Carving out the landscape of relativistic transport

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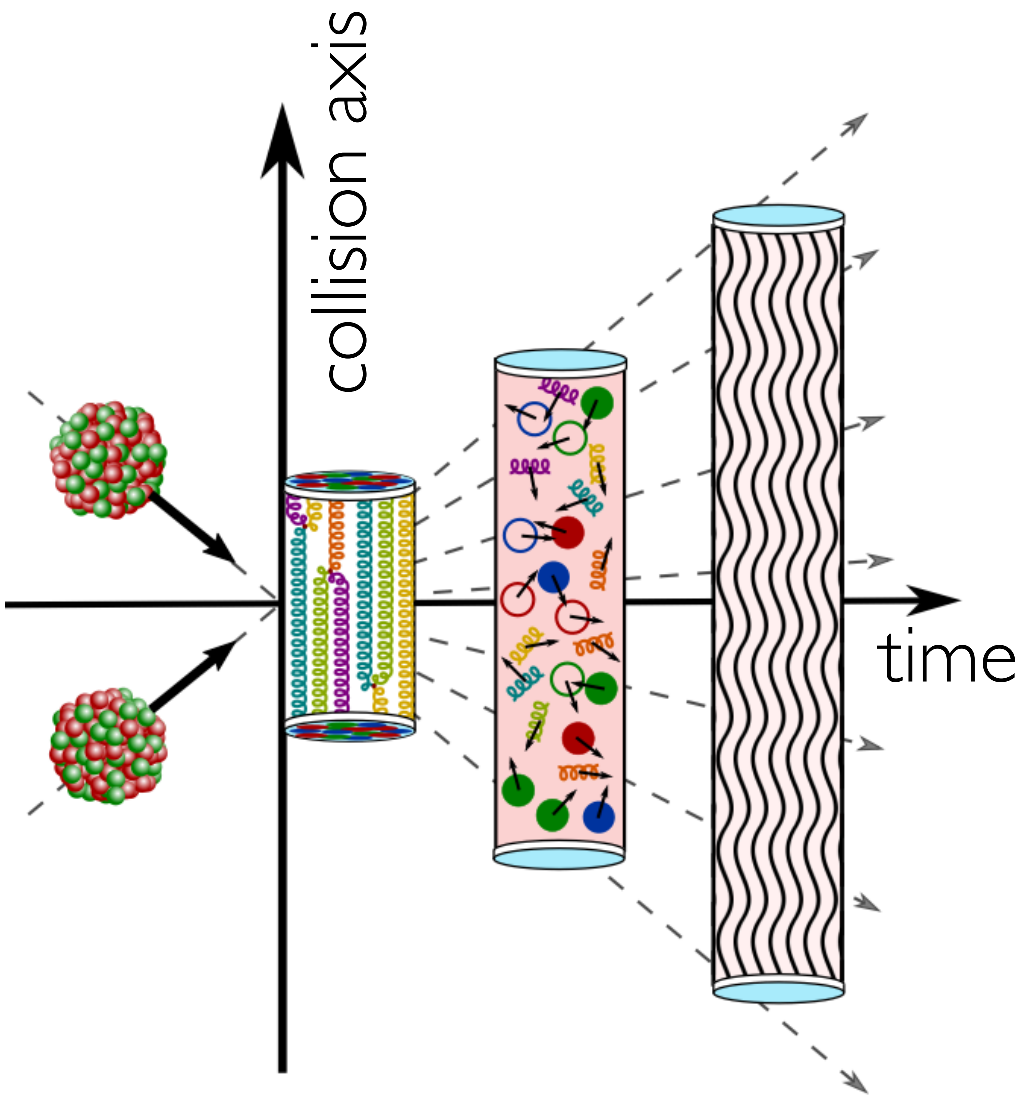
2003.07368 with Jefferson, Spaliński and Svensson

2212.07434 and **2305.07703** with Serantes, Spaliński and Withers

Introduction

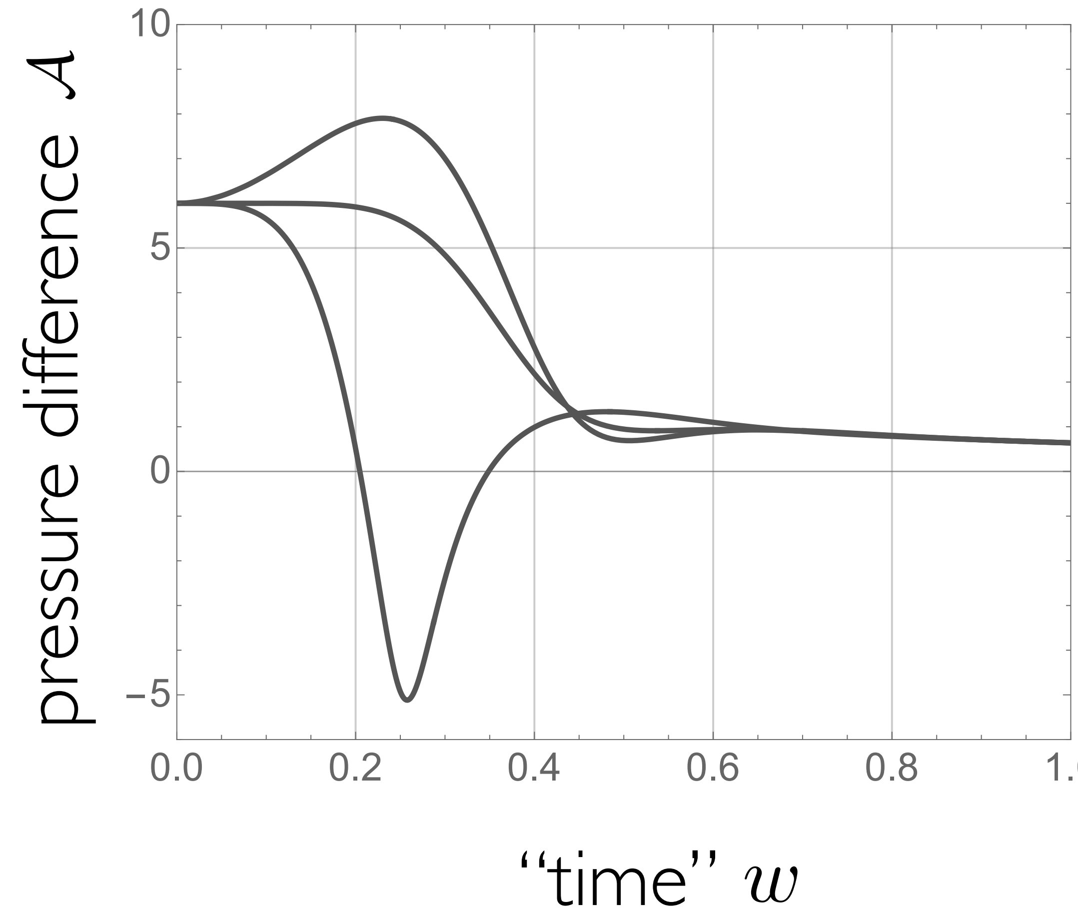
Hydro and ab initio simulations

heavy-ion collisions
at RHIC and LHC



2005.12299
with Berges, Mazeliauskas & Venugopalan

behaviour in
of theoretical models
(here: holographic Bjorken flow)



1103.3452 with Janik & Witaszczyk

Hydro works ≡ its constitutive relations hold

General $T^{\mu\nu}$ has 10 functions of 4 variables freedom subject to $\nabla_\mu T^{\mu\nu} = 0$

Relativistic hydro: $T^{\mu\nu}$ is systematically approx. in terms of 4 functions only

$$T^{\mu\nu} = \mathcal{E}(T) u^\mu u^\nu + \mathcal{P}(T) \left(g^{\mu\nu} + u^\mu u^\nu \right) + \pi^{\mu\nu}$$

@ conformality

$$\pi^{\mu\nu} = -\eta(T)\, \nabla^{\langle\mu} u^{\nu\rangle} - \zeta(T)(g^{\mu\nu} + u^\mu u^\nu)\nabla_\alpha u^\alpha + \mathcal{O}(\nabla^2)$$

shear term

bulk term

@ conformality:

2 order: 5 terms

0712.2451 by Baier et al.

3 order: ~20 terms

1507.02461 by Grozdanov & Kaplis

Two manifestations of constitutive relations

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Bjorken: $$\frac{\pi^T{}_T - \pi^L{}_L}{\mathcal{P}(T)} = \frac{a_1}{\tau T} + \frac{a_2}{(\tau T)^2} + \ldots$$

Linear response theory:

sound waves $$\omega(p) = \sum_{n=0}^{\infty} \alpha_{2n+1} p^{2n+1} + i \sum_{n=1}^{\infty} \beta_{2n} p^{2n}$$

shear mode $$\omega(p) = i \sum_{n=1}^{\infty} \beta_{2n} p^{2n}$$
(also charge diffusion)

Meta questions

What does it mean for relativistic hydro to work?

Are there fundamental bounds on transport coefficients in relativistic hydro?

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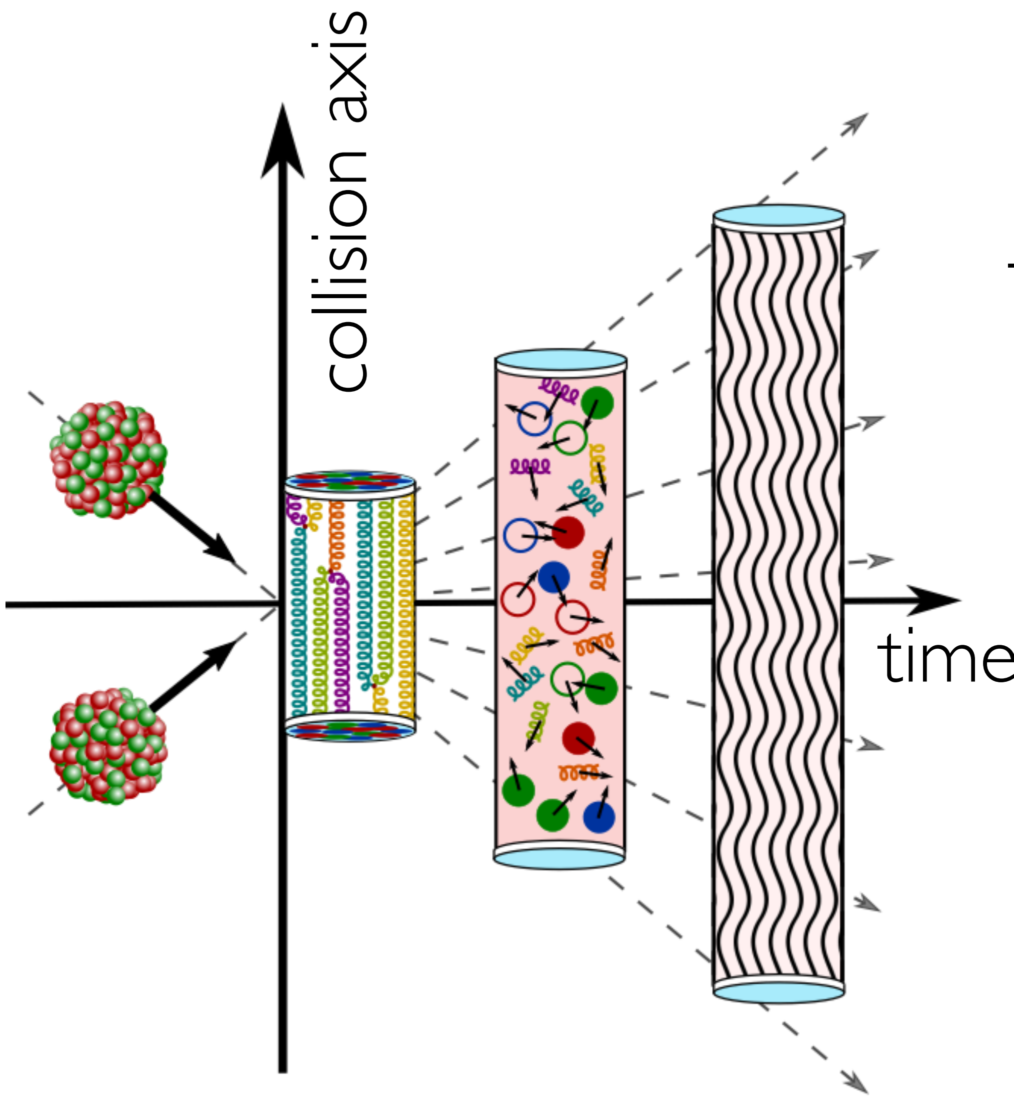

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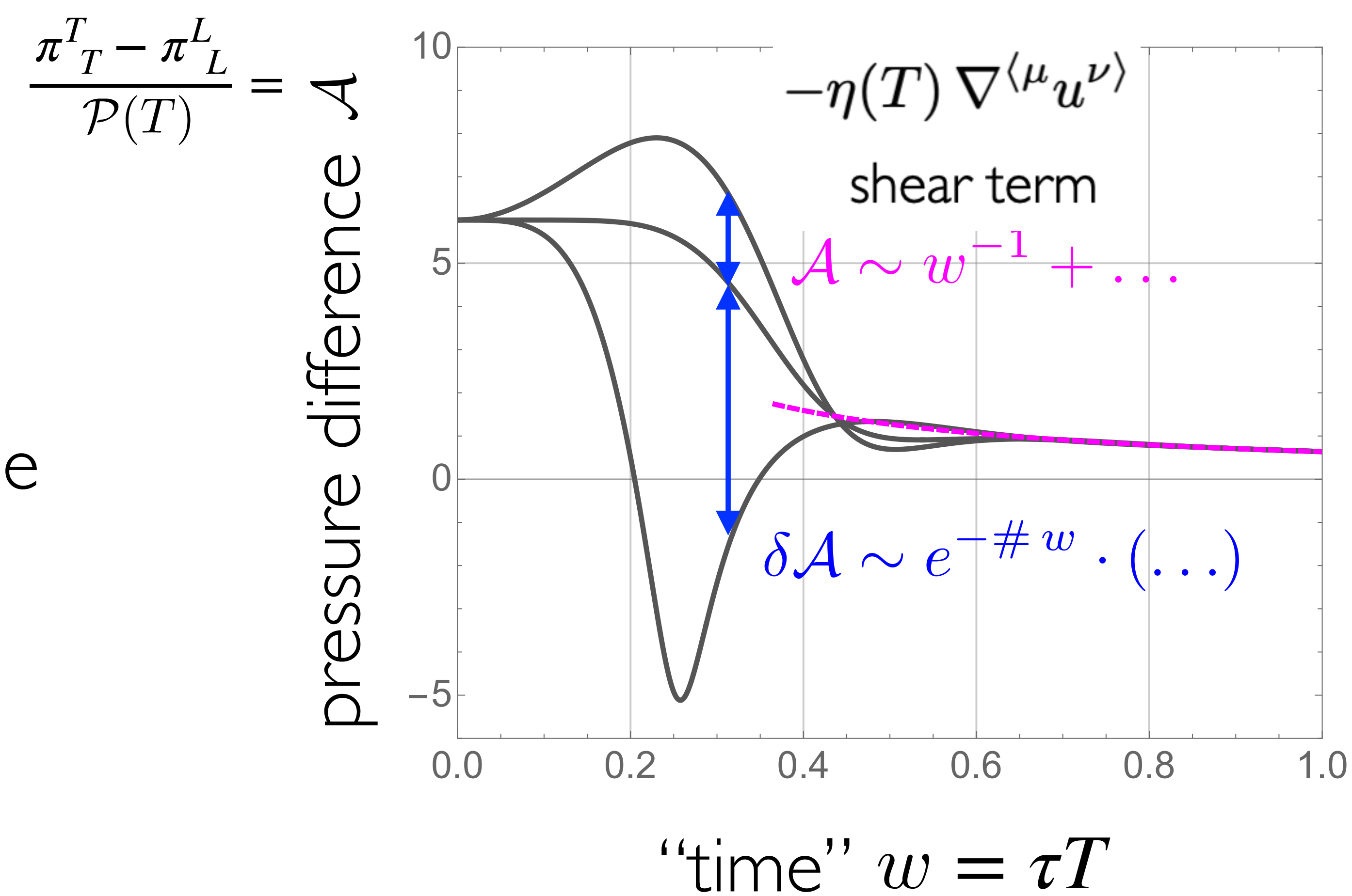

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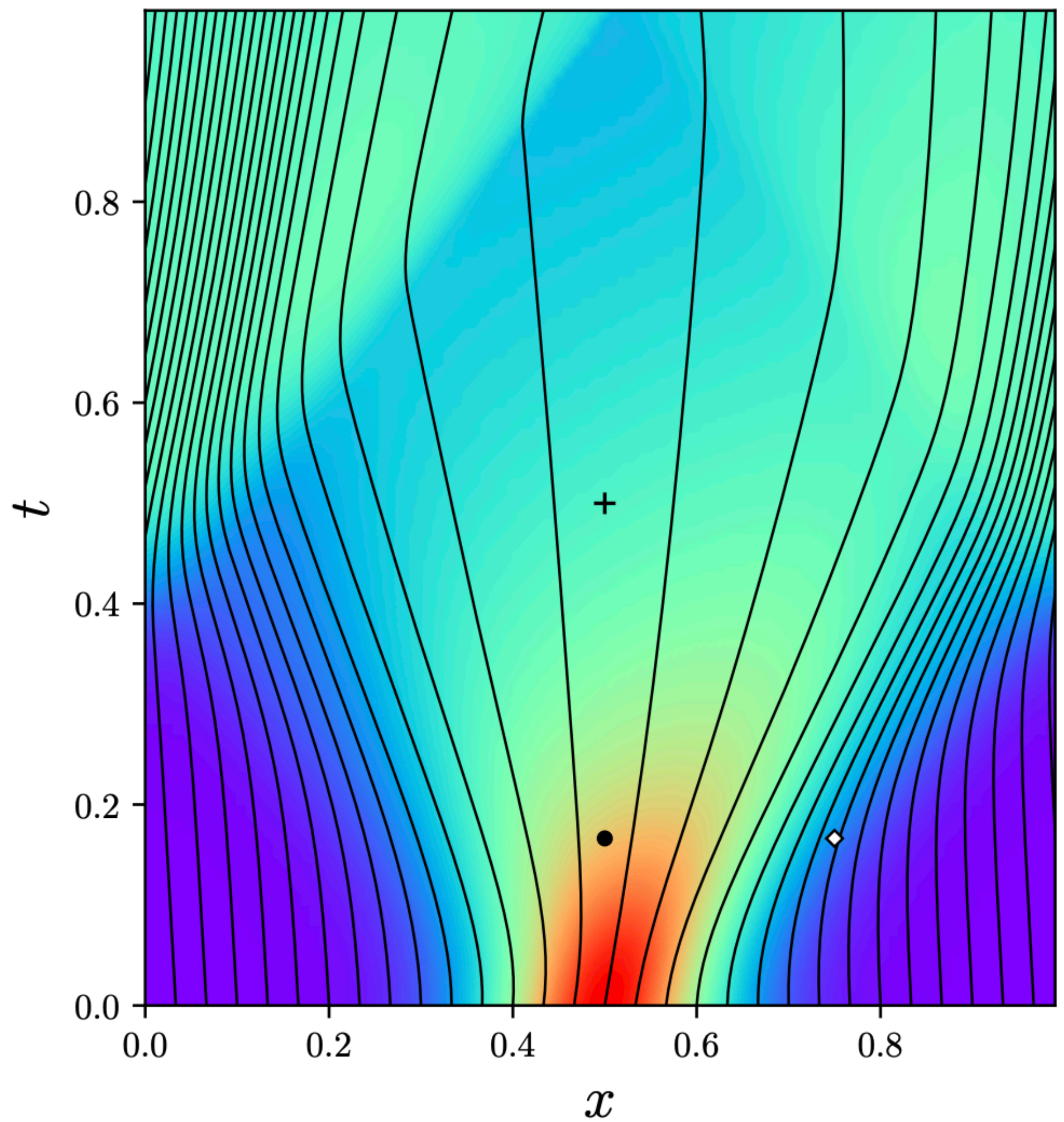
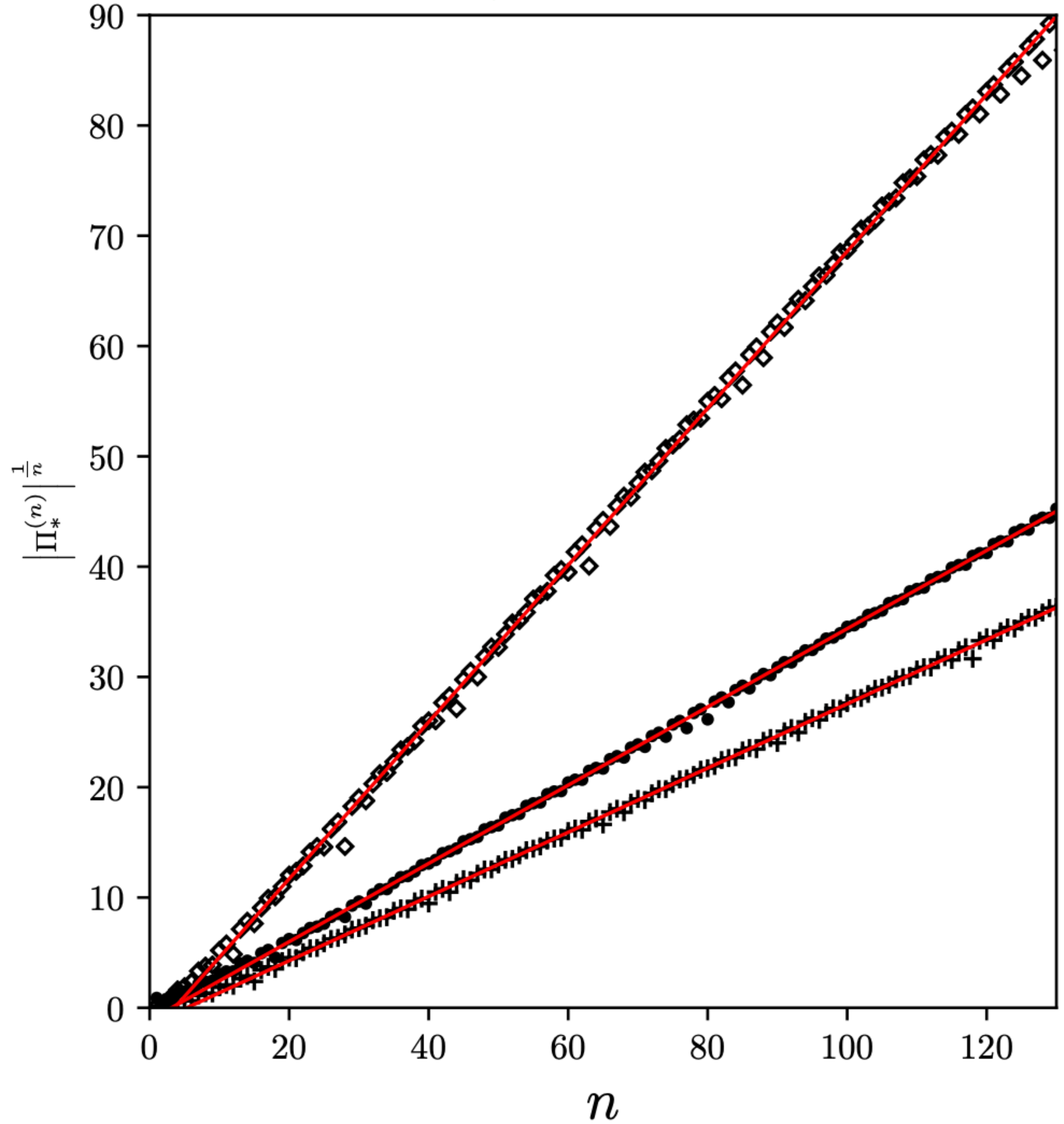
Hydro constitutive relations at higher orders





Hydro constitutive relations generally diverge factorially on-shell

1302.0697 with Janik, Witaszczyk; **1503.07514** with Spaliński;
2110.07621 with Serantes, Spaliński, Svensson, Withers

There is no unique resummation, just optimal truncations

1503.07514 with Spaliński; **2112.12794** with Serantes, Spaliński, Svensson and Withers

What is far from equilibrium relativistic hydro?

1503.07514 with Spaliński

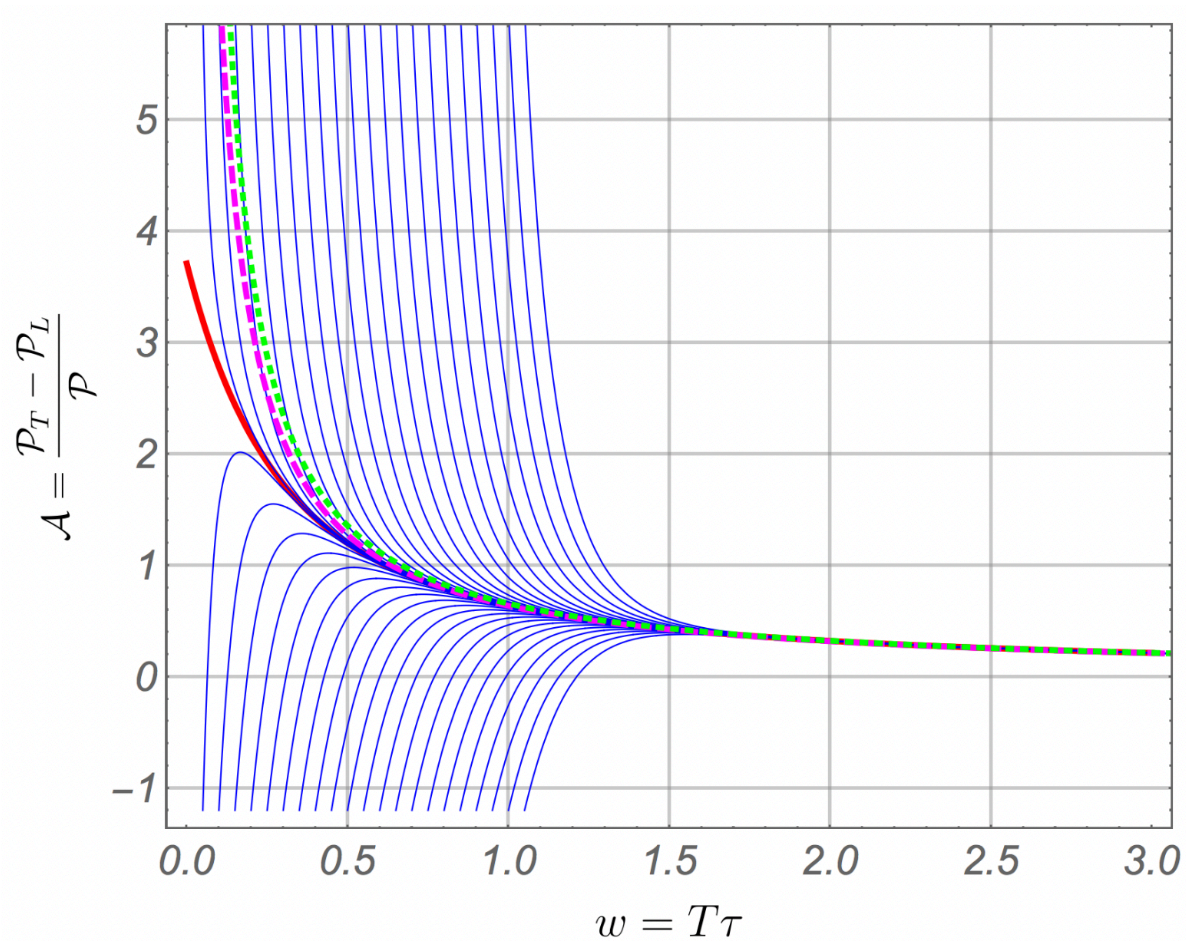
Possibility I:

Resum gradients + extra stuff (= transseries) or use optimal truncation

Possibility II:

Relativistic hydrodynamics far from equilibrium = a **dynamical attractor**

conformal
Israel-Stewart:

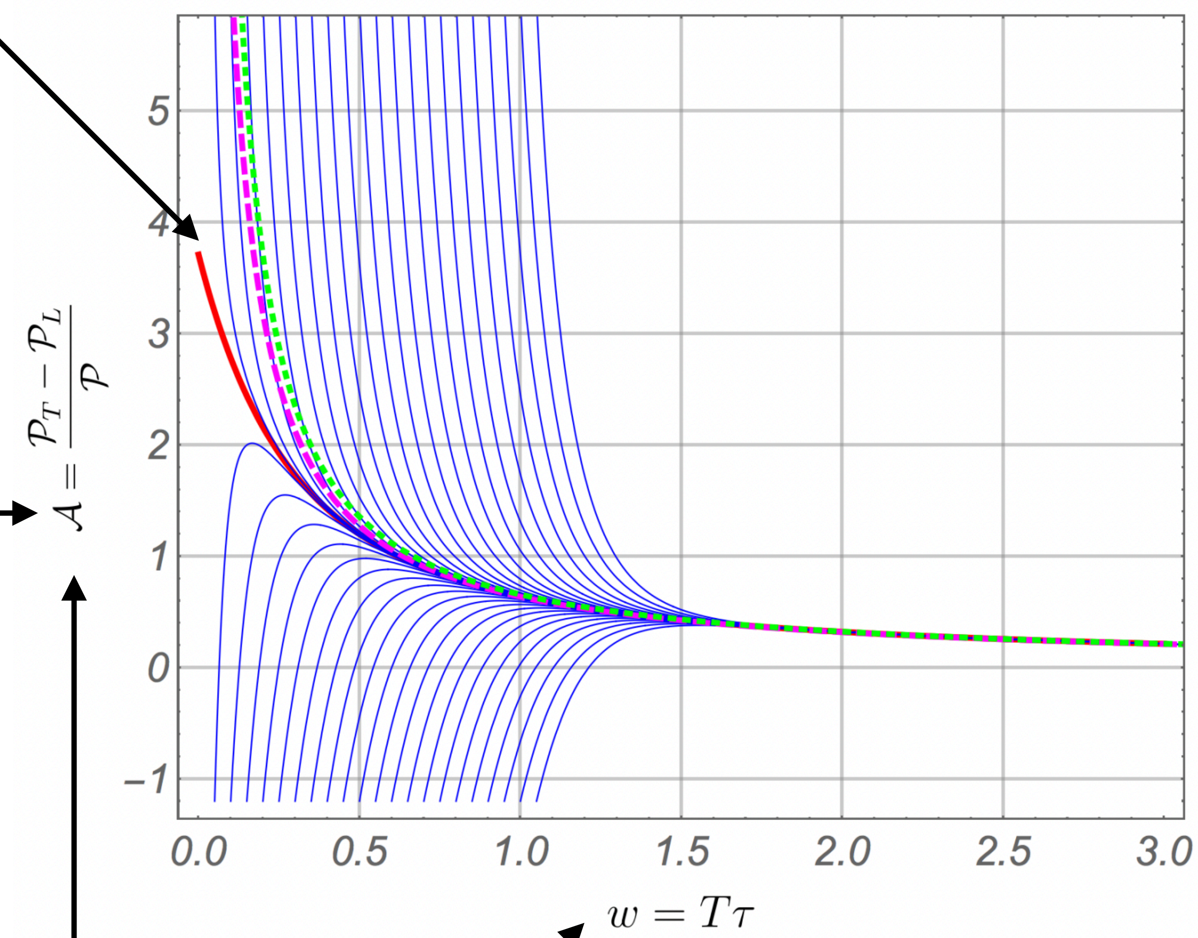


Why we need to go beyond $\mathscr{A}(w)$?

2003.07368 with Jefferson, Svensson, Spaliński

We should not rely on the behavior at $w = 0$ to identify the attractor

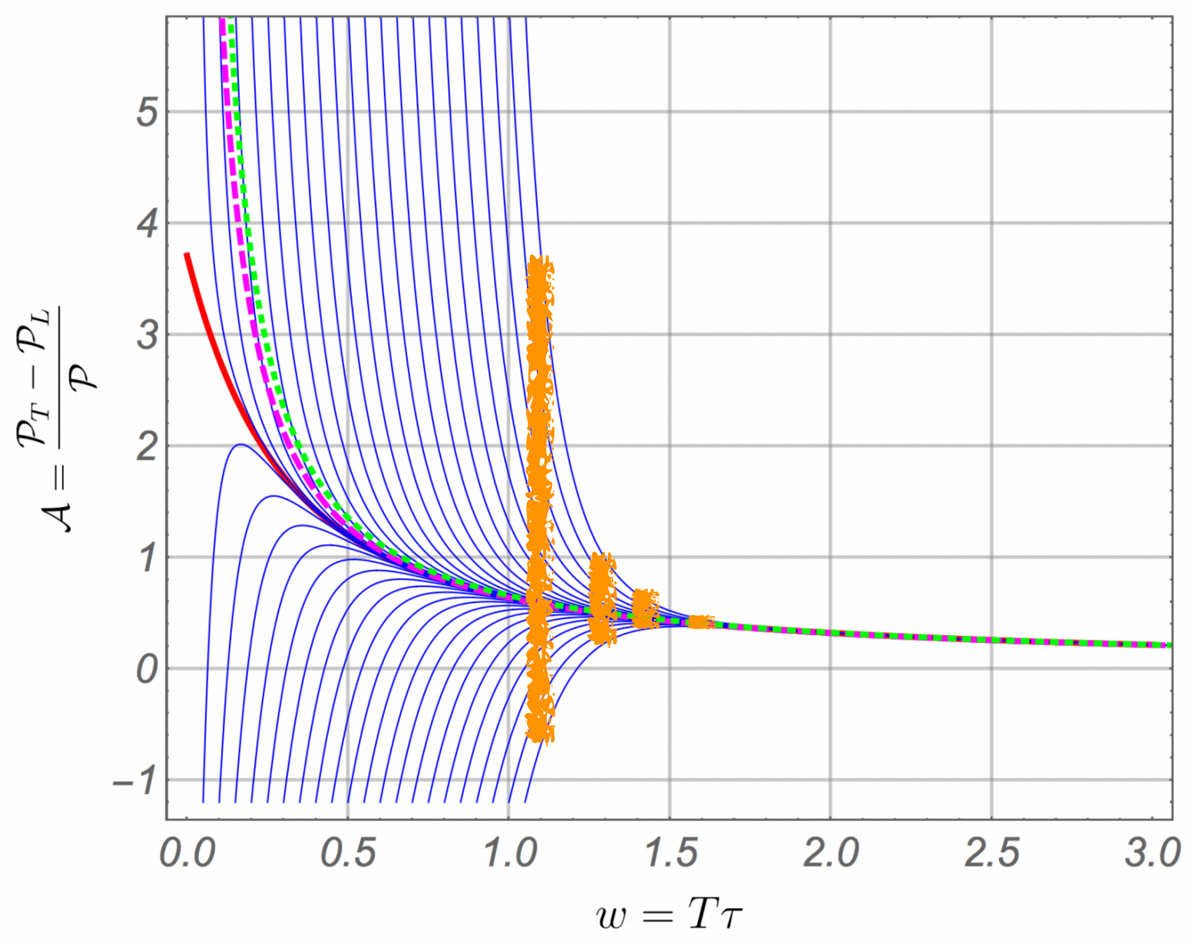
In kinetic theory and holography $\mathscr{A}$ at a given w does not fully specify the state



what are $\mathscr{A}$ and w in a less symmetric dynamics?

Reinterpreting the hydro attractor

2003.07368 with Jefferson, Svensson, Spaliński



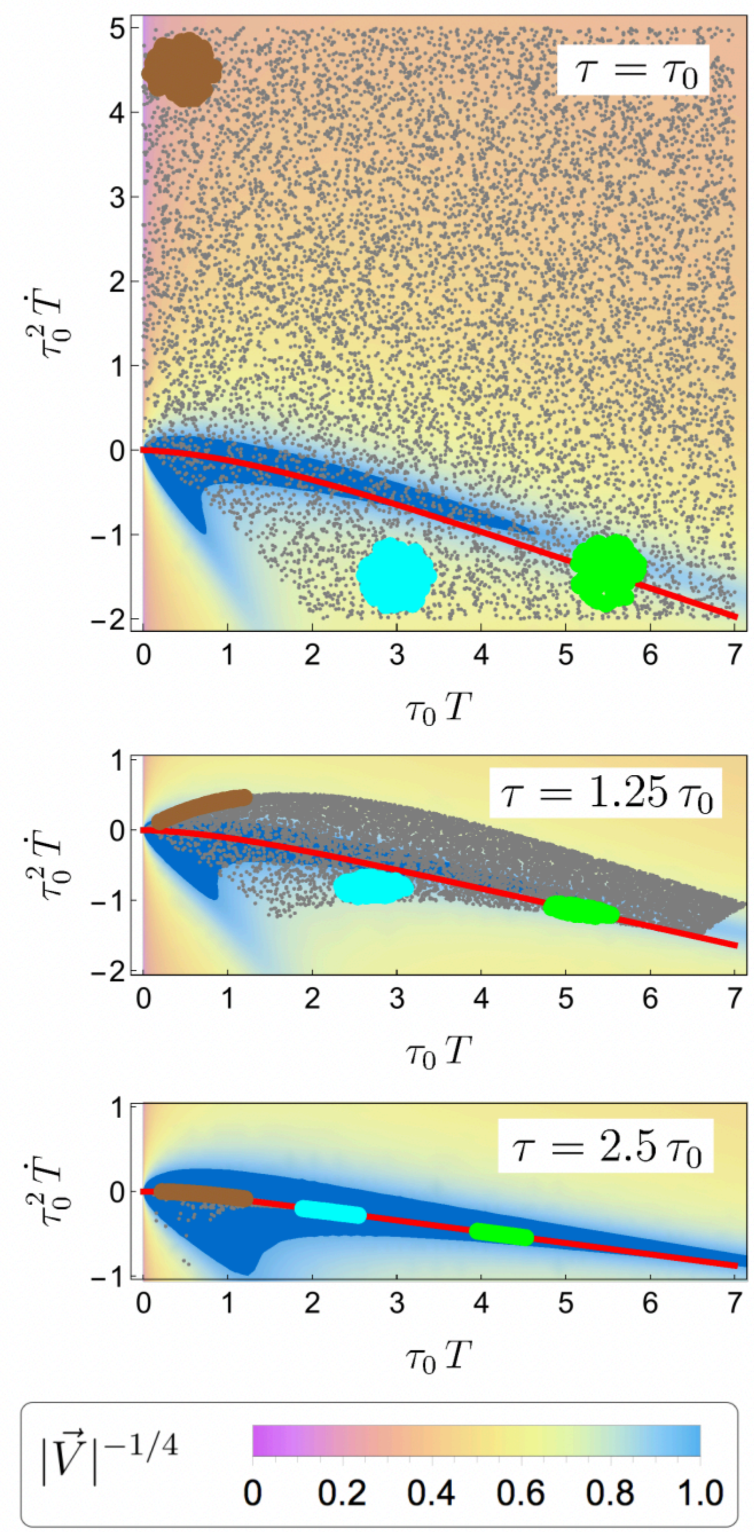
1-dimensional spread in A of some subset of states at a fixed value of w

as w grows →

becomes effectively 0-dimensional

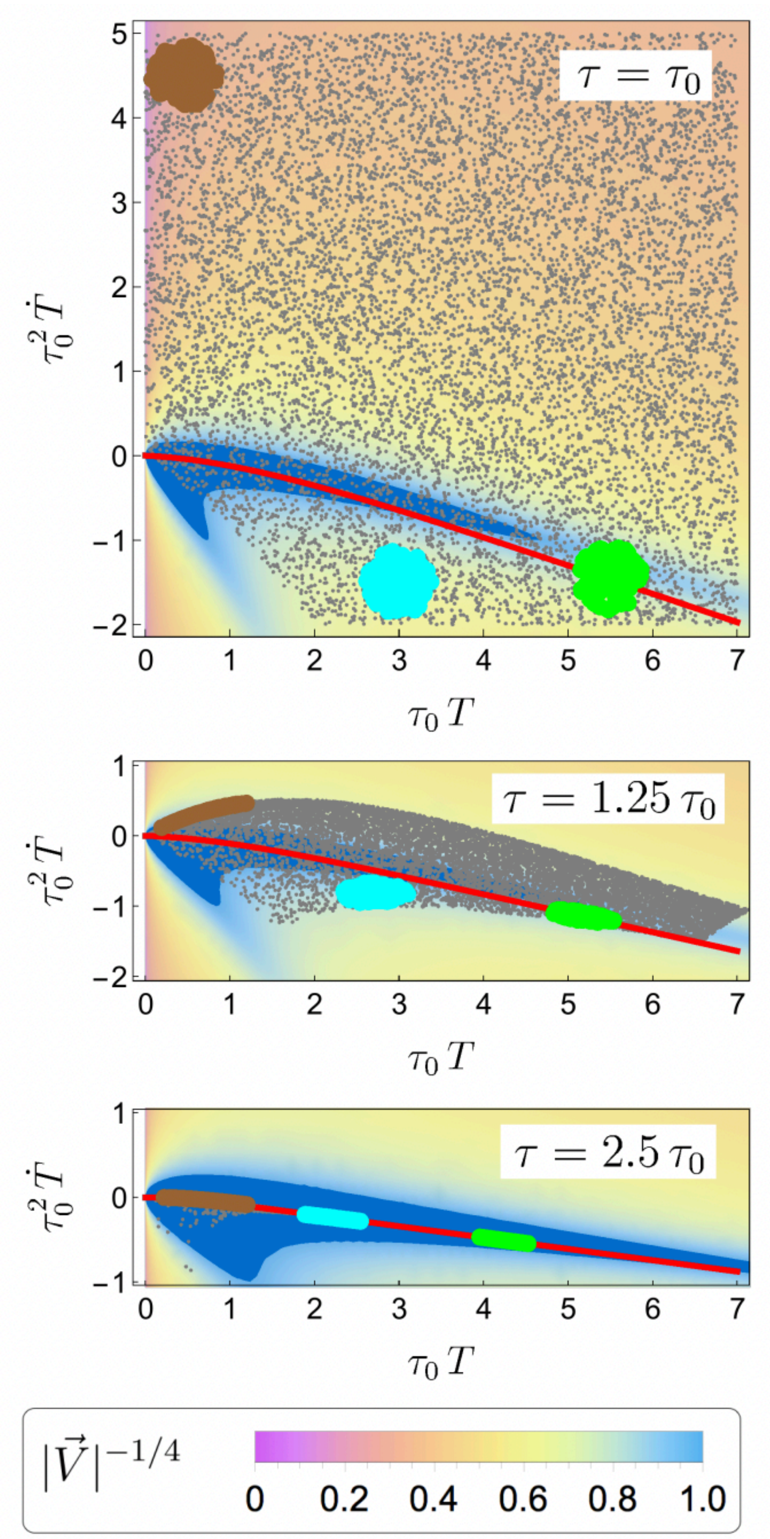
The hydro attractor in (an effective) phase space

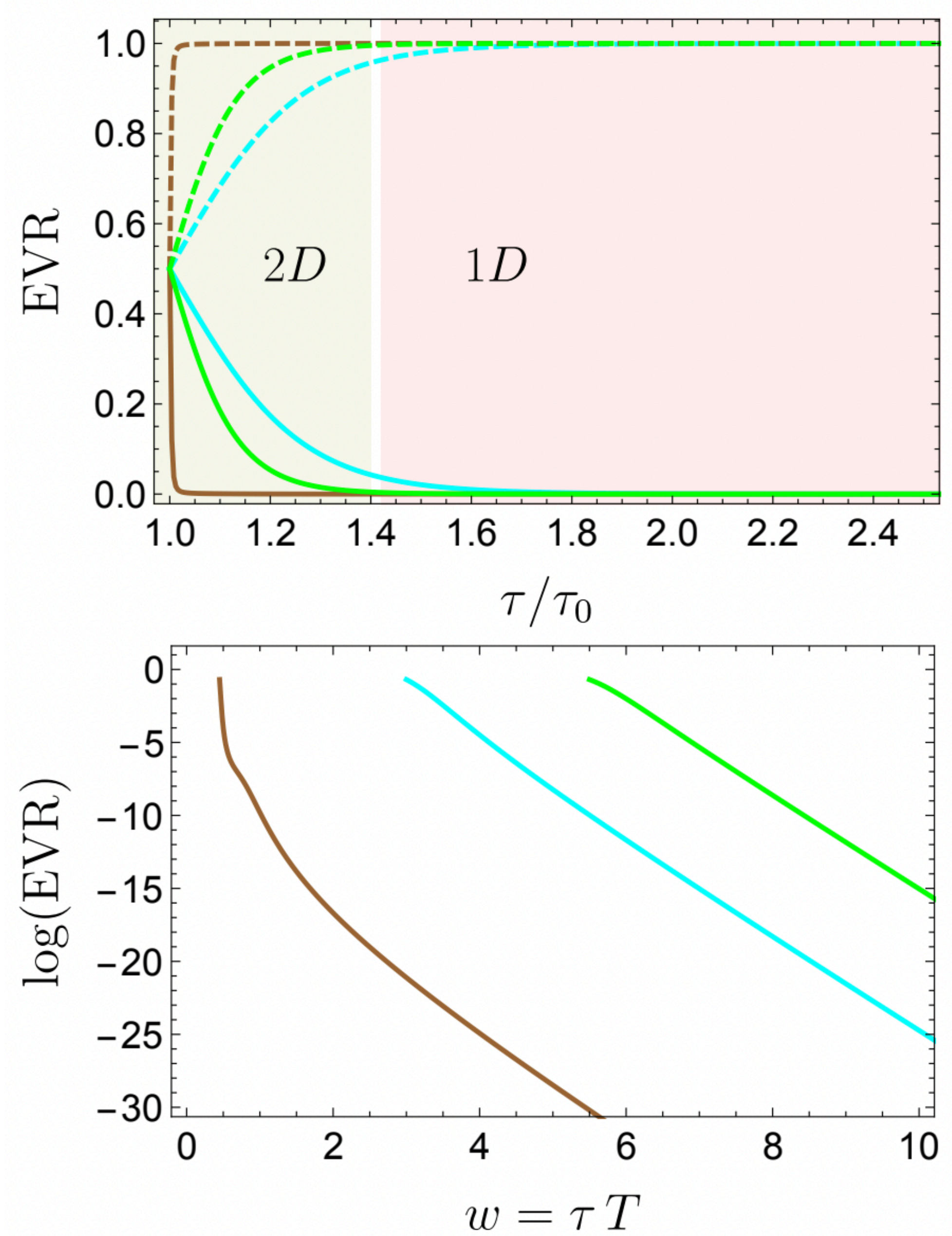
2003.07368 with Jefferson, Svensson, Spaliński



Principal component analysis for attractors

2003.07368 with Jefferson, Svensson, Spaliński





Are there fundamental bounds on transport coefficients in relativistic hydro?

Predominant transport philosophy

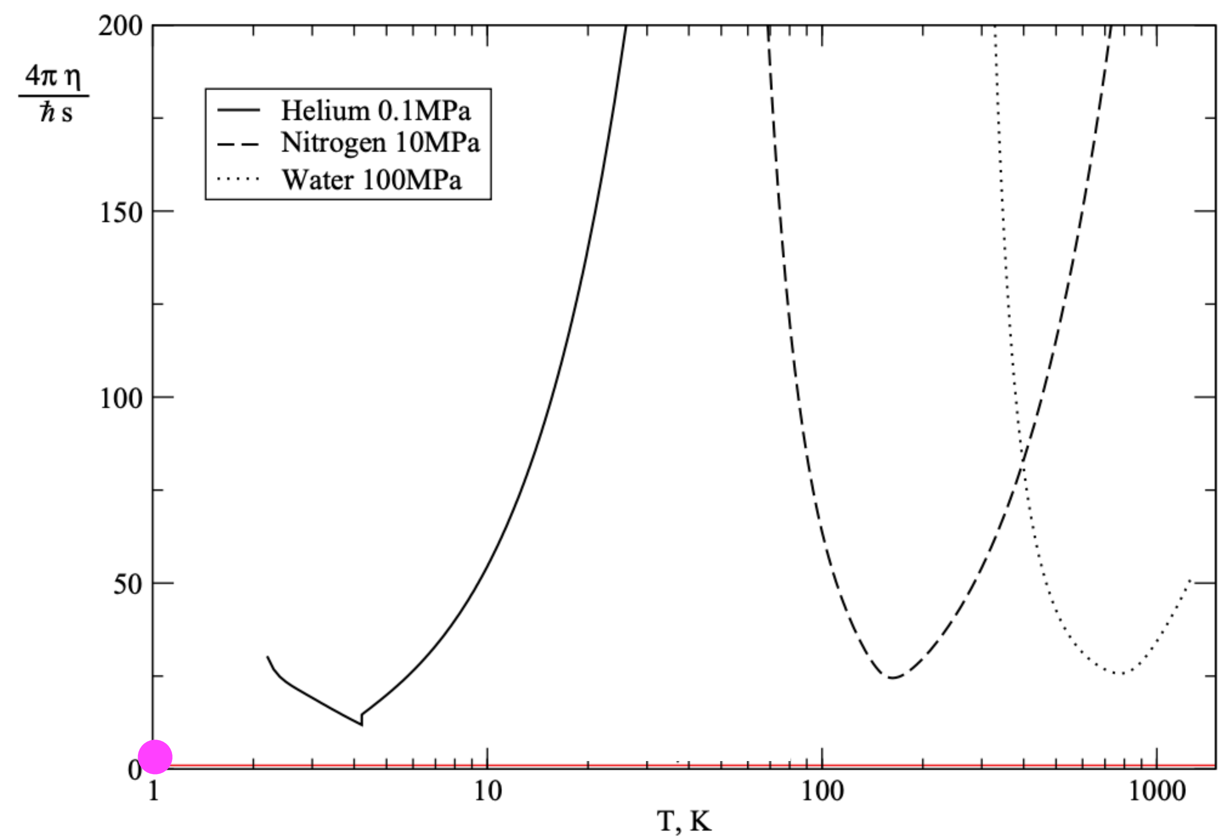
Mostly compute first and second order transport for various microscopics

But also the KSS bound conjecture

$$\frac{\eta}{s} \geq \frac{1}{4\pi}$$

:

hep-th/0405231 by Kovtun, Son, Starinets



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$$\frac{\eta}{s} \overset{?}{\gtrsim} \mathcal{O}\left(\frac{1}{4\pi}\right)$$

0812.2521 by Buchel, Myers, Sinha

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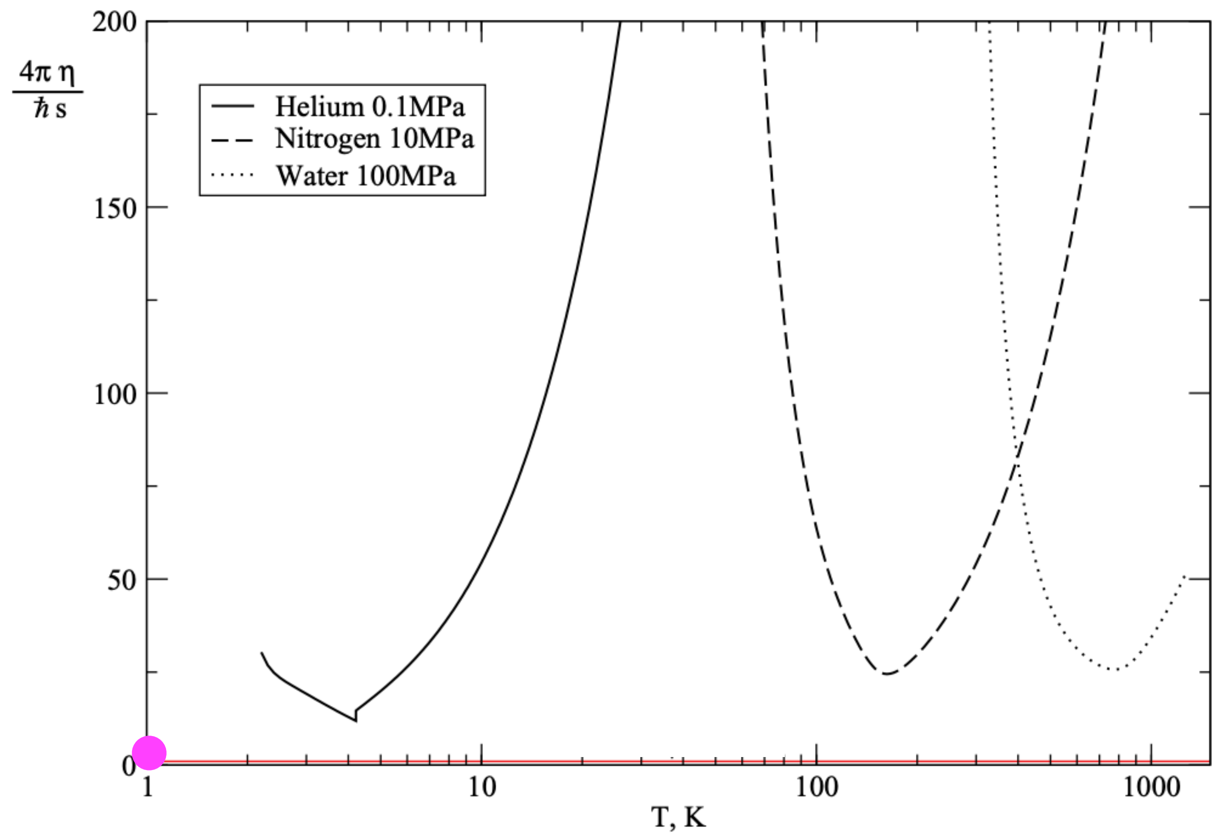

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New philosophy: bootstrapping transport

Hydrodynamic dispersion relations $\omega(p)$ appear as single poles of retarded correlators of conserved currents ($T^{\mu\nu}$, charge / particle number current)

Microscopic causality (Green's function support in the future lightcone) demands

$$-\mathrm{Im}\,\omega(p) \;+|\mathrm{Im}\,p| \geq 0 \quad \text{(in conventions } G_R(\omega,\vec{p}) \sim \int_{-\infty}^{\infty} dt \int d^3x\, e^{i\omega t - i\vec{p}\vec{x}}\, G_R(t,\vec{x})\text{)}$$

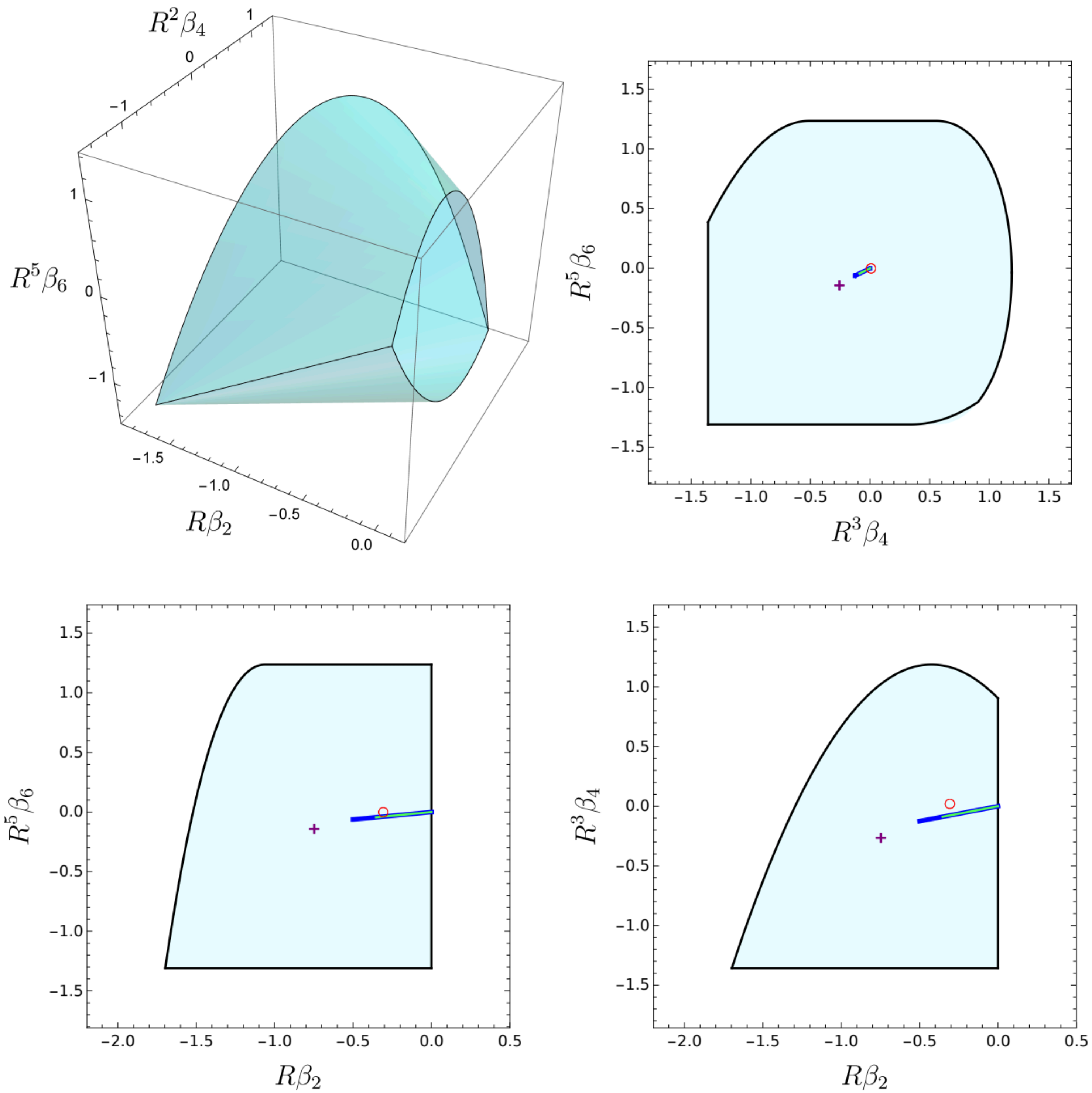
Introducing complex p leads to infinitely many independent inequalities

Bootstrap: using these inequalities to constrain transport coefficients

2212.07434 and **2305.07703** with Serantes, Spaliński and Withers

The hydrohedron: causally allowed transport

2305.07703 with Serantes, Spaliński and Withers



+ holographic *N*=4 SYM

○ conformal RTA Boltzmann

conformal Israel-Stewart

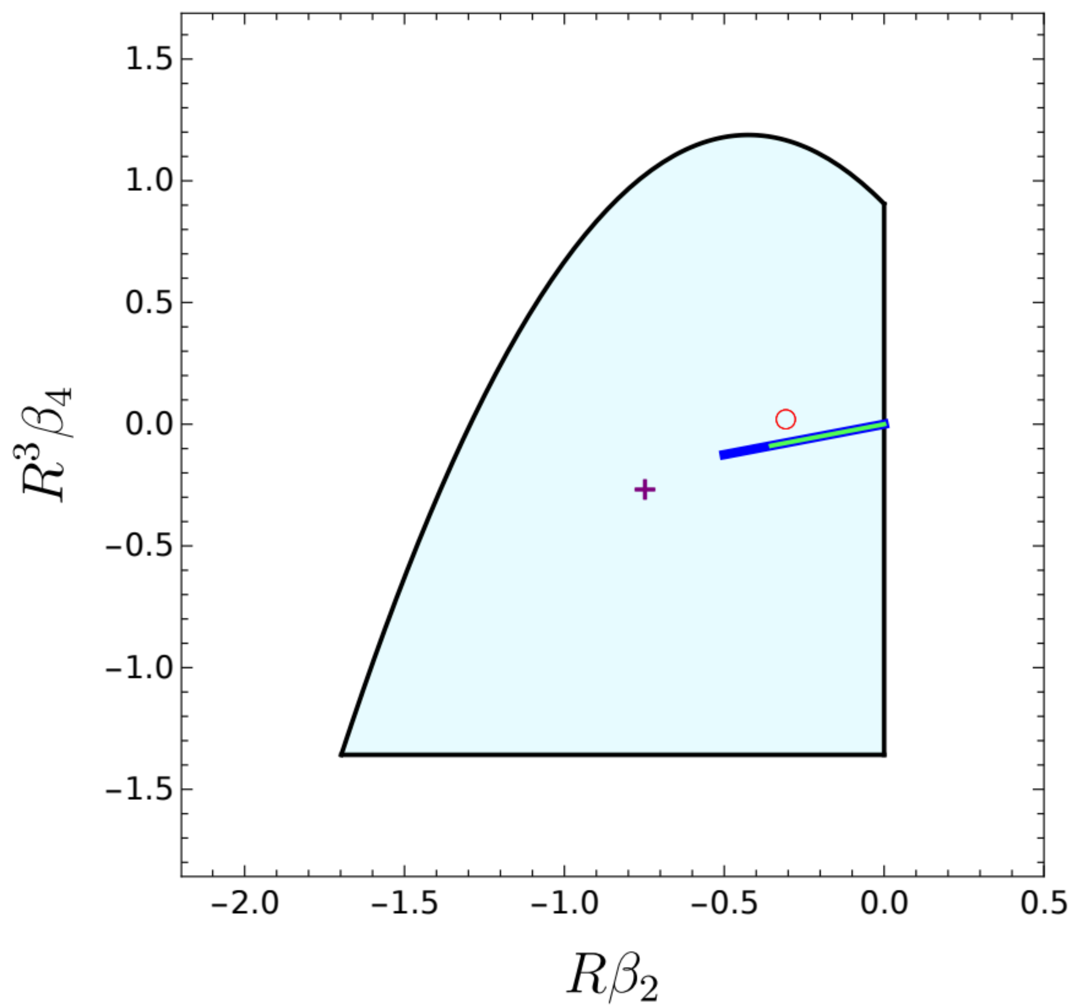
conformal BDNK

Comments on the hydrohedron

2305.07703 with Serantes, Spaliński and Withers

Hydrohedron has a universal shape regardless of a theory (~~fluctuations~~)

Axes normalized in terms of finite ($-\mathrm{Im}\,\omega(p)\ +|\mathrm{Im}\,p|\geq 0$) convergence radius $\boldsymbol{R}$



Causality does not lead to a nontrivial shear viscosity bound: $-R\beta_2 \equiv \dfrac{\eta}{s}\dfrac{R}{T} \geq 0$

Outlook

Outlook

In the past 10 years a lot of progress on understanding hydro constitutive relations near and far from local thermal equilibrium

This talk:

Data driven detection of hydro attractors as dimensionality reduction offers prospects to study them outside their native highly symmetric setting

2003.07368 with Jefferson, Spaliński and Svensson

Causality constraints hydrodynamics much more than thought to date and leads to a first generation of robust bounds on transport

2212.07434 and **2305.07703** with Serantes, Spaliński and Withers

Extra material

(added after the talk in response to one of questions)

Derivation of the causality inequality

Causality in a relativistic system: $G_R(t,\vec{x}) = 0$ for $t < |\vec{x}|$ as well as $t < 0$

This strongly constrains $G_R(\omega,\vec{p}) \sim \int_{-\infty}^{\infty} dt \int d^3x\, e^{i\omega t - i\vec{p}\vec{x}}\, G_R(t,\vec{x}) = \int_0^{\infty} dt \int_{|\vec{x}|<t} d^3x\, e^{i\omega t - i\vec{p}\vec{x}}\, G_R(t,\vec{x})$

as the singularities of $G_R(\omega,\vec{p})$ cannot lie where the Fourier integral converges

Let's look at the integrand for complex ω and p: $e^{-\mathrm{Im}\omega\, t + \mathrm{Im}p\, x\cos\theta} e^{i\mathrm{Re}\omega\, t - i\mathrm{Re}p\, x\cos\theta}\, G_R(t,\vec{x})$

Assuming $G_R(t,\vec{x})$ does not explode exp in time, we get for the convergence

$$e^{t\left(-\mathrm{Im}\omega + \mathrm{Im}p\,\frac{x\cos\theta}{t}\right)<0} \longrightarrow -\mathrm{Im}\,\omega + |\mathrm{Im}\,p| < 0$$

So all singularities (modes) $\omega(p)$ must obey $-\mathrm{Im}\,\omega(p) + |\mathrm{Im}\,p| \geq 0$