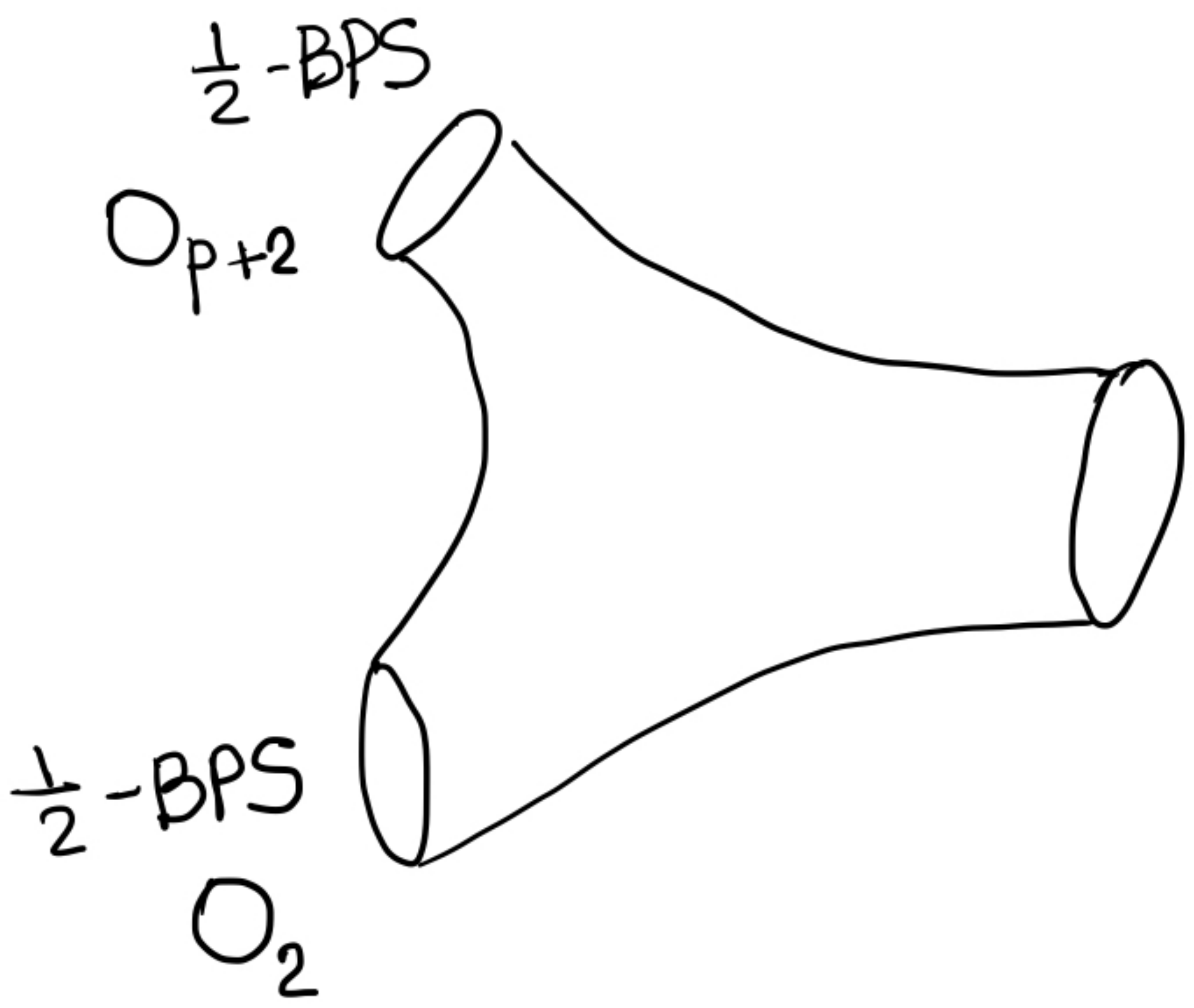
CFT Data of Stringy Operators of N = 4 Super-Yang-Mills at Strong Coupling

Nika Sokolova
PhD Student at KCL London

Eurostrings 2024
University of Southampton

Based on arXiv: 2310.06041 **and on upcoming work in collaboration with Julius Julius**

Probing Excited Strings on AdS$_5 \times S_5$



Long: $[\Delta, \ell, \ell, 0, p, 0]$

spin R-charge

"Stringy operators"

Anomalous Dimension

$\Delta = 2\sqrt{\delta}\,\lambda^{1/4} + \dots$

string mass level $\delta = 1, 2, 3 \dots$

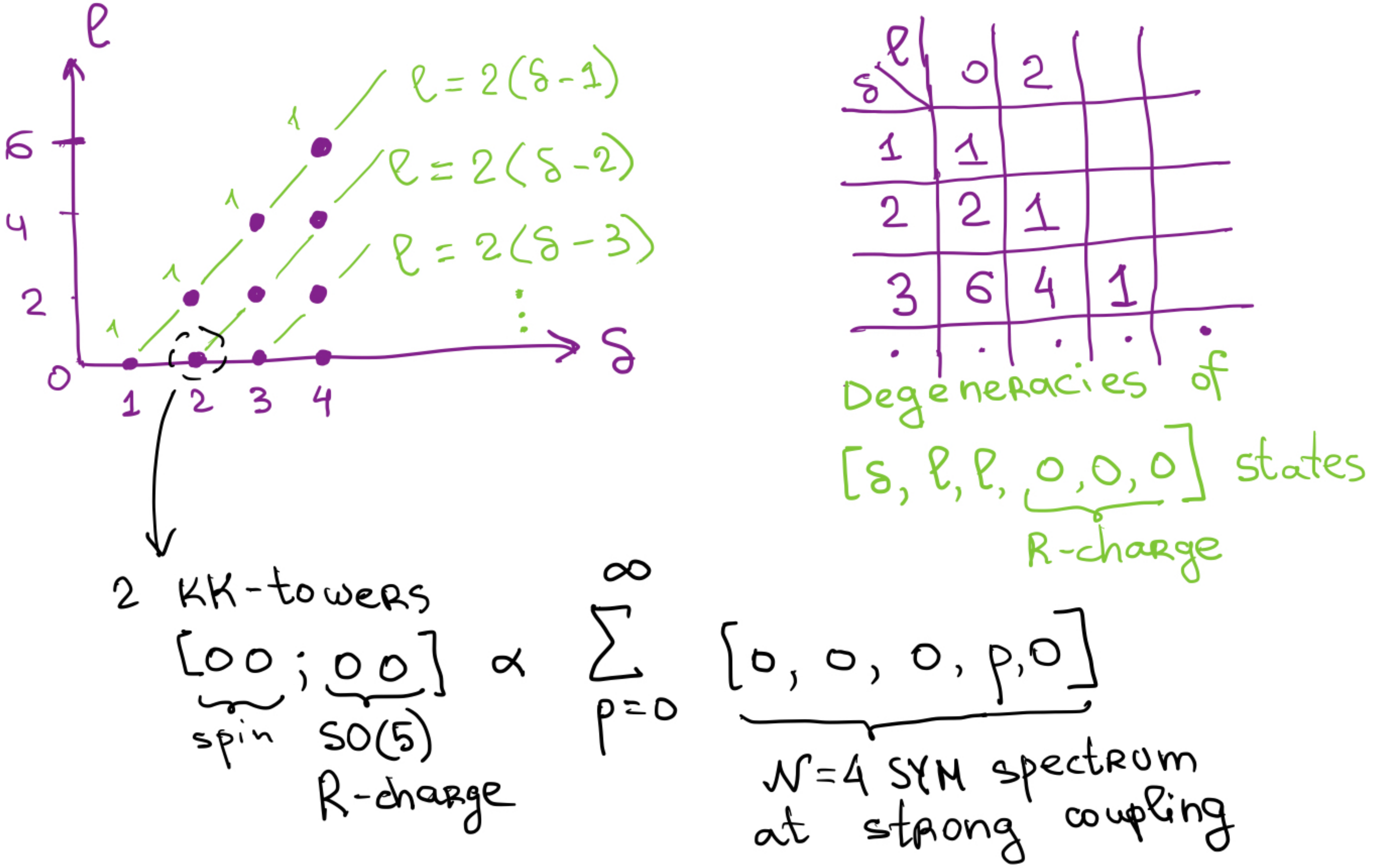
Strong coupling in the planar limit

$\lambda \equiv g_{YM}^2 N \qquad g \equiv \frac{\sqrt{\lambda}}{4\pi}$

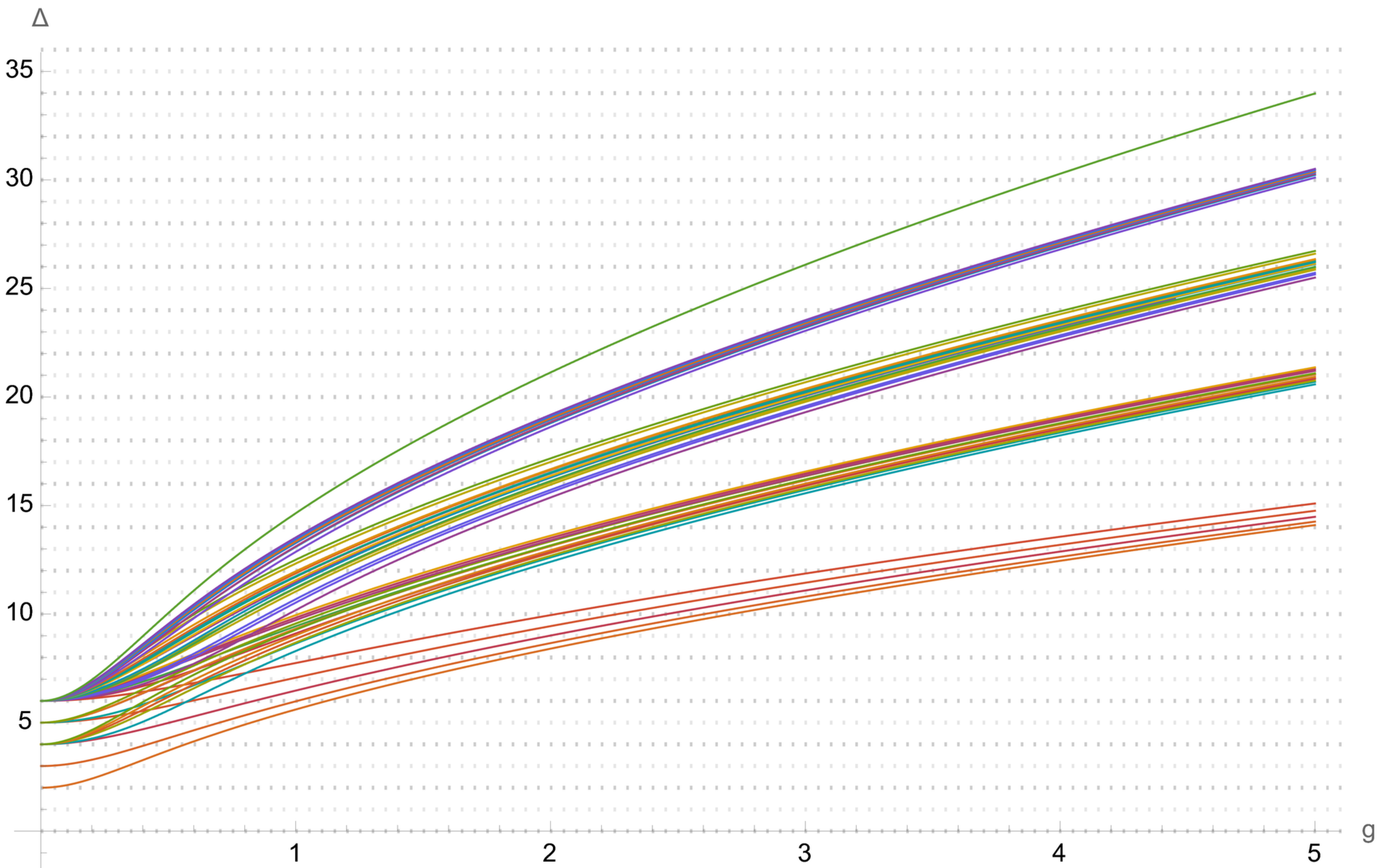
We find the spectrum numerically for many stringy operators at $\delta = 1,2,3$ and we would like to use this diverse spectrum information to unmix the degeneracies of the exchanged strings.

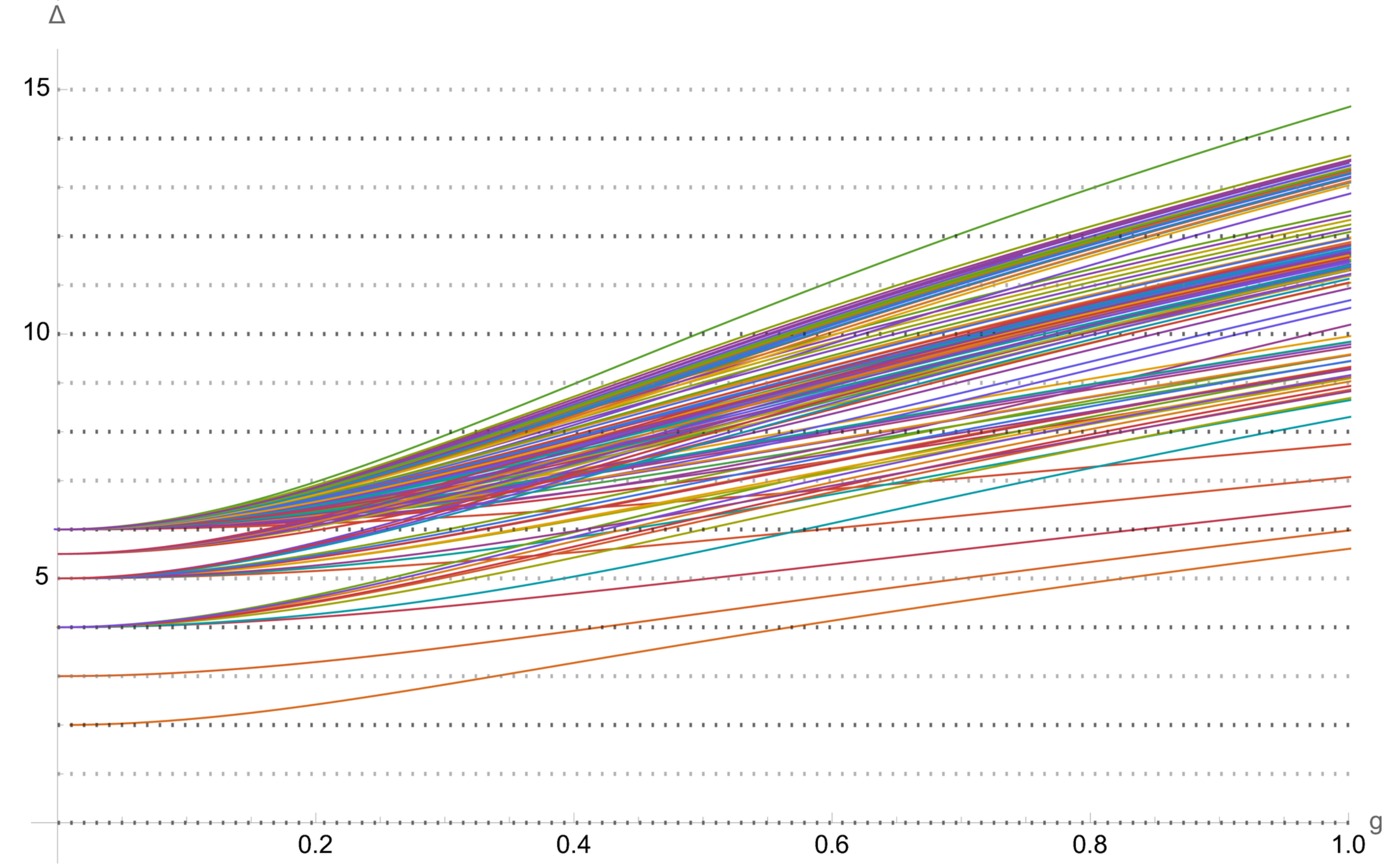
Degeneracies of the Stringy Spectrum

It is possible to count states with given δ in the flat space limit by counting the representations of $SO(9)$, the massive little group of $\mathbb{R}^{1,9}$ [Alday, Hansen, Silva '23].



Numerical Spectrum from QSC





[Gromov, Hegedus, Julius, NS ’23]

- At strong coupling spectrum forms “bands” with slopes corresponding to “string mass levels” [Gubser, Klebanov, Polyakov ’98]: $\Delta \simeq 2\sqrt{\delta}\lambda^{\frac{1}{4}}$.
- Every “band” with $\delta = 1,2,3\ldots$ has infinitely many states of $\mathcal{N} = 4$ SYM (finite number of KK towers).
- Degeneracies of states with the same δ and ℓ are lifted. The sub-leading coefficient d_1 for the most of the states is determined: $\Delta \simeq 2\sqrt{\delta}\lambda^{1/4} - 2 + \dfrac{d_1}{\sqrt{\delta}\lambda^{1/4}}$ which turns out to be a simple rational number.

States from can be continued to much higher g coupling [Ekhammar, Gromov, Ryan’ 24].

Deciphering Kaluza-Klein Towers

[Gromov, Hegedus, Julius, NS '23]

- Task: sort states into KK-towers
- Notice: subleasing Casimir j_1 is a constant for states in a KK-tower

$$J^2 \simeq 2\,\delta\sqrt{\lambda} + j_1 + \frac{j_2}{\delta\sqrt{\lambda}}$$

- Idea: Use this to classify
- Conjecture:

$$d_1 = \frac{p^2}{4} + \frac{p}{4}\left(q_1 + q_2 + 4\right) + \frac{j_1}{2} + \frac{1}{16}\Big[16 - 2\,\ell_1\,(\ell_1+2) - 2\,\ell_2\,(\ell_2+2) + 3\,q_1\,(q_1+4) + 3\,q_2\,(q_2+4) + 2\,q_1\,q_2\Big]$$

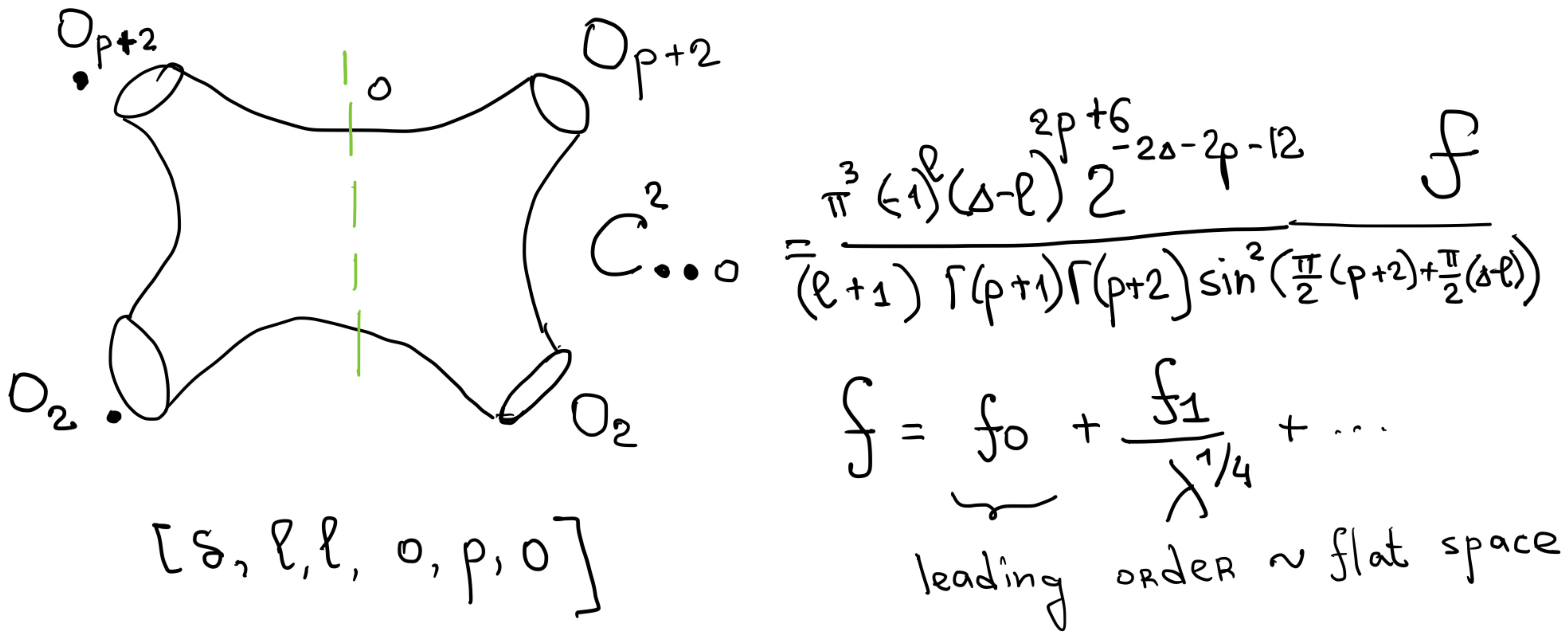
St. No.	State ID	$[\ell_1\ \ell_2\ q_1\ p\ q_2]$	Δ_0	$\Delta_0\ +\#\ g^2$	δ	$2\sqrt{\delta}\lambda^{1/4}-2+\frac{\#}{\sqrt{\delta}}\frac{1}{\lambda^{1/4}}$	d_1	j_1	Type	Degs.
3	$_4[0\ 0\ 2\ 2\ 2\ 2\ 0\ 0]_1$	[0 0 0 0 0]	4	$13-\sqrt{41}$	2	1.9999999	2	2	I	
4	$_4[0\ 0\ 2\ 2\ 2\ 2\ 0\ 0]_2$	[0 0 0 0 0]	4	$13+\sqrt{41}$	2	8.0000	8	14	I	
5	$_4[0\ 0\ 3\ 2\ 2\ 1\ 0\ 0]_1$	[0 0 1 0 1]	4	12	2	4.000000	4	2	I	
6	$_4[0\ 0\ 3\ 3\ 1\ 1\ 0\ 0]_1$	[0 0 0 2 0]	4	$10+2\sqrt{5}$	2	4.9999998	5	2	I	
13	$_5[0\ 0\ 3\ 3\ 3\ 1\ 0\ 0]_1$	[0 0 0 0 2]	5	6.788897449	2	4.25		2.00	III	15
15	$_5[0\ 0\ 4\ 2\ 2\ 2\ 0\ 0]_1$	[0 0 2 0 0]	5	6.788897449	2	4.25		2.00	III	13
17	$_5[0\ 0\ 3\ 3\ 2\ 2\ 0\ 0]_1$	[0 0 0 1 0]	5	5.527864045	2	3.2500000	$\frac{13}{4}$	2	I	
18	$_5[0\ 0\ 3\ 3\ 2\ 2\ 0\ 0]_2$	[0 0 0 1 0]	5	14.47213595	2	9.249999	$\frac{37}{4}$	14	I	
21	$_5[0\ 0\ 4\ 3\ 2\ 1\ 0\ 0]_1$	[0 0 1 1 1]	5	10	2	5.75007	$\frac{23}{4}$	2	II	22
22	$_5[0\ 0\ 4\ 3\ 2\ 1\ 0\ 0]_2$	[0 0 1 1 1]	5	10	2	5.750	$\frac{23}{4}$	2	II	21
24	$_5[0\ 0\ 4\ 4\ 1\ 1\ 0\ 0]_2$	[0 0 0 3 0]	5	12	2	7.24999996	$\frac{29}{4}$	2	I	
97	$_6[0\ 0\ 5\ 3\ 3\ 1\ 0\ 0]_2$	[0 0 2 0 2]	6	6.491188584	2	7.00000	7	2	I	
99	$_6[0\ 0\ 4\ 4\ 3\ 1\ 0\ 0]_1$	[0 0 0 1 2]	6	5.395626364	2	6.000	6	2	III	102
102	$_6[0\ 0\ 4\ 4\ 3\ 1\ 0\ 0]_1$	[0 0 2 1 0]	6	5.395626364	2	6.000	6	2	III	99
107	$_6[0\ 0\ 4\ 4\ 2\ 2\ 0\ 0]_3$	[0 0 0 2 0]	6	10.67351551	2	11.000000	11	14	I	
109	$_6[0\ 0\ 4\ 4\ 2\ 2\ 0\ 0]_5$	[0 0 0 2 0]	6	4.524563121	2	5.00000	5	2	I	
116	$_6[0\ 0\ 5\ 4\ 2\ 1\ 0\ 0]_2$	[0 0 1 2 1]	6	8	2	8.000	8	2	II	117
117	$_6[0\ 0\ 5\ 4\ 2\ 1\ 0\ 0]_3$	[0 0 1 2 1]	6	8	2	8.000	8	2	II	116
119	$_6[0\ 0\ 5\ 5\ 1\ 1\ 0\ 0]_2$	[0 0 0 4 0]	6	9.780167472	2	10.00000000	10	2	I	

- (green) belong to one of the 2 [0 0; 0 0], with the value of j_1 breaking the ambiguity,
- (blue) belong to [0 0; 0 2], (yellow) belong to [0 0; 2 0],
- (white) belong to either [0 0; 2 0], [0 0; 0 2] but it can't be identified uniquely.

States with $\delta = 2,\ \ell_1 = \ell_2 = 0$

AdS Virasoro-Shapiro Amplitude Constraints

Curvature corrections ($R \propto \frac{1}{\lambda^{1/4}}$) to AdS Virasoro-Shapiro Amplitude can be successfully computed [Alday, Hansen, Silva, Fardelli, Nocchi' 22 onwards]. See Tobias's talk featuring AdS Veneziano amplitude.



As a byproduct of the VSA computations, we have constraints on the CFT data in the form

$$\langle f_0 \rangle_{\ell=2(\delta-n)} \equiv \sum_{I=1}^{N_p} f_0^I = g_n^{(0)}(\delta),\ \langle f_0\ d_1 \rangle_{\ell=2(\delta-n)} \equiv \sum_{I=1}^{N_p} f_0^I d_1^I = g_n^{(1)}(\delta, p),\ \langle f_0\ d_1^2 \rangle_{\ell=2(\delta-n)} \equiv \sum_{I=1}^{N_p} f_0^I d_1^I = g_n^{(2)}(\delta, p)$$

where we sum over N_p degeneracies of states with same (δ, ℓ).

“Unmixing” the Conformal Data

[Julius and NS ’23]

As we know the relation between d_1 and j_1, it is possible to rewrite the average formulas in terms of j_1. For example

$$\langle f_0 \rangle_{\ell=2(\delta-2)} = \frac{r_1(\delta)}{3}(2\delta^2 + 3\delta - 8),$$

$$\langle f_0\ j_1 \rangle_{\ell=2(\delta-2)} = \frac{r_1(\delta)}{9}(30\delta^4 + 7\delta^3 - 147\delta^2 + 212\delta - 120).$$

$\delta \backslash \ell$	0	1
1	[00;00]	
2	2[00;00] + [00;20]	[00;00]

2 degenerate KK-towers with $j_1^{[00]_1} = 2$, $j_1^{[00]_2} = 14$

1 KK-tower with $j_1^{[20]_1} = 2$

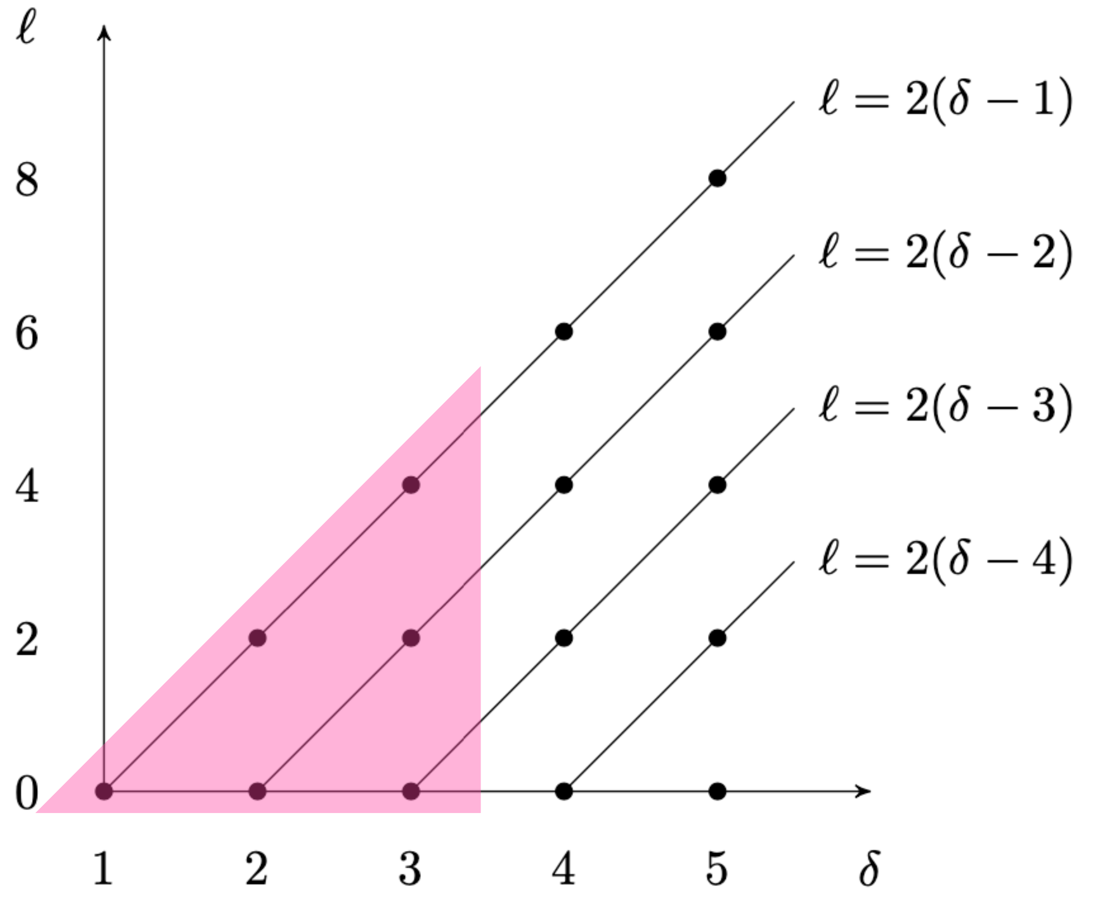
Unmixing gives us $f_0^{[00]_1} = -f_0^{[20]_1}$, $f_0^{[00]_2} = \frac{1}{4}$.

Positivity of OPE coefficients gives us $f_0^{[00]_1} = f_0^{[20]_1} = 0$

Patterns for the Structure Constants

[Julius and NS '23]

Let us list the extracted set of the leading order structure constants f_0 for the towers from the table.



$\delta \backslash \ell$	0	2	4
1	$\mathrm{KK}_{[0\ 0]}$		
2	$2\,\mathrm{KK}_{[0\ 0]} + \mathrm{KK}_{[2\ 0]}$	$\mathrm{KK}_{[0\ 0]}$	
3	$6\,\mathrm{KK}_{[0\ 0]} + 2\,\mathrm{KK}_{[1\ 0]} + 4\,\mathrm{KK}_{[2\ 0]} + \mathrm{KK}_{[4\ 0]}$	$4\,\mathrm{KK}_{[0\ 0]} + 3\,\mathrm{KK}_{[2\ 0]}$	$\mathrm{KK}_{[0\ 0]}$

$\delta = 2,\ \ell = 0$

$$f_0^{[0\ 0]_1} = f_0^{[2\ 0]_1} = 0 \quad f_0^{[0\ 0]_2} = \frac{1}{4}$$

$\delta = 3,\ \ell = 2$

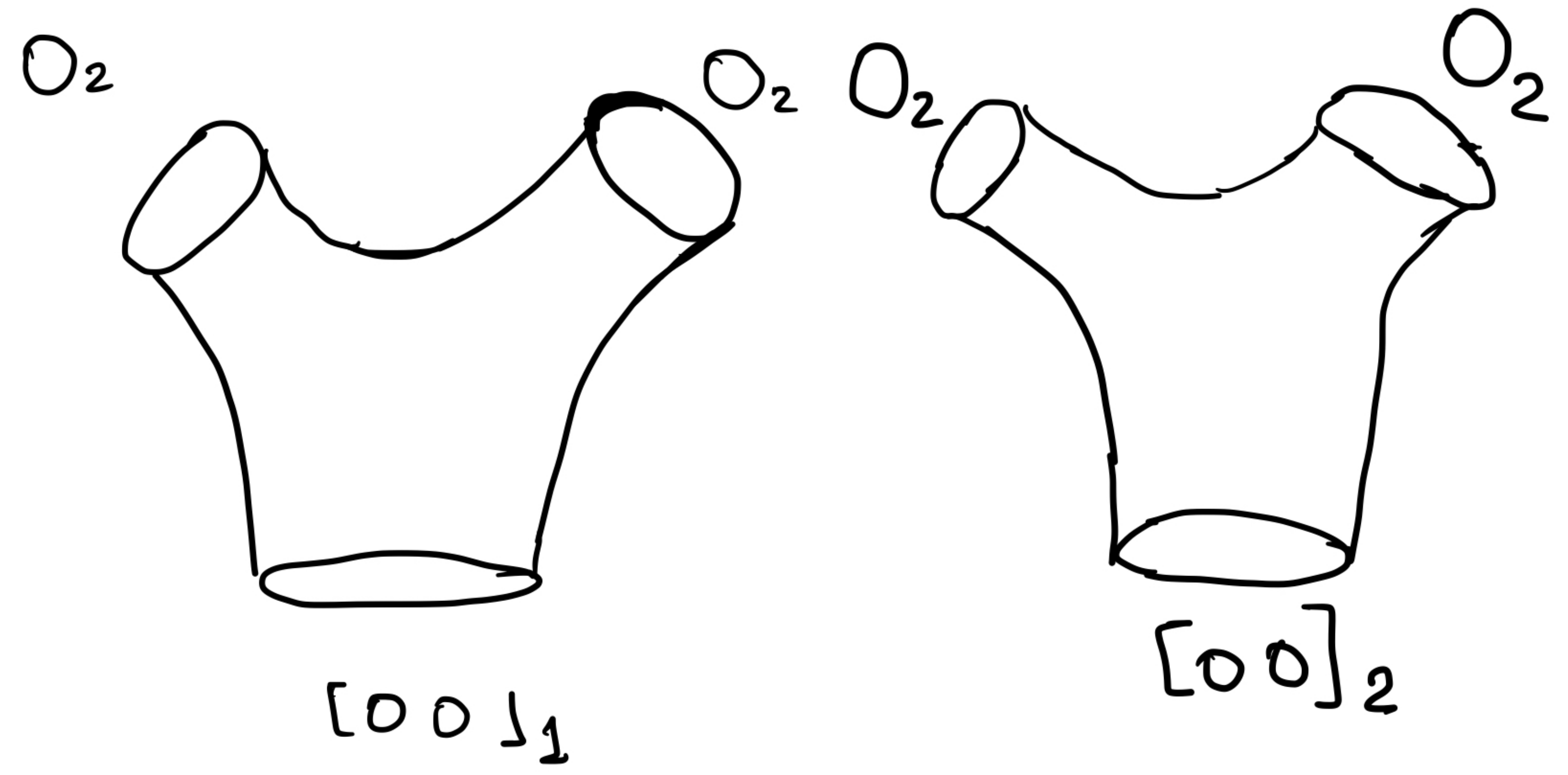
$$f_0^{[0\ 0]_1} = f_0^{[2\ 0]_1} = f_0^{[2\ 0]_2} = f_0^{[2\ 0]_3} = 0 \quad f_0^{[0\ 0]_2} = \frac{243}{1024} \quad f_0^{[0\ 0]_3} = f_0^{[0\ 0]_4} = \frac{135}{1024}$$

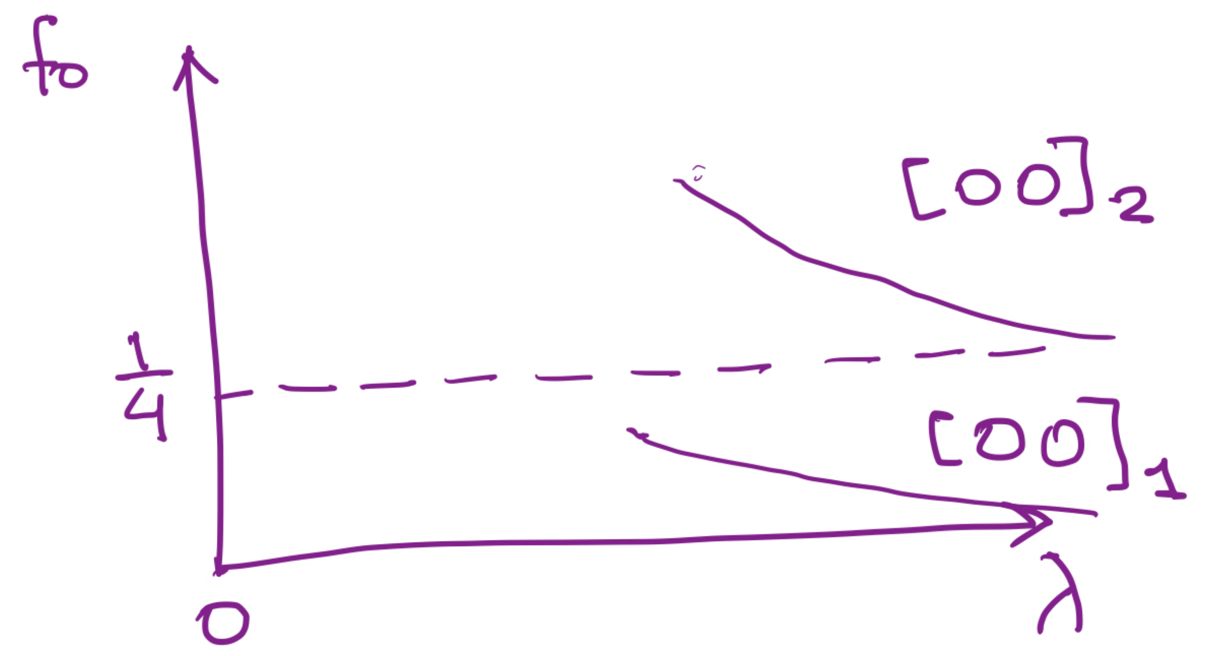
$\delta = 3,\ \ell = 0$

$$f_0^{[0\ 0]_1} = f_0^{[0\ 0]_2} = f_0^{[0\ 0]_3} = f_0^{[0\ 0]_6} = f_0^{[1\ 0]_1} = f_0^{[1\ 0]_2} = f_0^{[2\ 0]_1} = f_0^{[2\ 0]_2} = f_0^{[2\ 0]_3} = f_0^{[2\ 0]_4} = f_0^{[4\ 0]_1} = 0 \quad f_0^{[0\ 0]_4} = \frac{25}{1024} \quad f_0^{[0\ 0]_5} = \frac{81}{1024}\,.$$

Patterns to notice: 1. Many vanishing structure constants at the leading order, 2. Remaining structure constants are rational numbers.

Questions and Puzzles



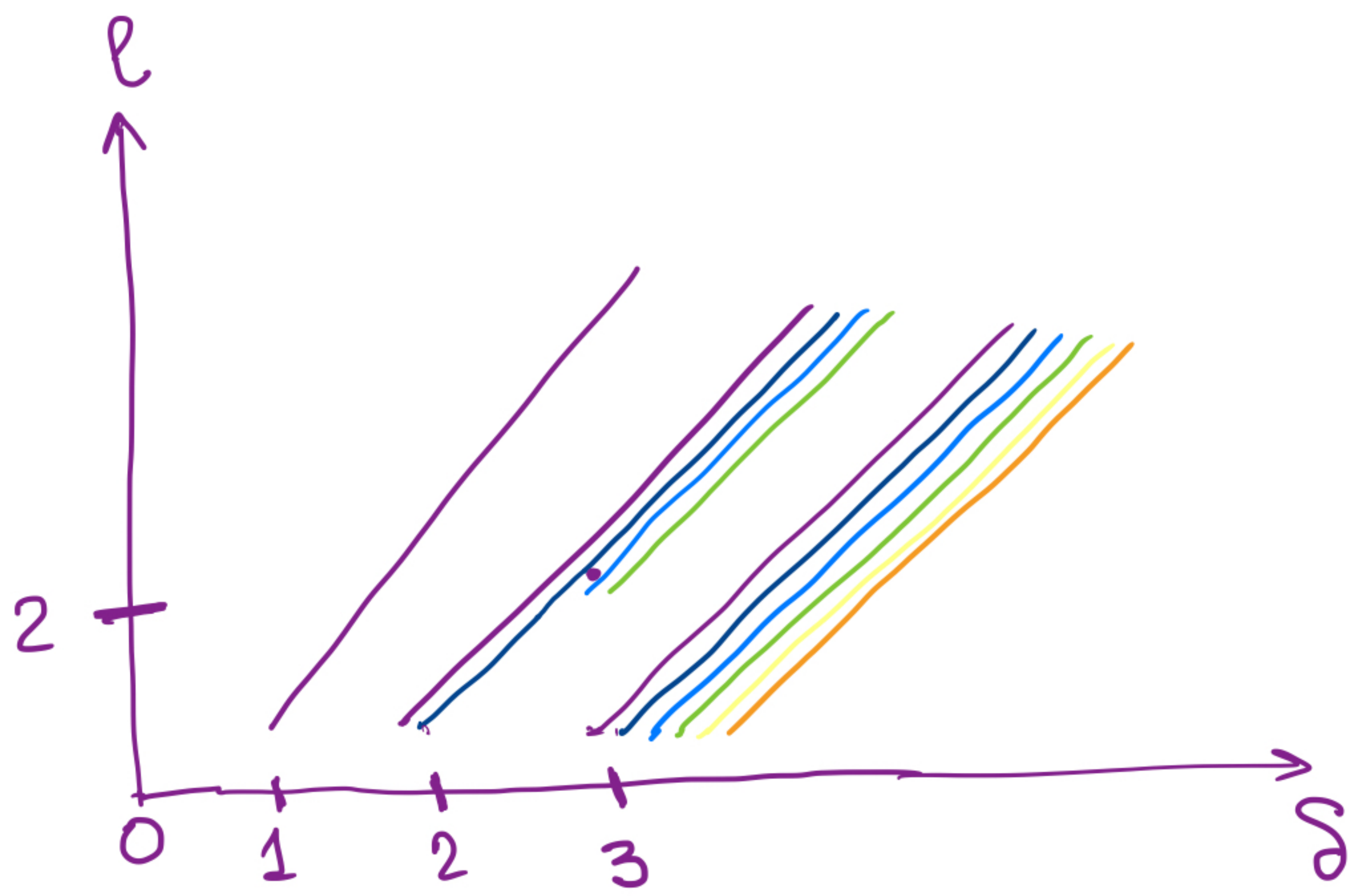


We see that OPE coefficient for one exchanged state is subleading to another at strong coupling. However, they are not at weak coupling.

1. What is the difference between two states? Hidden quantum numbers?
2. What is the selection rule for the states? Is it related to the "hidden" 10 D symmetry in flat space? How to formulate such a symmetry? [Aprile, Drummond, Heslop, Paul'17], [Caron-Huot, Trinh'18], [Caron-Huot, Coronado'21]
3. It is possible to compute these 3-point function in the flat space string theory [Minahan '12], [Bargheer, Minahan, Pereira '13], [Minahan, Pereira '14]. How to compute for higher exited states with $\delta = 2,3\ldots$? Is it possible to distinguish degenerate states?
4. We can determine constraints of the form $\langle f_0 \; j_1^n \rangle$ for many states. Does it help to compute the next curvature correction of the VSA? Does it give insights at the flat space S-matrix?

Bootstrapping Regge Trajectories I

[Julius, NS, upcoming]



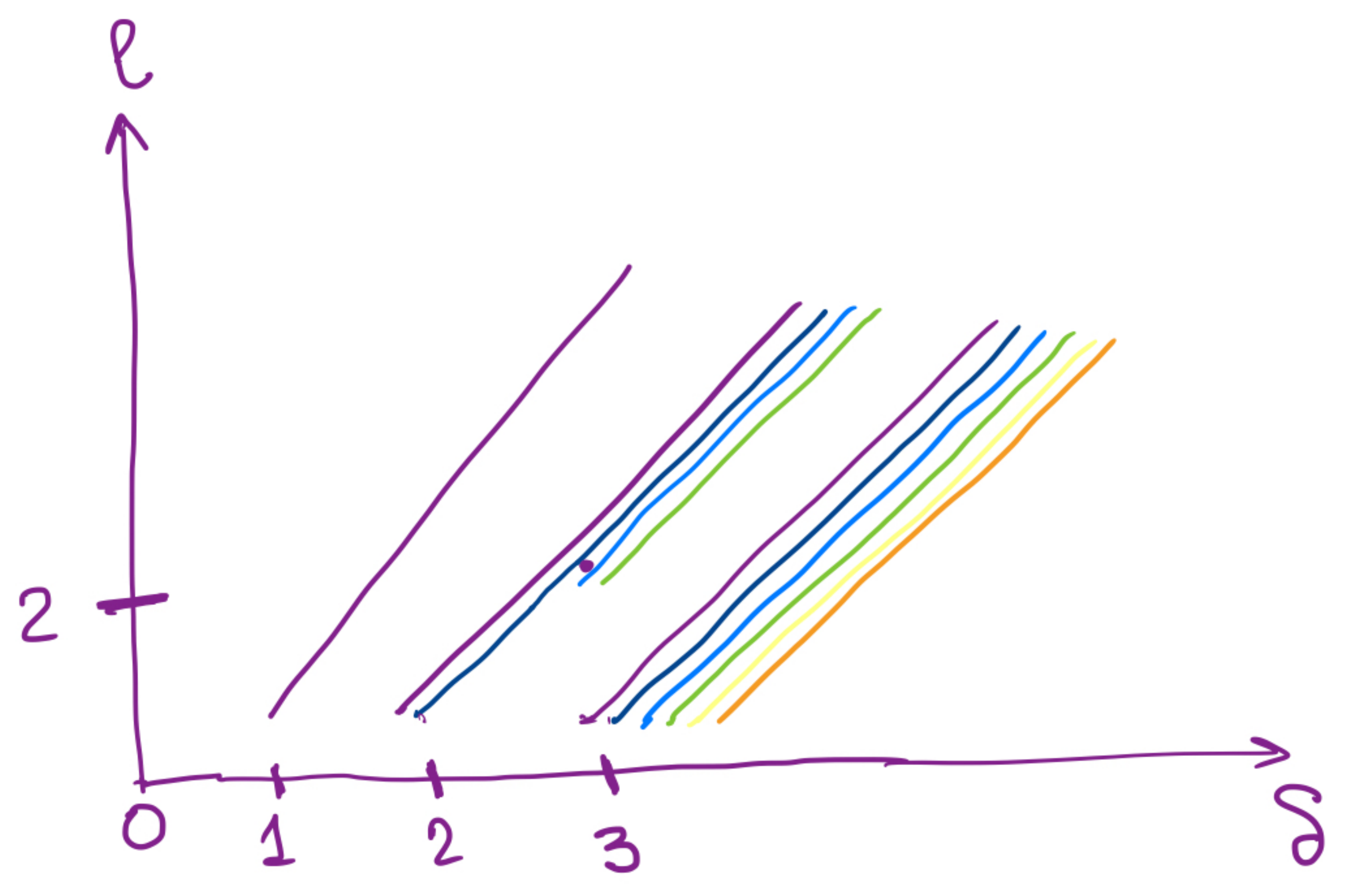
- We would like to introduce non-integer δ and to 'track' where do the states go once continued: states can be assigned to Regge trajectories. The leading trajectory has $\ell = 2(\delta - 1)$, the subleading has $\ell = 2(\delta - 2)$ etc.
- We assign to a Regge trajectory a quadratic Casimir which is now a function of δ. For example, the leading trajectory is known for a while as $j_1(\delta) = 5\delta^2 - 3\delta$ [Basso, Gromov, Tseytlin etc], [Alfimov, Gromov, Sizov, Levkovich-Maslyuk etc]. The other trajectories can be found using the QSC applied for the higher Regge trajectories, and for the subleading Regge trajectory it is possible to get for the first two trajectories:

$$j_{1;1} = 5\delta^2 - 3\delta,$$
$$j_{1;2} = 5\delta^2 - 9\delta$$

- We assume that the sub-leading Casimir has in general the following ansatz: $j_{1;m} = a_m\delta^2 + b_m\delta + c_m, \quad m = 3,4.$
- The goal is to obtain leading reduced OPE coefficients f_0 for the whole trajectories as functions of δ.

Bootstrapping Regge Trajectories II

[Julius, NS, upcoming]



- For the four sub-leading Regge trajectories we have four functions to find: $f_{0;1}(\delta), f_{0;2}(\delta), f_{0;3}(\delta), f_{0;4}(\delta)$.
- In principle, we have three average constraints for it: $\langle f_0 \rangle, \langle f_0 j_1 \rangle, \langle f_0 j_1^2 \rangle$, so we get $f_{0;1}, f_{0;3}, f_{0;4}$ as functions of $f_{0;2}, j_{1;1}, j_{1;2}, j_{1;3}, j_{1;4}$. We use the following features to find the solution:

1. We analyse the large-δ limit of the solutions and impose non-negativity of f_0 at this limit.
2. We impose known information at integer points and the fact that some OPE coefficients should vanish for consistency.
3. This gives us seven cases of possible solutions for the set of $f_{0;1}, f_{0;2}, f_{0;3}, f_{0;4}, j_{1;3}, j_{1;4}$ in terms of some free parameters.
4. We analyse the small-δ limit and impose some ad-hoc assumptions to further reduce the number of free parameters.
5. Finally, only one case is consistent with non-negativity and independently obtained integrability data.

Final result and Discussion

- After performing all the steps, we get the following consistent solution:

$$j_{1;3} = 5\,\delta^2 - \frac{19}{3}\delta + 2,\ j_{1;4} = 5\,\delta^2 - \frac{37}{4}\delta + 5,$$

$$f_{0;1} = \frac{\delta^2(23\delta - 25)}{(5\delta - 4)(5\delta - 3)}, f_{0;2} = 0, f_{0;3} = \frac{5(\delta - 2)(\delta - 1)\delta(35\delta + 33)}{3(5\delta - 3)(35\delta - 36)}, f_{0;4} = \frac{32(\delta - 3)(\delta - 2)(\delta - 1)}{(5\delta - 4)(35\delta - 36)}.$$

- Key takeway 1: $f_{0;2}$ is identically zero on its trajectory, a) more evidence for a hidden symmetry b) hints that in all other cases also, whenever we get a 0 at the bottom of a trajectory, that the whole trajectory may vanish.

- Key takeaway 2: Can use our results to construct constraints of the form $\langle f_0 j_1^n \rangle$ for any n, which should contain non-trivial information about the $1/R^{2n}$curvature correction to VSA, so this can be used in the program of [Alday, Hansen, Silva, Fardelli, Nocchi].

- Key takeaway 3: Method totally general and potentially applicable to other cases, especially where spectral data is not clearly available, e.g. Veneziano amplitude or ABJM.