Mass spectra of meson nonet (π, K, η, η') and the related QCD phase transitions under external magnetic field

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Outlines

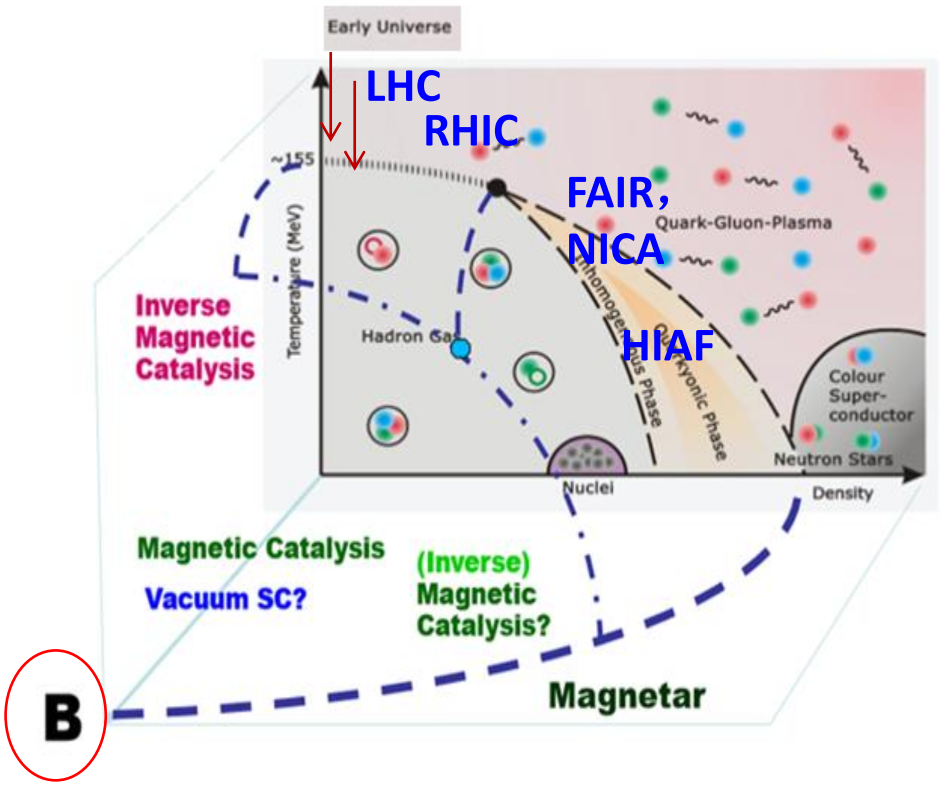
1 **Motivation**

2 **NJL model @ eB**

3 **Mesons and phase transitions**

4 **Summary and outlook**

QCD phase structure



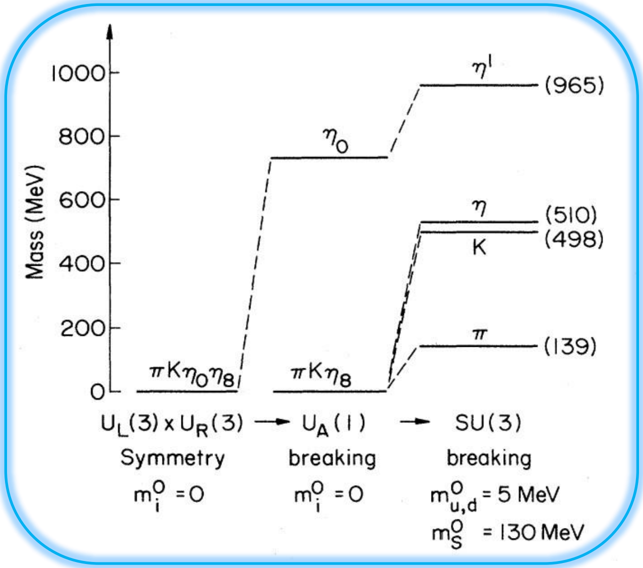
- chiral restoration
  (chiral symmetry)
- deconfinement
  (center symmetry)
- QCD superconductor /superfluid
  (color/isospin symmetry)
- $U_A(1)$ restoration
  ……

methods： LQCD;
effective models:(P)NJL,DS,(P)QM…

1 Study QCD phase transitions by mesons

- Landau's theory
- **symmetry**
- Goldstone's theorem

order parameters	QCD phase transitions	Goldstone bosons
chiral condensates	chiral restoration	π^0, K^0
pion/kaon condensate	Pion/Kaon superfluid	$\pi^\pm, K^\pm$
topological susceptibility	U_A (1) restoration	η, η'



methods: LQCD;
effective models

2 Framework: NJL model @ eB

Mesons and QCD phase transitions @eB

Quark level: LQCD, (P)NJL, DS equation

Meson level: (P)QM, chiral perturbation theory

2 Nambu--Jona-Lasinio model

$\mathbf{B} = (0, 0, B)$

SU(2) NJL model

$$\mathcal{L} = \bar{\psi}\left(i\gamma_\nu D^\nu - m_0\right)\psi + \frac{G}{2}\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\vec{\tau}\psi\right)^2\right]$$

SU(3) NJL model

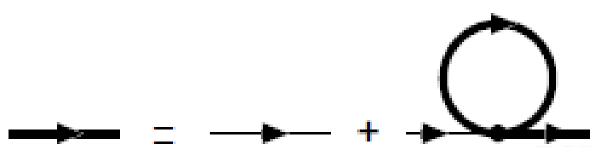
$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - \hat{m}_0)\psi + \mathcal{L}_S + \mathcal{L}_{KMT},$$

$$\mathcal{L}_S = G\sum_{\alpha=0}^{8}[(\bar{\psi}\lambda_\alpha\psi)^2 + (\bar{\psi}i\gamma_5\lambda_\alpha\psi)^2],$$

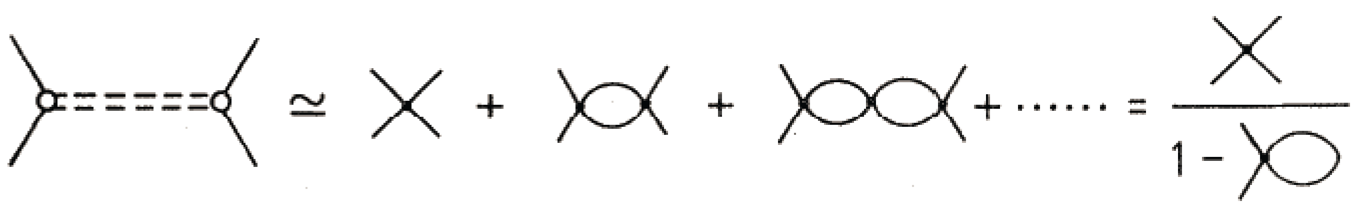
$$\mathcal{L}_{KMT} = -K[\det\bar{\psi}(1+\gamma_5)\psi + \det\bar{\psi}(1-\gamma_5)\psi].$$

idea:

(1) Quarks： mean field



(2) Mesons： RPA resummation (quantum fluctuation)



quark propagator @eB
= conserved Ritus momentum + Ritus transformation

$$S_f(x,y) = i\sum_{n=0}^{\infty}\int \frac{d\tilde{p}}{(2\pi)^3} e^{-i\tilde{p}\cdot(x-y)} P_n(x_1,p_2) D_f(\bar{p}) P_n(y_1,p_2),$$

$$P_n(z,q) = \frac{1}{2}\left[g_n^{s_f}(z,q) + I_n g_{n-1}^{s_f}(z,q)\right] + \frac{is_f}{2}\left[g_n^{s_f}(z,q) - I_n g_{n-1}^{s_f}(z,q)\right]\gamma^1\gamma^2,$$

$$D_f^{-1}(\bar{p}) = \gamma\cdot\bar{p} - m, \quad \bar{p} = (p_0, 0, -s_f\sqrt{2|Q_f B|n}, p_3)$$

$$\mathbf{B} = (0,0,B)$$

$$\tilde{p} = (p_0, 0, p_2, p_3) \qquad I_n = 1 - \delta_{n0}$$

Ritus momentum

$$g_n^{s_f}(z,q) = \phi_n(z - s_f q/|Q_f B|)$$

$$\phi_n(z) = \left(2^n n!\sqrt{\pi}|Q_f B|^{-1/2}\right)^{-1/2} \times e^{-z^2|Q_f B|/2} H_n\left(z/|Q_f B|^{-1/2}\right)$$

refs: Ritus, Ann. Phys. 69, 555 (1972); Leung, Wang,Nucl. Phys. B747, 266 (2006);
Elizalde, Ferrer, InceraAnn. Phys. 295, 33 (2002) .

Construct mesons in conserved momentum space

Y.X.Wang, **S.J. Mao**, PRD96, 034004(2017);99, 056005(2019)

Fourier transformation

neutral mesons

$$\mathcal{D}_M(k) = \int d^4(x-y)e^{ik\cdot(x-y)}\mathcal{D}_M(x,y),$$

$$\Pi_M(k) = \int d^4(x-y)e^{ik\cdot(x-y)}\Pi_M(x,y),$$

conserved momentum

$$k = (\omega, k_1, k_2, k_3)$$

charged mesons

$$\mathcal{D}_M(x,y) = \sum_{n=0}^{\infty}\int \frac{d\tilde{k}}{(2\pi)^3}e^{-i\tilde{k}\cdot(x-y)}f_n(x^1,k_2)\mathcal{D}_M(\bar{k})f_n^*(y^1,k_2)$$

where $\tilde{k} = (k_0, 0, k_2, k_3)$, $\zeta = (|QB|)^{1/2}(x^1 - s_Q k_2/|QB|)$

$$\mathcal{D}_M(\bar{k}) = -\left(k_0^2 - k_3^2 - (2k_n+1)|QB| - m_M^2\right)^{-1},$$

$$f_n(x^1,k_2) = \left(\frac{\sqrt{|QB|}}{2^n n!\sqrt{\pi}}\right)^{1/2} e^{-\zeta^2/2}H_n(\zeta) \; .$$

f_n, eigenfunction of the magnetized static KG Eq.

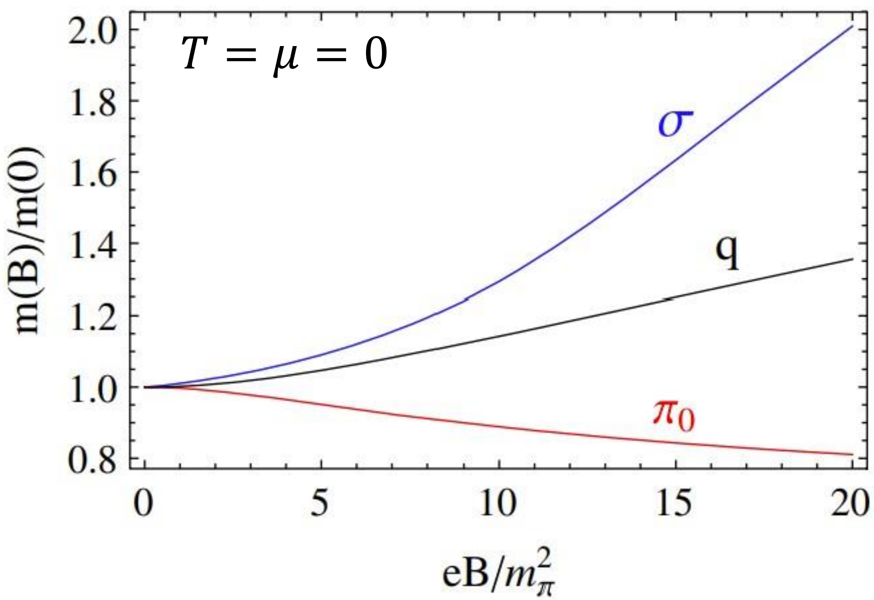
3 Results and discussion

3.1 π_0 ---- chiral restoration phase transition

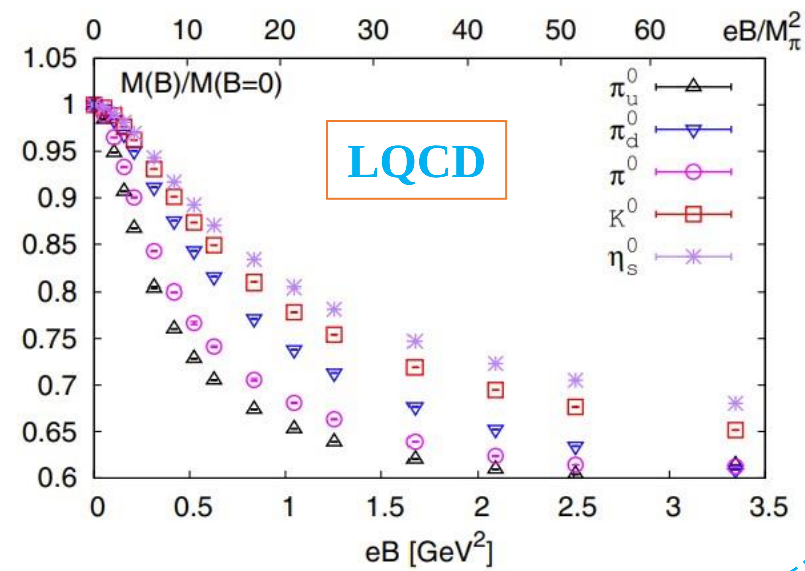
3.2 $\pi_\pm$ ---- pion superfluid phase transition

3.3 Kaons, η& η' ---- chiral, $U_A(1)$

3.1 π_0 & chiral restoration phase transition



q: order parameter
π_0: Goldstone mode
σ : Higgs mode

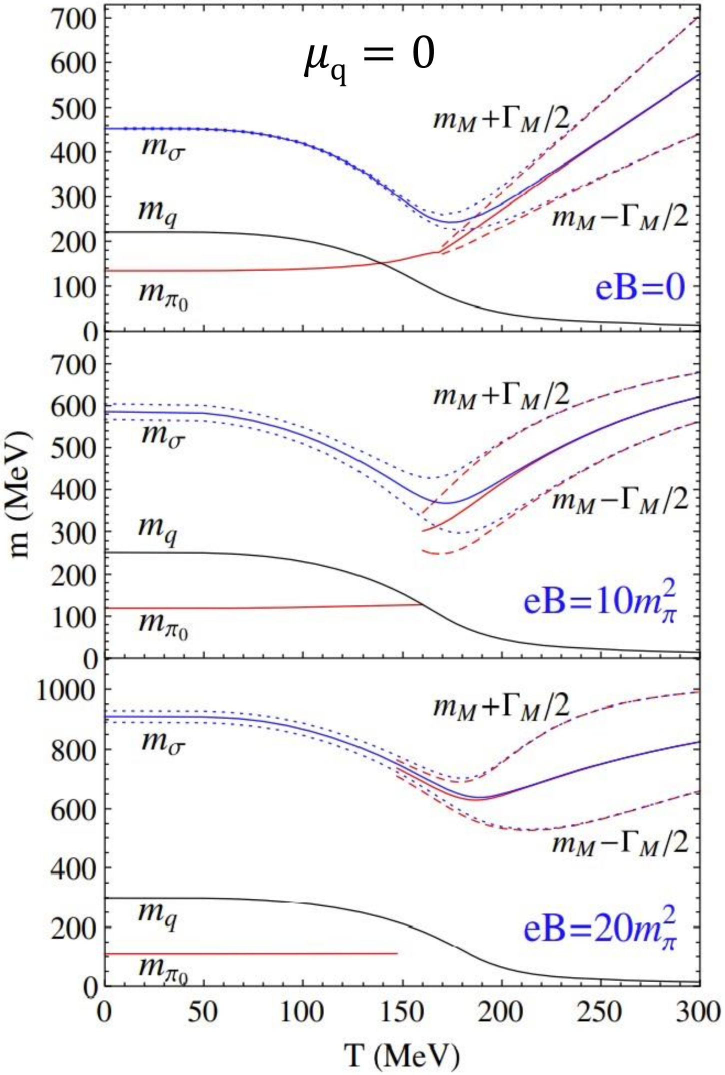


π_0 mass decreases with eB.

S.J. Mao, et al, PRD 96, 034004(2017)

H.T. Ding, et al, PRD 104, 014505(2021)

3.1 π_0 & chiral restoration phase transition

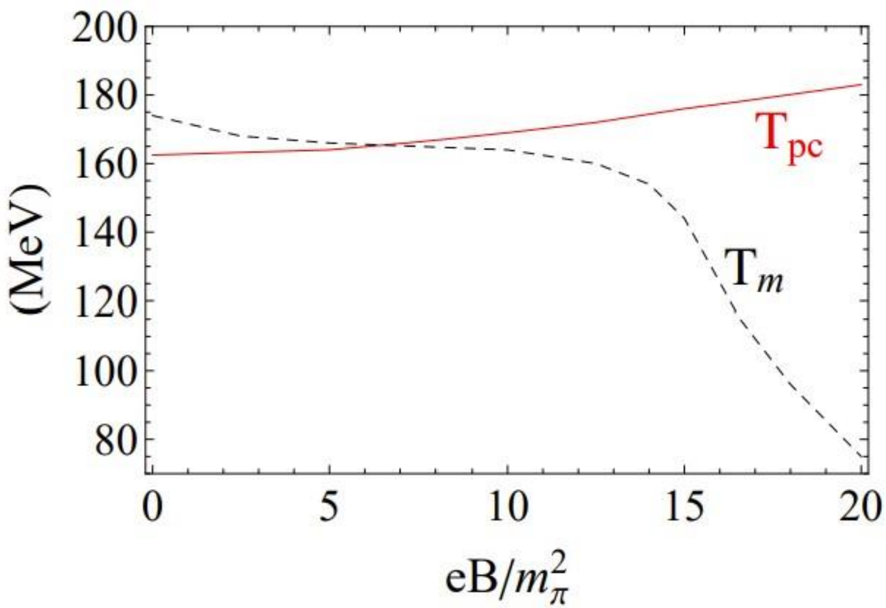


S.J. Mao, et al, PRD 96, 034004(2017)

π_0 mass jump @eB at Mott transition

$$m_\pi = 2m_q \rightarrow T_m$$

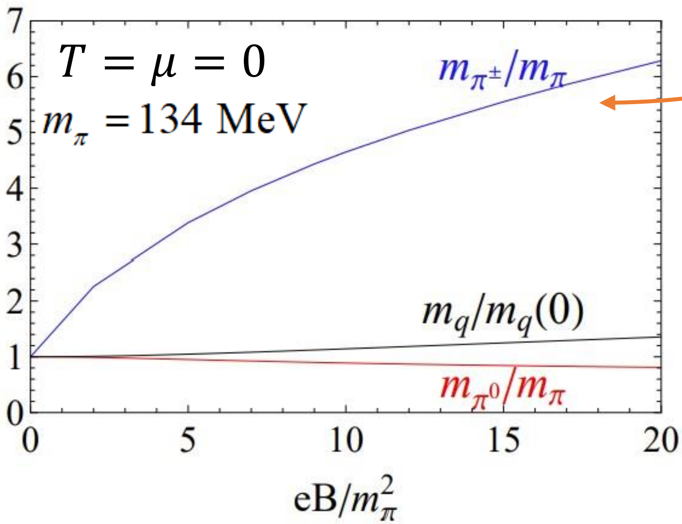
$$\frac{\partial^2 m_q}{\partial T^2} = 0 \rightarrow T_{pc}$$



S.J. Mao, CPC45, 021004(R) (2021)

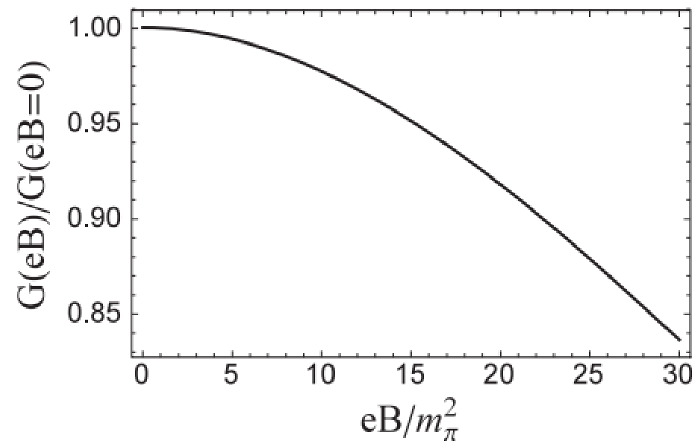
3.2 Charged pion @ eB

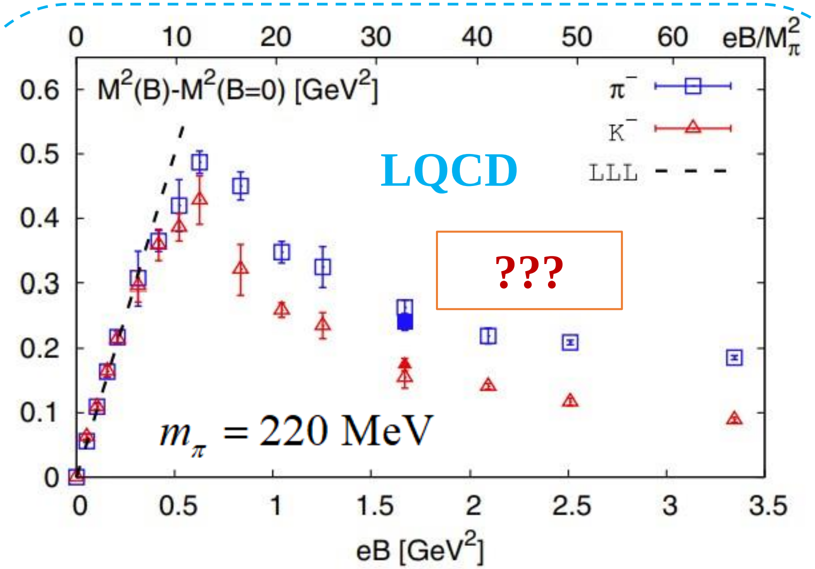
3.2 π_+ & pion superfluid phase transition



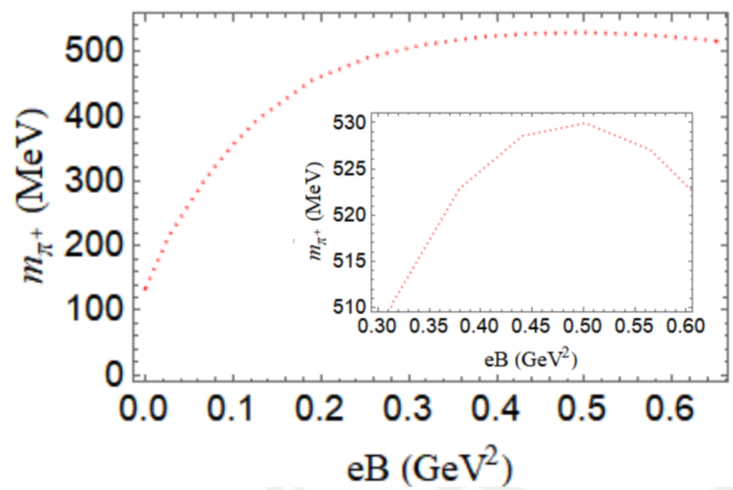
π_+ mass increases $(u\bar{d})$

S.J. Mao, PRD99, 056005 (2019)





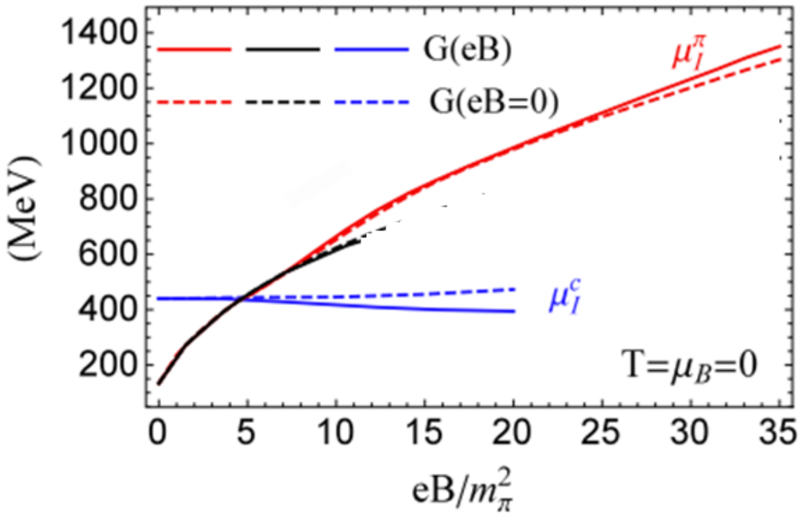
H.T. Ding, et al, PRD 104, 014505(2021)



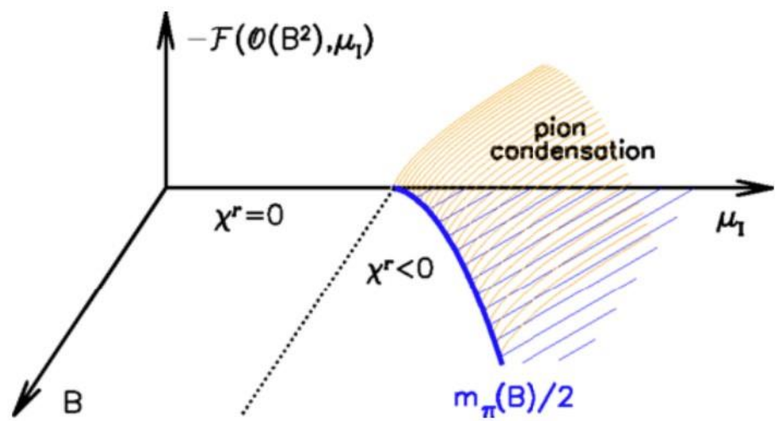
3.2 π_+ & pion superfluid phase transition

S.J. Mao, PRD102, 114006(2020);106.094017(2022).

Goldstone's theorem: $M_{\pi^+} = 0 \Rightarrow$ **pion superfluid phase transition**

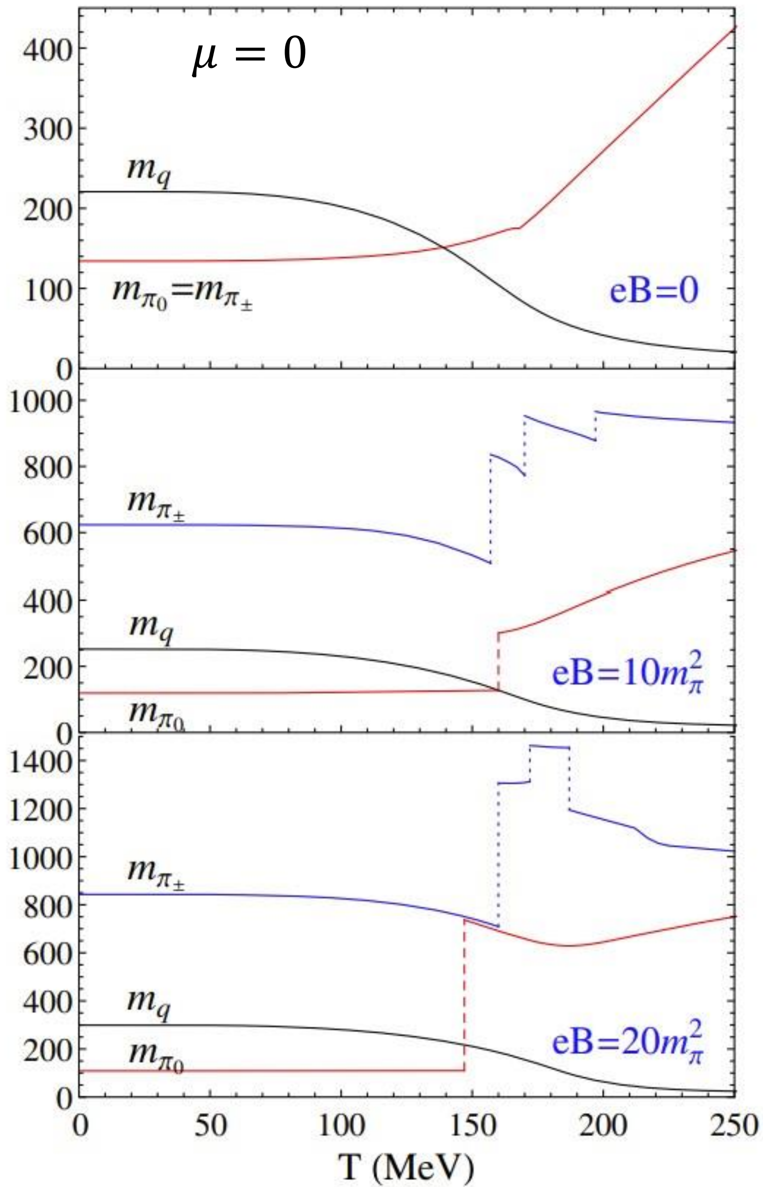


LQCD
PRD90,094501(2014)

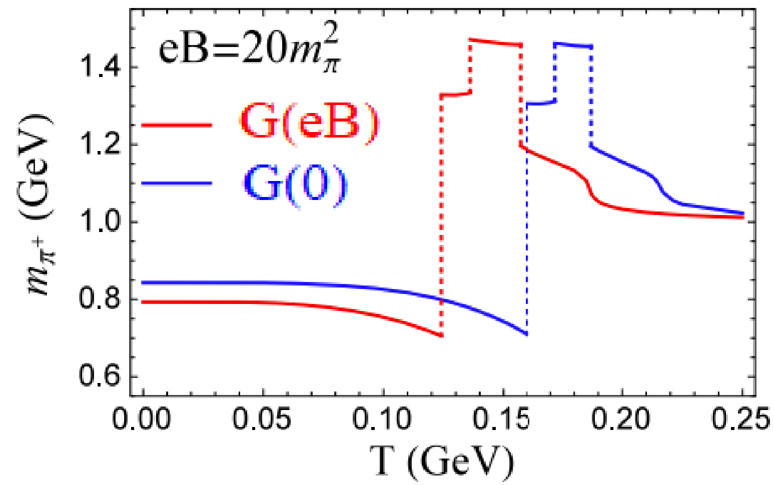


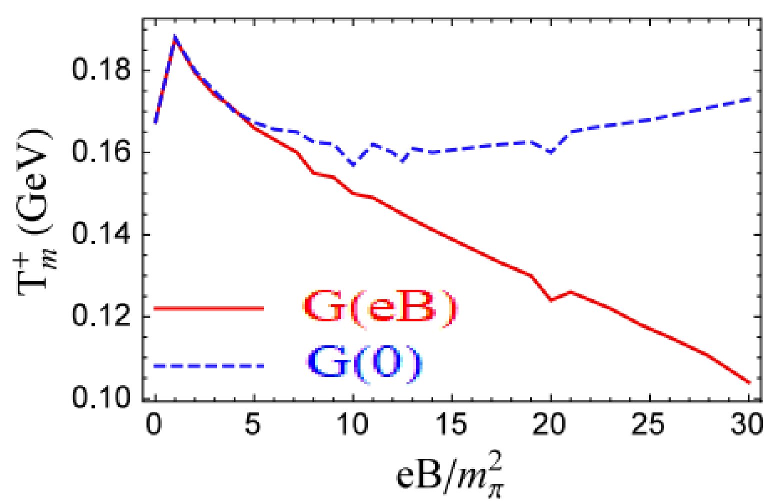
3.2 π_+ @ T and eB

S.J. Mao, PRD99, 056005 (2019); PRD108 054001(2023)



π_+ mass jump @eB at Mott transition



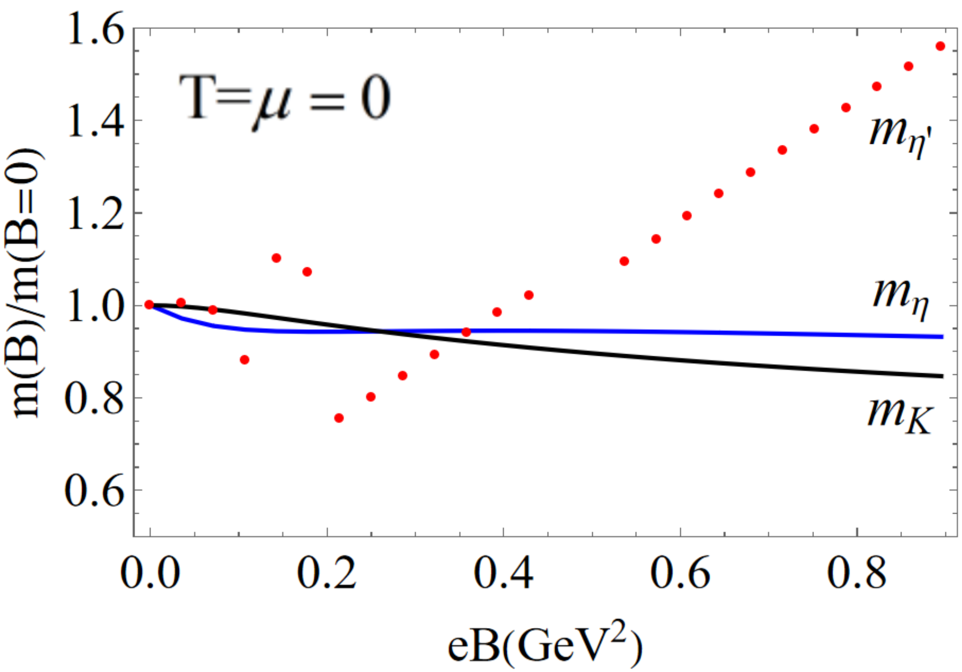


3.3 Kaon、η、 η’ @ eB

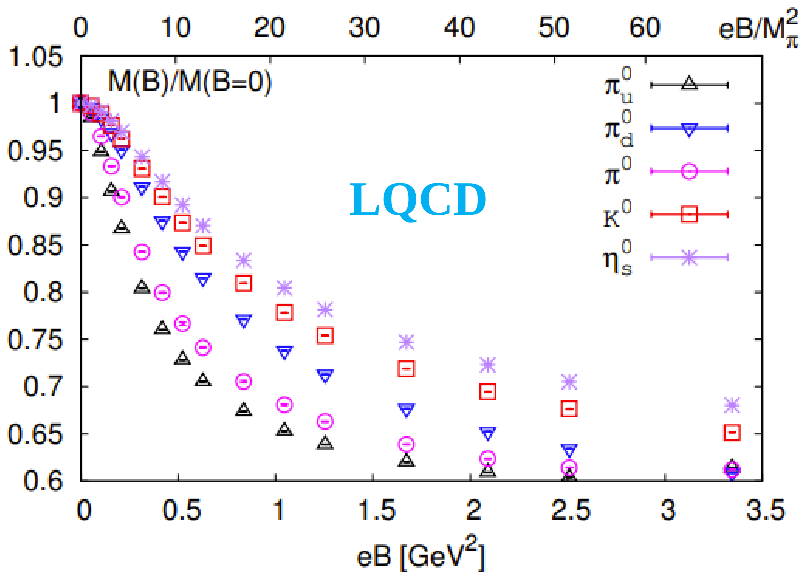
three-flavor NJL

chiral, Kaon superfluid, $U_A(1)$ phase transitions

3.3 K_0, η, η' mass spectra



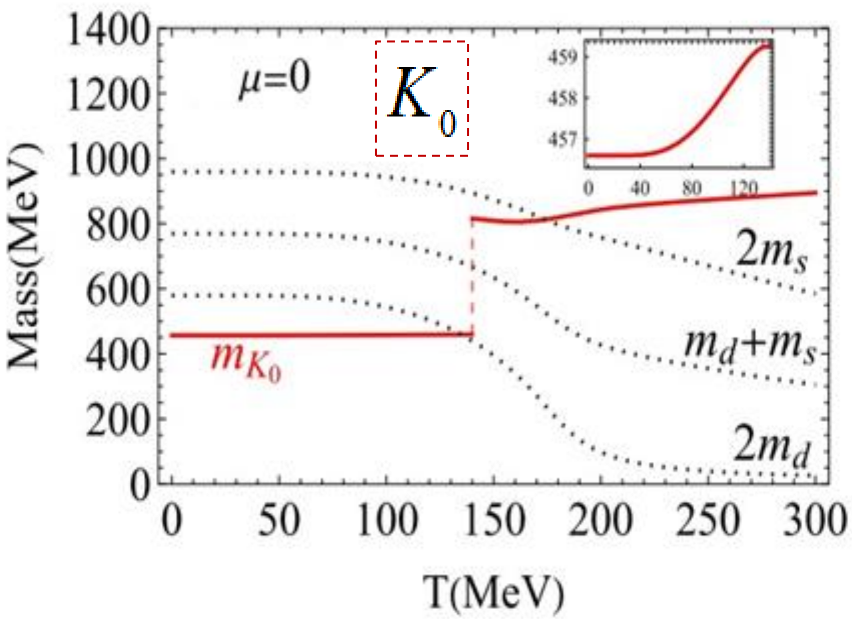
J.Mei, T.Xia, **S.J.Mao**, PRD107, 074018(2023)

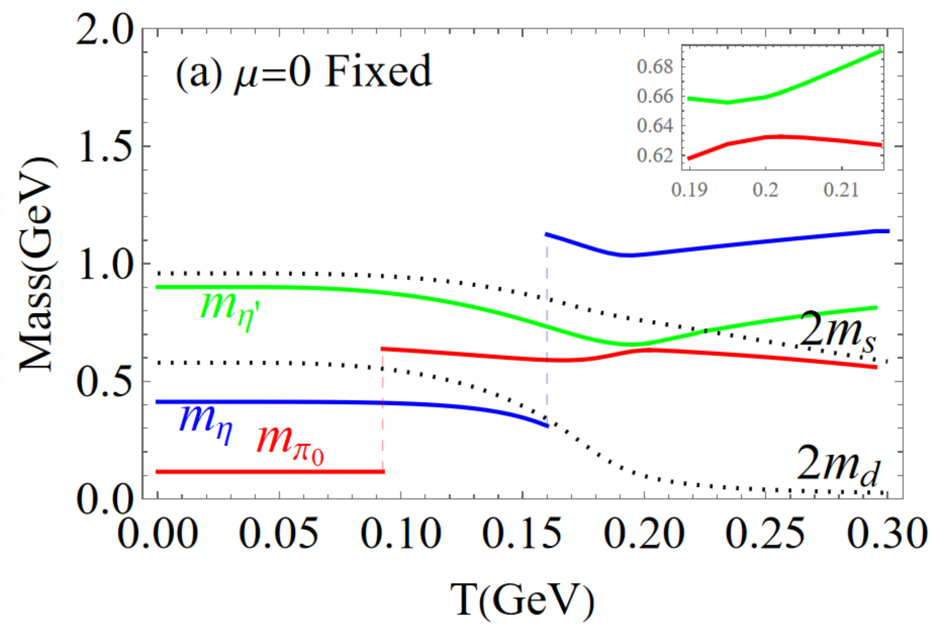


PRD104, 014505(2021)

3.3 K_0, η, η' mass spectra @ T

$$eB = 20 m_\pi^2$$





Mott transition

$$m_M = \sum quark\ mass$$

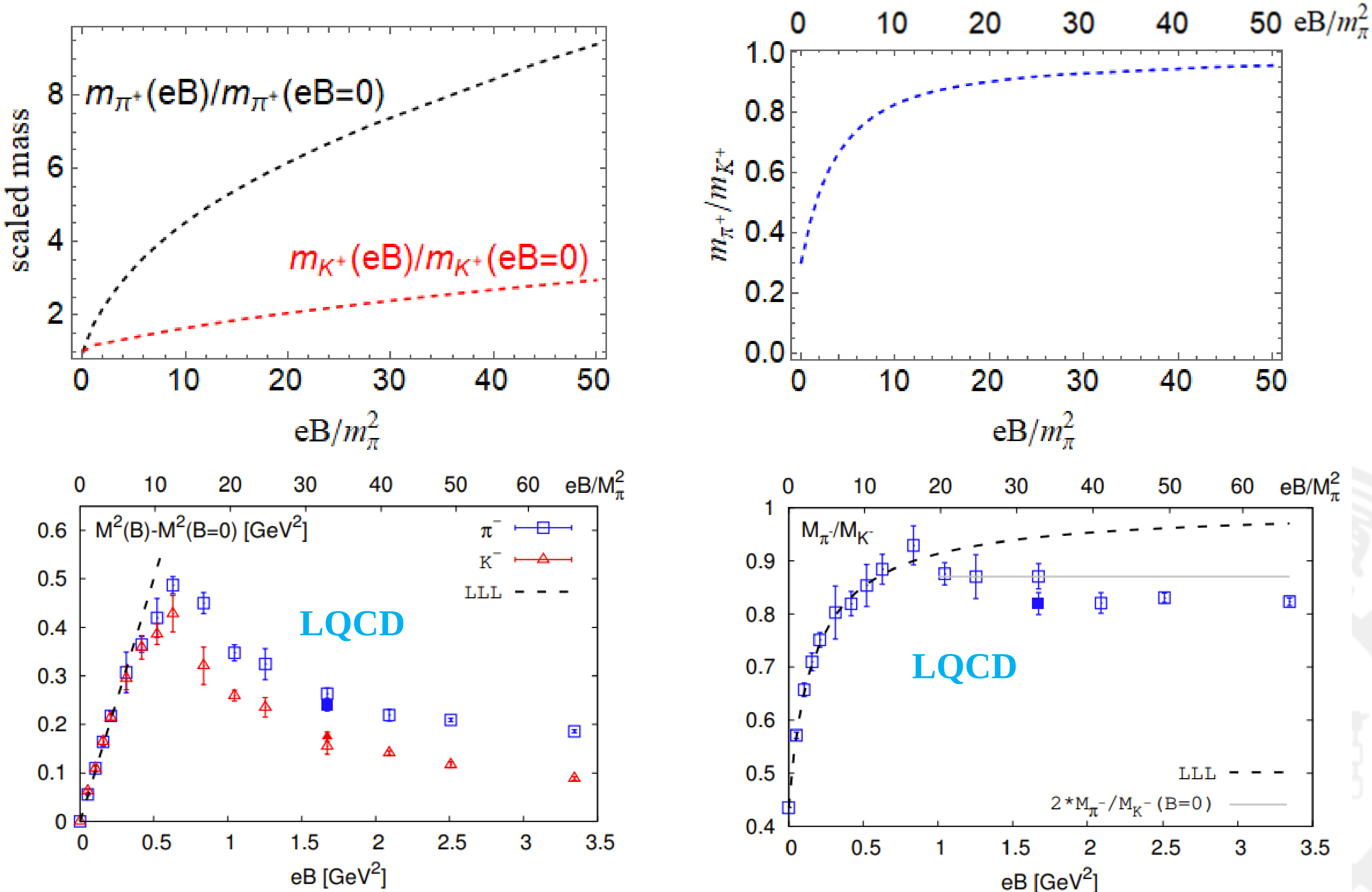
- **mass jumps @eB。**
- **observable?**

$U_A(1)$ restoration @eB under progress

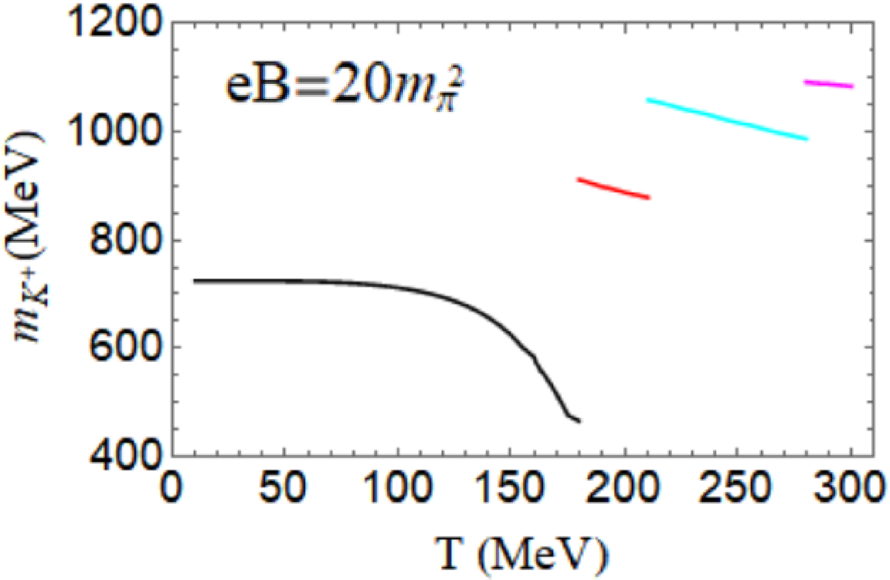
J.Mei, T.Xia, **S.J.Mao**, PRD107, 074018 (2023)

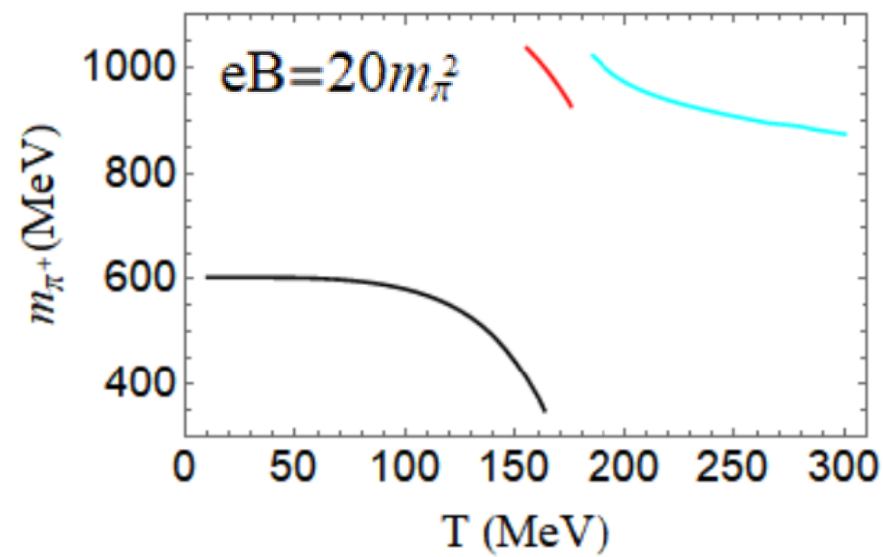
3.3 $K^{\pm}$ Mass spectra @ $T = \mu = 0$

G(eB)



3.3 $K^{\pm}$ Mass spectra@T $\mu = 0$

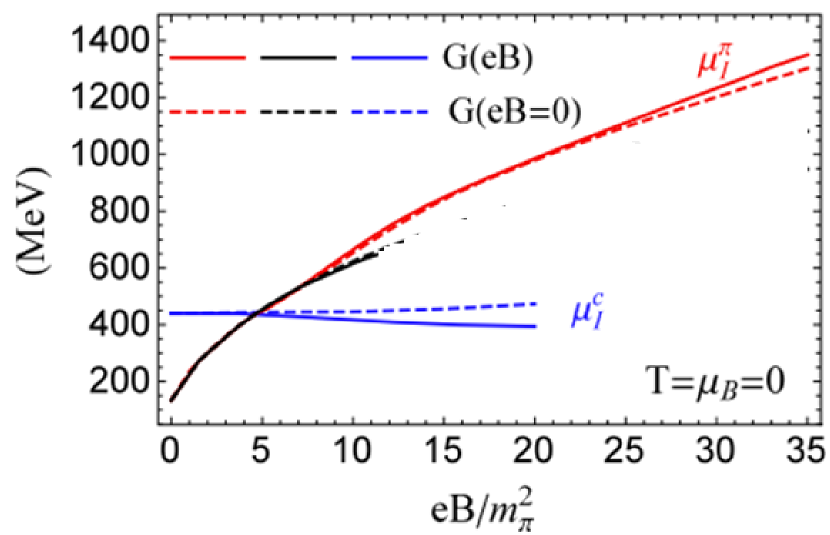




$$m_{K^+}(\mu_s = \mu_s^c) = 0$$

$\Rightarrow$ Kaon superfluid @eB

under progress…



4 Summary and outlook

Summary：

QCD phase transition @ meson level

① π^0, K^0 mass jump： chiral restoration

② Massless π^+, K^+： pion/Kaon superfluid

③ K, η, η'： $U_A(1)$ restoration

Outlook：

① Baryon decuplet @eB

② Observables of mass jumps @eB

仙交大欢迎你！

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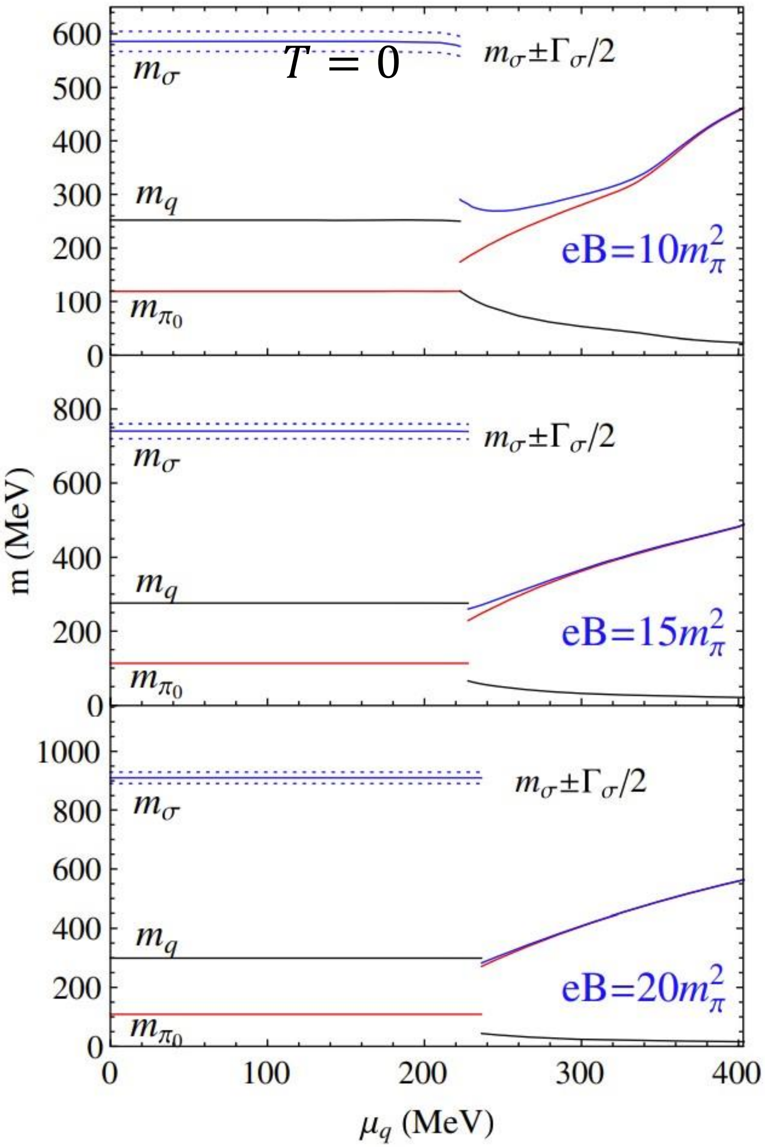
3.1 π_0 & chiral restoration phase transition



Mass jumps with 1st chiral restoration phase transition

No Mott transition (Pauli blocking)

$$m_M > 2m_q \quad \text{and} \quad m_M > 2\mu_q = 2/3\mu_B.$$

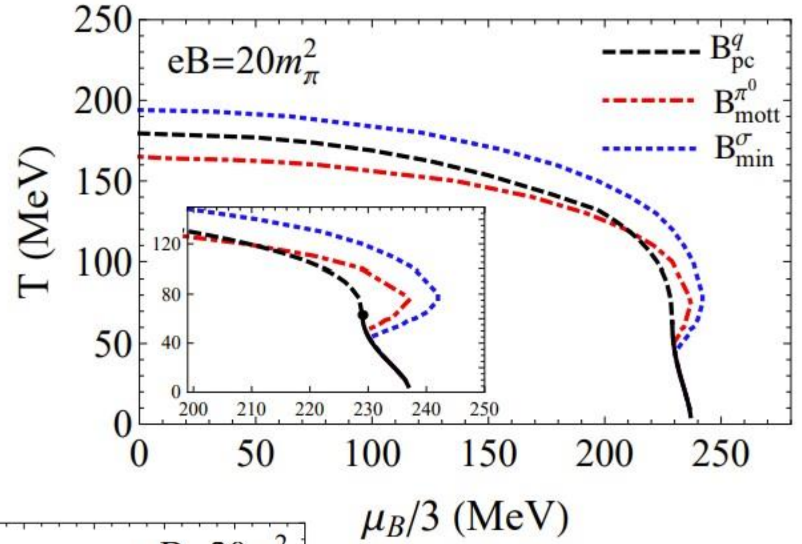
S.J. Mao, et al, PRD 96, 034004(2017)

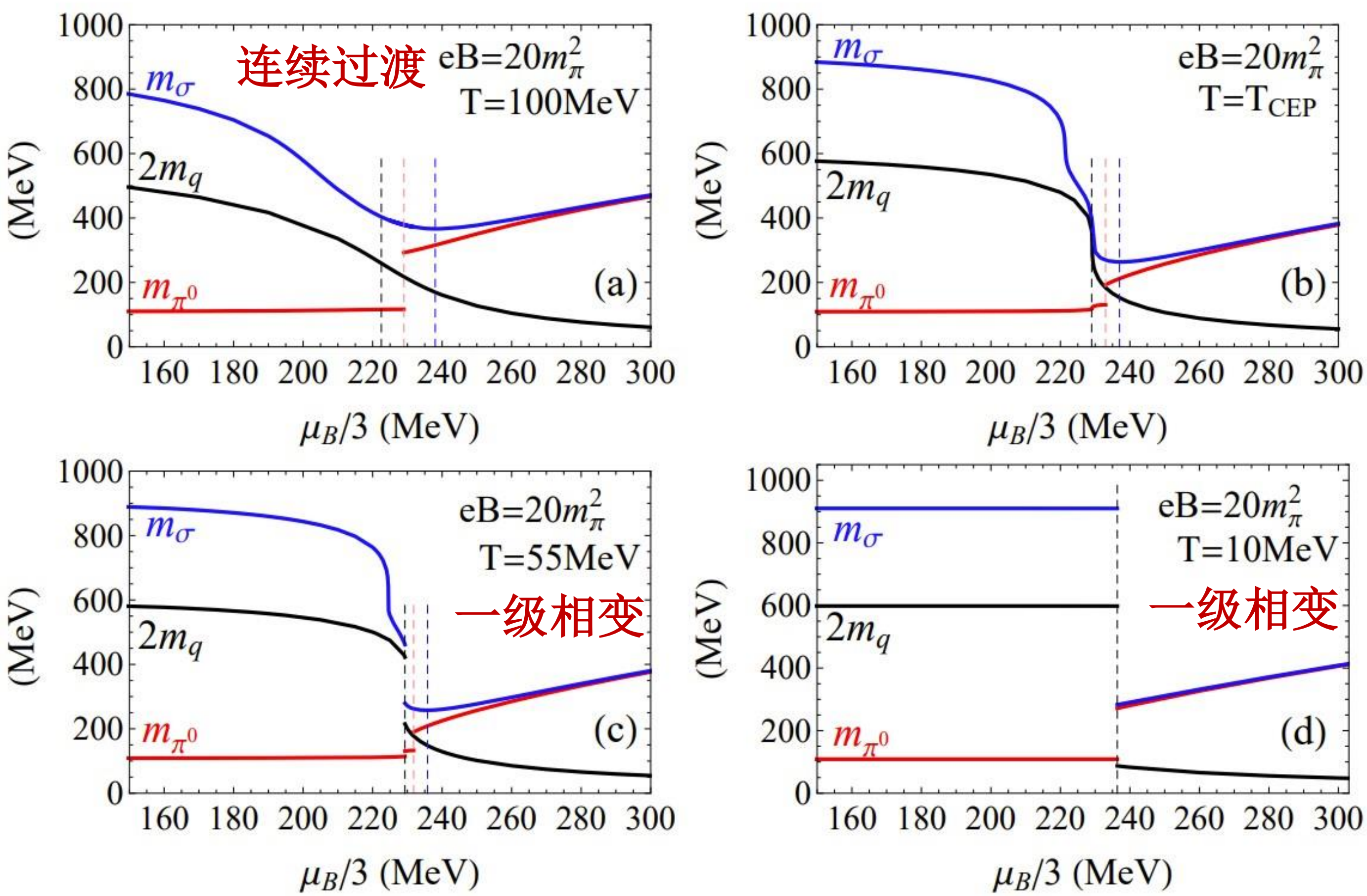
$T-\mu$平面

S.J. Mao, et al, Chin. Phys. C (2022) accepted

介子谱多次跳变：
寻找CEP和手征相边界

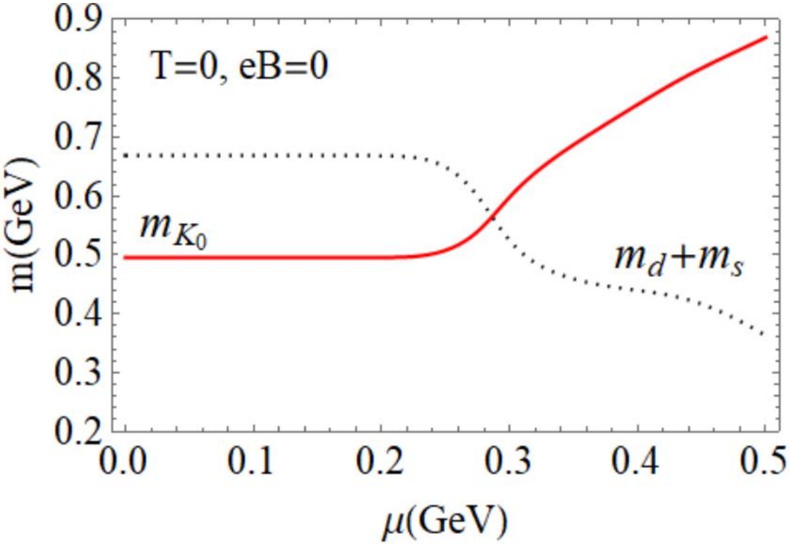
相图：

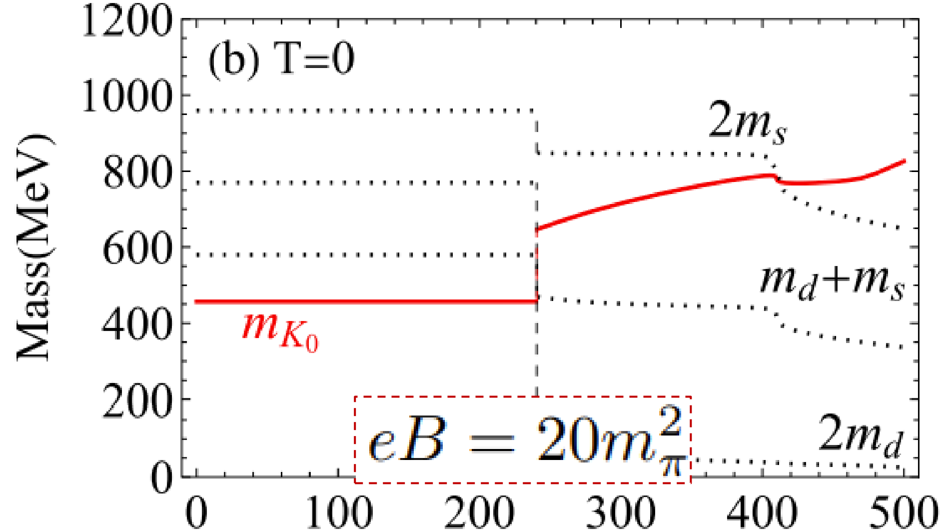


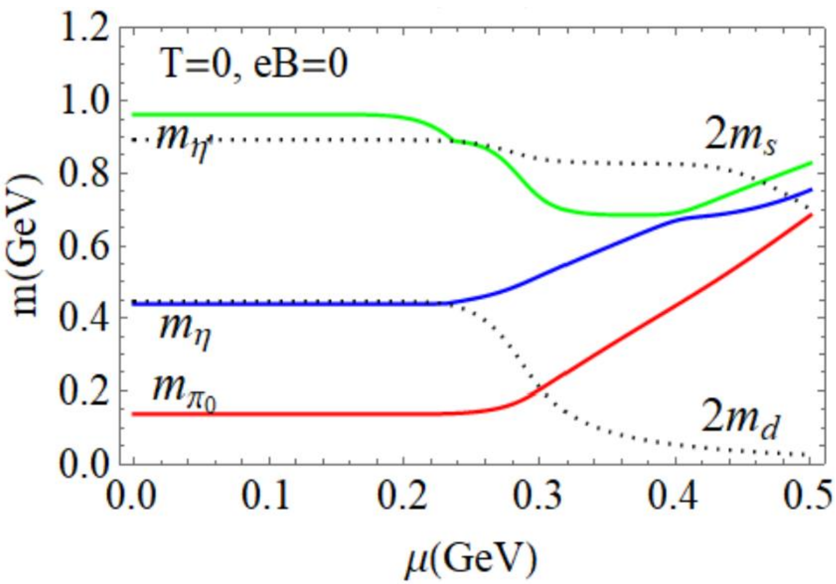


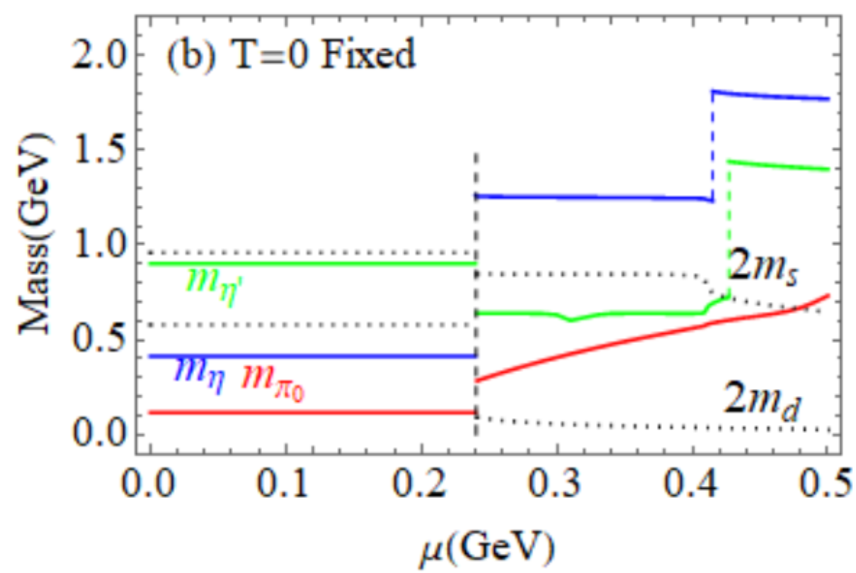
3.1 mass spectra@ baryon chemical potential μ

$eB = 20m_\pi^2$



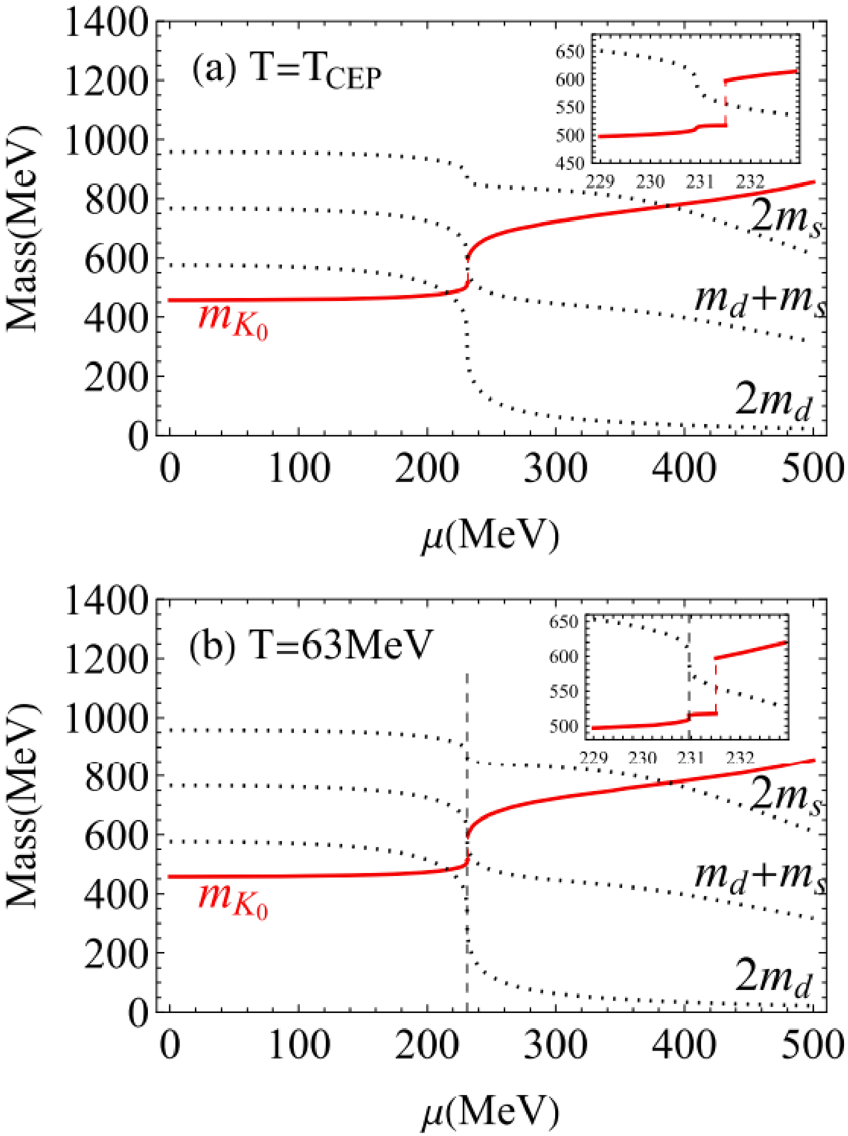


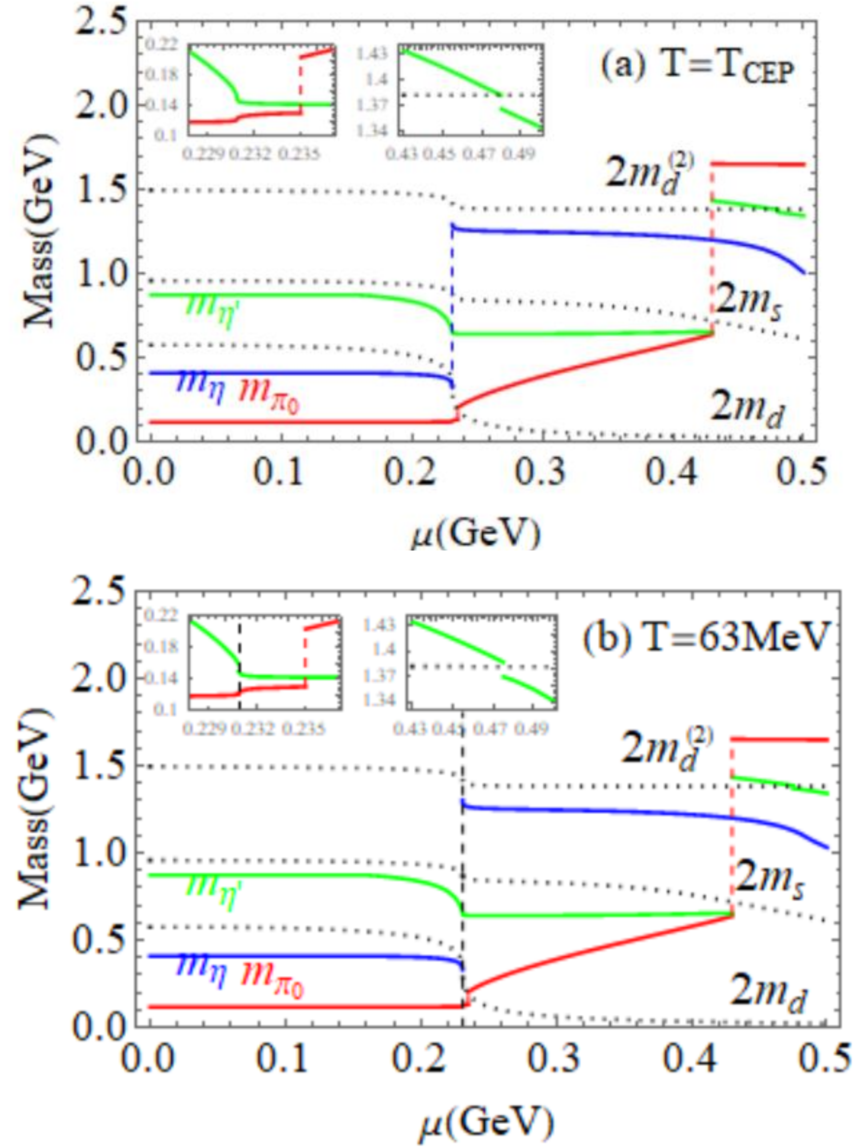




3.1 mass spectra @CEP

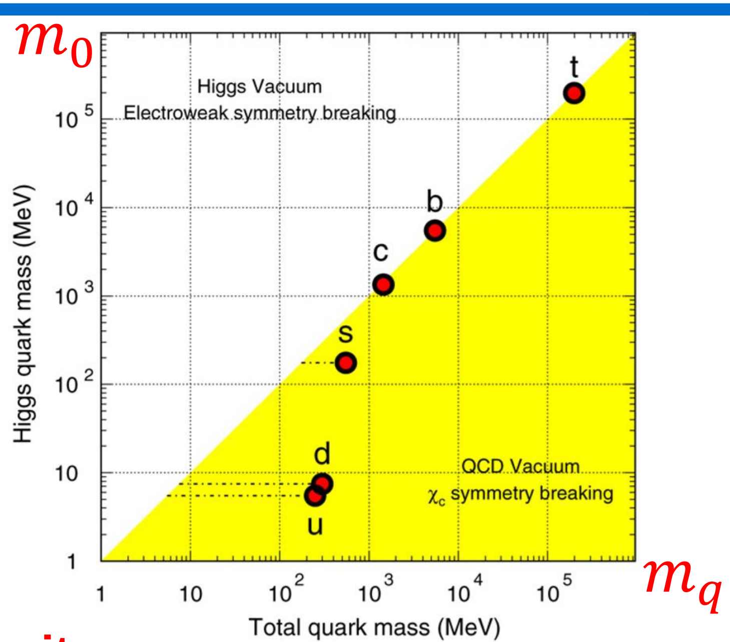
$eB = 20m_\pi^2$





1 Chiral symmetry

Light quark (hadron) mass is controlled by chiral symmetry.



- Order parameter: quark mass
- Goldstone modes;

X.Zhu, M.Bleicher, S.Huang, K.Schweda, H.Stoecker, N.Xu, P.Zhuang, PLB647, 366(2007)

● **chiral limit:**

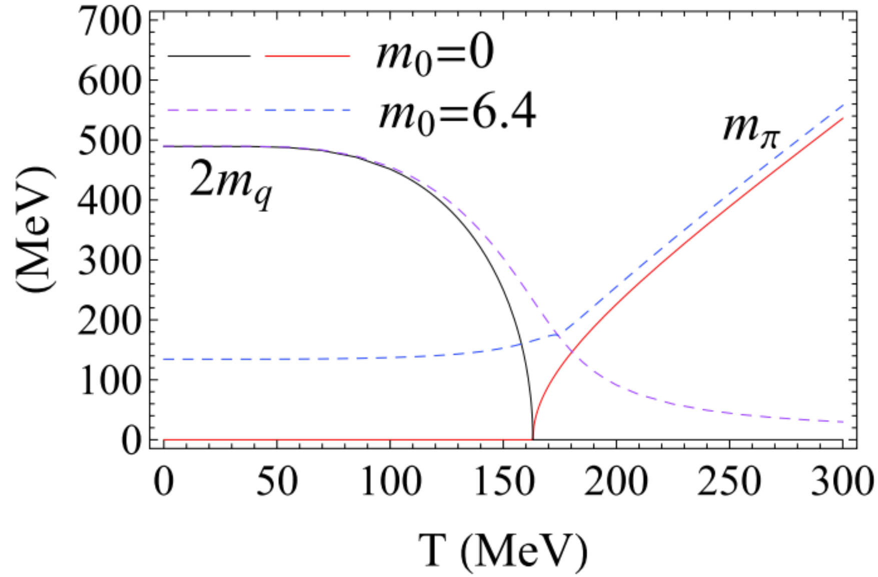
$m_0 = 0 \ \rightarrow \ m_\pi = 0$(Goldstone boson)

chiral phase transition

● **physical case**:

$m_0 \neq 0 \ \rightarrow \ m_\pi \neq 0(= 134\text{MeV})$
(pseudo-Goldstone boson)

chiral crossover



1 QCD phase structure

Study QCD phase transitions by mesons

- Spontaneous breaking of symmetries: order parameters
- Goldstone modes: collective modes (hadrons)

Meson nonet