Chiral kinetic theory in curved space reinterpreted and radiative corrections

Shu Lin

Sun Yat-Sen University

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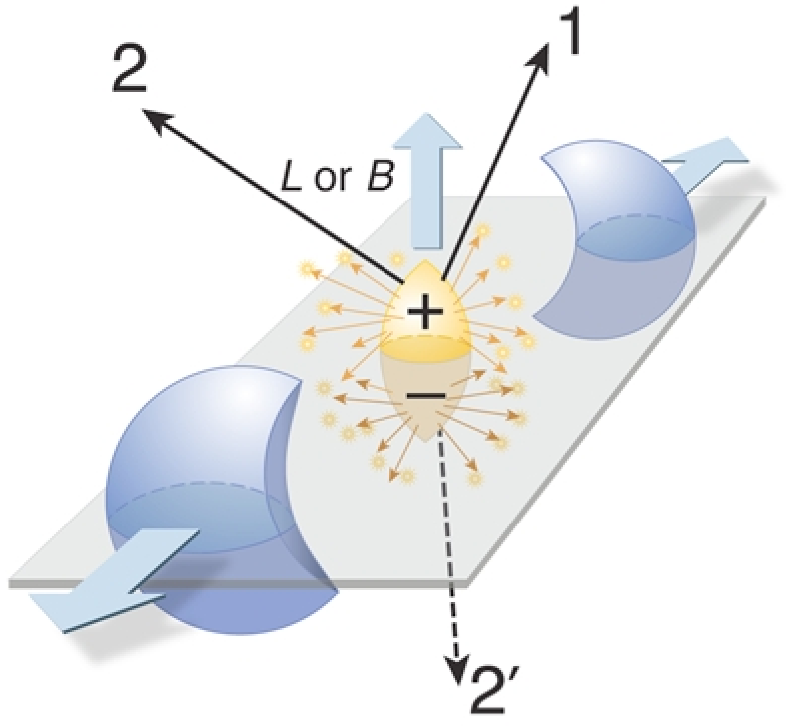
Outline

- Spin polarization in heavy ion collisions
- Lessons from quantum(chiral) kinetic theory and limitations
- Radiative corrections to spin coupling to EM fields
- Subtlety in mimicking off-equilbrium state by metric perturbion on equilibrium state
- Radiative corrections to spin coupling to hydro-gradients.
- Conclusion and outlook

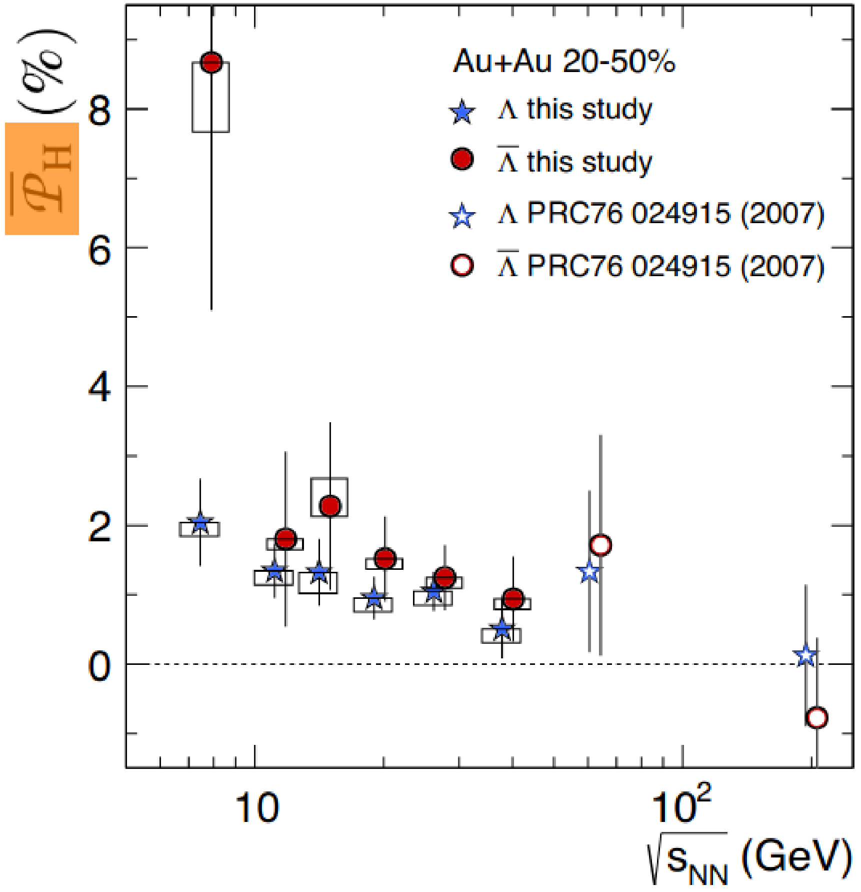
global spin polarization in heavy ion collisions



$$L_{ini} \sim 10^5 \hbar \rightarrow S_{final}$$

Liang, Wang, PRL 2005, PLB 2005

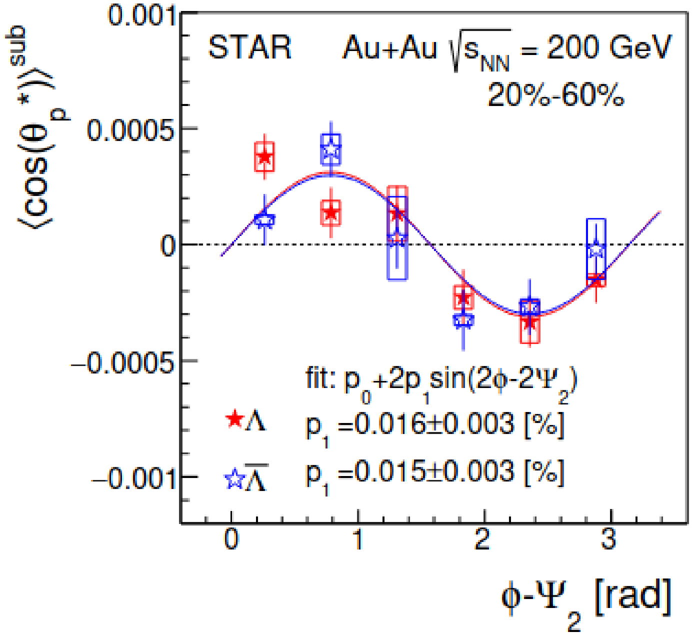
Esumi's talk



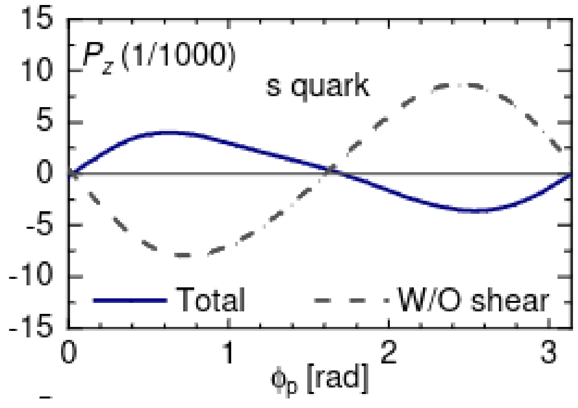
STAR collaboration, Nature 2017

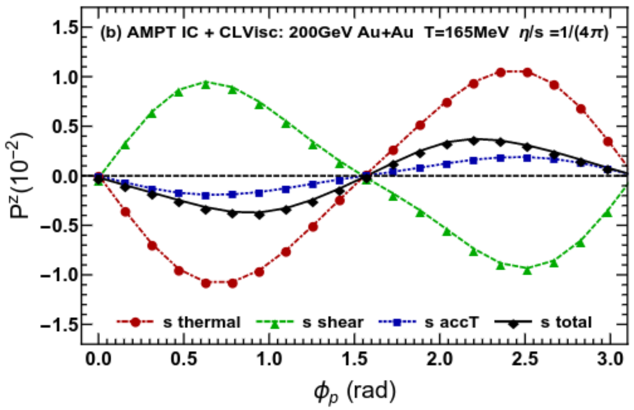
$$e^{-\beta(H_0 - \mathbf{S}\cdot\omega)}$$

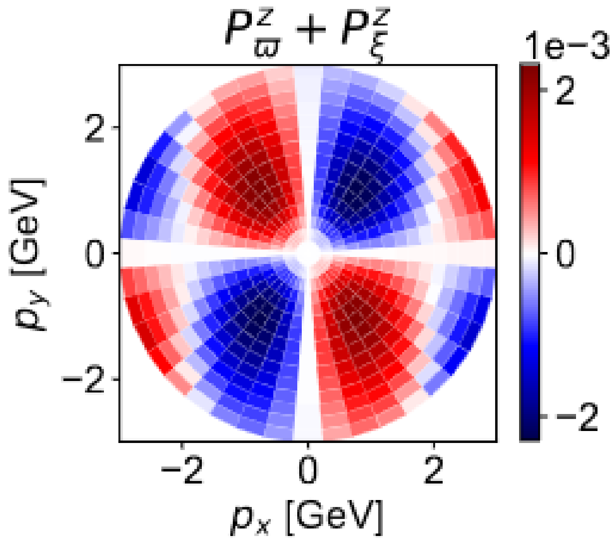
local spin polarization in heavy ion collisions



STAR collaboration, PRL 2019







Fu, Liu, Pang, Song, Yin, PRL 2021
Becattini, et al, PRL 2021
Yi, Pu, Yang, PRC 2021

$$\mathcal{P}^i \sim \omega^i \qquad \mathcal{P}^i \sim \epsilon^{ijk}\hat{p}_j\hat{p}_l\sigma_{kl}$$

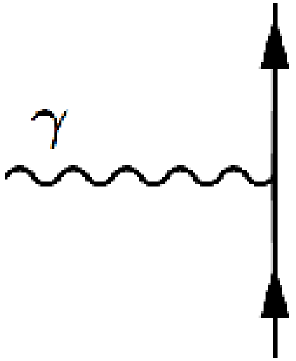
vorticity+shear

Spin polarization in heavy ion collisions

for $S = \frac{1}{2}$ particle

$S_i \sim B_i$

$S_i \sim \epsilon^{ijk} \hat{p}_j E_k$

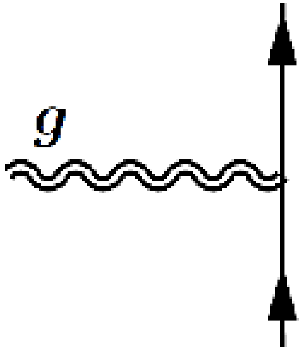


external EM fields

Liu, Yin, 2020

$S_i \sim \omega_i$

$S_i \sim \epsilon^{ijk} \hat{p}_j \hat{p}_l \sigma_{kl}$



off-equilibrium state: hydro gradient (mimicked by metric)

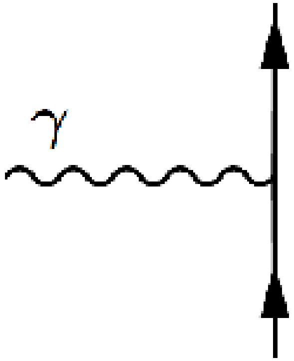
Spin polarization from correlation functions

Wigner function

$$S^{<}_{\alpha\beta}(X=\frac{x+y}{2},P)=\int d^4(x-y)e^{iP\cdot(x-y)/\hbar}\left(-\langle\bar{\psi}_\beta(y)\psi_\alpha(x)\rangle\right)$$

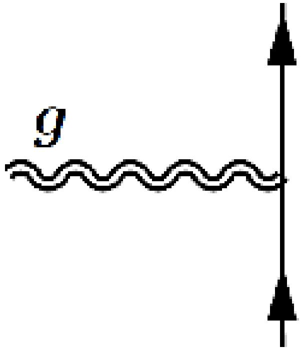
- Spin polarization in EM fields

$$\langle S^{<}(X,P)\rangle_{\text{eq},A_\mu}$$



- Spin polarization in off-equilibrium state: hydro gradient

$$\langle S^{<}(X,P)\rangle_{\text{off-eq}}=\langle S^{<}(X,P)\rangle_{\text{eq},h_{\mu\nu}}$$



$A_\mu,\ h_{\mu\nu}$ slow-varying $\partial_X \ll P$

Quantum (chiral) kinetic theory

$$\frac{i}{2}\partial\!\!\!/ S^< + P\!\!\!/ S^< = \frac{i}{2}\left(\Sigma^> S^< - \Sigma^< S^>\right)$$

diag part → spin-averaged Boltzmann equation

off-diag part → spin evolution equation

Hidaka, Pu, Wang, Yang, PPNP 2022

Yamamoto's talk

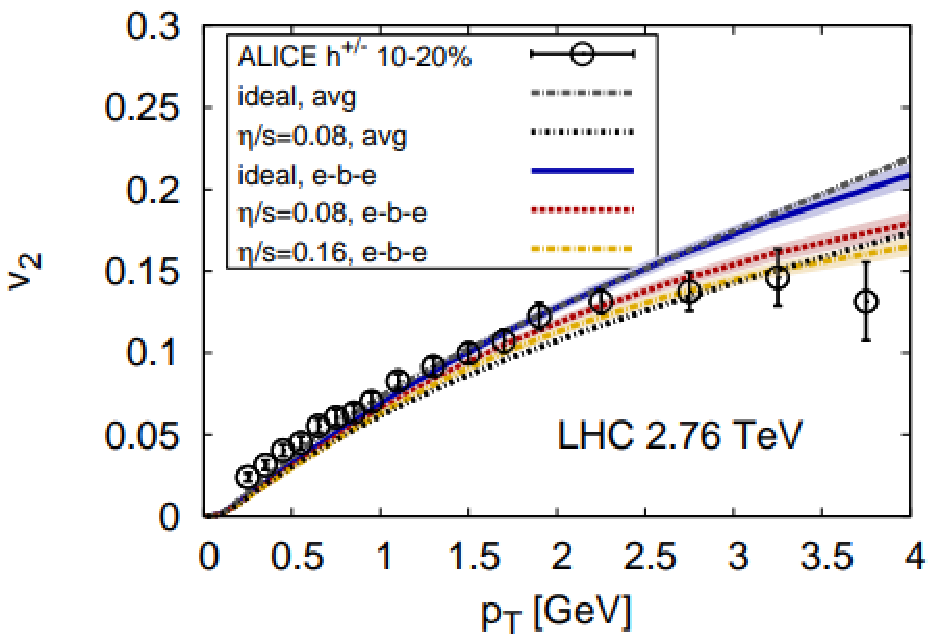
$$S^< = \frac{1}{4}\left(\gamma^\mu V_\mu + \gamma^5\gamma^\mu A_\mu\right) \propto \delta(P^2)$$ up to $O(\partial)$

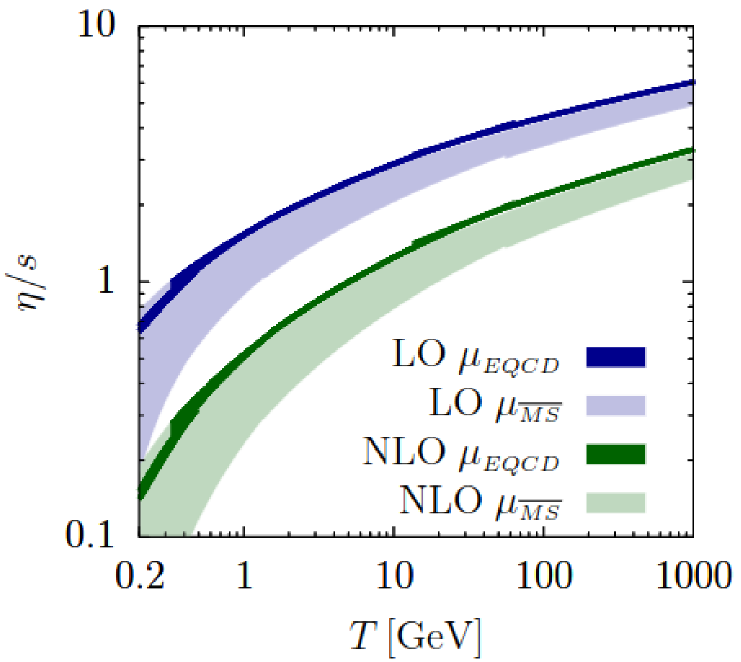
spin polarization

$$S^< = \frac{1}{4}\left[\left(1+\gamma^5\right)\gamma^\mu R_\mu + \left(1-\gamma^5\right)\gamma^\mu L_\mu\right]$$

Limitation of CKT

Phenomenology implementation based on free theory,
but correction in coupling can be significant and crucial

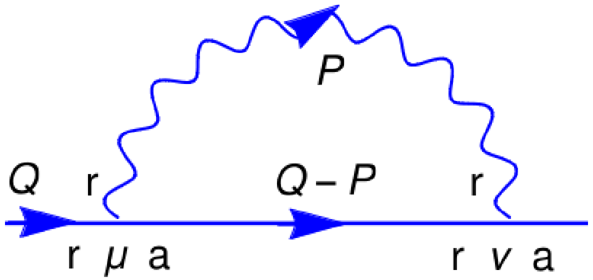




phenomenology $\frac{\eta}{s} \simeq 0.08$

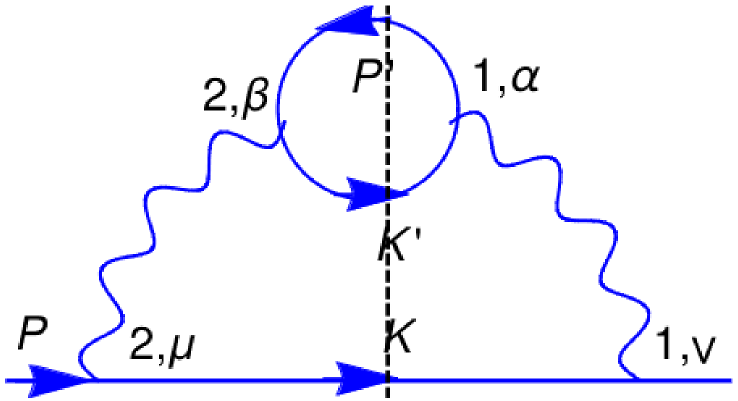
kinetic theory Arnold, Moore, Yaffe 2003
Ghiglieri, Moore, Teaney 2018

How to include coupling corrections?



correction to dispersion, usually ignored

this talk

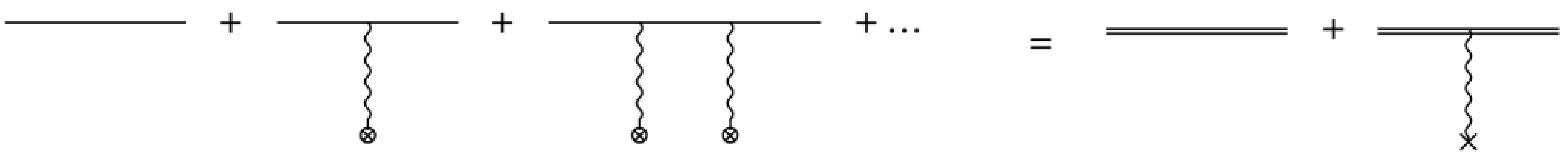


collision term, effect after resummation $O\left(\frac{\partial}{g^4}\right)$ Gagnon, Jeon, 2006

e.g. $\eta \sim \frac{1}{g^4}$

collisional contribution to spin-shear coupling:
SL, Wang, 2022, 2024

Equivalence of CKT to tree diagrams: EM fields



gauge link

motion modification by EM fields

for right-handed particle

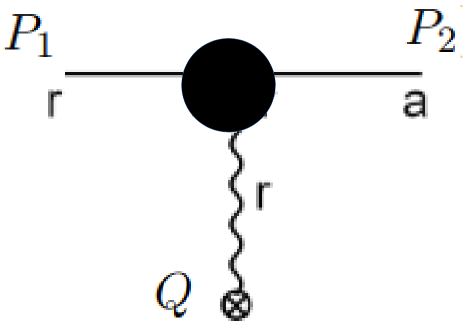
$$S^{<\mu} = -2\pi \left[\delta(P^2) P^\mu \tilde{f}(p_0) + \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} P_\nu F_{\alpha\beta} \delta'(P^2) \tilde{f}(p_0) \right]$$

equilibrium distribution unchanged

SL, Tian, 2023

Structure of radiative correction in medium

$$\Gamma^\mu = F_0 u^\mu + F_1 \hat{p}^\mu + F_2 \frac{i\epsilon^{\mu\nu\rho\sigma} u_\nu P_\rho Q_\sigma}{2(P\cdot u)^2}$$

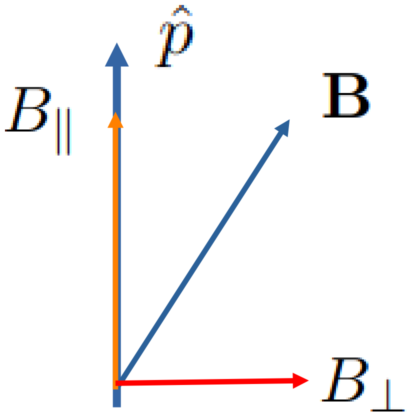


u^μ medium frame vector

$$S^{<0} = 2\pi F_2 p B_{\|} \delta'(P^2) f(p_0)$$

$$S^{<i} = 2\pi \left[F_0 \epsilon^{ijk} E_j p_k + F_1 p_0 B_\perp^i + F_2 B_{\|} p^i\right] \delta'(P^2) f(p_0)$$

spin Hall effect — spin-perpendicular magnetic coupling — spin-parallel magnetic coupling

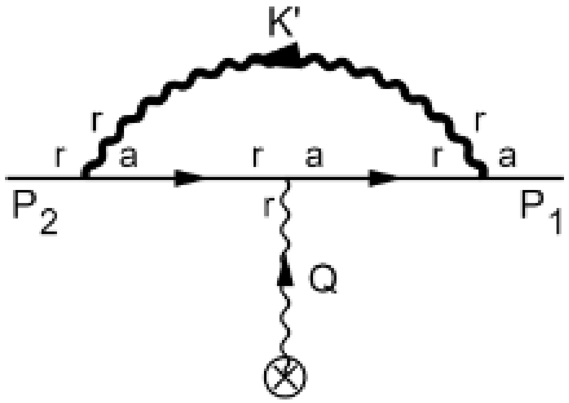


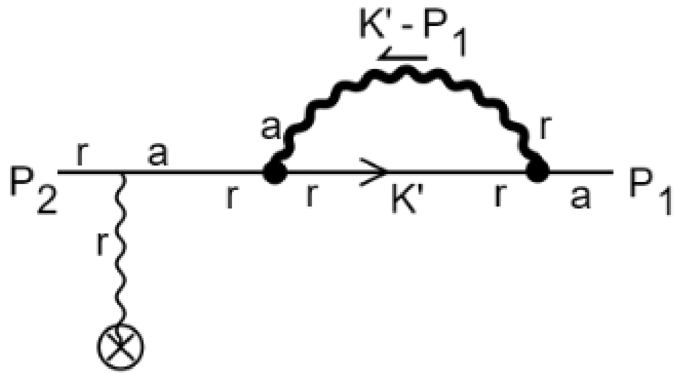
In vacuum $F_0 = F_1 = F_2 = 1$

In medium: lift of degeneracy possible

SL, Tian, 2023

One-loop correction to electromagnetic FF





Types of corrections:
vertex correction +
self-energy correction

$$\delta F_0 = \frac{2m_f^2}{p^2} X + \frac{m_f^2}{p^2}\left(1 - ln\frac{2p^2}{m_f^2}\right),$$

spin Hall effect

$$\delta F_1 = \frac{2m_f^2}{p^2}(X-1) + \frac{m_f^2}{p^2}\left(1 - ln\frac{2p^2}{m_f^2}\right),$$

spin-perpendicular
magnetic coupling

$$\delta F_2 = \frac{2m_f^2}{p^2} X + \frac{m_f^2}{p^2}\left(1 - ln\frac{2p^2}{m_f^2}\right),$$

spin-parallel
magnetic coupling

SL, Tian, 2023

$$X(p,T)$$

FF as renormalized couplings

CKT solution for off-equilibrium state

$$S^{<} = \frac{1}{4}\left[\,(1+\gamma^5)\,\gamma^\mu R_\mu + (1-\gamma^5)\,\gamma^\mu L_\mu\right]$$

$$R^\mu = -2\pi\delta(P^2)\left(P^\mu f_n + \frac{\epsilon^{\mu\nu\rho\sigma}P_\rho n_\sigma}{2P\cdot n}\partial_\nu f_n\right)$$

Hidaka, Pu, Yang 2016

n^μ arbitrary frame vector

$n^\mu \rightarrow u^\mu$

$$f\left(\frac{P\cdot u(X)}{T(X)}\right) \quad \Longrightarrow \quad S^i \sim \left(\beta\omega^i + \epsilon^{ijk}\hat{p}_l\hat{p}_k\beta\sigma_{jl} + \partial_i\beta\right)$$

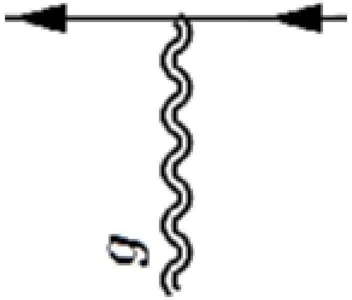
Hidaka, Pu, Yang 2017

Yi, Pu, Yang 2019

degenerate couplings to vorticity, shear, T-grad

Equilibrium state perturbed by metric: vertex+gauge link

perturbation I: motion modified by metric



$$S^<(X,P) = \int d^4y \sqrt{-g(X)} e^{-iP\cdot y} \langle \bar{\psi}_\beta(X+\frac{y}{2}) \psi_\alpha(X-\frac{y}{2}) \rangle$$

Gao, Huang, Mameda, Liu 2018

$$\bar{\psi}(X+\frac{y}{2}) = \bar{\psi}(X) \exp(\frac{y}{2}\cdot \overleftarrow{D}) \quad \psi(X-\frac{y}{2}) = \exp(-\frac{y}{2}\cdot D)\psi(X)$$

gravitational gauge link

$$D_\mu = \partial_\mu^X + \frac{1}{4}\omega_{\mu,ab}\gamma^{ab} - \Gamma^\lambda_{\mu\nu} y^\nu \partial^y_\lambda$$

perturbation II: rotation of spinor by spin connection

Perturbations can't change equilibrium distribution!

SL, Tian, to appear

Equilibrium state perturbed by metric: CKT

$$\gamma^\mu = e^\mu_a \gamma^a \qquad \bar{\psi} = \psi^\dagger \gamma^{\hat{0}}$$

μ curved index

a flat index

Gao, Huang, Mameda, Liu 2018

$$S^< = \frac{1}{4}\left[\,(1+\gamma^5)\,\gamma^a R_a + (1-\gamma^5)\,\gamma^a L_a\right]$$

Clifford algebra in flat basis

$$R^a = -2\pi\delta(P^2)\left(P^a f_n + \frac{\epsilon^{abcd} n_d}{2P\cdot n} e^\mu_b D_\mu (P_c f_n)\right)$$

f_n : equilibrium distribution

- agree with vertex+gravitational gauge link
- can't give spin hydro gradient coupling!

$$D_\mu = \partial^X_\mu + \Gamma^\lambda_{\mu\nu}\frac{\partial}{\partial P_\nu} P_\lambda$$

$$\langle S^<(X,P)\rangle_{\text{off-eq}} \overset{?}{=} \langle S^<(X,P)\rangle_{\text{eq},h_{\mu\nu}}$$

SL, Tian, to appear

Failure in mimicking off-equibrium by metric perturbation

What we have

$$R^a = -2\pi\delta(P^2)\left(P^a f_n + \frac{\epsilon^{abcd} n_d}{2P\cdot n} e_b^\mu D_\mu (P_c f_n)\right)$$

f_n : equilibrium distribution

$$D_\mu = \partial_\mu^X + \Gamma_{\mu\nu}^\lambda \frac{\partial}{\partial P_\nu} P_\lambda$$

What we want

$$R^a = -2\pi\delta(P^2)\left(P^a f_n + \frac{\epsilon^{abcd} P_c n_d}{2P\cdot n} \partial_b f_n(X)\right)$$

f_n : off-equilibrium distribution

Radiative correction expect to affect the Christoffel term only

SL, Tian, to appear

Hydrodynamics from metric perturbation

$$G^R_{\pi_i\pi_j} = \left(\delta_{ij} - \hat{k}_i\hat{k}_j\right)\frac{\eta k^2}{i\omega - \gamma_\eta k^2} + \hat{k}_i\hat{k}_j\frac{(\epsilon+p)(k^2c_s^2 - i\omega\gamma_s k^2)}{\omega^2 - k^2c_s^2 + i\omega\gamma_s k^2}$$

$$G^R_{\epsilon\epsilon} = \frac{(\epsilon+p)k^2}{\omega^2 - k^2c_s^2 + i\omega\gamma_s k^2}$$

$$\omega \to 0 \qquad G^R_{\pi_i\pi_j} \to -\delta_{ij}(\epsilon+p) \quad G^R_{\epsilon\epsilon} \to -\frac{\epsilon+p}{c_s^2}$$

$$\delta\pi_i = h_{0j}G^R_{\pi_i\pi_j} = -(\epsilon+p)\delta_{ij}h_{0j} \qquad \delta\epsilon = \frac{1}{2}h_{00}G^R_{\epsilon\epsilon} = -\frac{(\epsilon+p)}{2c_s^2}h_{00}$$

$$\Rightarrow \quad h_{0i} = -v^i \quad h_{00} = -2\frac{\delta T}{T}$$

- Off-equilibrium state reached after equilibration
- Interaction needed in principle, can be dropped in practice

An equilibrated state

$$R^a = -2\pi\delta(P^2)\left(P^a f_n + \frac{\epsilon^{abcd}n_d}{2P\cdot n}e^\mu_b D_\mu(P_c f_n)\right)$$

$$D_\mu = \partial^X_\mu + \Gamma^\lambda_{\mu\nu}\frac{\partial}{\partial P_\nu}P_\lambda$$

$$f\left(\frac{p_a e^a_\mu u^\mu}{T}\right)$$

$$u^\mu = (g_{00}^{-1/2}, 0, 0, 0)$$

equilibrium state in curved space

$$u^a = u^\mu e^a_\mu$$

off-equilibrium state in flat space

Proper choice of vielbein realizes off-equilibrium state

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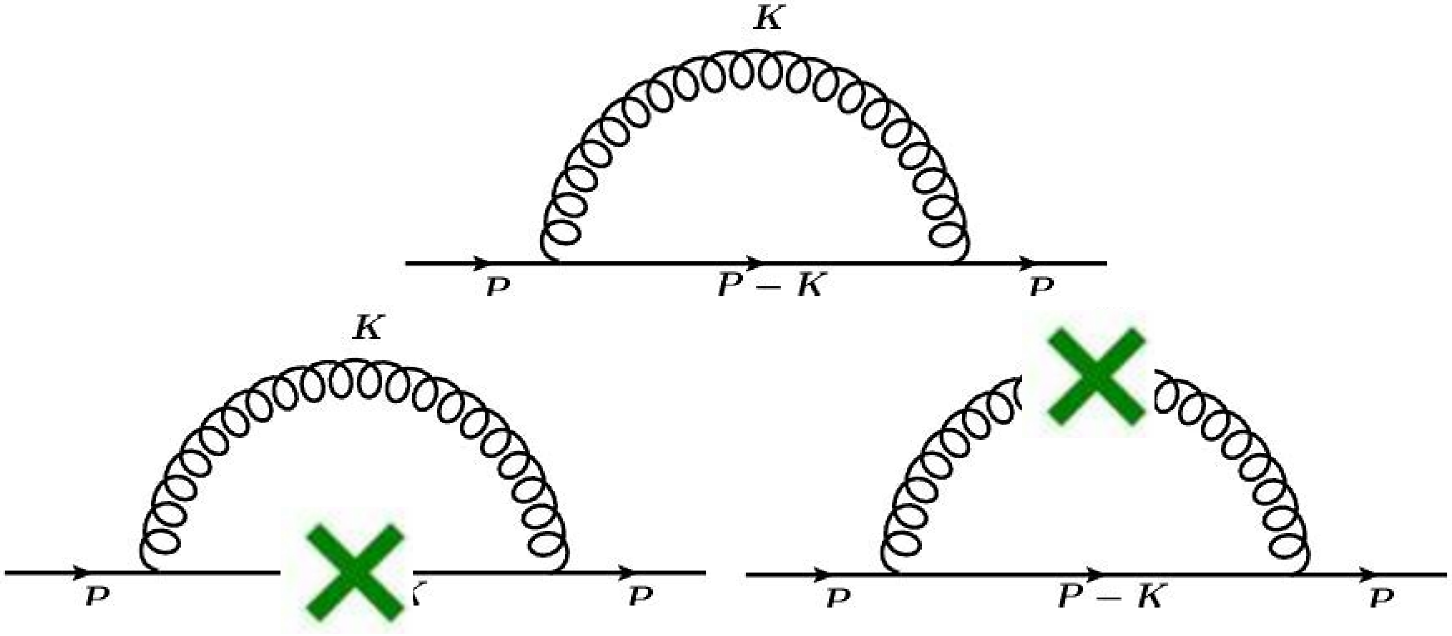
Radiative corrections to self-energy



off-equilibrium propagators for quark/gluon known from CKT

Hidaka, Pu, Yang 2017

Huang, Mitkin, Sadofyev, Speranza 2020

Hattori, Hidaka, Yamamoto, Yang 2020

SL 2020

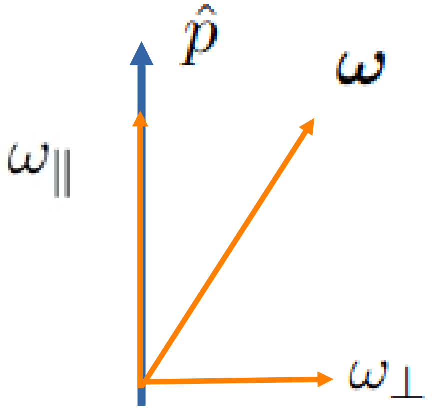
- Equilbrium self-energy doesn’t lead to polarization
- Off-equilibrium correction to self-energy contributes to polarization in addition to CKT results

SL, Tian, to appear

Polarization & damping rates

$$i\frac{\Sigma_{ar}(P)}{g^2} = \gamma^5\gamma_0\omega\cdot\hat{p}A_0(p,T) + \gamma^5\gamma_k\omega_{\parallel}^k A_{\parallel}(p,T) + \gamma^5\gamma_k\omega_{\perp}^k A_{\perp}(p,T)$$

$$+\,\epsilon^{ijk}\gamma^5\gamma_k\hat{p}_i\hat{p}_l\sigma_{jl}B(p,T) + \epsilon^{ijk}\gamma^5\gamma_k\hat{p}_i\partial_j\beta C(p,T)$$



A, B, C: complex functions

$$S_{ra}^R = \frac{i(P_\mu - iA_\mu)\bar{\sigma}^\mu}{(P-iA)^2} \qquad S_{ra}^L = \frac{i(P_\mu + iA_\mu)\sigma^\mu}{(P+iA)^2}$$

- lift of degeneracy in coupling to vorticity, shear, T-grad
- splitting of damping rates for R&L-handed particles

magnetic analog:

Dong, SL, 2024

SL, Tian, to appear

Conclusion

- Equilibrium state perturbed by metric fails to describe off-equilbrium state
- An equilibrated state in curved space describes off-equilbrium state
- Off-equilibrium correction to self-energy contributes to polarization
- Lift of degeneracy of spin coupling to vorticity, shear, T-grad
- Splitting of damping rate of R&L-handed particles

Outlook

- Quantum kinetic theory with self-energy correction

Thank you!

$$\left(2k^{\mu} - \frac{\partial \mathrm{Re}\Sigma}{\partial k_{\mu}}\right) \frac{\partial \rho}{\partial X^{\mu}} + \frac{\partial \mathrm{Re}\Sigma}{\partial X_{\mu}} \frac{\partial \rho}{\partial k^{\mu}} = -\left\{\Gamma, \mathrm{Re}\, G_R\right\}_{P.B.}$$

$$\rho(k, X) = \frac{\Gamma(k, X)}{\left(k^2 - m^2 - \Sigma^{\delta}(X) - \mathrm{Re}\, \Sigma_R(k, X)\right)^2 + \left(\Gamma(k, X)/2\right)^2}$$