Pressure and speed of sound in two-flavor color-superconducting quark matter at next-to-leading order

Andreas Geißel

Technische Universität Darmstadt

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TECHNISCHE UNIVERSITÄT DARMSTADT

[based on AG, T. Gorda, and J. Braun '24]

QCD Phase Diagram

temperature T

lattice

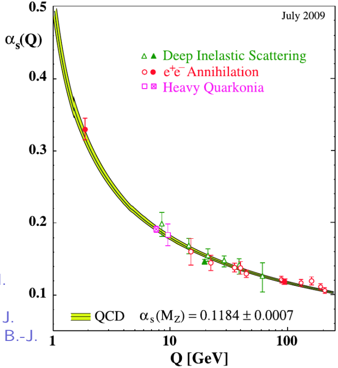
[H. T. Dings et al. (HotQCD) '19; S. Borsanyi et al. '20; F. Cuteri et al. '22; O. Philipsen '23; ...]

functional methods

[J. Bernhardt and C. S. Fischer '23; K.Fukushima, J. M. Pawlowski et al. '21; M. Drews and W. Weise '15; R.-A.Tripold, C. Jung, L. v. Smekal, J. Wambach, '21; J. Braun, F. Rennecke et al. '23; K. Otto, M. Oertel, and B.-J. Schäfer '19 and '20; W. Fu, F. Rennecke al. '23; ...]

χEFT

[K. Hebeler '20; E. Epelbaum, H.W. Hammer, and Ulf-G. Meißner '08; . . .]



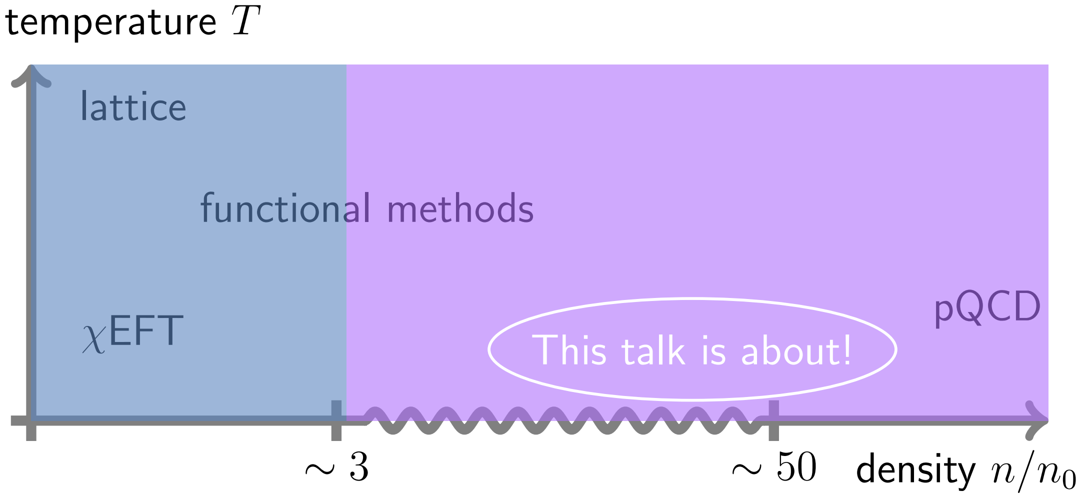
[Bethke '09]

pQCD

[A. Kurkela, P. Romatschke, and A. Vuorinen '10, A. Kurkela et al. '14; T. Gorda et al. '18; . . .]

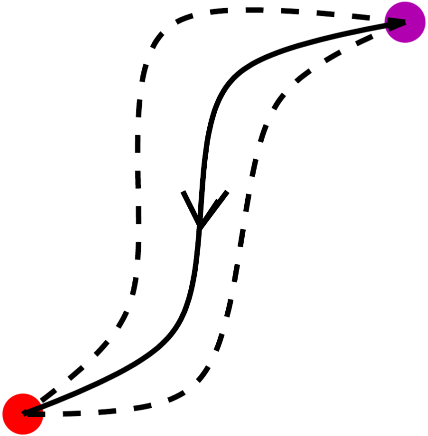
~ 3 ~ 50 density n/n_0

QCD Phase Diagram



Degrees of Freedom

[Wetterich '93; (figure adapted from) Gies '06]



$\Gamma_{k=\Lambda} = S$

$\Gamma_{k=0} = \Gamma \rightarrow$ pressure, expectation values, ...

Degrees of Freedom

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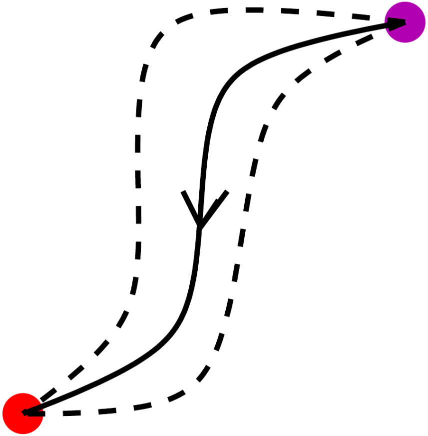
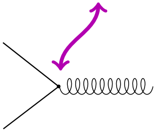


$$\Gamma_{k=\Lambda} = S = \int_x \bar{\psi}\left(i\partial\!\!\!/ - i\gamma_0\mu\right)\psi + g\bar{\psi}A\!\!\!/\psi + \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu}$$

$(N_\mathrm{f} = 2)$



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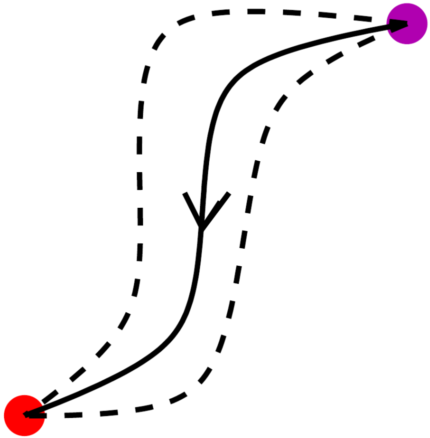

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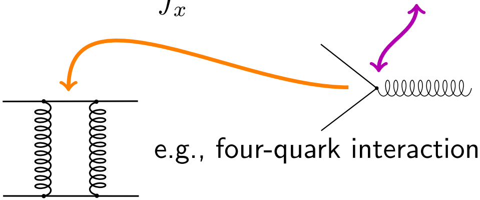

e.g., four-quark interactions

$\Gamma_{k=0} = \Gamma \rightarrow$ pressure, expectation values, . . .

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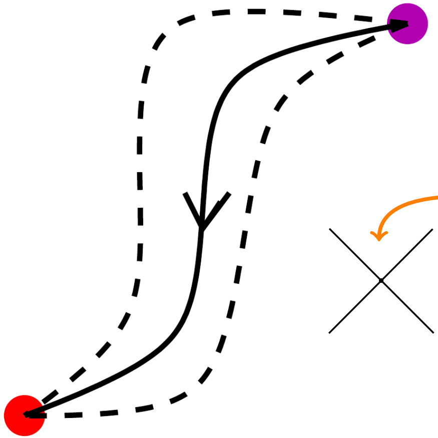

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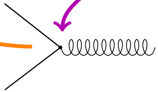

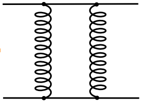

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resonant four-quark interaction (at high densities)

[M. Leonhardt et al. '19; J. Braun and B. Schallmo '21, B. Schallmo, AG, J. Braun (in prep.)]

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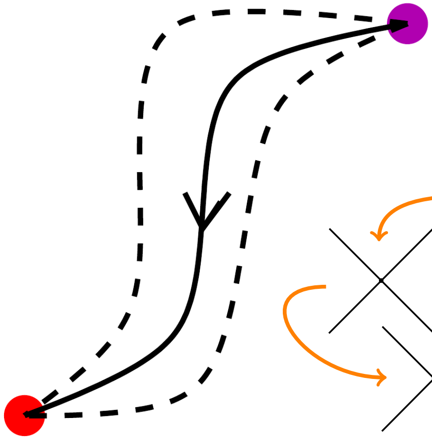

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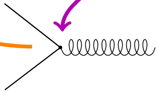

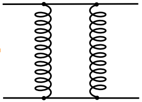

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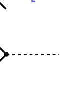

auxiliary diquark field Δ_a (Hubbard-Stratonovich)

$\Gamma_{k=0} = \Gamma \to$ pressure, expectation values, ...

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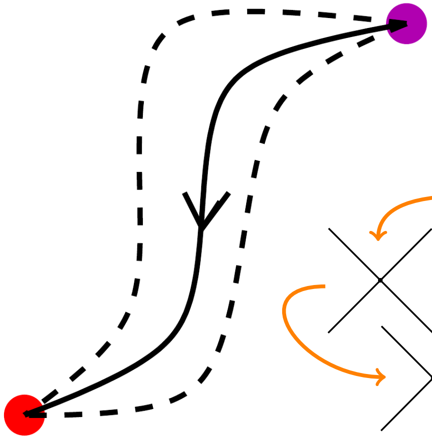

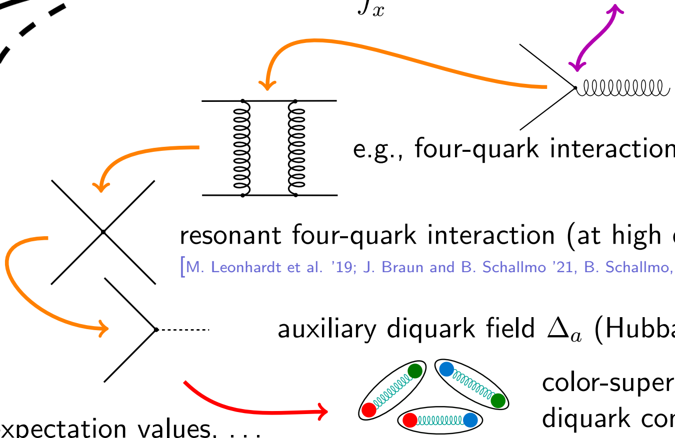

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auxiliary diquark field Δ_a (Hubbard-Stratonovich)

color-superconducting gap

diquark condensate

$\Gamma_{k=0} = \Gamma \rightarrow$ pressure, expectation values, . . .

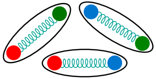
[R. Rapp et al. '98; M. G. Alford et al. '98; T. Schäfer and F. Wilczek '99, J. Berges and K. Rajagopal '99; M. Buballa '05]

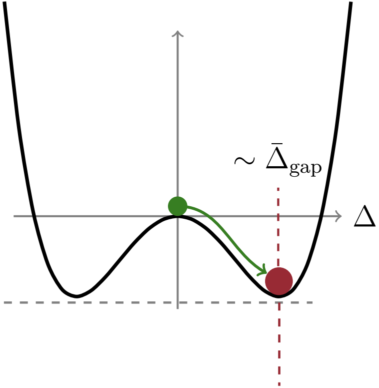
Color Superconductivity

$\mathrm{U}(1)_V$ symmetry breaking

quark cooper pairs

$\Delta_a \sim \langle \psi^T \mathcal{C} \gamma_5 \tau_2 \epsilon_a \psi \rangle$





ground state lowered

diquark condensate

2-flavor diquark condensate is chirally symmetric

Expansion of the Pressure

[J. Braun, AG, and B. Schallmo '22]

- effective action Γ in the presence of a color superconducting condensate:

$$p = -\frac{\Gamma}{V_4}\bigg|_{\mu,\,\Delta=\bar{\Delta}_{\mathrm{gap}}} + \mathrm{const.}$$

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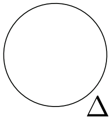
expansion about vanishing gap

$$p = p_{\mathrm{free}}\left(\gamma_0(g) + \gamma_1(g)\left(\frac{|\bar{\Delta}_{\mathrm{gap}}|}{\mu}\right)^2 + \ldots\right)$$

- expanding the γ_i perturbatively
- gap implicitly depends on g

One Loop

- zeroth order of the γ_i's
- the gap enters the fermionic propagator



- minimize the effective potential
- expansion of the pressure yields

$$p^{1-\text{loop}} = p_{\text{free}} \left(1 + 2 \left(\frac{|\bar{\Delta}_{\text{gap}}|}{\mu} \right)^2 + \dots \right)$$

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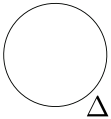


- minimize the effective potential
- expansion of the pressure yields

like in the diquark model

ground state is phase with the highest pressure

$$p^{1-\mathrm{loop}} = p_{\mathrm{free}} \left(1 + 2 \left(\frac{|\bar{\Delta}_{\mathrm{gap}}|}{\mu} \right)^2 + \ldots \right)$$

One Loop

- What about the gluons?

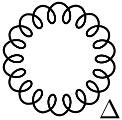
- diquarks carry color charge

- Anderson-Higgs mechanism
  $\Longrightarrow$ some gluons become massive

Covariant derivative gives

$$\sim A^a_\mu T^a_{cd} \bar{\Delta}_d A^b_\mu T^b_{ce} \bar{\Delta}^*_e$$

mass term for the gluon

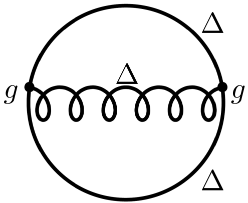


$$\Longrightarrow p \sim |\bar{\Delta}_{\text{gap}}|^4$$

- higher order in $|\bar{\Delta}_{\text{gap}}|$

Two Loops

- next order in loop expansion $\Longrightarrow$ sunset diagram
- gap enters all propagators



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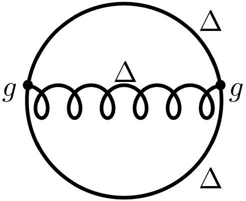


$$\Longrightarrow \; p^{2-\text{loop}} = p_{\text{free}} \left(-\frac{g^2}{2\pi^2} + 1.09(4)\, g^2 \left(\frac{|\bar{\Delta}_{\text{gap}}|}{\mu} \right)^2 + \dots \right)$$

Expansion of the Pressure

[AG, T. Gorda, and J. Braun '24]

double expansion

$$p \approx p_{\text{free}} \left(1 - \frac{g^2}{2\pi^2} + \left(2 + 1.09(4)\, g^2\right) \left(\frac{|\bar{\Delta}_{\text{gap}}|}{\mu} \right)^2 \right)$$

$\bar{\Delta}_{\text{gap}}(g)$

free pressure

pert. contribution

gapped quark loop

gapped 2-loop contribution

Expansion of the Pressure

[AG, T. Gorda, and J. Braun '24]

$N_\mathrm{f} = 2$

double expansion

$$p \approx p_\mathrm{free} \left(1 - \frac{g^2}{2\pi^2} + \left(2 + 1.09(4)\, g^2\right) \left(\frac{|\bar{\Delta}_\mathrm{gap}|}{\mu} \right)^2 \right)$$

$\bar{\Delta}_\mathrm{gap}(g)$

free pressure

pert. contribution

gapped quark loop

gapped 2-loop contribution

Expansion of the Pressure (3 flavor)

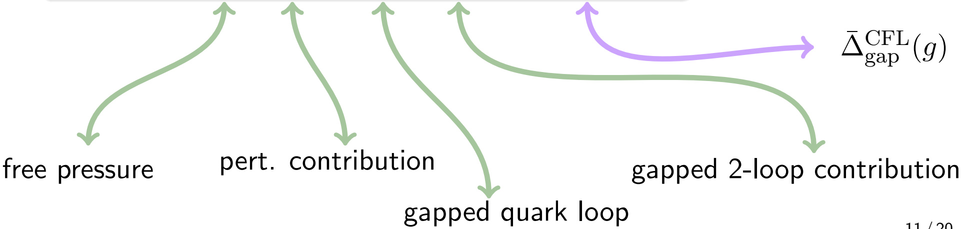
[M. Alford, K. Rajagopal, and F. Wilczek '98] [AG, T. Gorda, and J. Braun (in prep.)]

- color flavor locking (CFL) assumed to be the true ground state

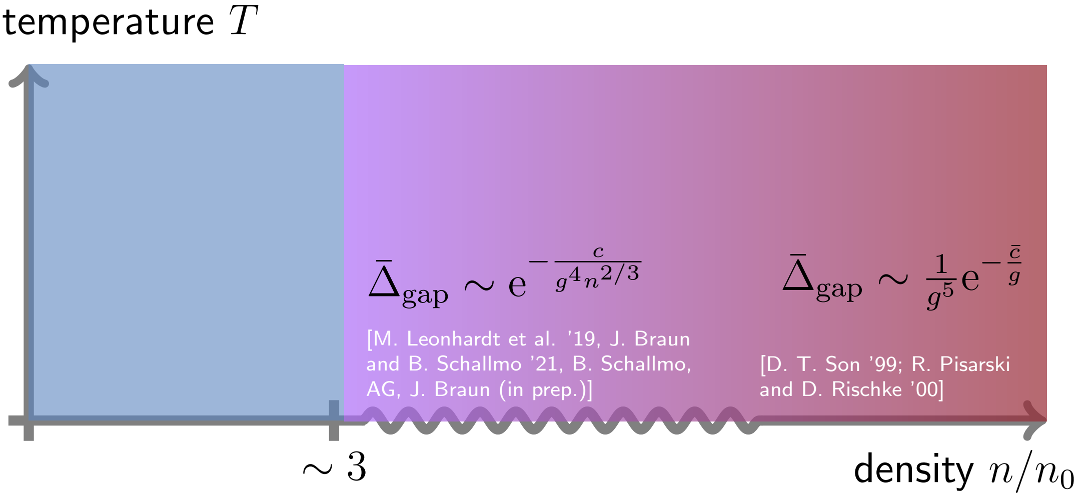
$N_{\mathrm{f}} = 3$

double expansion

$$p \approx p_{\mathrm{free}} \left(1 - \frac{g^2}{2\pi^2} + (4 + ??g^2) \left(\frac{|\bar{\Delta}^{\mathrm{CFL}}_{\mathrm{gap}}|}{\mu} \right)^2 \right)$$



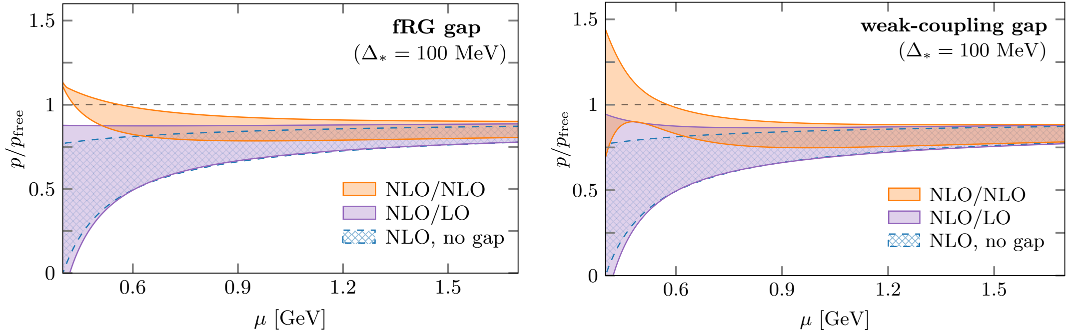
Diquark Gap



temperature T
$\bar{\Delta}_{\mathrm{gap}} \sim \mathrm{e}^{-\frac{c}{g^4 n^{2/3}}}$
[M. Leonhardt et al. '19, J. Braun and B. Schallmo '21, B. Schallmo, AG, J. Braun (in prep.)]
$\bar{\Delta}_{\mathrm{gap}} \sim \frac{1}{g^5}\mathrm{e}^{-\frac{\bar{c}}{g}}$
[D. T. Son '99; R. Pisarski and D. Rischke '00]
~ 3
density n/n_0

Pressure

[AG, T. Gorda, and J. Braun '24]



- gap effects become more important for lower densities
- NLO/NLO correction of the same order as NLO/LO contribution
- specific scaling of the gap less relevant for the pressure

Another way of characterizing dense matter

- Speed of sound

$$c_\mathrm{s}^2 = \frac{\partial p}{\partial \varepsilon} = \frac{1}{\mu}\left(\frac{\partial p}{\partial \mu}\right) \Bigg/ \left(\frac{\partial^2 p}{\partial \mu \partial \mu}\right)$$

- Causality

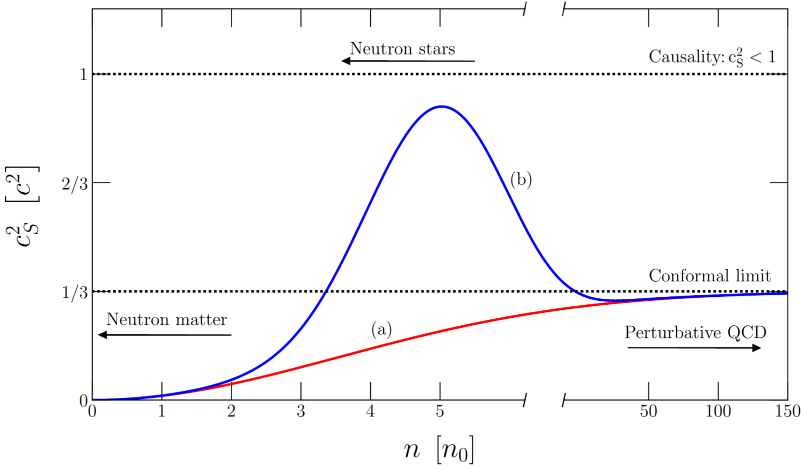
$$c_\mathrm{s}^2 \leq 1$$

- Thermodynamic stability

$$c_\mathrm{s}^2 \geq 0$$

- Speed of sound is a measure for the **stiffness** of the equation of state. **Stiffness** is needed to prevent a neutron star from collapsing to a black hole.

Speed of Sound

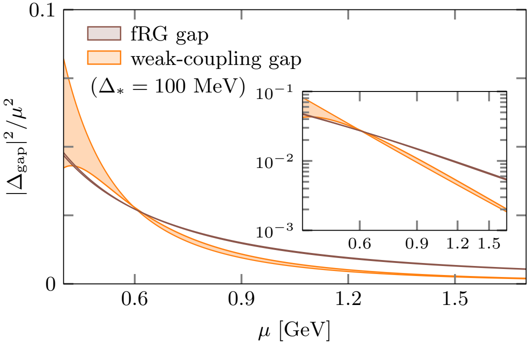


[I. Tews et al. '18]

Speed of Sound Scaling

[AG, T. Gorda, and J. Braun '24]

- assuming the gap scales like $\bar{\Delta}_{\rm gap} \sim \mu^{\sigma}$, with $-1 < \sigma < 1$



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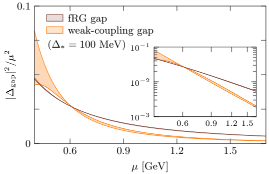
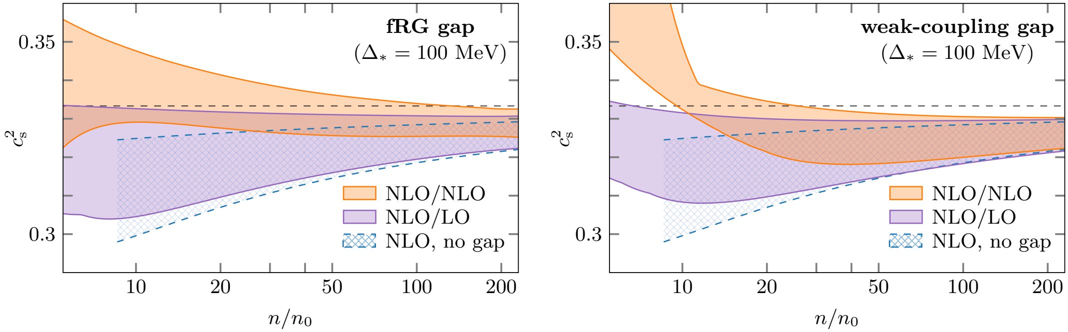


scaling law for the speed of sound

$$c_{\rm s}^2 \approx \frac{1}{3} - c_0 g^4 + \gamma_1^{c_{\rm s}} \left(\frac{|\bar{\Delta}_{\rm gap}|}{\mu} \right)^2 \quad \text{with} \quad c_0, \gamma_1^{c_{\rm s}} > 0$$

Speed of Sound

[AG, T. Gorda, and J. Braun ’24]



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Speed of Sound

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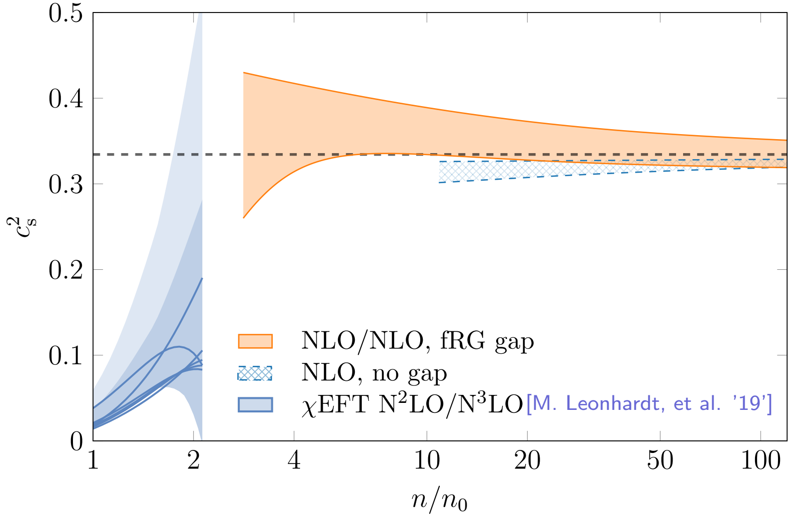

Take Home Message

A color-superconducting **gap** suggests a **maximum** in the speed of sound at supranuclear densities.

At even higher densities, the speed of sound again **crosses** the conformal limit and approaches it from **below**.

Thank you for your attention!

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