Real-time dynamics of pseudo-Goldstone and critical mode in QCD

xQCD2024 IMP, in Lanzhou, China

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Based on: **arXiv:2403.03503**
arXiv:24xx.xxxxx in preparation

July 19, 2024

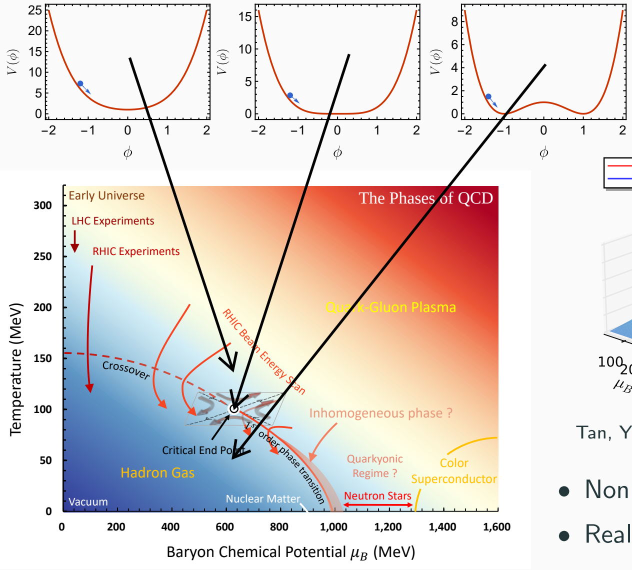
Dalian University of Technology

Outline

1. Introduction
2. Damping of pseudo-Goldstone near critical points
3. Relaxation dynamics of critical mode near CEP
4. Summary

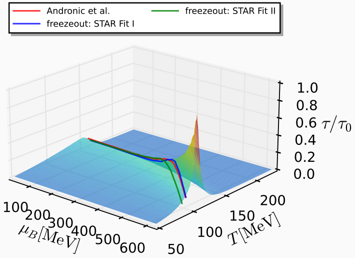
Introduction

QCD Phase Diagram



Critical slowing down with the relaxation time: $\tau = \xi^z f(k\xi)$

z:dynamic critical exponent



Tan, Yin, Chen, Huang, WF, in preparation

- Nonperturbative approach
- Real-time description

Functional renormalization group approach

$$Z_k[J] = \int \mathcal{D}\varphi \exp\left(-S[\varphi] + J\cdot\varphi - \Delta S_k[\varphi]\right)$$

where$\Delta S_k[\varphi] = \frac{1}{2}\int \frac{d^d q}{(2\pi)^d}\varphi(-q)R_k(q)\varphi(q)$
The scale dependent effective action:

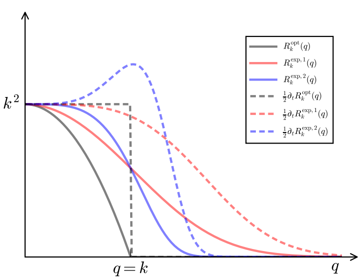
$$\Gamma_k[\phi] = -\ln Z_k[J] + \int d^d x J(x)\phi(x) - \Delta S_k[\phi]$$

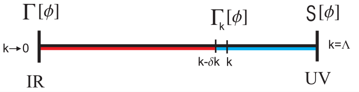
Wetterich Equation :

$$k\partial_k \Gamma_k[\phi] = \frac{1}{2}\mathrm{STr}\left[\left(\Gamma_k^{(2)}[\phi] + R_k\right)^{-1} k\partial_k R_k\right]$$

The scale evolution equation for n- point function(one loop exact)

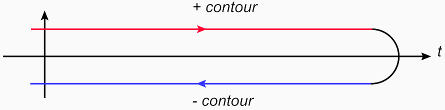
$$\partial_t \Gamma_k^{(n)}[\phi] = \tilde{\partial}_t \begin{pmatrix} \text{all one - loop correction} \\ \text{diagrams of } \Gamma_k^{(n)}[\phi] \end{pmatrix}$$





- Derivative expansion
- Vertex expansion

fRG in Keldysh path integral



$$\varphi_c = \frac{1}{\sqrt{2}}(\varphi_+ + \varphi_-)\,, \quad \varphi_q = \frac{1}{\sqrt{2}}(\varphi_+ - \varphi_-)$$

The flow equation in the closed time path :

$$\partial_\tau \Gamma_k[\Phi] = \frac{i}{2}\,\mathrm{Tr}\left[(\partial_\tau R_k^*)\,(\Gamma_k^{(2)}[\Phi] + R_k)^{-1}\right]$$
$$\equiv \frac{i}{2}\,\mathrm{Tr}\left[(\partial_\tau R_k^*)\,G_k\right]$$

- Propagator

$$G_k = \begin{pmatrix} G_k^K & G_k^R \\ G_k^A & 0 \end{pmatrix}$$

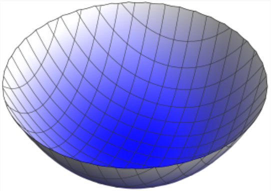
- Regulator

$$R_k = \begin{pmatrix} 0 & R_k^A \\ R_k^R & R_k^F \end{pmatrix}$$

$G^R = (G^A)^*$ and the Regulator $R_k^F = 0$. We use frequency-independent flat regulator to maintain the causal structure:
$R_k^R(\omega, \boldsymbol{q}) = R_k^A(\omega, \boldsymbol{q}) = -Z_k(k^2 - \boldsymbol{q}^2)\theta(k^2 - \boldsymbol{q}^2)$

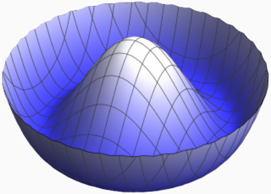
Damping of pseudo-Goldstone near critical points

Symmetry breaking of chiral phase transition



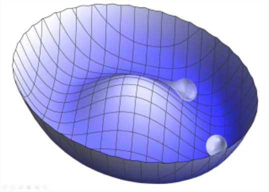
$SU(2)_L \times SU(2)_R \simeq O(4)$

Symmetric phase
Two-flavor QCD with
$m_q \to 0$



$O(4) \to O(3) \times Z_2 \;\; (\vec{\pi}, \sigma)$

Spontaneous symmetry breaking
π- Goldstone bosons with $m_\pi = 0$



Approximate symmetry spontaneous breaking
m_q is small but $\neq 0$
or $c \neq 0$
π- pseudo-Goldstone bosons with $m_\pi \neq 0$

$O(N)$ Effective action on the S-K contour

$$\Gamma[\phi_c, \phi_q] = \int d^4x \left(Z_a^{(t)} \, \phi_{a,q} \, \partial_t \, \phi_{a,c} - Z_a^{(i)} \, \phi_{a,q} \, \partial_i^2 \, \phi_{a,c} + V'(\rho_c) \, \phi_{a,q} \, \phi_{a,c} - 2 \, Z_a^{(t)} \, T \, \phi_{a,q}^2 - \sqrt{2} c \, \sigma_q \right)$$

Universal damping of pseudo-Goldstone

The real-time dispersion relation of pseudo-Goldstone:

$$\omega(\boldsymbol{p}) = \pm c_s \sqrt{m_\varphi^2 + \boldsymbol{p}^2} - \frac{i}{2}\left[\Omega_\varphi + (D_Q + D_\varphi)\,\boldsymbol{p}^2\right] + \cdots$$

Ω_φ: relaxation/damping rate, D_φ: Goldstone diffusivity,

D_Q:charge diffusivity, m_φ: screening mass, c_s:speed of sound

Universal relation for pseudo-Goldstone damping

$$\frac{\Omega_\varphi}{m_\varphi^2} \simeq D_\varphi + \mathcal{O}\left(m_\varphi^2/T^2\right)$$

Holographics:

Amoretti, Areán, Goutéraux, Musso, PRL 123 (2019) 211602;
Amoretti, Areán, Goutéraux, Musso, JHEP 10 (2019) 068;
Ammon et al., JHEP 03 (2022) 015;
Cao, Baggioli, Liu, Li, JHEP 12 (2022) 113

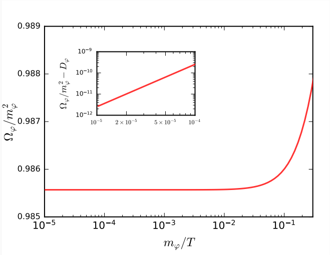
Hydrodynamics:

Delacrétaz, Goutéraux, Ziogas, PRL 128 (2022) 141601

EFT:

Baggioli, Phys. Rev. Res. 2 (2020) 022022;
Baggioli, Landry, SciPost Phys. 9 (2020) 062

$T = 0.3T_c$



$$\frac{\Omega_\varphi}{m_\varphi^2} - D_\varphi(T) \propto \left(\frac{m_\varphi}{T}\right)^\alpha$$

$$\alpha = 2.00021(2)$$

Universal damping of pseudo-Goldstone near the critical point

The relaxation/damping rate in Model A:

$$G^R_{\varphi\varphi}(\omega, q) = \frac{1}{-\mathrm{i}Z^{(t)}_\varphi \omega + Z^{(i)}_\varphi \left(q^2 + m_\varphi^2\right)} \rightarrow \quad \frac{\Omega_\varphi}{m_\varphi^2} = \frac{Z^{(i)}_\varphi}{Z^{(t)}_\varphi}$$

$$\frac{\Omega_\varphi}{m_\varphi^2} \propto m_\varphi^{\Delta_\eta}, \;\; \Delta_\eta \equiv \eta_t - \eta > 0$$

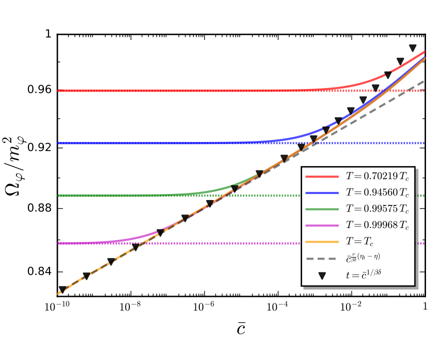
$$\Delta_\eta \simeq 0.0172$$

GMOR also breaks down with(only relates to the static properties):

$$m_\varphi^2 \propto c^{\frac{2\nu}{\beta\delta}} \simeq c^{0.806}$$

mean-field theory with:

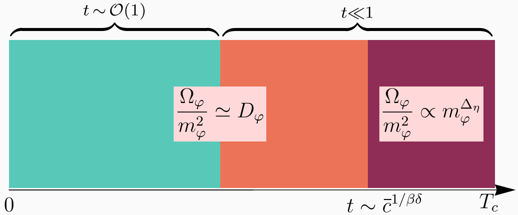
$$m_\varphi^2 \simeq c^{2/3}$$



Critical region of chiral phase transition:

$$m_{\pi 0} \lesssim 0.1 \sim 1\mathrm{MeV}$$

Universal damping of pseudo-Goldstone near the critical point



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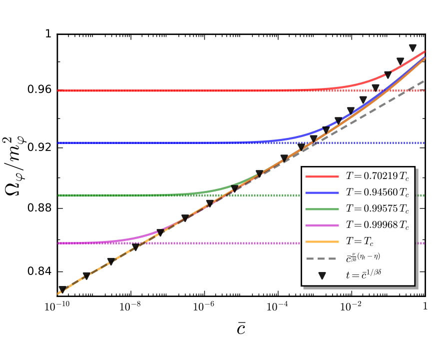


Large N limit

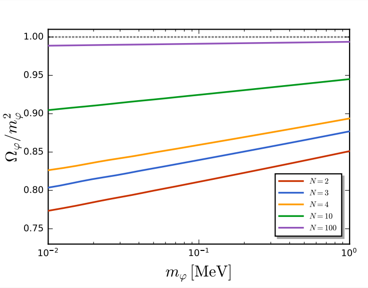
Neglect sigma mode in the fRG flow equation we have:

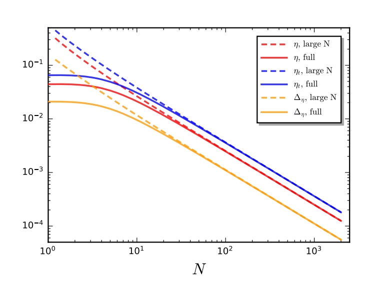
$$\eta = \frac{5}{N-1}\frac{(1+\eta)(1-2\eta)^2}{(5-\eta)(2-\eta)^2}$$

$$\eta_t = \frac{1}{9(N-1)}\frac{(1-2\eta)^2\left(13+15\eta-2\eta^3\right)}{(2-\eta)^2}$$

In the large N limit

$\eta \propto 1/N$,$\eta_t \propto 1/N$ and $\Delta_\eta \propto 1/N$





Relaxation dynamics of critical mode near CEP

QCD-asisted Langevin transport model

Effective action on the S-K contour

$$\Gamma[\phi_c,\phi_q]=\int \mathrm{d}^4x\Big(Z_\phi^{(t)}\sigma_q\partial_t\sigma_c-Z_\phi^{(i)}\sigma_q\partial_i^2\sigma_c+2U'(\sigma_c)\sigma_q-2Z_\phi^{(t)}T\sigma_q^2\Big)$$

Langevin equation:

$$Z_\phi^{(t)}\partial_t\sigma-Z_\phi^{(i)}\partial_i^2\sigma+U'(\sigma)=\xi$$

with classical noise:

$$\langle\xi(t,\boldsymbol{x})\xi(t',\boldsymbol{x}')\rangle=2Z_\phi^{(t)}T\delta(t-t')\delta(\boldsymbol{x}-\boldsymbol{x}')$$

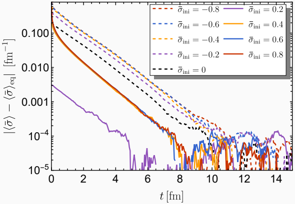
relaxation ansatz:

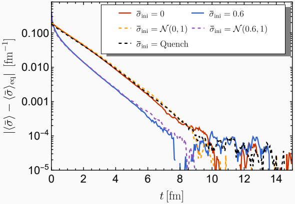
$$\langle\bar\sigma(t)\rangle=A\mathrm{e}^{-\frac{t}{\tau}}+\langle\bar\sigma(t)\rangle_{\mathrm{eq}}$$

Langevin simulations are performed on the GPUs using Julia with CUDA.jl and DifferentialEquations.jl packages.

Lattice size: $32\times32\times32$

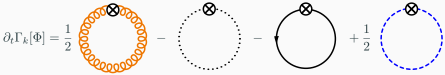
Lattice spacing : $0.1\mathrm{fm}$





Inputs from first principal fRG-QCD

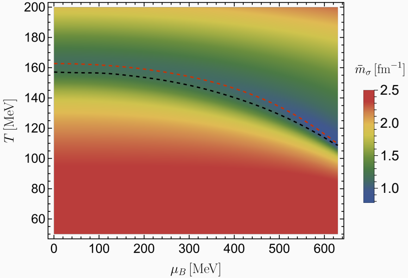
Flow of fRG-QCD(static properties):



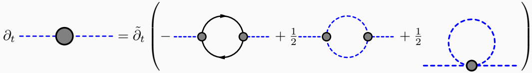
Fu,Pawlowski,Rennecke, PRD.101.054032

$$U'(\sigma) = \frac{\delta \Gamma_{\rm QCD}[\Phi]}{\delta \sigma}\bigg|_{\substack{\sigma(x)=\sigma \\ \tilde{\Phi}=\tilde{\Phi}_{\rm EoM}}}$$

$$\Gamma^{(2)}_{\sigma\sigma} = \frac{\delta^2 \Gamma_{\rm QCD}[\Phi]}{\delta\sigma\delta\sigma}\bigg|_{\Phi=\Phi_{\rm EoM}}, \quad Z^{(i)}_{\phi} = \frac{\partial \Gamma^{(2)}_{\sigma\sigma}(p_0, \boldsymbol{p})}{\partial \boldsymbol{p}^2}\bigg|_{\substack{p_0=0 \\ \boldsymbol{p}=0}}$$

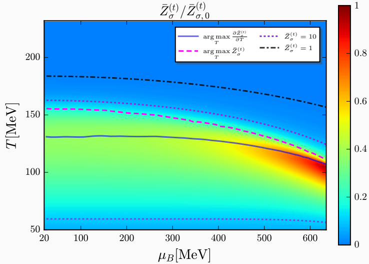


Flow of two point function:

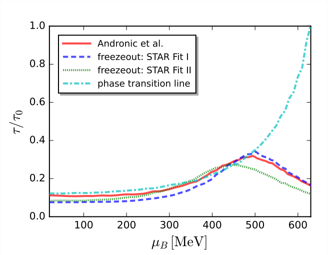


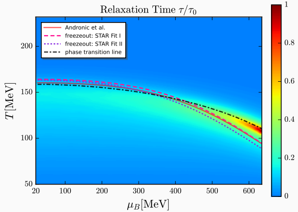
$$\Gamma^{(2)}_{\sigma\sigma,\rm R}(\omega, \boldsymbol{p}) = \lim_{\epsilon\to 0^+} \Gamma^{(2)}_{\sigma\sigma}\left(p_0 = -\mathrm{i}(\omega + \mathrm{i}\epsilon), \boldsymbol{p}\right)$$

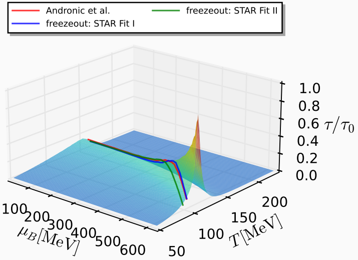
$$Z^{(t)}_{\phi} = \lim_{|\boldsymbol{p}|\to 0} \lim_{\omega\to 0} \frac{\partial}{\partial\omega} \mathrm{Im}\Gamma^{(2)}_{\sigma\sigma,\rm R}(\omega, \boldsymbol{p})$$



Relaxation time on the QCD phase diagram



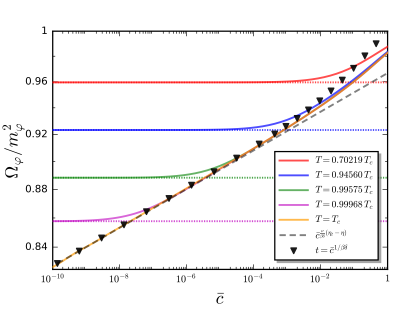


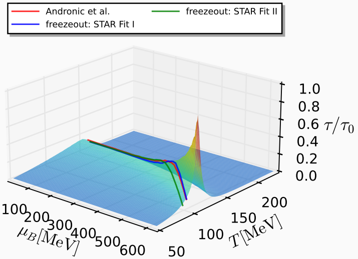


- Once away from the CEP the relaxation time quickly drops which indicates a small critical region around the CEP.
- The critical slowing down and nonequilibrium fluctuations have limited effects on the observables in the freezeout lines.

Summary

Summary





- The pseudo-Goldstone damping has a universal relation $\frac{\Omega_\varphi}{m_\varphi^2} \simeq D_\varphi + \mathcal{O}\left(m_\varphi^2/T^2\right)$ in the hydrodynamic regime while breaks down to $\frac{\Omega_\varphi}{m_\varphi^2} \propto m_\varphi^{\Delta_\eta},\ \Delta_\eta > 0$ in the critical regime.
- The relaxation time quickly drops when away from the CEP and the relaxation time is relatively small in the freeze-out line thus the effect of critical slowing down is mild on the chemical freeze-out curve.