The shear viscosity of parton matter under two-body scatterings

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Outline

1) Motivation
2) Comparison of η and η/s from different methods
3) Application to parton matter in the AMPT model
4) Conclusions

Mainly based on
Noah MacKay & ZWL,
Eur Phys J C 82, 918 (2022)

National
Science
Foundation

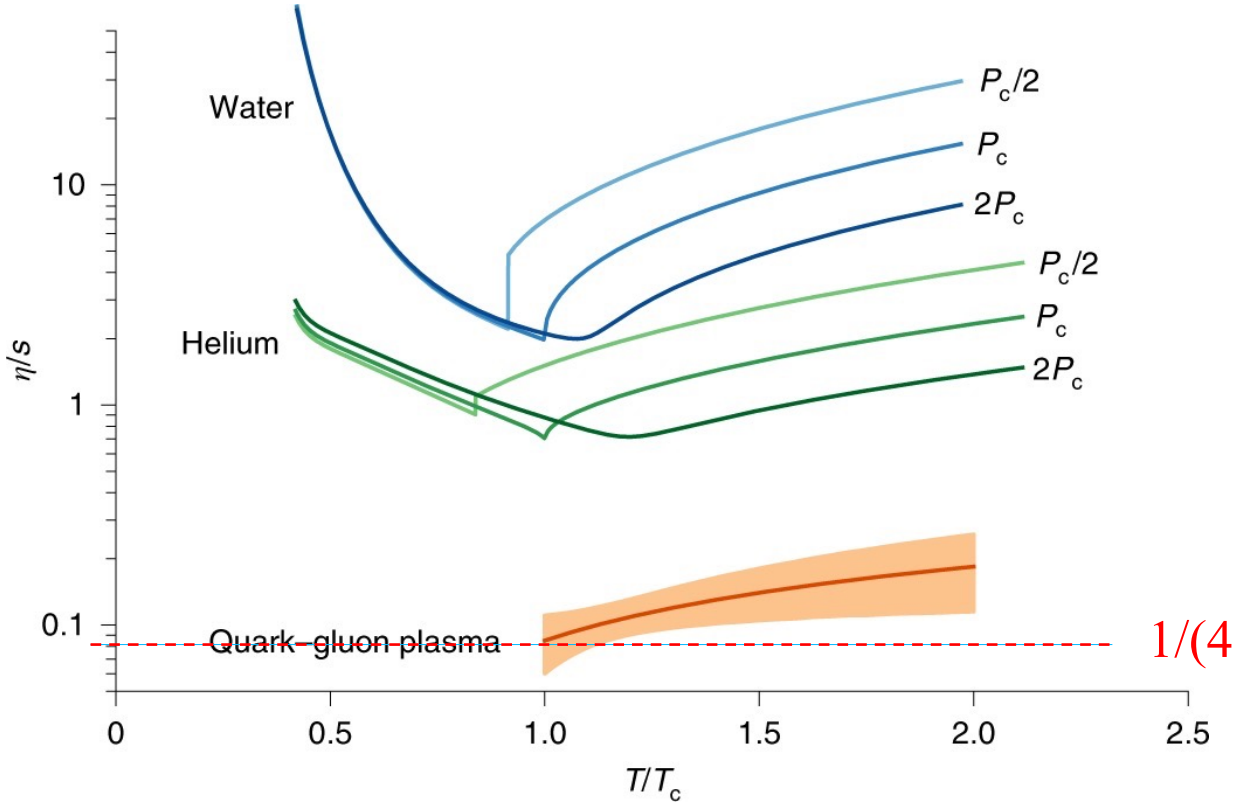
1) Motivation

Shear viscosity η is an important property of the quark–gluon plasma:

$$T^{\mu\nu} = eu^{\mu}u^{\nu} - (p + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

η *or* η/s: is an input function to viscous hydrodynamics;
is generated by interactions in kinetic theory: relation?

η/s can be extracted from data/model comparisons:



Bernhard, Moreland & Bass,
Nature Phys (2019)

$1/(4\pi)$ bound from AdS/CFT:

Kovtun, Son & Starinets,
Phys Rev Lett (2005)

Isotropic versus forward-angle scatterings

For parton 2-to-2 elastic scatterings:

- Isotropic scattering: $\frac{d\sigma}{d\Omega} = constant = \frac{\sigma}{4\pi}$

- Forward-angle scattering:
  As the example, we take the parton cross section used in AMPT/ZPC/MPC:

$$\frac{d\sigma}{d\hat{t}} = \frac{9\pi\alpha_s^2}{2}(1+a)\frac{1}{(\hat{t}-\mu^2)^2}$$

$a \equiv \frac{\mu^2}{\hat{s}}$ is added to obtain a $\hat{s}$-independent cross section: $\sigma = \frac{9\pi\alpha_s^2}{2\mu^2}$

This is based on the pQCD gg-gg cross section:

$$\frac{d\sigma}{d\hat{t}} \propto 3 - \frac{\hat{t}\,\hat{u}}{\hat{s}^2} - \frac{\hat{s}\,\hat{u}}{\hat{t}^2} - \frac{\hat{s}\,\hat{t}}{\hat{u}^2}$$ + screening mass μ

Isotropic versus forward-angle scatterings

Transport cross section σ_{tr} often appears in shear viscosity expressions:

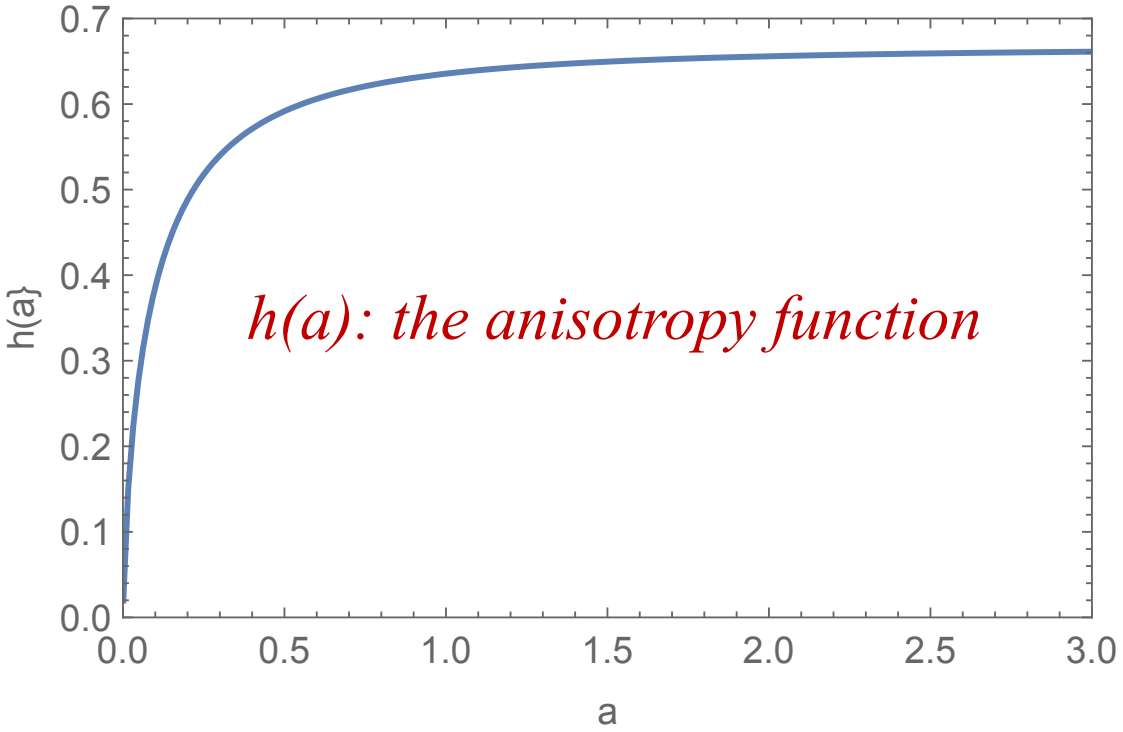
$$\sigma_{tr} \equiv \int d\sigma \sin^2 \theta_{cm}$$

θ_{cm}: scattering angle in the 2-parton CM frame

Molnar & Gyulassy, Nucl Phys A (2002)

- Isotropic scattering: $\sigma_{tr} = \frac{2}{3}\sigma$
- Forward-angle scattering: $\sigma_{tr} = 4a(1+a)\left[(1+2a)\, ln\left(1+\frac{1}{a}\right) - 2\right]\sigma$

$$\equiv h(a)\,\sigma$$



$$a \equiv \frac{\mu^2}{\hat{s}}$$

$\ll 1$ very forward, $h(a) \to 0$: *scatterings are less effective*

$\gg 1$ ~isotropic, $h(a) \to 2/3$

Isotropic versus forward-angle scatterings

Thermal average:

even if σ is a constant, σ_{tr} is not since it depends $a \equiv \frac{\mu^2}{\hat{s}}$.

For a parton matter in thermal equilibrium at temperature T,
the thermal average (for Boltzmann statistics) is Kolb & Raby, Phys Rev D (1983)

$$\langle \sigma_{tr} \rangle = \frac{\sigma}{32} \int_0^\infty du \, [\, u^4 K_1(u) + 2u^3 K_2(u)] \, h\left(\frac{w^2}{u^2}\right)$$
$$\equiv \sigma \, h_0(w)$$

K_n: Bessel functions

$w \equiv \frac{\mu}{T}, u \equiv \frac{\sqrt{\hat{s}}}{T}$

$h_0(w)$ is just an average of the anisotropy function $h(a)$,
$h_0(w) \to 0$ for $w \ll 1$
$h_0(w) \to 2/3$ for $w \gg 1$

2) Comparison of η and η/s from different methods

Analytical:

- Israel–Stewart (IS) method: $\eta^{IS} = \frac{6T}{5\sigma}$ for isotropic scatt.

  Huovinen & Molnar, Phys Rev C (2009)

- Navier–Stokes (NS) method: $\eta^{NS} \approx 1.2654\frac{T}{\sigma}$ for isotropic scatt.

  de Groot, van Leeuwen & Weert book (1980)

- Relaxation time approximation (RTA) & modified version (RTA*):

$$\eta^{RTA} = \frac{4T}{5\sigma} \qquad \eta^{RTA*} = \frac{6T}{5\sigma}$$ for isotropic scatt.

Anderson & Witting, Physica (1974) — Plumari, Puglisi, Scardina & Greco, Phys Rev C (2012)

- Chapman–Enskog (CE) method: $\eta^{CE} = \frac{T\,\gamma_0^2}{10\,c_{00}}$, ... Wiranata & Prakash, Phys Rev C (2012)

Numerical:

- Green–Kubo relation: $\eta = \frac{V}{T}\int_0^\infty dt < \bar{\pi}^{xy}(t+t')\bar{\pi}^{xy}(t') > = \frac{4}{15}\varepsilon\tau$

τ: relaxation time extracted from correlation <…>(t)

2) Comparison of η and η/s from different methods

Analytical:

- Israel–Stewart (IS) method: $\eta^{IS} = \frac{4T}{5\langle\sigma_{tr}\rangle} = \frac{4T}{5\sigma\, h_0(w)}$

Huovinen & Molnar, Phys Rev C (2009)

- Navier–Stokes (NS) method: $\eta^{NS} \approx 0.8436 \frac{T}{\langle\sigma_{tr}\rangle}$

generalized to anisotropic scatt. using σ_{tr} instead of σ

- Relaxation time approximation (modified version RTA*):

$$\eta^{RTA*} = \frac{4T}{5\langle\sigma_{tr} v_{rel}\rangle}$$

Plumari, Puglisi, Scardina & Greco, Phys Rev C (2012)

- Chapman–Enskog (CE) method: $\eta^{CE} = \frac{T\,\gamma_0^2}{10\, c_{00}}$, ...

Wiranata & Prakash, Phys Rev C (2012)

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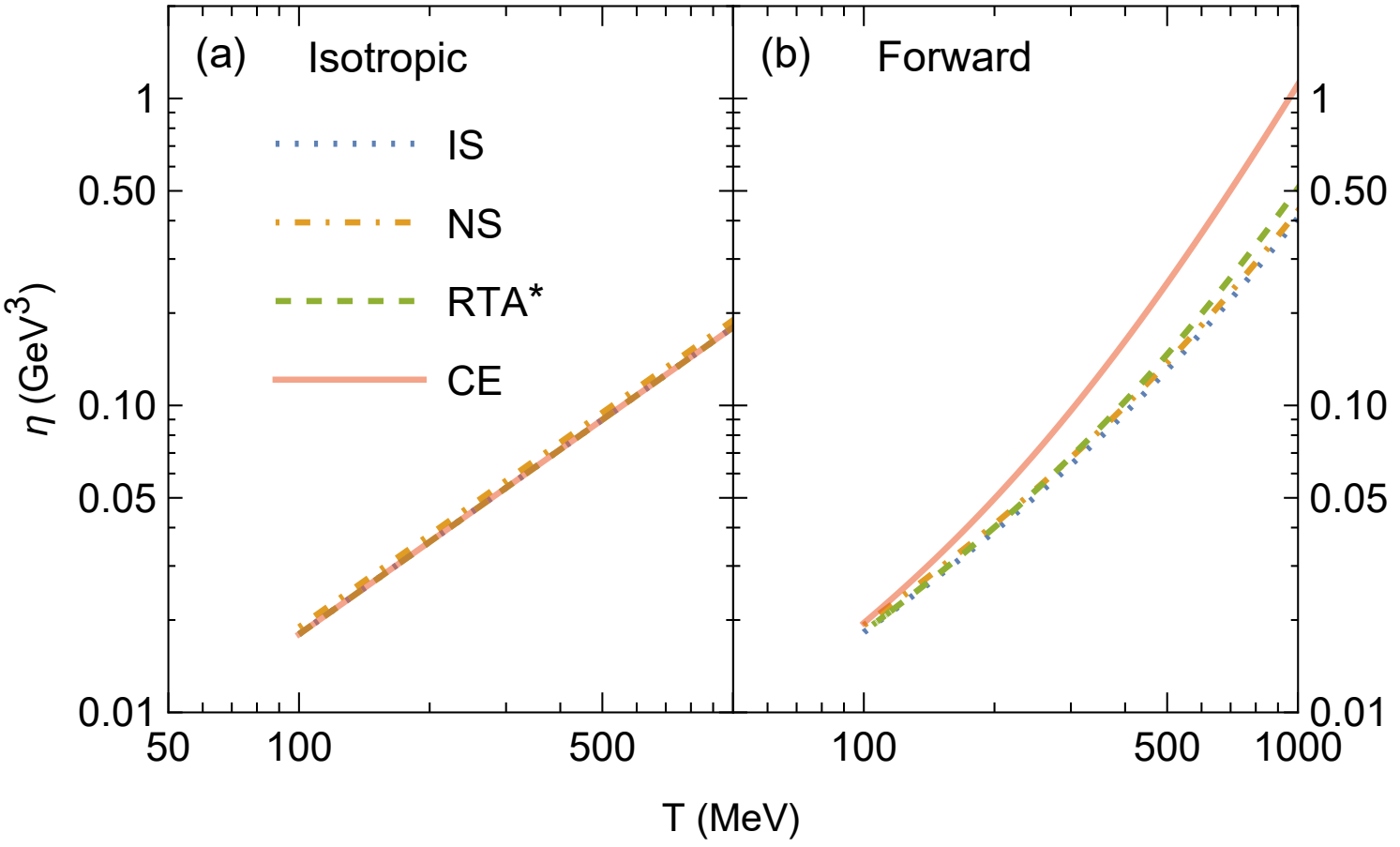
2) Comparison of η and η/s from different methods

Analytical results of η MacKay & ZWL, Eur Phys J C (2022)

for massless gluons & σ=2.6 mb (or μ~0.7GeV at $\alpha_s \approx 0.47$):



- For isotropic scatterings:

IS=RTA*=CE

≈NS (~5% higher)

- For forward scatterings:

IS≈RTA*≈NS < CE *mostly*

T<<μ → almost isotropic

2) Comparison of η and η/s from different methods

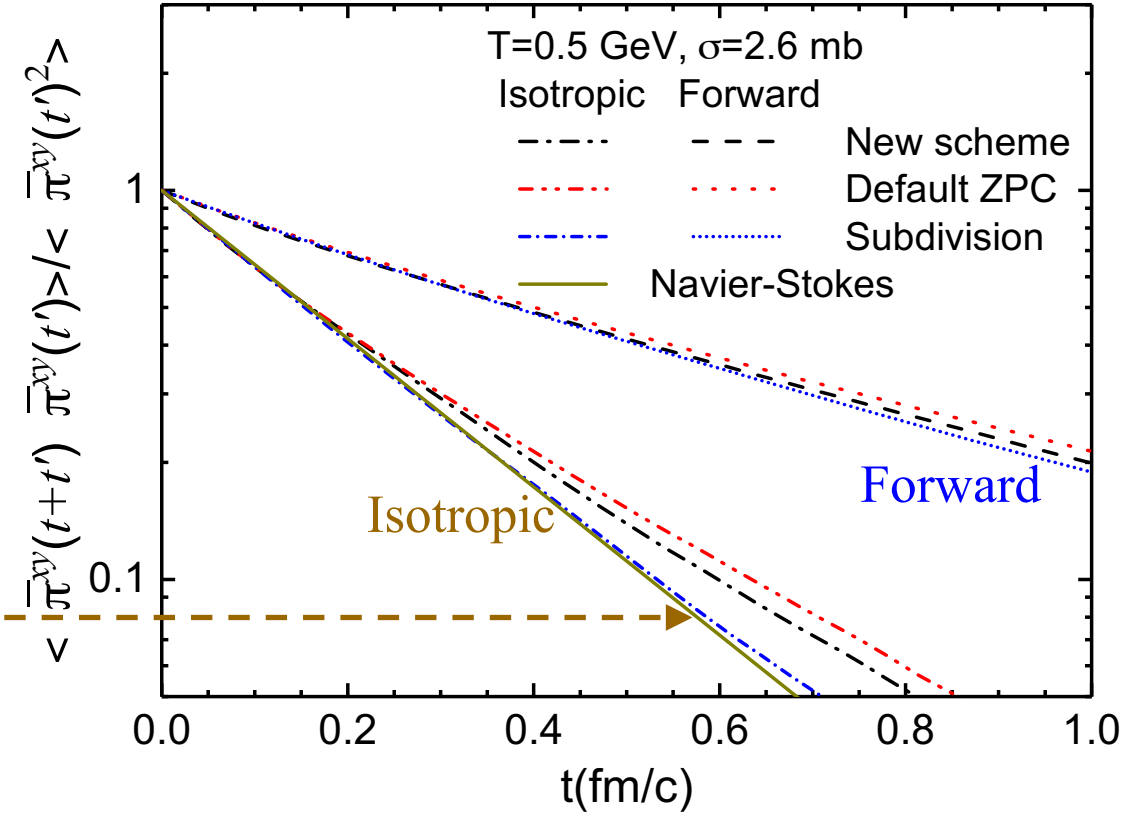
Q: which analytical result of η is accurate?

A: compare with numerical results from Green-Kubo:

$$\eta = \frac{V}{T}\int_0^\infty dt < \bar{\pi}^{xy}(t+t')\bar{\pi}^{xy}(t') > = \frac{4}{15}\varepsilon\tau$$

With ZPC parton cascade, we have calculated η of gluons in a box with the Green-Kubo relation.

Subdivision (*with factor* $l=10^6$): results should be accurate (*no causality violation from cascade*), agree well with NS expectation for isotropic scatterings.

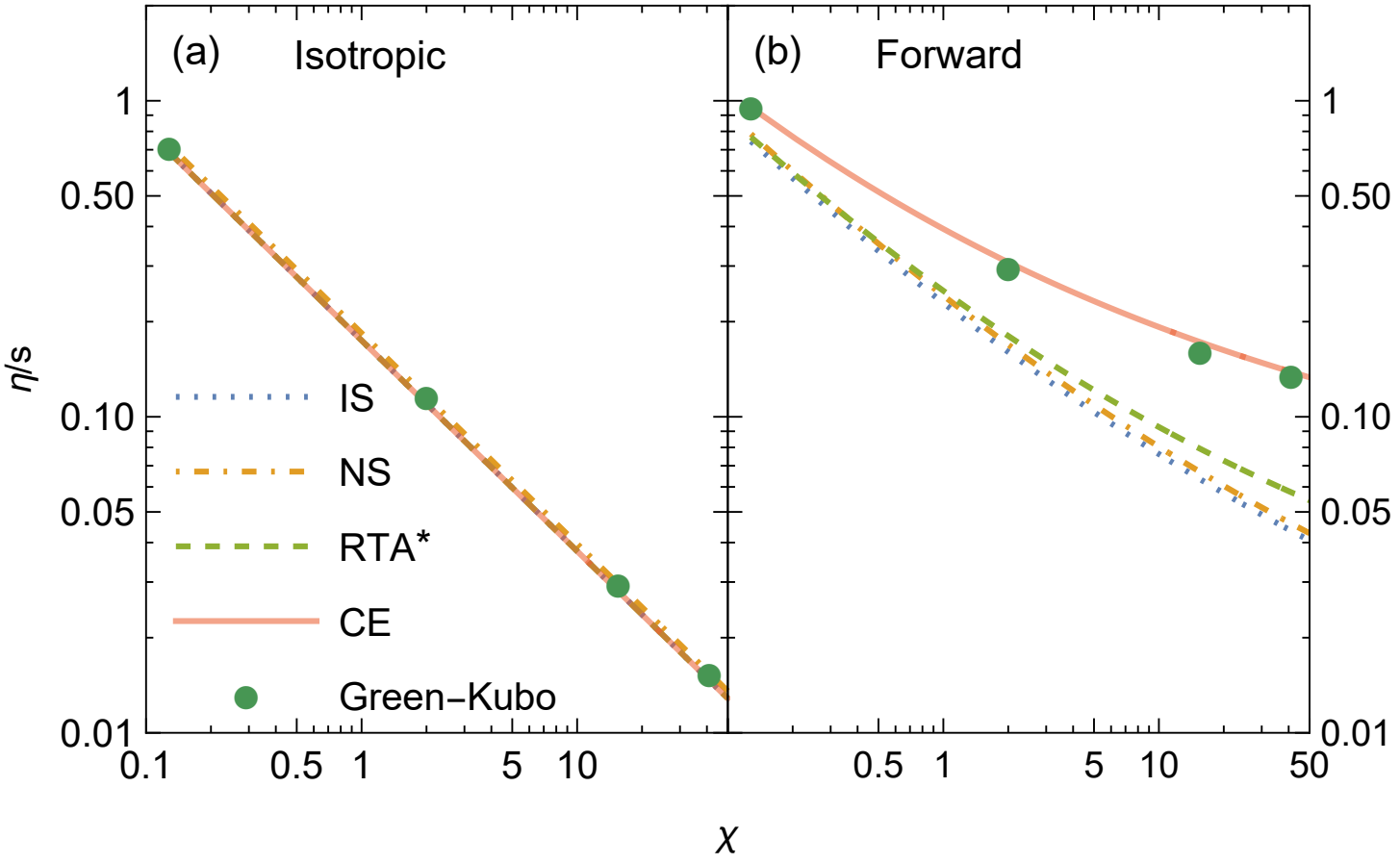


Zhao, Ma, Ma & ZWL, Phys Rev C (2020)

2) Comparison of η and η/s from different methods

Compare 4 analytical methods
with Green-Kubo results for η/s versus χ:

MacKay & ZWL, Eur Phys J C (2022)



χ (opacity parameter)
≡ interaction radius/mfp

$$\chi = \sqrt{\frac{\sigma}{\pi}}/\lambda = n\sqrt{\frac{\sigma^3}{\pi}}$$

Zhang, Gyulassy & Pang,
Phys Rev C (1998)

- For isotropic scatterings:
  all methods agree well.
- For anisotropic scatterings:
  CE agrees well with Green-Kubo;
  but the other analytical methods
  are not accurate.

2) Comparison of η and η/s from different methods

The fact that Green-Kubo agrees with CE
(but not with RTA*) has been shown in
Plumari, Puglisi, Scardina & Greco, Phys Rev C (2012)
despite two typos in the eta formulae.

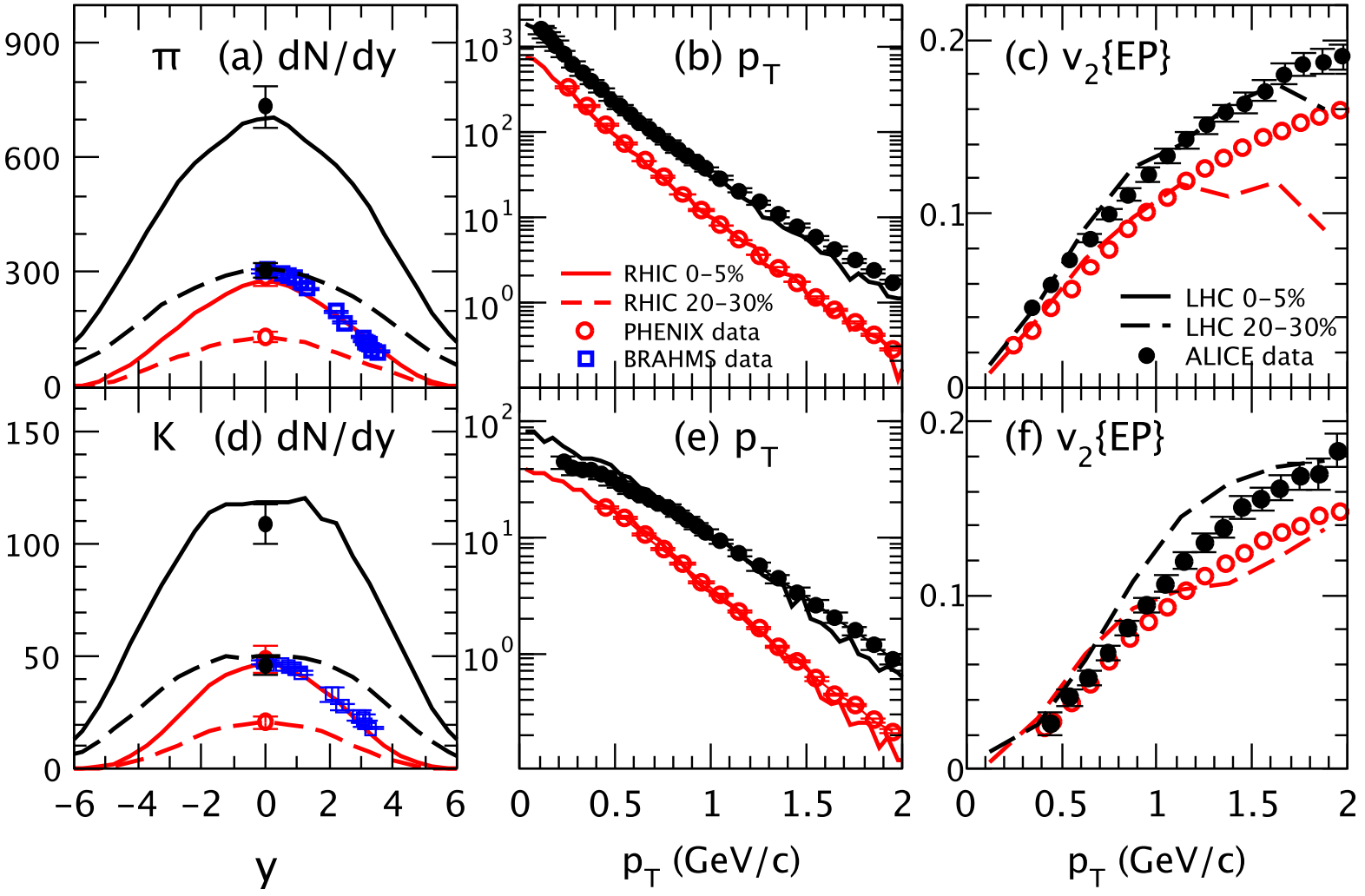
Wiranata, Koch, Prakash & Wang, Phys. Rev. C 88 (2013)
extended the CE formula to a mixture of hadrons under elastic scatterings.

3) Application to parton matter in the AMPT model

We now apply the Chapman–Enskog (CE) method to study
η and η/s of the parton matter in the string melting AMPT model for A+A.

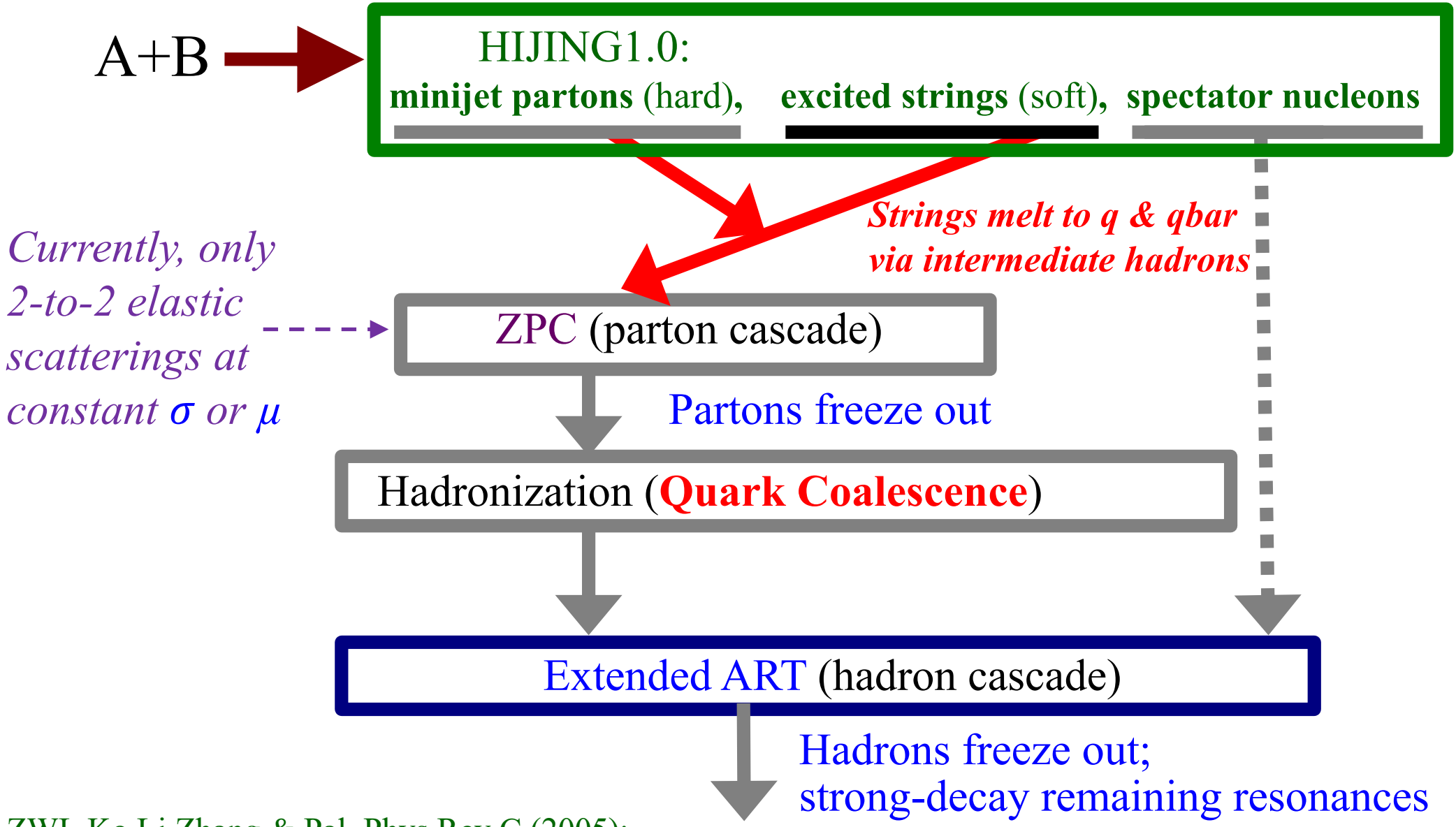
The AMPT model can reasonably describe
the bulk matter observables at low p_T in A+A collisions:

ZWL, Phys Rev C (2014)



3) Application to parton matter in the AMPT model

Structure of the String Melting version of AMPT:



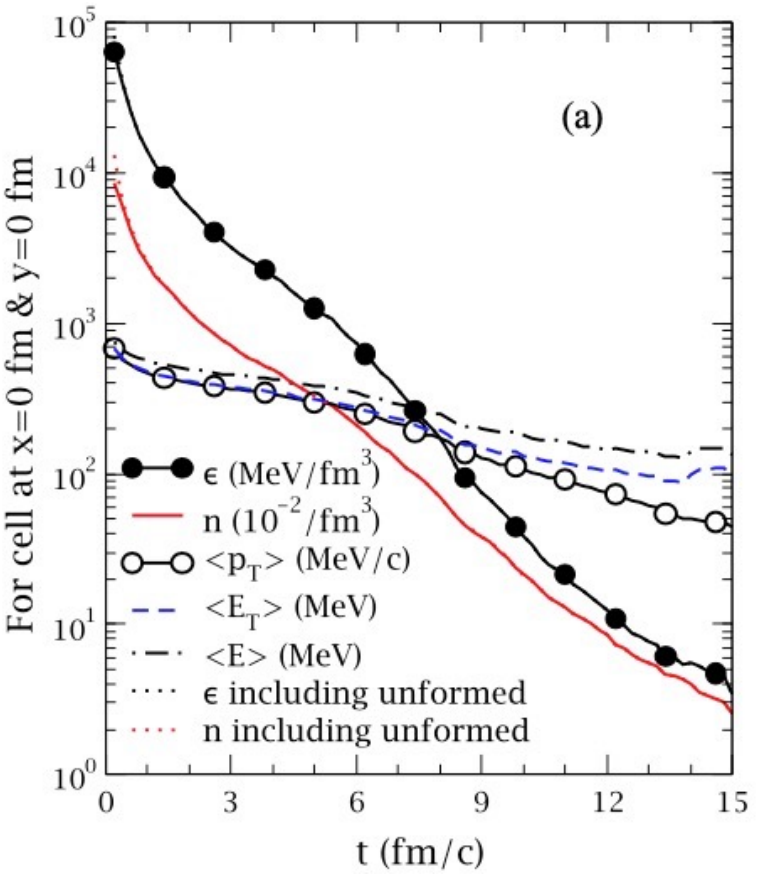
ZWL,Ko,Li,Zhang & Pal, Phys Rev C (2005);
ZWL & Zheng, Nucl Sci Tech (2021);
https://myweb.ecu.edu/linz/ampt/

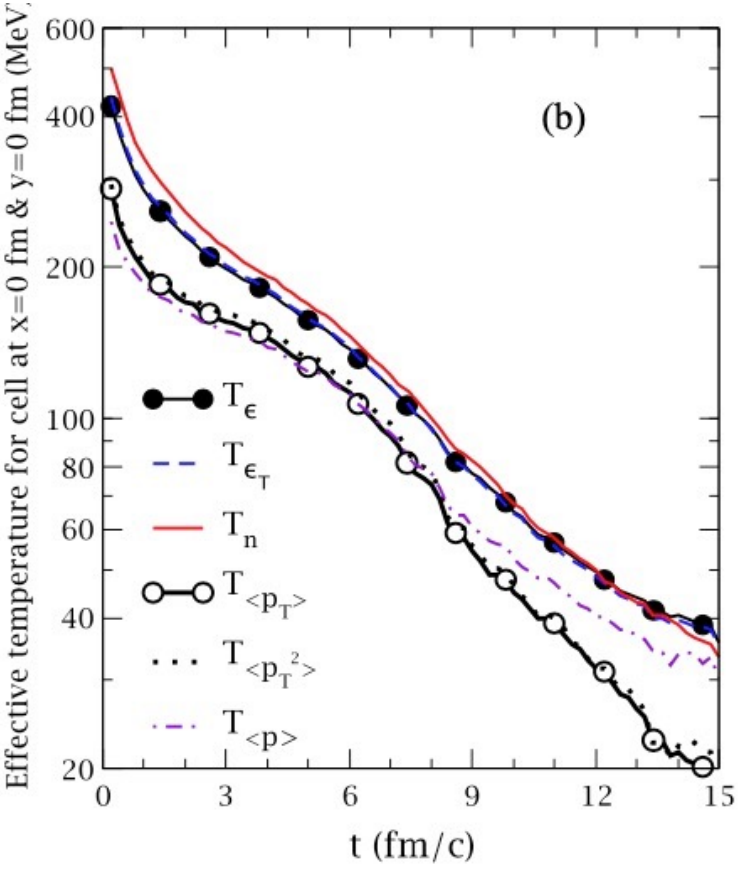
3) Application to parton matter in the AMPT model

For parton matter in the center cell,
we have extracted the effective temperatures.

ZWL, Phys Rev C (2014)

For example, central Au+Au at 200A GeV:





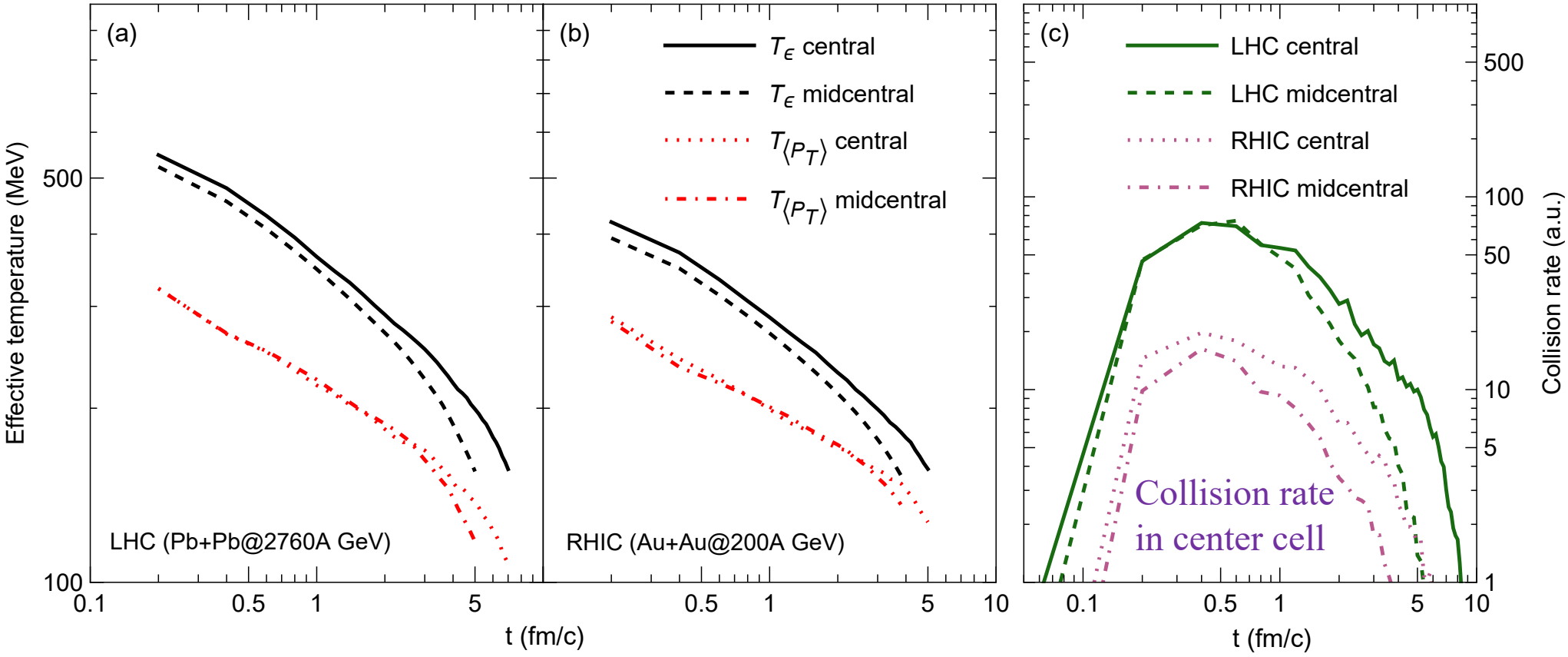
using $\varepsilon = \frac{3g_B}{\pi^2} T_\varepsilon^4$, $T_{\langle p_T \rangle} = \frac{4}{3\pi} \langle p_T \rangle$, ...

$T_{\langle p_T \rangle} < T_\varepsilon$ → the parton matter is not in chemical equilibrium.

3) Application to parton matter in the AMPT model

We have extracted effective temperatures $T_{\langle p_T \rangle}$ & T_{ε} of the center cell for 4 different collision systems:

ZWL, Phys Rev C (2014)



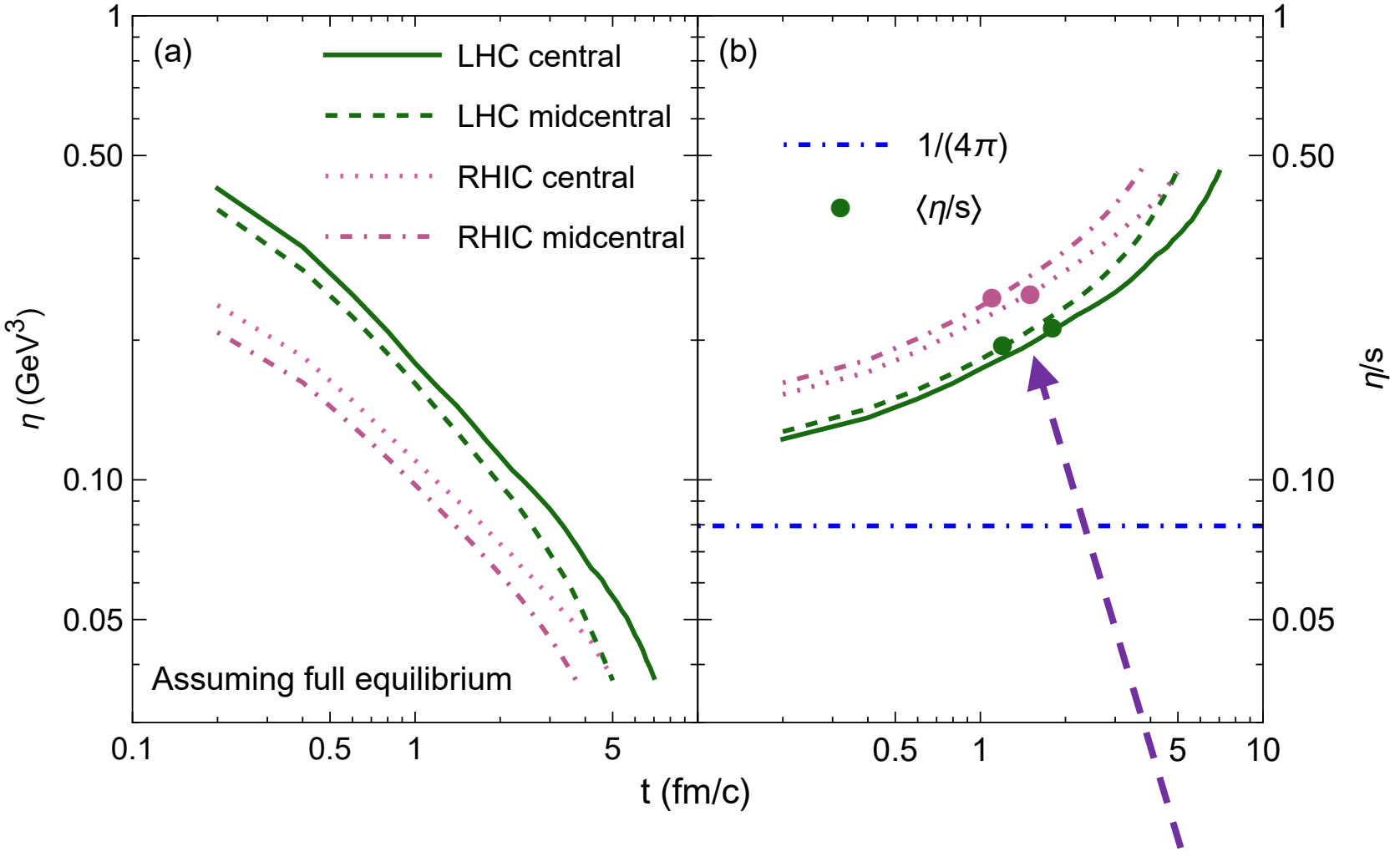
MacKay & ZWL, Eur Phys J C (2022)

We use these temperatures to calculate η and η/s of the center cell

(a volume around mid-pseudorapidity with 1 fm^2 in transverse area)

3) Application to parton matter in the AMPT model

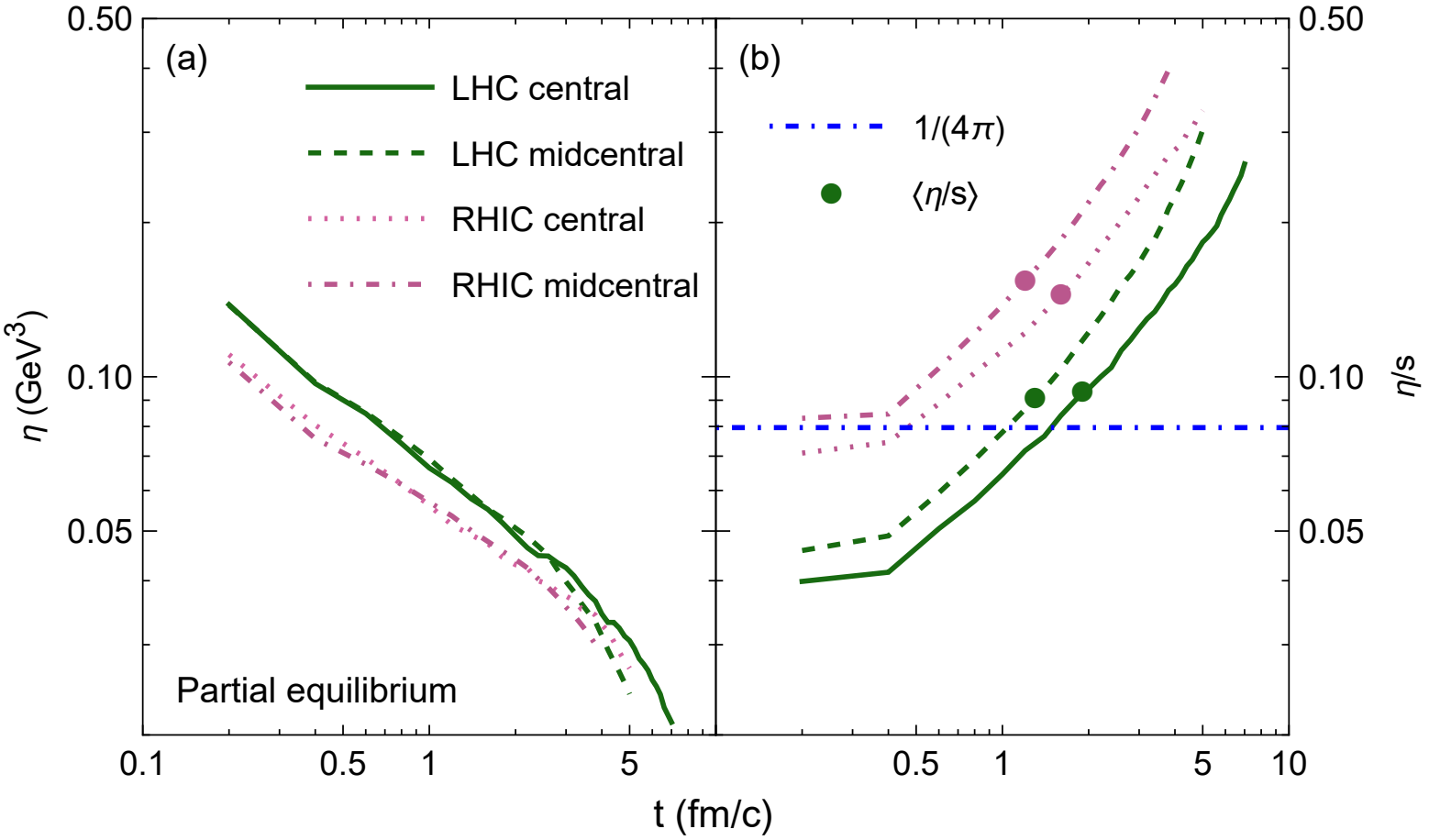
- When treating the matter as a QGP in full equilibrium (N_f=3), we use temperature T_ε to calculate both η and s:



- Time-averaged value weighted by collision rates: $<\eta/s>$ is quite small.
- Temperature dependence of η/s is "wrong", due to constant σ

3) Application to parton matter in the AMPT model

- When treating the matter as a QGP in partial chemical equilibrium, we use temperature $T_{\langle p_T \rangle}$ to calculate η but use T_ε to calculate s, since η is determined by momentum transfer but not density:



$<\eta/s>$ is even smaller

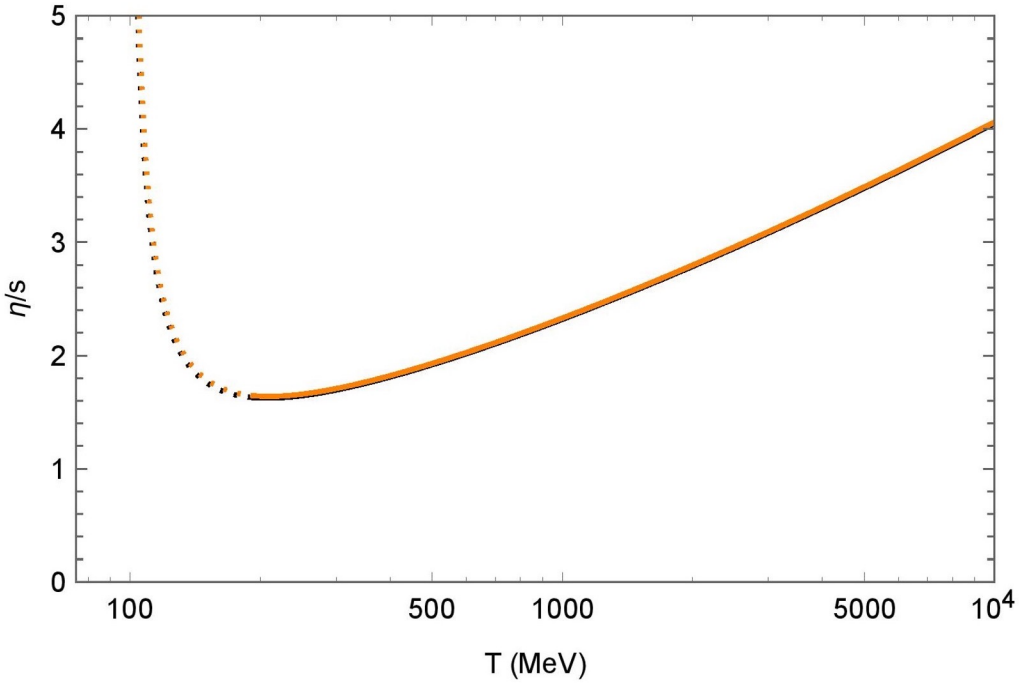
- η & η/s are lower in partial equilibrium due to $T_{\langle p_T \rangle} < T_\varepsilon$:
  lower T (*at constant* μ) makes scattering more isotropic and effective

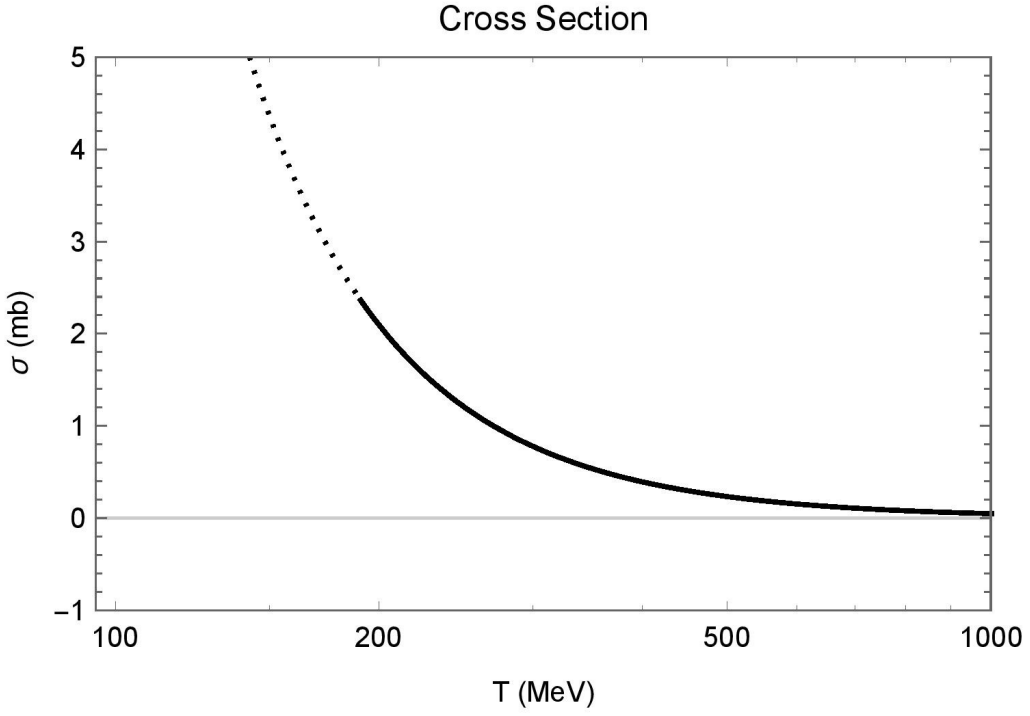
3) Application to parton matter in the AMPT model

pQCD with $\mu \propto gT$:

$$\frac{\eta}{s} \approx \frac{5.12}{g^4 \ln(2.42/g)}$$

Arnold, Moore & Yaffe, JHEP (2003);
Csernai, Kapusta & McLerran, Phys Rev Lett (2006)





If we use $\mu \propto gT$

→ $\sigma \propto 1/\mu^2$ will be larger at lower T
→ $\eta/s \propto T/\sigma$ will have the expected T- and t-dependences
→ a direction to improve ZPC/AMPT

Conclusions

- The Chapman–Enskog (CE) method gives accurate expression of η for parton matter under 2-to-2 scatterings

- Applying the CE method, $<\eta/s>$ for parton matter in the center cell of high energy A+A collisions from the AMPT model is very small at (1-3)/(4π)

- T-dependence or time-dependence of η/s in AMPT is opposite to pQCD expectation, because of the constant σ or screening mass μ
- This problem can be resolved by adopting $\mu \propto gT$; this improvement will lead to a better ZPC/AMPT as a dynamical model for non-equilibrium studies

Thank you!