Modulated instabilities and the AdS_2 point in dense holographic matter

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Work in collaboration with T. Demircik and M. Järvinen
based in 2405.02399

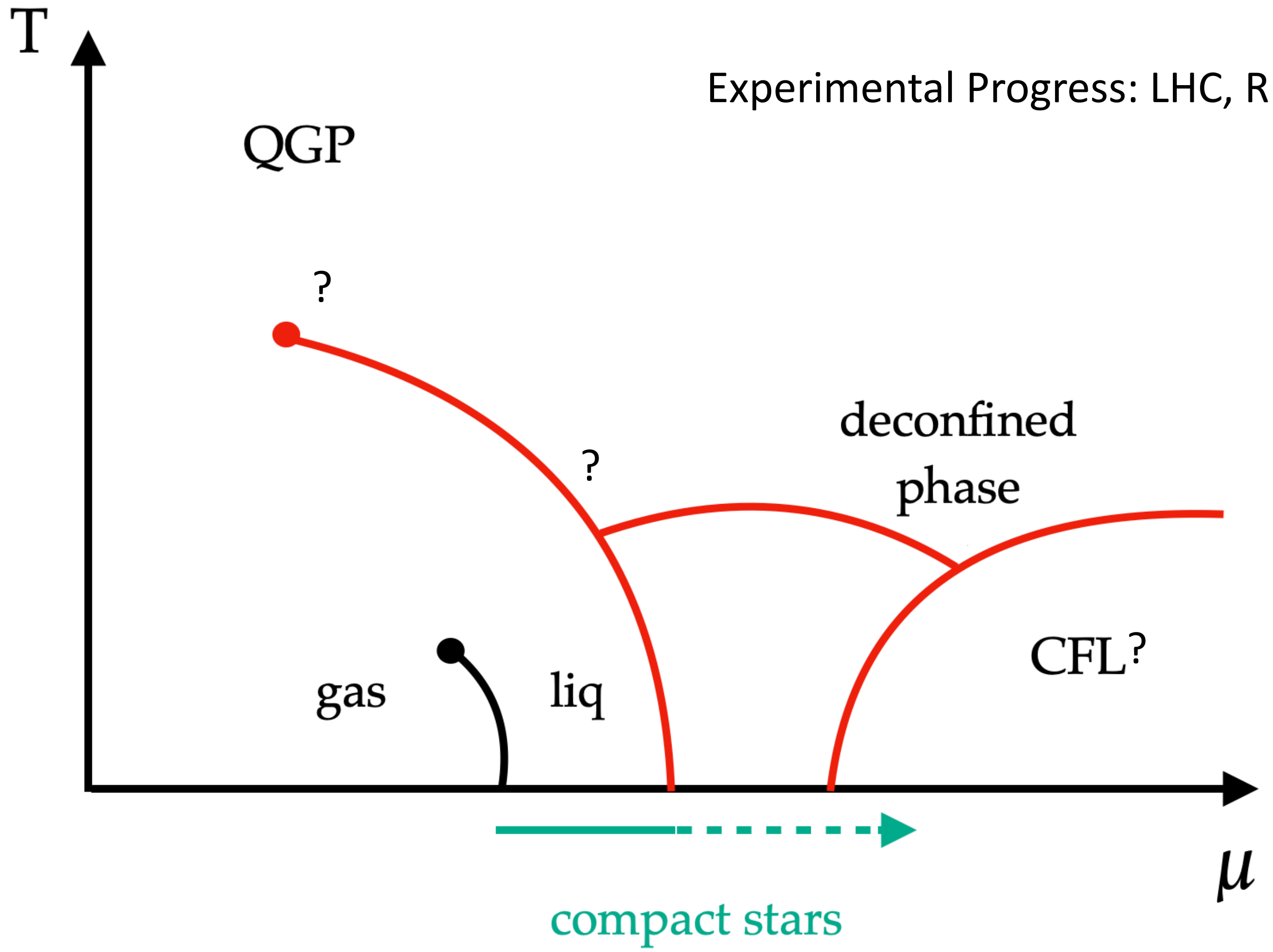
Plan

- Motivation
- Introduction to holographic model
- Striped instabilities
- Approach to the AdS_2 point
- Summary and future work

Description of the phase diagram

We are interested in the strongly coupled regime of QCD at low temperature and high densities at the deconfined phase.

Experimental Progress: LHC, RHIC, FAIR,



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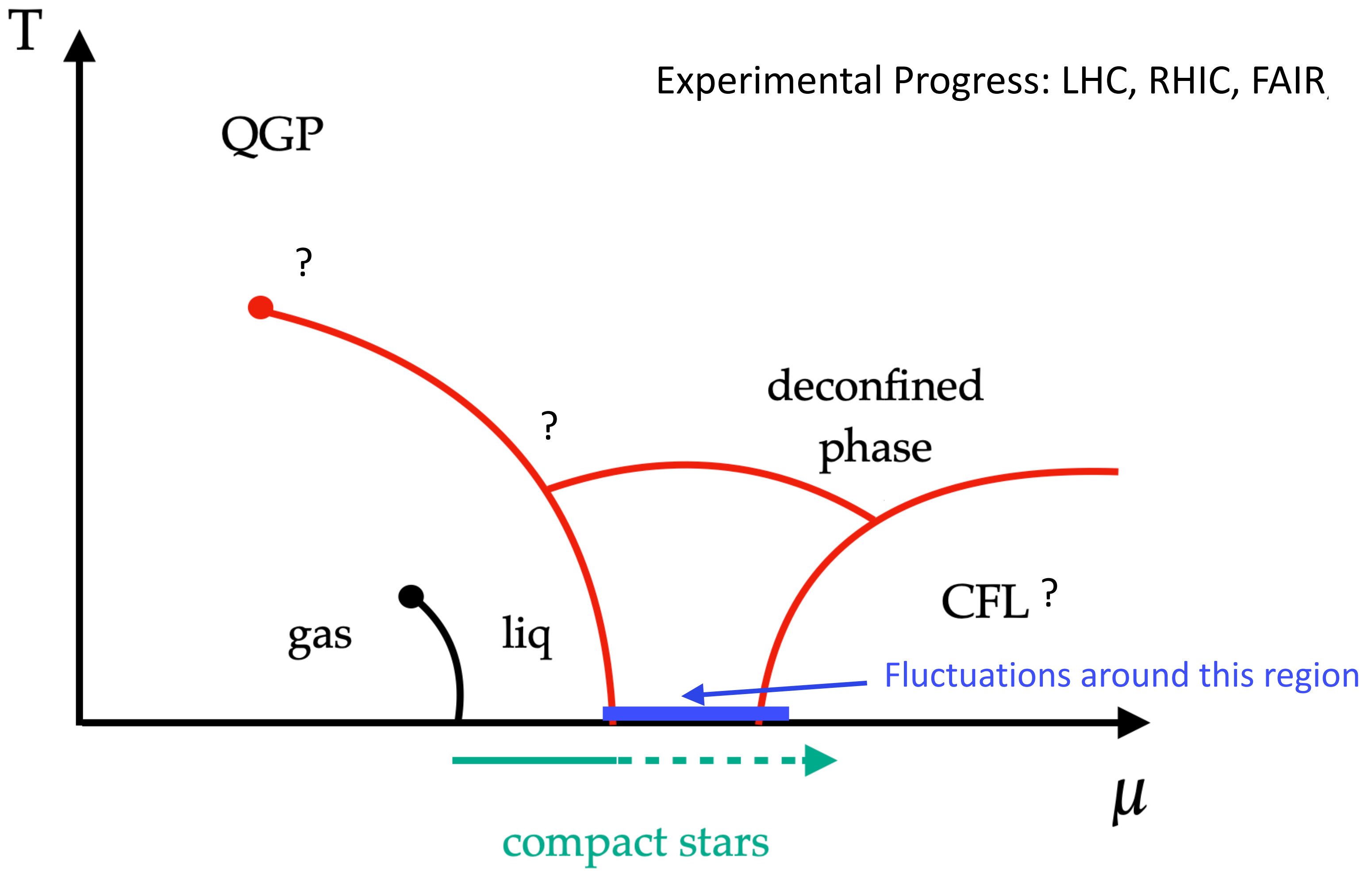


Stability at large densities

Our main motivation: instabilities in QCD

- In gravitational solutions dual to n+1 dim field theories at finite charge density, an AdS_2 x $\mathbb{R}^n$ geometry appears in the extremal limit of black holes.
- In the dual QFT, there is a new quantum critical regime at T = 0

Alho, Järvinen , Kajantie, Kiritsis, Rosen, Tuominen

- Such AdS_2 solutions are unstable as AdS_2 has a more restrictive BF bound.

- May compete with color superconducting/color-flavor locked phases

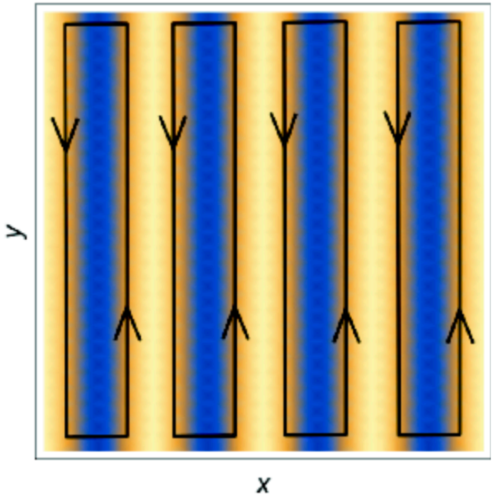
Modulated instabilities in holography

Spatial modulation often appears in condensed matter

Typical example (Ooguri-Park): instability of gauge fields in the bulk

Domokos, Harvey 0704.1604
Nakamura, Ooguri, Park 0911.0679; Ooguri, Park 1007.3737

- Induced by a Chern-Simons (CS) term (needed to implement axial and flavour anomalies on the field theory)
- Field theory ground state has modulated persistent currents



Also appear in top-down constructions:

- Witten-Sakai-Sugimoto model

Chuang, Dai, Kawamoto, Lin, Yeh 1004.0162; Ooguri, Park

- D3-D7 and D2-D8 models, with 2+1 dimensional dual

Bergman, Jokela, Lifschytz, Lippert. ; Jokela, Järvinen, Lippert

The V-QCD framework

- A bottom-up holographic model for to describe a QCD-like theory: SU(N_c) Yang-Mills coupled to fundamental matter.

- Full backreaction from flavor sector by Keeping $\frac{N_f}{N_c}$ fixed.

- The aim is to provide a general framework to modeling the physics of QCD as closely as possible with holography.

- Apart from confinement and chiral symmetry breaking, it describes, among other things, the hadron spectra, as well as physics at finite T and μ.

- In particular we can get the equation of state at finite T and μ and also compute the bulk viscosity.

C R J, Hoyos, Jokela, Järvinen et al. , *Phys.Rev.Lett.* 133 (2024) 7, 071901

Our Set-up

We can define: $\mathbf{A}_{L\,MN} = g_{MN} + w(\phi) F^{(L)}_{MN}$ A_L and A_R are N_f x N_f matrices in flavor space.

The action of the model is: $S = S_g + S_f + S_{\rm CS}$ with:

$$S_g = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left[R - \frac{4}{3} g^{MN} \partial_M \phi \partial_N \phi + V_g(\phi) \right]$$

$$S_f = -\frac{1}{32\pi G_5 N_c} \mathrm{Tr} \int d^4x\, dr \, \left(V_f(\lambda, T^\dagger T) \sqrt{-\det \mathbf{A}_L} + V_f(\lambda, T T^\dagger) \sqrt{-\det \mathbf{A}_R} \right)$$

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where $\lambda = e^{\phi}$ is identified as the 't Hooft coupling near the boundary.

The V-QCD dictionary

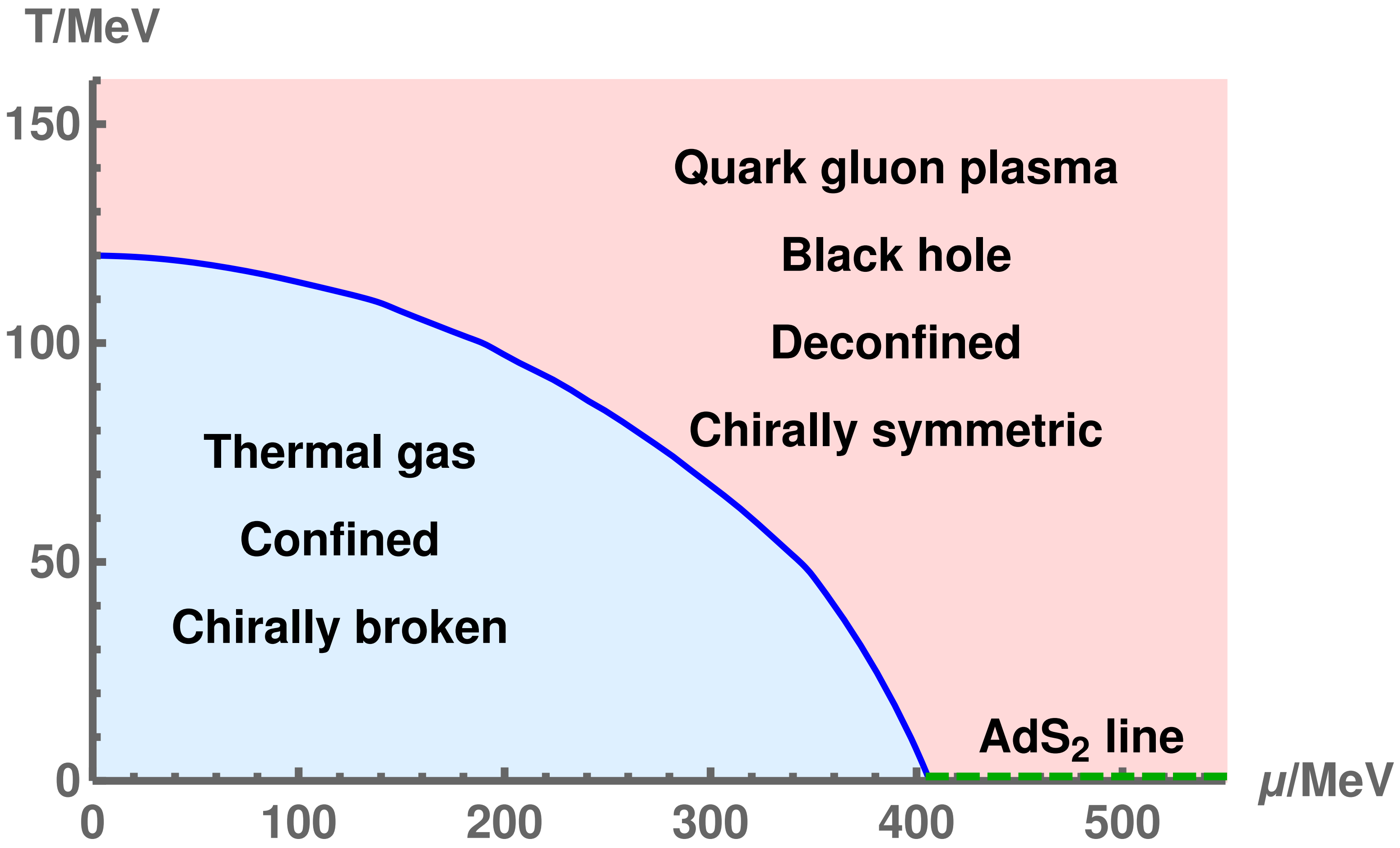
- The dilaton ϕ is dual to the F^2 operator, where F is the gluon field in QCD. The corresponding source is the coupling of QCD.
- The (bulk) metric is dual to the energy-momentum tensor $T_{\mu\nu}$ of QCD. The source is the metric of the field theory.
- The axion a is dual to $F \wedge F$, and the source is the θ-angle.
- The tachyon T^{ij}, where $i, j = 1 \ldots N_f$ are the flavor indices, is dual to the quark bilinear $\bar{\psi}^i \psi^j$. The source is therefore the quark mass matrix.
- The left and right handed gauge fields $A^{ij}_{L/R\,\mu}$ are dual to the chiral currents $\bar{\psi}^i (1 \pm \gamma_5) \gamma_\mu \psi^j$. The sources are various chemical potentials (for temporal components) and external fields (for spatial components).

$$S_a = -\frac{M_p^3}{2} \int d^5x \sqrt{-\det g}\, Z(\phi) \left[N_c \partial_M \hat{a} - \mathrm{Tr} \left(A_M^L - A_M^R \right) \right]^2$$

The V-QCD phase diagram

- Solving numerically regular black hole (BH) and horizonless geometries and computing free energies
- μ = quark number chemical potential = source for $\mathrm{Tr}(A_{L\,t} + A_{R\,t})$
- I will focus in the BH phase

  in the rest of the talk



The Chern-Simons term

$$S_{\rm CS} = \frac{iN_c}{24\pi^2}\int {\rm Tr}\Big[-iA_L\wedge F_L\wedge F_L + \frac{1}{2}A_L\wedge A_L\wedge A_L\wedge F_L + \frac{i}{10}A_L\wedge A_L\wedge A_L\wedge A_L\wedge A_L - (L\leftrightarrow R)\Big]$$

Writing $V=A_L+A_R/2$, and as the only non zero component in the background is $V_t(r)=\Phi(r)$ the CS term can be written as:

$$S_{\rm CS} = \frac{N_c}{2\pi^2}\int \Phi dt\wedge {\rm Tr}\left[d\tilde{A}\wedge d\tilde{V}\right]$$

Quasinormal modes in Holography

Modes classified according to helicity (rotations keeping q fixed)

– *helicity* 0 –

Abelian: $\delta\hat{V}_t, \delta\hat{V}_z, \delta\hat{A}_t, \delta\hat{A}_z, \delta g_{tz}, \delta g_{tt}, \delta g_{zz}, \delta g_{xx} + \delta g_{yy}, \delta\phi, \delta\tau, \delta\hat{a}$

GIVs: $\{Z_0, \hat{E}_V, Z_\phi\}$ $\{Z_a, \hat{E}_A\}$ $\{\delta\tau\}$

Non-Abelian: $\delta V_t^a, \delta V_z^a, \delta A_t^a, \delta A_z^a$

GIVs: $\{E_V^a\}$ $\{E_A^a\}$

– *helicity* ± 1 –

Abelian: $\delta\hat{V}_\alpha, \delta\hat{A}_\alpha, \delta g_{t\alpha}, \delta g_{z\alpha}$

GIVs: $\{Z_1^\pm, \delta\hat{V}^\pm, \delta\hat{A}^\pm\}$

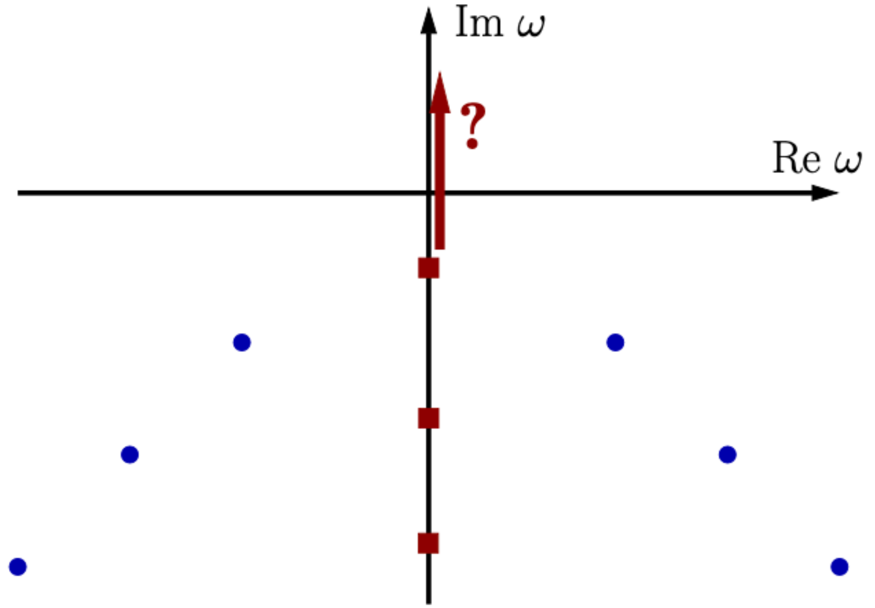
Non-Abelian: $\delta V_\alpha^a, \delta A_\alpha^a$

GIVs: $\{\delta A_{L\alpha}{}^a\}$ $\{\delta A_{R\alpha}{}^a\}$

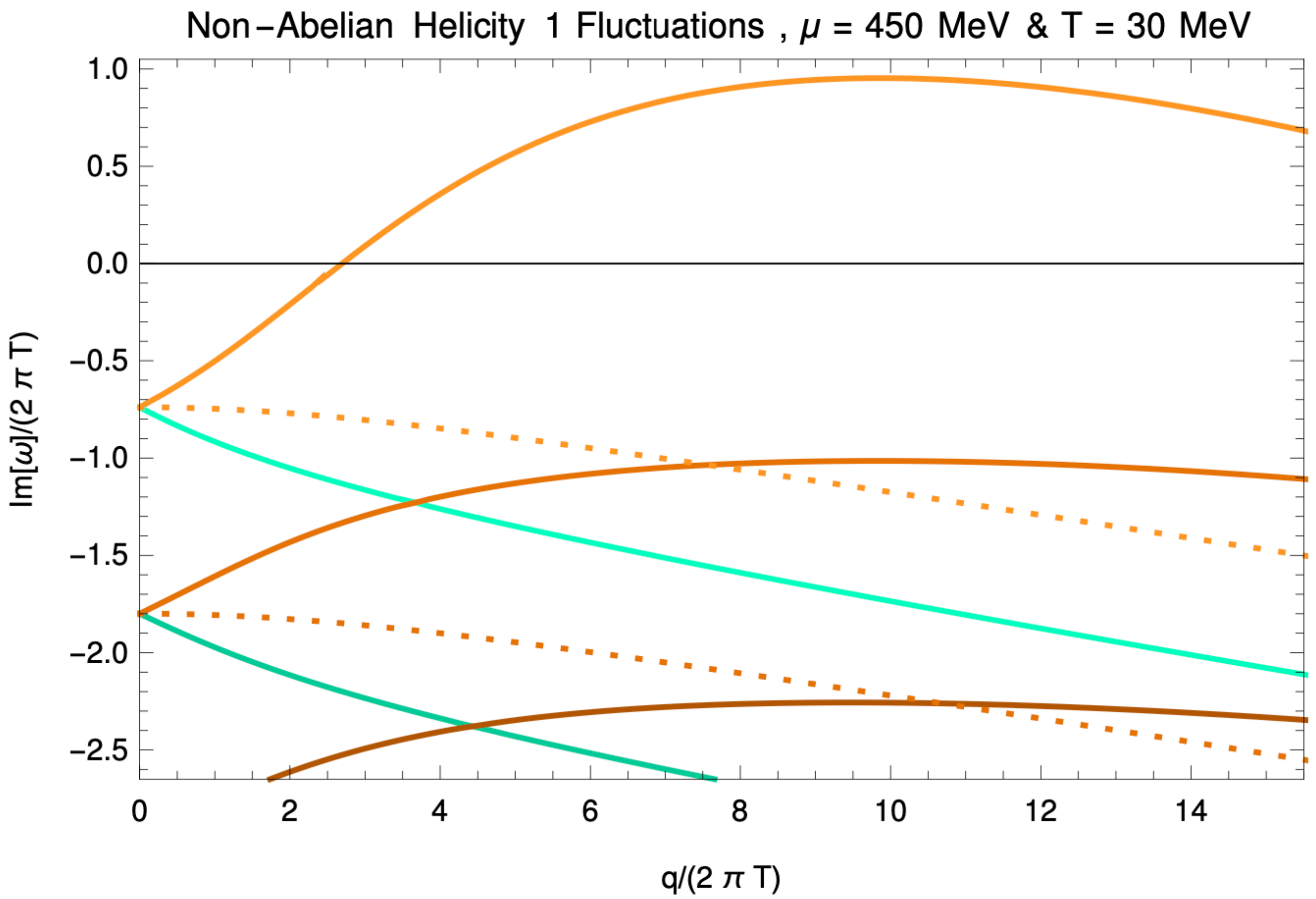
– *helicity* ± 2 –

$\delta g_{xy}, \delta g_{xx} - \delta g_{yy}$

GIVs: $\{Z_2^\pm\}$



Instability

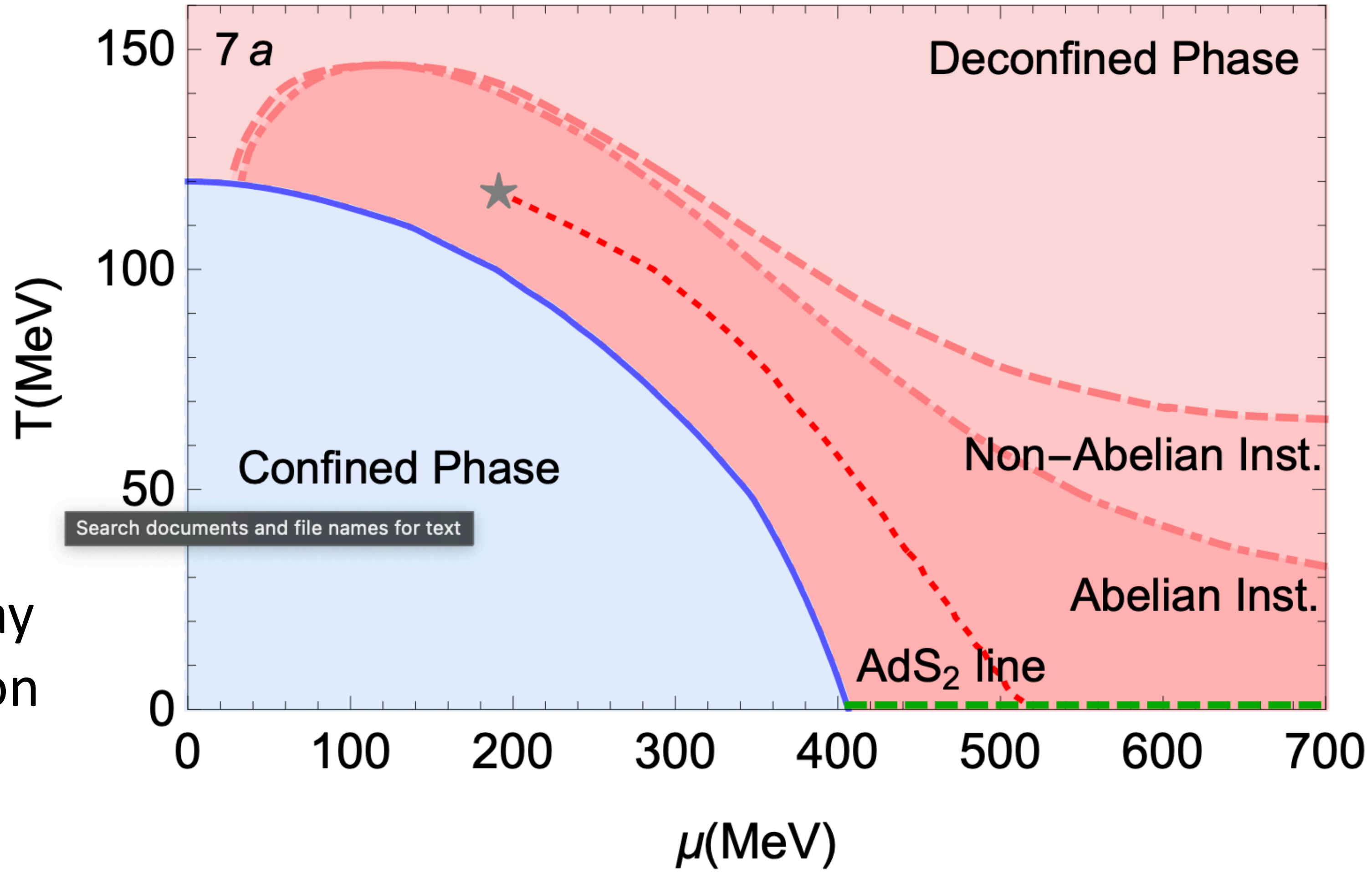


Result in the non-Abelian sector:

- Plain Ooguri-Park instability
- Disappears if the CS term is turned off (dotted curves)

Region with instability

- Strong instability when parameters fitted to QCD
- Surprise: extends to very low densities.
- Caveat: holographic description may be unreliable in the transition region



Approach to AdS_2

Geometric description of the flow:

Large T: "regular" black holes

- AdS_5 (UV) $\to$ horizon (IR)

Small T:

- AdS_5 (UV) $\to$ Small AdS_2 BH $\times R^3$ (IR)

Zero T:

- AdS_5 (UV) $\to AdS_2 \times R^3$ (IR)

The endpoint of the flow (AdS_2) is μ-independent

Faulkner, Liu, McGreevy, Vegh 0907.2694
Alho, Jarvinen, Kajantie, Kiritsis, Rosen, Tuominen;
Hoyos, Jokela, Jarvinen, Subils, Tarrio, Vuorinen

Expansions around the IR and UV known analytically in all cases

QNMs in the AdS_2 region: zoom into $T \to 0$

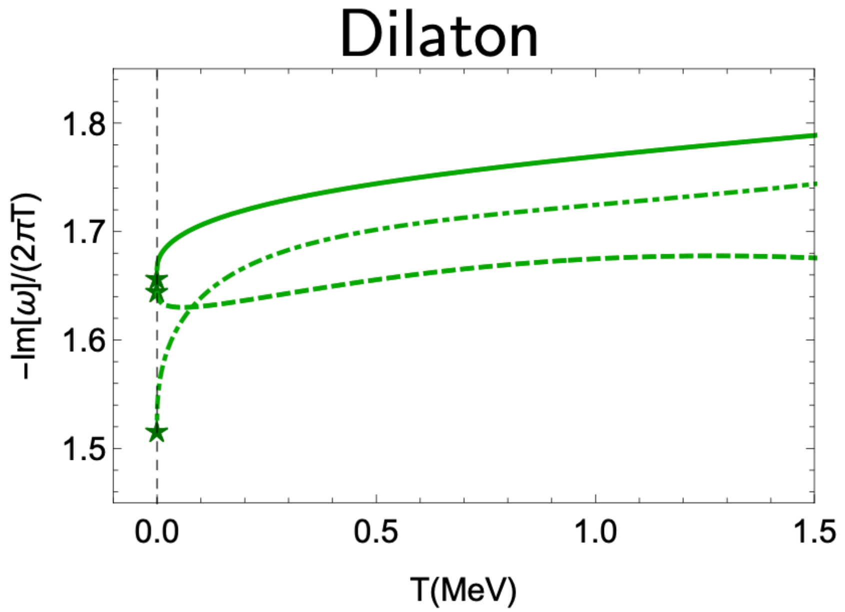
All imaginary "AdS_2 modes" arrange into towers as $T \to 0$, (Δ depends on the fluctuation)

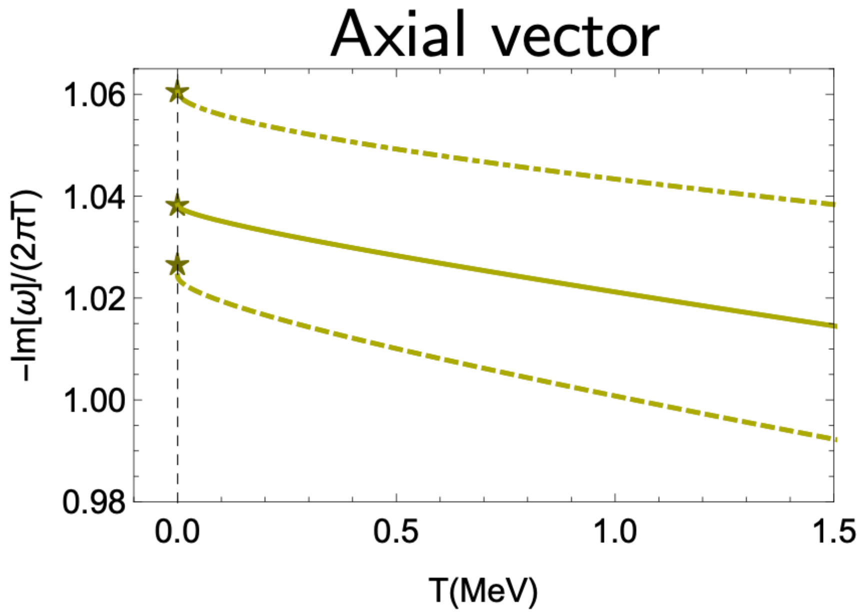
$$\frac{\omega}{2\pi T} \longrightarrow -i\,(\Delta + n)\ , \qquad n = 0, 1, 2, \ldots$$

Faulkner, Iqbal, Liu, McGreevy, Vegh 1101.0597

Integer Δ: "universal" modes (metric $\Delta = 0$ or 1, vector $\Delta = 2$)

Δ is known analytically: obtained by considering fluctuations for the AdS_2 metric





Summary

- A strong Ooguri-Park instability observed
  - It extends to low density, where the model is determined through fit to QCD lattice data
  - Expected to compete with color superconducting phases (to be added)

- We've analyzed the QNM for all sectors as a function of momentum, temperature and for different fixed values of μ, and found the behavior at finite momentum of the modes.

- Detailed study of the approach to zero temperature and the AdS_2 region
  - Universal breakdown of hydrodynamics (in the unstable phase)

Thank you all for your attention.

Comparison to data

V-QCD is constrained by fixing the potentials requiring qualitative agreement with QCD

- With a choice of Vg, with asymptotics similar to noncritical string theory, the glue sector, IHQCD, confines and produces a mass gap for glueballs

- Fit to lattice data (Equation of State and baryon number susceptibility) near $\mu=0$

Gürsoy, Kiritsis, Mazzanti, Nitti; Järvinen, Jokela Remes

- UV expansions at small λ should match pQCD

Gürsoy, Kiritsis ; Järvinen, Kiritsis

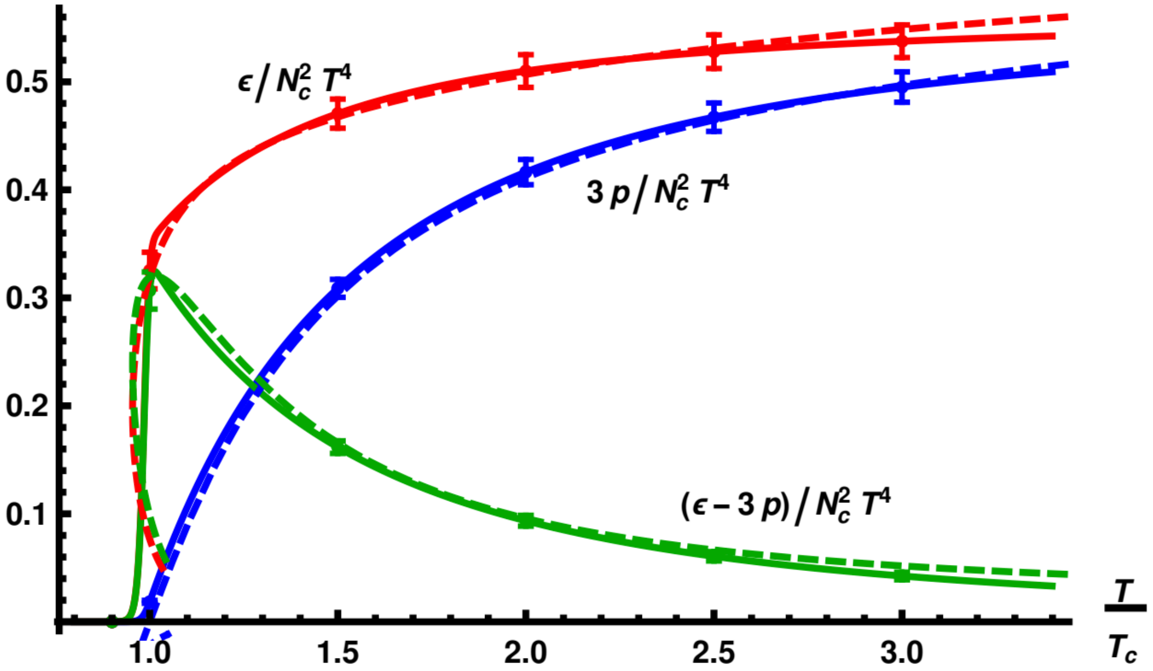
- Extrapolated V-QCD for cold quark matter agrees with known constraints for the equation of state from astrophysical measurements **Järvinen, Jokela Remes**

Fitting of VQCD to Lattice data

Having determined the asymptotics of the potentials both at small and large λ, one needs to tune the potentials in the middle so that the model agrees quantitatively with QCD

The main available data are lattice data for the thermodynamics of QCD (at small density) and experimental data for QCD spectrum

The potentials are tuned so that both the zero temperature vacuum properties (glueball spectra) and finite temperature thermodynamics both agree remarkably well with results from lattice simulations.

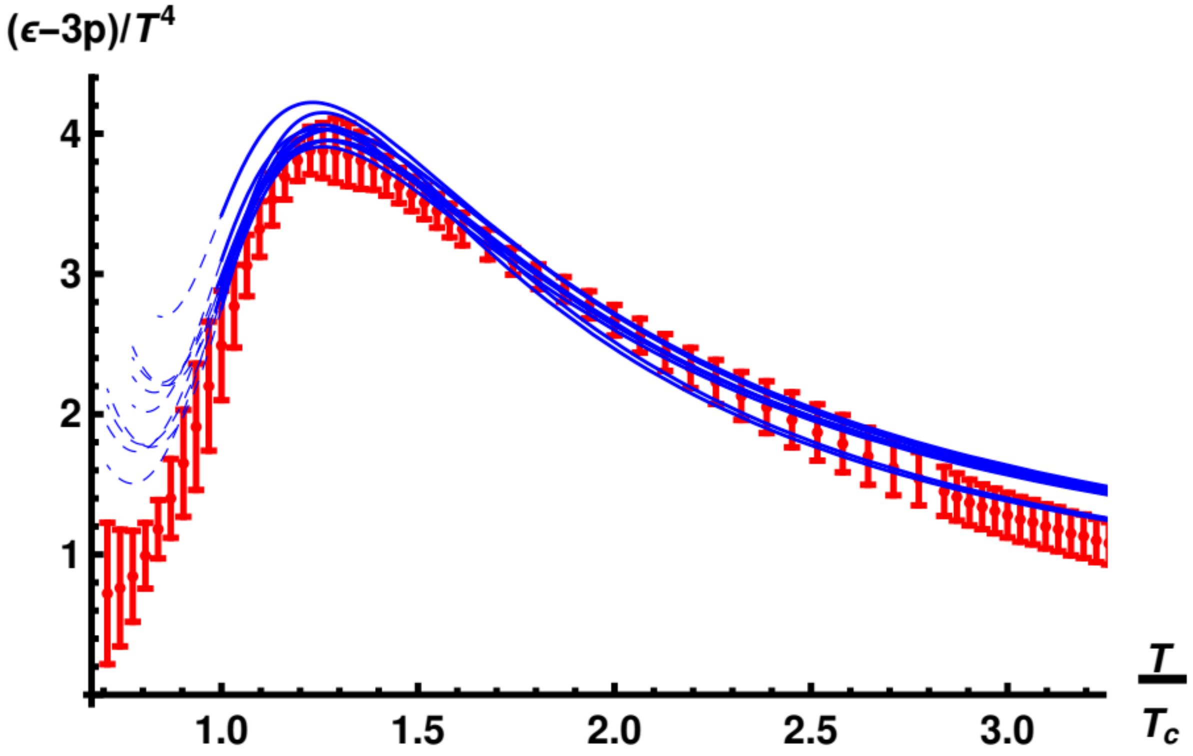


Fitting $V_g(\lambda)$ to the large Nc lattice data for pure Yang Mills. Red, blue, and green curves are for energy density, pressure, and interaction measure.

Lattice data taken from: M. Panero, "Thermodynamics of the QCD plasma and the large-N limit," *Phys. Rev. Lett.* **103** (2009) 232001, 0907.3719.

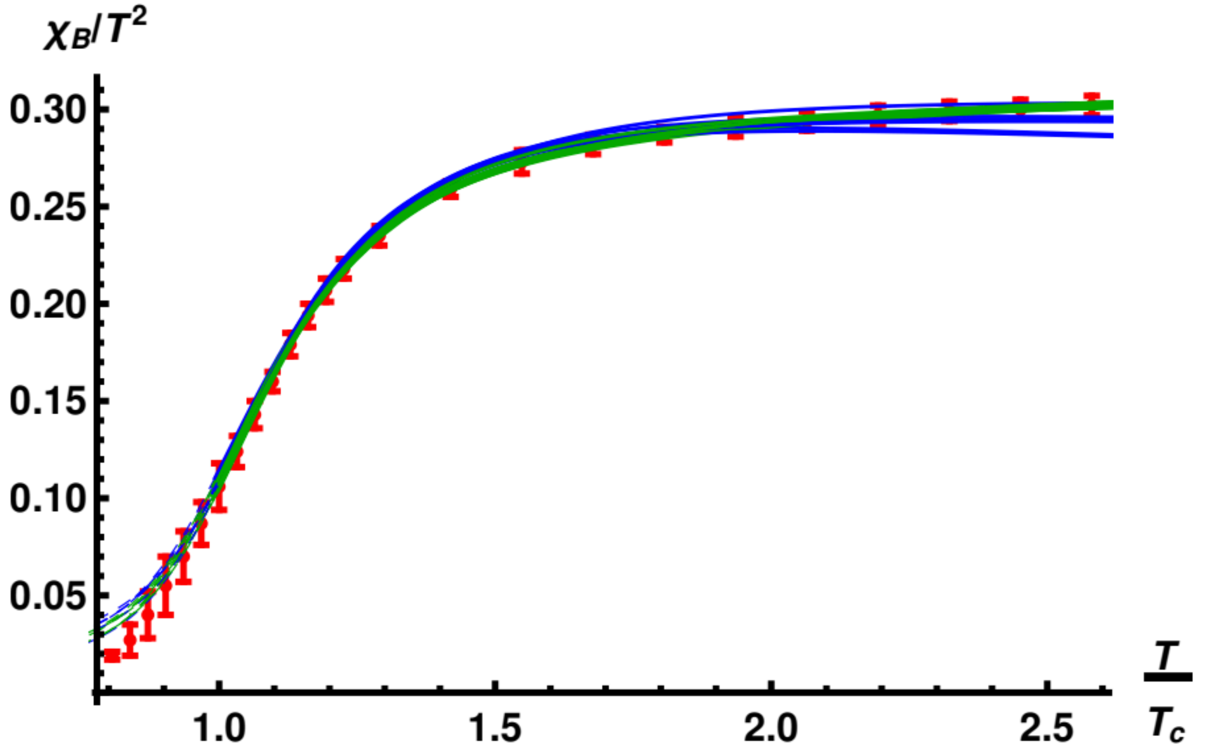
N. Jokela, M. Jarvinen, and J. Remes,

Fitting of VQCD to Lattice data



Fitting $V_{f0}(\lambda)$ to the QCD lattice data for the interaction measure. The red dots and error bars show the lattice data, and blue curves are V-QCD fits.

S. Borsanyi, Z. Fodor, S. D. Katz, S. Krieg, C. Ratti, and K. Szabo, "Fluctuations of conserved charges at finite temperature from lattice QCD," *JHEP* **01** (2012) 138, 1112.4416.



Fitting $w(\lambda)$ to the QCD lattice data for the cumulant χB. The red dots and error bars show the lattice data, and blue and green curves are V-QCD fits.

S. Borsanyi, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg, and K. K. Szabo, "Full result for the QCD equation of state with 2+1 flavors," *Phys. Lett. B* **730** (2014) 99–104, 1309.5258.

Method for localization of QNM

QNM spectrum comes from solving an ODE eigenvalue problem

We use the pseudospectral method to find the QNM. The solution is approximated as a weighted sum of a set of basis functions –> translate the problem to a system of algebraic equations.

The essence of pseudospectral method is the discretization of differential equations to turn it to numerically solvable matrix equations

We locate the poles of $\ln\left(\,|\,\det M\,|\,\right)$ where M is a matrix constructed with the coefficient of the fluctuation equations, the field fluctuations and the matrix of differential operators on the grid.

Definition of the scale Λ

Usually in bottom-up models of QCD one makes sure that the leading behavior of the bulk fields near the boundary agrees with the leading UV dimensions of the dual operators.

Here we also require that the first few quantum corrections, and the RG flow imposed by them, agrees with the near-boundary holographic RG flow of the bulk fields

The boundary conditions for the tachyon T are such that it vanishes near the boundary. It also turns out that the gauge field is irrelevant for the boundary behavior of the metric

Setting T= 0 in the action the geometry is determined by the effective potential $V_{\text{eff}} = V_g - xV_f$

For the geometry to be asymptotically AdS at the boundary we need. $V_{\text{eff}}(\lambda) \to \frac{12}{\ell^2}$ as $\lambda \to 0$

Definition of the scale Λ

Assuming a Taylor series around. λ= 0 we get that the near boundary asymptotics of the geometry (with $A \sim \log(\mu)$) is AdS5 with logarithmic corrections

$$A(r) = -\log\frac{r}{\ell_0} + \frac{4}{9\log(r\Lambda)} \quad + \frac{\left(\frac{95}{162} - \frac{32v_2}{81v_1^2}\right) + \left(-\frac{23}{81} + \frac{64v_2}{81v_1^2}\right)\log(-\log(r\Lambda))}{(\log(r\Lambda))^2} + \mathcal{O}\left(\frac{1}{(\log(r\Lambda))^3}\right)$$

$$\frac{v_1\lambda(r)}{\lambda_0} = -\frac{8}{9\log(r\Lambda)} + \frac{\left(\frac{46}{81} - \frac{128v_2}{81v_1^2}\right)\log(-\log(r\Lambda))}{(\log(r\Lambda))^2} + \mathcal{O}\left(\frac{1}{(\log(r\Lambda))^3}\right).$$

The source term of the dilaton λ has become logarithmically flowing instead of a constant, and the value of the source is now identified with the scale $\Lambda = \Lambda_{UV}$. This scale defines the units for all dimensionful quantities

AdS_2 regions in holography

Generic phenomenon: $AdS_2 \times \mathbb{R}^n$ appears in the extremal limit of charged black holes

- AdS_2 point $\leftrightarrow$ Emergence of a quantum critical region of the field theory, controlled by a (0+1) dimensional IR CFT

Faulkner, Liu, McGreevy, Vegh 0907.2694

- Near AdS_2 points, as $T \to 0$ one obtains universal results for the equilibration rates

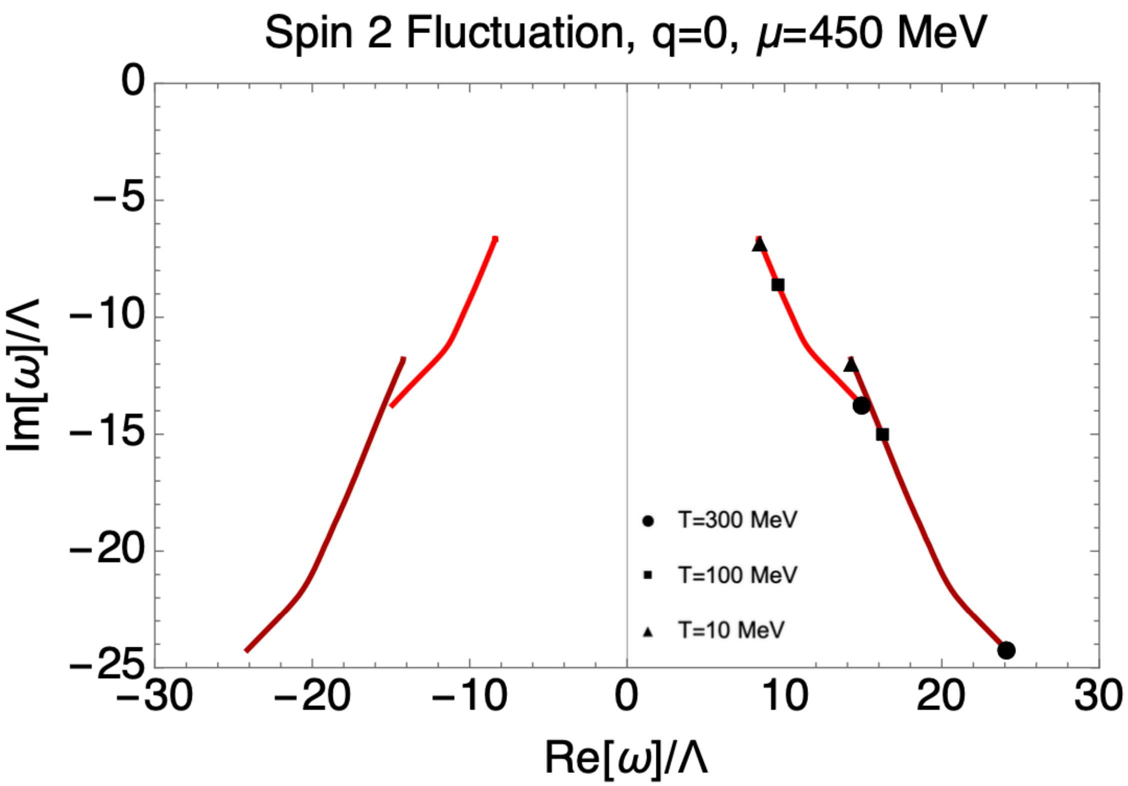
Grozdanov, Kovtun, Starinets, Tadic 1904.01018, 1904.12862

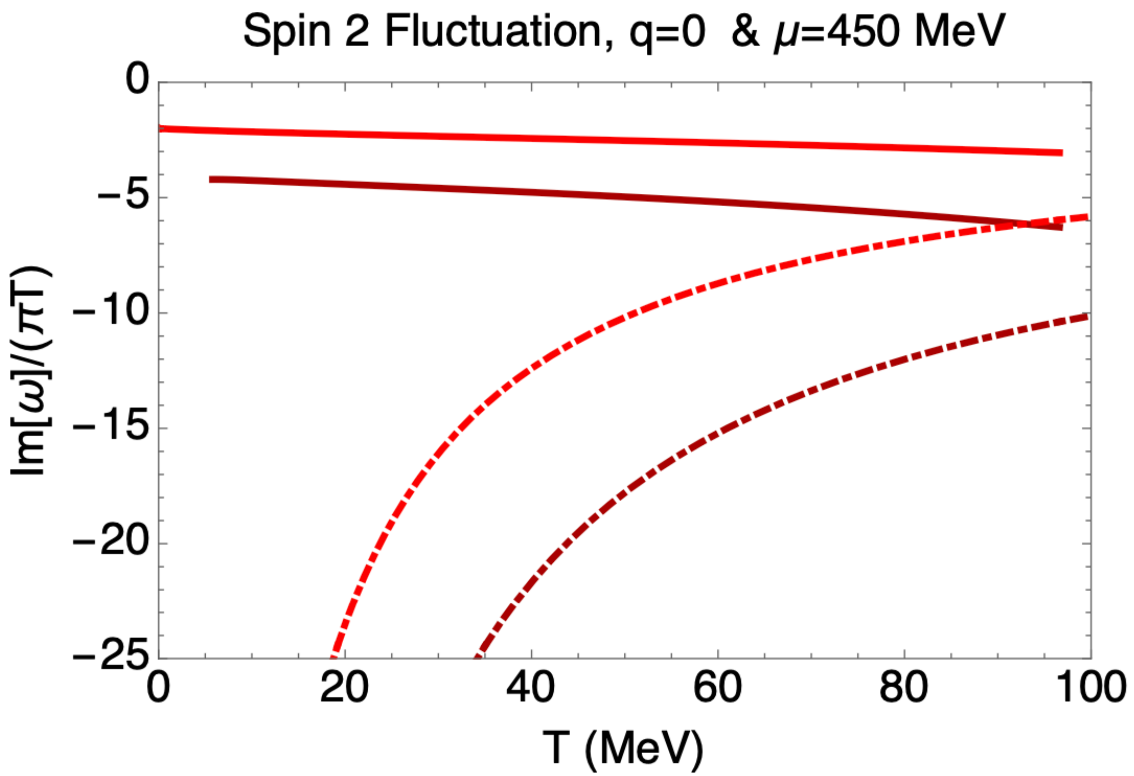
- Breakdown of hydrodynamics is caused by modes associated to the AdS_2 region, as a consequence the critical frequency is set by data coming from IR

$$\omega_{\rm eq} = 2\pi \Delta T \ , \qquad k^2_{\rm eq} = \frac{\omega_{\rm eq}}{D}$$

Areán, Davison, Goutéraux, Suzuki 2011.12301

QNM modes as a function of T





As T is lowered:

- Complex modes obey Imω ~ μ ~ Λ (= the UV scale)
- Imaginary "AdS_2" modes obey Im ω ~ T
- ⇒ imaginary modes closer to real axis, more important