Recent results on the far-from-equilibrium phase of quark-gluon matter

Kirill Boguslavski

TECHNISCHE UNIVERSITÄT WIEN

Institute for Theoretical Physics
TU Wien, Austria

FWF Der Wissenschaftsfonds.

20th International Conference
on QCD in Extreme Conditions

Lanzhou,
July 19, 2024

Table of Contents

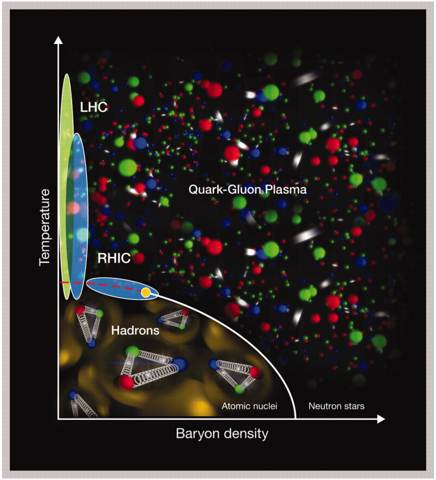
1. Motivation
2. Universality during initial stages
3. Hard probes and transport coefficients
4. Excitations and a new transport peak
5. Conclusion

Table of Contents

1. Motivation
2. Universality during initial stages
3. Hard probes and transport coefficients
4. Excitations and a new transport peak
5. Conclusion

QCD phase diagram

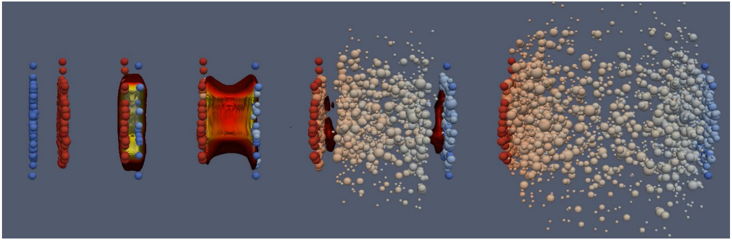
- High T or density:
  Quark-Gluon plasma (QGP)
- Low temperature T: hadrons
- *Early Universe:* High to low T
- *Similarly on Earth:*
  QGP in heavy-ion collisions
  (LHC at CERN, RHIC at BNL)



Jacak, Müller, Science 6092, 310 (2012)

Colliders reproduce QGP from first instants of Early Universe

Stages in heavy-ion collisions



MADAI collaboration

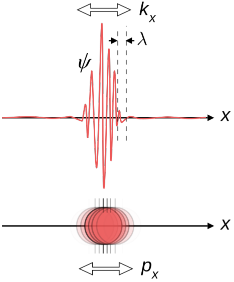
- High-energy collisions ⇒ **QGP** created
- Cooling during evolution, go through different **phases**
  ⇒ pre-equilibrium QGP (initial stages) → fluid QGP → hadrons
- **Pre-equilibrium QGP:** testing the very nature of quantum physics
  ⇒ Gluons first as (classical) waves → scatterings of (quasi-)particles

Goals

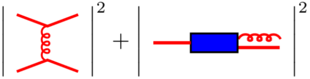
Learn about **real-time** properties of QCD in **extreme conditions**

Pre-equilibrium QGP: descriptions (for couplings $g^2 \ll 1$)

- initially: classical-statistical simulations, 'Glasma'
  - $\Rightarrow$ large gluon fields $A \sim 1/g$, full initial conditions
  - $\Rightarrow$ nonlinear dynamics of interacting classical waves
  - $\Rightarrow$ Valid while occupancies large $f(t,p) \approx \langle AA \rangle p \gg 1$



- then, as energy decreases (dilution): kinetic theory
  - $\Rightarrow$ Boltzmann equation for f

$(\partial_t + \boldsymbol{v} \cdot \boldsymbol{\nabla}) f =$ 

$$\frac{\partial f_{\vec{p}}}{\partial \tau} - \frac{p_z}{\tau} \frac{\partial f_{\vec{p}}}{\partial p_z} = -\mathcal{C}^{2\leftrightarrow 2}[f_{\vec{p}}] - \mathcal{C}^{1\leftrightarrow 2}[f_{\vec{p}}]$$

Arnold, Moore, Yaffe, JHEP 01, 030 (2003)

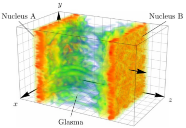
Quantum wave particle duality, approximative descriptions

classical fields $A(t, \vec{x})$ ('waves') $\rightarrow$ interacting particle distribution $f(t, \vec{p})$

Strong initial fields: classical-statistical lattice simulations

- Initial state: Glasma – large longitudinal fields

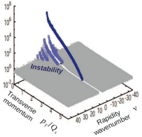
  McLerran, Venugopalan (1999); Krasnitz, Venugopalan (1999, 2000, 2001); Krasnitz, Nara, Venugopalan (2001, 2003); Lappi (2003, 2006, 2011); Lappi, McLerran (2006); Schenke, Tribedy, Venugopalan (2012); Gelfand, Ipp, Müller (2016, 2017); . . .



Ipp, Müller (2017)

- Plasma instabilities – from boost-invariant Glasma to highly occupied (mainly gluonic) plasma

  Mrowczynski (1993); Arnold, Lenaghan, Moore (2003); Romatschke, Strickland (2003); Romatschke, Venugopalan (2006); Attems, Rebhan, Strickland (2012); Fukushima, Gelis (2012); Berges, KB, Schlichting, (2012, 2013); Epelbaum, Gelis (2013); . . .



Berges, Schenke, Schlichting, Venugopalan (2014)

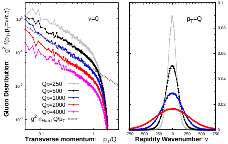
- Classical self-similar attractor far from equilibrium – universal dynamics of over-occupied plasma

  $\Rightarrow$ agrees with 1. stage of 'bottom-up' scenario

  Berges, KB, Schlichting, Venugopalan (2013, 2014); Kurkela, Zhu (2015); . . .

  $\Rightarrow$ Far-from-equilibrium universality class with scalars

  Berges, KB, Schlichting, Venugopalan (2015); . . .



Berges, KB, Schlichting, Venugopalan (2013)

Bottom-up thermalization: QCD kinetic theory

- When quasiparticles have formed:
  Kinetic theory becomes applicable

  Note: Assumes narrow excitations in spectral functions, which may not be true at low momenta for strong anisotropy

  KB, Kurkela, Lappi, Peuron (2018, 2019, 2021)

- Bottom-up thermalization: Baier, Mueller, Schiff, Son (2001)
  1. Classical attractor (see above)
  2. Anisotropy freezes
  3. Radiational breakup

- QCD effective kinetic theory (EKT) simulations

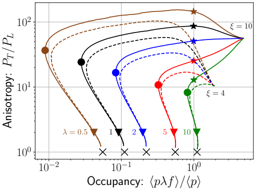
  Arnold, Moore, Yaffe (2003); Kurkela, Zhu (2015); Kurkela, Mazeliauskas (2019);

$$-\frac{\partial f_{\vec{p}}}{\partial \tau} = \mathcal{C}^{1\leftrightarrow 2}[f_{\vec{p}}] + \mathcal{C}^{2\leftrightarrow 2}[f_{\vec{p}}] - \frac{p_z}{\tau}\frac{\partial f_{\vec{p}}}{\partial p_z}$$

- EKT: smooth transition to hydrodynamics;
  KoMPoST: EKT + $\delta T^{\mu\nu}(\tau, \vec{x})$ perturbations

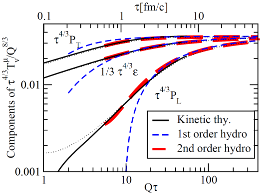
  Kurkela, Zhu (2015); Kurkela, Mazeliauskas, Paquet, Schlichting, Teaney (2018)

Bottom-up evolution



Kurkela, Zhu (2015); KB, Kurkela, Lappi, Lindenbauer, Peuron (2023)

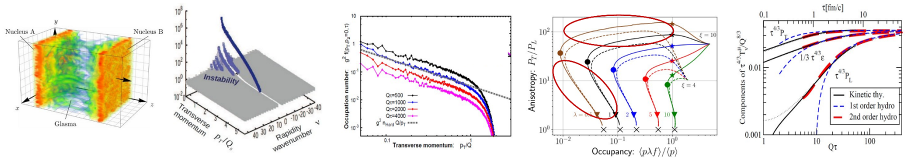
Onset of hydro



Kurkela, Zhu (2015)

Initial stages in heavy-ion collisions (weak-g^2 perspective)

$Q_s\tau = 0^+$ | $Q_s\tau = \ln^2(\alpha_s^{-1})$ | $Q_s\tau = \alpha_s^{-3/2}$ | $Q_s\tau = \alpha_s^{-13/5}$



Glasma | Instabilities | Bottom-up stage 1 | Bottom-up stages 2 and 3 | Onset of hydrodynamics

$$\underbrace{D_\mu F^{\mu\nu} = J^\nu}_{\text{classical-statistical simulations}}$$

$$\underbrace{-\frac{\partial f_{\vec p}}{\partial \tau} = \mathcal{C}^{1\leftrightarrow 2}[f_{\vec p}] + \mathcal{C}^{2\leftrightarrow 2}[f_{\vec p}] - \frac{p_z}{\tau}\frac{\partial f_{\vec p}}{\partial p_z}}_{\text{QCD effective kinetic theory simulations}}$$

hydrodynamics ...

Interesting research questions

- What is universal about the initial stages in heavy-ion collisions?
- Can we use heavy quarks, jets and quarkonia to observe dynamics?
- What is the form of excitations in the pre-equilibrium QGP evolution?

Table of Contents

1. Motivation
2. Universality during initial stages
3. Hard probes and transport coefficients
4. Excitations and a new transport peak
5. Conclusion

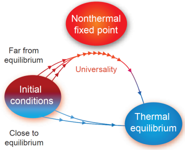
What is universal about initial stages?

Stage 1 in bottom-up scenario

Nonthermal fixed point, links to different systems (e.g., cold atoms)

Universality: Berges, Rothkopf, Schmidt (2008); Piñeiro Orioli, KB, Berges (2015); Berges, KB, Schlichting, Venugopalan (2015); Walz, KB, Berges (2017); Chantesana, Piñeiro Orioli, Gasenzer (2018); KB, Piñerio Orioli (2020); . . .

Cold-atom exp.: Prüfer et al., Nature (2018); Erne et al., Nature (2018); Glidden et al., Nature Phys. (2021); Gazo et al. (2023)



- ★ Initial over-occupancy ⇒ may approach attractor
- ★ System 'forgets' initial conditions
- ★ Self-similar universal dynamics

$$f(t,p) = t^{\alpha} f_s(t^{\beta} p)$$

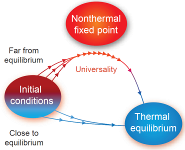
What is universal about initial stages?

Stage 1 in bottom-up scenario

Nonthermal fixed point, links to different systems (e.g., cold atoms)

Universality: Berges, Rothkopf, Schmidt (2008); Piñeiro Orioli, KB, Berges (2015); Berges, KB, Schlichting, Venugopalan (2015); Walz, KB, Berges (2017); Chantesana, Piñeiro Orioli, Gasenzer (2018); KB, Piñerio Orioli (2020); . . .

Cold-atom exp.: Prüfer et al., Nature (2018); Erne et al., Nature (2018); Glidden et al., Nature Phys. (2021); Gazo et al. (2023)



- ★ Initial over-occupancy ⇒ may approach attractor
- ★ System 'forgets' initial conditions
- ★ Self-similar universal dynamics

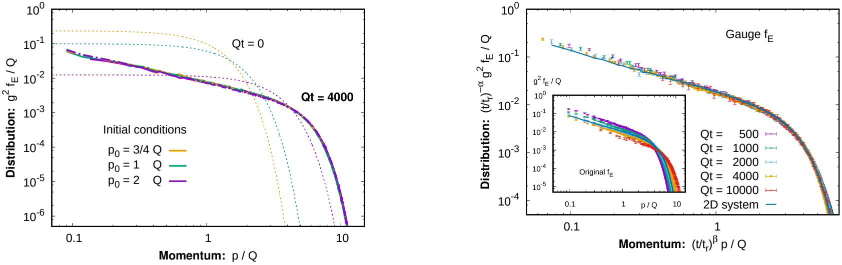
$$f(t,p) = t^{\alpha} f_s(t^{\beta} p)$$

Recent developments on universality

- **Prescaling** (Schmied, Mikheev, Gasenzer (2019); Mazeliauskas, Berges (2019); Heller, Mazel., Preis (2024))
- **Adiabatic hydrodyn.** (Brewer, Yan, Yin (2019); Rajagopal, Scheihing-Hitschfeld, Steinhorst (2024))
- **Hydrodynamic attractors** (M. Heller's talk; **Limiting:** KB, Kurkela, Lappi, Lindenbauer, Peuron ('23))
- **Gauge inv. condens.** (Berges, KB, Mace, Pawlowski ('20); Berges, KB, Butler, de Bruin, Pawlowski ('24))

Example: gluon plasmas (isotropic)

Figures: attractor in 2+1D; KB, Kurkela, Lappi, Peuron, PRD 100, 094022 (2019)



- Gluonic initial $f_g(t{=}0, p \lesssim Q) \sim \frac{1}{g^2} \gg 1$ often approach attractors

$$f(t,p) = (Qt)^{\alpha} f_s\left((Qt)^{\beta} p\right)$$

- Universal exponents insensitive to details of initial conditions
  - ✓ 2+1D: $\beta = -1/5$, $\alpha = 3\beta$, KB, Kurkela, Lappi, Peuron (2019)
  - ✓ 3+1D: $\beta = -1/7$, $\alpha = 4\beta$, Berges, Scheffler, Sexty (2009); Kurkela, Moore (2011, 2012); Berges, Schlichting, Sexty (2012); Schlichting (2012); Berges, KB, Schlichting, Venugopalan (2014); ...
- Great control ⇒ attractors useful to understand dynamical properties

Table of Contents

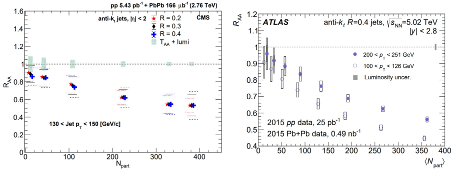
1. Motivation
2. Universality during initial stages
3. Hard probes and transport coefficients
4. Excitations and a new transport peak
5. Conclusion

Hard probes show signatures of QGP

Hard probes are modified while traversing the QGP

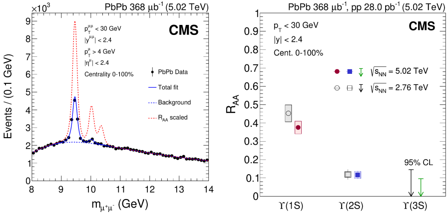
Examples: heavy quarks, quarkonia ($q\bar{q}$), jets ($p \gg T$)

Jet energy loss / suppression



CMS Collaboration, PRC (2017) ; ATLAS Collaboration, PLB (2019)

Bottomonium suppression

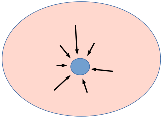


Transport coefficients from pre-equilibrium QGP

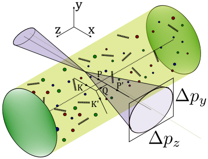
Jets, heavy (c, b) quarks: potential for signatures of initial stages

medium interactions $\Rightarrow$ QGP properties encoded in observables

- Quarks/jets get 'kicks' $\dot{p}_i(\tau) = \mathcal{F}_i(\tau)$
- Heavy-quark diffusion coefficient $\kappa_i = \frac{\mathrm{d}}{\mathrm{d}\tau}\langle p_i^2 \rangle$
  $\Rightarrow$ heavy quark (c, b), small momentum $p \ll M$



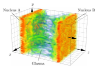
- κ enters Lindblad eq. for quarkonium dynamics
  Brambilla, Escobedo, [Soto], [Strickland], Vairo, [v.d. Griend, Weber] (2016, 2021)
  $\Rightarrow$ describe suppression of bottomonium ($b\bar{b}$ states)
- Jet quenching parameter $\hat{q}_i = \frac{\mathrm{d}}{\mathrm{d}\tau}\langle p_{\perp,i}^2 \rangle$
  $\Rightarrow$ jet with high momentum $p \gg Q_s, T$
- They encode also pre-equilibrium dynamics



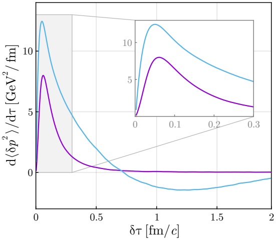
KB, Kurkela, Lappi, Lindenbauer, Peuron (2023)

Mrowczynski (2018); Ruggieri, Das (2018); Sun, Coci, Das, Plumari, Ruggieri, Greco (2019); Ipp, Müller, Schuh (2020); KB, Kurkela, Lappi, Peuron (2020); Khowal, Das, Oliva, Ruggieri (2022); Carrington, Czajka, Mrowczynski (2020, 2022); Avramescu, Baran, Greco, Ipp, Müller, Ruggieri (2023); KB, Kurkela, Lappi, Lindenbauer, Peuron (2023); Du (2023); Barata, Hauksson, Lopez, Sadofyev (2024); ...

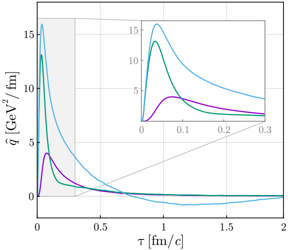
κ and $\hat{q}$ during Glasma phase



κ_i of beauty quarks



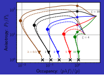
$\hat{q}_i$ of jets



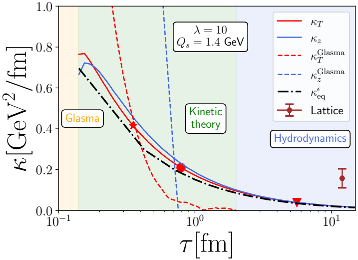
Avramescu, Baran, Greco, Ipp, Müller, Ruggieri PRD 107, 114021 (2023); 2307.07999

- Classical-statistical simulations of hard probes in the **Glasma** phase
  - Extraction of κ_i and $\hat{q}_i$
    Ipp, Müller, Schuh (2020); KB, Kurkela, Lappi, Peuron (2020); Carrington, Czajka, Mrowczynski (2022); Khowal, Das, Oliva, Ruggieri (2022); Avramescu et al. (2023); ...
  - Here simulations via Wong's equations in Glasma (particle-in-cell)
  - Large values, anisotropic $\kappa_z > \kappa_T$ and $\hat{q}_z > \hat{q}_y$ (z is beam direction)
  - Why are they large? Why do κ_z and $\hat{q}_z$ become negative?

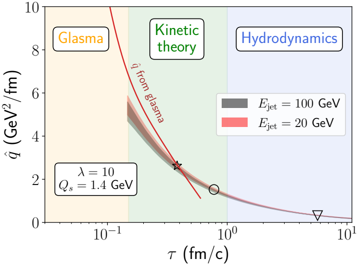
κ and $\hat{q}$ during kinetic regime



κ of heavy quarks



$\hat{q}$ for jet quenching



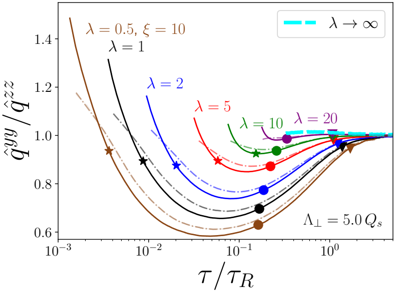
KB, Kurkela, Lappi, Lindenbauer, Peuron, for κ PRD [2303.12520];
for $\hat{q}$: Phys. Lett. B (2024) [2303.12595], PRD [2312.00447]

- $\hat{q}$ smoothly connects Glasma and hydro, κ not so much
- Mostly the same ordering $\kappa_z > \kappa_T$ and $\hat{q}_z > \hat{q}_y$

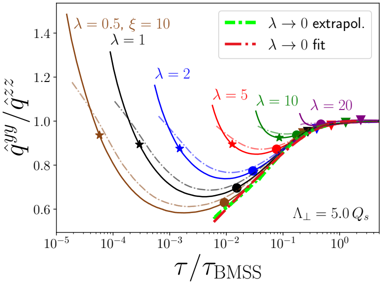
- Impact on jet energy loss (BDMPS-Z), jet polarisation, substructure?
  (with F. Lindenbauer, A. Altenburger, L. Hörl + J. Barata, A. Sadofyev)
- Phenomenological impact of $\kappa_i(\tau)$ on heavy quarks/quarkonia?

Limiting attractors

Rescaling with $\tau_R = \frac{4\pi\,\eta/s(\alpha_s)}{T_\epsilon(\tau)}$



Rescaling with $\tau_{\rm BMSS} = \alpha_s^{-13/5}/Q$



KB, Kurkela, Lappi, Lindenbauer, Peuron, Phys. Lett. B (2024) [2312.11252]

- Limiting attractors from extrapolating coupling $\lambda = 4\pi N_c \alpha_s$
- Hydrodynamic lim. attr. ($\lambda \to \infty$): very good description of P_L/P_T
- Bottom-up lim. attr. ($\lambda \to 0$): early description of $\hat{q}^{yy}/\hat{q}^{zz}$, κ_T/κ_z

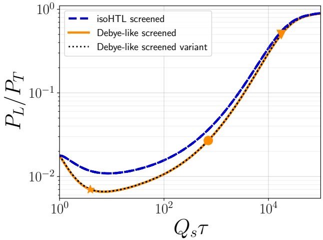
$\Rightarrow$ some observables better described with bottom-up even at $\lambda \sim \mathcal{O}(10)$

Improving QCD kinetic theory for gluonic plasmas

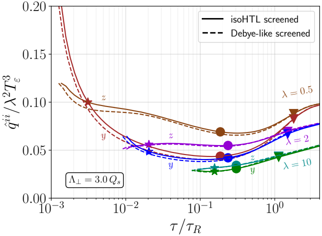
Does soft-gluon exchange matter (improving screening prescription)?

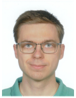
KB, Lindenbauer, [2407.09605]

Yes, at early times



Not really for $\hat{q}$





Lindenbauer

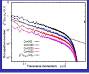
$$\mathcal{C}^{2\leftrightarrow 2}[f_{\vec p}] = \frac{1}{4|\vec p|\nu_g}\int_{kp'k'} |\mathcal{M}|^2 (2\pi)^4 \delta^{(4)}(P+K-P'-K')\left(f_{\vec p} f_{\vec k}(1+f_{\vec p'})(1+f_{\vec k'}) - f_{\vec p'} f_{\vec k'}(1+f_{\vec p})(1+f_{\vec k})\right)$$

- Screening in $\frac{|\mathcal{M}|^2}{4\lambda^2 d_A} = 9 + \frac{(t-s)^2}{\underline{u^2}} + \frac{(s-u)^2}{\underline{t^2}} + \frac{(u-t)^2}{s^2}$ (Arnold, Moore, Yaffe (2003))
- Debye-like: self-energy approx. by m_D (Abraao York, Kurkela, Lu, Moore (2014))
- isoHTL: using full (isotropic) HTL (Braaten, Pisarski (1990))

Table of Contents

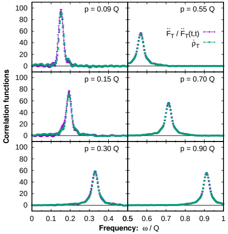
1. Motivation
2. Universality during initial stages
3. Hard probes and transport coefficients
4. Excitations and a new transport peak
5. Conclusion

What excitations drive the dynamics in the QGP?

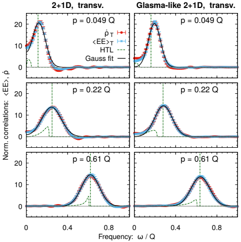


Study microscopics of the Quark-Gluon plasma

- *Spectral functions* $\rho(t,\omega,p) \sim \langle[\hat{A},\hat{A}]\rangle$ encode excitation spectrum!
- Compute $\langle EE\rangle$ in class.-stat. + algorithm for ρ (KB, Kurkela, Lappi, Peuron (2018))
- Generalized FDR observed $\langle EE\rangle \sim \omega\rho$

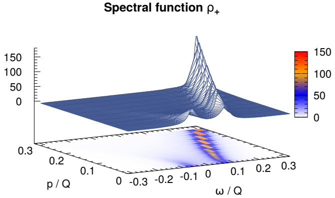


Gluonic 3+1D



Gluonic 2+1D

KB, Kurkela, Lappi, Peuron (2018, 2019, 2021)



Fermionic 3+1D

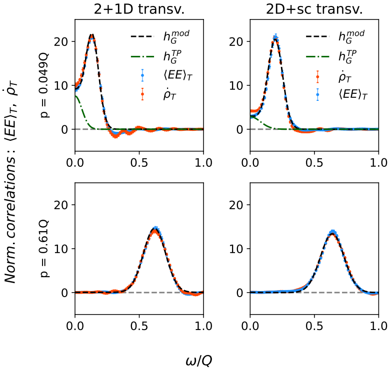
KB, Lappi, Mace, Schlichting (2022)

- Gauge fixed: temporal $A_0 = 0$ + Coulomb-type $\partial^j A_j\big|_t = 0$

Gluonic 2+1D: New transport peak in Glasma-like systems

L. Backfried, KB, P. Hotzy, *in preparation*

Transv. polarization (w.r.t. $\vec{p}$)



Models (non-exp. geometry)

- 2+1D: Yang-Mills $S_{\mathrm{YM}}^{\mathrm{2D}}$
- 2D+sc: $S_{\mathrm{YM}}^{\mathrm{2D}}$ + adj. scalar A_z

$\Rightarrow$ Glasma-like but at classical attractor + Minkowski

extending [KB, Kurkela, Lappi, Peuron (2019, 2021)]

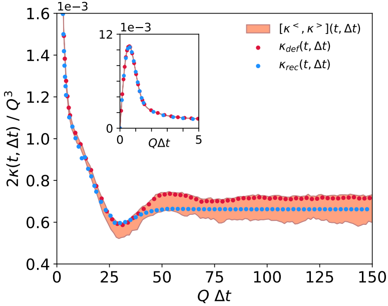
Backfried

Hotzy

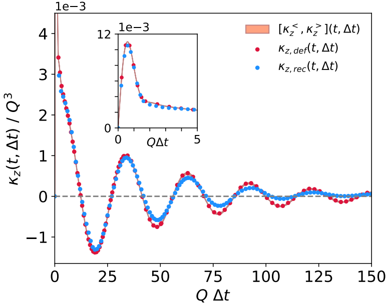
- HTL perturbation theory breaks down $\Rightarrow$ broad Gaussian excitations
- New transport peak h_G^{TP} at $\omega = 0$ for $p \lesssim m_D$ $\Rightarrow$ nonperturbative!

Heavy-quark diffusion coefficients in 2+1D plasmas

2+1D gluonic 2κ



Glasma-like scalar κ_z



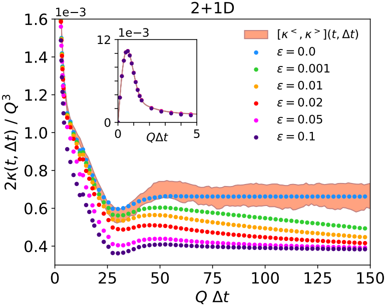
$$2\kappa(t,\Delta t) \approx \frac{g^2}{N_c}\int_t^{t+\Delta t} \mathrm{d}t' \langle EE\rangle(t,t',\Delta\vec{x}{=}0)\,, \qquad \Rightarrow \text{gauge invariant}$$

$$\approx \frac{g^2}{N_c}\int_{\vec{p}}\int_{-\infty}^{\infty}\frac{\mathrm{d}\omega}{2\pi}\,\frac{\sin(\omega\Delta t)}{\omega}\sum_{\alpha=T,L}\langle EE\rangle_\alpha(t,\omega,p)$$

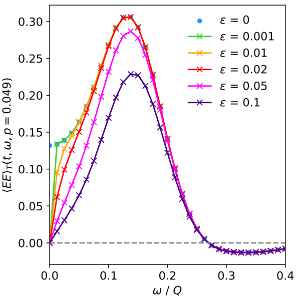
- Gauge-fixed correlators $\langle EE\rangle_\alpha(t,\omega,p)$ reconstruct evolution
- Qualitatively similar to Glasma: 2κ finite (diffusive), κ_z around 0

Manipulate correlations ⇒ study impact

2+1D gluonic 2κ



Suppress low ω of $\langle EE \rangle_T$

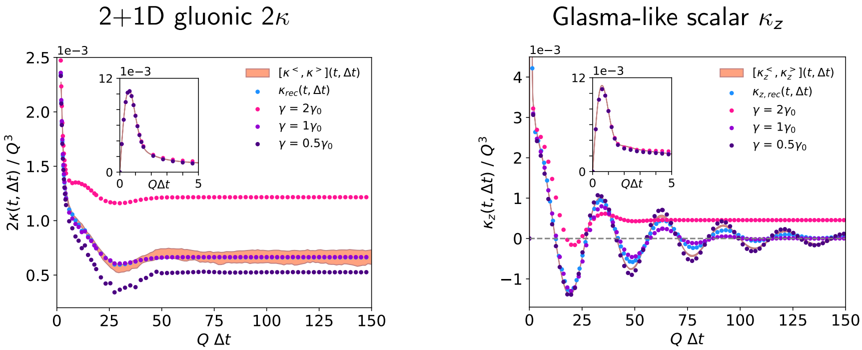


$$2\kappa(t,\Delta t) \approx \frac{g^2}{N_c} \int_{\vec{p}} \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi} \frac{\sin(\omega \Delta t)}{\omega} \sum_{\alpha=T,L} \langle EE \rangle_\alpha(t,\omega,p)$$

- Significant impact on late-Δt evolution ⇒ evidence of transport peak!

- Preliminary: transport peak also in Glasma (KB, Hotzy, Müller, *in progress*)
  ⇒ enhanced transport coefficients, relevance for initial stages?

Manipulate correlations ⇒ study impact II



$$2\kappa(t, \Delta t) \approx \frac{g^2}{N_c} \int_{\vec{p}} \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi} \frac{\sin(\omega \Delta t)}{\omega} \sum_{\alpha = T, L} \langle EE \rangle_\alpha (t, \omega, p)$$

- γ is peak width ⇒ 2κ requires broad $\langle EE \rangle_T$ and κ_z narrow $\langle EE \rangle_z$
- We also demonstrate: scalars are enhanced at low $p \lesssim m_D$

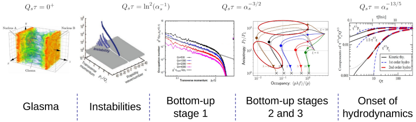
Table of Contents

1. Motivation
2. Universality during initial stages
3. Hard probes and transport coefficients
4. Excitations and a new transport peak
5. Conclusion

Conclusion

- Initial stages of the QGP

  ⇒ waves vs. particles



- Universal attractors during evolution

  ⇒ Help to predict and generalize dynamics

- Hard probes (jets, heavy quarks, quarkonia): medium interactions

  ⇒ Coefficients κ, $\hat{q}$ in Glasma and bottom-up $\rightarrow$ large, anisotropic
  ⇒ EKT: Limiting attractors useful, screening prescription less relevant
  ⇒ Can they unravel signatures of pre-equilibrium dynamics?

- Excitations in Glasma-like plasmas

  ⇒ Evidence of new tranport peak, broad gluonic and narrow scalar peaks
  ⇒ Significant impact on transport coefficients

Thank you for your attention!

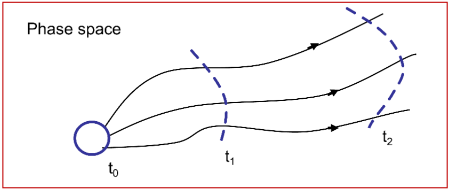
Backup slides

Classical-statistical simulations

- At initial time t_0 set (quantum) initial conditions (IC):

$\Rightarrow$ Choose $\langle AA \rangle$, $\langle EE \rangle$ in x or p space

- Approximate quantum dynamics with classical EOMs $D_\mu F^{\mu\nu} = 0$
  - $\Rightarrow$ Gauge co-variant lattice formulation using links $U_j(x) = e^{ig\, a_j\, A_j(x)}$
- Obtain observables at t by averaging over trajectories (same IC)

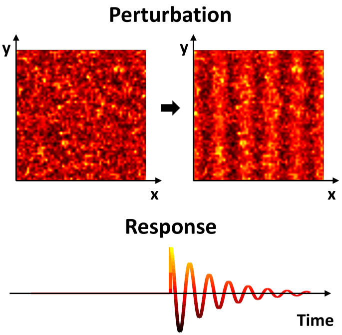


- Valid if occupancies large $f(t,p) \approx \langle AA \rangle p \approx \langle EE \rangle / p \gg 1$
- Applicability limited to earliest times!

Quasiparticles? Extract gluon spectral function ρ

Classical-statistical $\mathrm{SU}(N_c)$ simulations + linear response theory

KB, Kurkela, Lappi, Peuron, *PRD 98, 014006 (2018)*



- Similar algorithm for fermions
- Split $A(t,\vec{x}) \mapsto A(t,\vec{x}) + \delta A(t,\vec{x})$ at t, perturb with plane wave $j_0(\vec{p})\,\delta\,(t'-t)$
- Response $\langle \delta A(t',\vec{p})\rangle = G_R(t',t,\vec{p})\,j_0(\vec{p})$
- Linearized EOM for $\delta A(t,\vec{x})$ such that Gauss law conserved (also in gauge-cov. formulation)

  Kurkela, Lappi, Peuron, *EUJC 76 (2016) 688*
- $\theta(t'-t)\ \boxed{\rho(t',t,p)} = G_R(t',t,p)$
- Fourier transform $\rho(\bar{t},\omega,p)$ $\quad (\bar{t} = \frac{1}{2}(t+t'))$

Very similar methods for scalars:

Aarts (2001); Piñeiro Orioli, Berges (2019); Schlichting, Smith, von Smekal (2020); KB, Piñeiro Orioli (2020); ...

Spectral and statistical correlation functions

- Equal-time correlator $\langle\{\hat{E}(t), \hat{E}(t)\}\rangle \propto f(t,p)$ is distribution
  $\Rightarrow$ But what are the relevant excitations?
- Knowledge of spectral function needed ($\dot{\rho} = \partial_t \rho,\ E = \partial_t A$)

$$\dot{\rho}(x,x') = \frac{i}{N_c^2 - 1}\left\langle \left[\hat{E}(x), \hat{A}(x')\right]\right\rangle$$

- Statistical correlator $\langle EE\rangle$ ($\equiv \ddot{F}$) in general independent of $\dot{\rho}$

$$\langle EE\rangle(x,x') = \frac{1}{2(N_c^2 - 1)}\left\langle \left\{\hat{E}(x), \hat{E}(x')\right\}\right\rangle$$

- Fourier transf. in $t - t'$ and $\vec{x} - \vec{x}'$ to frequency ω and momentum $\vec{p}$
  Approximation: normally at fixed $\bar{t} = \frac{1}{2}(t + t')$, we hold $t \approx \bar{t}$
- In classical-statistical simulations

$$\langle EE\rangle(t,t',p) = \frac{1}{N_c^2 - 1}\left\langle E(t,\vec{p})E^*(t',\vec{p})\right\rangle$$

- Gauge: temporal $A_0 = 0$ + Coulomb-type $\partial^j A_j\big|_t = 0$

Perturbative computation: HTL results

- Hard loop (HTL) framework applicable for $m_D/\Lambda \ll 1$;
  in thermal equ. for $g \sim m_D/T \ll 1$, Braaten, Pisarski (1990); Blaizot, Iancu (2002); ...
- In 3+1D $m_D^2 = 4N_c \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \frac{g^2 f(t,p)}{p} \sim g^2 f \Lambda^2 \Rightarrow$ HTL applicable
- In 2+1D soft-soft interactions important

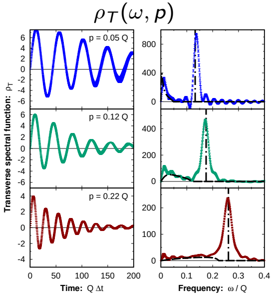
$$m_D^2 \approx d_{\mathrm{pol}} N_{\mathrm{c}} \int \frac{\mathrm{d}^2 p}{(2\pi)^2} \frac{g^2 f(t,p)}{\sqrt{m^2 + p^2}} \sim g^2 f \Lambda \ln\left(\frac{\Lambda}{m_D}\right)$$

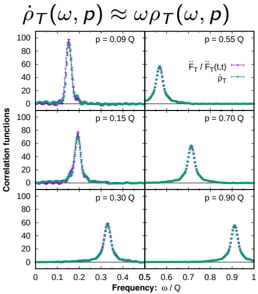
$\Rightarrow$ HTL breaks down already at soft scale $p \sim m_D$

- Comparison to HTL still useful to extract nonperturbative features
- Quasiparticles in $\rho^{\mathrm{HTL}}(\omega, p)$ as $\sim \delta\big(\omega - \omega_\alpha^{\mathrm{HTL}}(p)\big)$
- All expressions depend only on m_D, computed consistently in HTL

Gluon ρ in 3+1D: compare with HTL perturbation theory

KB, Kurkela, Lappi, Peuron, PRD 98, 014006 (2018)





- Narrow Lorentzian q.p. peaks
  (position $\omega(p)$, width $\gamma(p)$)
- HTL at LO (black dashed)
  describes main features well
- Landau cut ($\omega < p$) and q.p. peak distinguishable

- Generalized fluctuation dissipation relation (FDR)

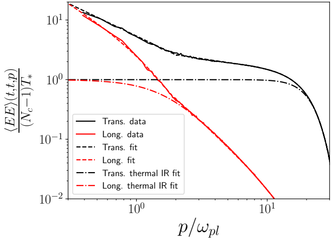
$$\frac{\langle EE\rangle_\alpha(t,\omega,p)}{\langle EE\rangle_\alpha(t,\Delta t{=}0,p)} \approx \frac{\dot\rho_\alpha(t,\omega,p)}{\dot\rho_\alpha(t,\Delta t{=}0,p)}$$

$\ddot F \equiv \langle EE\rangle$, $\alpha = T, L$ polarizations

Deep infrared (IR) of 3+1D gluonic plasmas

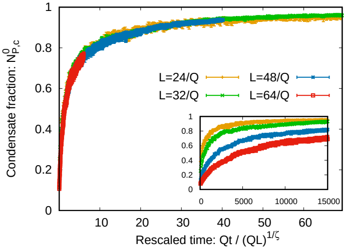
Can we understand the infrared ($p \ll Q$) of the pre-equilibrium QGP?

Excess of IR gluons in $\langle EE \rangle$ correlator for $p \lesssim \omega_{\rm pl}$



KB, Kurkela, Lappi, Peuron, JHEP 09 (2020) 077

Gauge-invariant condensation via spatial Wilson and Polyakov loops



Berges, KB, Mace, Pawlowski, PRD 102 (2020) 034014

Berges, KB, Butler, de Bruin, Pawlowski, PRD (2024)

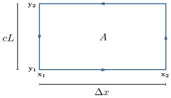
Vision

- *Connections:* IR gluon excess, gauge-inv. cond., B.E. cond. in scalars?

Bose-Einstein condensation in scalars far from equilibrium: Berges, Sexty (2012); Piñerio Orioli, KB, Berges (2015)

Berges, KB, Mace, Pawlowski, PRD 102, 034014 (2020)

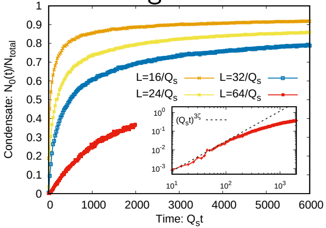
Teaser: another universal (?) phenomenon of scalars and gluons



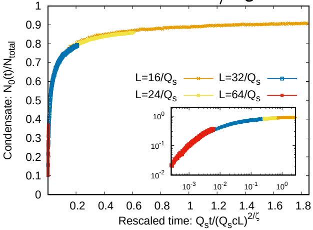
(Spatial) Wilson loop: $W(\Delta x, cL, t) = \frac{1}{N_c} \mathrm{Tr}\, \mathcal{P} e^{-i g \int_{\mathcal{C}[\Delta x, cL]} \mathcal{A}_i(\vec{z},t)\, dz_i}$

Condensate fraction: $\frac{N_0(t, cL)}{N_{\mathrm{total}}} \equiv \frac{1}{(cL)^d} \int_0^{cL} d^d \Delta x \, \langle W(\Delta x, cL, t) \rangle$

Original



Rescaled as t/t_c



- ‘Condensate formation time’ $t_c \sim L^{2/\zeta}$ with universal $\zeta = 0.54 \pm 0.09$
- Based on self-similar evolution of Wilson loops

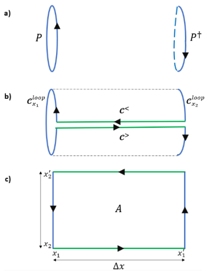
Mace, Schlichting, Venugopalan, PRD 93, 074036 (2016); Berges, Mace Schlichting, PRL 118, 192005 (2017)

Gauge-invariant condensation for different order parameters

- Correlations of spatial Polyakov loops P like $\langle W \rangle$

  Berges, KB, Butler, de Bruin, Pawlowski, PRD (2024)

- Order parameters via $\langle W \rangle \sim \langle PP \rangle \sim \langle \phi\phi \rangle$
- Far-from-equilibrium gauge-inv. condensation

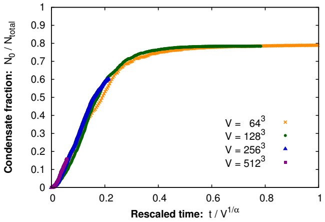


- Bose-Einstein condensation in scalars

  Berges, Sexty (2012); Piñerio Orioli, KB, Berges (2015)

$$\frac{N_0^{\phi}(t)}{N_{\text{total}}^{\phi}} = \frac{1}{V}\int_0^L d^dx \, \frac{\langle\{\phi(t,x),\phi^\dagger(t,0)\}\rangle}{\langle\{\phi(t,0),\phi^\dagger(t,0)\}\rangle}$$

- $t_c \sim L^{1/\beta}$, universal $\beta = 0.55 \pm 0.03$



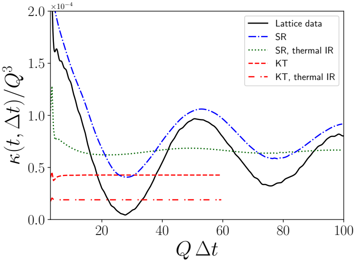
$\Rightarrow$ very similar, intriguing phenomenon, but still many questions

Gauge-invariant observation of IR gluon excess in 3+1D

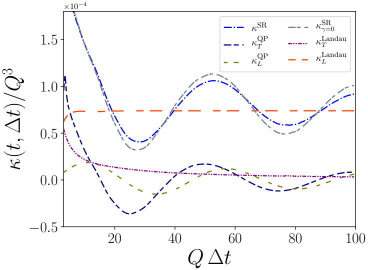
KB, Kurkela, Lappi, Peuron, JHEP 09 (2020) 077

$$\kappa\,(t,\Delta t) \approx \frac{g^2}{3N_c}\int \frac{\mathrm{d}^3 p}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{\mathrm{d}\omega}{2\pi}\,\frac{\sin(\omega\,\Delta t)}{\omega} \times \left[2\langle EE\rangle_T(t,t,p)\frac{\dot\rho_T(t,\omega,p)}{\dot\rho_T(t,t,p)} + \langle EE\rangle_L(t,t,p)\frac{\dot\rho_L(t,\omega,p)}{\dot\rho_L(t,t,p)}\right]$$

Total $\kappa\,(t,\Delta t) \equiv \sum_i \kappa_i\,(t,\Delta t)$



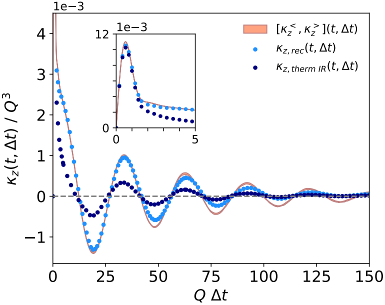
Components $\kappa_i\,(t,\Delta t)$



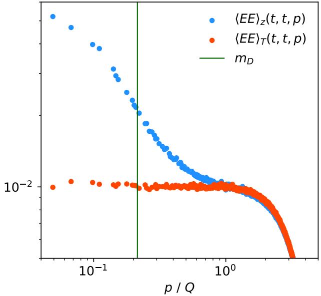
- Oscillations with $\omega_{\rm pl}$ due to QP excitations, sign of IR excess
- Heavy quarks, quarkonia, jets encode nonthermal dynamics of QGP!

2+1D: Manipulate correlations ⇒ study impact III

Glasma-like scalar κ_z



Equal-time correlators $\langle EE \rangle_\alpha(t, t, p)$



$$\kappa_z(t, \Delta t) \approx \frac{g^2}{N_c} \int_{\vec{p}} \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi} \frac{\sin(\omega \Delta t)}{\omega} \langle EE \rangle_z(t, t, p) \frac{\dot{\rho}_z(t, \omega, p)}{\dot{\rho}_z(t, t, p)}$$

- If no infrared excess of scalars, smaller oscillations
  ⇒ evidence of infrared enhancement!