Triplet Track Fit For The ATLAS Event Filter

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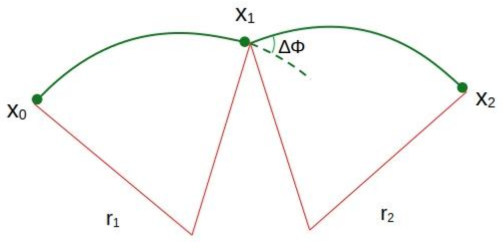
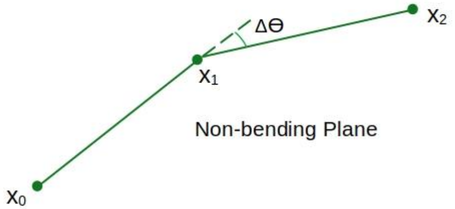
Triplet Track Fit

- Track fitting - adapting parameters of a track model to measurements
  - Models describe trajectory in magnetic field, materials, energy loss.
  - Fitting also takes uncertainties into account.
- Why the *triplet* representation ?
  - At least three hits required to measure track parameters.
  - Less complex than ‘quadruplets’.
  - Kinematics overconstrained ➔ filter bad combinations with fit quality.
  - Can also be used in track seeding.
- There exists an analytical solution for fitting triplets with Multiple Coulomb scattering (MS) and hit position uncertainties ✅



Bending Plane



Non-bending Plane

General Triplet Track Fit

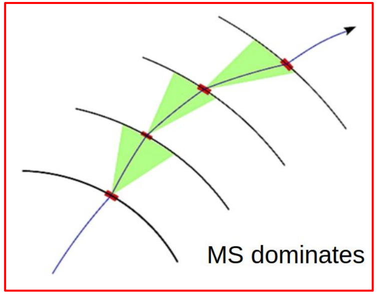
- Generalized from MS-only triplet fit [1] to include hit uncertainties [2]
  - MS-only fit implemented on GPUs for Mu3e [3].
- Fit quality for a triplet:

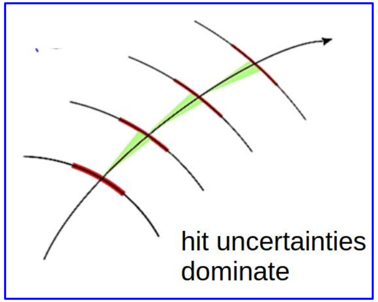
$$\chi^2(p,\vec{\delta}_k) = \frac{\Delta\Theta_{\mathrm{MS}}(p,\vec{\delta}_k)^2}{\sigma^2_{\mathrm{MS}}(p)} + \sin^2(\hat{\theta})\frac{\Delta\Phi_{\mathrm{MS}}(p,\vec{\delta}_k)^2}{\sigma^2_{\mathrm{MS}}(p)} + \sum_{k=0}^{2}(\vec{\delta}_k)^\top V_k^{-1}\vec{\delta}_k$$

$$\vec{\delta}_k = \vec{r}_{\mathrm{fit},k} - \vec{r}_{\mathrm{meas},k}$$

$\Delta\Theta_{\mathrm{MS}} = \Delta\Theta - \Delta\Theta_{\mathrm{hit}};\ \Delta\Phi_{\mathrm{MS}} = \Delta\Phi - \Delta\Phi_{\mathrm{hit}}$ with apparent kinks $\Delta\Theta,\ \Delta\Phi$

- χ^2 minimized with respect to momentum p & hit position shifts $\vec{\delta}_k$

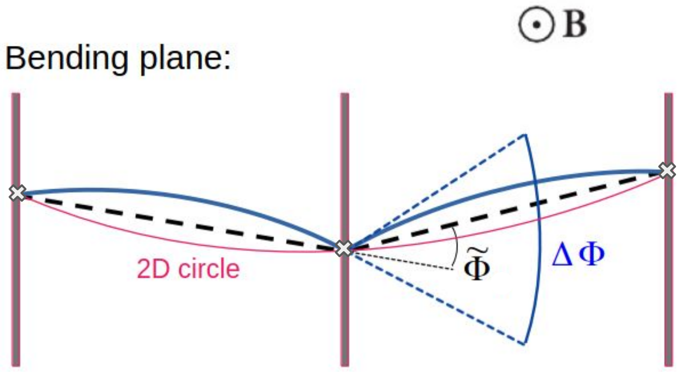




[1]: Berger, Kozlinskiy, Kiehn, & Schöning, *A new three-dimensional track fit with multiple scattering* 2017
[2]: Schöning, *A* A General Track Fit Based on Triplets
[3]: Mu3e Collaboration, *Technical design of the phase-I Mu3e experiment* 2021

Triplet Parameters

Kink angles are given by transcendental equations - start with an approximate solution connecting three hits with a circle in the transverse plane



Linearized kink angle:

$$\Delta\Phi = \tilde{\Phi} + \rho_\Phi c_{3D} \qquad c_{3D} = R_{3D}{}^{-1} = \frac{|q|B}{p}$$

Kink angle for vanishing curvature ($c_{3D} = 0$)

Dependance of kink angle on curvature

…similarly for the scattering angle in non-bending plane

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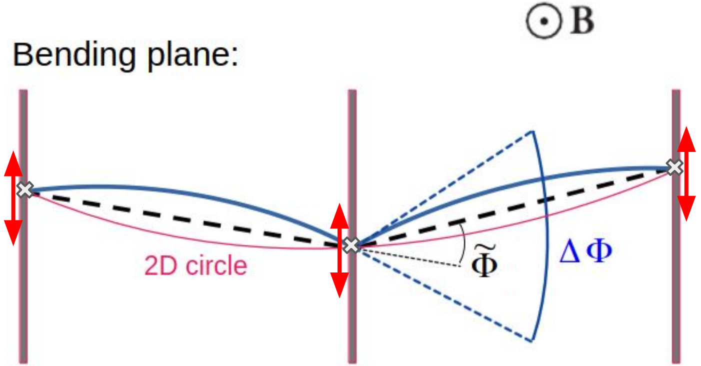

$$\Delta\Phi_{\text{hit}} = \sum_{k=1}^{3} \overrightarrow{h_{\Phi\,k}} \cdot \overrightarrow{\delta_k}$$

with $\overrightarrow{h_{\Phi\,k}} = \text{grad}\,\Delta\Phi(\overrightarrow{x_k})$

Contributions to kink angles from position uncertainties taken into account using directional derivatives of kink angles w.r.t hit position shifts.

Linearized kink angle:

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Kink angle for vanishing curvature (c_{3D}= 0)

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$$\Delta\Theta_{\text{MS}} = \Delta\Theta - \Delta\Theta_{\text{hit}};\ \Delta\Phi_{\text{MS}} = \Delta\Phi - \Delta\Phi_{\text{hit}}$$

with apparent kinks $\Delta\Theta,\ \Delta\Phi$

…similarly for the scattering angle in non-bending plane

χ^2 Minimization

$$
\begin{aligned}
\chi^2(c_{3\mathrm{D}};\vec{\boldsymbol{\delta}}) \;=\; & \sum_{\text{triplet } j}^{n_{\text{hit}}-2} \frac{1}{\sigma^2_{\text{MS},j}} \left(\tilde{\Theta}_j + \rho_\Theta\, c_{3\mathrm{D}} - \Delta\Theta_{\text{hit},j}(\vec{\boldsymbol{\delta}})\right)^2 \\
+ & \sum_{\text{triplet } j}^{n_{\text{hit}}-2} \frac{\sin^2 \hat{\vartheta}_j}{\sigma^2_{\text{MS},j}} \left(\tilde{\Phi}_j + \rho_\Phi\, c_{3\mathrm{D}} - \Delta\Phi_{\text{hit},j}(\vec{\boldsymbol{\delta}})\right)^2 \\
+ & \sum_{\text{hit } k=0}^{n_{\text{hit}}-1} \frac{{\delta_{k_i}}^2}{{\sigma_{k_i}}^2}
\end{aligned}
$$

Global fit χ^2 :

$$
\begin{aligned}
\chi^2(c_{3\mathrm{D}};\vec{\boldsymbol{\delta}}) \;=\; & \left(\boldsymbol{\Psi} + \boldsymbol{\rho}\, c_{3\mathrm{D}} - \vec{\boldsymbol{H}}\vec{\boldsymbol{\delta}}\right)^T \boldsymbol{D}_{\text{MS}} \left(\boldsymbol{\Psi} + \boldsymbol{\rho}\, c_{3\mathrm{D}} - \vec{\boldsymbol{H}}\vec{\boldsymbol{\delta}}\right) \\
+ & \vec{\boldsymbol{\delta}}^T\, \vec{\vec{\boldsymbol{D}}}_{\text{hit}}\, \vec{\boldsymbol{\delta}}
\end{aligned}
$$

Triplet parameters from linearization

Hit position derivatives of kink angles

Multiple scattering uncertainty matrix

Hit position uncertainty matrix

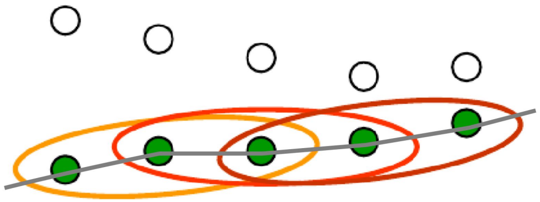
Hit position shifts

Minimization involves a matrix inversion.

General Triplet Track Fit

- The fit is factorized into two steps#:
  - Local fit of individual triplets.
  - Global fit of all connected triplets in a track - requires a matrix inversion.

- Several advantages of the two step procedure:
  - Local fit of individual triplets can be done in parallel.
  - Triplets with bad fit qualities can be rejected early to reduce combinatorics.
  - Global fit is completely detector independent.
    - Triplet parameters absorb detector dependencies (geometry, magnetic field).
  - Benefit from using compatibility of triplets (fitted p, q) before combination.

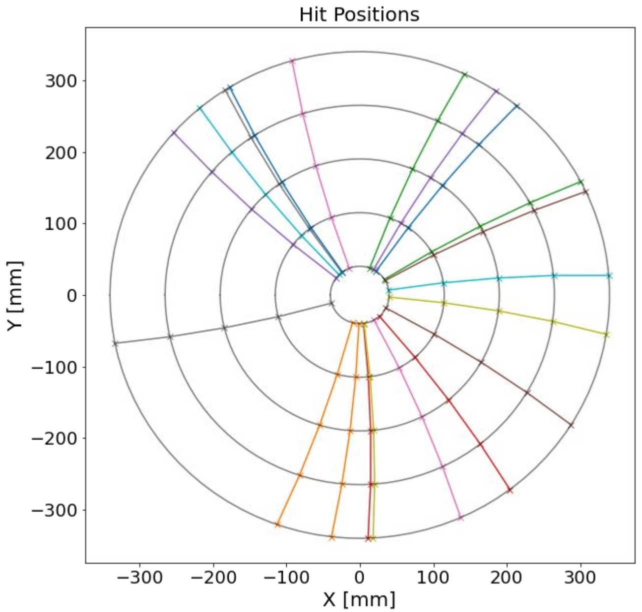


#: The full local fit is not necessary for doing the global fit - only the calculation of triplet parameters and hit position derivatives of the kink angle, introduced before.

Toy Simulation

- Developed for standalone testing of the fits
  - Minimal setup: provides tight control over results.
- Particles propagated through uniform, solenoidal B-field.
- Detector with 5 equally spaced barrel layers (axis parallel to B-field).

Generated particles	Position	(0,0,0) - beamspot of size 45 mm along z
	Azimuthal angle (Φ)	Uniform in [0, 2π]
	Polar angle (ϴ)	70° (η= 0.36)
	Charge	+1
Simulated uncertainties	Multiple scattering	x/X_0 = 2% (per layer)
	Hit uncertainties	12 µm in sensor transverse directions



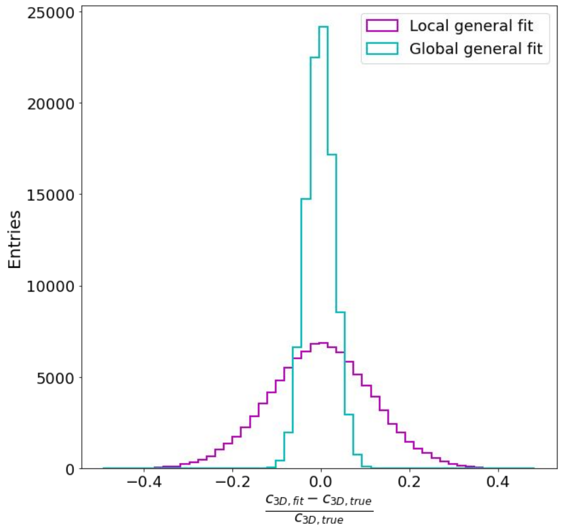
In bending plane

Results: Implementation on CPU

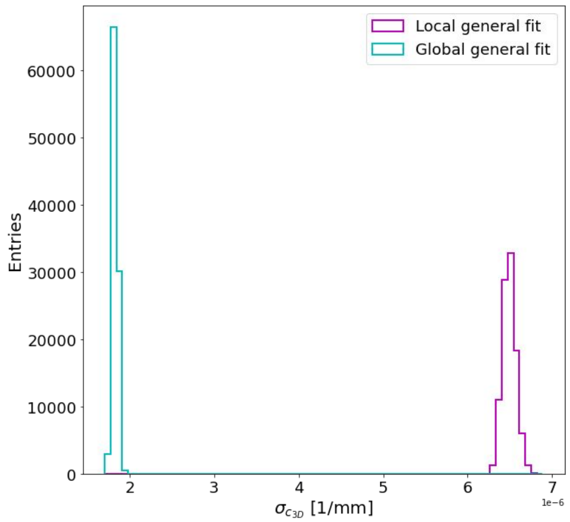
Curvature: $c_{3D} = {R_{3D}}^{-1} = \frac{|q|B}{p}$ to avoid numerical instabilities at high momenta

p_T = 10 GeV

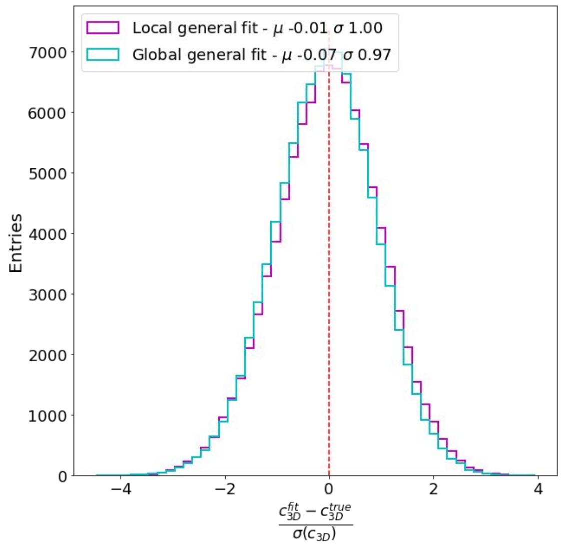
Relative resolution



Fit uncertainty



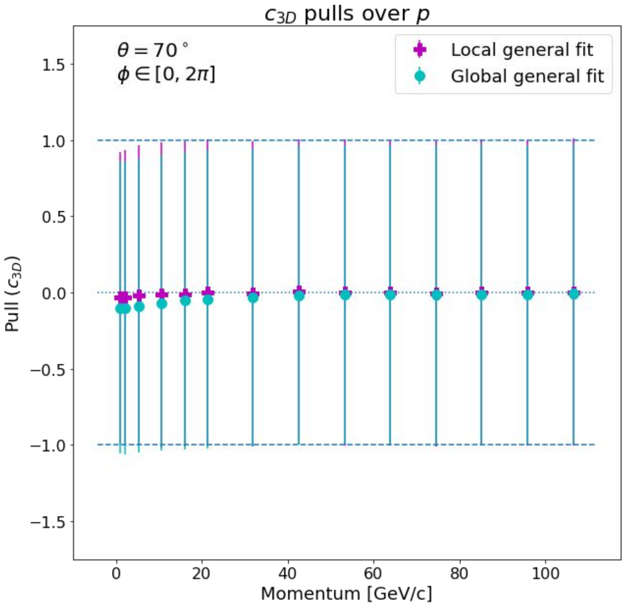
Pull distribution



Plots for local fit everywhere are for one triplet; the global fit combines all (three) triplets on track.

Pull distribution with 0 mean and 1 sigma implies correct estimation of curvatures and uncertainties.

Curvature Pulls

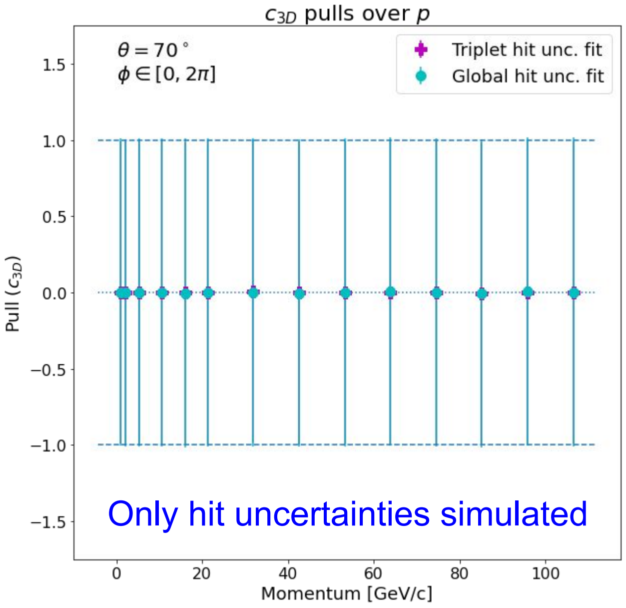


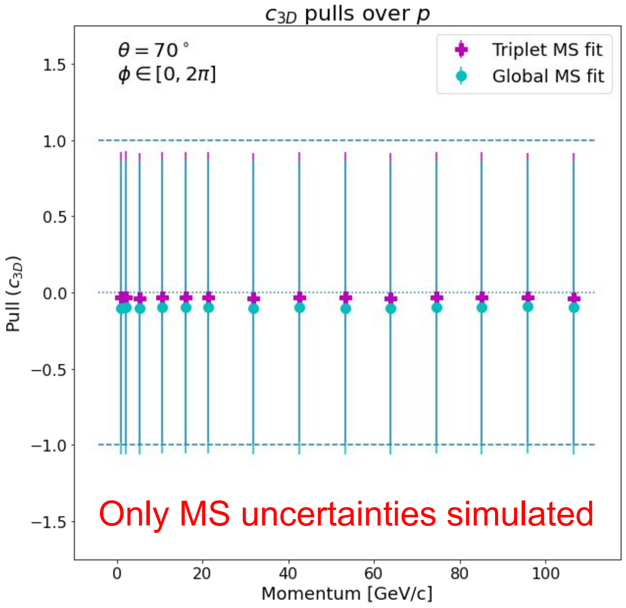
Correct estimation of curvatures and uncertainties over a large momentum range.

Dominating MS effects at low momenta → correlation between curvatures and uncertainties → bias in pull distributions.

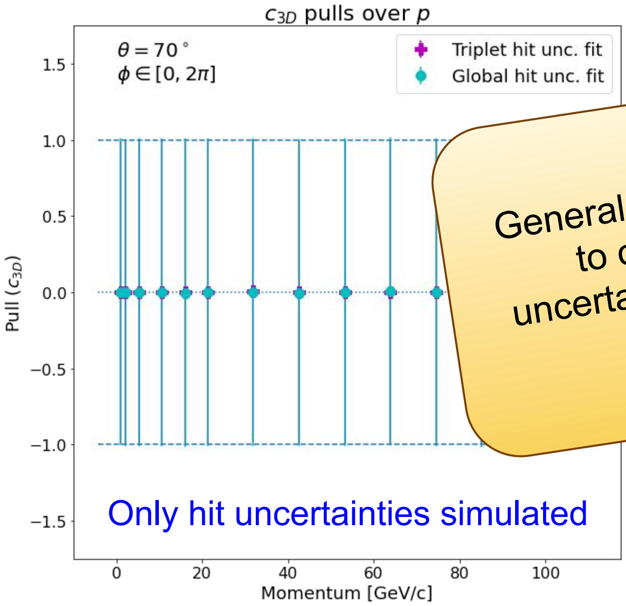
➤ Well understood and can be corrected.

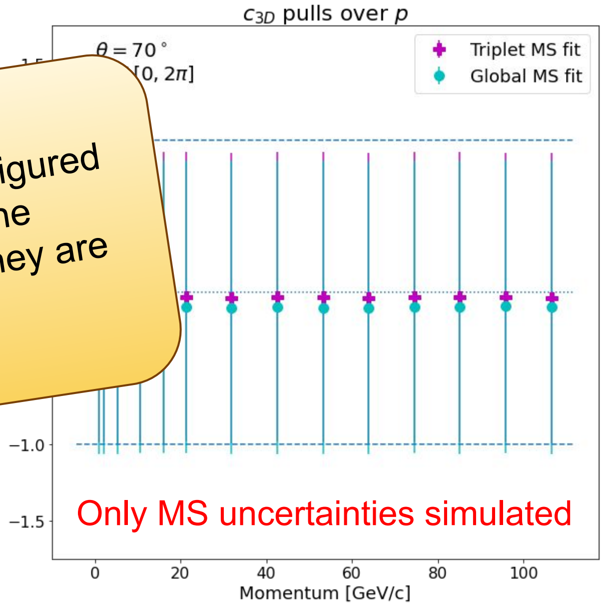
Curvature Pulls





Curvature Pulls





General fit can be configured to only fit one of the uncertainties where they are dominating

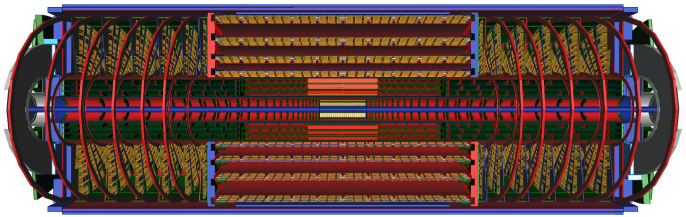
Summary & Outlook

- General tripet track fit with Multiple scattering and hit position uncertainties has been developed.
    - Plan to exploit massive parallelism by using GPUs ➔ for fast track fitting in ATLAS EF.
- The triplet fit was tested using standalone simulation.
- Correct estimation of curvatures and uncertainties observed over a large momentum range.
- Gain from using MS-only triplet fits, wherever possible.
- Next steps: Integrating in `traccc`; study performance in ITk.

Backup

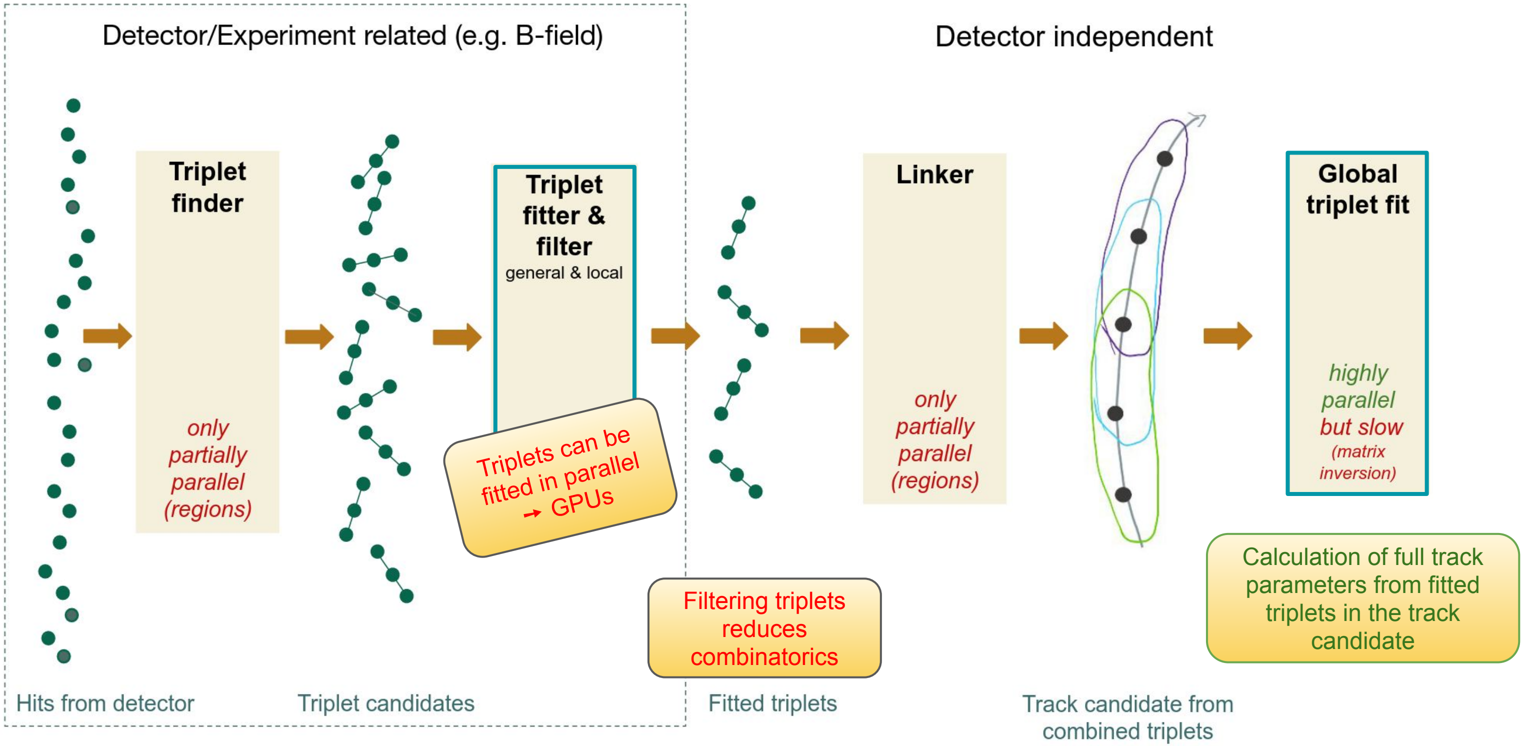
ATLAS Event Filter Tracking

- Series of upgrades underway to prepare for Hi-Lumi LHC
  - Inner Detector (ID) to be replaced with all-Si Inner Tracker (ITk).
  - Trigger and DAQ systems have to cope with increased data rate and event complexity.
- Event Filter (EF) to reduce 1 MHz Level-0 rate to 10 kHz.
  - Running offline-like algorithms on commercial hardware.
  - Possibly using accelerators (GPUs, FPGAs) to gain performance.
- Tracking in Event Filter (EF Tracking)
  - Crucial to maintain sensitivity while keeping trigger rates under control.
  - Regional and full-scan tracking at 1 MHz and 150 kHz respectively.
  - Most expensive computationally.

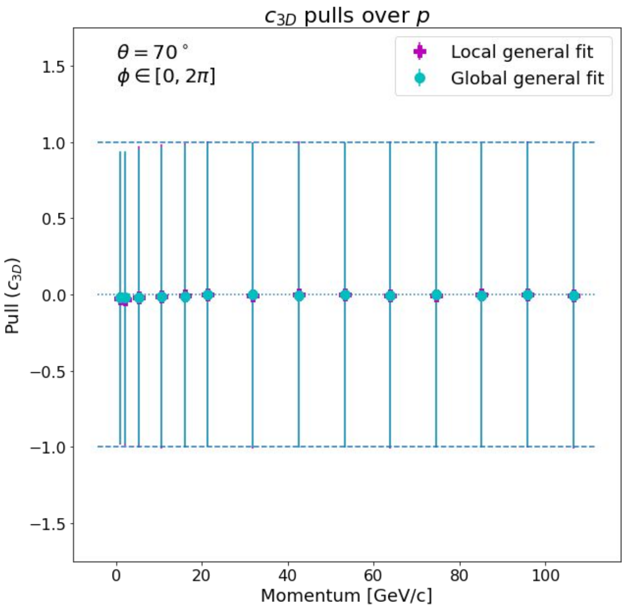


ATLAS Collaboration, *Expected tracking and related performance with the updated ATLAS Inner Tracker layout at the High-Luminosity LHC* - ATL-PHYS-PUB-2021-024

Triplet Fit: Hits to Tracks



Curvature Pulls



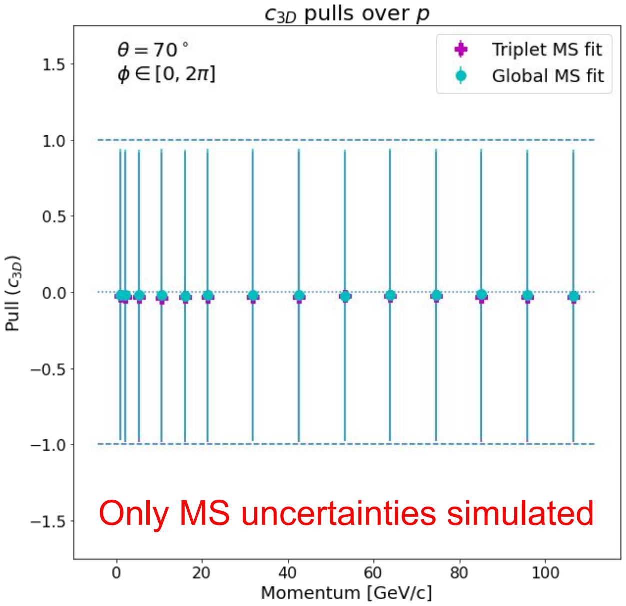
Correct estimation of curvatures and uncertainties over a large momentum range.

Bias goes away if the ‘correct’ MS uncertainties - estimated using the curvature from the global fit - are used, instead of those estimated using the linearization/local fits.

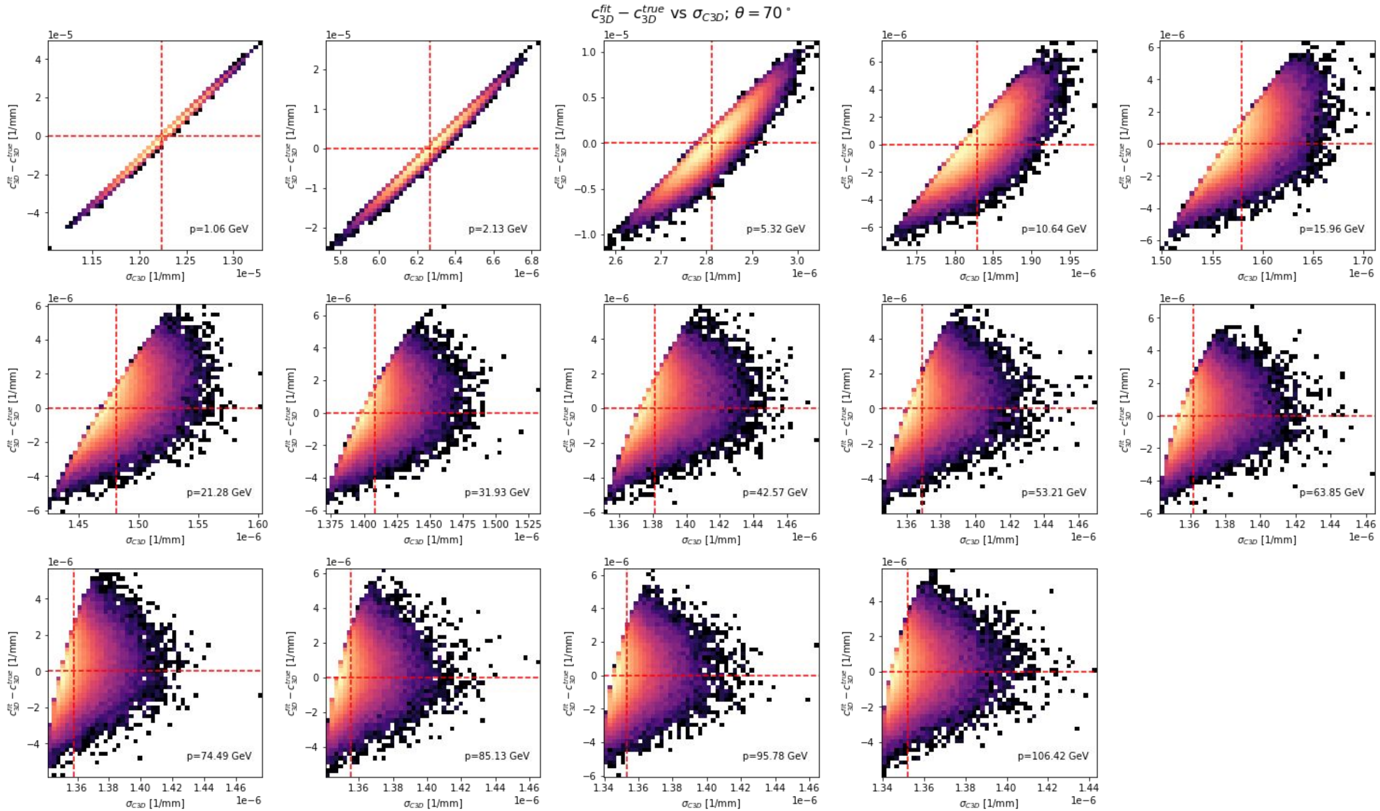
➞ iterative!

Curvature Pulls

With MS uncertainties estimated from global fit curvatures.



Pull Bias



c_{3D} and $\sigma(c_{3D})$ strongly correlated at low momenta (dominating MS uncertainties)

<u>Two effects</u>:

- pulls for a single triplet have $\mu < 0$ (negative residuals divided by smaller σ-s)

- statistical effect: combining triplets make the biases larger