Nuclear Science Computing Center at CCNU

Baryon electric charge correlation as a magnetometer of QCD

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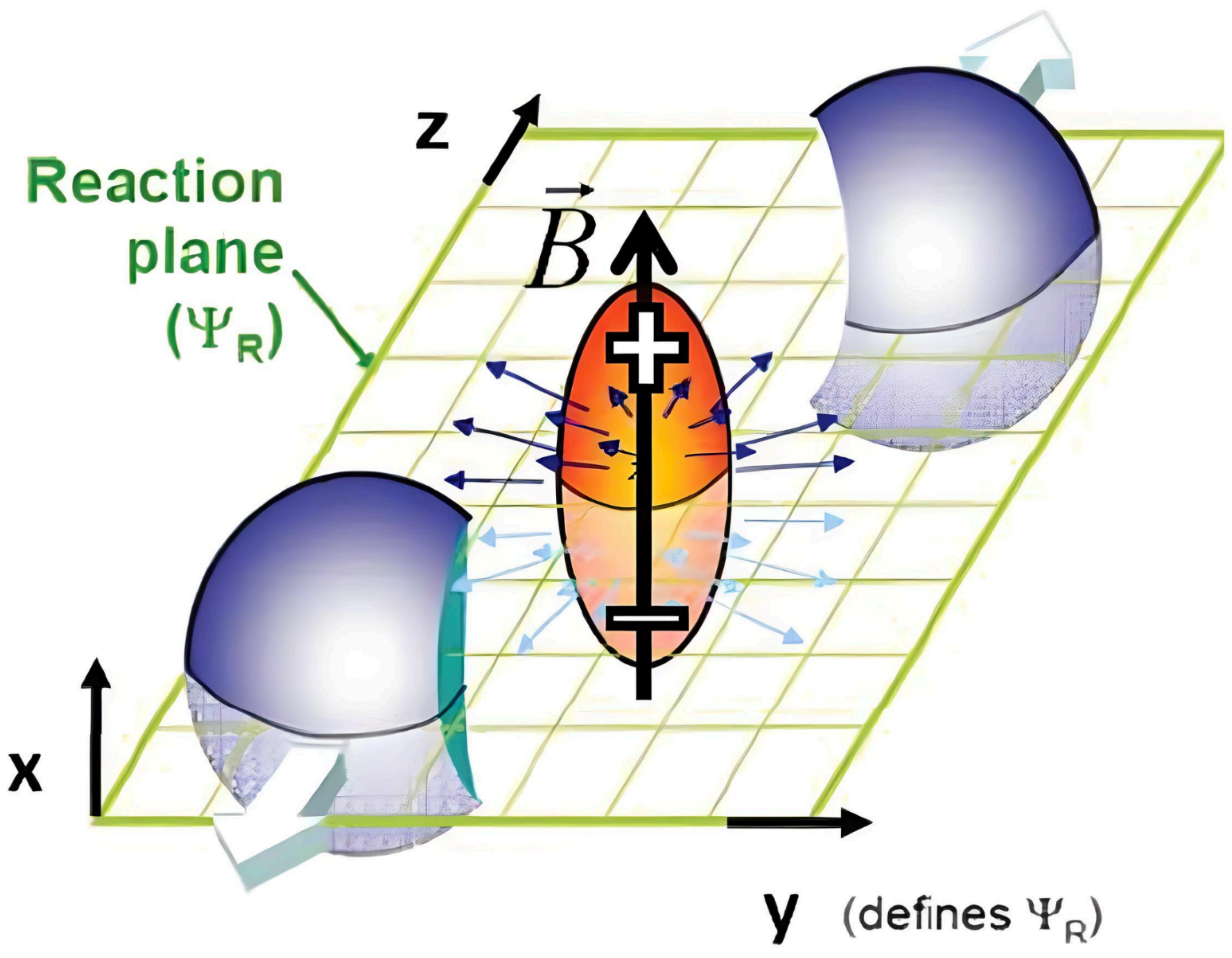
Based on Phys. Rev. Lett. **132**, 201903 (2024) and work in progress

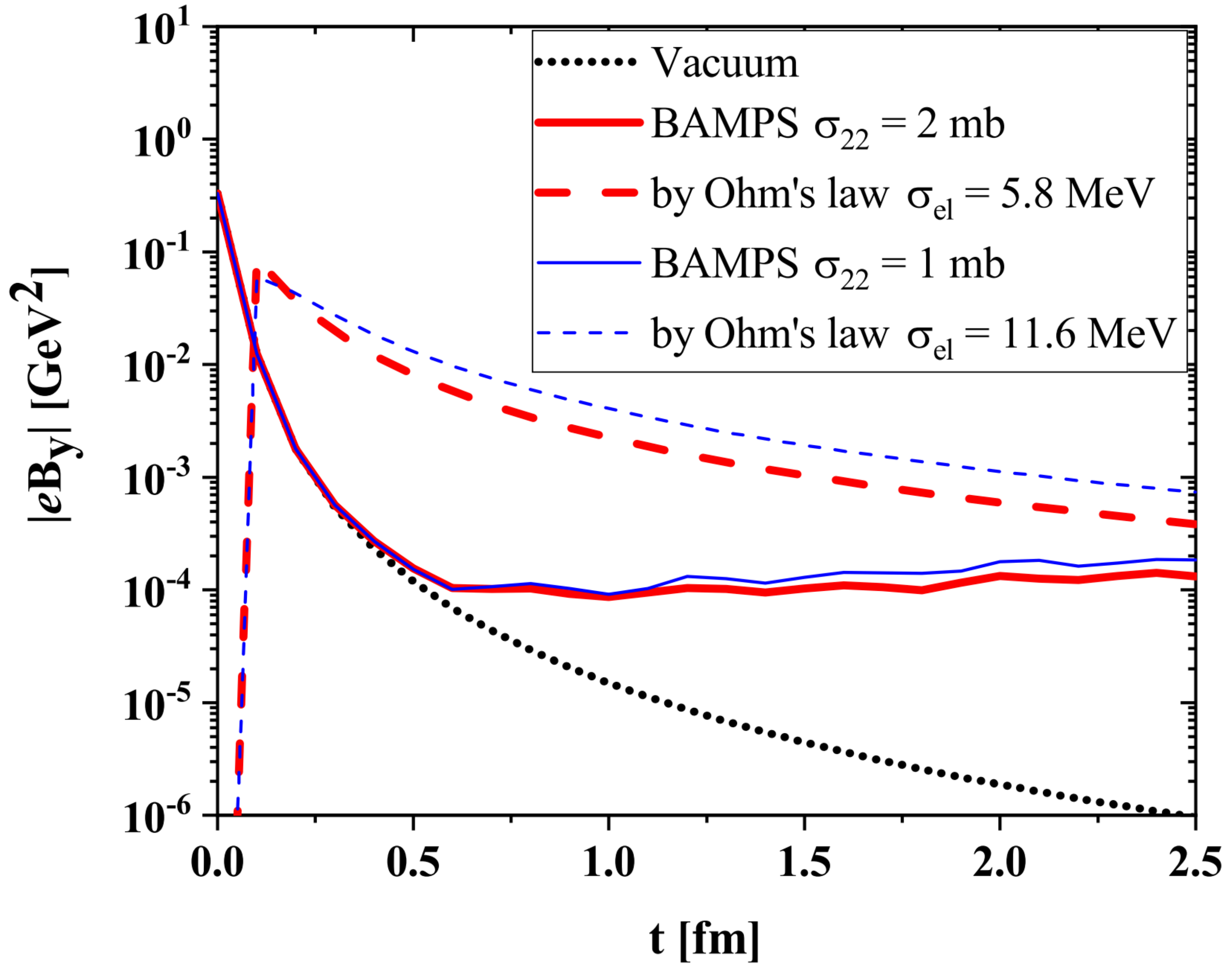
In collaboration with H.-T. Ding, A. Kumar, S.-T. Li and J.-H. Liu

The 20th International Conference on QCD in Extreme Conditions (XQCD 2024)
2024, July 17 - 19, 2024 @ LanZhou

Strong magnetic fields in heavy-ion collisions



W.-T. Deng et al. Phys.Rev.C 85 (2012) 044907



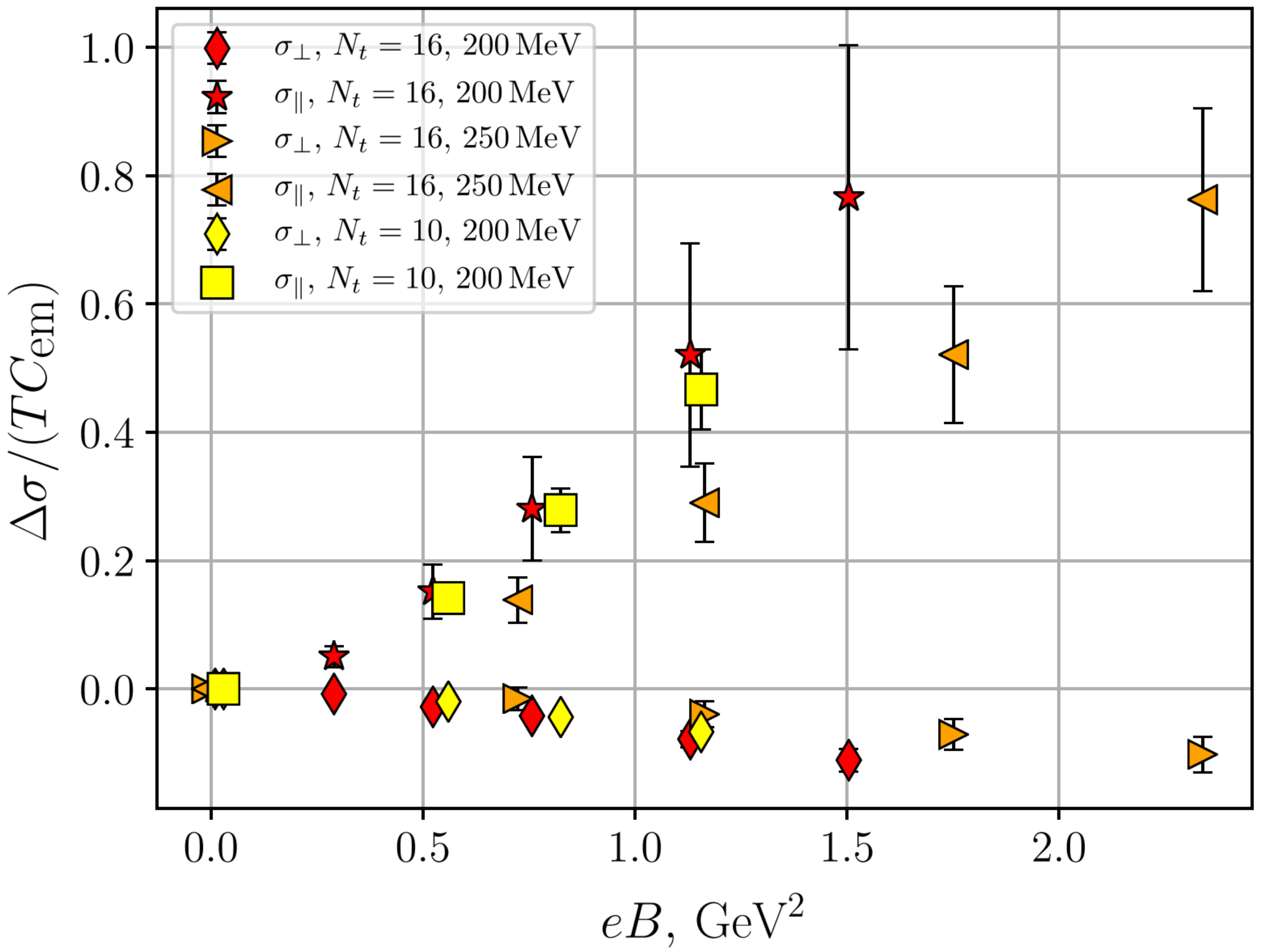
Z. Wang et al. Phys.Rev.C 105 (2022) L041901

$eB_{\tau=0} \sim 5\, M_\pi^2$ in RHIC $\quad eB_{\tau=0} \sim 70\, M_\pi^2$ in LHC

Whether strongly decaying magnetic field has impact on the final state of the collision?

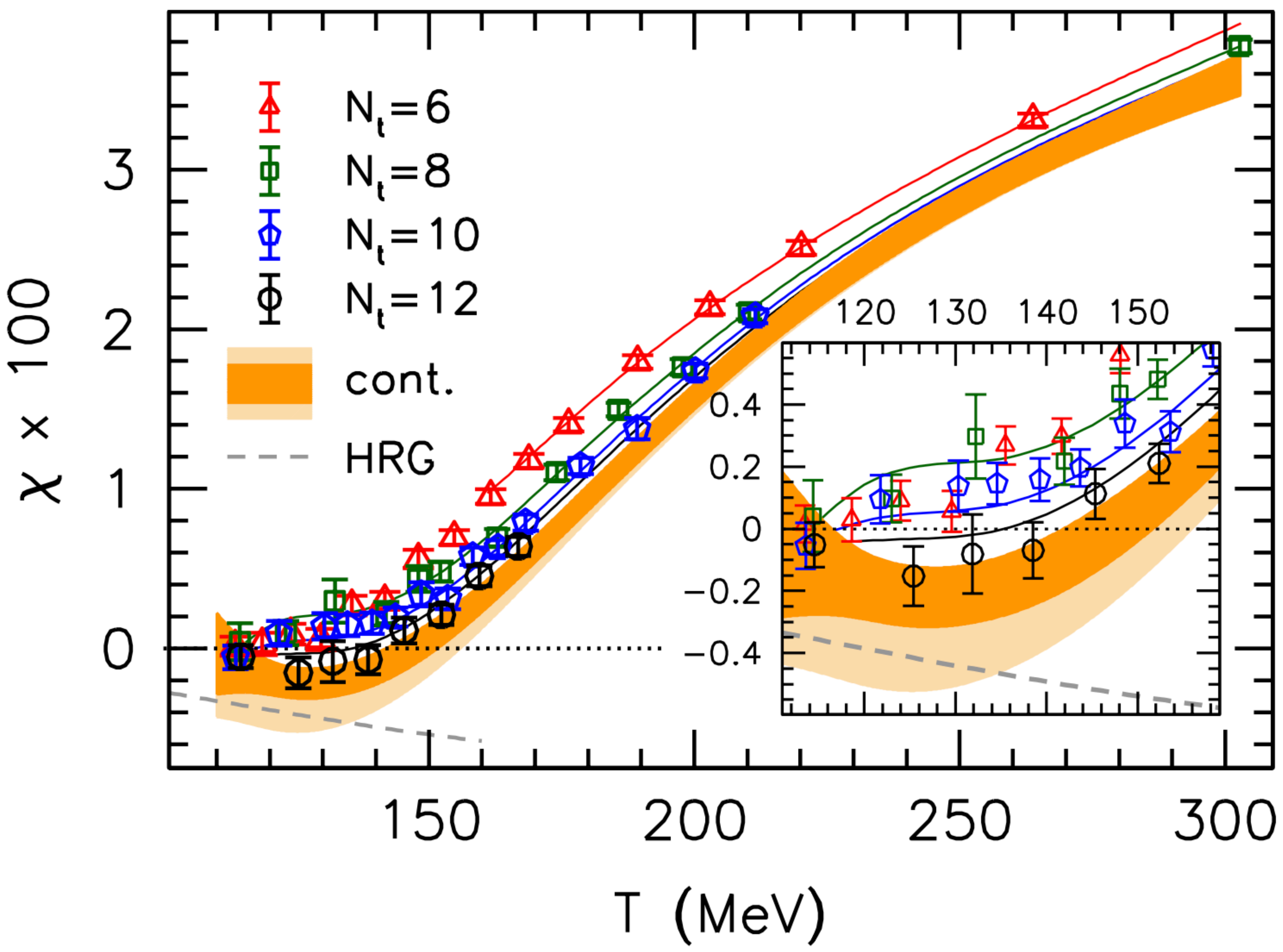
Electromagnetic conductivity and type of magnetism of QGP

Electromagnetic conductivities at non-zero magnetic fields



N. Astrakhantsev et al., PRD 102 (2020) 054516

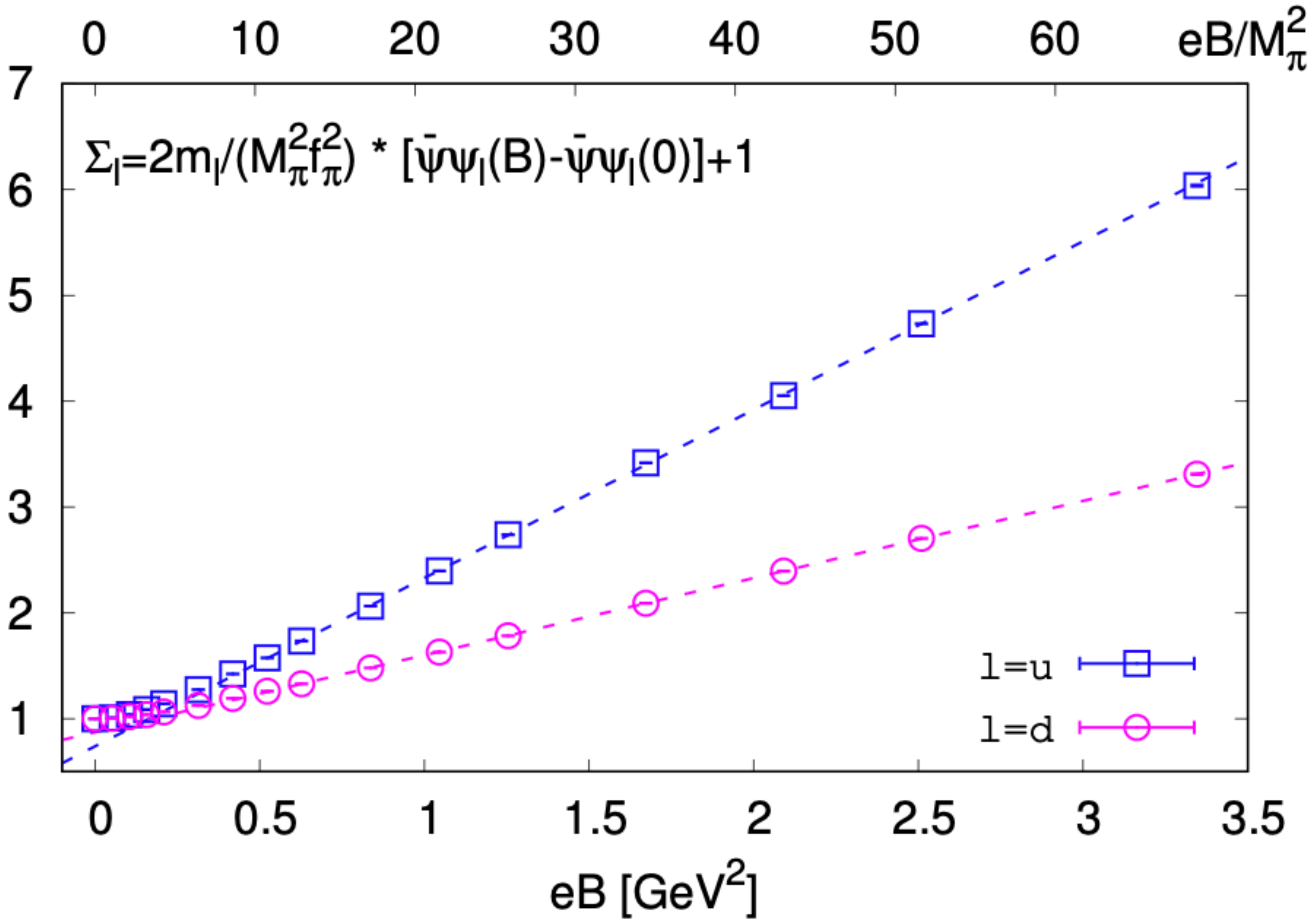
Magnetic susceptibility at non-zero magnetic fields



G. Bali et al., JHEP 07 (2020) 183

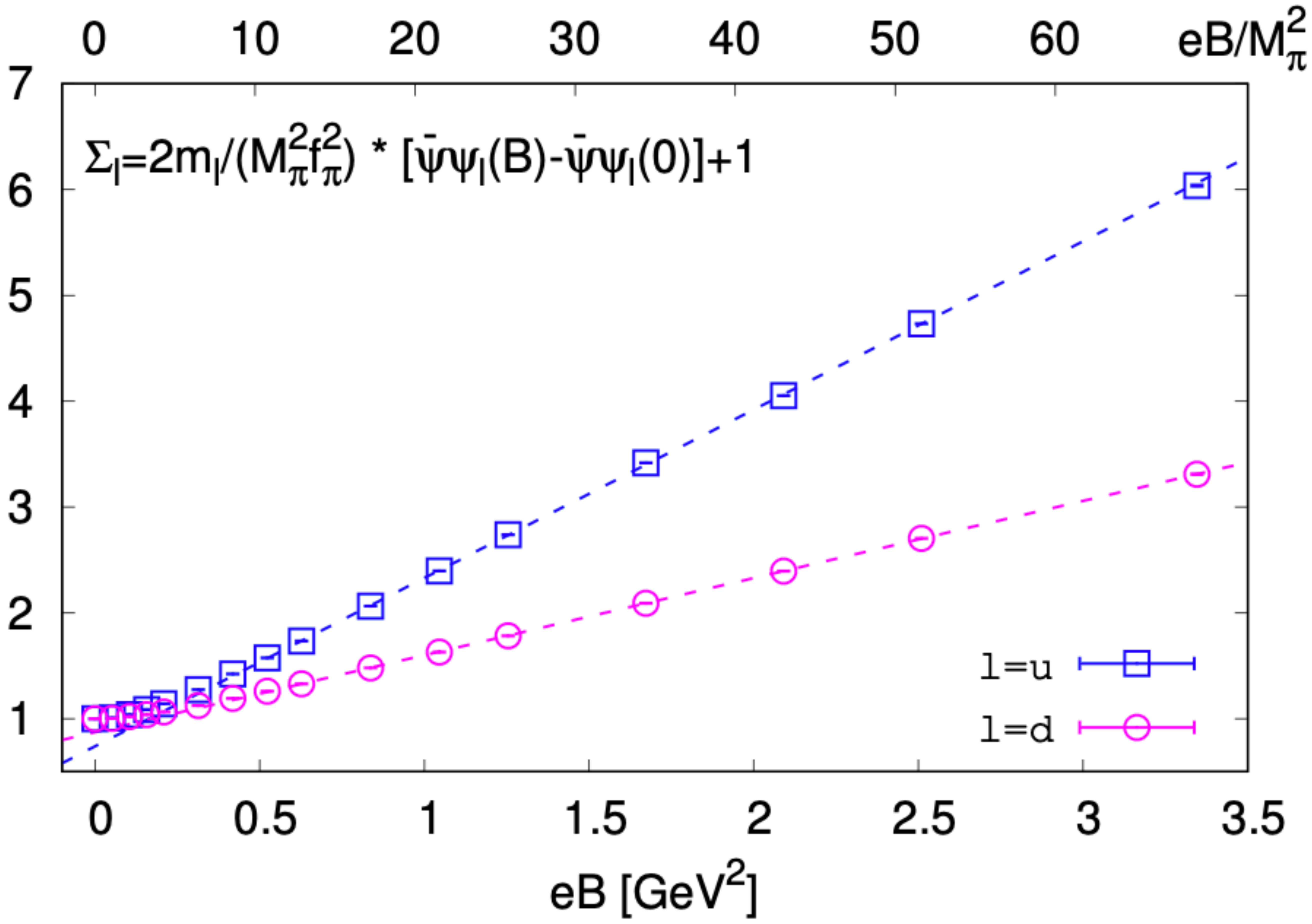
Questions: What observables are suitable as probes for magnetic fields in HIC?

Isospin symmetry breaking at $eB \neq 0$ manifested in chiral condensates



H.-T.Ding, S.-T. Li, A. Tomiya, X.-D. Wang and Y. Zhang, Phys.Rev.D 104 (2021) 1, 014505

- *u* quarks and *d* quarks condensates are clearly different in strong magnetic fields
- Signal of isospin symmetry breaking



H.-T.Ding, S.-T. Li, A. Tomiya, X.-D. Wang and Y. Zhang, Phys.Rev.D 104 (2021) 1, 014505

A clear effect but Not accessible in HIC experiments!

Fluctuations of net baryon number, electric charge and strangeness

Taylor expansion of the QCD pressure:

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln \mathcal{Z}\left(T, V, \hat{\mu}_u, \hat{\mu}_d, \hat{\mu}_s\right) = \sum_{i,j,k=0}^{\infty} \frac{\chi_{ijk}^{\mathrm{BQS}}}{i!j!k!} \left(\frac{\mu_{\mathrm{B}}}{T}\right)^i \left(\frac{\mu_{\mathrm{Q}}}{T}\right)^j \left(\frac{\mu_{\mathrm{S}}}{T}\right)^k$$

C. Allton et al., Phys.Rev. D 66 (2002) 074507

Taylor expansion coefficients at $\mu = 0$ are computable in LQCD

$$\chi_{ijk}^{uds} = \left.\frac{\partial^{i+j+k} p/T^4}{\partial\left(\mu_u/T\right)^i \partial\left(\mu_d/T\right)^j \partial\left(\mu_s/T\right)^k}\right|_{\mu_{u,d,s}=0}$$

$$\chi_{ijk}^{\mathrm{BQS}} = \left.\frac{\partial^{i+j+k} p/T^4}{\partial\left(\mu_{\mathrm{B}}/T\right)^i \partial\left(\mu_{\mathrm{Q}}/T\right)^j \partial\left(\mu_{\mathrm{S}}/T\right)^k}\right|_{\mu_{\mathrm{B,Q,S}}=0}$$

$$\mu_u = \frac{1}{3}\mu_{\mathrm{B}} + \frac{2}{3}\mu_{\mathrm{Q}}$$

$$\mu_d = \frac{1}{3}\mu_{\mathrm{B}} - \frac{1}{3}\mu_{\mathrm{Q}}$$

$$\mu_s = \frac{1}{3}\mu_{\mathrm{B}} - \frac{1}{3}\mu_{\mathrm{Q}} - \mu_{\mathrm{S}}$$

LQCD: H.-T.Ding, F. Karsch, S.Mukherjee, Int. J. Mod. Phys. E 24 (2015) no.10, 1530007
Exp.: X.-F. Luo & N. Xu, Nucl. Sci. Tech. 28 (2017) 112

μ_{Q} and μ_{S} can expanded as,

$$\hat{\mu}_{\mathrm{Q}} = q_1\hat{\mu}_{\mathrm{B}} + q_3\hat{\mu}_{\mathrm{B}}^3 + \mathcal{O}(\hat{\mu}_{\mathrm{B}}^5)$$

$$\hat{\mu}_{\mathrm{S}} = s_1\hat{\mu}_{\mathrm{B}} + s_3\hat{\mu}_{\mathrm{B}}^3 + \mathcal{O}(\hat{\mu}_{\mathrm{B}}^5)$$

with $n_{\mathrm{Q}}/n_{\mathrm{B}} = \mathit{constant}$ and $n_{\mathrm{S}} = 0$

Pressure caused by $\mu_{\mathrm{B}} \neq 0$ at $eB \neq 0$:

$$\frac{\Delta P}{T^4} = \frac{P(T, eB, \hat{\mu}_{\mathrm{B}}) - P(T, eB, 0)}{T^4} = \sum_{k=1}^{\infty} P_{2k}(T)\hat{\mu}_{\mathrm{B}}^{2k}$$

A. Bazavov et al. Phys. Rev. D 95 (2017) 054504

Lattice Setup

- Highly improved staggered fermions and a tree-level improved Symanzik gauge action
- $N_f = 2 + 1$
- Lattice sizes : $32^3 \times 8,\ 48^3 \times 12$
- $m_s^{\text{phy}}/m_l = 27,\ M_\pi \approx 135\ \text{MeV}$
- T window : $(144\ \text{MeV},\ 165\ \text{MeV})$, i.e. $(0.9T_{pc},\ 1.1T_{pc})$
- eB window: $0 \leq eB < 45M_\pi^2$

$$eB = \frac{6\pi N_b}{N_x N_y} a^{-2}, \quad N_b = [0,32]$$

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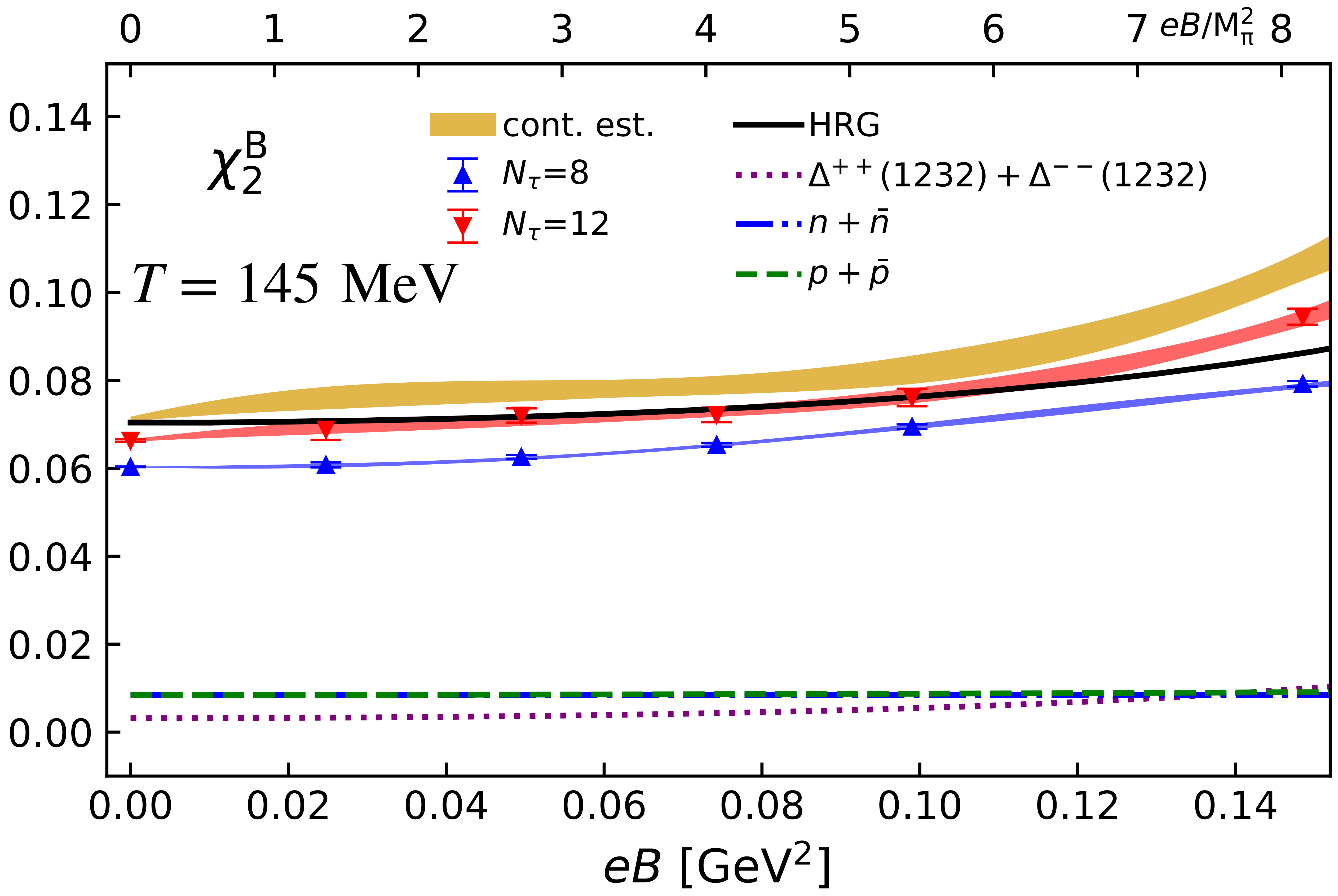
Baryon number fluctuations at $T = 145$ MeV



H.-T. Ding, J-B. Gu et al. Phys. Rev. Lett. ***132****, 201903 (2024)*

- χ_2^B increases $\sim 45\%$ at $eB \sim 8M_\pi^2$

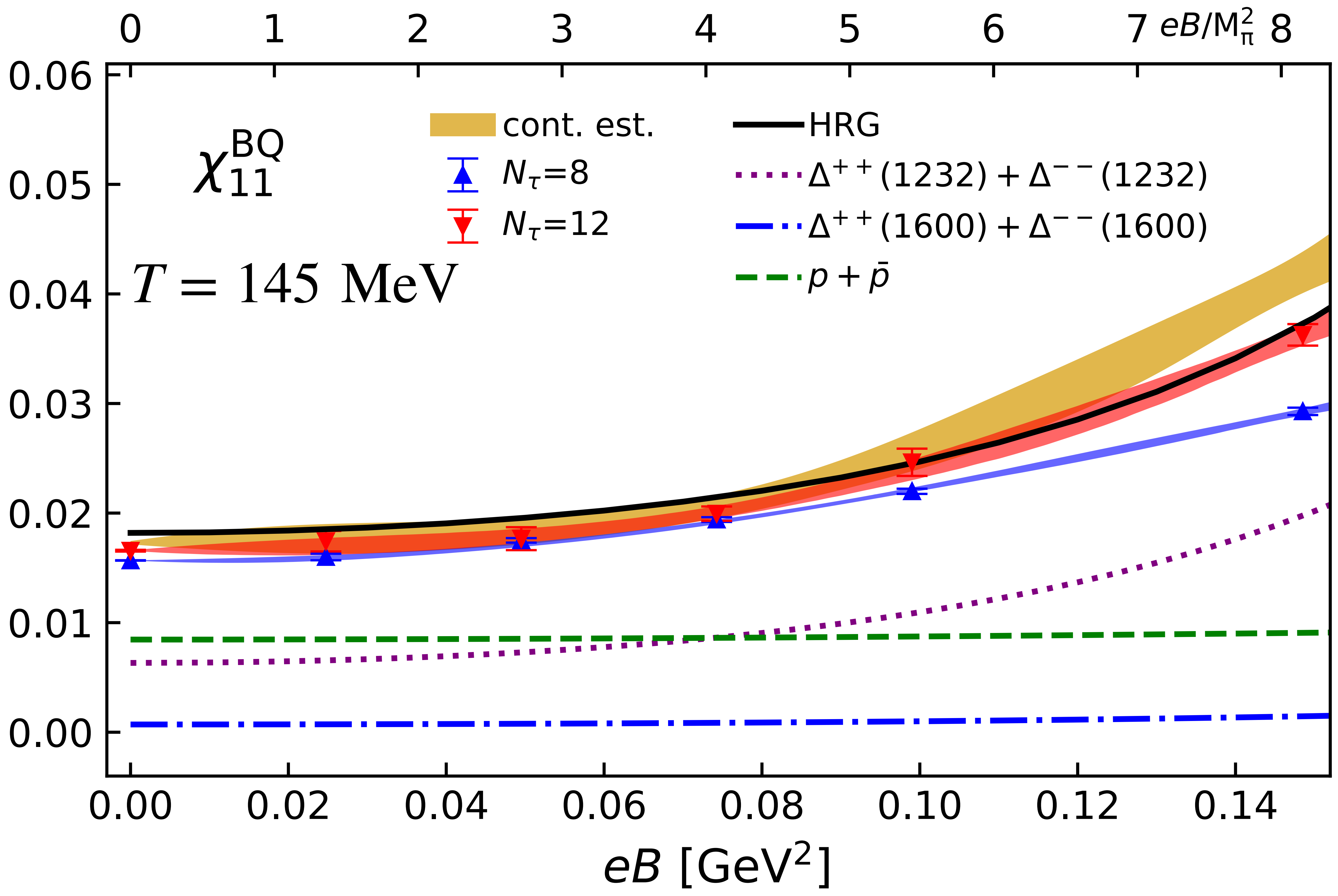
- Hadron Resonance Gas model (HRG): Pressure arising from charged hadrons ($eB \neq 0$):

$$\frac{p_c^{M/B}}{T^4} = \frac{|q_i| B}{2\pi^2 T^3} \sum_{s_z=-s_i}^{s_i} \sum_{l=0}^{\infty} \varepsilon_0 \sum_{n=1}^{\infty} (\pm 1)^{n+1} \frac{e^{n\mu_i/T}}{n} K_1\left(\frac{n\varepsilon_0}{T}\right)$$

where $\varepsilon_0 = \sqrt{m_i^2 + 2|q_i| B (l + 1/2 - s_z)}$,

K_1 is the first-order modified Bessel function of the second kind

Baryon electric charge correlation at $T = 145$ MeV



H.-T. Ding, J-B. Gu et al. Phys. Rev. Lett. ***132****, 201903 (2024)*

- ✦ χ_{11}^{BQ} increases ~ **140%** at $eB \sim 8M_\pi^2$
- ✦ The results of HRG model are consistent with LQCD up to $eB \sim 6M_\pi^2$
- ✦ $\Delta^{++}(1232)$ and $\Delta^{--}(1232)$ give most of the contributions of magnetic field dependence of χ_{11}^{BQ}
- ✦ $\Delta^{++}(1232)$ and $\Delta^{--}(1232)$ are not measurable in HIC experiments

Proxy construction based on the HRG

$\Delta^{++}(1232) \rightarrow p + \pi^+$: branching ratio almost **100%** !

HRG: Fluctuations expressed in terms of stable hadronic states:

$$\chi_{ijk}^{BQS}\left(T, \hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S\right) = \sum_R B_R^i Q_R^j S_R^k \frac{\partial^l p_R/T^4}{\partial \hat{\mu}_R^l}$$

net- B : $\tilde{p} + \tilde{n} + \tilde{\Lambda} + \tilde{\Sigma}^+ + \tilde{\Sigma}^- + \tilde{\Xi}^0 + \tilde{\Xi}^- + \tilde{\Omega}^-$

net- Q : $\tilde{\pi}^+ + \tilde{K}^+ + \tilde{p} + \tilde{\Sigma}^+ - \tilde{\Sigma}^- - \tilde{\Xi}^- - \tilde{\Omega}^-$

net- S : $\tilde{K}^+ + \tilde{K}^0 - \tilde{\Lambda} - \tilde{\Sigma}^+ - \tilde{\Sigma}^- - 2\tilde{\Xi}^0 - 2\tilde{\Xi}^- - 3\tilde{\Omega}^-$

B_R, Q_R, S_R are the baryon number, electric charge and strangeness of the species R

R. Bellwied et al. ,Phys. Rev. D 101, 034506 (2020)

In HIC, fluctuations are related to the variance or covariance of net-multiplicity for Identified π, K, p

STAR, Phys.Rev.C 100 (2019) 1, 014902 ; STAR, Phys.Rev.C 105(2019) 2, 029901

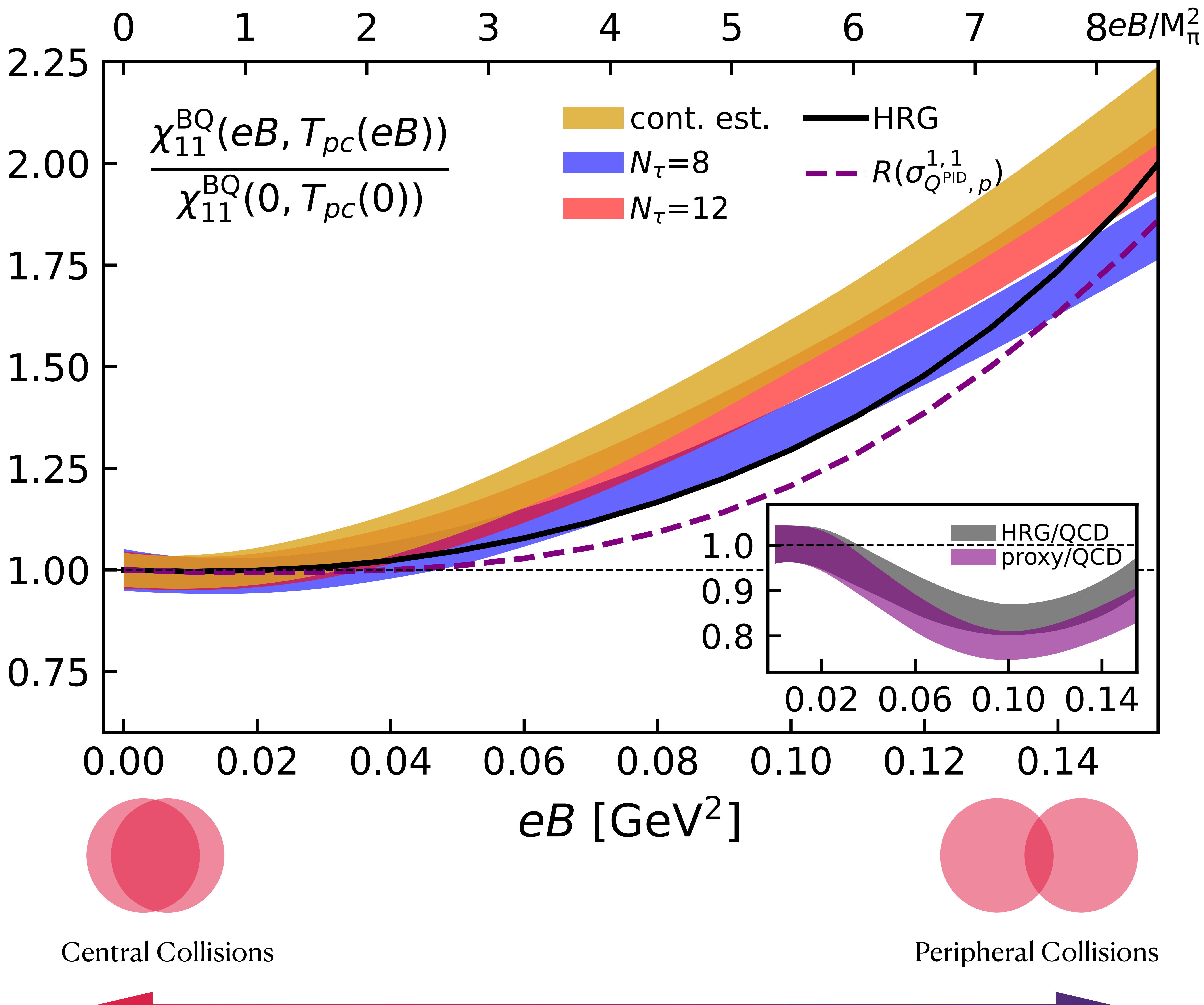
$\sigma^{1,1}_{Q^{PID},p}$ as proxy for χ^{BQ}_{11}:

$$\sigma^{1,1}_{Q^{PID},p} = \sum_R \left(P_{R\rightarrow\tilde{p}}\right)\left(P_{R\rightarrow Q^{PID}}\right)\frac{\partial^2 p_R/T^4}{\partial \hat{\mu}_R^2} + \frac{\partial^2 p_{\tilde{p}}/T^4}{\partial \hat{\mu}_{\tilde{p}}^2}$$

where $P_{R\rightarrow i}$ represents number of particle i produced by particle R after the **entire decay chain**,
Q^{PID} : $\tilde{\pi}^+$, $\tilde{K}^+$, $\tilde{p}$

In proxy, contributions from all resonance decays are considered!

Proxy for χ_{11}^{BQ} along the transition line



H.-T. Ding, J-B. Gu et al. Phys. Rev. Lett. **132**, 201903 (2024)

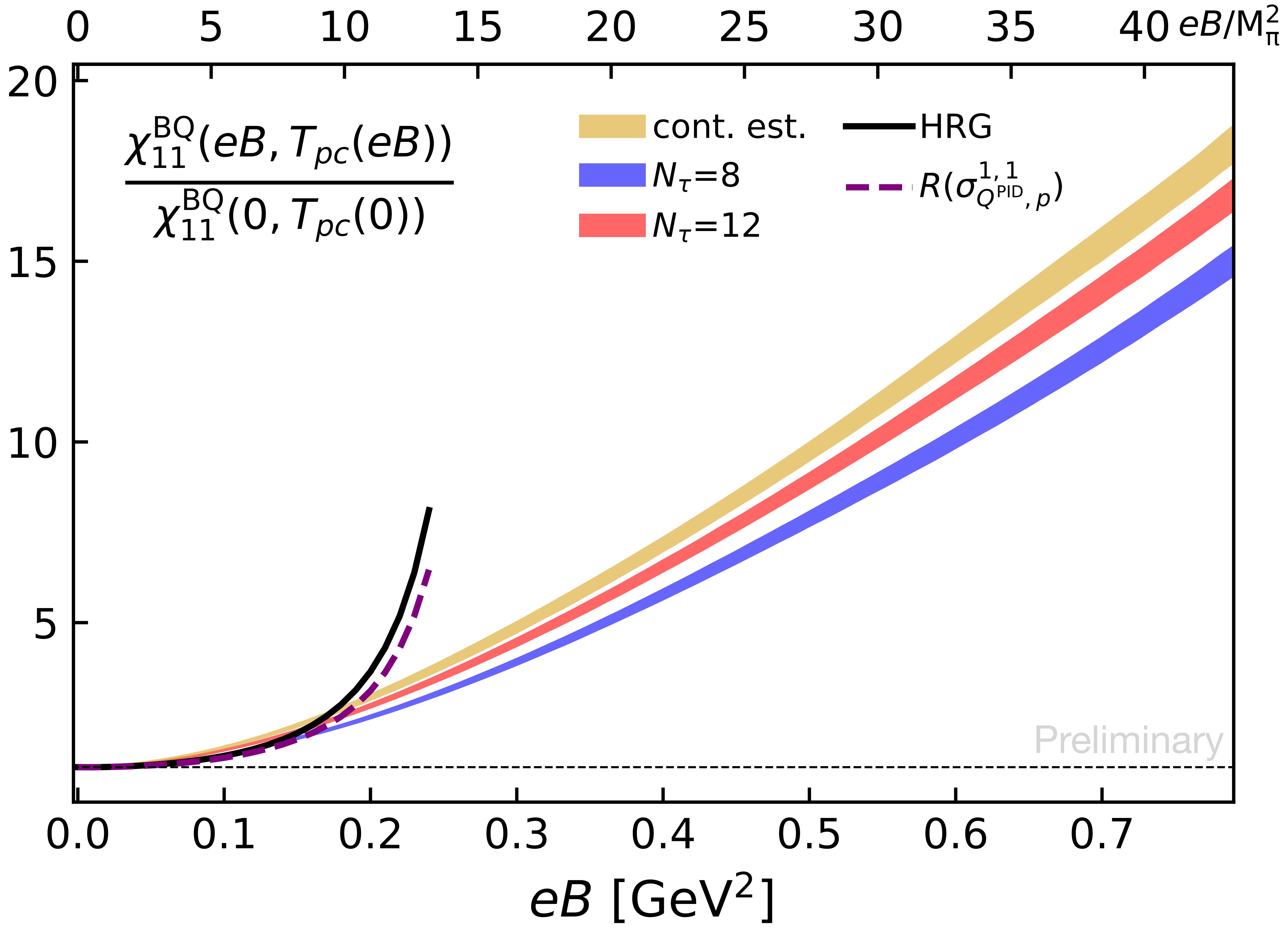
- At $eB \simeq 8M_\pi^2$, ratio of $\chi_{11}^{BQ} \sim$ **2.1**

$$R(\sigma^{1,1}_{Q^{PID},p}) = \sigma^{1,1}_{Q^{PID},p}(eB)/\sigma^{1,1}_{Q^{PID},p}(eB=0)$$

- The proxy $R(\sigma^{1,1}_{Q^{PID},p})$ can represent **80～85%** of the LQCD results

- $R(\sigma^{1,1}_{Q^{PID},p})$ is a reasonable proxy for χ_{11}^{BQ}

χ_{11}^{BQ} along the transition line in the large magnetic field range



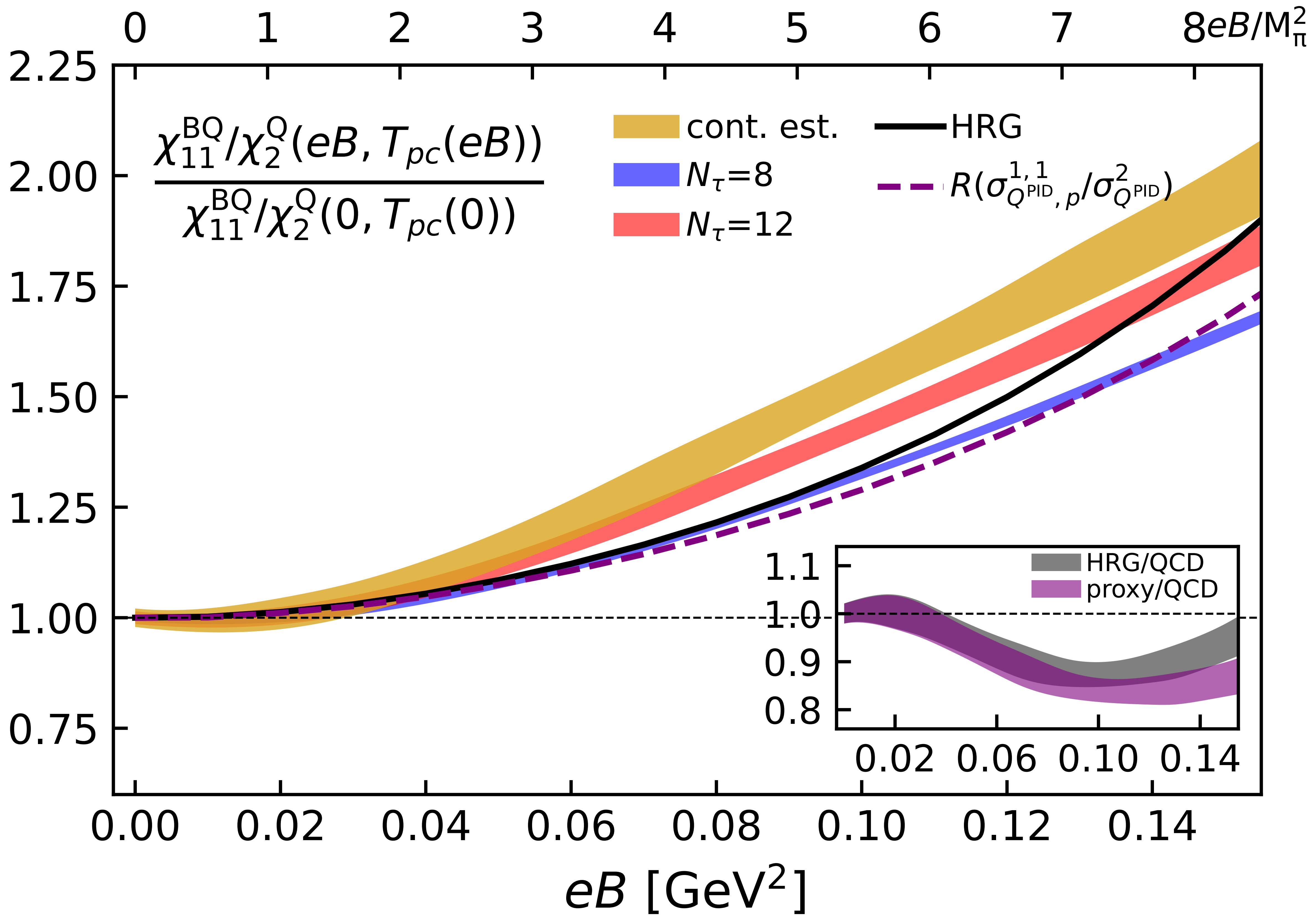
✦At $eB \simeq 40M_\pi^2$, ratio of $\chi_{11}^{BQ} \sim$ **17**

$$R(\sigma^{1,1}_{Q^{PID},p}) = \sigma^{1,1}_{Q^{PID},p}(eB)/\sigma^{1,1}_{Q^{PID},p}(eB=0)$$

✦At $eB > 10M_\pi^2$, the HRG and proxy are breaking down

H.-T. Ding, J.-B. Gu, A. Kumar and S.-T. Li work in progress

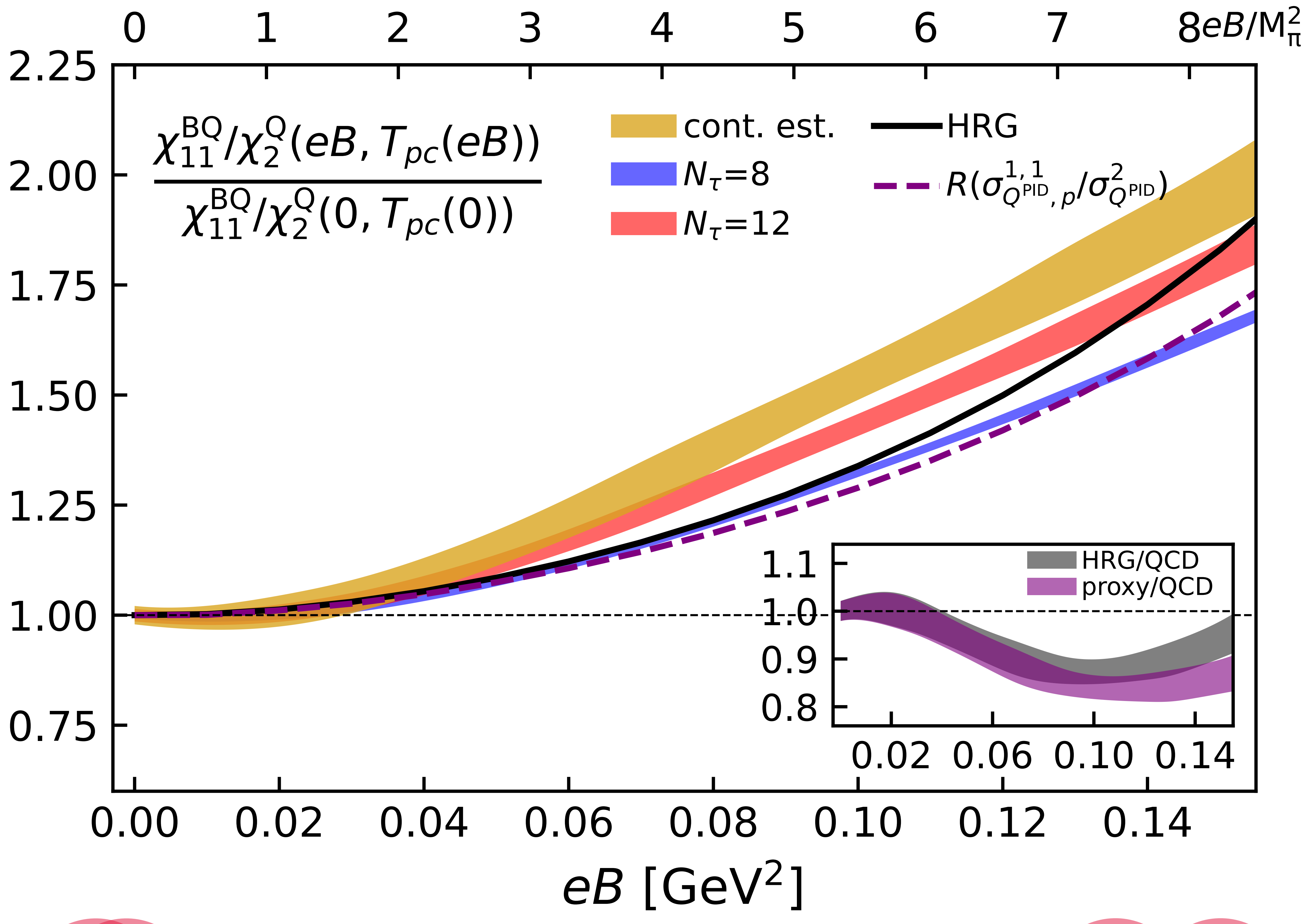
Proxy for $\chi_{11}^{\mathrm{BQ}}/\chi_2^{\mathrm{Q}}$ along the transition line

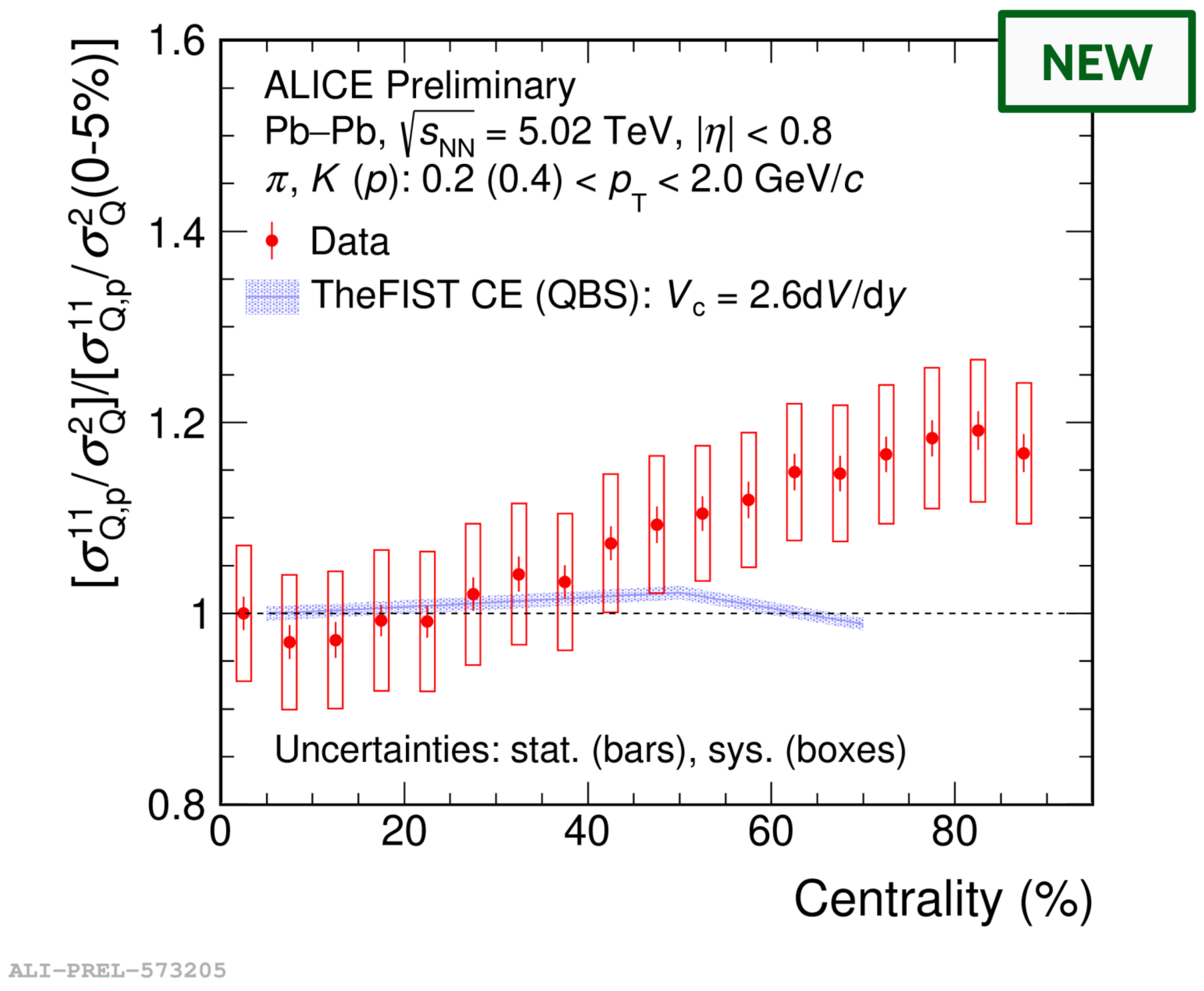


- At $eB \simeq 8M_\pi^2$, ratio of $\chi_{11}^{\mathrm{BQ}}/\chi_2^{\mathrm{Q}} \sim \mathbf{1.9}$
- The proxy $R(\sigma^{1,1}_{Q^{PID},p}/\sigma^2_{Q^{PID}})$ can represent ~**85%** of the LQCD results
- $R(\sigma^{1,1}_{Q^{PID},p}/\sigma^2_{p})$ is a reasonable proxy for $\chi_{11}^{\mathrm{BQ}}/\chi_2^{\mathrm{Q}}$

H.-T. Ding, J-B. Gu et al. Phys. Rev. Lett. ***132****, 201903 (2024)*

Proxy for $\chi_{11}^{BQ}/\chi_2^{Q}$ compare with experiments





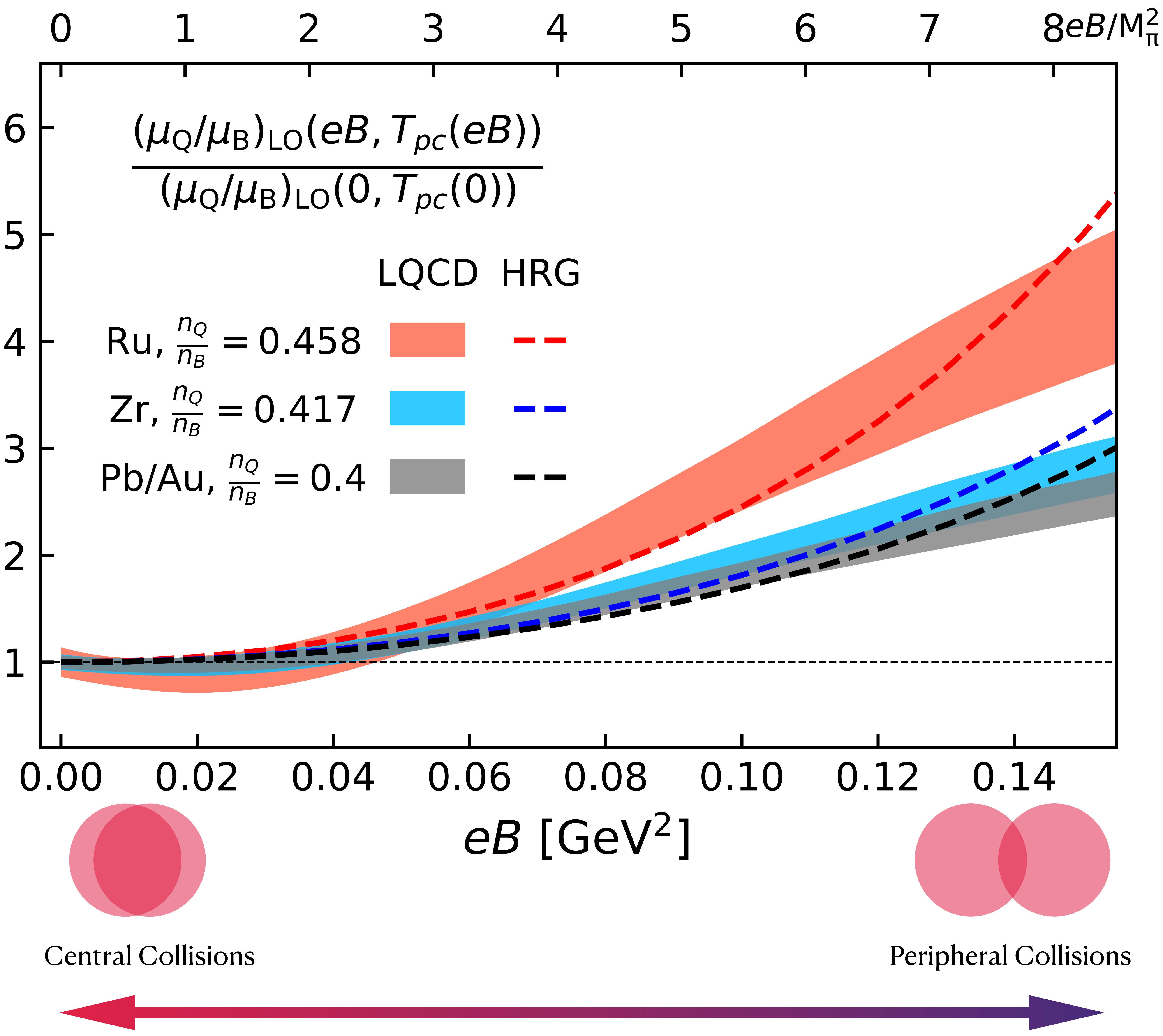
Central Collisions

Peripheral Collisions

S. Saha et al. (ALICE Collaboration) @ SQM 2024

H.-T. Ding, J-B. Gu et al. Phys. Rev. Lett. ***132****, 201903 (2024)*

Dependence of $(\mu_Q/\mu_B)_{LO}$ on the magnetic field



H.-T. Ding, J-B. Gu et al. Phys. Rev. Lett. **132**, 201903 (2024)

$$\mu_Q/\mu_B = q_1 + q_3\,\hat{\mu}_B^2 + \mathcal{O}(\hat{\mu}_B^4)$$

$$q_1 = \frac{r\left(\chi_2^B\chi_2^S - \chi_{11}^{BS}\chi_{11}^{BS}\right) - \left(\chi_{11}^{BQ}\chi_2^S - \chi_{11}^{BS}\chi_{11}^{QS}\right)}{\left(\chi_2^Q\chi_2^S - \chi_{11}^{QS}\chi_{11}^{QS}\right) - r\left(\chi_{11}^{BQ}\chi_2^S - \chi_{11}^{BS}\chi_{11}^{QS}\right)}$$

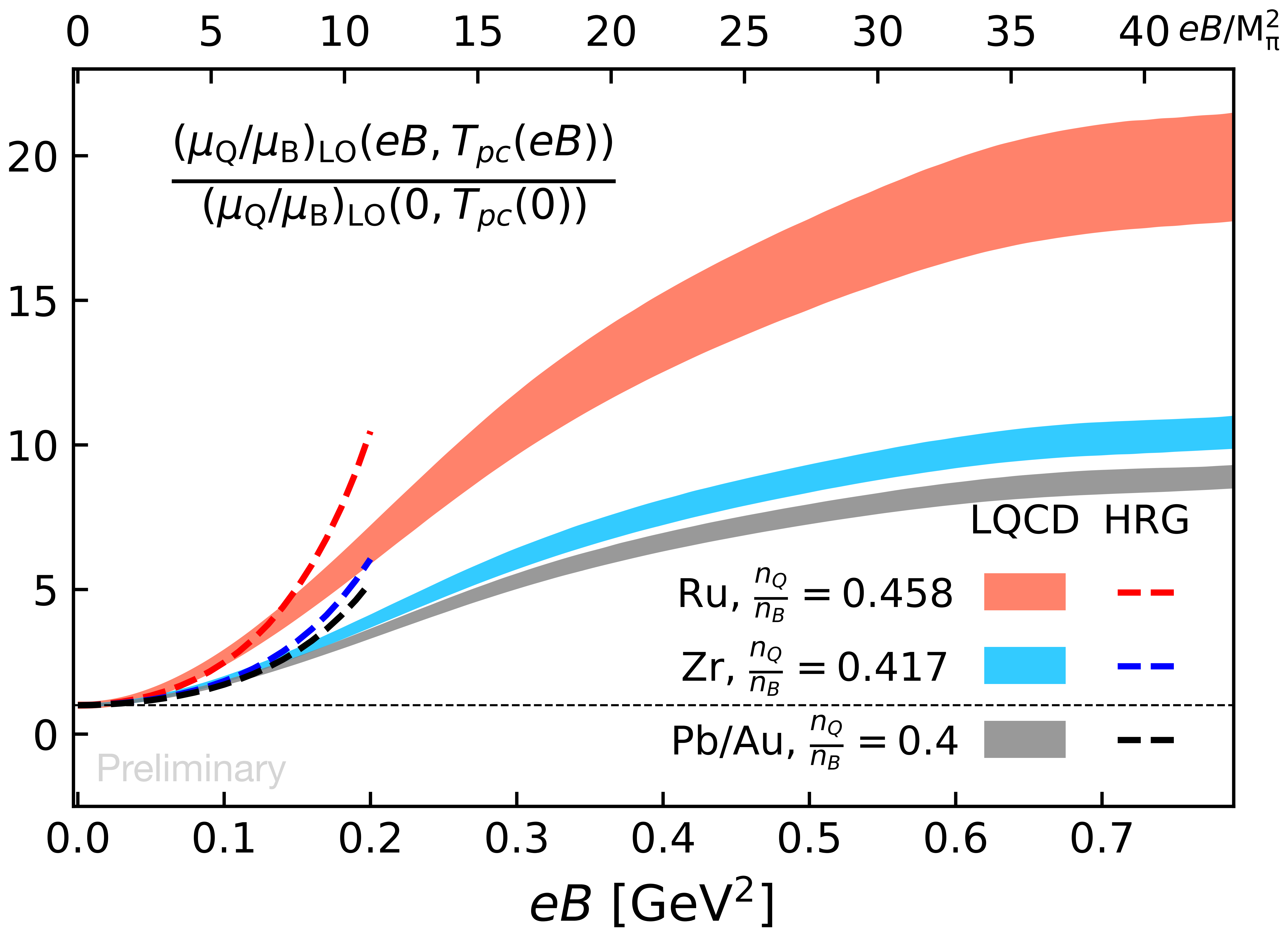
$$r = n_Q/n_B$$

✦At $eB \simeq 8M_\pi^2$,

Ratio of $(\mu_Q/\mu_B)_{LO}$ for Pb, Au, Zr ~2.4

Ratio of $(\mu_Q/\mu_B)_{LO}$ for **Ru ~ 4**

Dependence of $(\mu_Q/\mu_B)_{LO}$ on the magnetic field in the large magnetic field range



$$\mu_Q/\mu_B = q_1 + q_3\,\hat{\mu}_B^2 + \mathcal{O}(\hat{\mu}_B^4)$$

$$q_1 = \frac{r\left(\chi_2^B\chi_2^S - \chi_{11}^{BS}\chi_{11}^{BS}\right) - \left(\chi_{11}^{BQ}\chi_2^S - \chi_{11}^{BS}\chi_{11}^{QS}\right)}{\left(\chi_2^Q\chi_2^S - \chi_{11}^{QS}\chi_{11}^{QS}\right) - r\left(\chi_{11}^{BQ}\chi_2^S - \chi_{11}^{BS}\chi_{11}^{QS}\right)}$$

$$r = n_Q/n_B$$

✦At $eB \simeq 40M_\pi^2$,

Ratio of $(\mu_Q/\mu_B)_{LO}$ for Pb, Au, Zr ~ 9

Ratio of $(\mu_Q/\mu_B)_{LO}$ for **Ru ~ 20**

H.-T. Ding, J.-B. Gu, A. Kumar and S.-T. Li work in progress

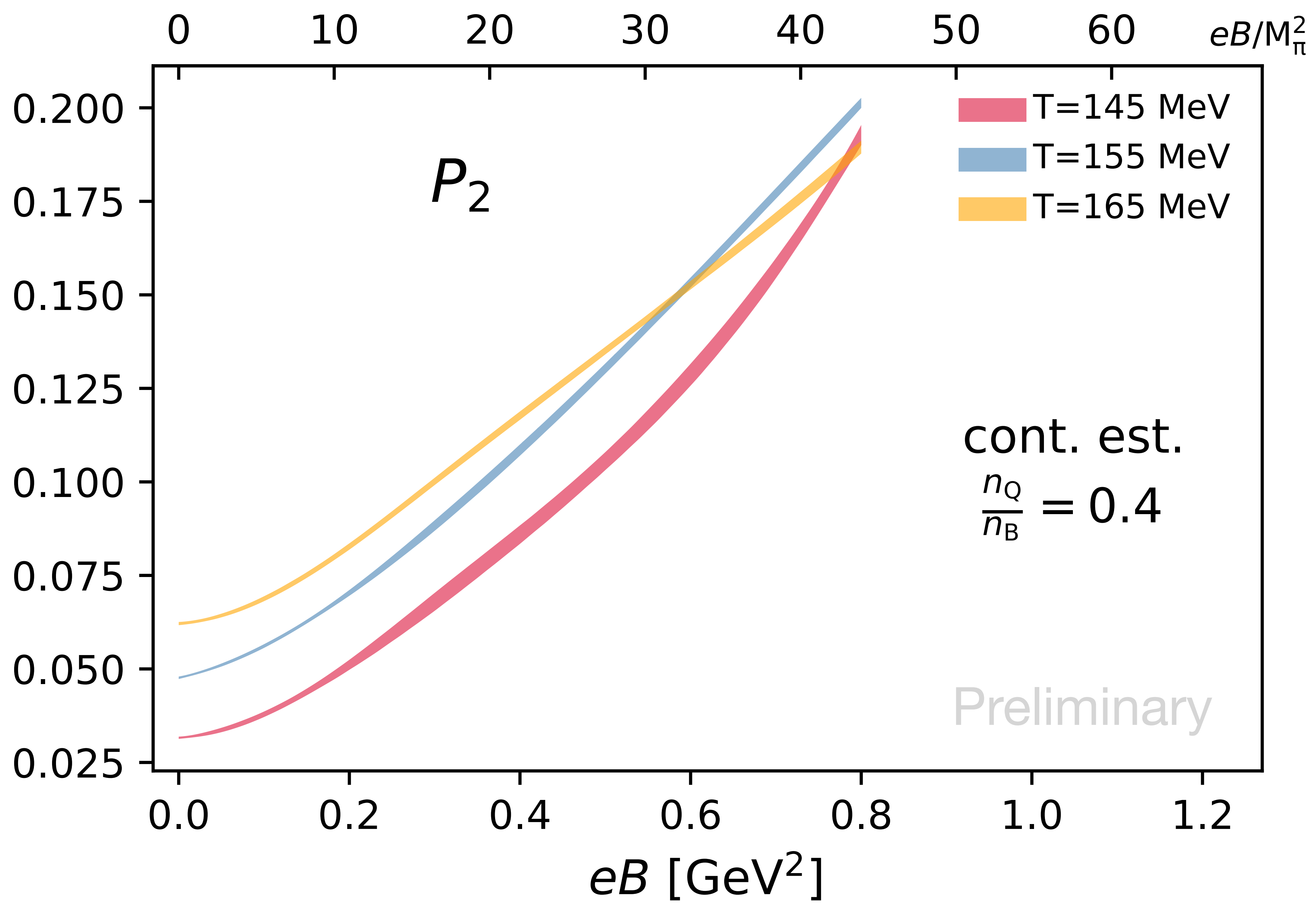
Equation of State at non-zero magnetic fields

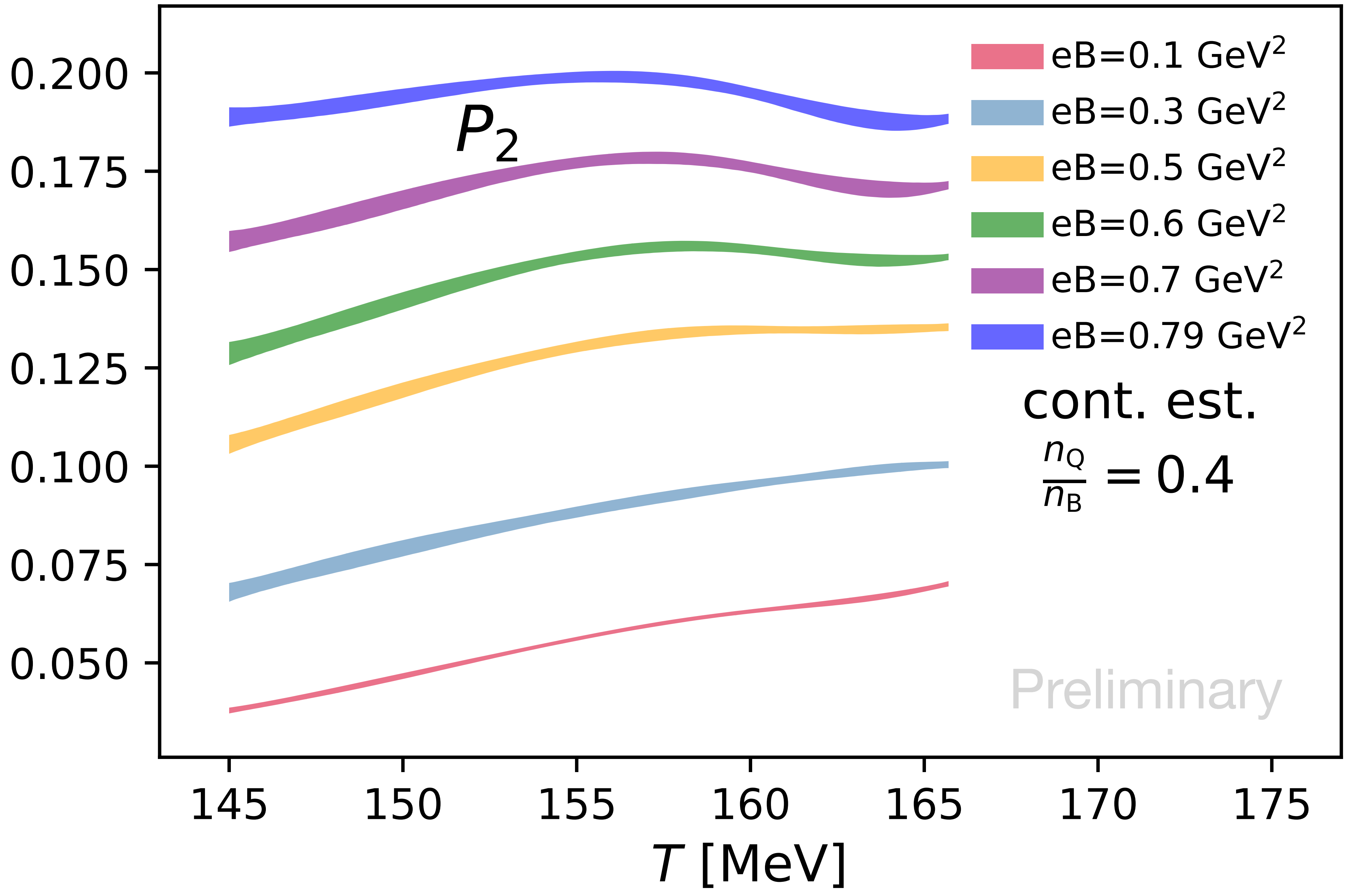
QCD Pressure: $$\frac{\Delta P}{T^4} = \frac{P(T, eB, \hat{\mu}_B) - P(T, eB, 0)}{T^4} = \sum_{k=1}^{\infty} P_{2k}(T)\hat{\mu}_B^{2k}$$

$$\hat{\mu}_Q/\hat{\mu}_B = q_1 + q_3\,\hat{\mu}_B^2 + \mathcal{O}(\hat{\mu}_B^4)$$

$$\hat{\mu}_S/\hat{\mu}_B = s_1 + s_3\,\hat{\mu}_B^2 + \mathcal{O}(\hat{\mu}_B^4)$$

Leading order: $$P_2 = \frac{1}{2!}\left(\chi_2^B + \chi_2^Q q_1^2 + \chi_2^S s_1^2\right) + \chi_{11}^{BQ} q_1 + \chi_{11}^{BS} s_1 + \chi_{11}^{QS} q_1 s_1$$





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See also poster presented by A. Kumar

Leading order Taylor series for energy and entropy densities

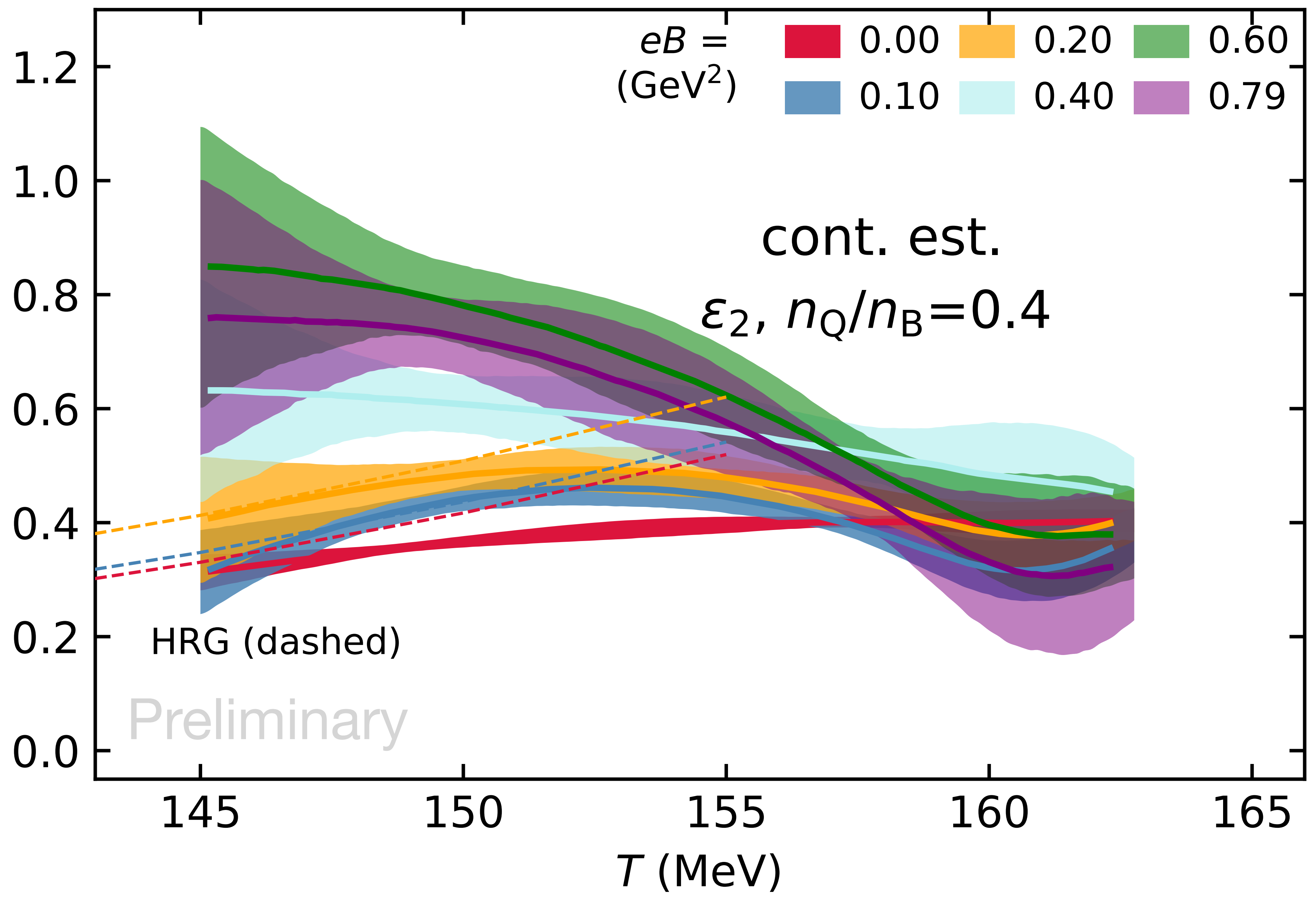
Energy densities:

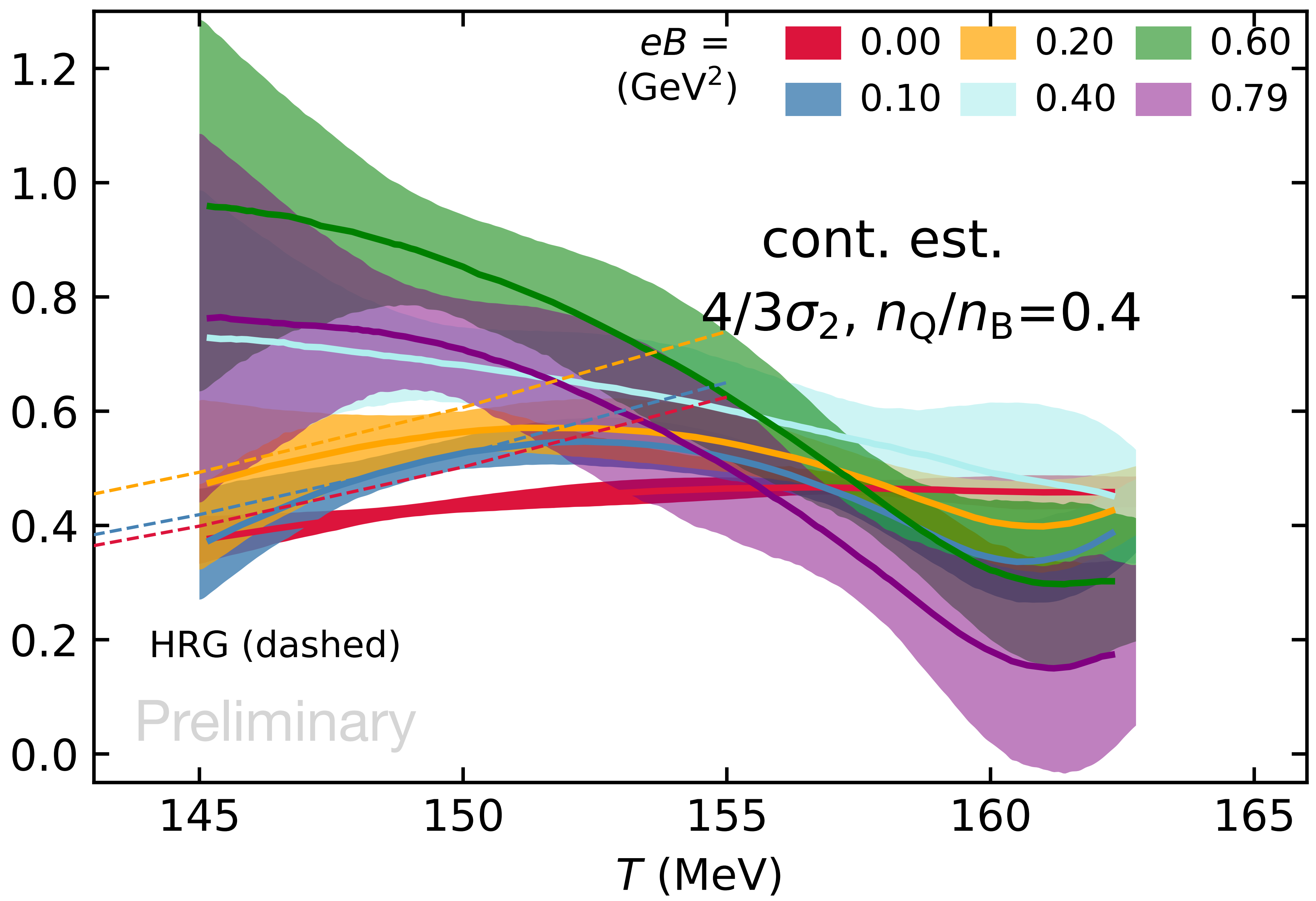
$$\frac{\epsilon\left(T, eB, \mu_B\right) - \epsilon(T, eB, 0)}{T^4} = \sum_{k=1}^{\infty} \epsilon_{2k}(T)\hat{\mu}_B^{2k}$$

Entropy densities:

$$\frac{\sigma\left(T, eB, \mu_B\right) - \sigma(T, eB, 0)}{T^3} = \sum_{k=1}^{\infty} \sigma_{2k}(T)\hat{\mu}_B^{2k}$$

Leading order: $\epsilon_2(T) = 3P_2 + TP_2' - rTq_1'N_1^B$ $\sigma_2 = \epsilon_2 + P_2 + TP_2' - \left(1 + rq_1\right) N_1^B$



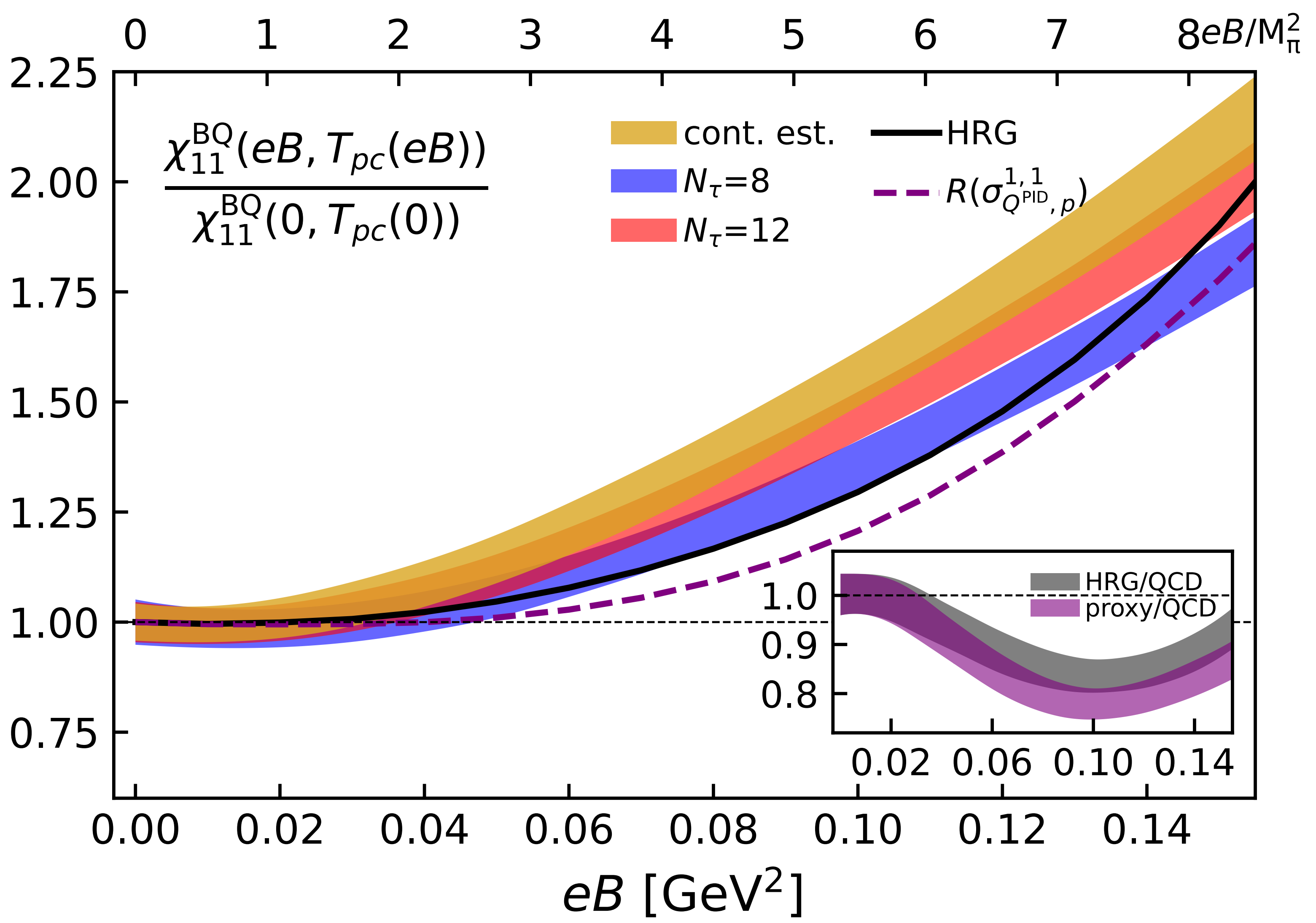


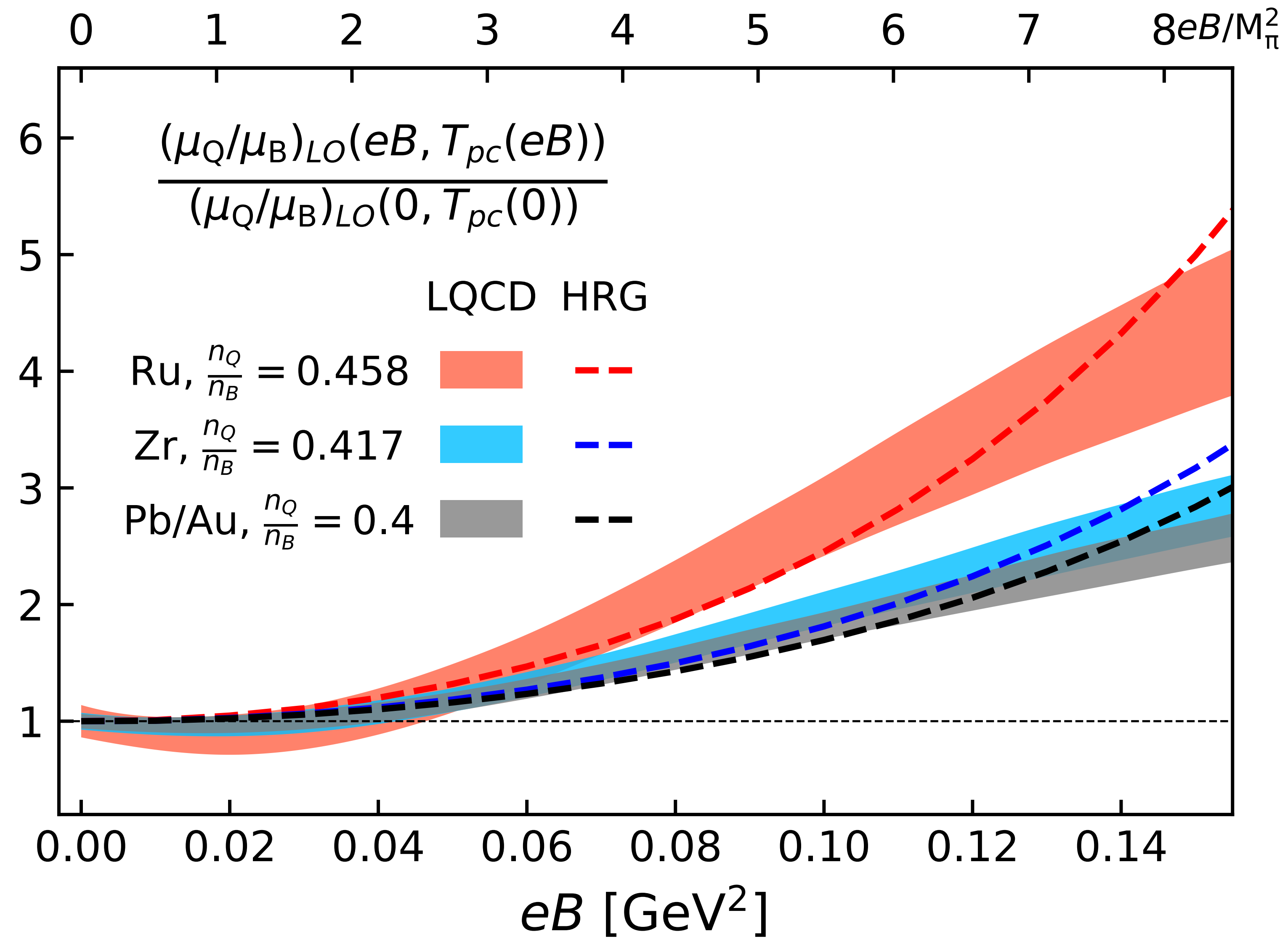
H.-T. Ding, J.-B. Gu, A. Kumar and S.-T. Li work in progress

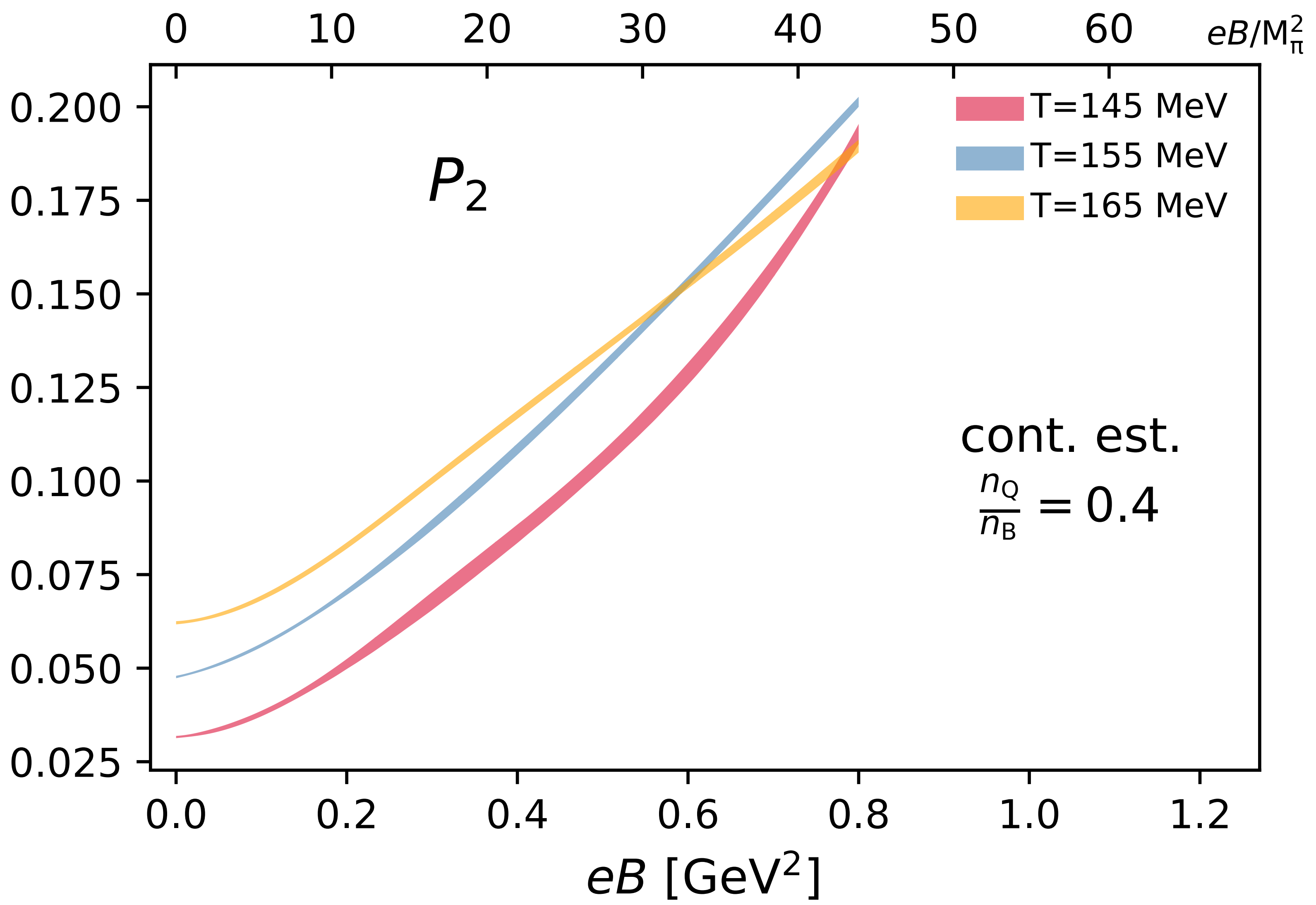
See also poster presented by A. Kumar

Summary

- QCD benchmarks are provided for the 2nd order fluctuations of conserved charges based on LQCD computation on N_τ= 8 and 12 lattices
- χ_{11}^{BQ} is strongly affected by eB, and a reasonable proxy is provided for measurement in HIC
- The $\mu_{\mathrm{Q}}/\mu_{\mathrm{B}}$ show a significant dependence on the magnetic field and is sensitive to the initial $n_{\mathrm{Q}}/n_{\mathrm{B}}$
- The results of the EoS in the magnetic field at nonzero μ_{B} in leading order are provided







Backup

Lattice QCD in strong magnetic fields

B pointing along the z direction

$$u_x\left(n_x, n_y, n_z, n_\tau\right) = \begin{cases} \exp\left[-iqa^2 B N_x n_y\right] & (n_x = N_x - 1) \\ 1 & \text{(otherwise)} \end{cases}$$

$$u_y\left(n_x, n_y, n_z, n_\tau\right) = \exp\left[iqa^2 B n_x\right]$$

$$u_z\left(n_x, n_y, n_z, n_\tau\right) = u_t\left(n_x, n_y, n_z, n_\tau\right) = 1$$

No sign problem !

Quantization of the magnetic field

$$q_u = 2/3\ e$$
$$q_d = -1/3\ e$$
$$q_s = -1/3\ e$$

→

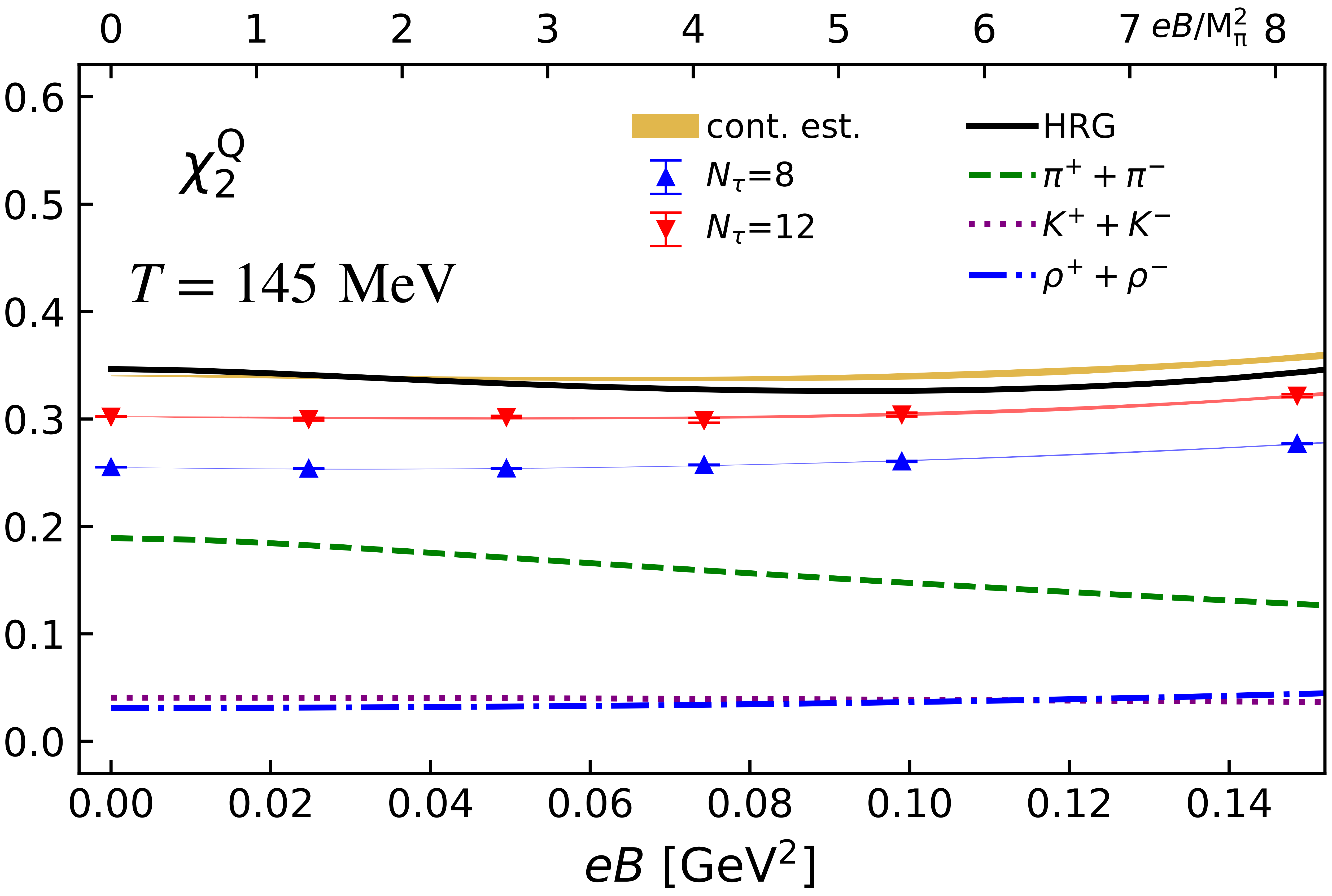
$$eB = \frac{6\pi N_b}{N_x N_y} a^{-2}$$

a is changed to get the targeted T, $T = \dfrac{1}{aN_\tau}$

- Statistics($eB \neq 0$): N_τ=8: ~40000 (#N_{rv} : 603)
  N_τ=12: ~5000 (#N_{rv} : 102 ~ 705)

Landau gauge
G.S. Bali, F. Bruckmann, G. Endrodi, Z. Fodor, S.D. Katz, S. Krieg et al., JHEP 02 (2012) 044.

Electric charge fluctuations at $T = 145$ MeV



H-T. D, J-B. Gu et al. Phys. Rev. Lett. **132**, 201903 (2024)

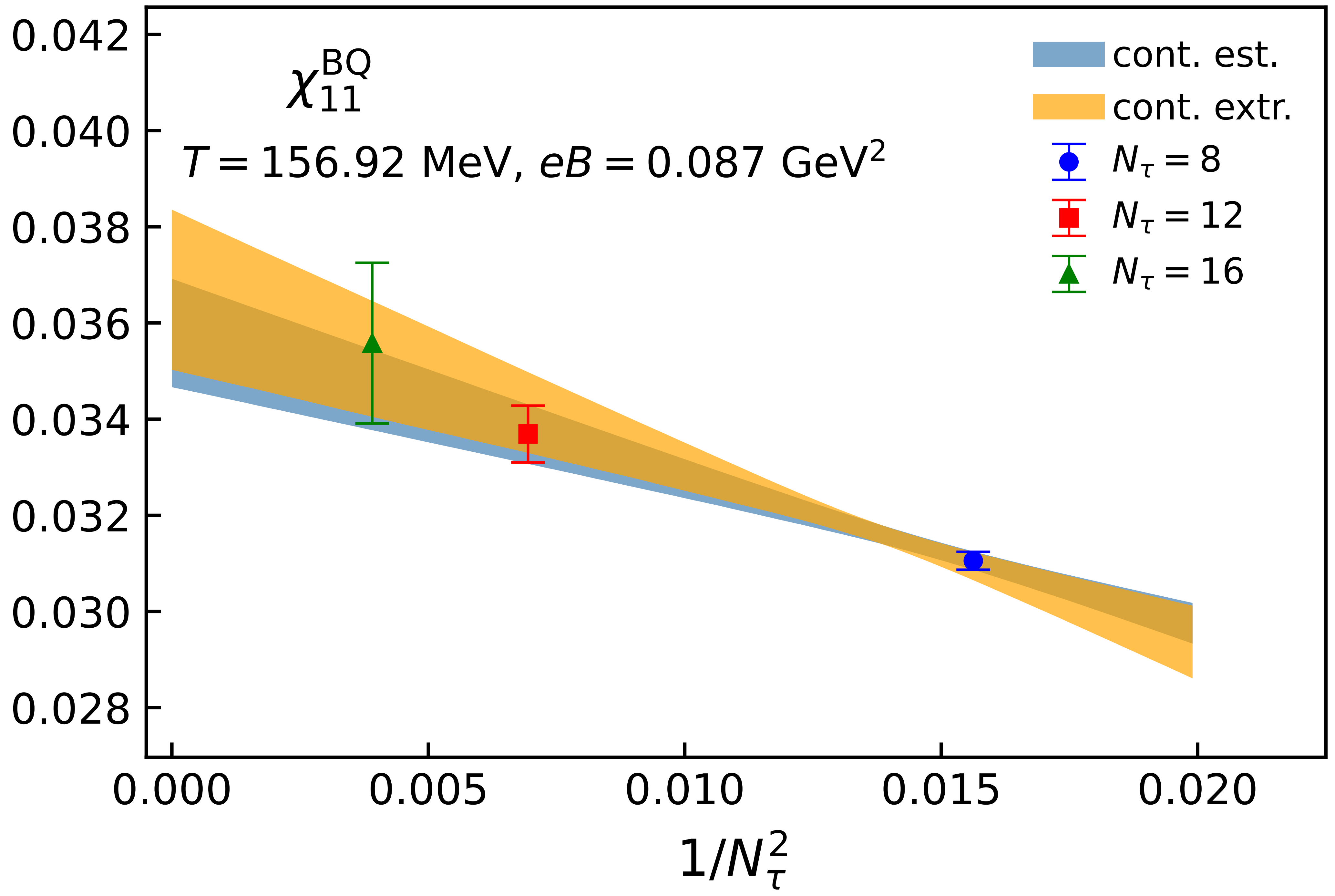
- χ_2^Q almost independent on eB
- Hadron Resonance Gas model (HRG): Pressure arising from charged hadrons ($eB \neq 0$):

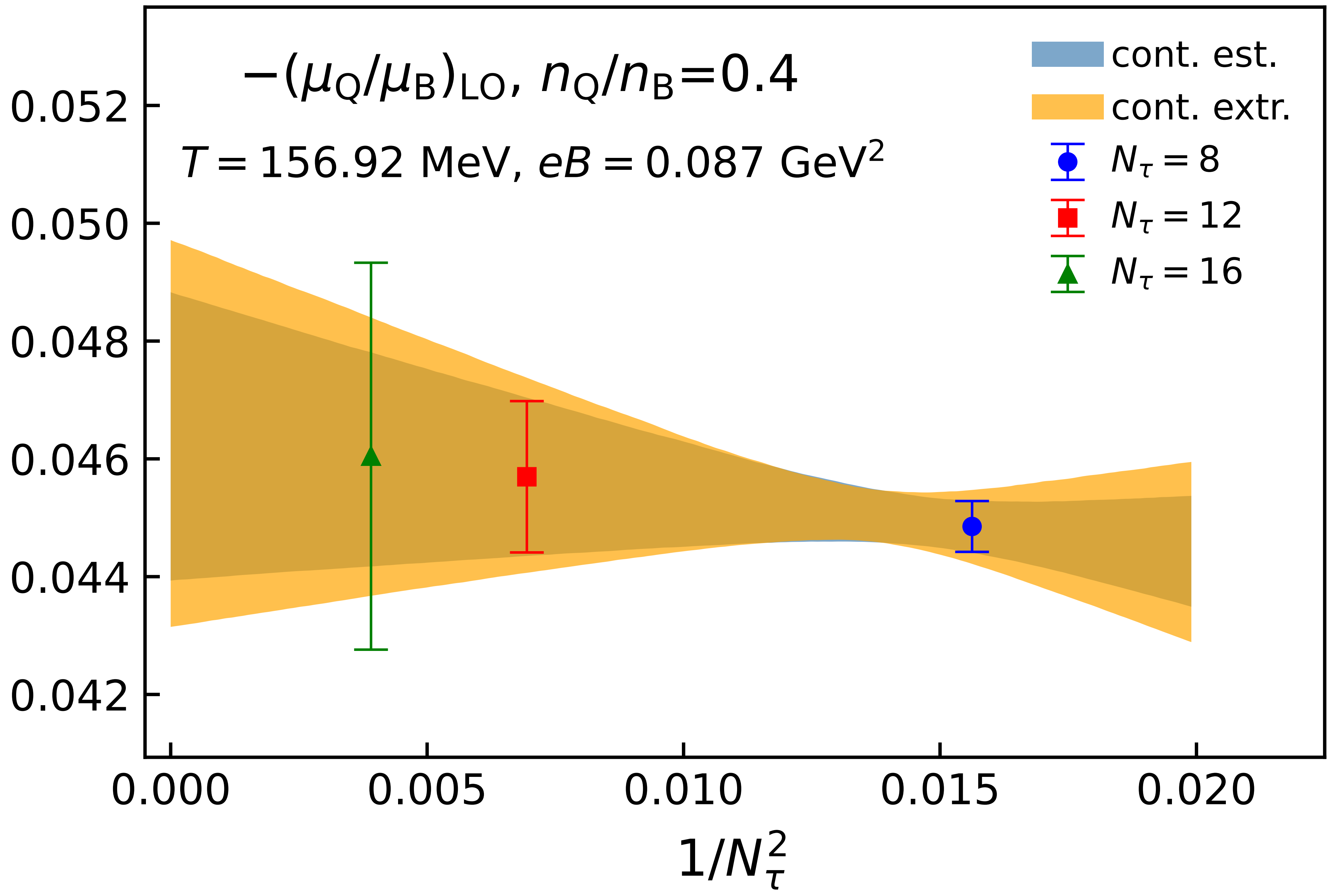
$$\frac{p_c^{M/B}}{T^4} = \frac{|q_i| B}{2\pi^2 T^3} \sum_{s_z=-s_i}^{s_i} \sum_{l=0}^{\infty} \varepsilon_0 \sum_{n=1}^{\infty} (\pm 1)^{n+1} \frac{e^{n\mu_i/T}}{n} K_1\left(\frac{n\varepsilon_0}{T}\right)$$

where $\varepsilon_0 = \sqrt{m_i^2 + 2|q_i| B (l + 1/2 - s_z)}$,

K_1 is the first-order modified Bessel function

Continuum estimate and extrapolation



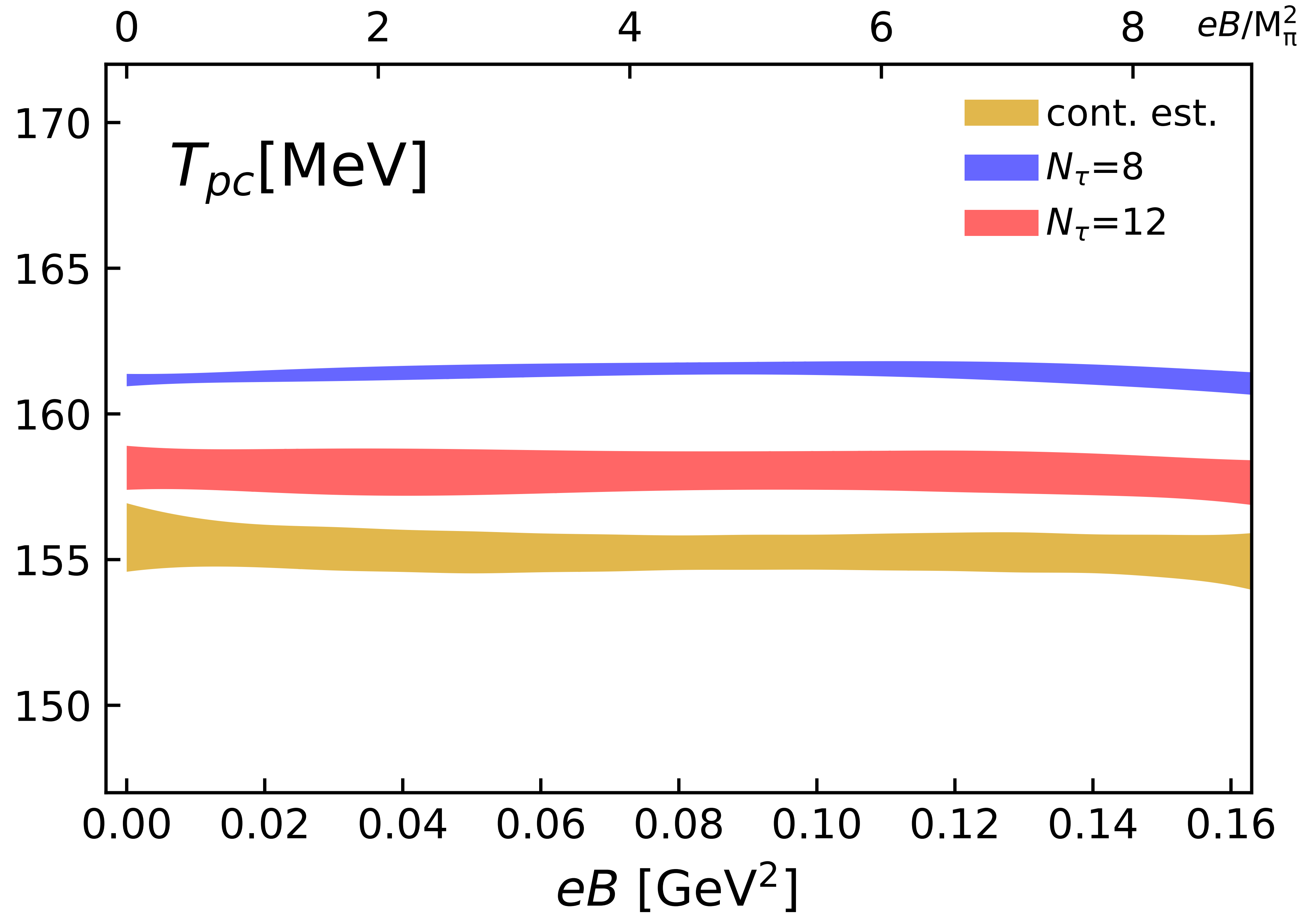


About 3000 additional configurations for $eB = 0.087$ GeV^2 and $T = 156.92$ MeV at $64^3 \times 16$

Ansatz: $1/N_\tau^2$

$$\mathcal{O}\left(T, eB, N_\tau\right) = \mathcal{O}(T, eB) + \frac{c}{N_\tau^2}$$

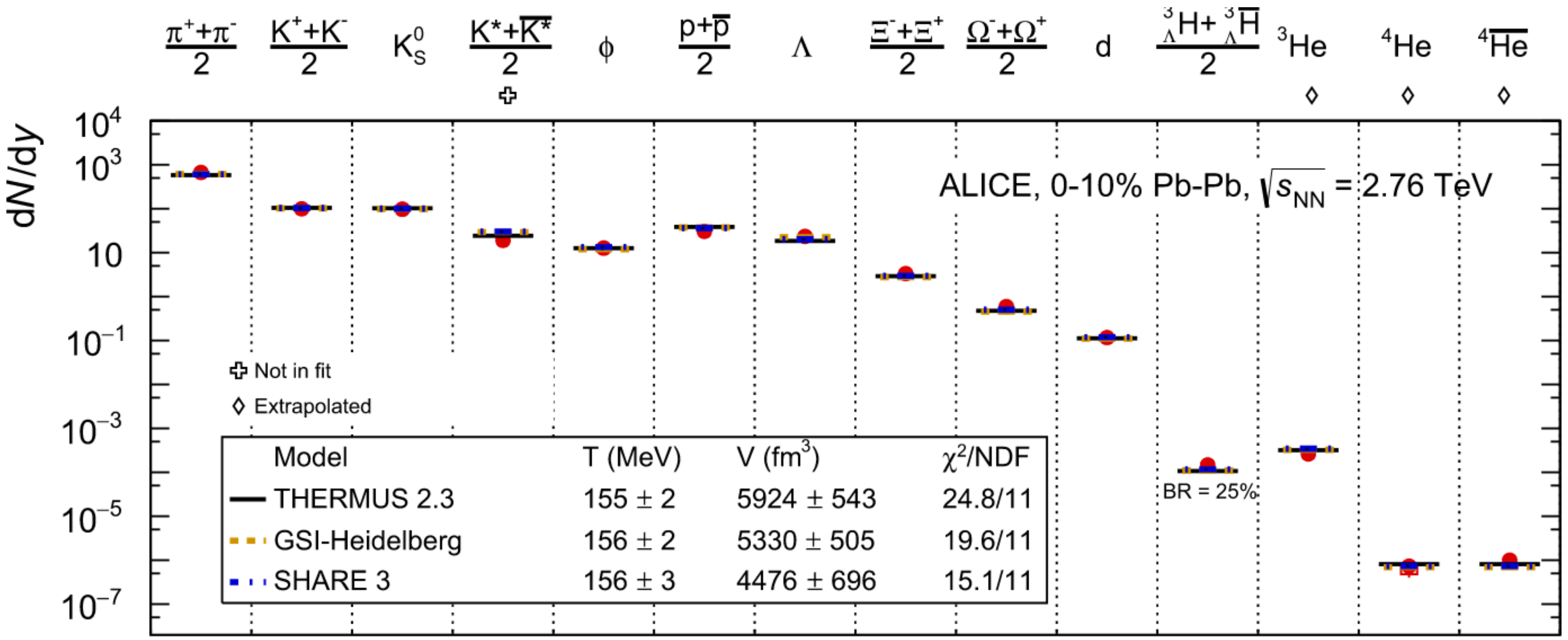
Transition line on $T - eB$ plane and $T_{\rm ch}$ in experiment



$$M = \frac{1}{f_K^4}\left[m_s\left(\langle\bar\psi\psi\rangle_u + \langle\bar\psi\psi\rangle_d\right) - \left(m_u + m_d\right)\langle\bar\psi\psi\rangle_s\right]$$

$$\chi_M(eB) = \frac{m_s}{f_K^4}\left[m_s\chi_l(eB) - 2\langle\bar\psi\psi\rangle_s(eB=0) - 4m_l\chi_{su}(eB=0)\right]$$

Finding the peak location of χ_M at each eB value to determine $T_{pc}(eB)$



$T_{\rm ch} \simeq 156$ MeV

ALICE , Nucl.Phys.A 971 (2018) 1–20

Proxy in experiment

✦ Conserved charges susceptibilities in experiment:

$$\chi_\alpha^2 = \frac{1}{VT^3}\kappa_\alpha^2, \quad \chi_{\alpha,\beta}^{1,1} = \frac{1}{VT^3}\kappa_{\alpha,\beta}^{1,1}$$

the second-order cumulants(κ) are the variance or covariance(σ) of the net-multiplicity N:

$$\kappa_\alpha^2 = \sigma_\alpha^2 = \langle(\delta N_\alpha - \langle\delta N_\alpha\rangle)^2\rangle$$

$$\kappa_{\alpha,\beta}^{1,1} = \sigma_{\alpha,\beta}^{1,1} = \langle(\delta N_\alpha - \langle\delta N_\alpha\rangle)(\delta N_\beta - \langle\delta N_\beta\rangle)\rangle$$

with $\delta N_\alpha = N_{\alpha^+} - N_{\alpha^-}$ and $\alpha, \beta = p, Q^{PID}, k$

- p : a proxy for the net-baryon
- k : a proxy for the net-strangeness
- Q^{PID}: identified π, k and p

STAR, Phys.Rev.C 100 (2019) 1, 014902

$$\sigma_{Q^{\mathrm{PID}},p}^{1,1} = \sigma_p^2 + \sigma_{p,\pi}^{1,1} + \sigma_{p,K}^{1,1}$$

$$\sigma_p^2 = \sum_R \left(P_{R\to\tilde{p}}\right)\left(P_{R\to\tilde{p}}\right)\frac{\partial^2 p_R/T^4}{\partial\hat{\mu}_R^2}$$

$$\sigma_{p,\pi}^{1,1} = \sum_R \left(P_{R\to\tilde{p}}\right)\left(P_{R\to\tilde{\pi}^+}\right)\frac{\partial^2 p_R/T^4}{\partial\hat{\mu}_R^2}$$

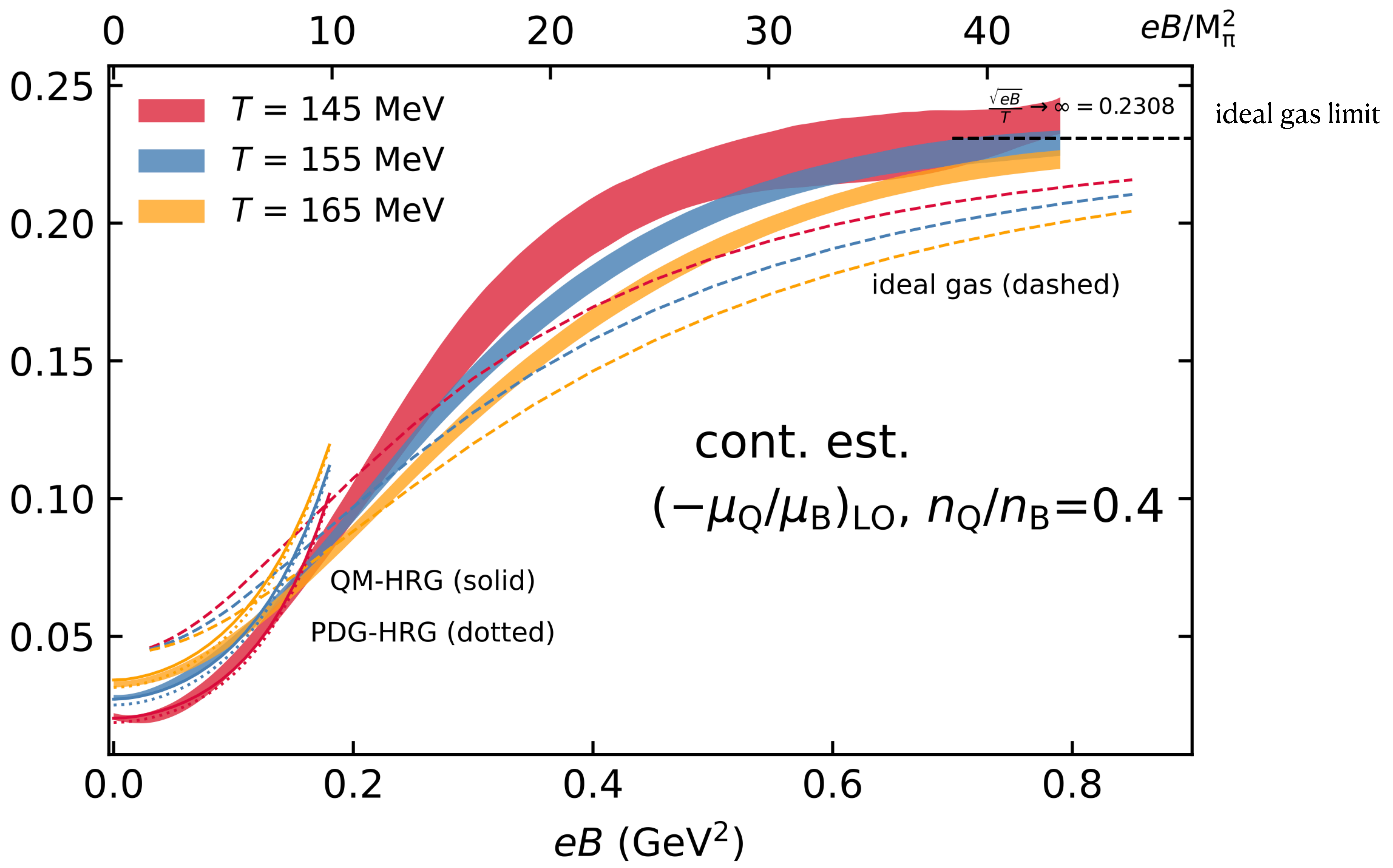
$$\sigma_{p,K}^{1,1} = \sum_R \left(P_{R\to\tilde{p}}\right)\left(P_{R\to\tilde{K}^+}\right)\frac{\partial^2 p_R/T^4}{\partial\hat{\mu}_R^2}$$

where $P_{R\to i} = \sum_\alpha \mathrm{N}_{R\to i}^\alpha n_{i,\alpha}^R$

$n_{i,\alpha}^R$: numbers of i produced by R in decay channel α

$\mathrm{N}_{R\to i}^\alpha$: Branching ratio of channel α

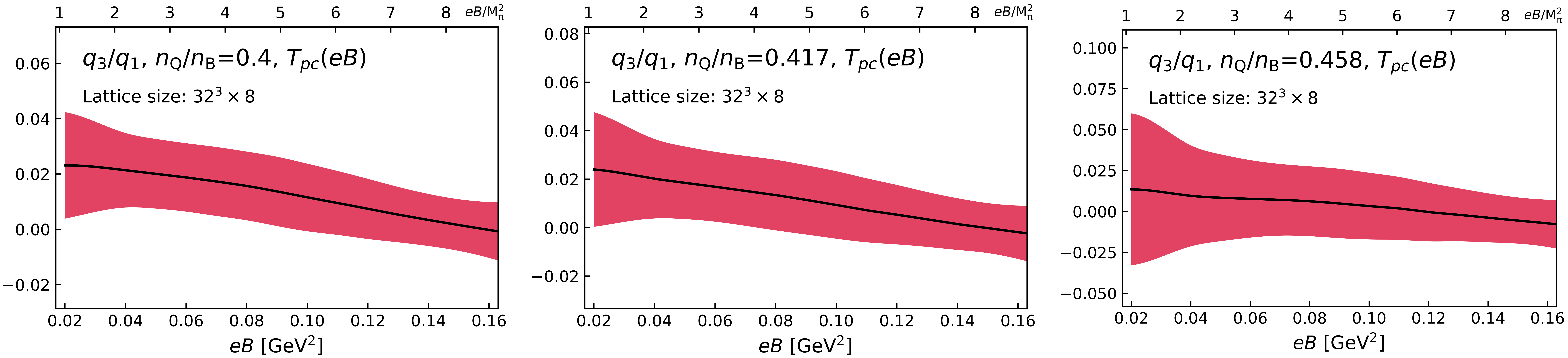
$(\mu_Q/\mu_B)_{LO}$ at different temperature



See also poster presented by A. Kumar

Dependence of $(\mu_Q/\mu_B)_{NLO}$ on the magnetic field

$$\hat{\mu}_Q/\hat{\mu}_B = q_1 + q_3\,\hat{\mu}_B^2 + \mathcal{O}(\hat{\mu}_B^4) \qquad \hat{\mu}_B = \mu_B/T$$



q_3/q_1 in all cases remains within 2%

The next-to-leading order correction is negligible!