Towards a “bottom-up” construction of kinetic theory with spin

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Spin observable probe QGP

- Heavy-ion collision (HIC) creates QCD matter in extreme condition

○ Spin observables in HIC

- Informative Λ hyperon polarization

Xin-Nian Wang, Zuo-Tang Liang, PRL05';
Becattini et al, Annals Phys 13'

- Vector mesons ($K^{\star}$,ϕ, J/ψ)

e.g. STAR, 2204.02302;
ALICE PRL 20', 2204.10171

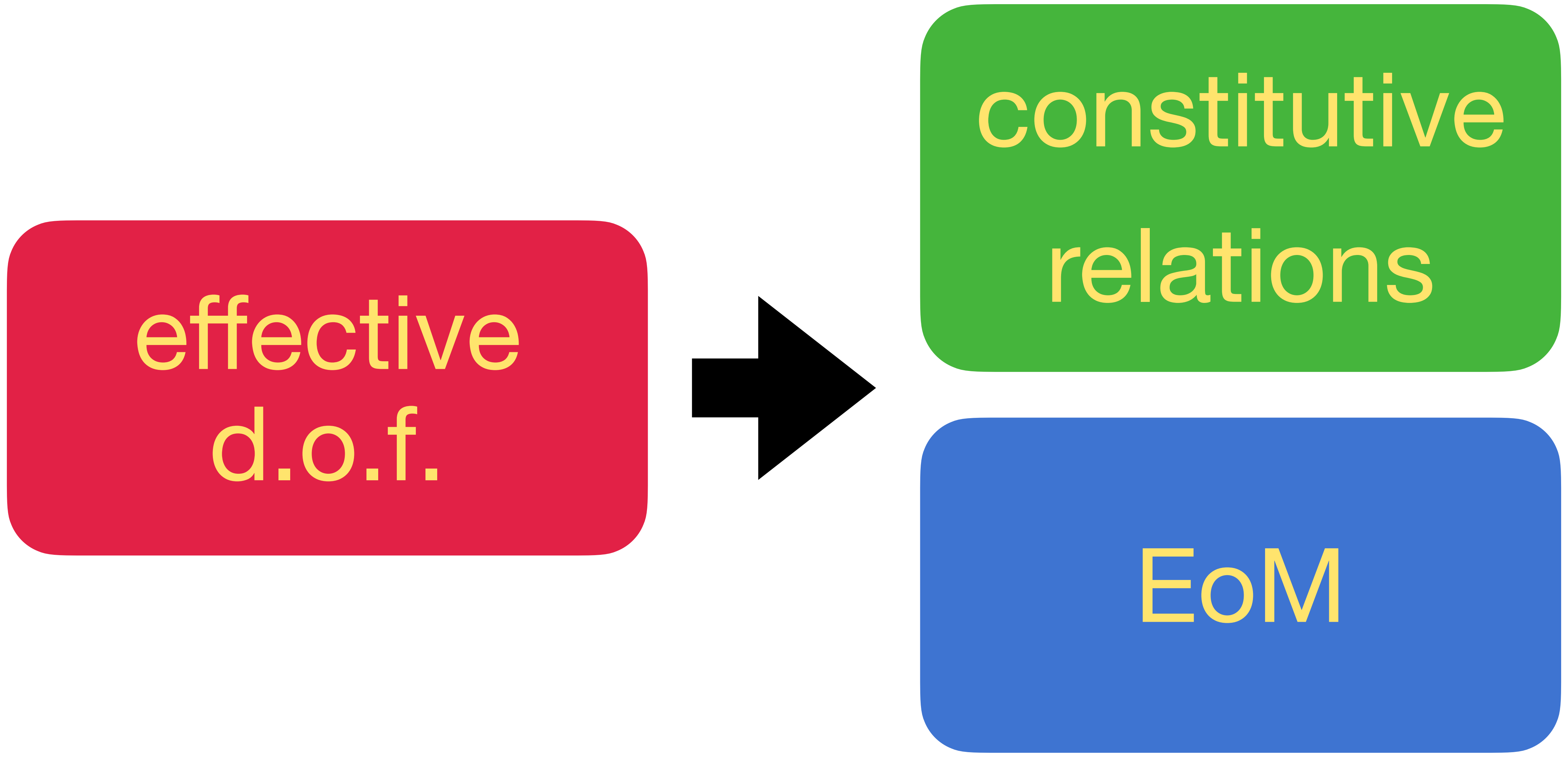
- an important way to probe the properties of QGP

Quantum Kinetic Theory

- Kinetic theory is an effective description of many-body system with spin
- “Top-down”: start from a microscopic theory and derive the EoM
- Modern view of effective theory by Landau:
  - Low-energy (IR) dynamics can be written down without details in the UV.
- Eg. hydrodynamics
- Can we construct the kinetic theory in a similar way?

“Bottom-up” Methodology

- Main steps



- Eg. hydrodynamics $\varepsilon, u^\mu \to T^{\mu\nu}[\varepsilon, u] = \varepsilon u^\mu u^\nu + P(\varepsilon)(g^{\mu\nu} + u^\mu u^\nu) + \mathcal{O}(\partial)$
- QKT : spin-average distribution $f(x, p)$, spin distribution $\boldsymbol{s}(x, p)$ (axial vector)

  not enough attention has been paid to the constitutive relations in QKT

Constitutive Relation

- Spirit of effective theory: write down all possible terms (obey symmetries) for observable $\hat{O}$ that constructed by f, s, momentum p and the external field ϕ (eg. EM field A_μ, gravity $h_{\mu\nu}$).

$$\hat{O} = \hat{O}[f, s, \phi, p]$$

- Assume $\tau_R^{-1} \ll \partial \ll \Lambda_{\rm eff}$ (typical energy scale of particles)

- Formulate it up to $\mathcal{O}(\partial)$ & $\mathcal{O}(\phi)$

Constitutive Relation

- Eg. electric current under $\boldsymbol{E}$:

$$\boldsymbol{J} = \underline{\boldsymbol{v}f + D\nabla f} + \underline{(\Delta \boldsymbol{J})_{\text{ext}}} + \mathcal{O}(\partial^2)$$

$$(\Delta \boldsymbol{J})_{\text{ext}} = \sigma \boldsymbol{E} \sim \mathcal{O}(\partial)$$

turn off $\boldsymbol{E}$, $(\Delta \boldsymbol{J})_{\text{ext}} = 0$; $\boldsymbol{v}f + D\nabla f$ still exists, which is a general relation.

- We can separate the constitutive relation into two parts

$$\hat{O}[f, s, \phi, p] = \hat{O}_{\text{dyn}}[f, s, p] + \hat{O}_{\text{ext}}[\phi, p]$$

dynamical & general — local/direct response to ϕ

Largely inspired by Jingyuan Chen, Dam T. Son, Annals Phys. 377 (2017)

Spin Current

$$\mathscr{A}^\mu = (v_\perp^\mu + c_1 n^\mu)(v \cdot s) + c_2 s^\mu + c_3 \epsilon^{\mu\nu\rho\sigma} n_\nu v_\rho \partial_\sigma f + (\Delta \mathscr{A})^\mu_{\text{ext}}$$

spin current parallel to $\boldsymbol{v}$

spin charge density

spin current in different directions with $\boldsymbol{v}$

momentum and off-equilibrium generate spin

eg. spin Hall effect $\boldsymbol{v} \times \boldsymbol{E}$

Covariant form: $\boldsymbol{s} \rightarrow s^\mu$

$\mathscr{A}^\mu$ can be derived from the Wigner function $W \sim \psi\bar{\psi}$, $\mathscr{A}^\mu \sim \mathrm{Tr}\{\gamma^\mu \gamma^5 W\}$

s^μ has only 3 physical d.o.f.

Gauge fixing: $n_\mu s^\mu = 0$, with a time-like vector field n^μ

Change of gauge fixing $\leftrightarrow$ redefinition of s^μ; $\mathscr{A}^\mu$ should not depend on gauge fixing.

f & $\boldsymbol{s}$ (4 d.o.f.) $\rightarrow$ observables

systematically consider all the possible effects which can induce spin

Massless EoM as a Heuristic

- left/right-handed distribution f_+, f_-

- the Boltzmann equation

$$\left[\partial_t + \boldsymbol{v} \cdot \partial_{\boldsymbol{x}} + \left(\boldsymbol{F}(\boldsymbol{x}) - \underline{\partial_{\boldsymbol{x}}\Phi_s(\phi)}\right) \cdot \partial_{\boldsymbol{p}}\right] f_s(t, \boldsymbol{x}; \boldsymbol{p}) = 0, \; (s = \pm\,)$$

- Φ_s: energy shift of different spin states generated by external field ϕ

Kinetic Equations

- EoM of f and s^μ

$$v^\mu \partial_\mu f + F_\mu(\phi)\partial_p^\mu f = 0$$

$$v^\nu \partial_\nu s^\mu + \Pi^\mu{}_\nu(\phi)\partial_p^\nu f = 0$$

$F_\mu(\phi)$ is the spin independent force generated by the external field;

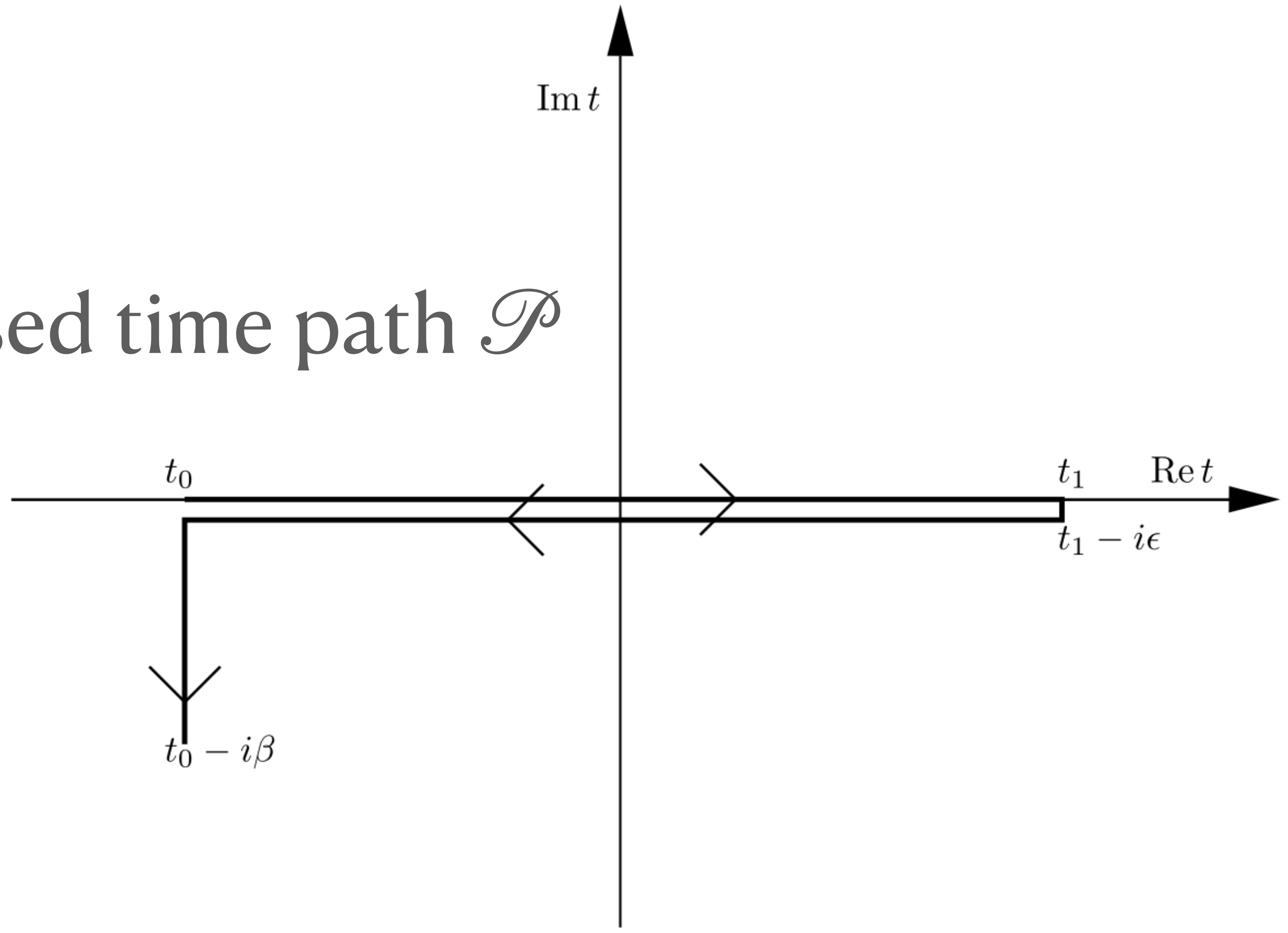
$\Pi^\mu{}_\nu(\phi)$ is an analogy of $\partial_{\boldsymbol{x}}\Phi_s(\phi)$. (combination of parity odd tensors constructed by the gradient of the external fields)

We had ignored the collision terms according to $\tau_R^{-1} \ll \partial \ll T_{\text{eff}}$

Spin Current from QFT

- Confirm the QKT by field theory calculation
- Schwinger-Keldysh formalism

Closed time path $\mathscr{P}$



$$W \sim \int_{\mathscr{P}} \mathscr{D}\psi \, \psi(x_+)\bar{\psi}(x_-) \, e^{iS[\psi,\phi]}$$

$$\mathscr{A}^\mu \sim \mathrm{Tr}\{\gamma^\mu \gamma^5 W\}$$ Expand it up to $\mathscr{O}(\partial)$ & $\mathscr{O}(\phi)$

$$W_\phi = W_0 + G_R \phi + L_\phi \phi, \quad \phi = A_\mu, h_{\mu\nu}$$

Equilibrium + Retarded Correlation + Gauge link

Non-locality

The retarded correlation function is non-local (can depend on the perturbation earlier than t)

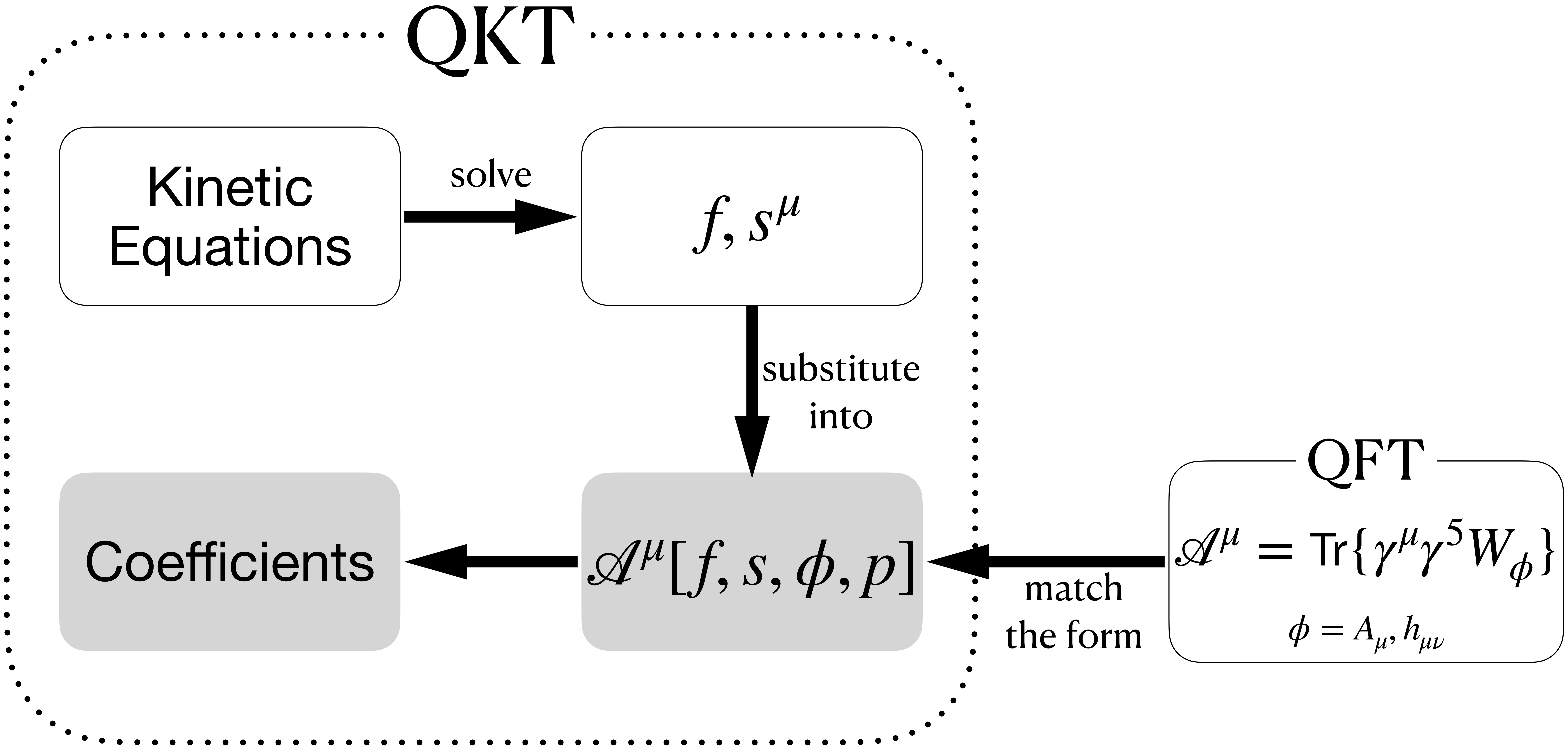
$$G_R \sim \langle \cdots \rangle \theta(t - t_0) \to \frac{(\dots)}{(p^0)^2 - E_{\boldsymbol{p}} + i\epsilon p^0} \qquad f, s^\mu \sim \frac{(\dots)}{\omega - \boldsymbol{k}\cdot\boldsymbol{v} + i\epsilon}$$

$v^\mu \partial_\mu$ in Fourier space

- The non-locality of linear response is absorbed by the distributions.

non-locality of G_R —absorbed by→ non-locality of f, s

Confirm QKT by QFT



Match the response to EM field

Using QED action in SK formalism

$$\mathscr{A}^\mu = (v_\perp^\mu + c_1 n^\mu)(v \cdot s) + c_2 s^\mu + c_3 \epsilon^{\mu\nu\rho\sigma} n_\nu v_\rho \partial_\sigma f + (\Delta\mathscr{A})^\mu_{\text{ext}}$$

$$c_1 = 1,\ c_2 = \frac{m^2}{(n \cdot p)^2},\ c_3 = \frac{1}{n \cdot p}$$

there are several terms in $(\Delta\mathscr{A})^\mu_{\text{ext}}$, eg. $\frac{f_0(p)}{(n \cdot p)^2} \tilde{F}^{\mu\nu} v_\nu^\perp \sim \boldsymbol{v} \times \boldsymbol{E}$ (spin Hall effect)

where $f_0(p)$ is the original distribution.

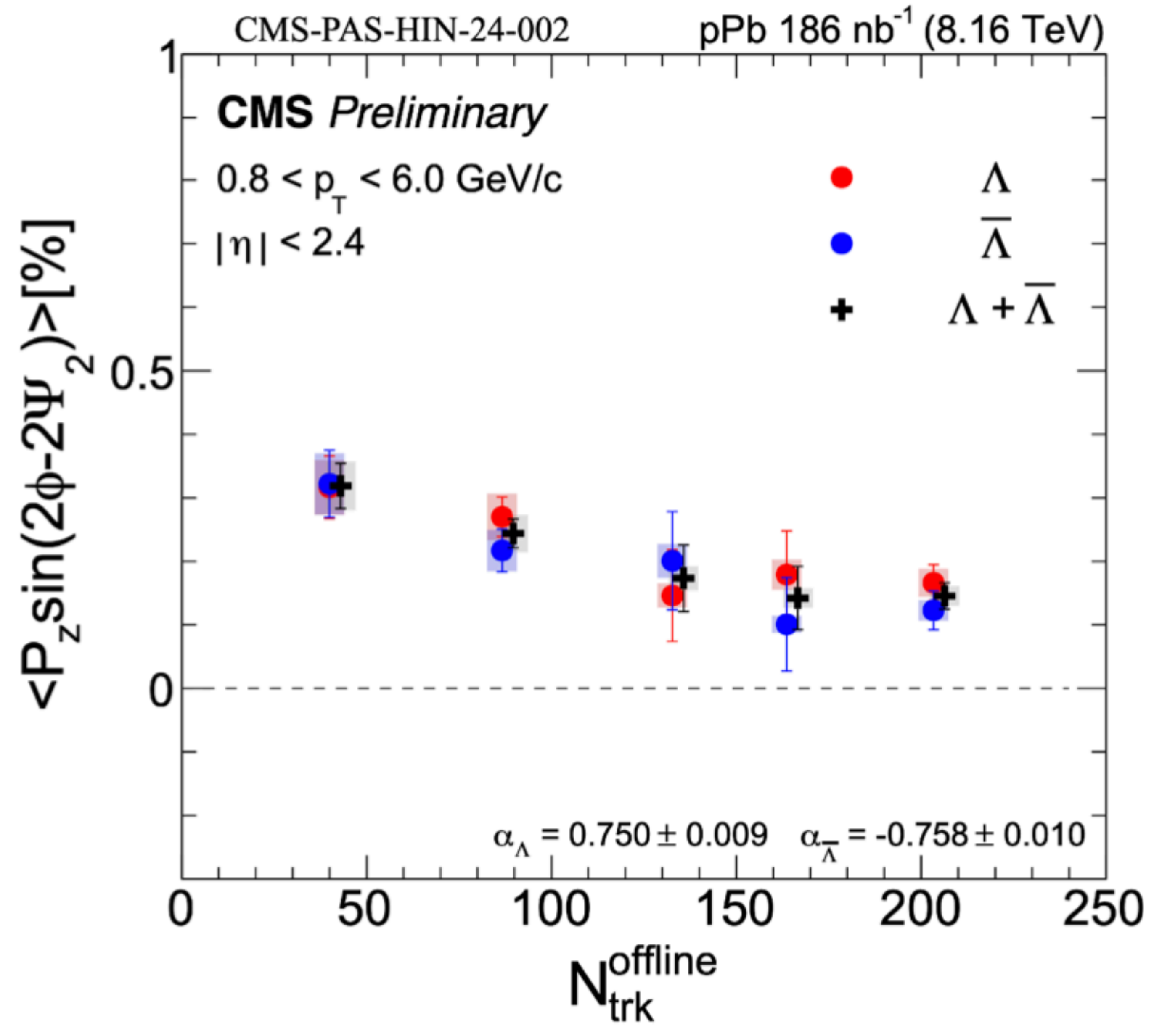
Spin generation in off-equilibrium

- Momentum and off-equilibrium distribution generate spin polarization (spin-motion correlation)

$$\mathscr{A} \propto v \times \partial f \quad \epsilon^{\mu\nu\rho\sigma} n_\nu v_\rho \partial_\sigma f$$

- Differential polarization is present even without hydro. flow.

Surprising p-Pb results



CMS at SQM 24'

Summary

- Effective "bottom-up" construction of QKT with spin

- D.o.f.: f & $\boldsymbol{s}$, 4 in total, less than traditional QKT;

- Observables: constitutive relations, systematically consider all the possible effects which can induce spin.

- The EoMs are simple to solve.

- Can be used in off-equilibrium system

THANK YOU!

Match the response to EM field

$$(\Delta\mathscr{A})^{\mu}_{\text{ext}} = (d_1 v^{\perp}_{\nu} + d_2 n_{\nu})\tilde{F}^{\mu\nu} + d_3(v_{\perp}^2 \Delta^{\mu\nu} - v^{\mu}_{\perp} v^{\nu}_{\perp})\tilde{F}_{\nu\rho} n^{\rho} + d_4 n^{\mu} \tilde{F}_{\nu\rho} v^{\nu}_{\perp} n^{\rho}$$

$$\tilde{F}^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$$

Linear response: $d_1 = \dfrac{f_0(p)}{(n\cdot p)^2},\ d_2 = 0,\ d_3 = \dfrac{f_0(p)}{(n\cdot p)^2},\ d_4 = 0$

$$f = \frac{iF_{\mu\nu} v^{\mu} \partial^{\nu}_{p} f_0}{v\cdot q + i\epsilon}, \quad s^{\mu} = \frac{i\Pi^{\mu}{}_{\nu} \partial^{\nu}_{p} f_0}{v\cdot q + i\epsilon}$$

$$\Pi^{\mu}{}_{\nu} = \frac{1}{2n\cdot p} \tilde{F}^{\mu\rho} n_{\rho} q_{\nu} \partial^{\nu}_{p} f_0(p)$$