Baryonic Vortex and Magnetic Field Generation

Zebin Qiu (Keio University)
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Regime: Low Energy Dense QCD

Assume spontaneous Chiral Symmetry Breaking

$$U(1)_V \times SU(N_f)_L \times SU(N_f)_R \rightarrow U(1)_V \times SU(N_f)_V$$

2-flavor Chiral Perturbation Theory (ChPT) of Pions

$$\Sigma = \sigma + i\tau^a \pi^a \in SU(N_f = 2)$$

$$\mathcal{L}_{\rm chiral} = \frac{f_\pi^2}{2} \mathrm{Tr}\left(D_\mu \Sigma^\dagger D^\mu \Sigma\right), \quad D_\mu \Sigma = \partial_\mu \Sigma - i\left(A_\mu + A_\mu^I\right)\left[\frac{\tau^3}{2}, \Sigma\right]$$

Isospin chemical potential $A_\mu^I = (\mu_I, \mathbf{0})$ + "dynamical" axial magnetic field $A_\mu = A_\phi(\rho)\hat{\phi}$ needs $O(p^4)$ terms:

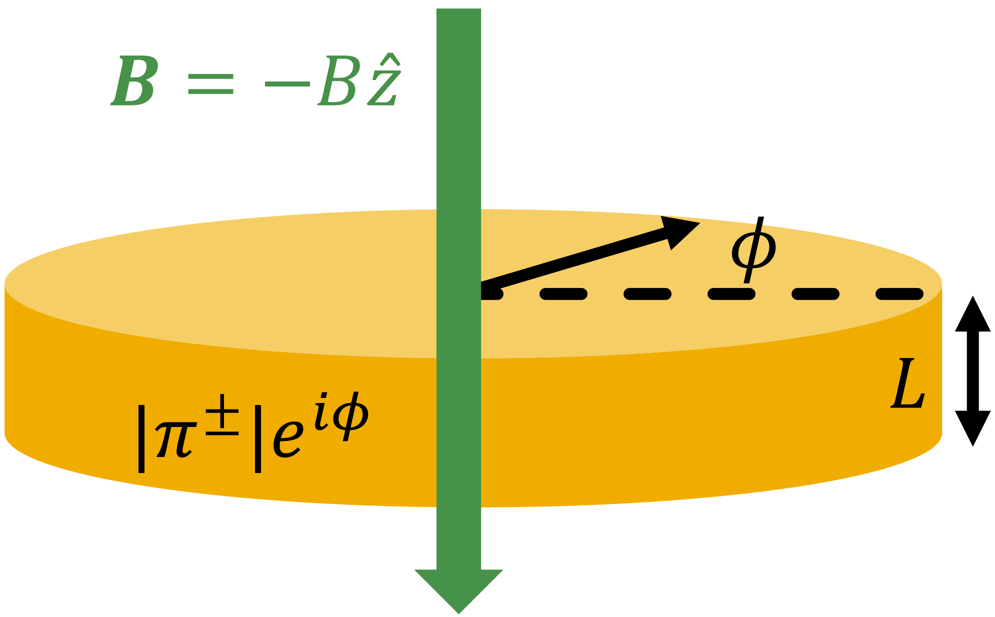
$$\mathcal{L}_{\rm EM} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \quad \mathcal{L}_{\rm Skyrme} = \frac{1}{32 s^2} \mathrm{Tr}\left(\left[\Sigma^\dagger D_\mu \Sigma, \Sigma^\dagger D_\nu \Sigma\right]\right)^2$$

μ_I: Charged Pion Vortex

Charged $\pi^{\pm} \xrightarrow{\mu_I}$ uniform condensate $\xrightarrow{\boldsymbol{B}}$ Superconductor (SC)

$$V_{\text{eff}} = -\frac{f_\pi^2}{2}\mu_I^2\left(\pi_1^2 + \pi_2^2\right)$$

Type II SC
under external
magnetic field



P. Adhikari, E. Leeser and J. Markowski, Mod. Phys. Lett. A 38 (2023) 2350078
M.S. Grønli and T. Brauner, Eur. Phys. J. C 82 (2022) 354 ……

$$B > B_{c1}^{\text{ext}} = T/\Phi_0;$$

$$T = E_{\text{vortex}}/L, \quad \Phi_0 = 2\pi/e$$

μ_B: Neutral Pion Domain Wall

Neutral $\pi^0 \xrightarrow{B}$ Domain Wall / Chiral Soliton Lattice (CSL)

$$\mathcal{L}_{\rm anom} = \frac{1}{8\pi^2}\epsilon^{\mu\nu\alpha\beta}\sum_i C_i \partial_\mu \psi_i A^{\rm B}_\nu F_{\alpha\beta}$$

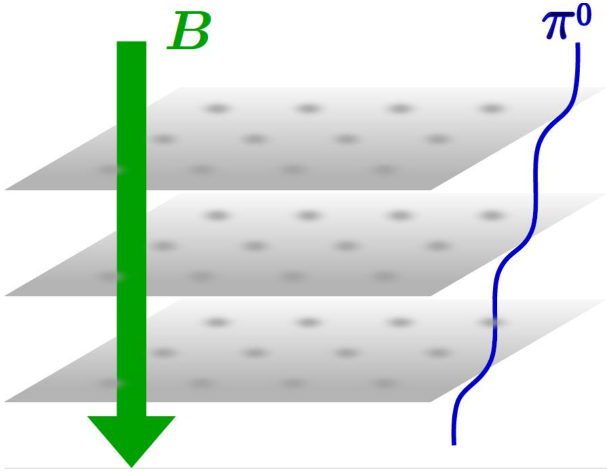
D.T. Son, M.A. Stephanov, and A.R. Zhitnitsky, Phys. Rev. Lett. 86, 3955 (2001);

Anomalous coefficients $U(1)_{\rm B}:\ A^{\rm B}_\nu = (\mu_B, 0)$ $U(1)_{\rm EM}:\ B$

Triangle Anomaly: two U(1) + one NG boson ψ_i ($\pi^0, \eta\ldots$)

$$\mathcal{H}_{\rm WZW} = -\frac{\mu_B B}{4\pi^2}\partial_z \pi^0$$

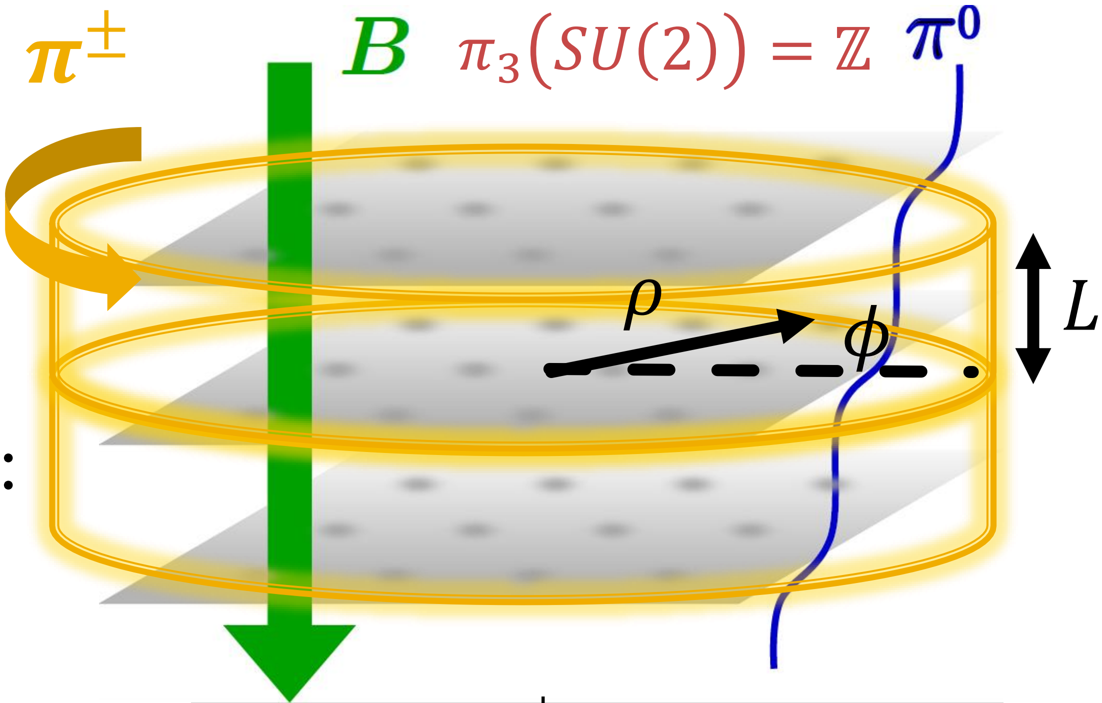
T. Brauner and N. Yamamoto, JHEP 04, 132 (2017).



$\mu_{I,B}$: Baryon as Vortex Skyrmion

Our scenario: Baryon at finite $\mu_{I,B}$ + Self-generated B

Ad hoc assumptions:
cylindrical symmetry,
chiral limit,
separation of variables.



Vortex-Skyrmion Ansatz:

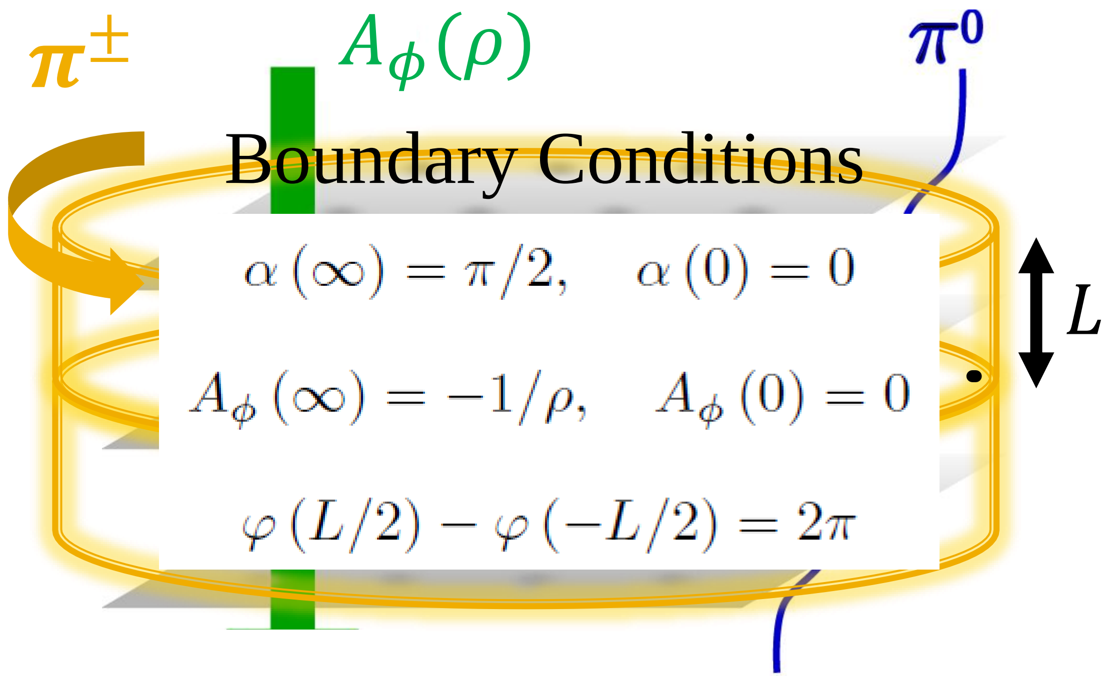
$$\Sigma(\rho, \phi, z) = \begin{pmatrix} e^{i\varphi(z)} \cos\alpha(\rho) & e^{-i\phi} \sin\alpha(\rho) \\ e^{i\phi} \sin\alpha(\rho) & e^{-i\varphi(z)} \cos\alpha(\rho) \end{pmatrix}$$

Outer: $\pi^{\pm}$ SC ring.
Inner: π^0 winding $\varphi(z)$.
Together: Baryon (Skrymion)

Baryon Number: Topological Charge

Gauged WZW term: coupling of U(1) with baryon current

$$\mathcal{L}_{\mathrm{WZW}} = \left(qA_\mu + A^B_\mu\right) j^\mu_B$$



Boundary Conditions

$$\alpha(\infty) = \pi/2, \quad \alpha(0) = 0$$

$$A_\phi(\infty) = -1/\rho, \quad A_\phi(0) = 0$$

$$\varphi(L/2) - \varphi(-L/2) = 2\pi$$

Baryon current (Goldstone-Wilczek current):

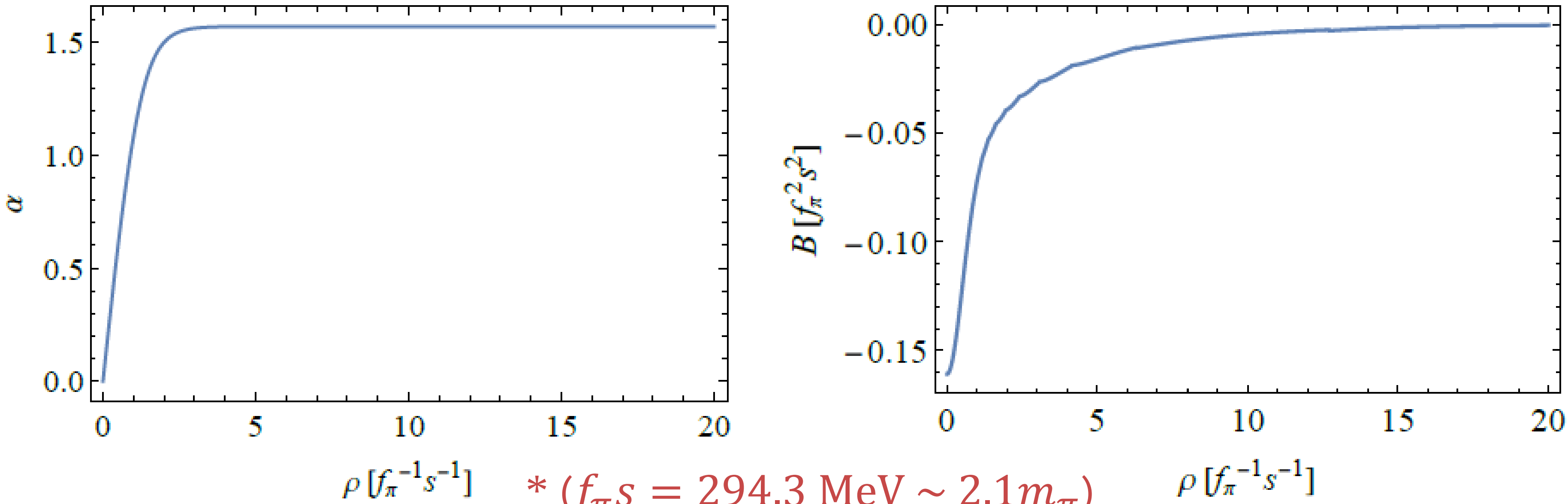
$$j_B = \star\frac{1}{24\pi^2}\mathrm{Tr}\left\{l \wedge l \wedge l + 3iQd\left[A \wedge (l - r)\right]\right\}$$

$$l = \Sigma^\dagger d\Sigma, \quad r = \Sigma d\Sigma^\dagger, \quad Q = \mathbb{I}/6 + \tau^3/2$$

$$\int d^3x\, j^0_B = N$$

Solution Profile at Finite μ_I

Result: $\varphi = kz$; $\alpha(\rho)$ and $B(\rho)$ solved numerically:



* $(f_\pi s = 294.3\ \text{MeV} \sim 2.1 m_\pi)$

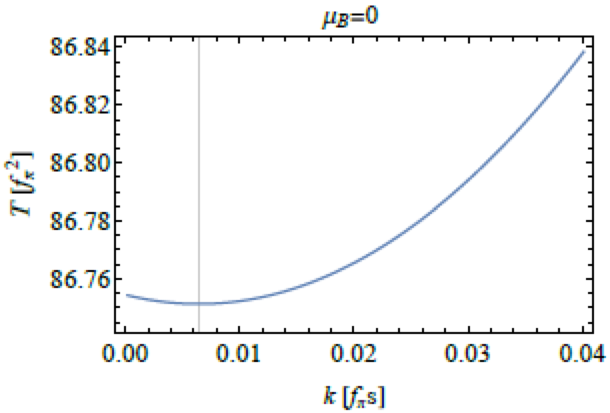
Figure 1. Vortex Skyrmion (left) and magnetic field (right) at $\mu_I = 0.5 f_\pi s$ and $k = 2.0 f_\pi s$.

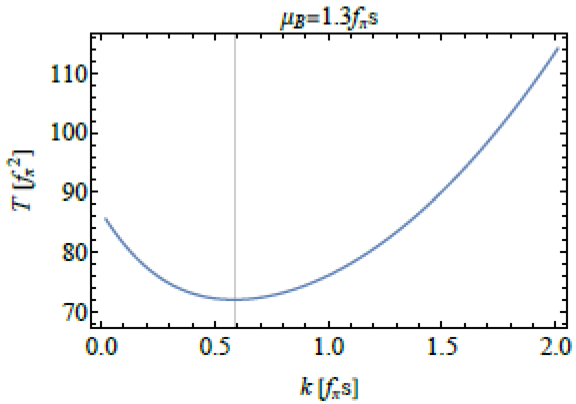
Baryon vortex carries a quantized magnetic flux $\int \rho d\rho\, B(\rho) = 1$ as type-II SC.

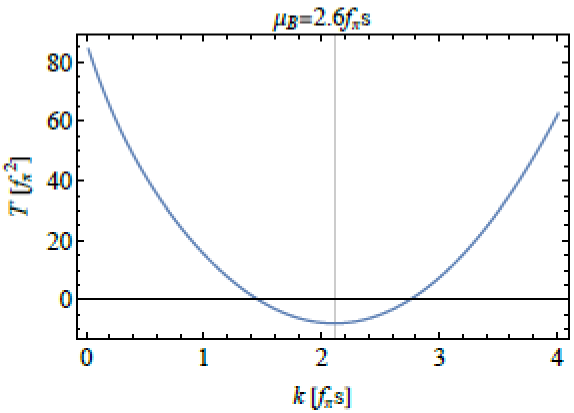
String Tension $T(k)$ at Varied μ_B

$k_v = \text{argmin}\, T(k)$ proves a stable solution.

k_v exists for any μ_B and $\mu_I > m_\pi$.



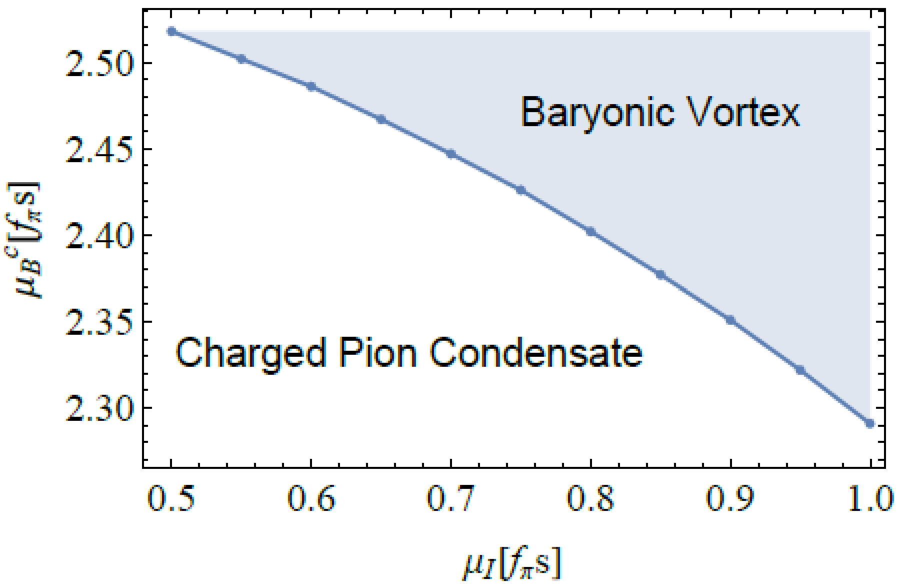


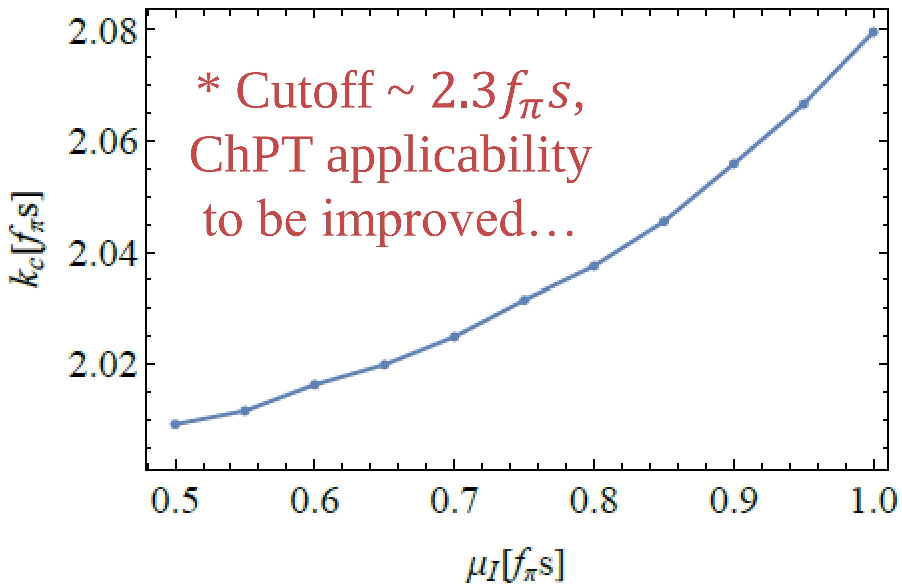


- $T(k_v) < T(0)$ "improves" $\pi^\pm$ (non-baryonic) vortex.
- When $\mu_B \uparrow$, $T(k_v) \downarrow$, until $\mu_B > \mu_B^c$, $T(k_v) < 0$, **ground state changes to the baryonic vortex.**

Critical μ_B^c Between Phases

Phase transition at $\mu_B^c(\mu_I)$ and $k_c = k_v(\mu_B^c,\ \mu_I)$.





- $\mu_B^c(\mu_I = m_\pi) \sim 743$ MeV $< m_N \sim 940$ MeV: density window for baryonic vortex to be ground state.
- $\mu_B > \mu_B^c$, (seed) **magnetic field can be spontaneously enhanced and preserved.**

Conclusion & Outlook

- Conclusion:
  A baryonic vortex can have lower energy than a homogeneous pion condensate. It carries a spontaneously generated quantized magnetic flux.

- Outlook:
  Pion Mass: finite condensate of both π^0 and $\pi^\pm$ on boundary, coupled ρ, z dependence of fields.
  Vortex Crystal: evaluation of magnetic field strength, inter-vortex interaction, realistic density scales
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