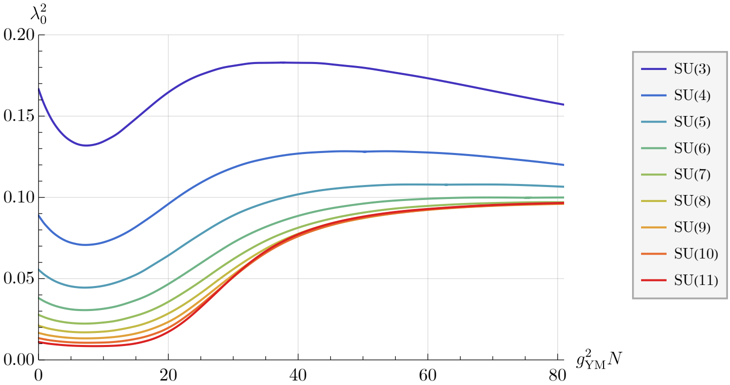

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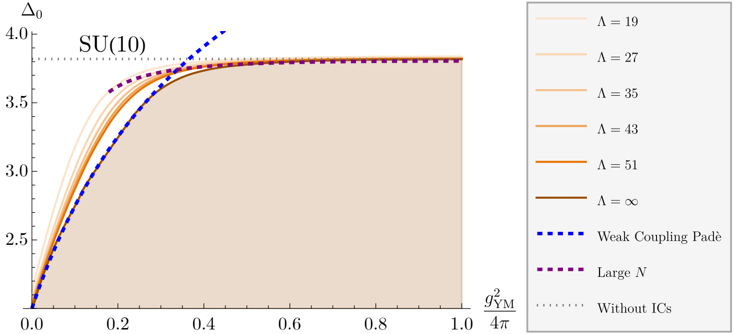
Bounds: Lowest Δ for $SU(10)$



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- Observe non-pert **level repulsion**, in between weak coupling for single trace and strong coupling for double trace.

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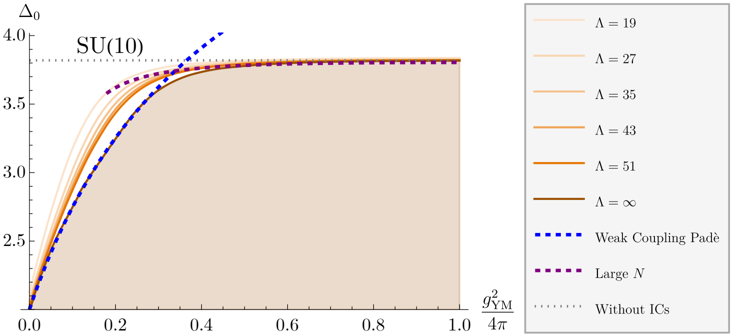

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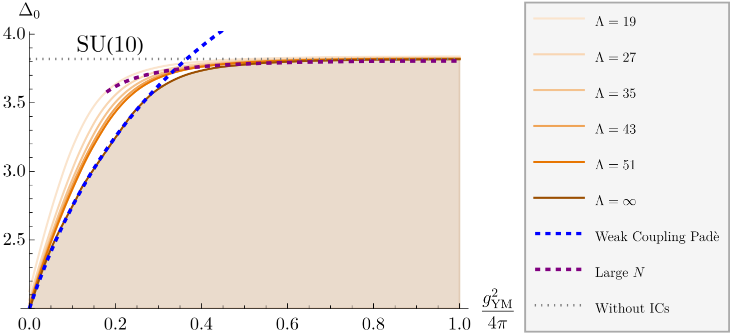

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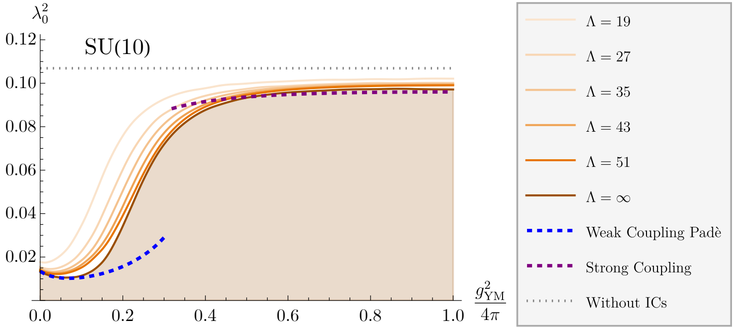

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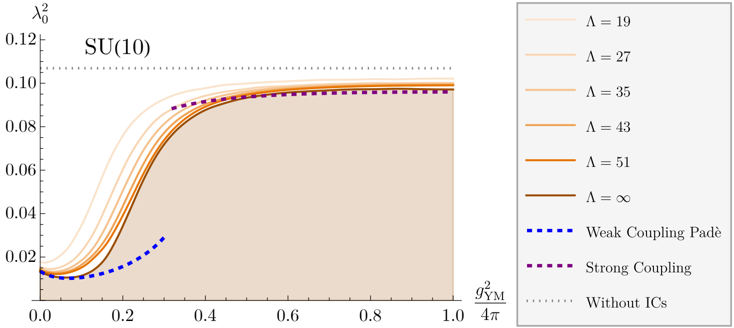

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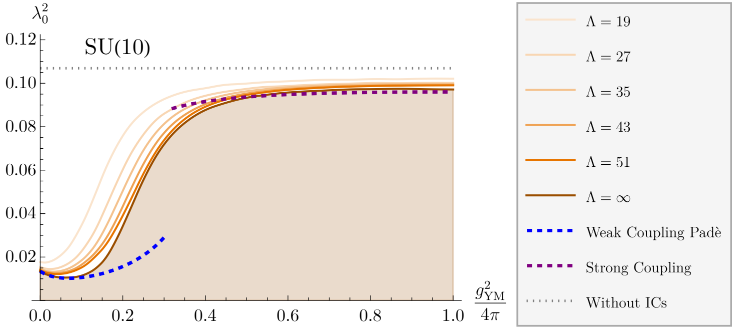

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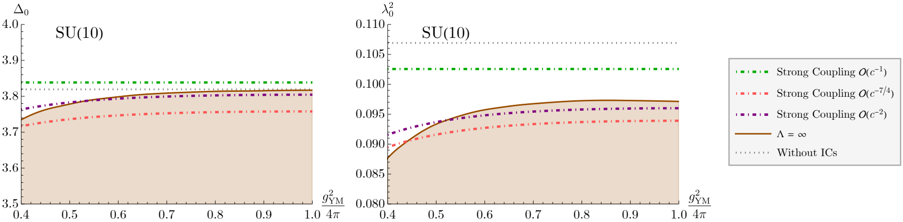
Sensitivity to stringy corrections



- For largish N (e.g. $SU(10)$), we see that analytic bootstrap result gets closer to bound as we include more $1/c$ corrections.
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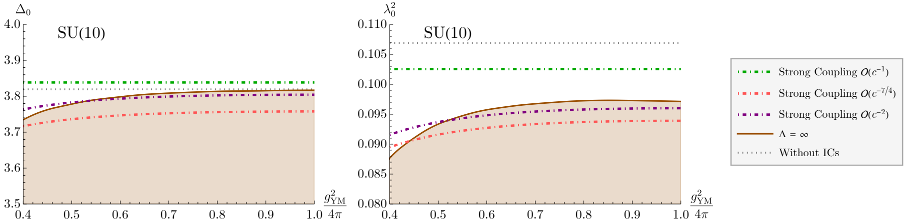

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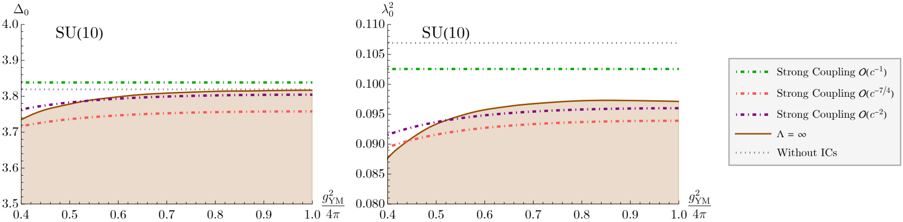

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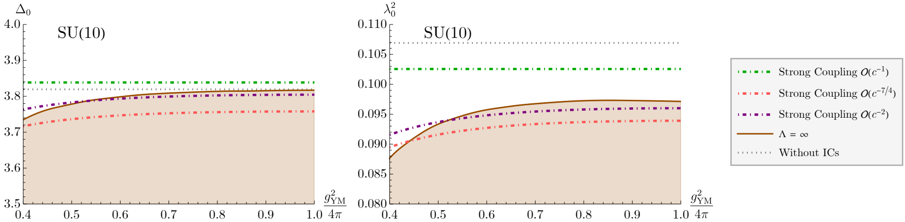

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3d ABJM stress tensor correlator

- Expand stress tensor superprimary correlator in superblocks:

$$\langle S^a(x_1)S^b(x_2)S^c(x_3)S^d(x_4)\rangle = \frac{1}{x_{12}^2x_{34}^2}\Big[\sum_{\Delta,\ell}\lambda^2_{\Delta,\ell}G^{abcd}_{\Delta,\ell}(x_i)+\sum_M \lambda_M^2 G_M^{abcd}(x_i)\Big]$$

  - Susy ward identities solved by superblocks, written as linear combos of 3d blocks in terms of structures $\sum_{i=1}^6 T_i^{abcd}\frac{G^i(U,V)}{x_{12}^2x_{34}^2}$.
  - Protected multiplets are nontrivial, include $(A,2)$ for odd spin, $(A,+)$ for even spin, $(B,2)$, $(B,+)$ and stress tensor for spin zero. Can bootstrap both upper and lower bounds (i.e. islands)!

- Theory specified by $c\sim N^{3/2}$ and $k=1,2$.
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Localization constraints

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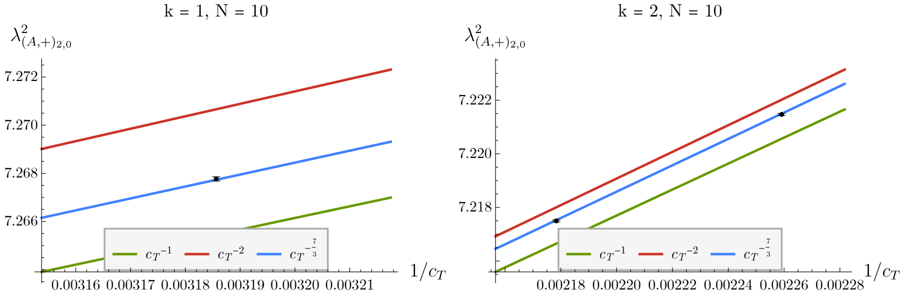
Bootstrap islands: Sensitivity to M-theory corrections



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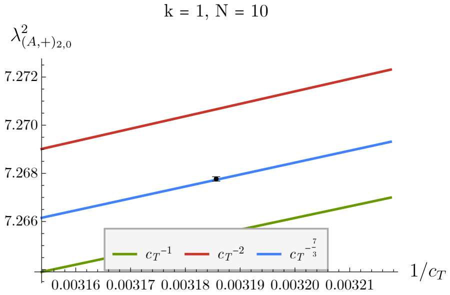

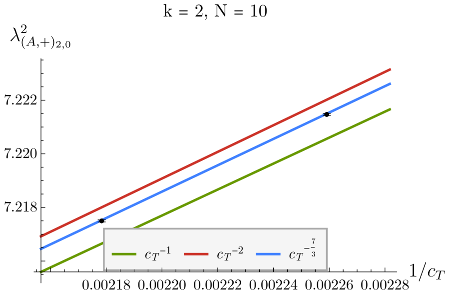

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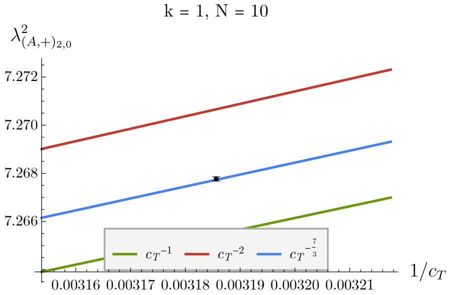

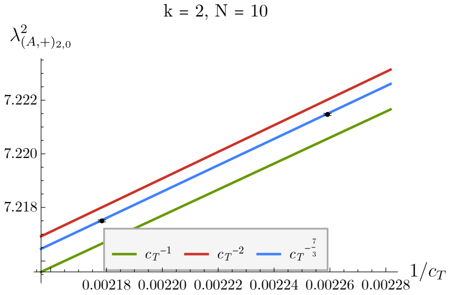

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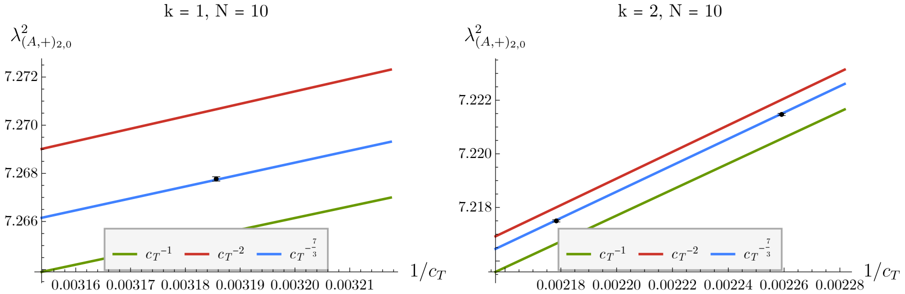

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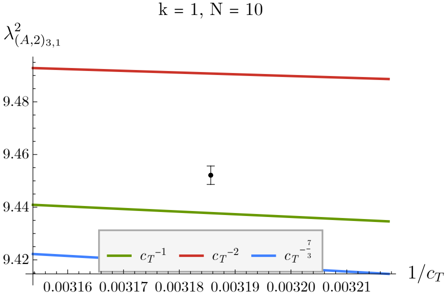
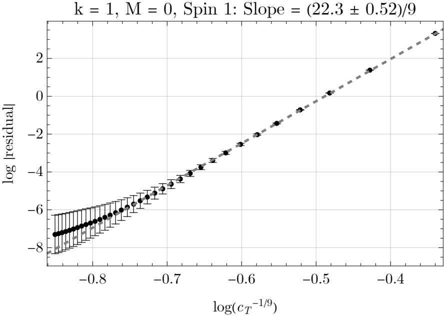
Bootstrap islands: Unprotected $D^8 R^4$ term





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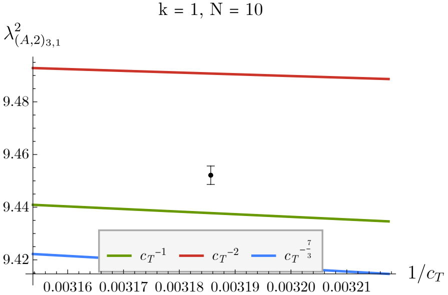

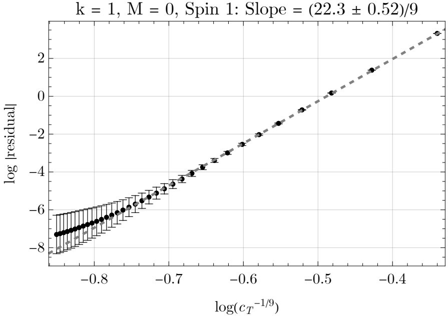

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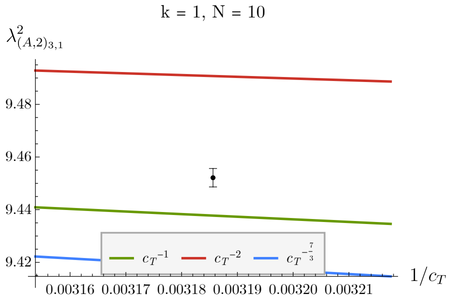

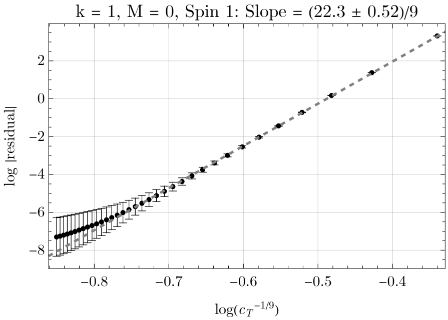

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Conclusion

- Bounds from bootstrap+localization give non-perturbative solution to holographic CFTs at finite N and coupling!
  - For $\mathcal{N} = 4$ SYM, evidence from comparing upper bound to both weak and strong coupling.
  - For ABJM, have rigorous upper and lower bounds (i.e. islands), and also matches strong coupling.
- Bounds accurate enough to read off lowest few protected corrections from string/M-theory to supergravity.
- For ABJM, can also read off prediction for unprotected $D^8 R^4$ correction to M-theory S-matrix!

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See you in London!