MIXING, MOATS AND MODULATIONS IN DENSE QCD MATTER

Fabian Rennecke

XQCD 2024

LANZHOU - 17/07/2024

PATTERNS

Regularly repeated arrangements; spatial modulations. Ubiquitous in nature

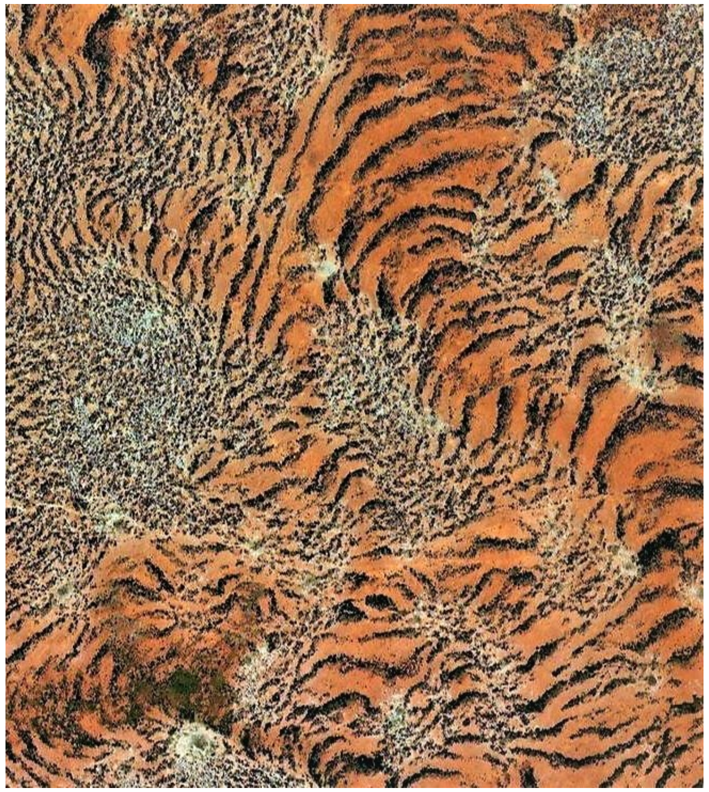
materials

[Robert Fotograf]

animals

[Maksymilian Rose]

vegetation



[Somali Water and Land Information Management]

PATTERN FORMATION

THE CHEMICAL BASIS OF MORPHOGENESIS

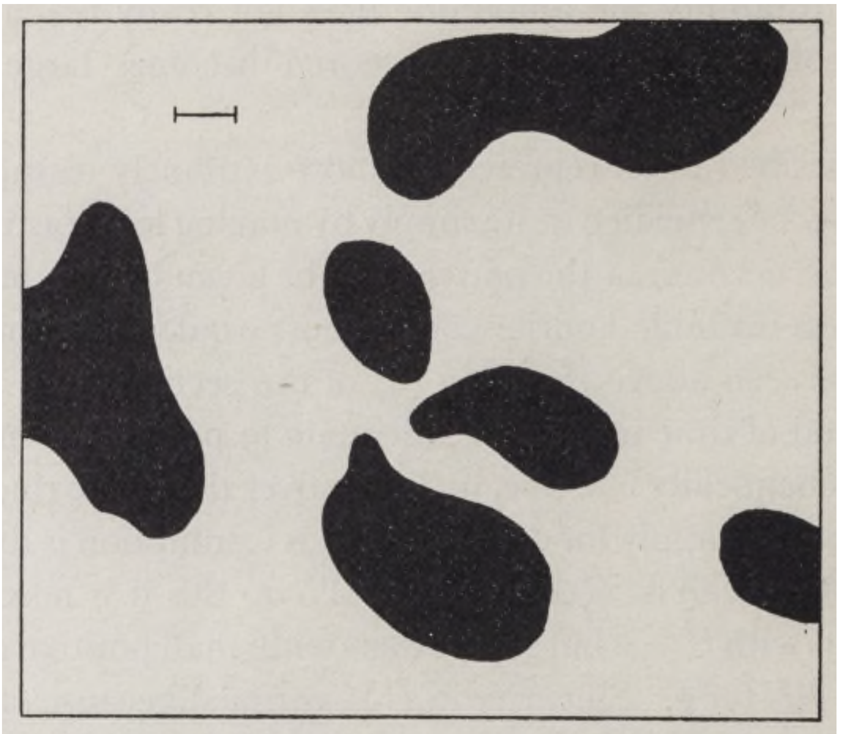
By A. M. TURING, F.R.S. *University of Manchester*

(*Received* 9 *November* 1951—*Revised* 15 *March* 1952)

It is suggested that a system of chemical substances, called morphogens, reacting together and diffusing through a tissue, is adequate to account for the main phenomena of morphogenesis. Such a system, although it may originally be quite homogeneous, may later develop a pattern or structure due to an instability of the homogeneous equilibrium, which is triggered off by random disturbances. Such reaction-diffusion systems are considered in some detail in the case of an isolated ring of cells, a mathematically convenient, though biologically unusual system.

reaction-diffusion equation

$$\partial_t \begin{pmatrix} \phi \\ \chi \end{pmatrix} = \begin{pmatrix} Z_\phi \vec{\nabla}^2 + M_\phi^2 & G_1 \\ G_2 & Z_\chi \vec{\nabla}^2 + M_\chi^2 \end{pmatrix} \cdot \begin{pmatrix} \phi \\ \chi \end{pmatrix}$$

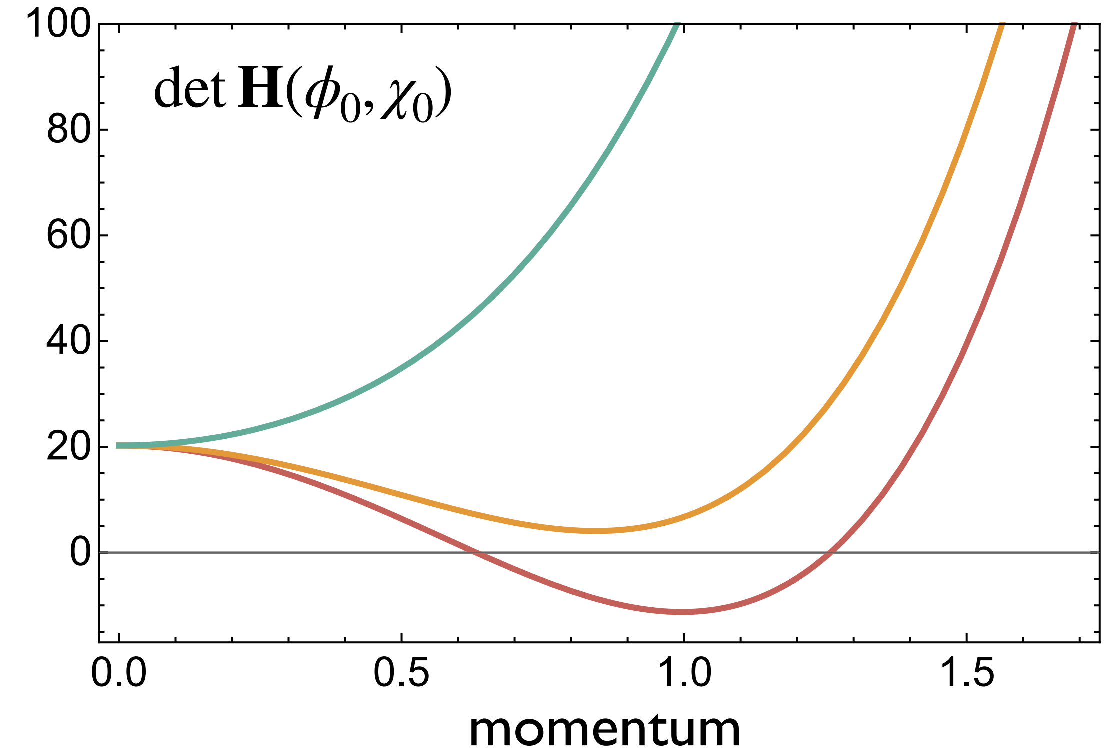


PATTERN FORMATION

Characterize possible solutions through "Hessian" $\mathbf{H}$.

Example: Brusselator model

$$\partial_t \begin{pmatrix} \phi \\ \chi \end{pmatrix} = \mathbf{H}(\phi, \chi) \cdot \begin{pmatrix} \phi \\ \chi \end{pmatrix}$$



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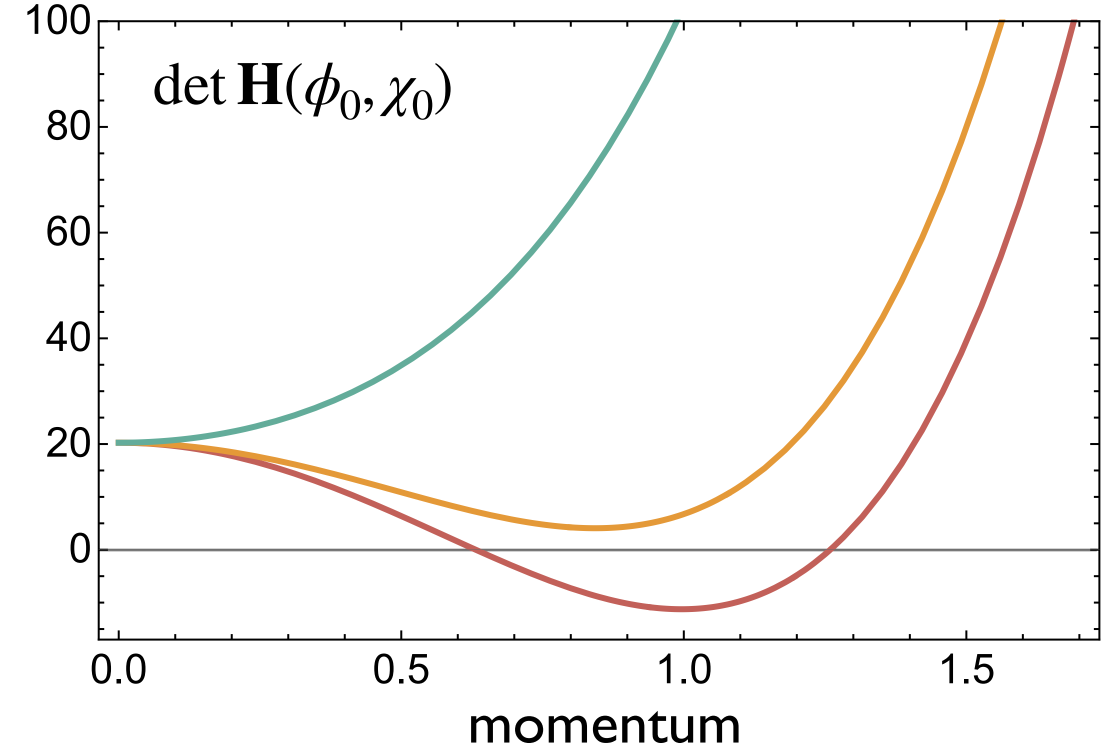
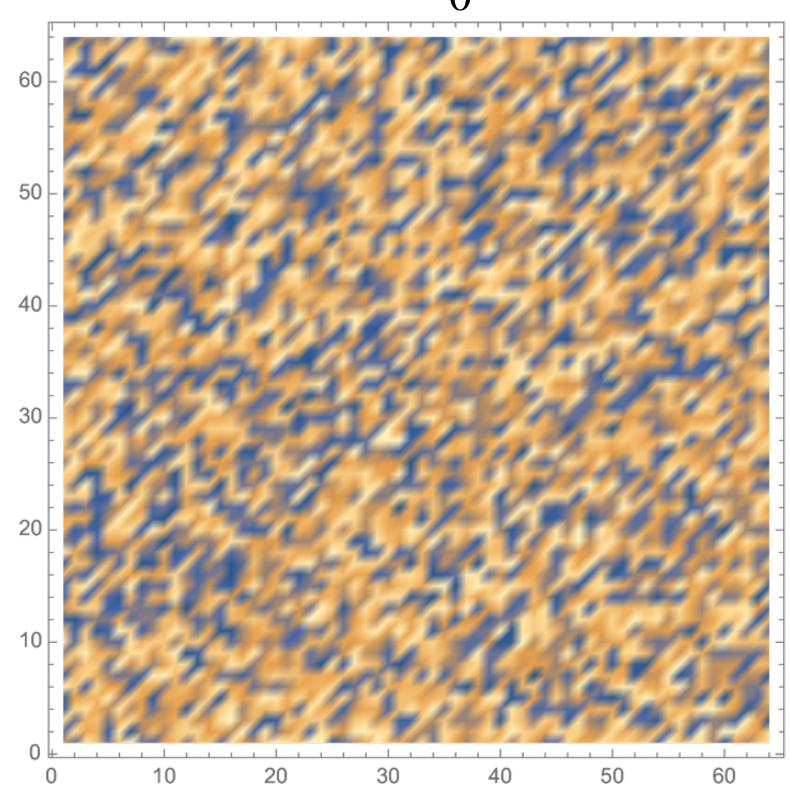


$$\partial_t \begin{pmatrix} \phi \\ \chi \end{pmatrix} = \mathbf{H}(\phi, \chi) \cdot \begin{pmatrix} \phi \\ \chi \end{pmatrix}$$

$t = t_0$

$\phi(x)$



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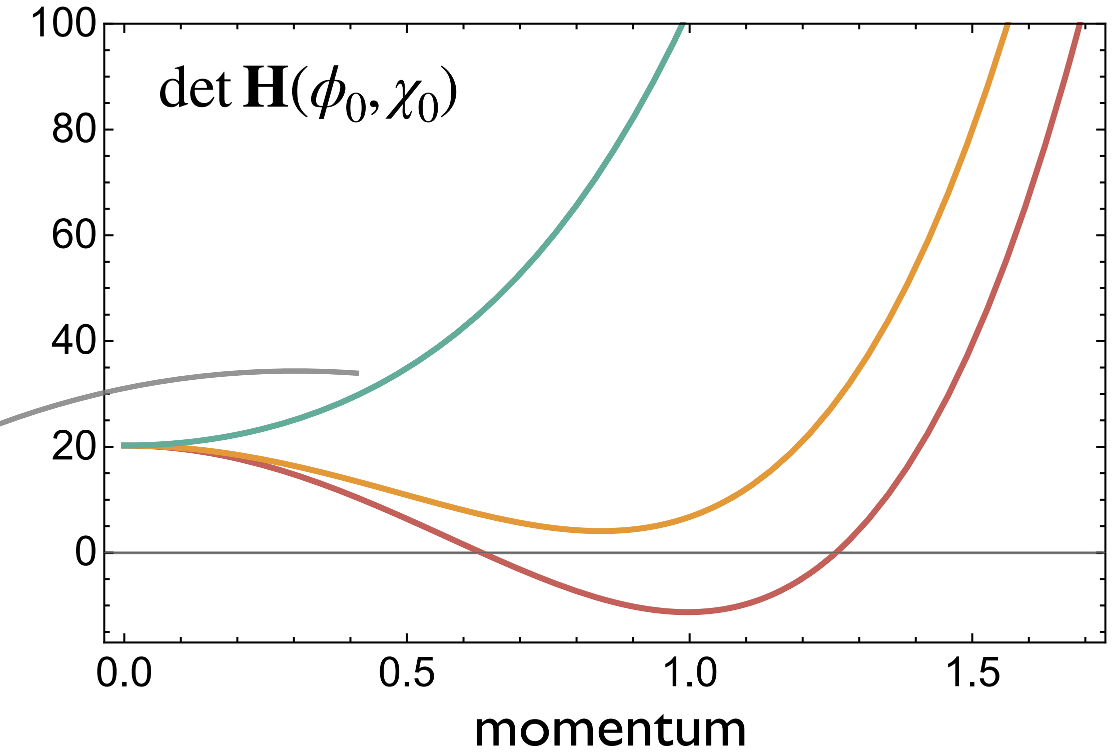

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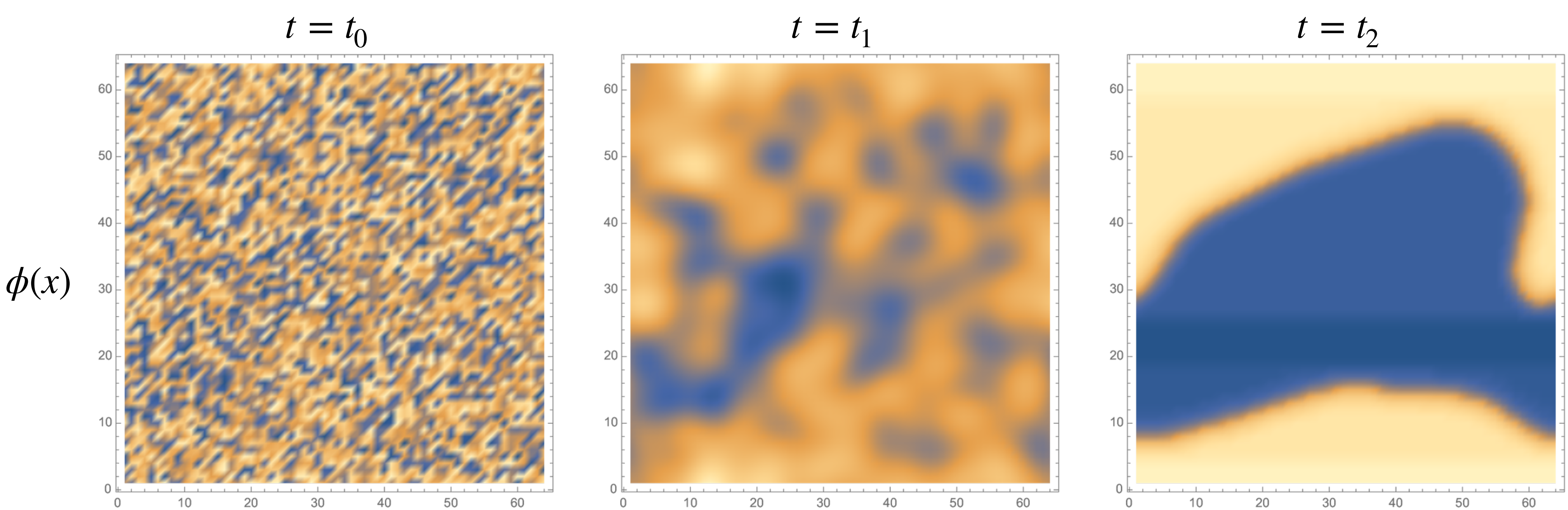
homogeneous phase



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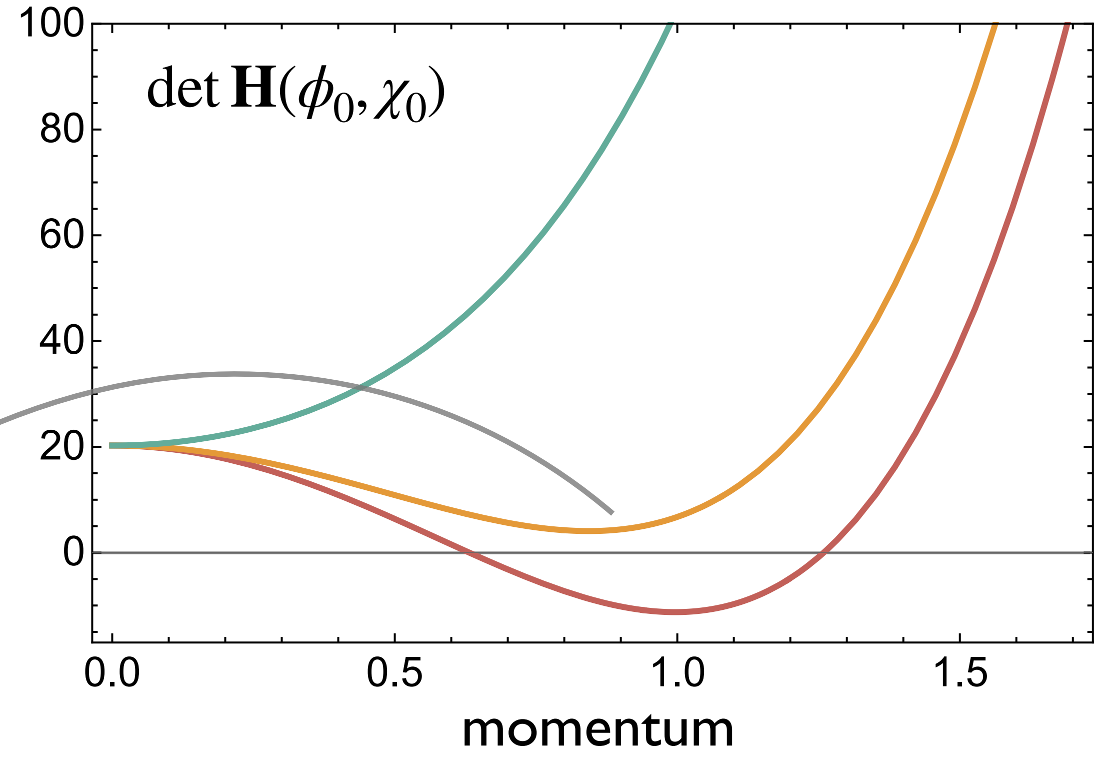
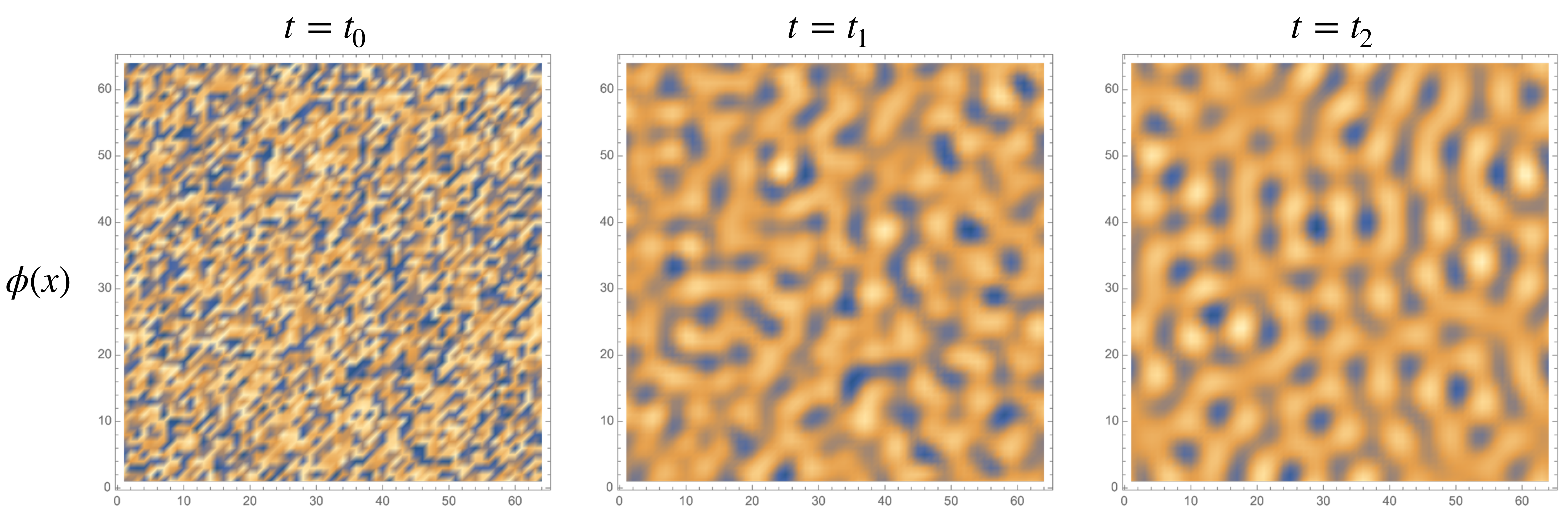


moat regime: "homogeneous pattern"

[Pisarski, FR (2021)]



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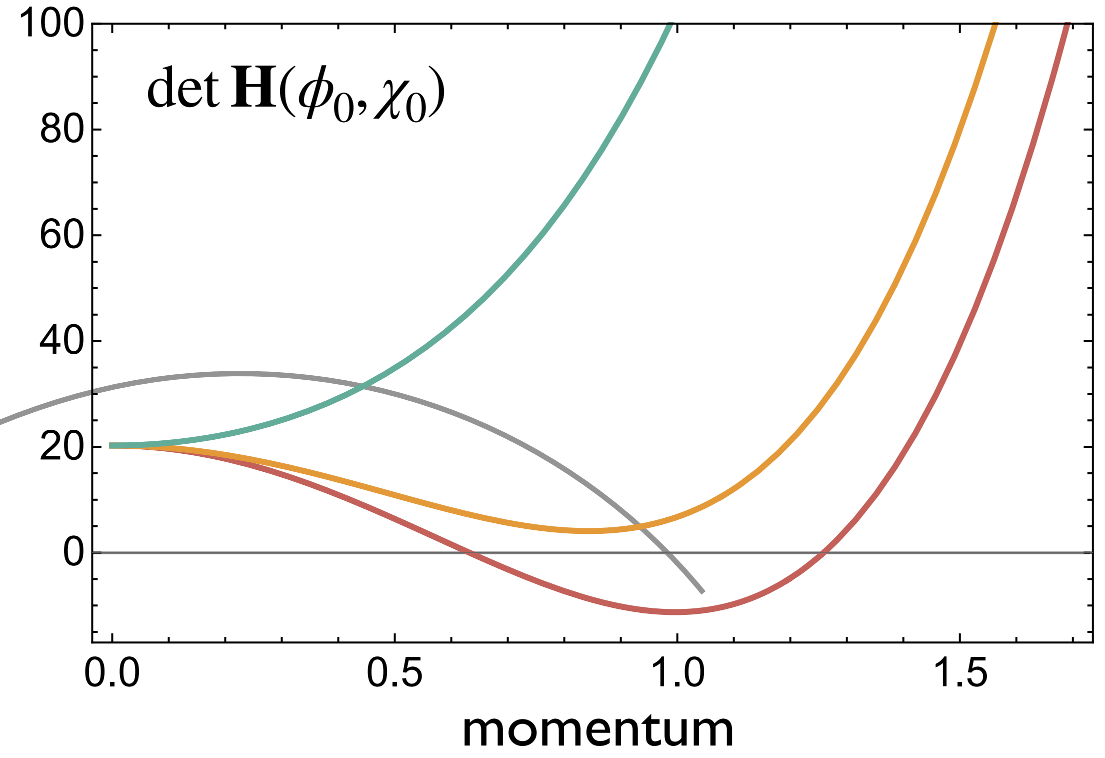
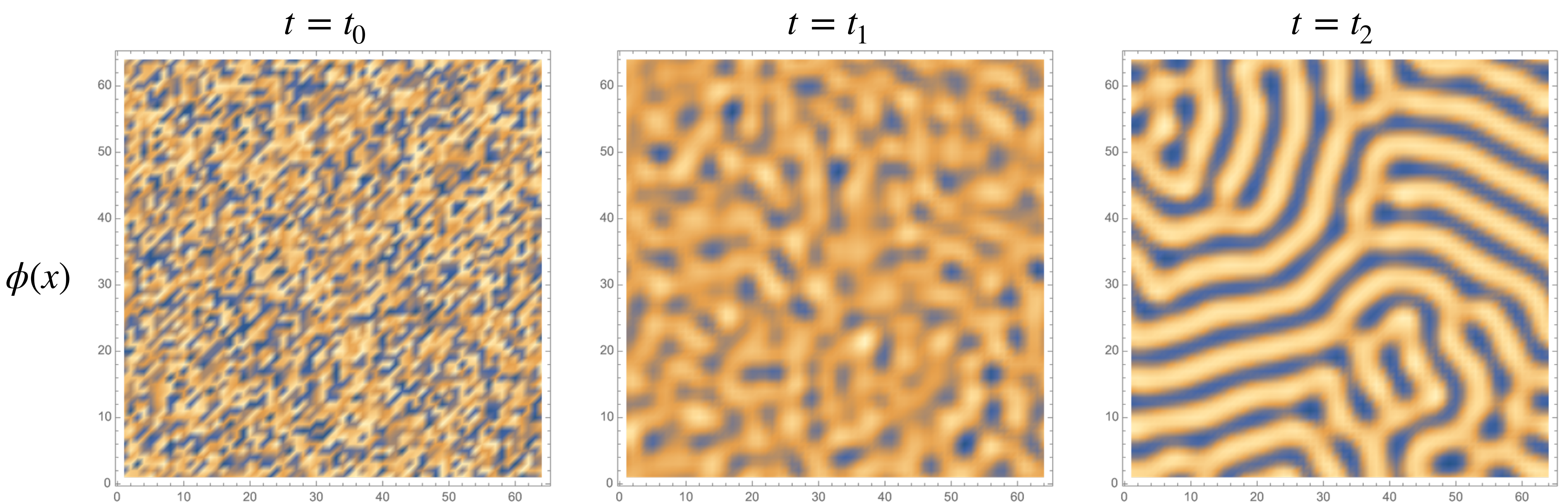


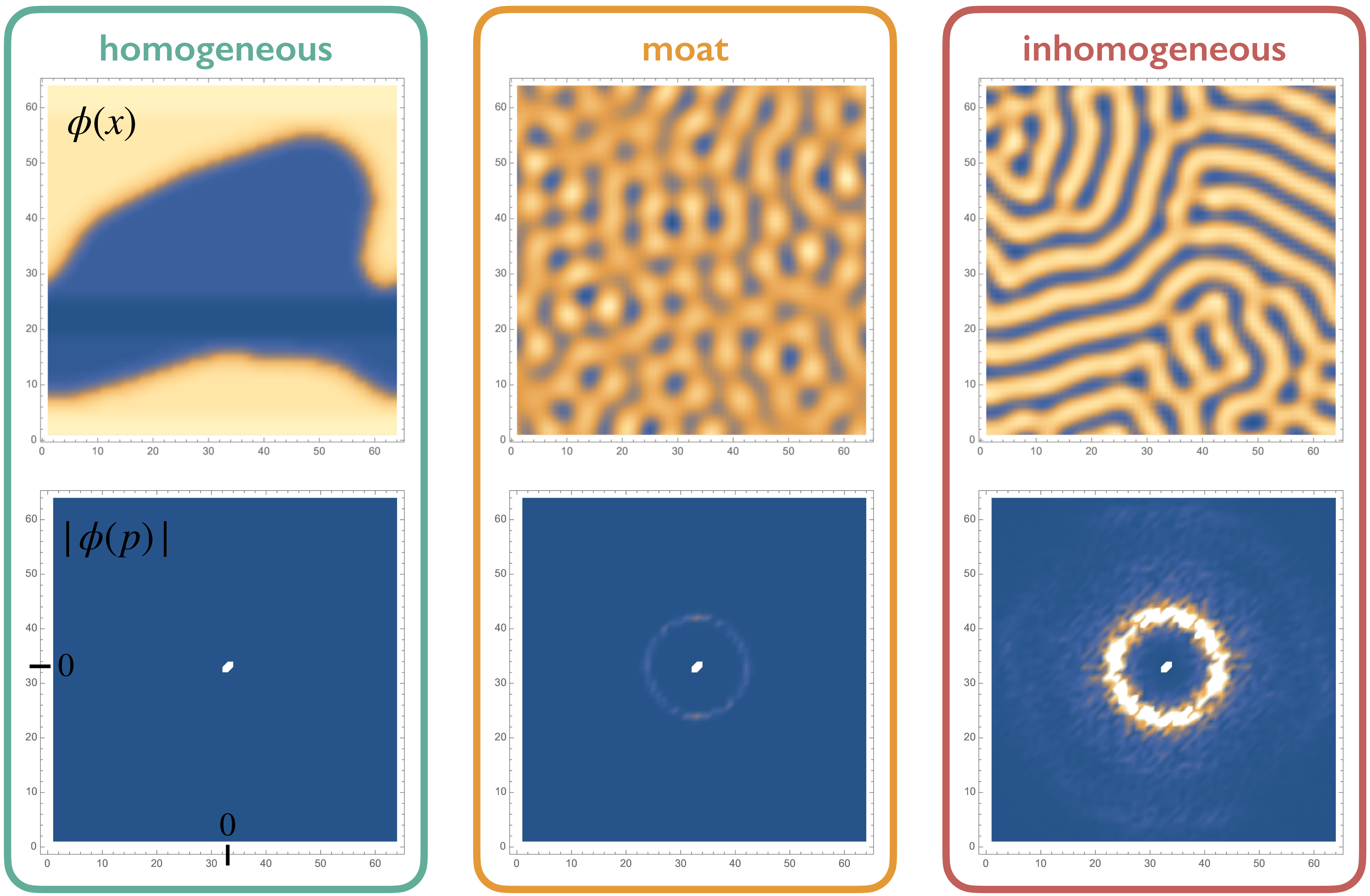
inhomogeneous phase : Turing pattern

instability of homogeneous ground state, $\det \mathbf{H}(\phi_0, \chi_0) \leq 0$



PATTERN FORMATION

in momentum space



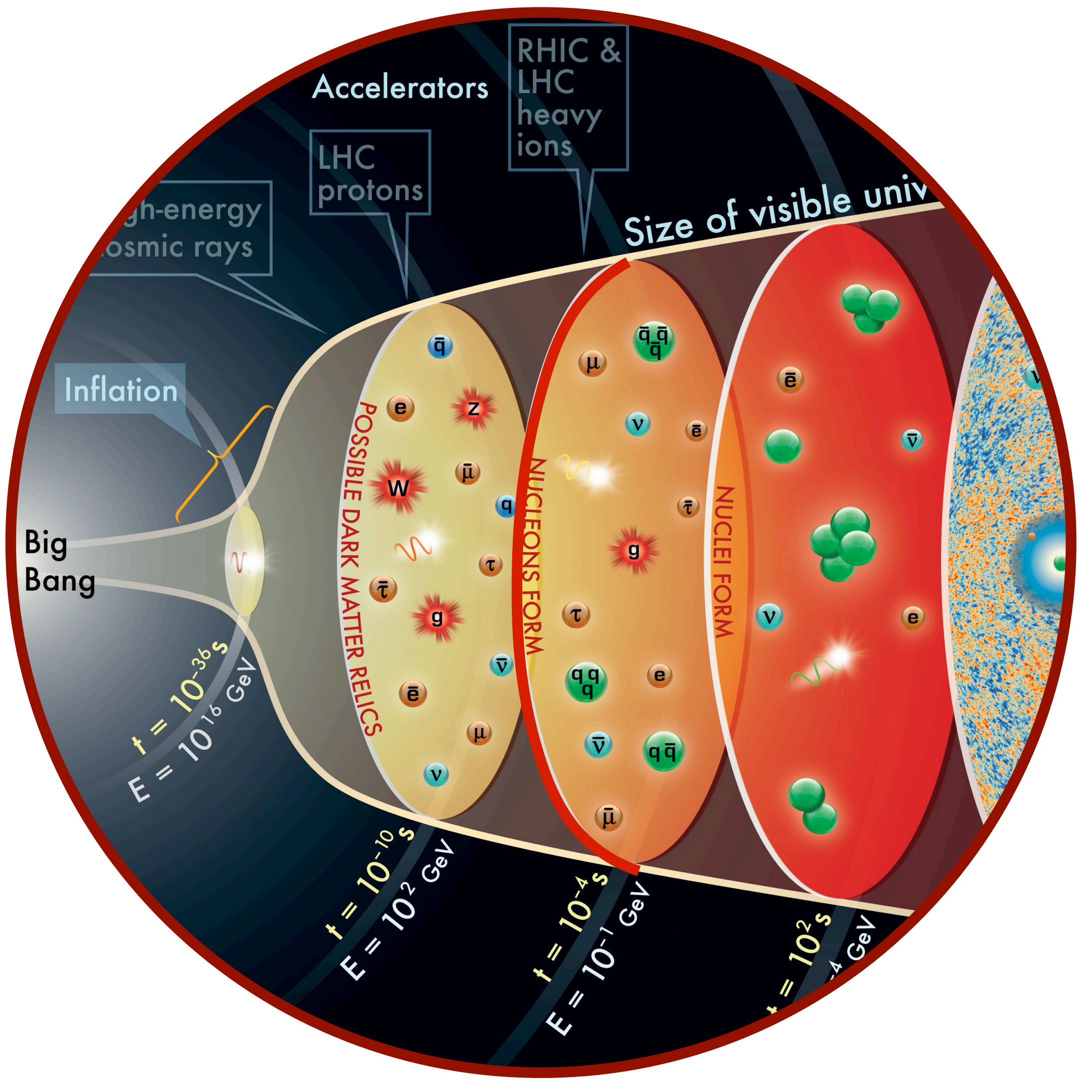
patterns

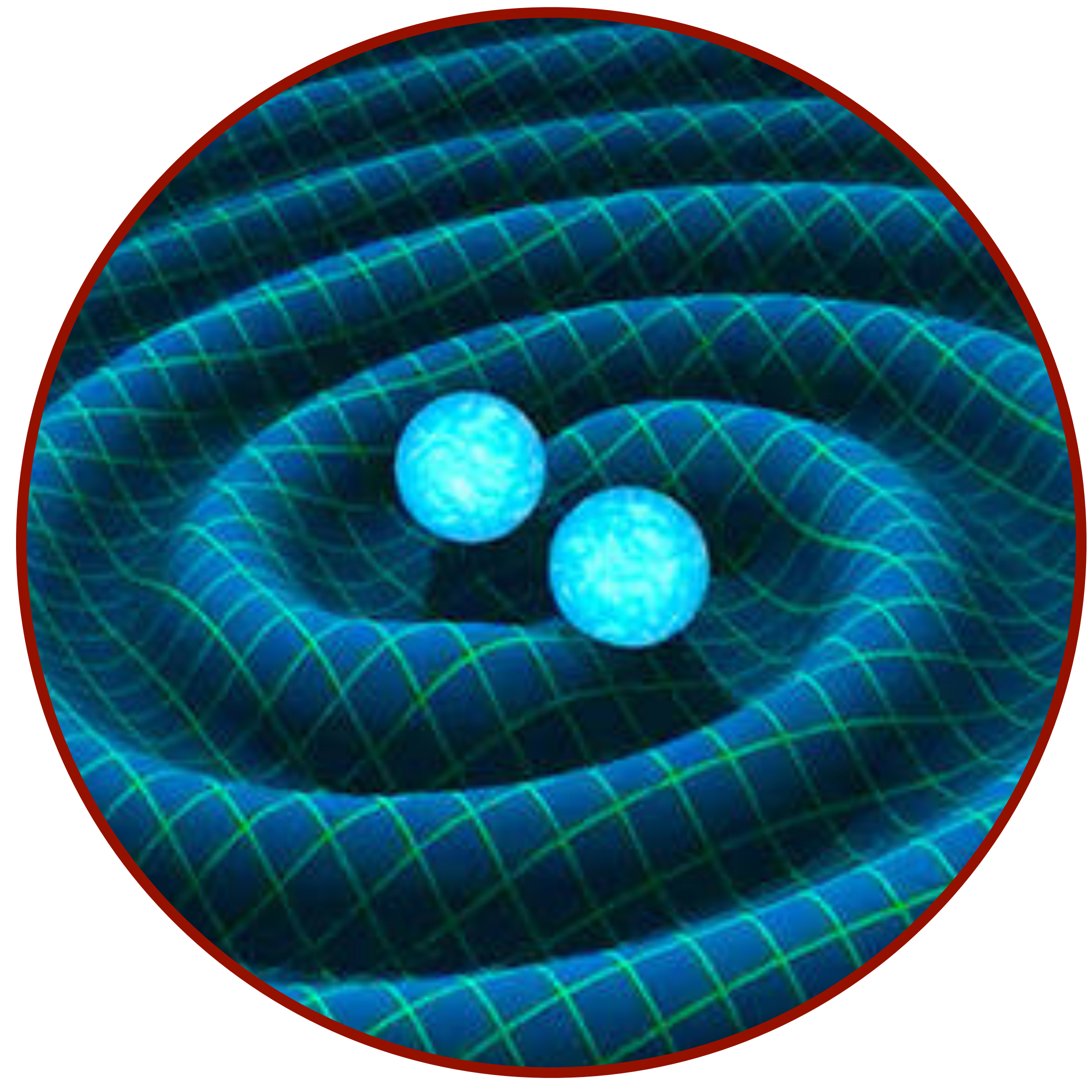
favored momentum (wavenumber)

PATTERNS IN EXTREME CONDITIONS

We are familiar with patterns under (relatively) normal conditions: flora, fauna, crystals, ...

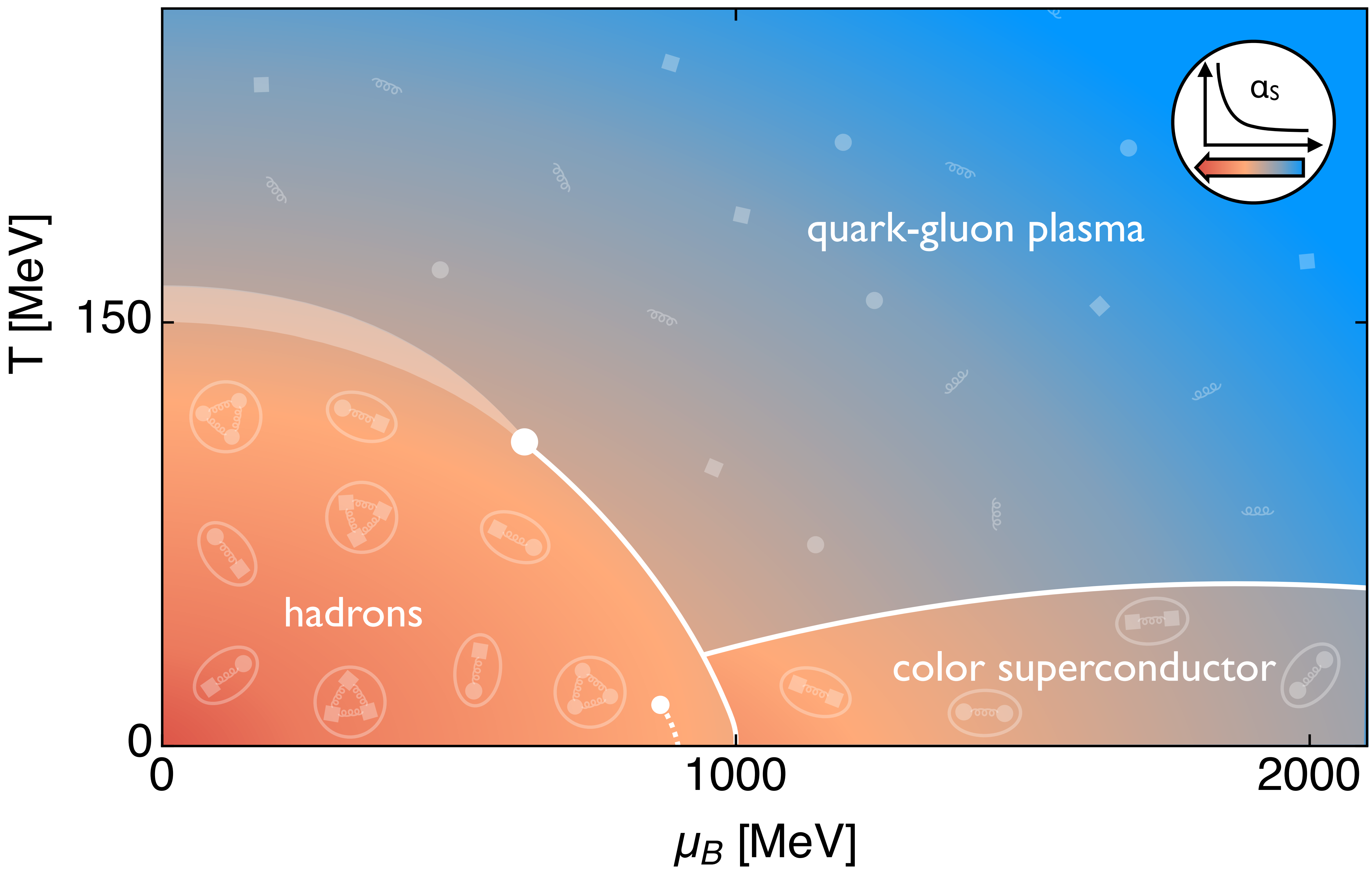
Can they also form under the most extreme conditions in the universe?





QCD PHASE DIAGRAM

in theory:



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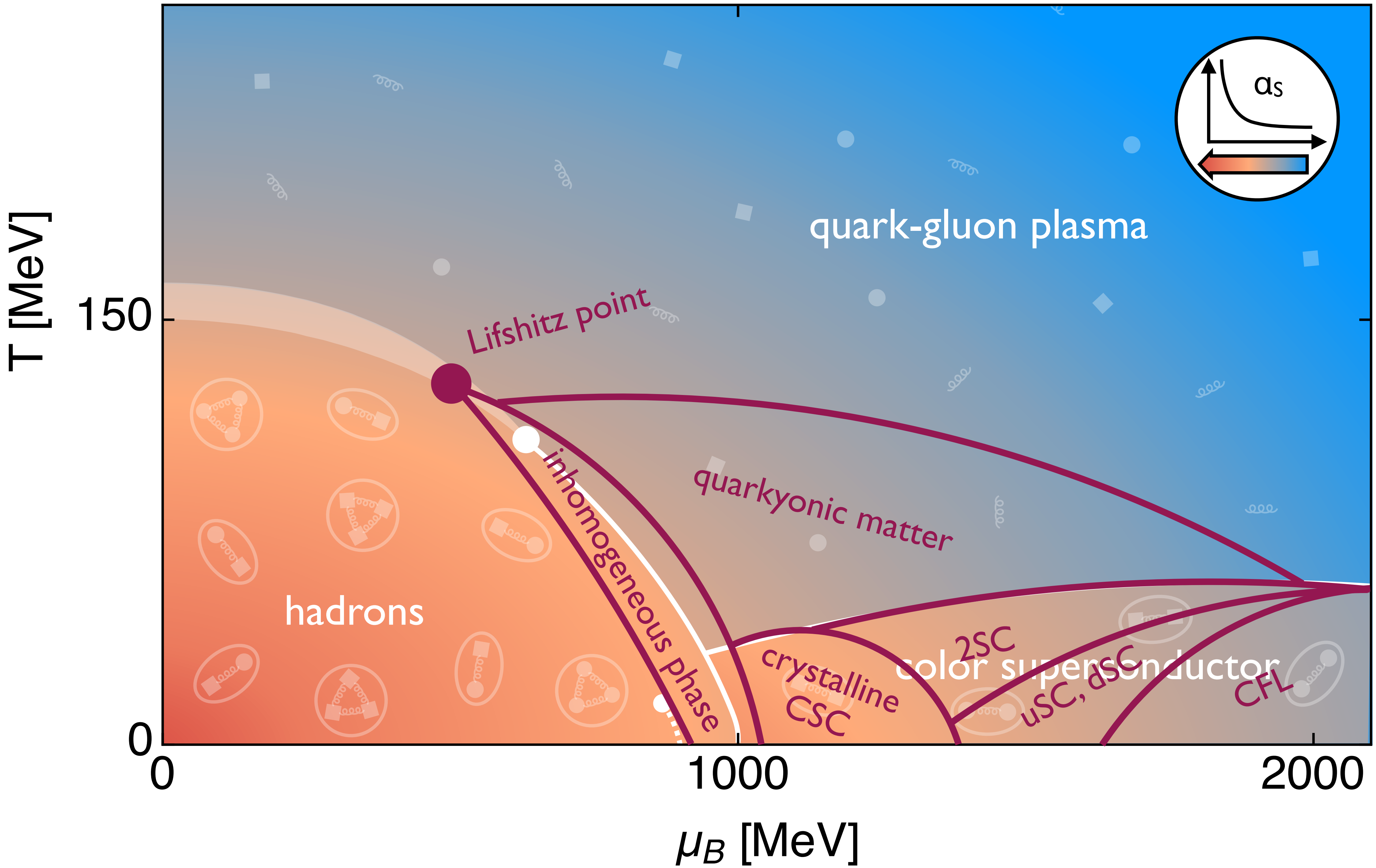

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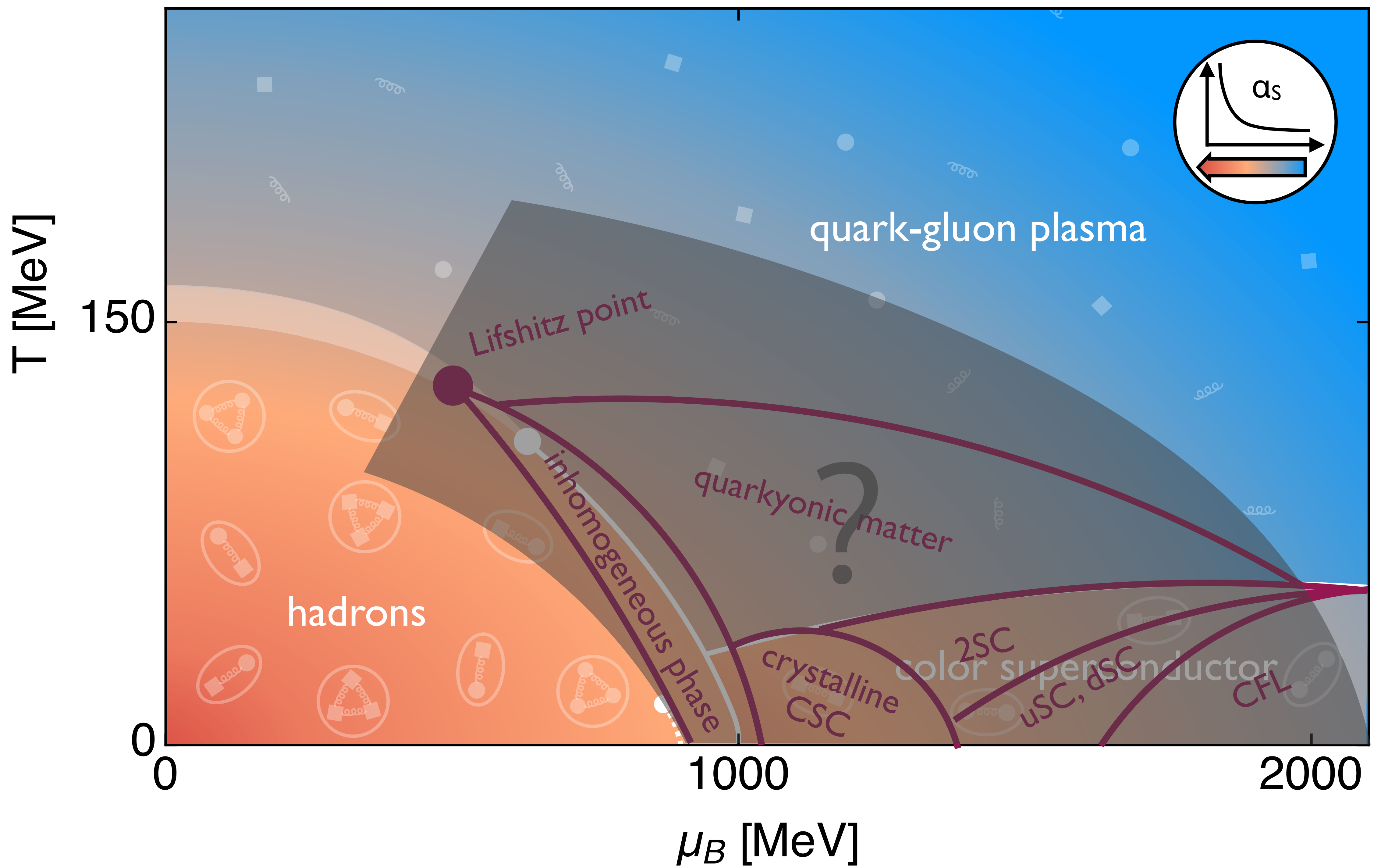
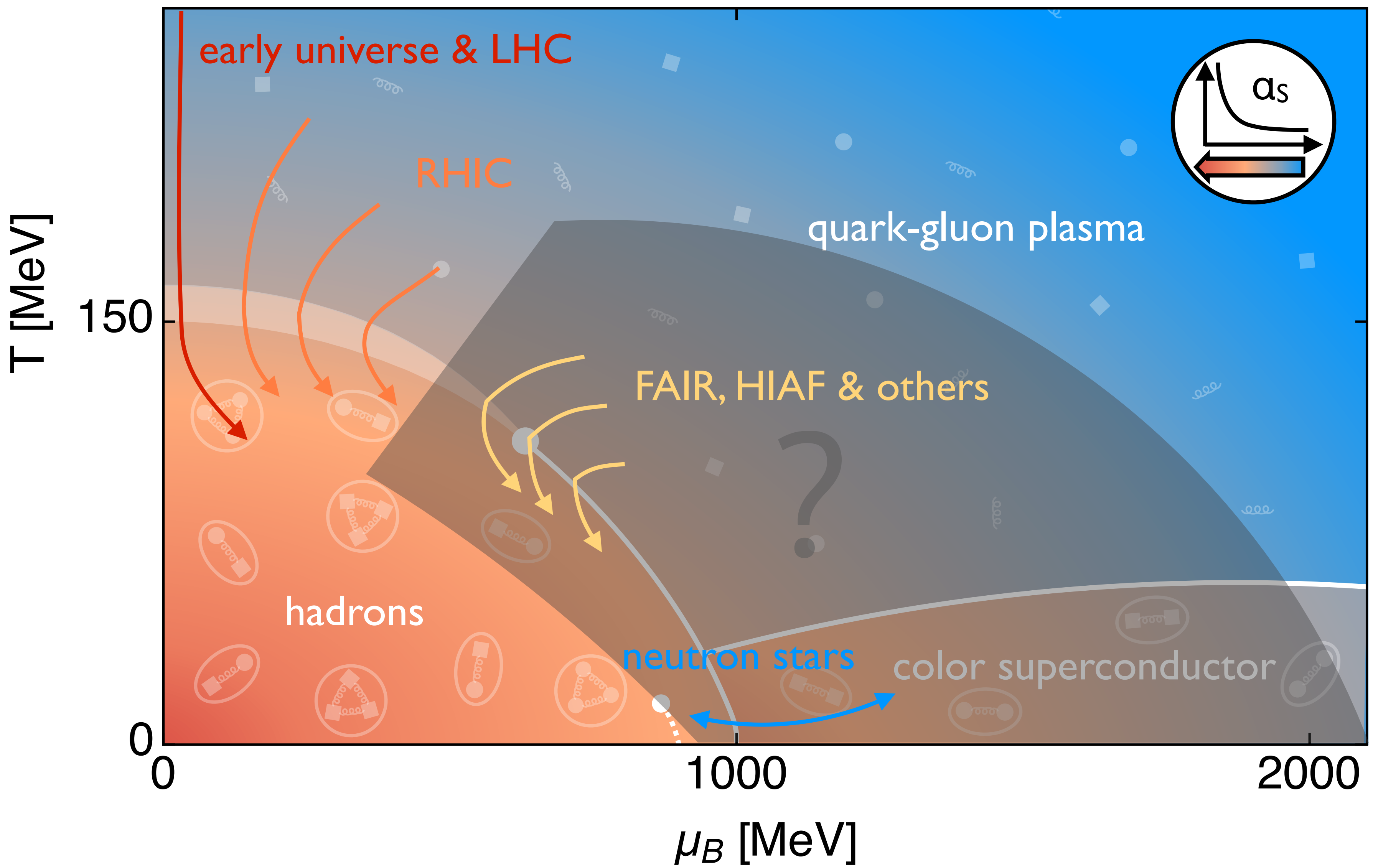


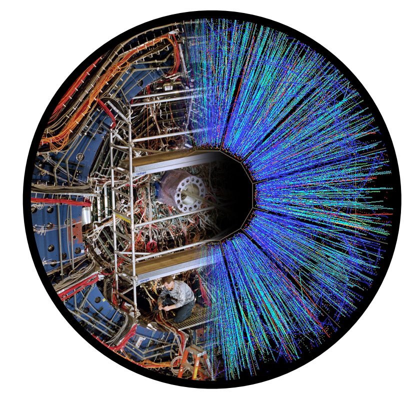
QCD PHASE DIAGRAM

in nature/experiment:

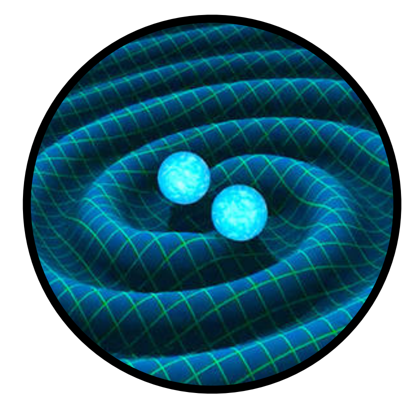


Experiments:

heavy-ion collisions



e.g. gravitational waves



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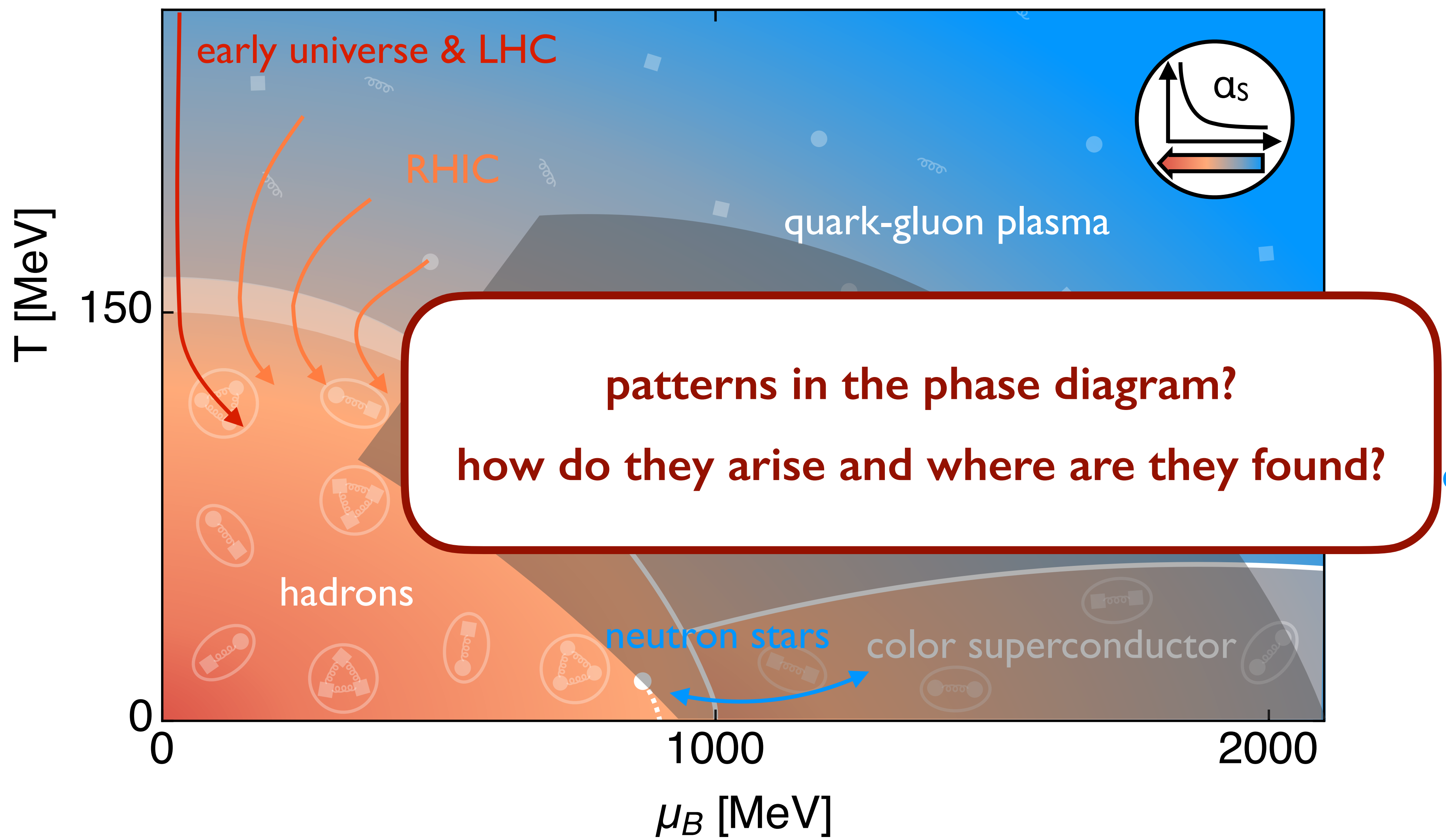
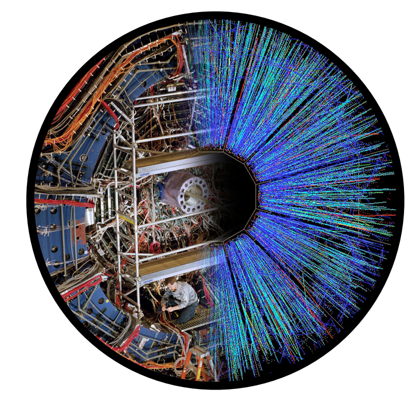


patterns in the phase diagram?

how do they arise and where are they found?

Experiments:

heavy-ion collisions



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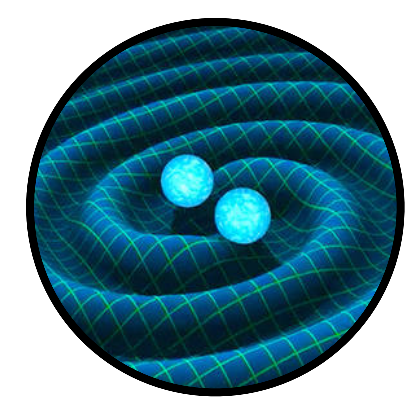
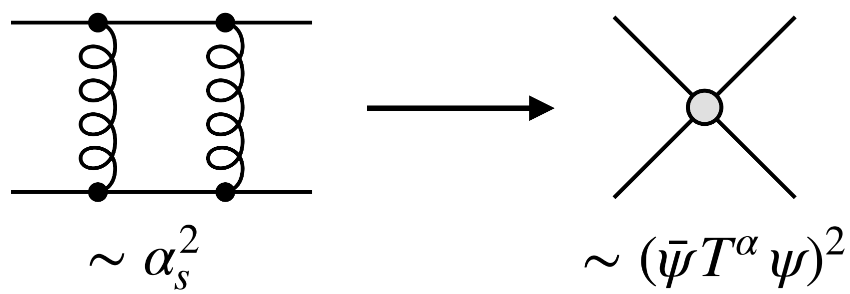


PATTERN FORMATION IN QCD

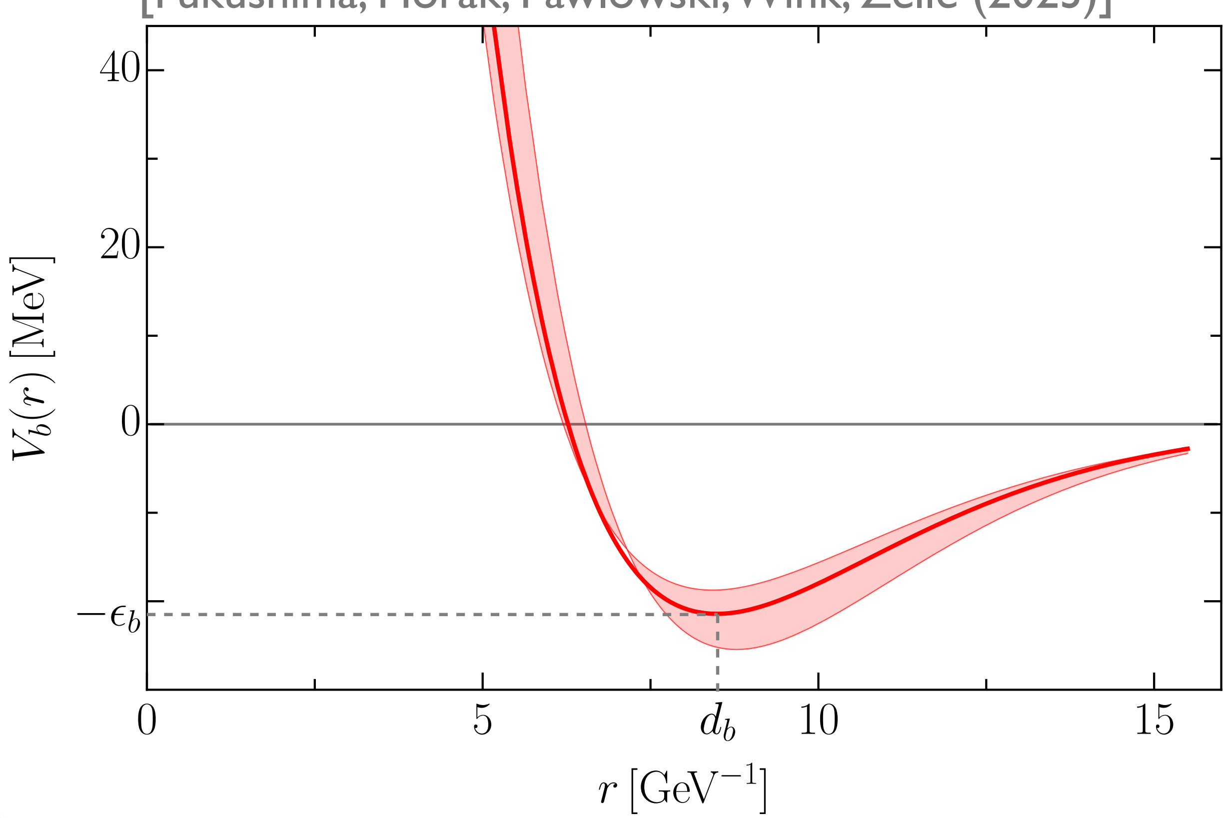
Intuition tells us there are two important ingredients for pattern formation:

competition between attractive and repulsive interactions

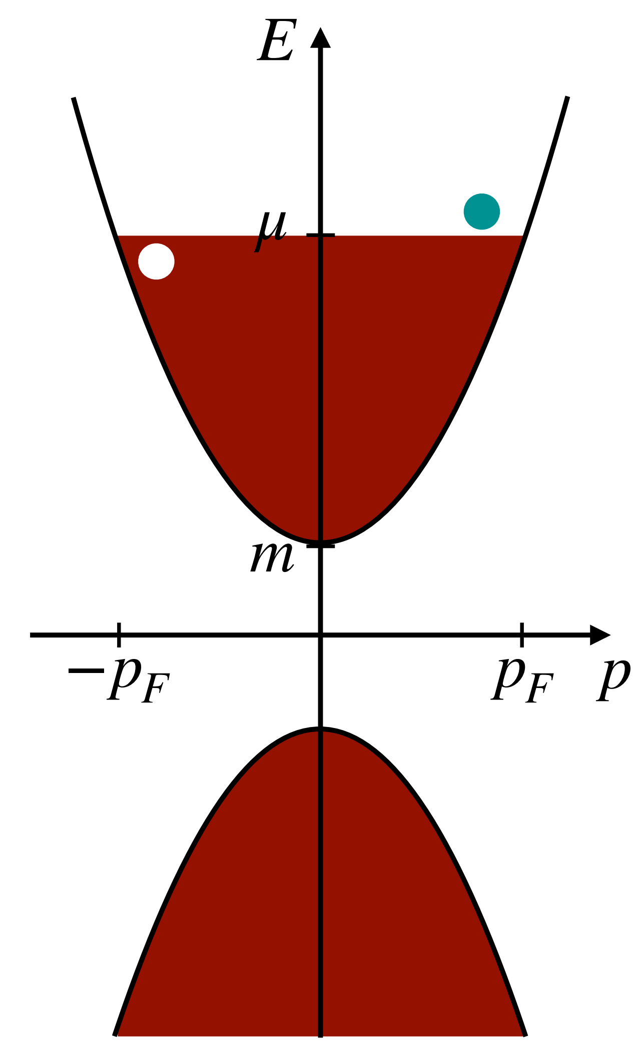


potential between quark-bilinears in the vector channel, $\bar{\psi}\gamma^0\psi$

[Fukushima, Horak, Pawlowski, Wink, Zelle (2023)]



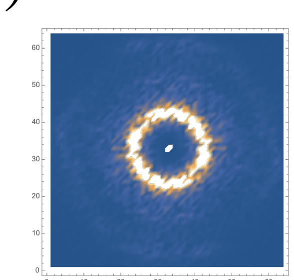
a large Fermi surface



net-momentum excitation around Fermi-surface

$$\bar{\psi}_+(p_F)\,\psi_+(-p_F)$$

spatial modulations
(Friedel oscillations)
[FR, Yin (in preparation)]



DENSE QCD MATTER AND CK SYMMETRY

QCD at finite density:

$$\mathscr{L}_{\text{QCD}} = \bar{q}\left(i\gamma^{\mu}D_{\mu} - m_q\right)q - \frac{1}{4}F_a^{\mu\nu}F_{\mu\nu}^a + i\mu\,\bar{q}\gamma^0 q$$

$$D_{\mu} = \partial_{\mu} - igA_{\mu}^a T^a$$

$$F_{\mu\nu}^a T^a = -\frac{i}{g}\left[D_{\mu}, D_{\nu}\right]$$

- charge conjugation symmetry (C) is broken at finite μ
- retains symmetry under CK (K: complex conjugation)

$$C:\quad \begin{aligned} \bar{q}q &\longrightarrow \bar{q}q \\ \bar{q}\gamma^{\mu}q &\longrightarrow -\bar{q}\gamma^{\mu}q \\ A^{\mu} &\longrightarrow -A^{\mu} \end{aligned}$$

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Intuition from CK-extended ϕ^4-theory:

$$\mathscr{L} = \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m^2\phi^2 - \lambda\phi^4 - \frac{1}{2}(\partial_i\omega^0)^2 - \frac{1}{2}m_\omega(\omega^0)^2 + ig\phi\omega^0$$

- scalar field ϕ, vector field ω^μ
- linear coupling between ϕ and ω^0: **mixing**
- imaginary coupling ig: **repulsion**
- possesses CK symmetry

Non-relativistic EoM is like the reaction-diffusion equation from earlier!

The system has two peculiar features:

non-Hermitian Hessian ($p_0 = 0$)

$$H = \begin{pmatrix} \vec{p}^2 + m^2 & -ig \\ -ig & \vec{p}^2 + m_\omega^2 \end{pmatrix}$$

modified dispersion of ϕ

from integrating-out ω^0

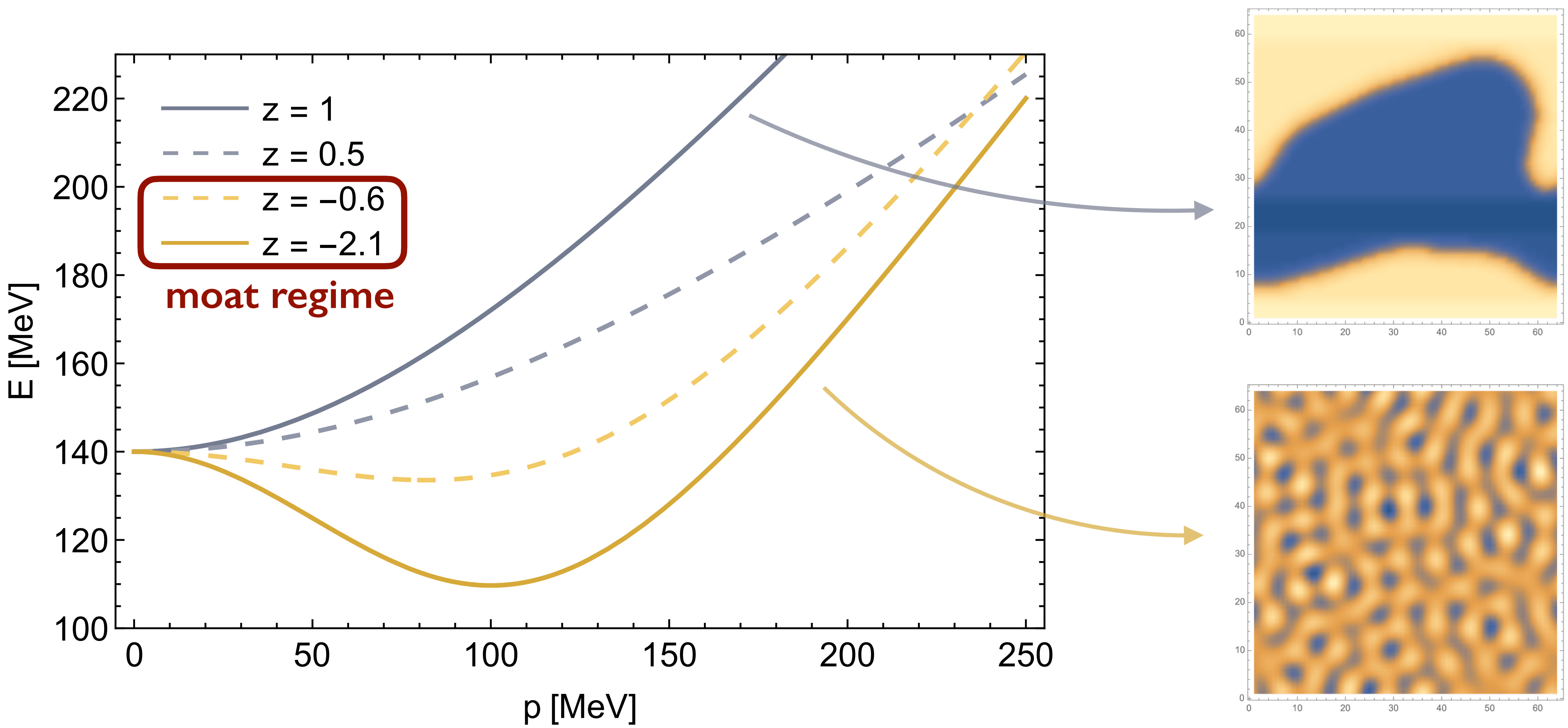
$$E^2(\vec{p}^2) = \vec{p}^2 + m^2 + \frac{g^2}{\vec{p}^2 + m_\omega^2}$$

[Schindler, Schindler, Medina, Ogilvie (2019)]

MODIFIED DISPERSION

in the small-momentum regime:

$$E^2(\vec{p}^{\,2}) = \underbrace{\left(1 - \frac{g^2}{m_\omega^4}\right)}_{z} \vec{p}^{\,2} + \frac{g^2}{m_\omega^6}\vec{p}^{\,4} + m^2 + \frac{g^2}{m_\omega^2} + \mathcal{O}\left(\vec{p}^{\,6}\right)$$



$$E^2 = \det H \longrightarrow$$ $z < 0$ for strong repulsive mixing: **moat regime**

[Pisarski, FR (2021)]

MODIFIED HESSIAN

Diagonalize to get physical degrees of freedom

$$H = \begin{pmatrix} \vec{p}^2 + m^2 & -ig \\ -ig & \vec{p}^2 + m_\omega^2 \end{pmatrix} \longrightarrow \begin{pmatrix} \vec{p}^2 + M_+^2 & 0 \\ 0 & \vec{p}^2 + M_-^2 \end{pmatrix}, \quad M_\pm = \frac{m^2 + m_\omega^2}{2} \pm \frac{1}{2}\sqrt{(m^2 - m_\omega^2)^2 - 4g^2}$$

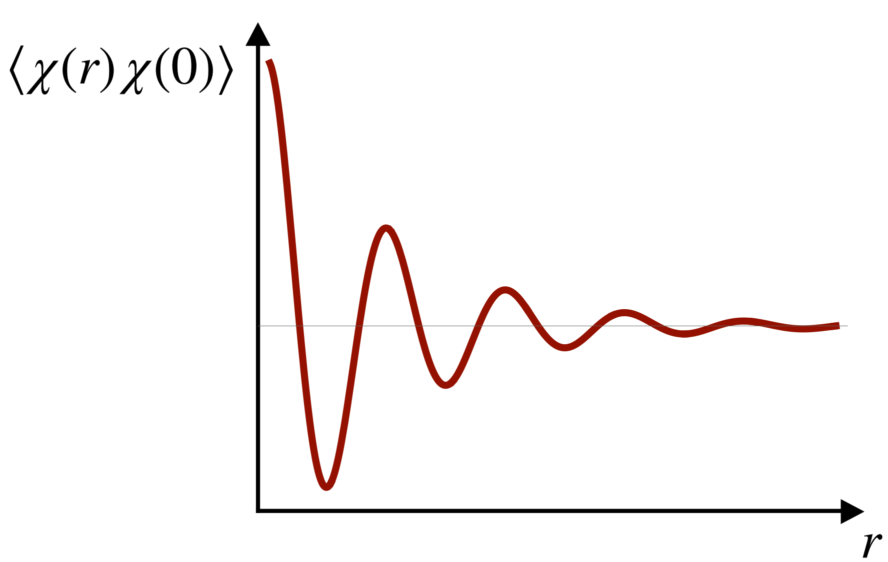
⟶ strong repulsive mixing: eigenvalues/masses come in complex conjugate pairs

(follows from CK symmetry)

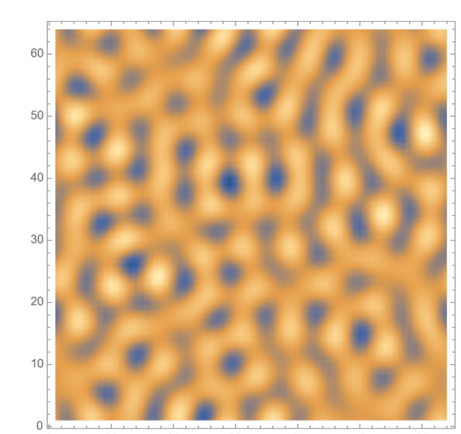
These masses determine screening properties of the physical fields

$$\lim_{r\to\infty} \langle \chi(r)\chi(0) \rangle \sim e^{-Mr}$$

- real M: ordinary exponential decay of disordered fields
- complex M: **spatial modulations** $\sim e^{-\mathrm{Re}[M]\, r} \sin\left(\mathrm{Im}[M]\, r\right)$



⟶ spatial modulations for strong repulsive mixing



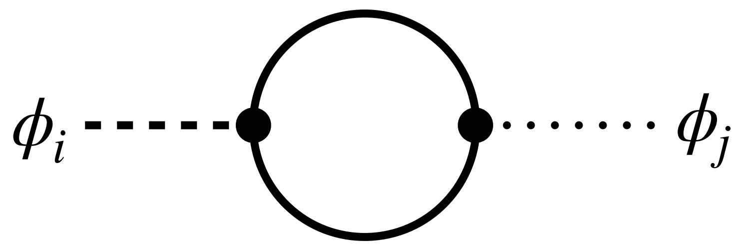
MIXING, MOATS AND MODULATIONS IN QCD

The intuition from this simple model is directly applicable to QCD!

MIXING, MOATS AND MODULATIONS IN QCD

Extensive repulsive mixing in QCD at finite density from fundamental interactions between quarks

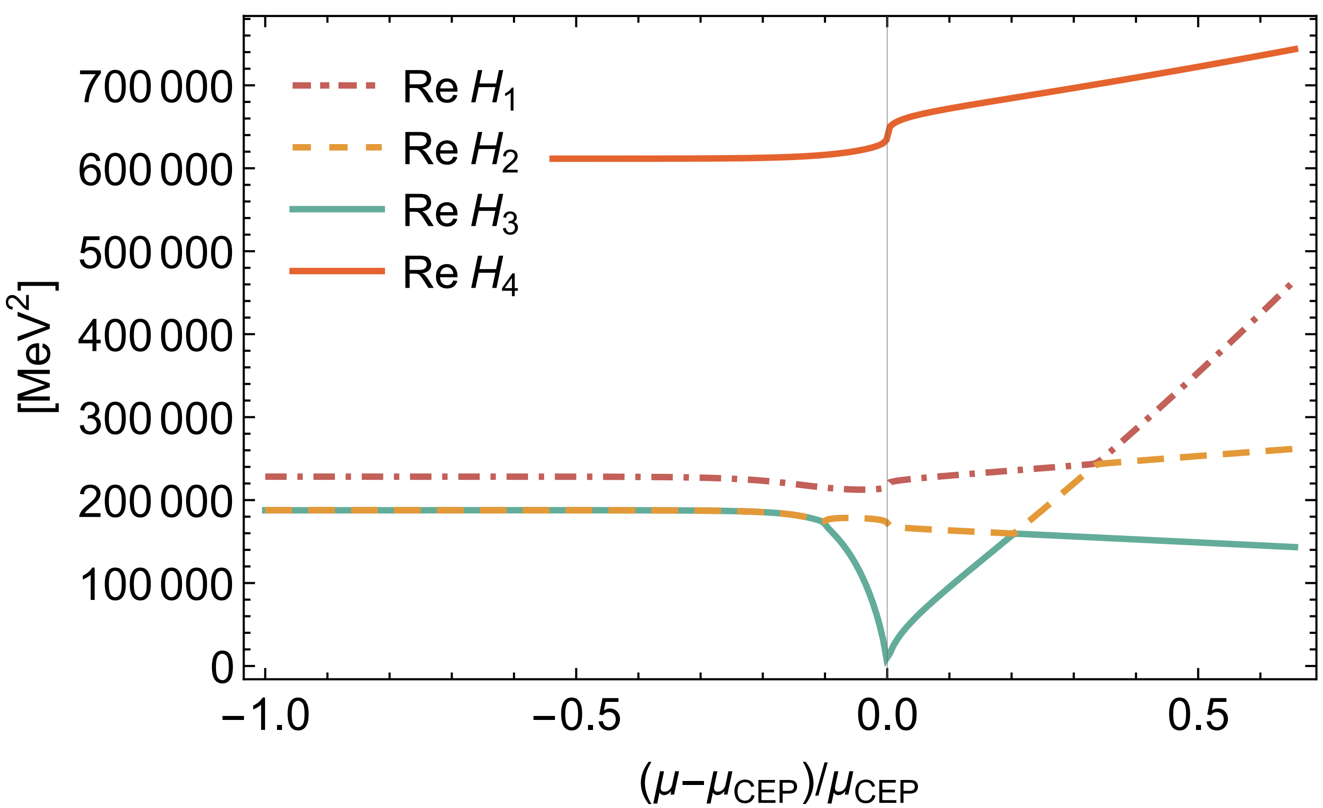
[Haensch, FR, von Smekal (2023)]

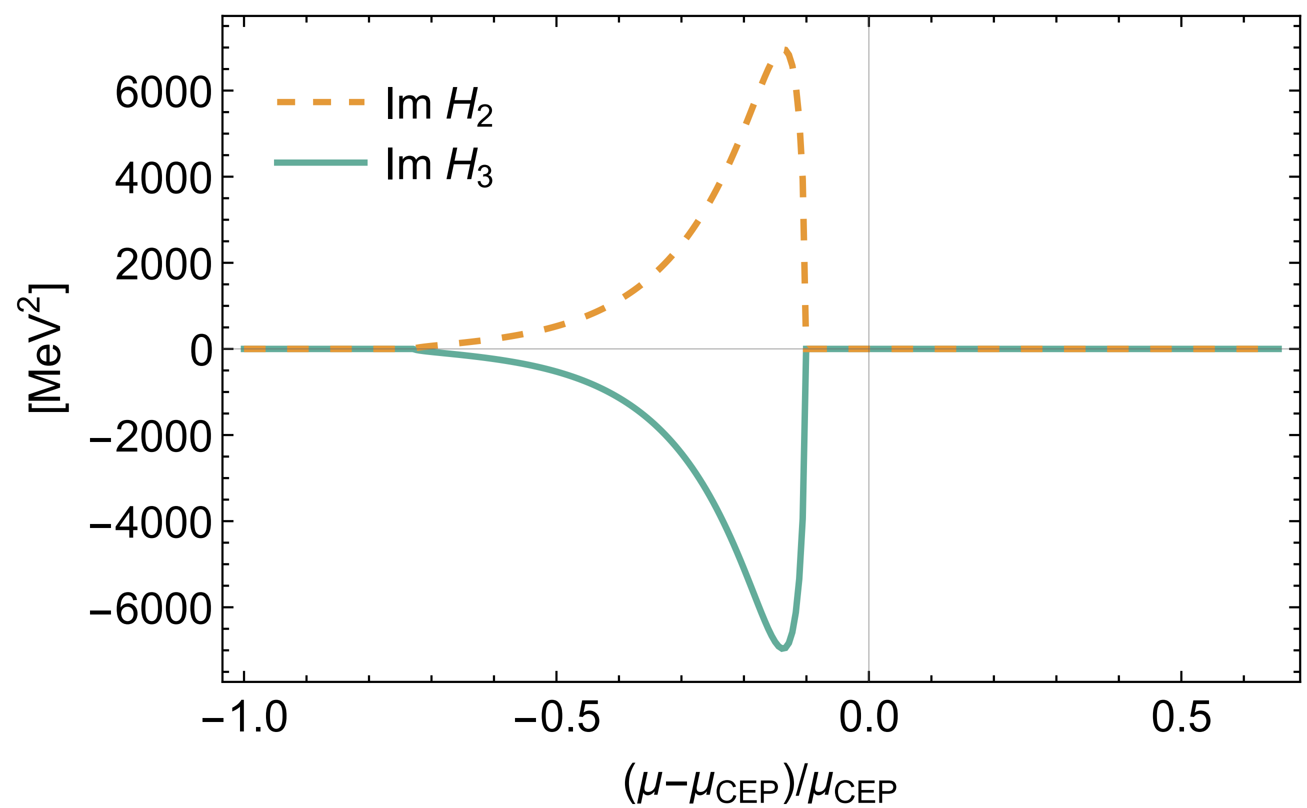


$\phi = (\sigma, \omega^0, L, \bar{L})$

- chiral condensate σ (chiral symmetry breaking)
- Polyakov loops $L, \bar{L}$ (confinement)
- density mode ω^0 (C-symmetry breaking)

Eigenvalues of the Hessian at $T = T_{\mathrm{CEP}}$ in a low-energy model (PQM model with vector repulsion)

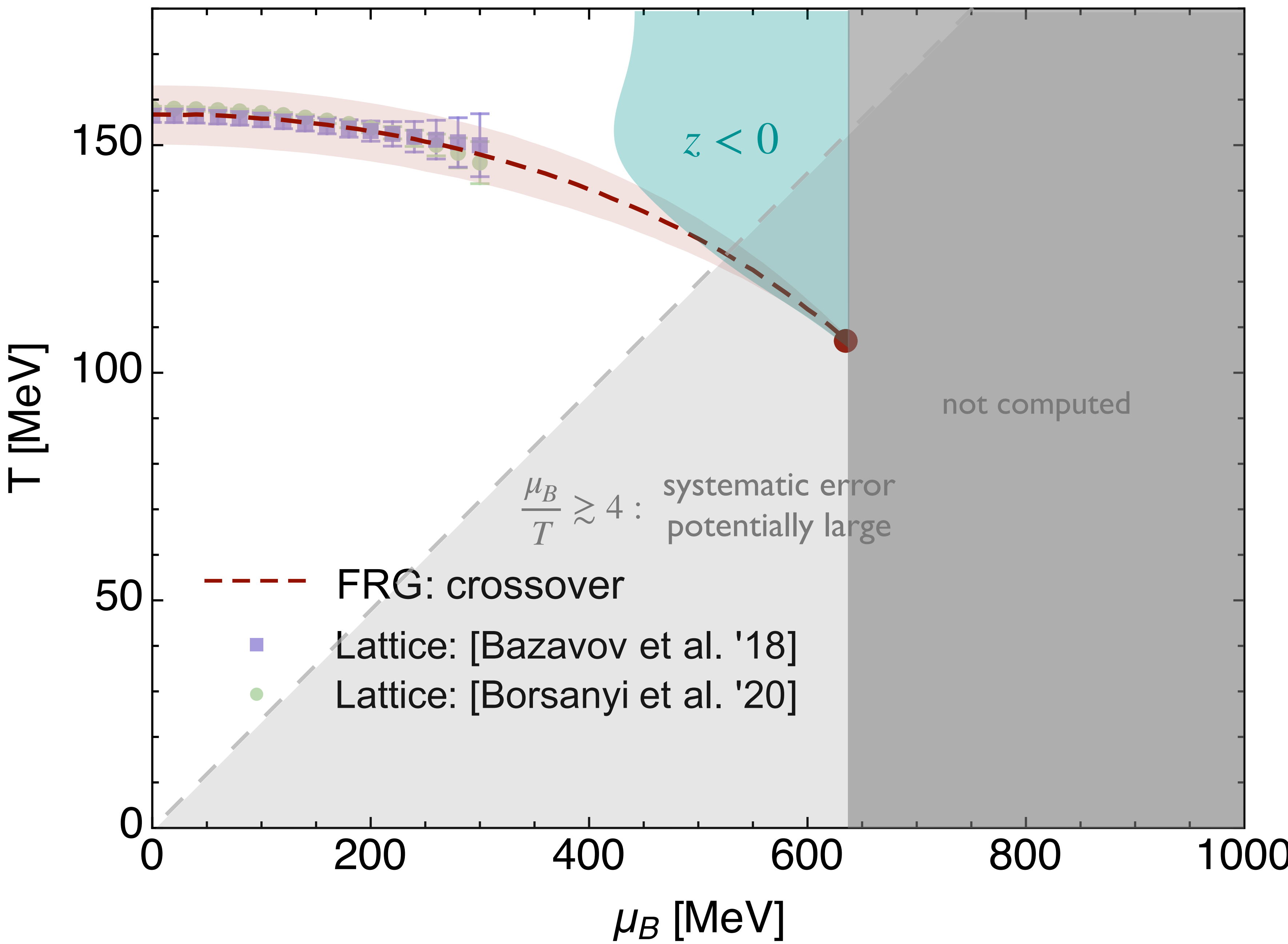




MIXING, MOATS AND MODULATIONS IN QCD

The QCD phase diagram

[Fu, Pawlowski, FR (2019)]



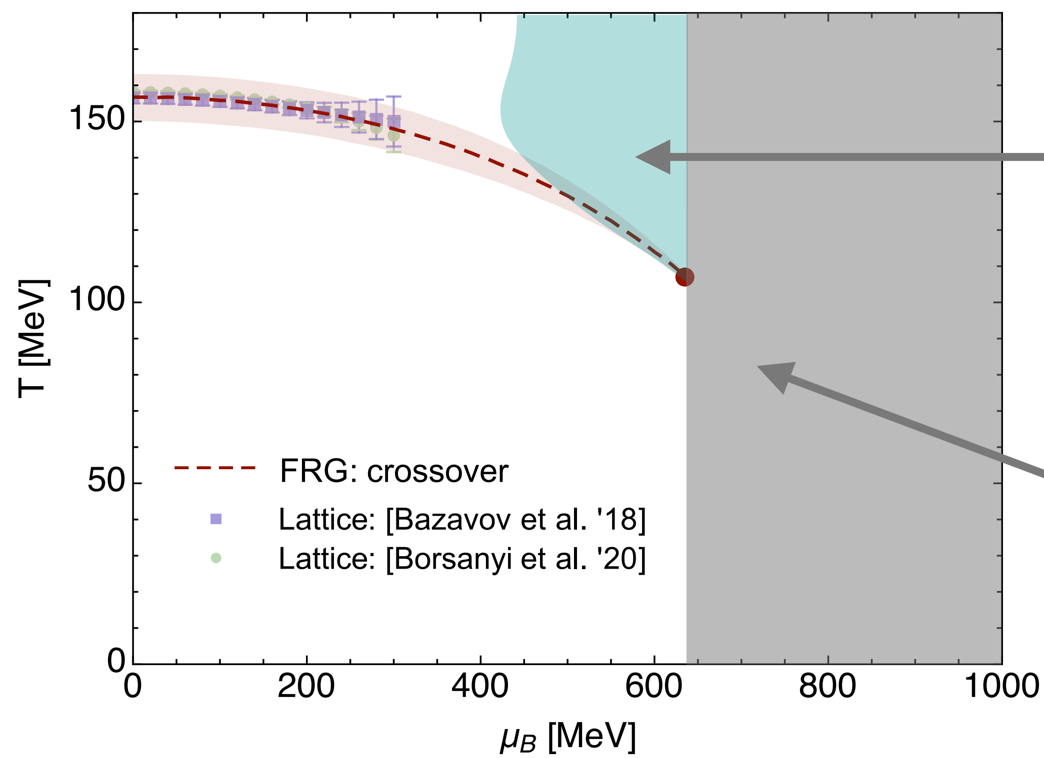
- direct calculation in QCD using the FRG
- CEP at $(T, \mu) = (107{,}635)\,\mathrm{MeV}$ consistent with DSE results and extrapolations of lattice data

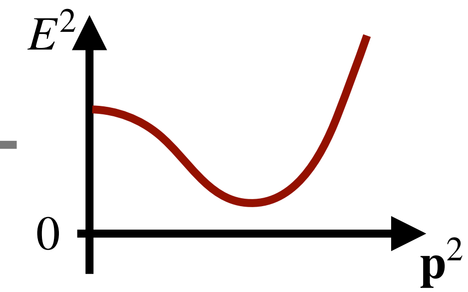
  [Gao, Pawlowski (2020+)]
  [Gunkel, Fischer (2021)]
  [Basar et al. (2023)]
  [Schmidt et al. (2024)]
- need to improve systematics for definitive statements

→ indication for extended region with $z < 0$ in QCD: moat regime

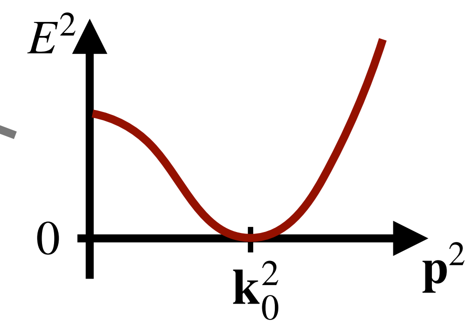
FROM HOMOGENEOUS TO TOURING PATTERNS

The energy gap might close at lower T and larger μ_B :





$E^2 > 0$ for all $\mathbf{p}^2$

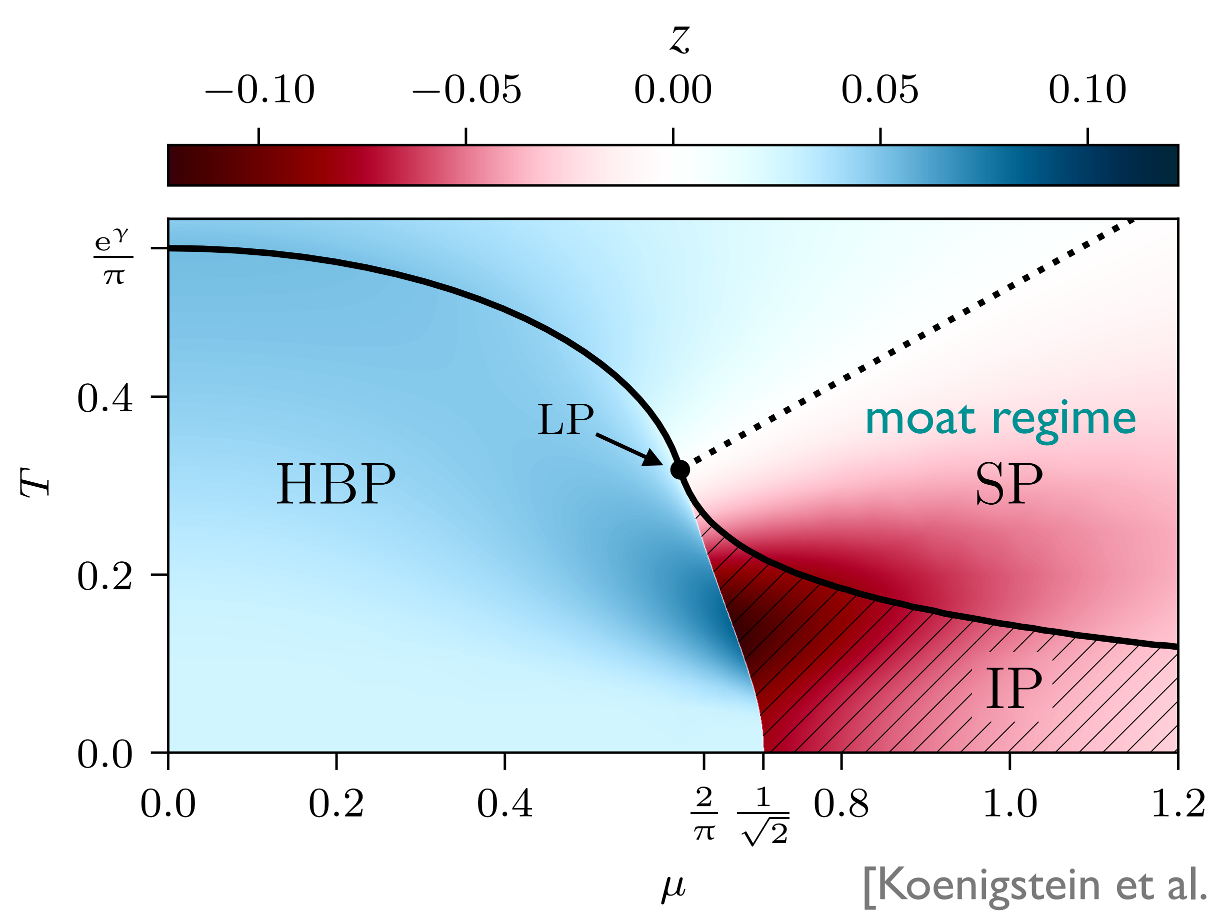


$E^2 = 0$ at $\mathbf{p}^2 > 0$:

Zero energy cost to condense particles with nonzero momentum k_0

→ instability towards formation of an **inhomogeneous condensate**

- Example: Gross-Neveu Model in 1+1 dim. at large N_f



[Koenigstein et al. (2021)]

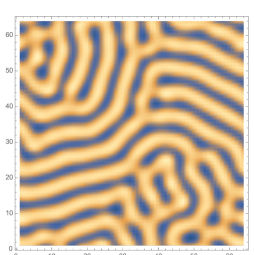
TYPES OF PATTERNS

Will an inhomogeneous instability automatically lead to Touring patterns?

No: formation of inhomogeneous phases depends on dynamics of soft (massless) modes.

→ fluctuation-induced instabilities of inhomogeneous phases

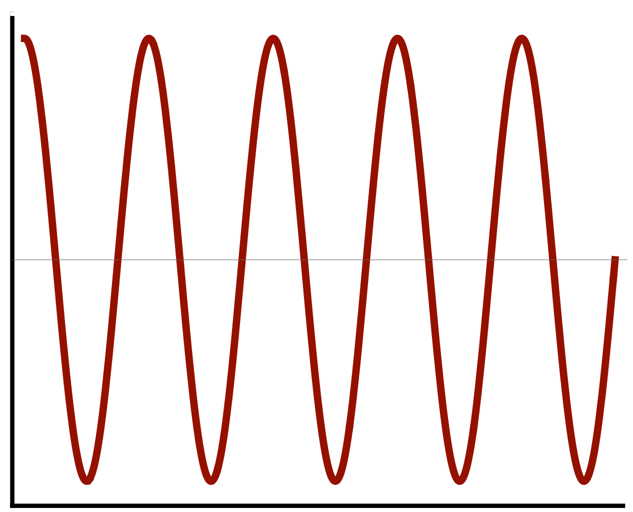
→ other types of phases possible (possibly without long-range order, but always patterned)



inhom. phase

no instability
(typical in mean-field)

$$\langle \phi(r)\phi(0)\rangle \sim \sin(k_0\, r)$$

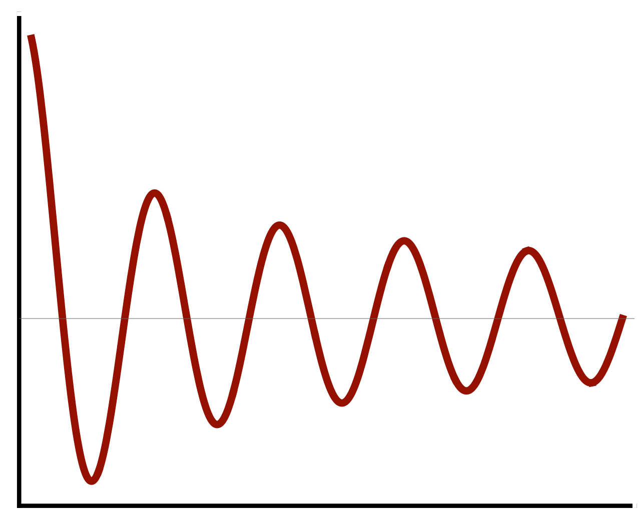


[Fukushima, Hatsuda, RPP 74 (2010)]
[Buballa, Carignano, PPNP 81 (2014)]

liquid crystal

Landau-Peierls instability
(Goldstones from spatial SB)

$$\langle \phi(r)\phi(0)\rangle \sim \sin(k_0 r)\, r^{-\alpha}$$

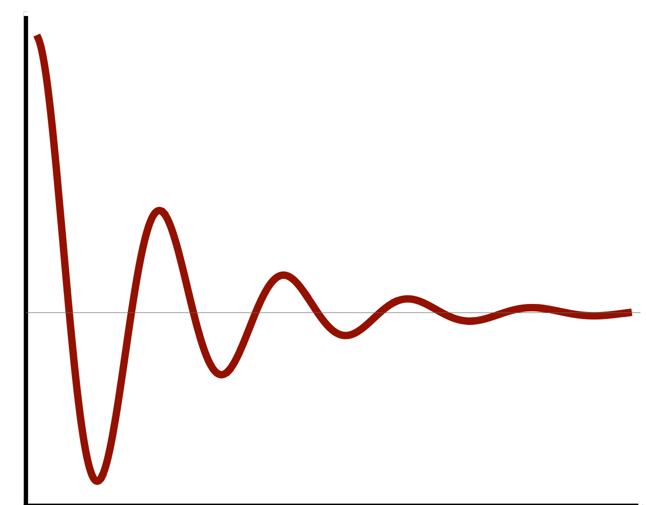


[Landau, Lifshitz, Stat. Phys. I, §137]
[Lee et al., PRD 92 (2015)]
[Hidaka et al., PRD 92 (2015)]

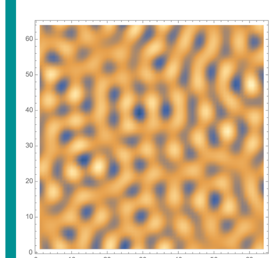
quantum pion liquid

PTV instability
(Goldstones from flavor SB)

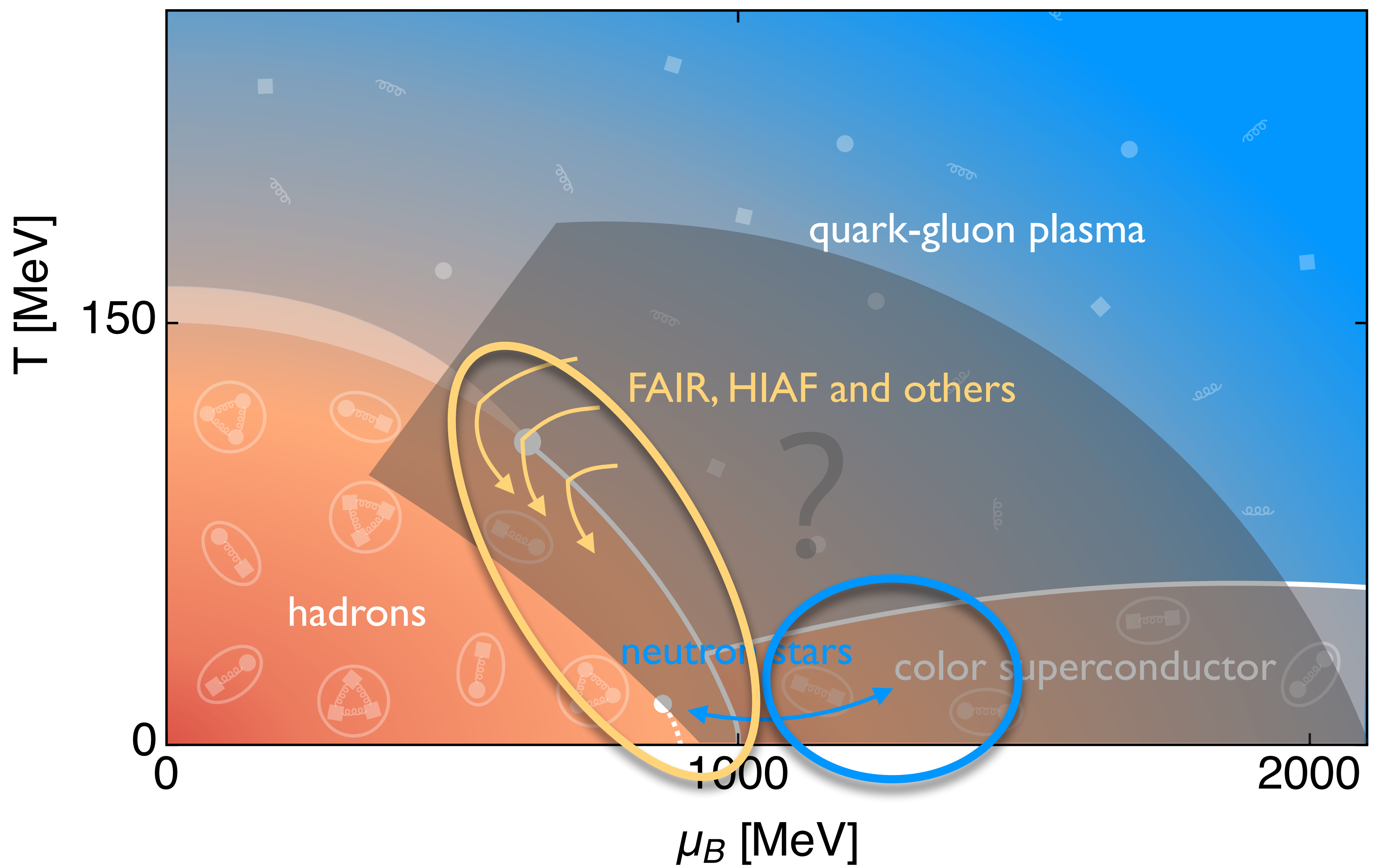
$$\langle \phi(r)\phi(0)\rangle \sim \sin(k_0 r)\, e^{-mr}$$



[Pisarski, Tsvelik, Valgushev, PRD 102 (2020)]
[Pisarski, PRD 103 (2021)]
[Valgushev, Winstel (2024)]



WHERE DO WE EXPECT PATTERNS?



patterns are expected in the "unknown" region of the phase diagram

this will be covered by future fixed target experiments

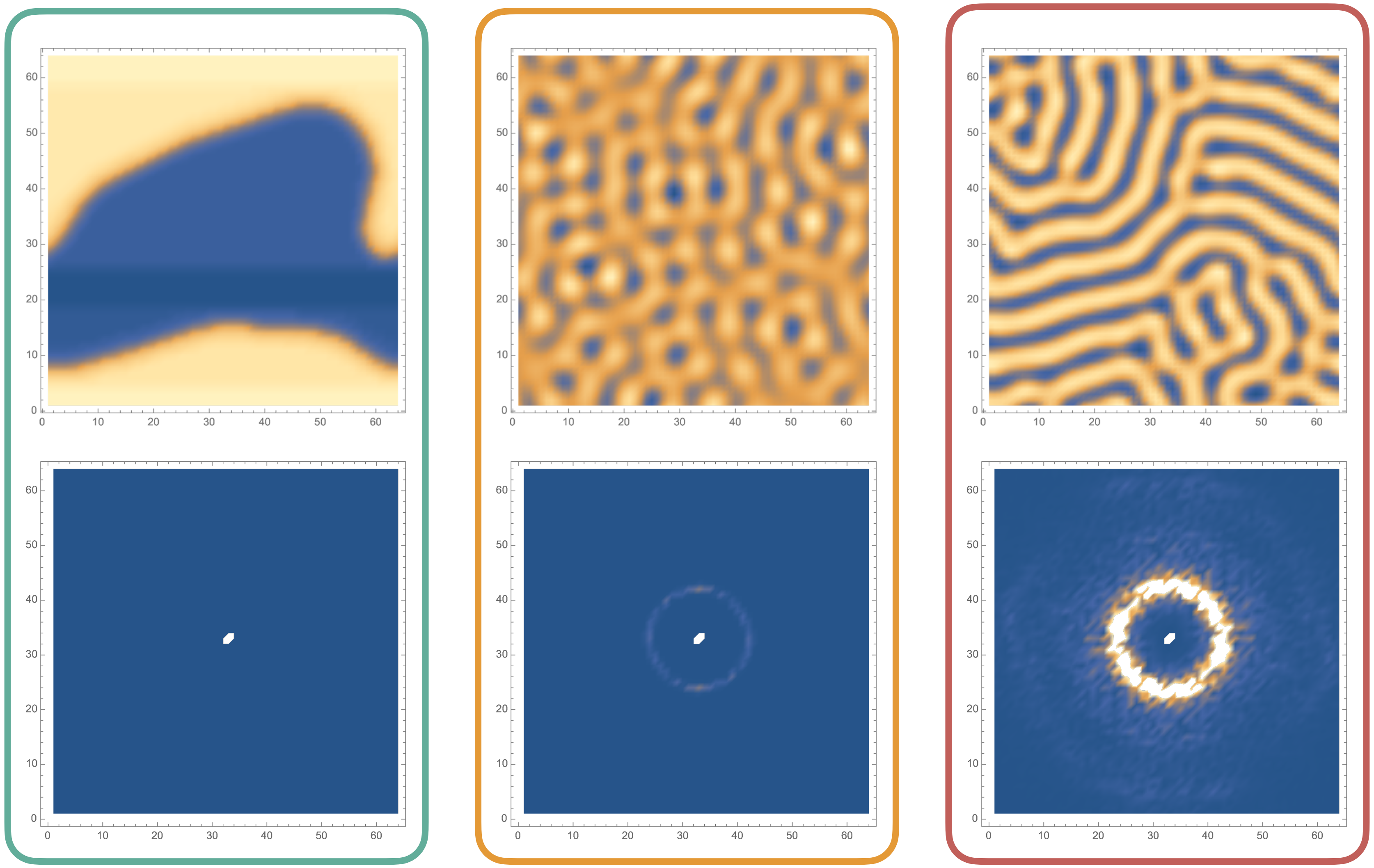
→ **search for patterns in heavy-ion collisions!**

SEARCH FOR PATTERNS IN HICS

intuitive idea: [Pisarski, FR (2021)]

Characteristic feature of patterns: modes with minimal energy at nonzero momentum

$\Rightarrow$ **enhanced particle production at nonzero momentum**



→ look for signatures in the **momentum dependence** of particle correlations

SPECTRA & INTERFERENCE

experiments count particles $\longrightarrow$ particle number correlations

- compute **particle spectra**, e.g.,

$$n_1(\mathbf{p}) = \omega_{\mathbf{p}} \langle \hat{N}_1 \rangle = \omega_{\mathbf{p}} \langle a^\dagger_{\mathbf{p}} a_{\mathbf{p}} \rangle$$

$$n_2(\mathbf{p}, \mathbf{q}) = \omega_{\mathbf{p}} \omega_{\mathbf{q}} \langle \hat{N}_1 \hat{N}_2 \rangle = \omega_{\mathbf{p}} \omega_{\mathbf{q}} \langle a^\dagger_{\mathbf{p}} a_{\mathbf{p}} a^\dagger_{\mathbf{q}} a_{\mathbf{q}} \rangle$$

- most elementary correlation: interference (follows from identical particles; no other fluctuations necessary)
- interference from two-particle scattering: encoded in n_2
- Gaussian approximation captures relevant effects:

$$\boxed{\begin{aligned} n_2(\mathbf{p}, \mathbf{q}) &\sim \langle a^\dagger_{\mathbf{p}} a_{\mathbf{p}} \rangle \langle a^\dagger_{\mathbf{q}} a_{\mathbf{q}} \rangle + \left| \langle a^\dagger_{\mathbf{p}} a_{\mathbf{q}} \rangle \right|^2 + \left| \langle a_{\mathbf{p}} a_{\mathbf{q}} \rangle \right|^2 \\ &= n_1(\mathbf{p})\, n_1(\mathbf{q}) + \left| n_1(\mathbf{p}, \mathbf{q}) \right|^2 + \left| \bar{n}_1(\mathbf{p}, \mathbf{q}) \right|^2 \end{aligned}}$$

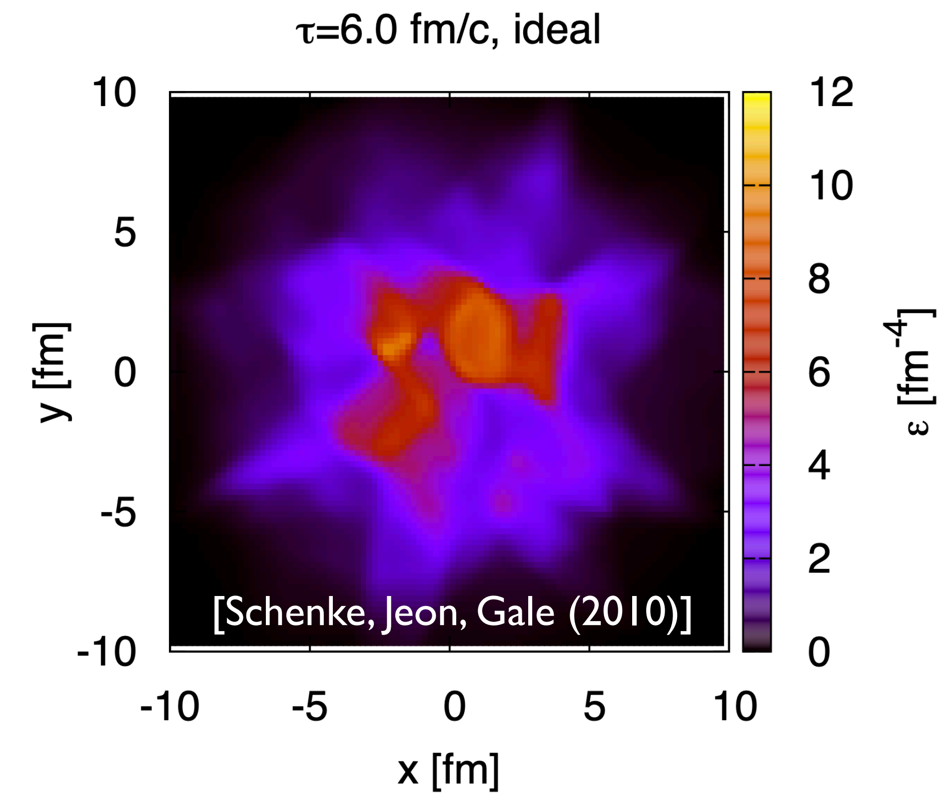
particle-particle interference
(Hanbury-Brown Twiss correlation)

particle-antiparticle interference
(negligible here)

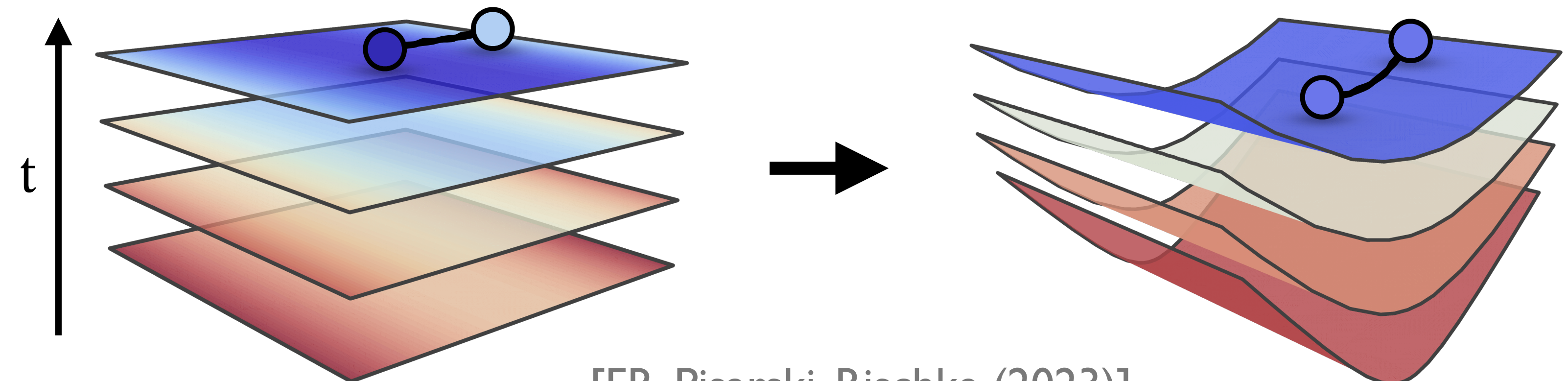
$\longrightarrow$ study interference in a moat regime

INTERFERENCE ON A HYPERSURFACE

To connect the fireball created in a HIC to the phase diagram, we need to fix T, μ: defines hypersurface Σ



→ consider correlations in appropriate **foliation of spacetime**



[FR, Pisarski, Rischke (2023)]

Interference in local thermal equilibrium (fluctuation-dissipation relation + sufficiently isotropic system)

average and relative pair momentum

single-particle distribution, e.g., Bose-Einstein

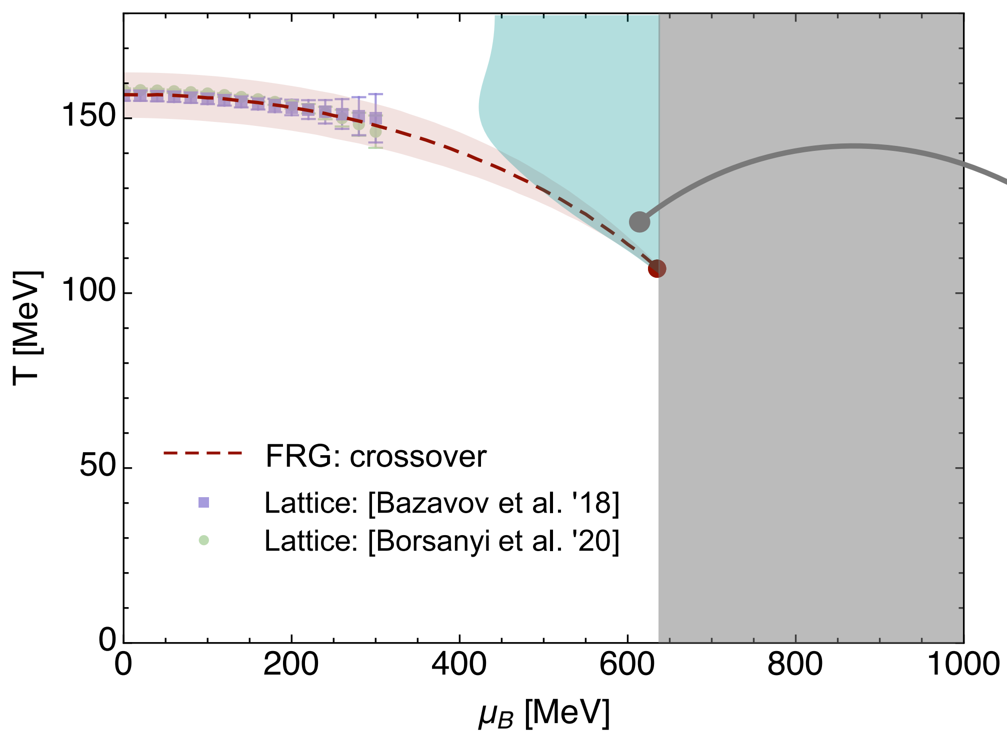
$$\boxed{n_1(\mathbf{P}, \boldsymbol{\Delta}\mathbf{P}) = \frac{1}{2}\int d\Sigma_X\, e^{-i\overline{\Delta P}\cdot X}\int \frac{dP_\parallel}{2\pi}\left[\left(P_\parallel + \overline{P}_\parallel\right)^2 - \frac{1}{4}\overline{\Delta P}_\parallel^2\right] f\left(X; P_\parallel, \mathbf{P}_\perp\right)\rho\left(X; P_\parallel, \mathbf{P}_\perp\right)}$$

average position

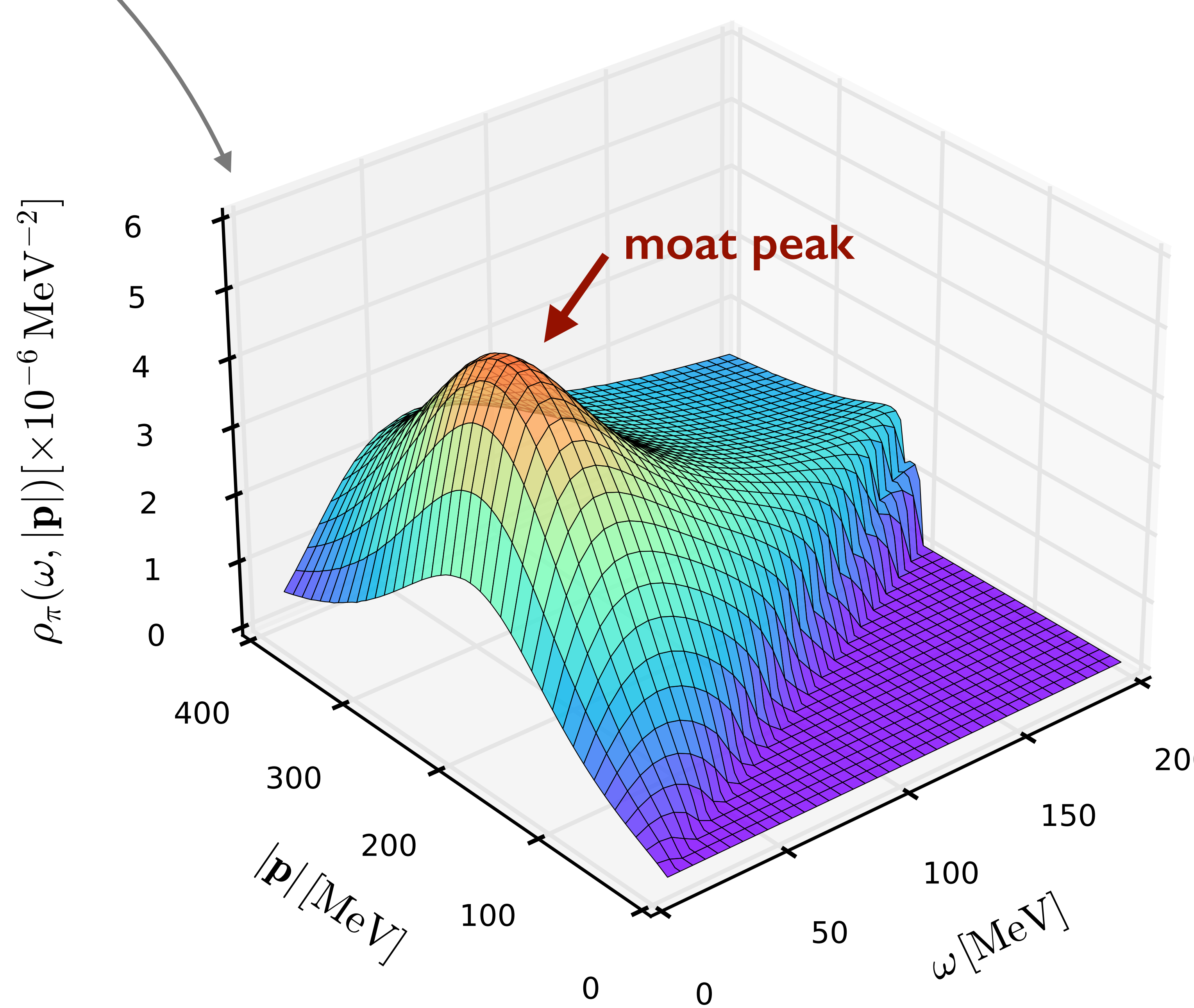
→ in-medium effects enter through P-dependence of the spectral function $\rho(x,y) = \langle[\phi(x), \phi(y)]\rangle$

SPECTRAL FUNCTION IN A MOAT REGIME

HBT correlation determined by spectral function



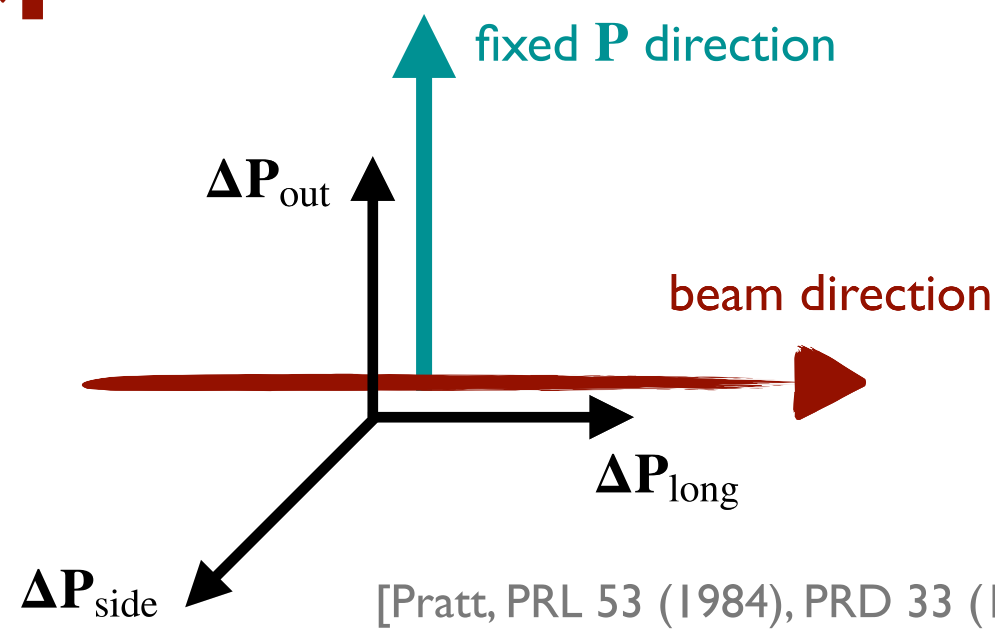
[Fu, Pawlowski, Pisarski, FR, Wen, Yin (in preparation)]



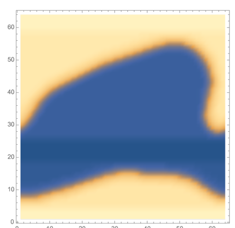
TWO-PARTICLE SPECTRUM

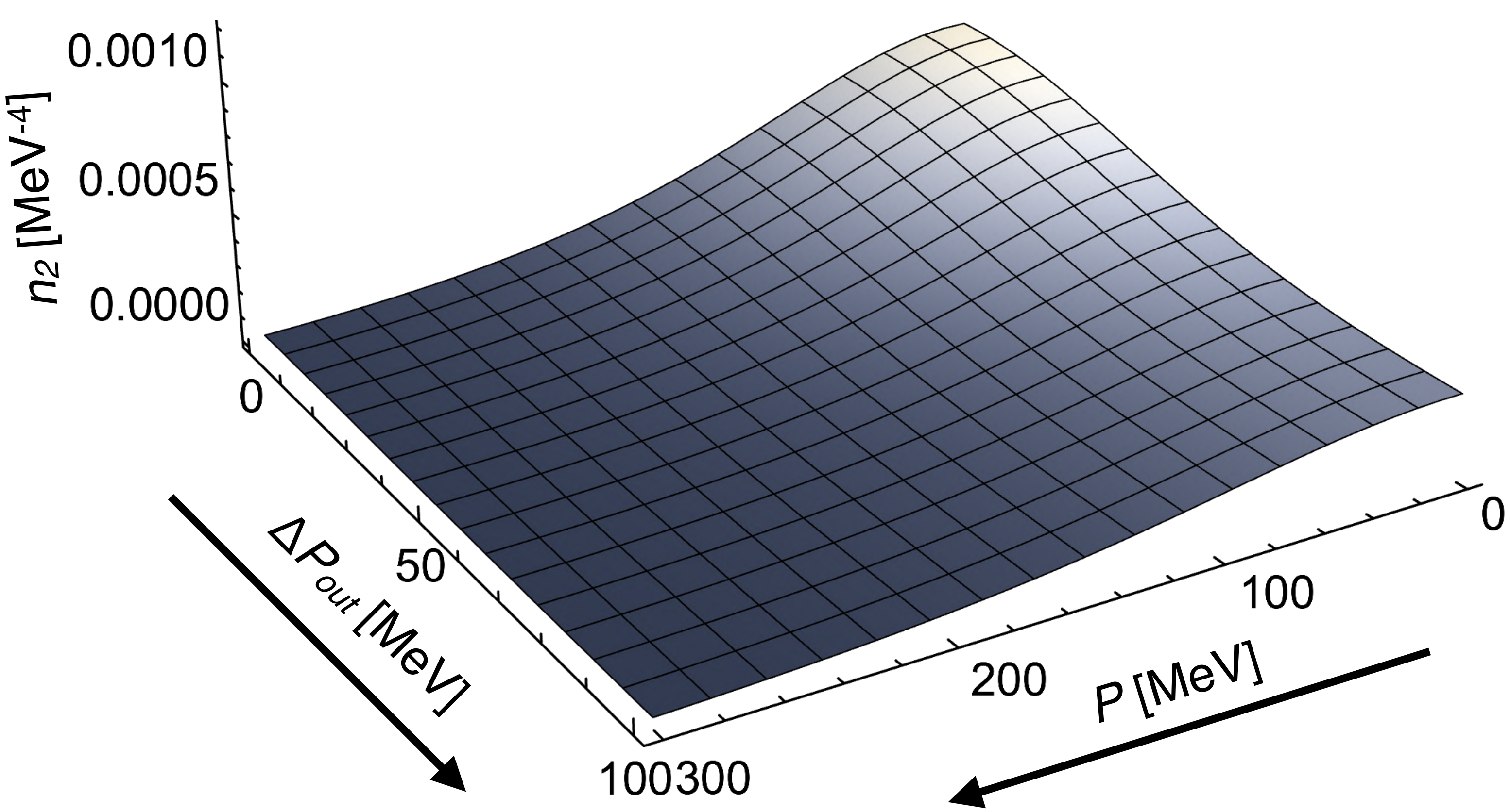
Compute in an illustrative model

- moat quasi-particle with $k_0 = 100\,\mathrm{MeV}$
- hypersurface at fixed proper time



[Pratt, PRL 53 (1984), PRD 33 (1986)]

normal phase: 



minimal energy leads to peak in spectral function

correlation peaks at $|\mathbf{P}| = 0$

TWO-PARTICLE SPECTRUM

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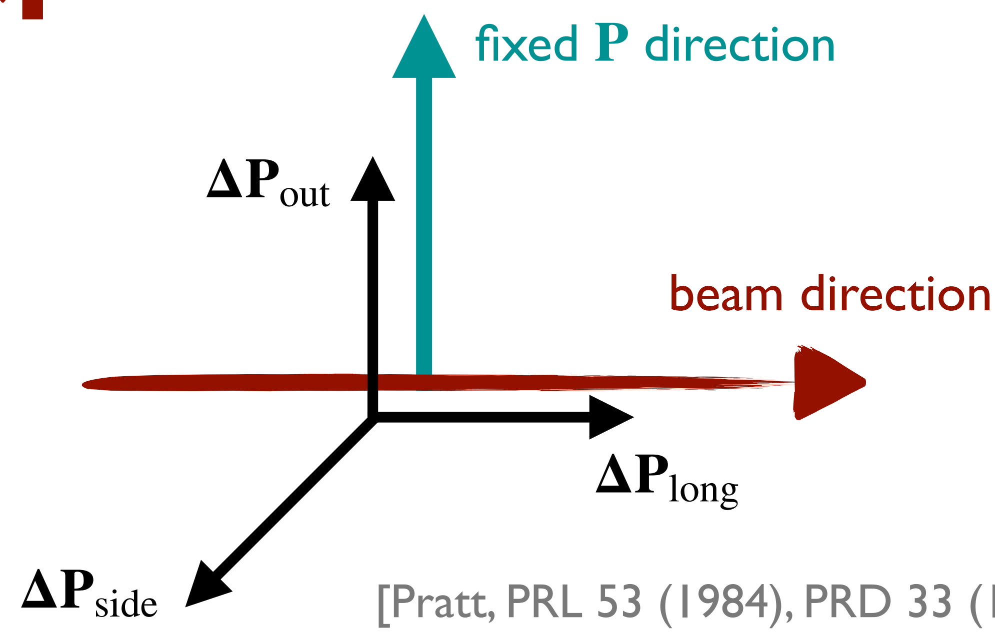
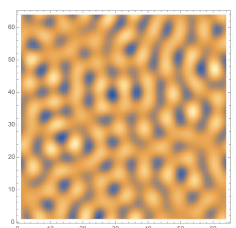
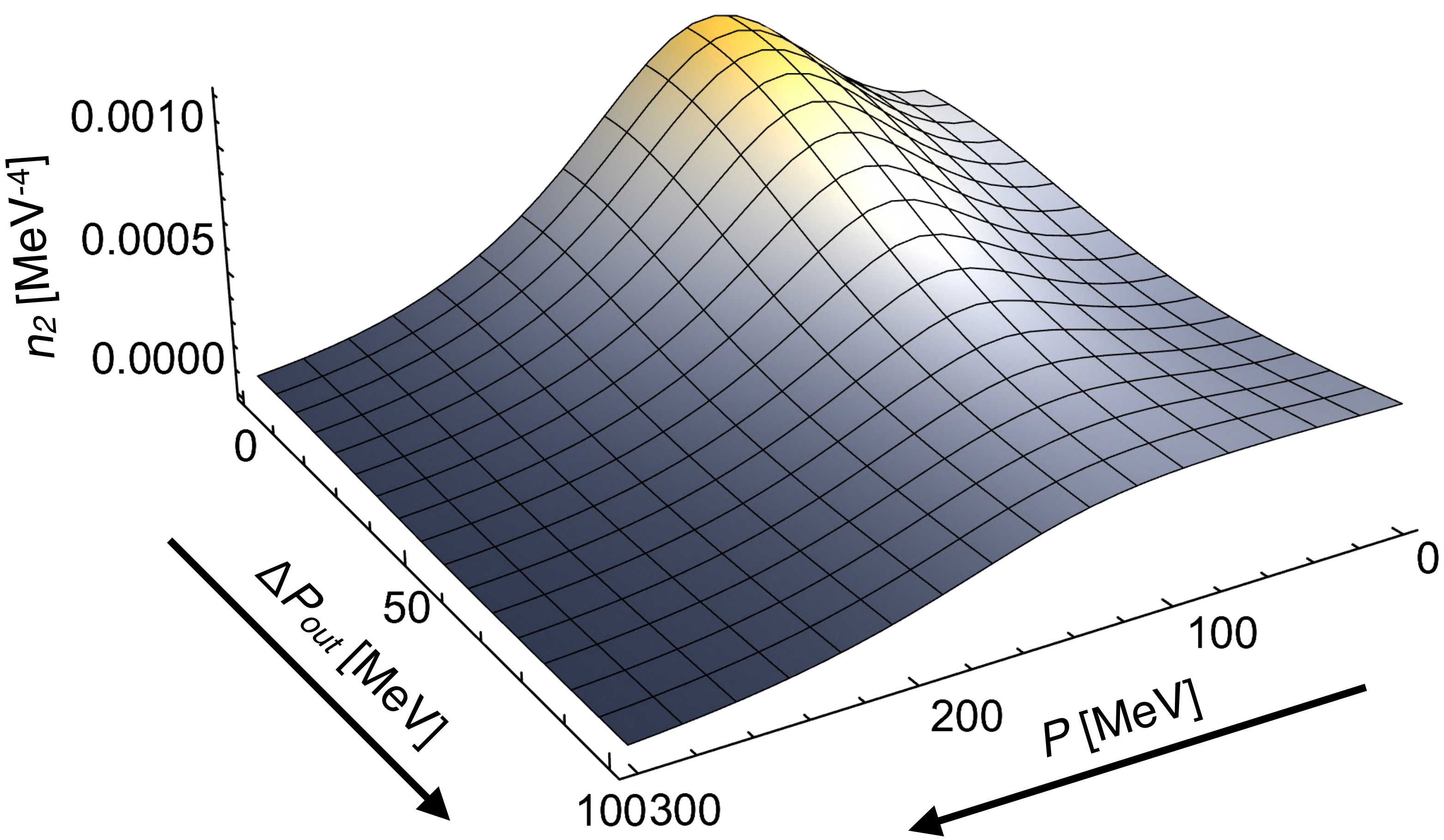


[Pratt, PRL 53 (1984), PRD 33 (1986)]

moat regime: 



minimal energy leads to peak in spectral function

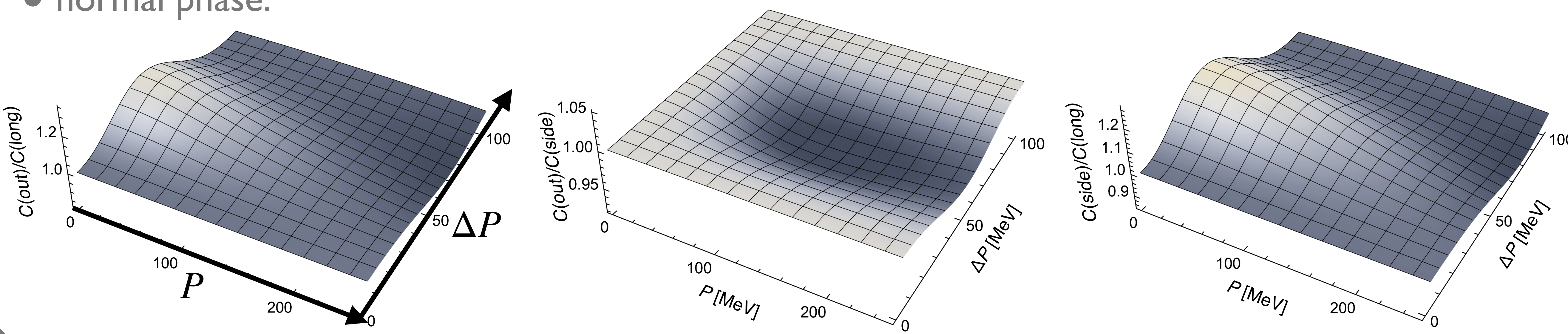
correlation peaks at $|\mathbf{P}| = k_0$

NORMALIZED TWO-PARTICLE CORRELATION

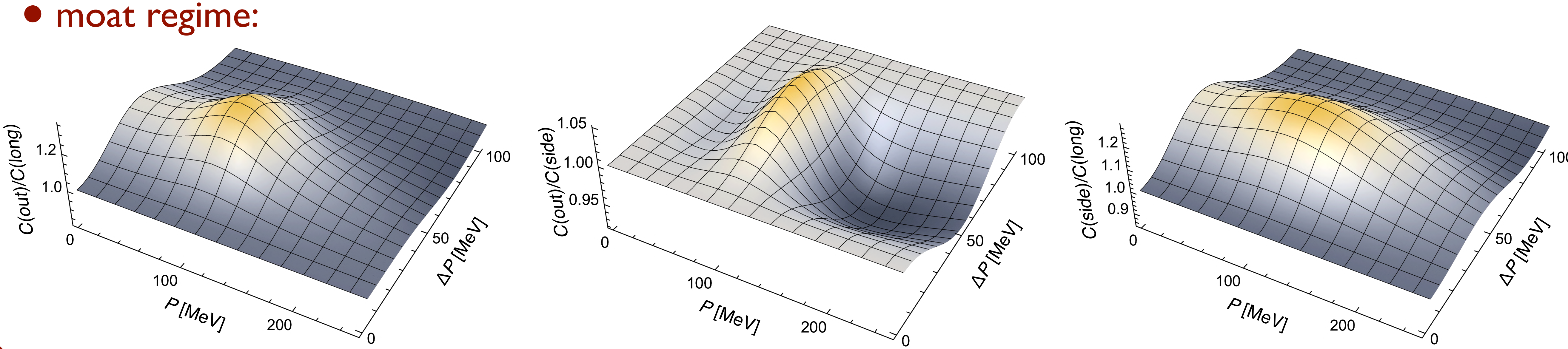
Usually measured in experiments: $$C(\mathbf{P},\mathbf{\Delta P}) = \frac{n_2(\mathbf{P},\mathbf{\Delta P})}{n_1\left(\mathbf{P}+\frac{1}{2}\mathbf{\Delta P}\right) n_1\left(\mathbf{P}-\frac{1}{2}\mathbf{\Delta P}\right)}$$

We propose to look at **ratios**: $C_{\text{out}}/C_{\text{long}}$, $C_{\text{out}}/C_{\text{side}}$ and $C_{\text{side}}/C_{\text{long}}$

- normal phase:



- moat regime:



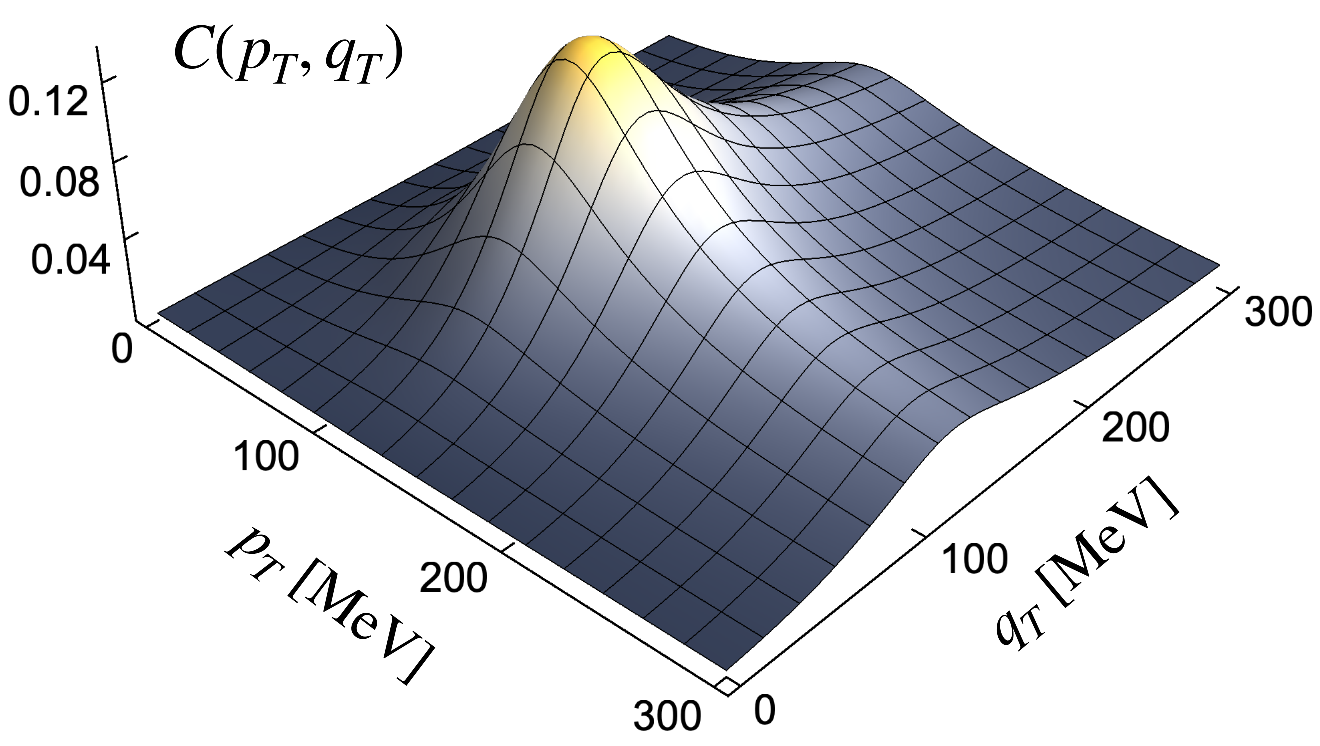
MORE SIGNALS FROM THE MOAT REGIME

Here: 2-particle correlation from identical particle interference in a moat regime

But qualitative result appears to be generic

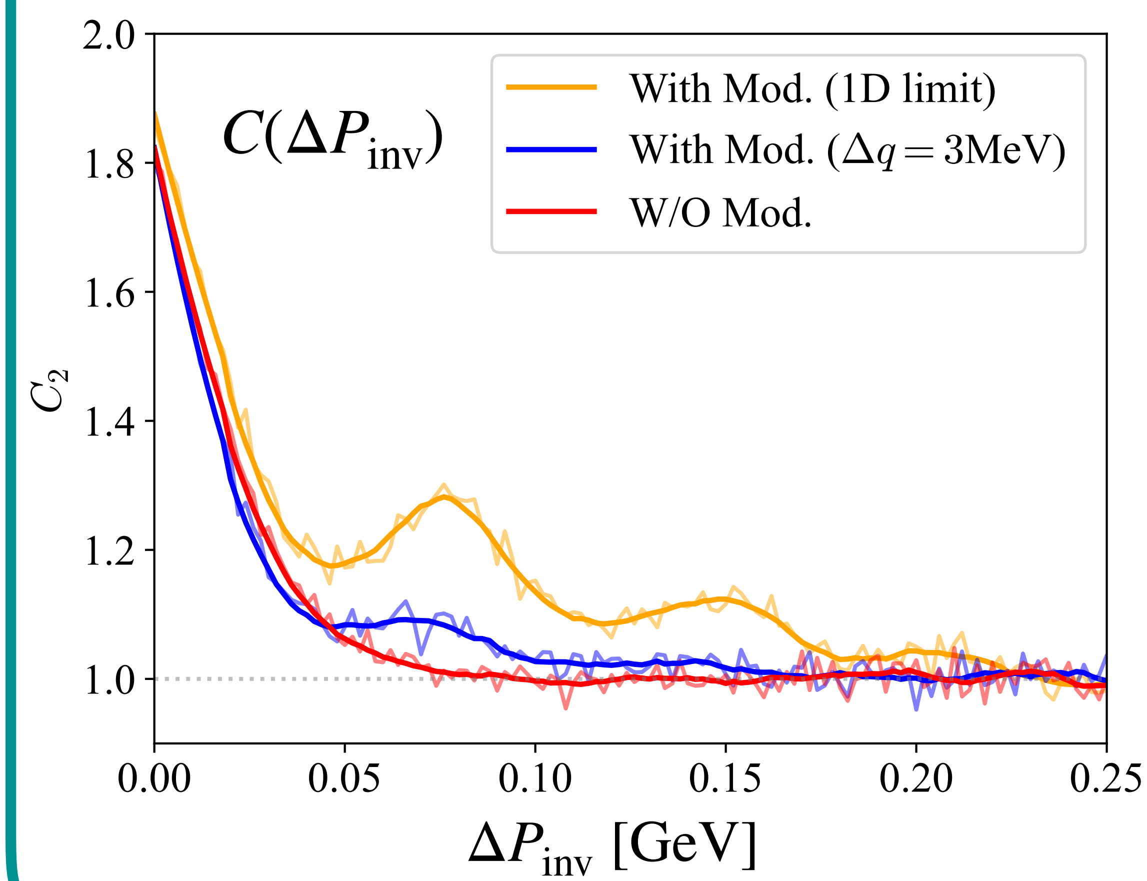
thermodynamic fluctuations in a moat regime

[Pisarski, FR (2021)]



interference from primordial inhomogeneity (with AMPT transport)

[Fukushima et al. (2023)]



- work in progress: dilepton production [Nussinov, Ogilvie, Pannullo, Pisarski, FR, Schindler, Winstel]

→ **peak position related to wavenumber of modulation**

SUMMARY

Mixing, moats and modulations likely in dense QCD matter

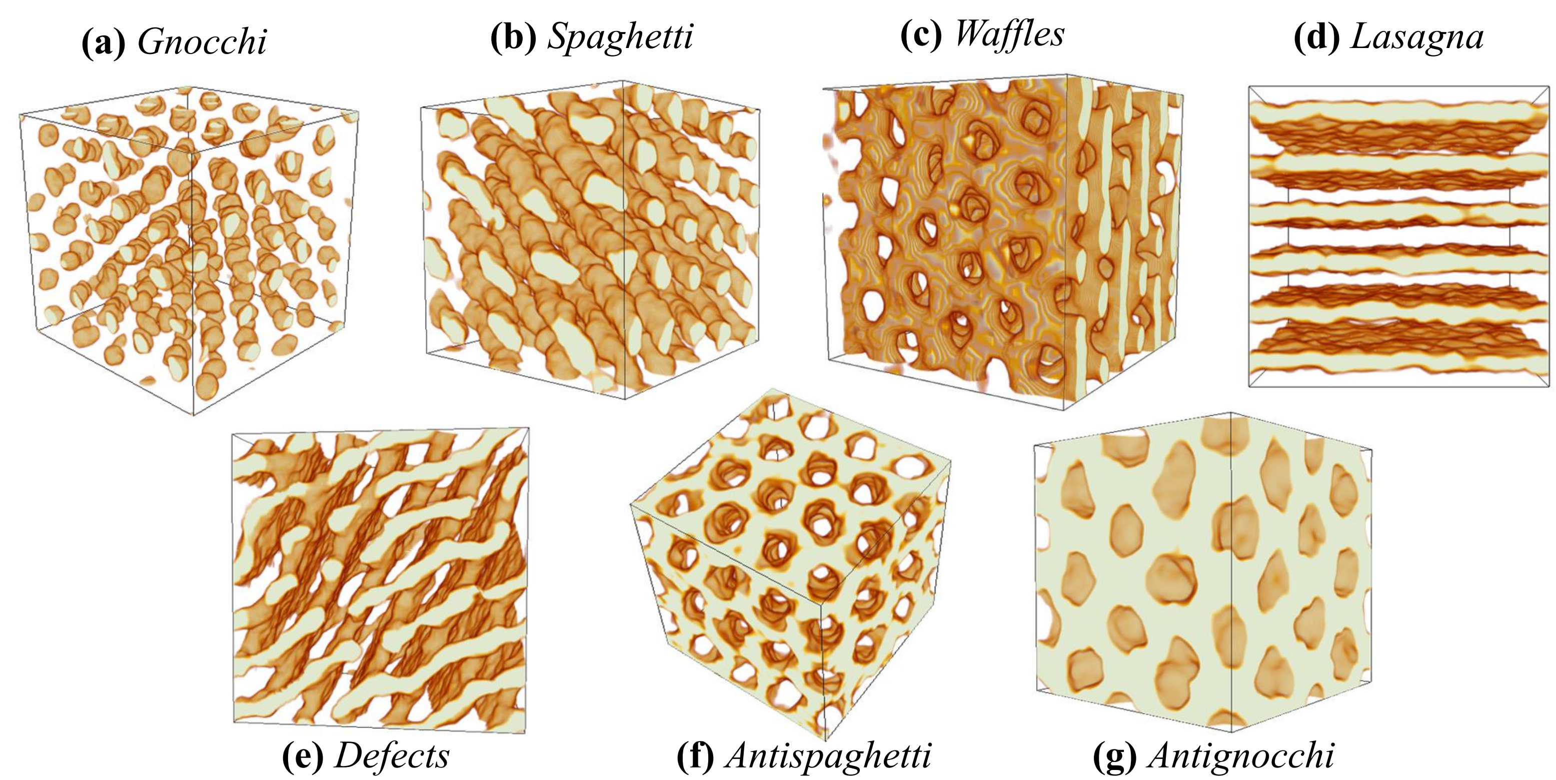
- directly related to pattern formation
- generic features of systems with C-breaking and competing attraction and repulsion

... expected to occur in HIC range

- characteristic enhancement of correlations at finite momentum
- need to understand (and find more) possible experimental signatures

... relevant for neutron stars

nuclear pasta [Caplan, Horowith (2017)]



... but details not well understood

mostly mean-field results, but fluctuations decide which phases are actually realized