A quarkyonic matter model

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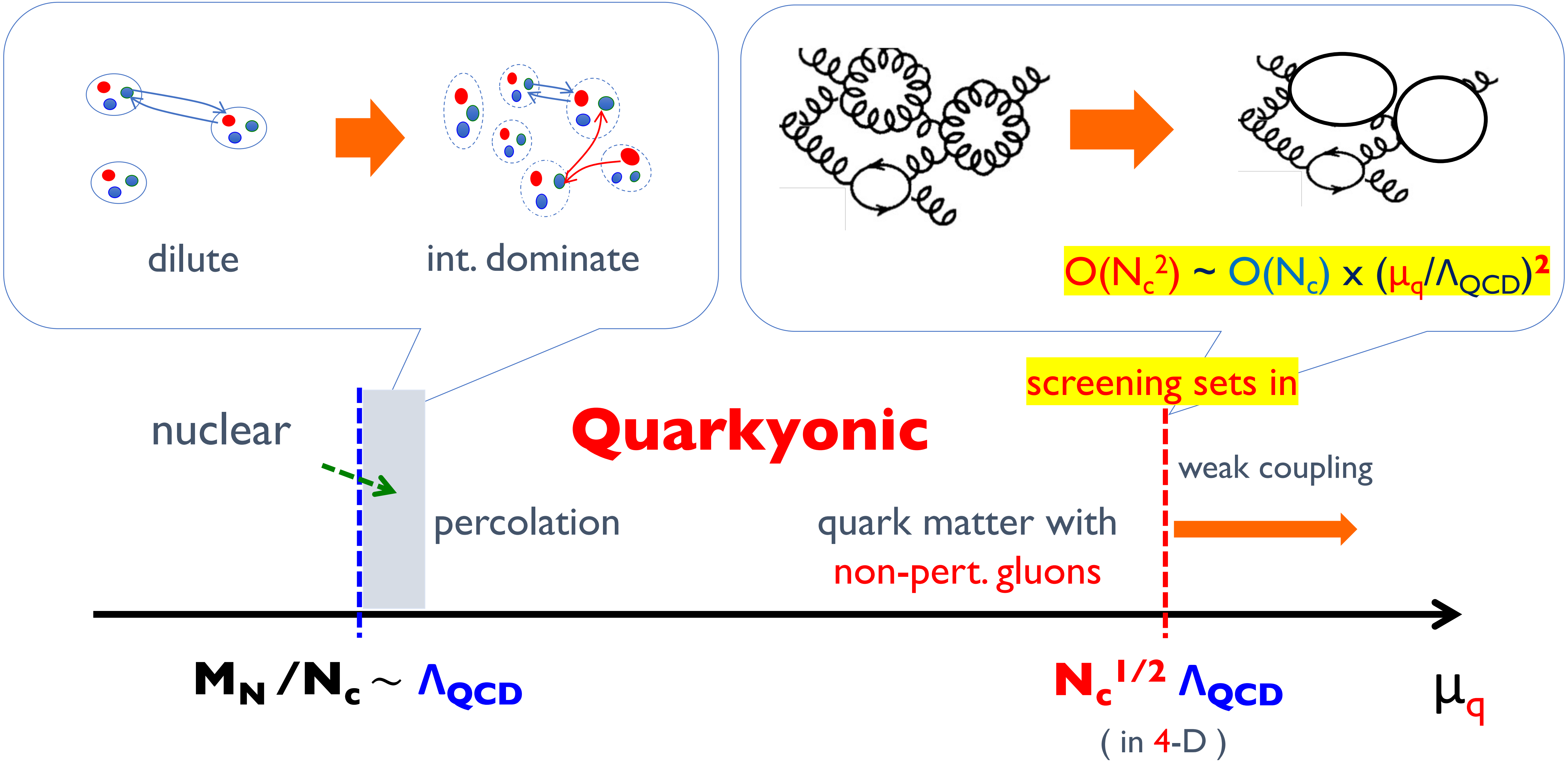
Refs) Baym-Hatsuda-TK-Powell-Song-Takatsuka, "QHC", review on neutron stars (2018)

TK, "Stiffening of matter in quark-hadron continuity" PRD (2021)

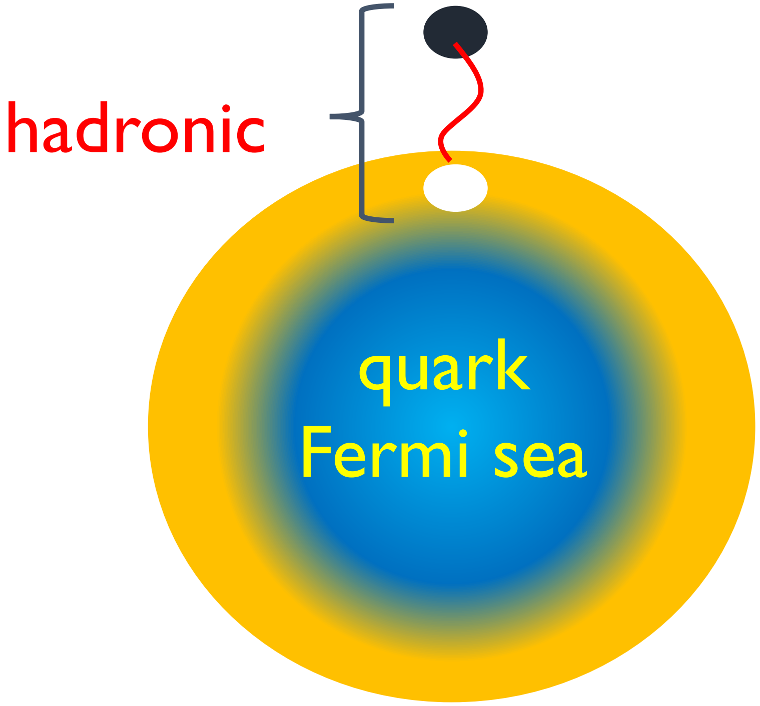
Fujimoto-TK-McLerran, "IdylliQ matter model" PRL (2024)

McLerran-Pisarski's "two-scale picture" [McLerran-Pisarski '07]



Quarkyonic matter

def := quark matter with confining gluons



impacts on

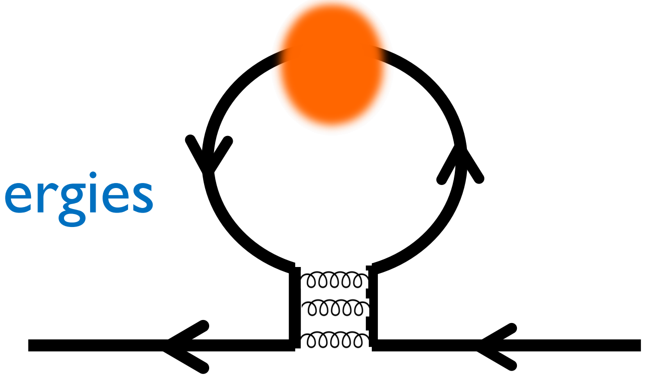
e.g.)
- entropy & transport properties
- gap **weakly** depending on μ
- phase structures

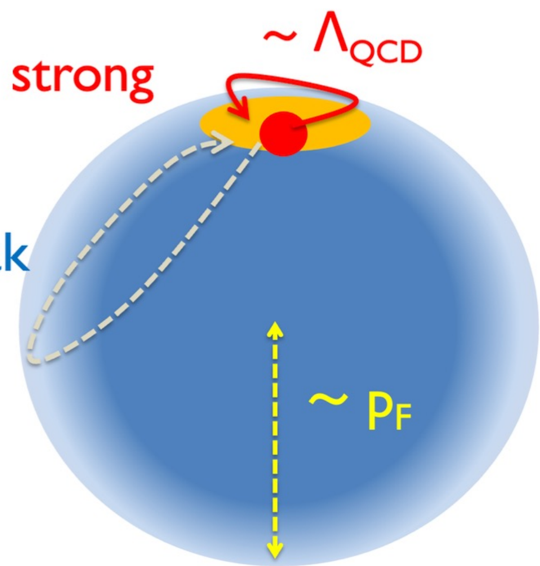
possible consequences (NOT definitions) :

- chiral symmetric but confining phase [Glozman+ '08]
- chiral spirals (inhomo. chiral) [TK+ '09, '10, '11]

& many other speculations

An application of concepts; gap-eq. & EOS





If IR gluons dominate

M or Δ ~ Λ_{QCD} (!)

(weak μ-dep.)

EOS

$$P(\mu) = c_0\mu^4 + c_2\Delta^2\mu^2 + c_4\Delta^4 + \cdots$$

$$\sim \; c_0\mu^4 + c_2'\underline{\Lambda_{QCD}^2}\mu^2 + c_4'\underline{\Lambda_{QCD}^4} + \cdots$$

non-perturbative

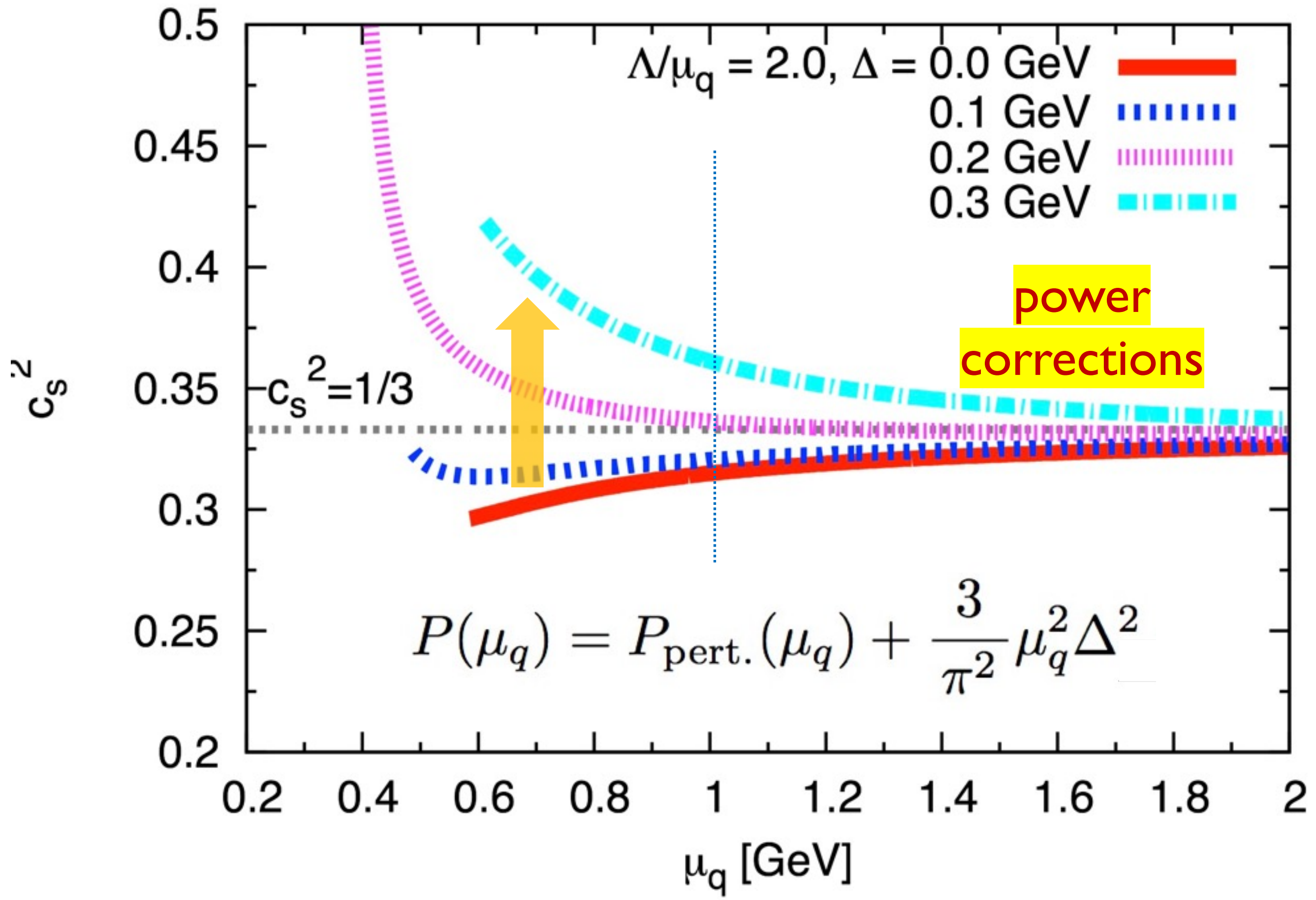
"**power corrections**"

[Shifman-Vainshtein-Zakharov, '78]

[TK-Powell-Song-Baym, '14]

An application of concepts; c_s^2 at high density

sound speed: $$c_s^2 = \frac{\partial P}{\partial \varepsilon} = \frac{2c_0\mu^2 + c_2\Delta^2}{6c_0\mu^2 + c_2\Delta^2} \geq \frac{1}{3}$$ (for $c_2 > 0$)



e.g. diquark pairing (CFL) terms

For Δ ~ **0.2** GeV ~ Λ_{QCD}

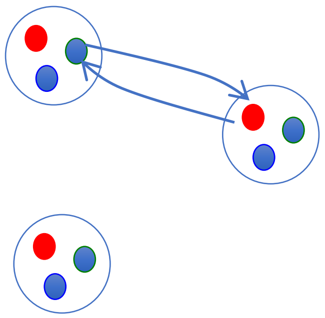
$(\Delta/\mu_q)^2 \sim$ **4 %** at μ ~ 1GeV

but qualitative trend changes

more important
toward lower density

Neutron Star matter ($n_0 = 0.16\ \mathrm{fm}^{-3}$) [Masuda+ '12; TK+ '14]

- few meson exchange
- nucleons only

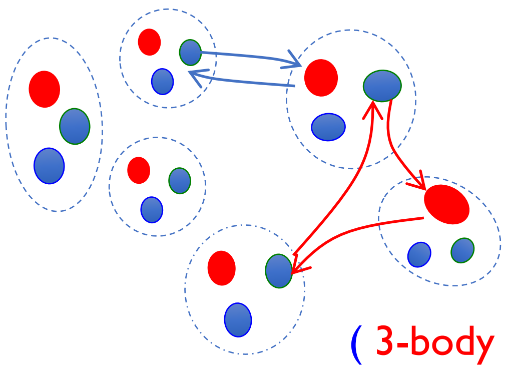


ab-initio nuclear cal.

laboratory experiments

steady progress

- many-quark exchange
- structural change,...
- hyperons, Δ, ...

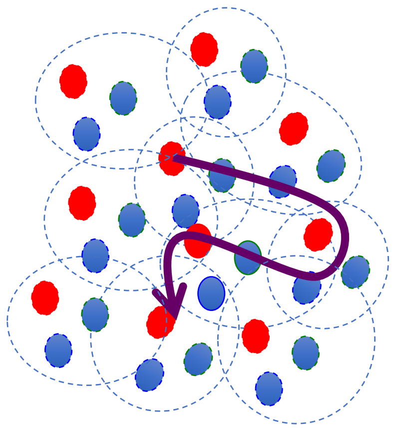


(3-body)

most difficult

(d.o.f ??)

- Baryons overlap
- Quark Fermi sea



strongly correlated

(d.o.f : quasi-particles??)

not explored well

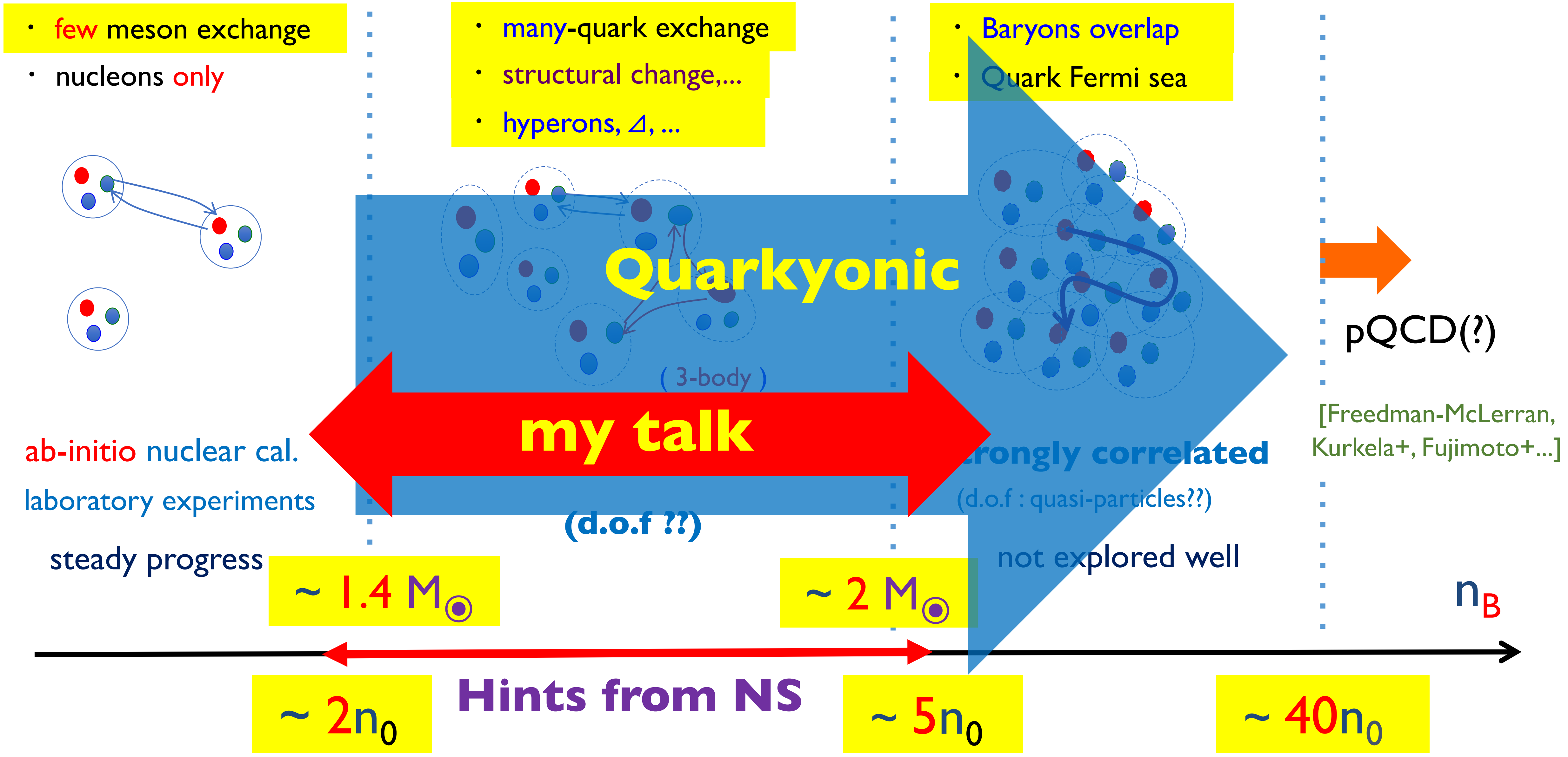
pQCD(?)

[Freedman-McLerran, Kurkela+, Fujimoto+...]

~ 1.4 $M_\odot$ ~ 2 $M_\odot$ n_B

~ $2n_0$ **Hints from NS** ~ $5n_0$ ~ $40n_0$

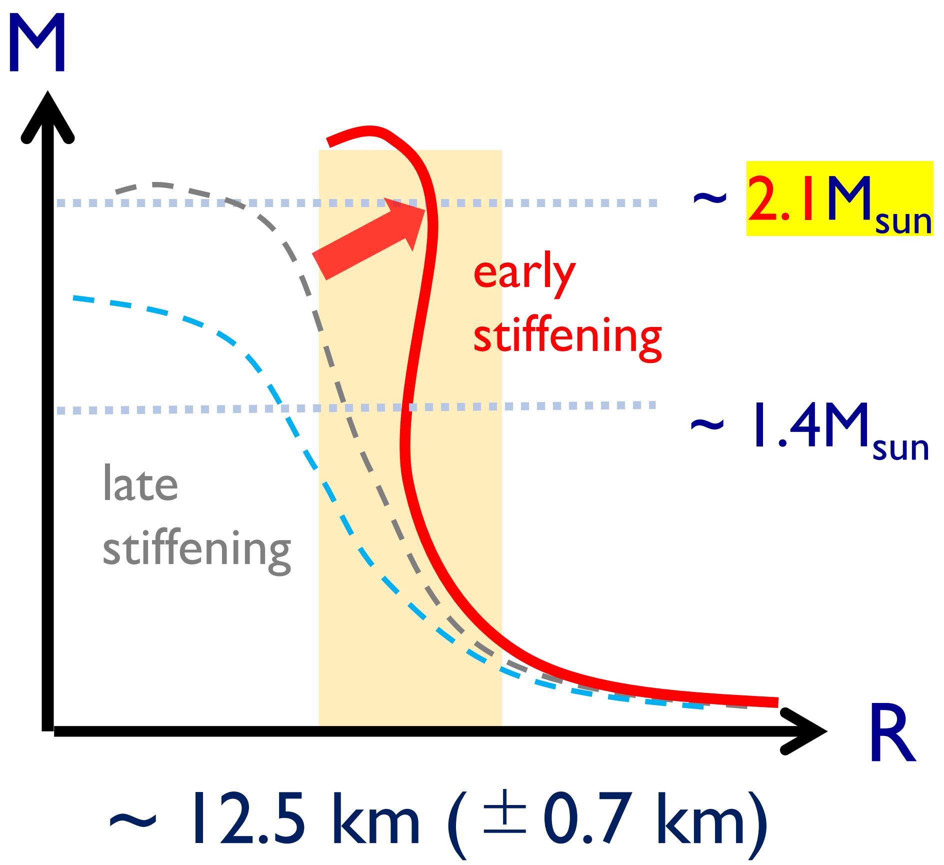
Neutron Star matter (n_0 = 0.16 fm^{-3}) [Masuda+ '12; TK+ '14]

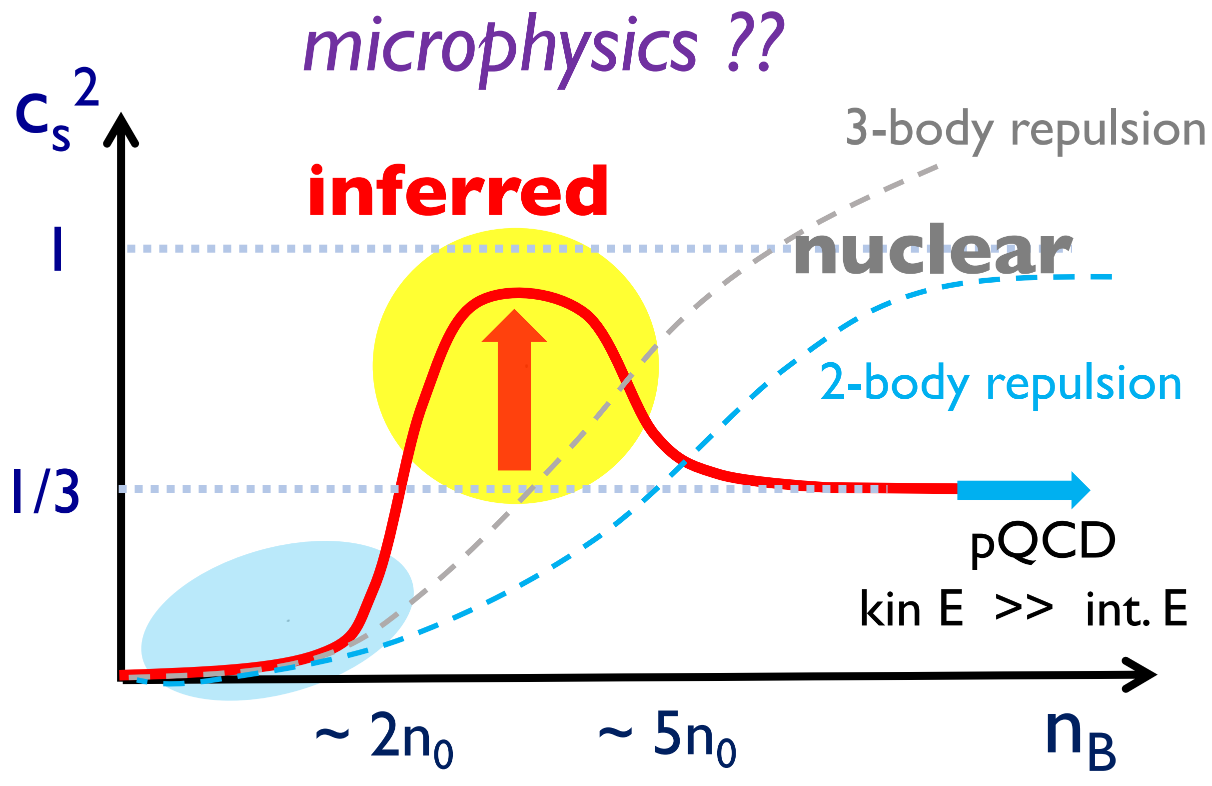


Implications from NS

NICER for 1.4 & 2.1 M_{sun} + **GW** + **nuclear** ($< \sim 1.5 n_0$)

→ $R_{1.4} \sim R_{2.1}$ (!)





IdylliQ model

= Ideal dual Quarkyonic model

Describe **single** physics in **two** languages (baryon/quark)

Powerful in transient regimes (2-5n_0)

Sum rules for occupation probabilities

cf) [TK '21]

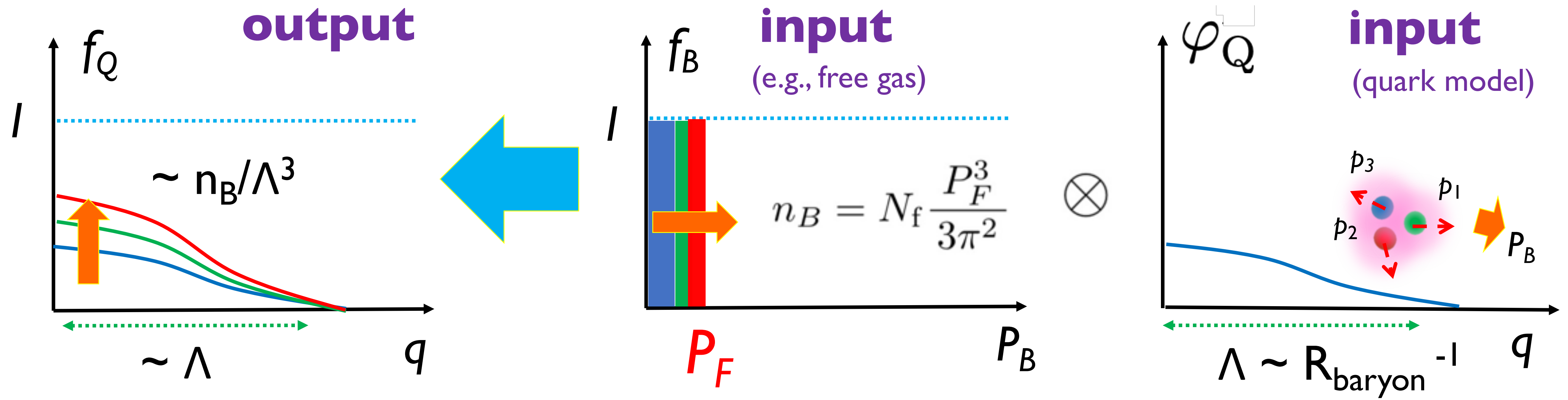
occupation **probability** of **quark** state with p

occupation **probability** of **baryon** state with P_B

quark mom. distribution **in a baryon**

$$f_Q(\mathbf{q}) = \int_{\boldsymbol{P}_B} f_B(\boldsymbol{P}_B)\,\varphi_Q^B(\mathbf{q} - \boldsymbol{P}_B/N_c)$$

e.g.) in **ideal** baryonic matter



An ideal model

[Fujimoto-TK-McLerran, PRL'24]

1) neglect interactions *except* confining forces

e.g.) 2-flavor hamiltonian: $$\varepsilon_{\rm B}[f_{\rm B}] = 4\int_k E_{\rm B}(k) f_{\rm B}(k)$$

2) keep using the same $\varphi_{\rm Q}$ (quarkyonic)

3) use a special quark distribution → sum rules analytically *invertible*

$$\varphi_{\rm 3d}(\boldsymbol{q}) = \frac{2\pi^2}{\Lambda^3}\frac{e^{-q/\Lambda}}{q/\Lambda} \qquad \hat{L} = -\boldsymbol{\nabla}^2 + \frac{1}{\Lambda^2} \qquad \hat{L}[\varphi(\boldsymbol{p}-\boldsymbol{q})] = \frac{(2\pi)^3}{\Lambda^2}\delta(\boldsymbol{p}-\boldsymbol{q})$$

nontrivial output

$$f_{\rm Q}(\mathbf{q}) = \int_{\boldsymbol{P}_B} f_{\rm B}(\boldsymbol{P}_B)\varphi_Q^B(\mathbf{q} - \boldsymbol{P}_B/N_{\rm c})$$

natural at ***low*** *density*

⇄

nontrivial output

$$f_{\rm B}(N_{\rm c}\boldsymbol{q}) = \frac{\Lambda^2}{N_{\rm c}^3}\hat{L}[f_{\rm Q}(\boldsymbol{q})]$$

natural at ***high*** *density*

Minimize energy with sum rule constraints

[Fujimoto-TK-McLerran, PRL'24]

constraint to fix n_B

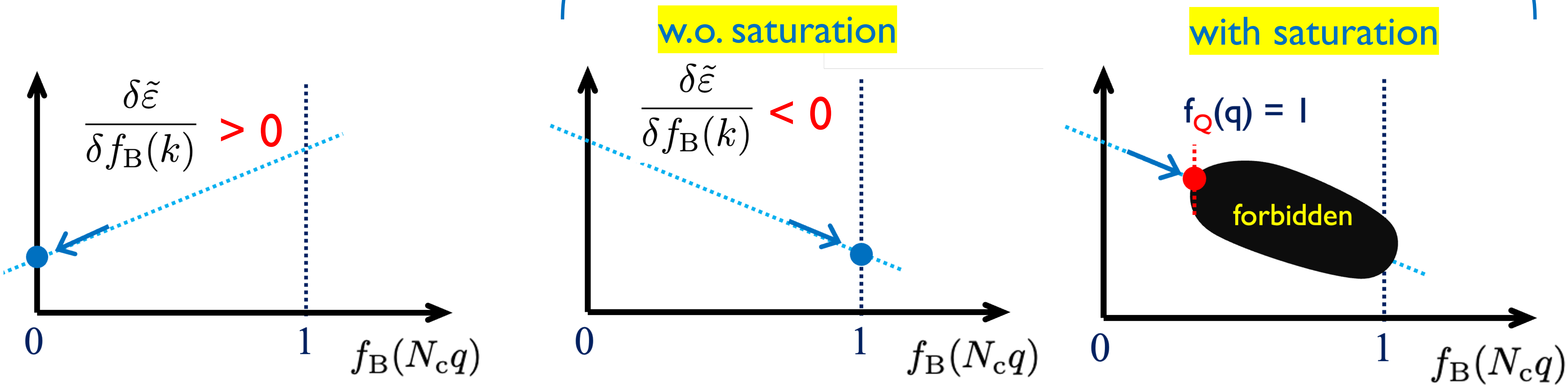
$$\tilde{\varepsilon} = \varepsilon_B[f_B] - \lambda_B n_B$$

$$E_B(k) = \sqrt{M_B^2 + k^2}$$

$$n_B = 4 \int_k f_B(k)$$

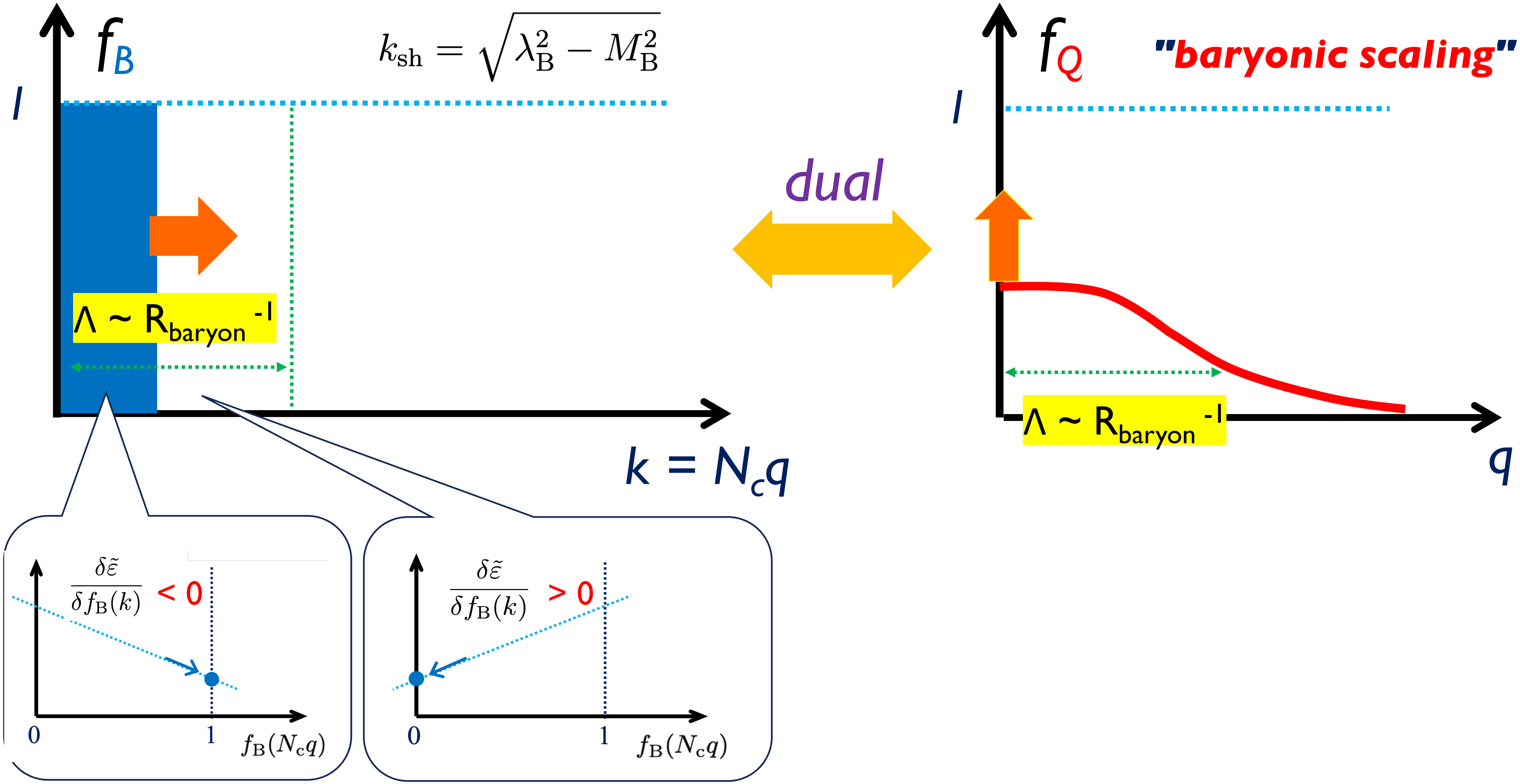
optimization: $$\frac{\delta \tilde{\varepsilon}}{\delta f_B(k)} = E_B(k) - \lambda_B$$ *at a given k*

$E_B(k) > \lambda_B$ $\qquad$ $E_B(k) < \lambda_B$



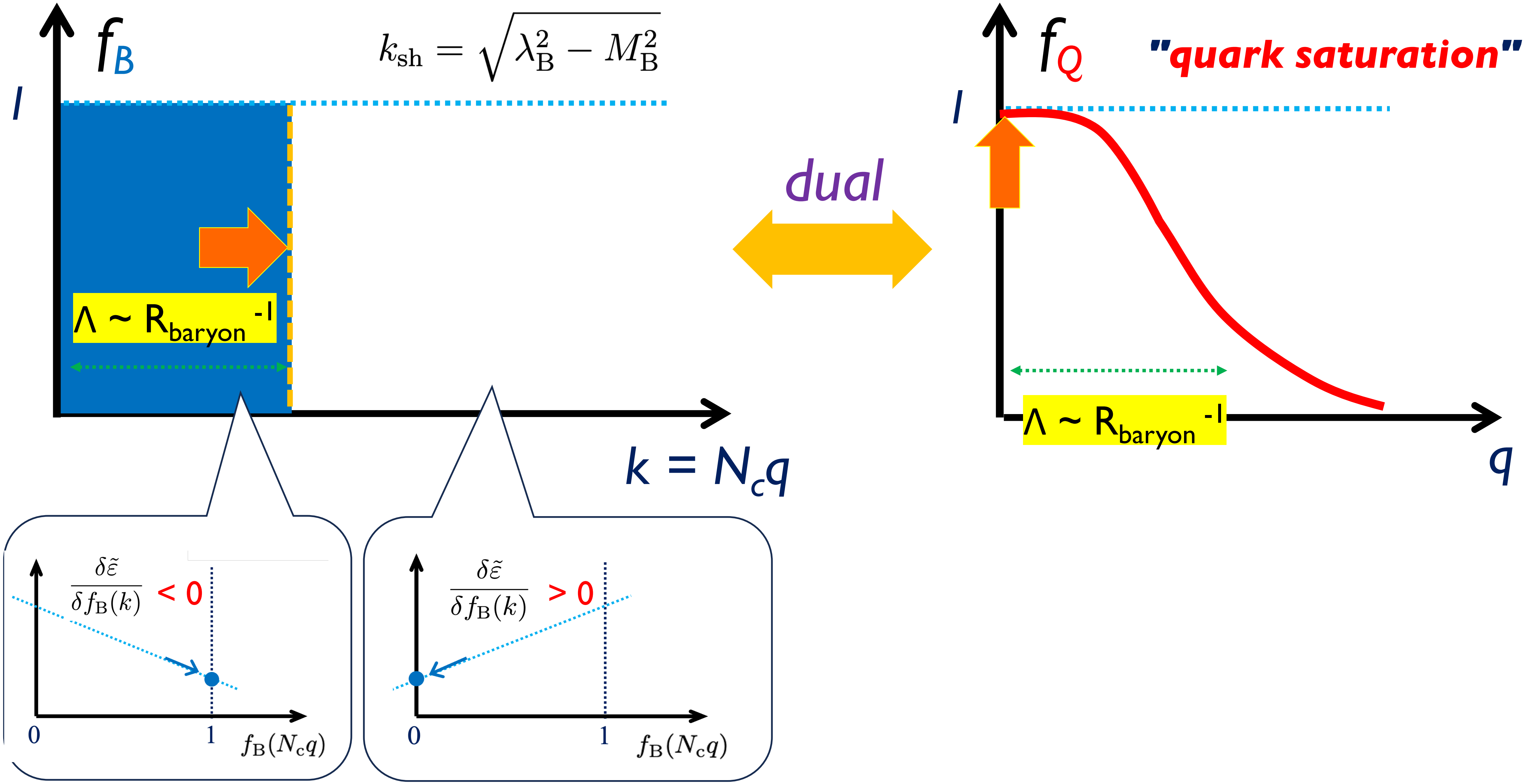
Solution (dilute regime)

[Fujimoto-TK-McLerran, PRL'24]



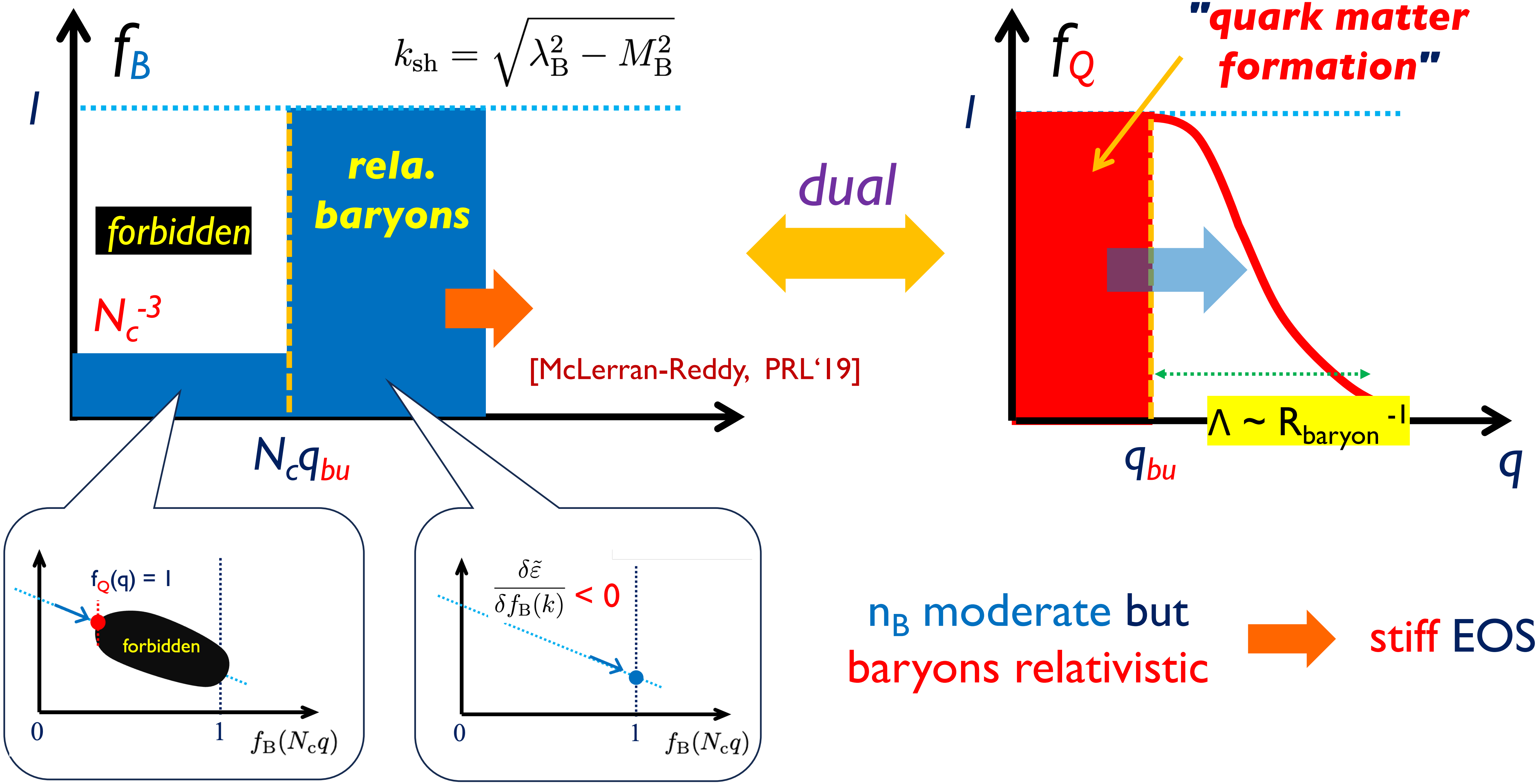
Solution (at saturation)

[Fujimoto-TK-McLerran, PRL'24]



Solution (post saturation)

[Fujimoto-TK-McLerran, PRL‘24]



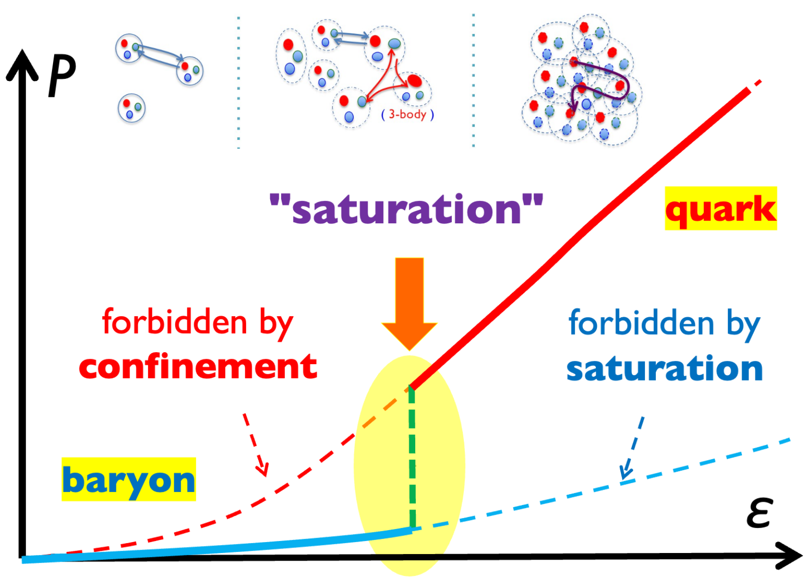
n_B moderate but baryons relativistic → stiff EOS

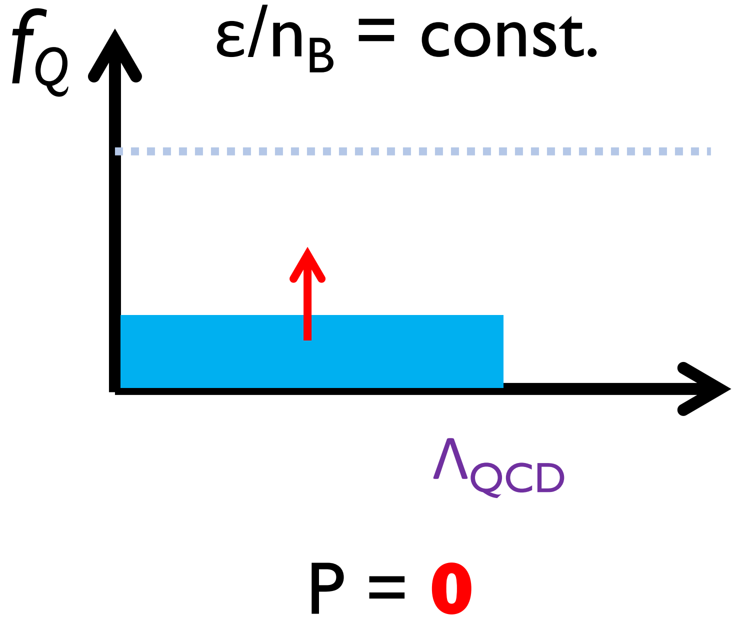
Stiffening in quark picture

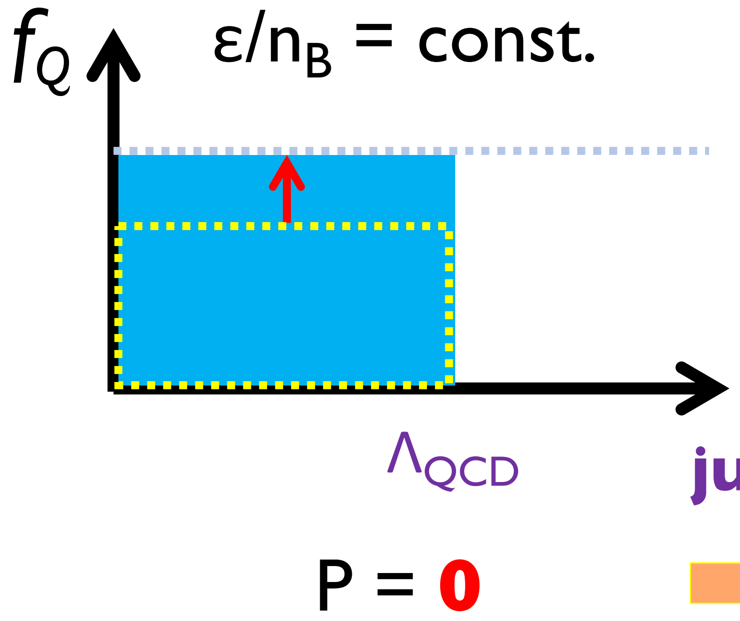
(very schematic)

$$\mathcal{P} = n_B^2 \frac{\partial}{\partial n_B}\left(\frac{\varepsilon}{n_B}\right)$$

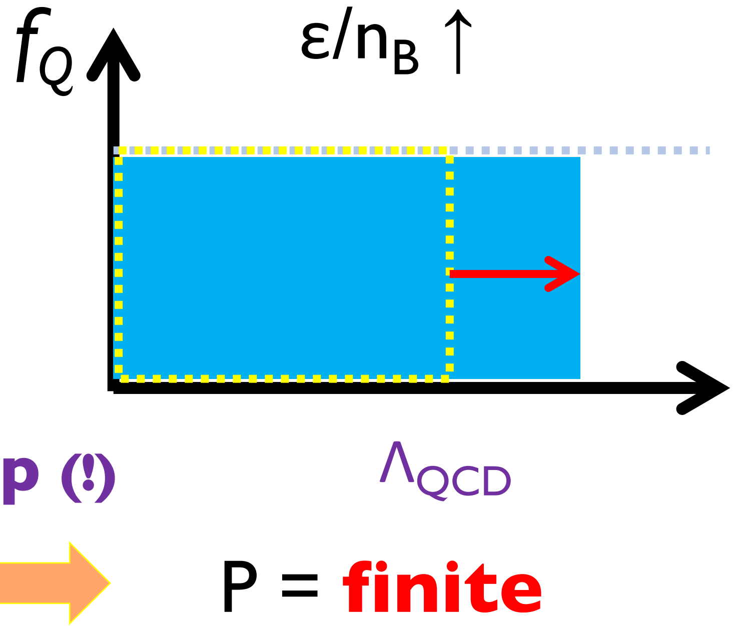
energy per particle







jump (!)

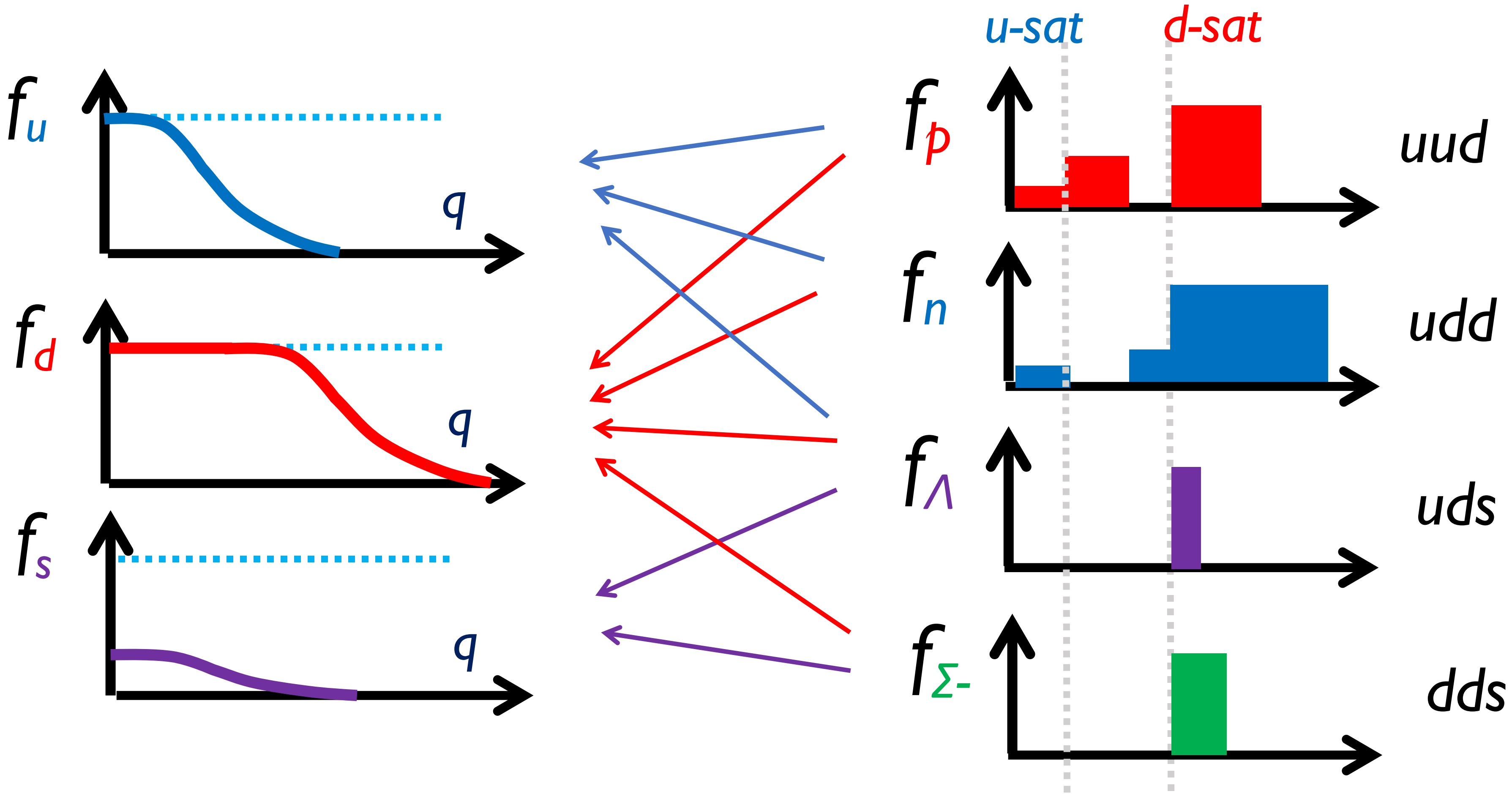


Multi-flavor extension

[Fujimoto-TK-McLerran, ’24, in preparation]

$$f_Q(\boldsymbol{q}) = \sum_{B=p,n,\Sigma,\cdots} N_Q^B \int_{\boldsymbol{k}} f_B(\boldsymbol{k}) \varphi\left(\boldsymbol{q} - \frac{\boldsymbol{k}}{N_c}\right)$$

Q = u, d, s



and so on...

saturation constraints

flavor correlations

Summary & Outlook

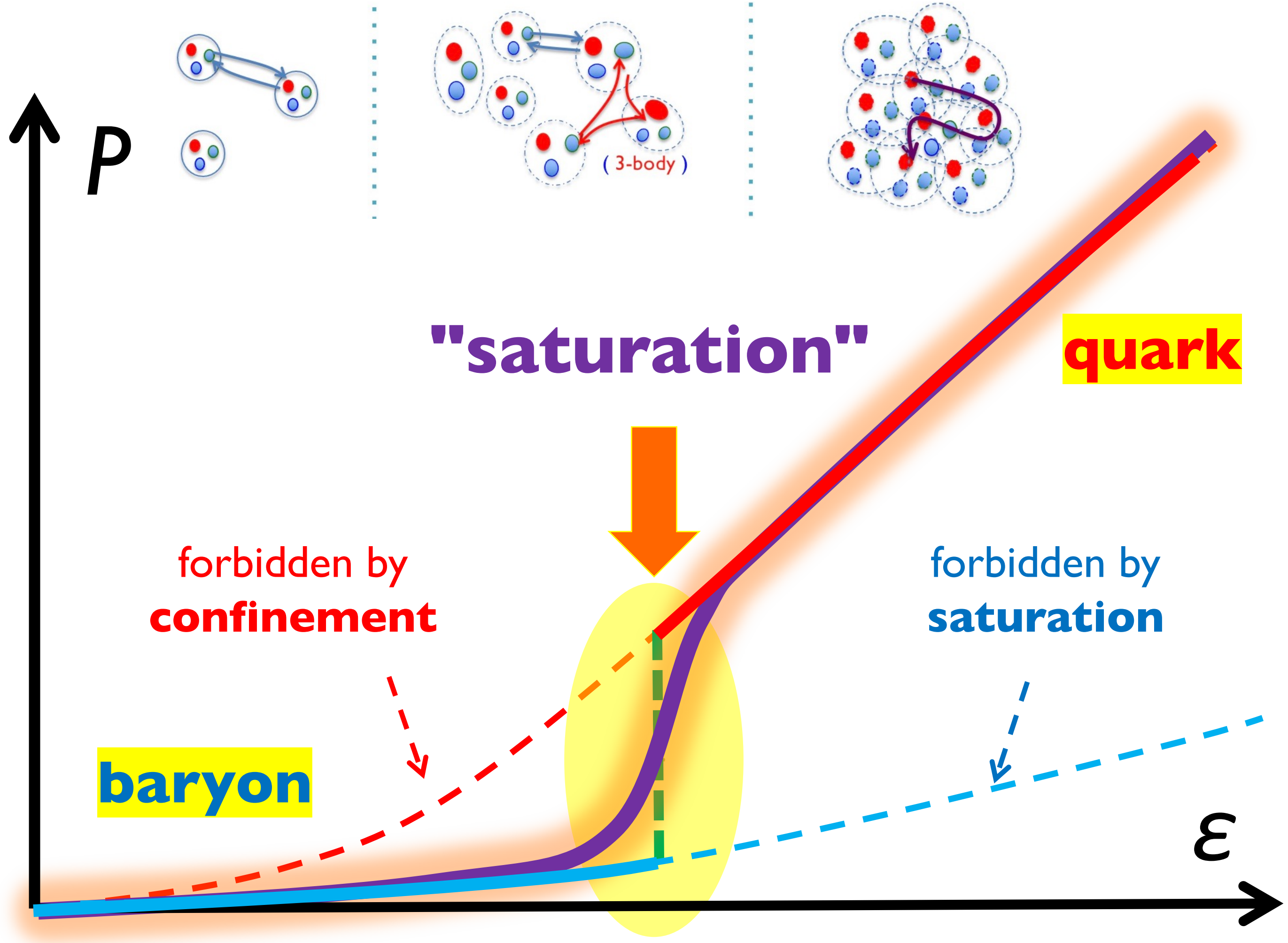
- *Quarkyonic matter* = quark matter with confining gluons
- *quark saturation* ➔ inevitable stiffening, c_s^2 peak
- the saturation occurs at ~ 2-3n_0 (!) (< ~ 5n_0 for baryon overlap)
- baryons are NOT independent; quark substructure constraint

p, n, Δ, ... cannot be freely put into the system at the same time

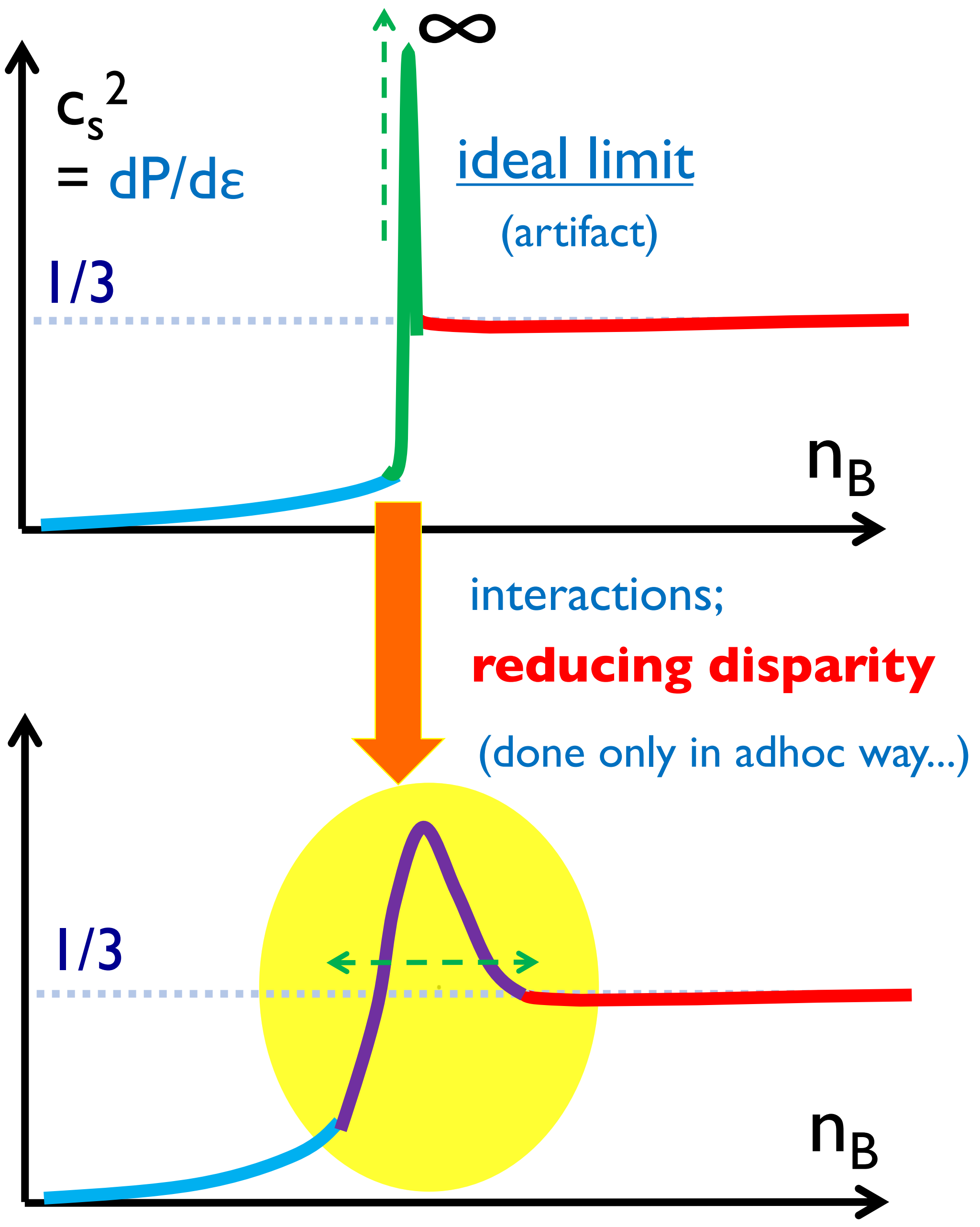
the hyperon puzzle to be solved [Fujimoto-TK-McLerran, in preparation]

Back Up

Peak in sound velocity

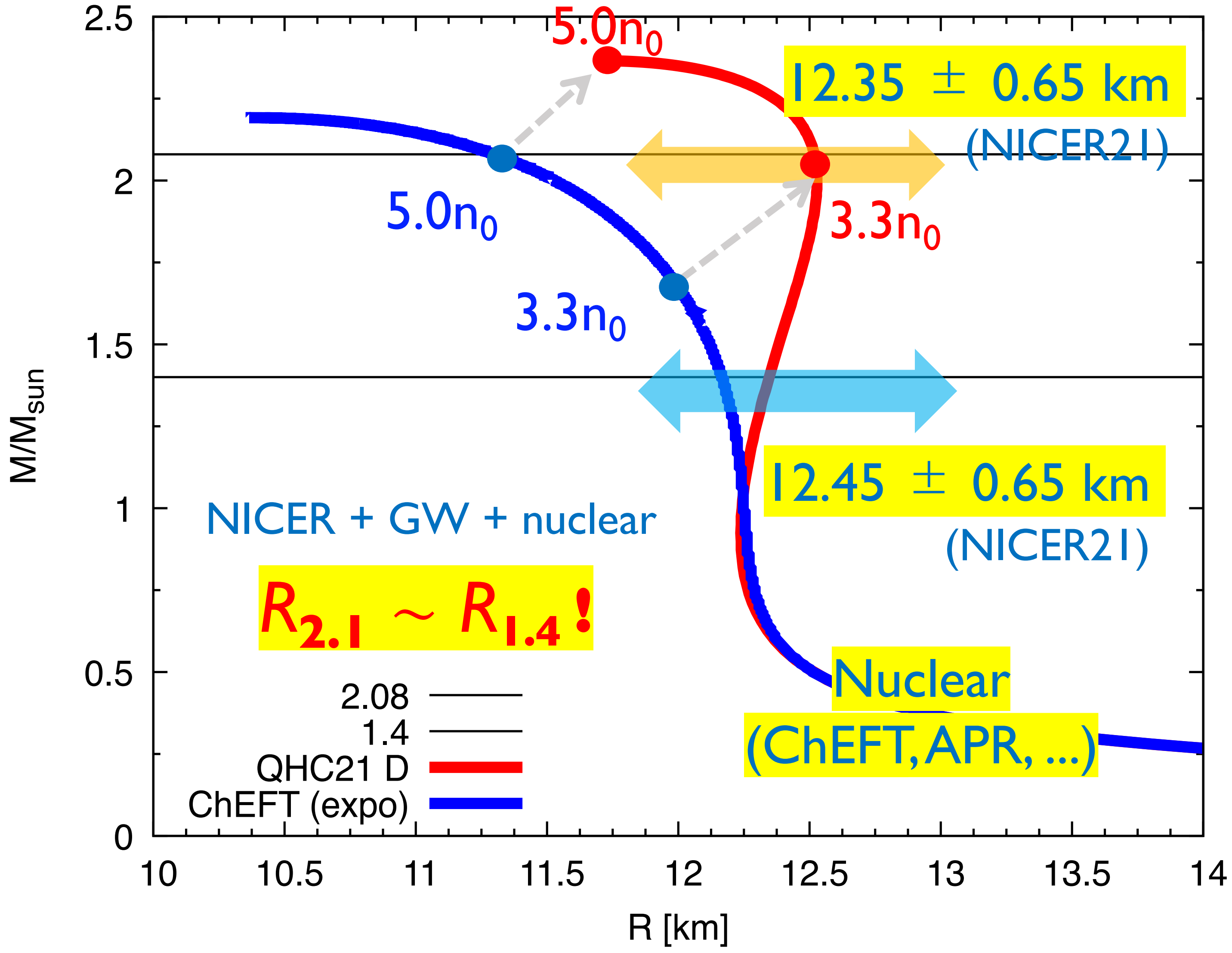


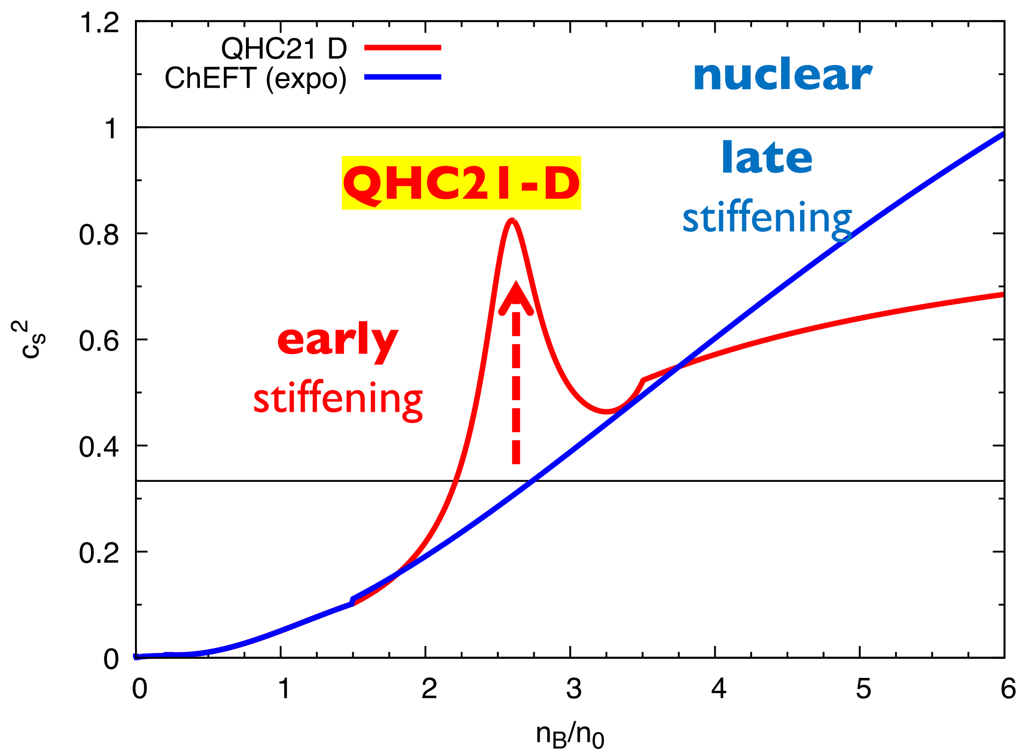
“inevitable” stiffening



Early vs late stiffening: QHC vs nucleonic

[TK-Hatsuda-Baym '21]





2-$3n_0$: already beyond purely nucleonic regime?

Nucleonic models & many-body forces

$$\varepsilon(n_B) = m_N n_B + a\frac{n_B^{5/3}}{m_N} + b n_B^{\alpha} \quad \Rightarrow \quad P = \frac{2}{3} a \frac{n_B^{5/3}}{m_N} + b(\alpha-1) n_B^{\alpha}$$

large (!) small (!) small (!)

$$\mathcal{P} = n_B^2 \frac{\partial}{\partial n_B}\left(\frac{\varepsilon}{n_B}\right)$$

⇒ p << ε (at least in dilute regime)

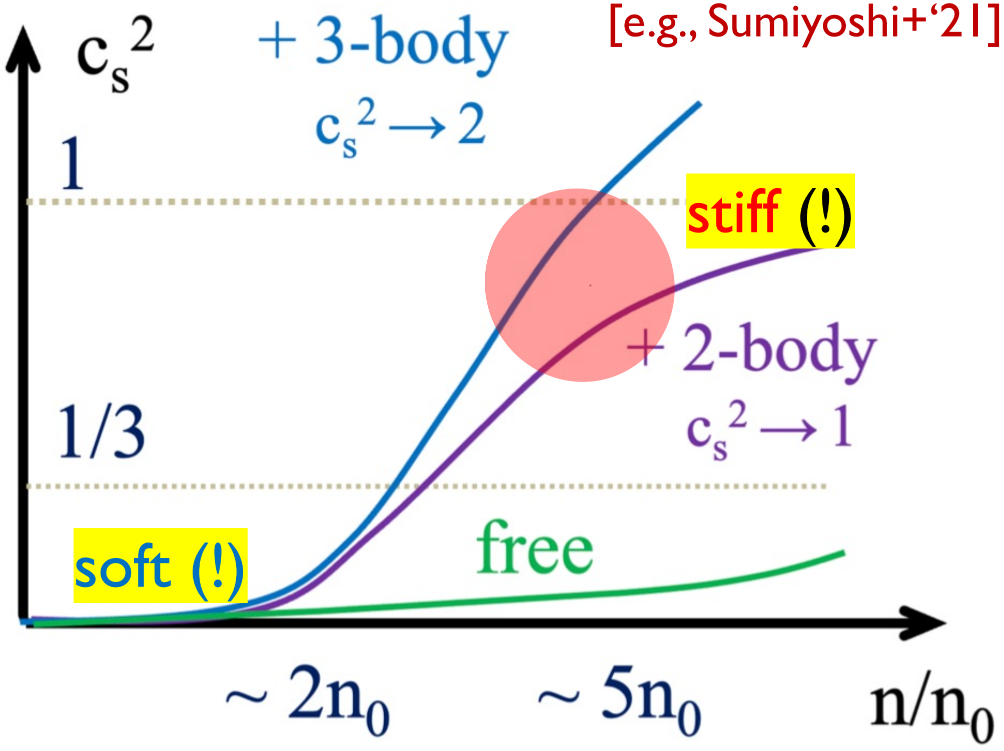
If interactions dominate (at large n_B):

$$P \sim (\alpha - 1)\varepsilon \;\rightarrow\; c_s^2 \sim (\alpha - 1)$$

2-body int. → α = 2 3-body int. → α = 3

- *causality & convergence ??*
- *stiffening occurs slowly (power growth)*

[e.g., Sumiyoshi+'21]



alternative: quark EOS

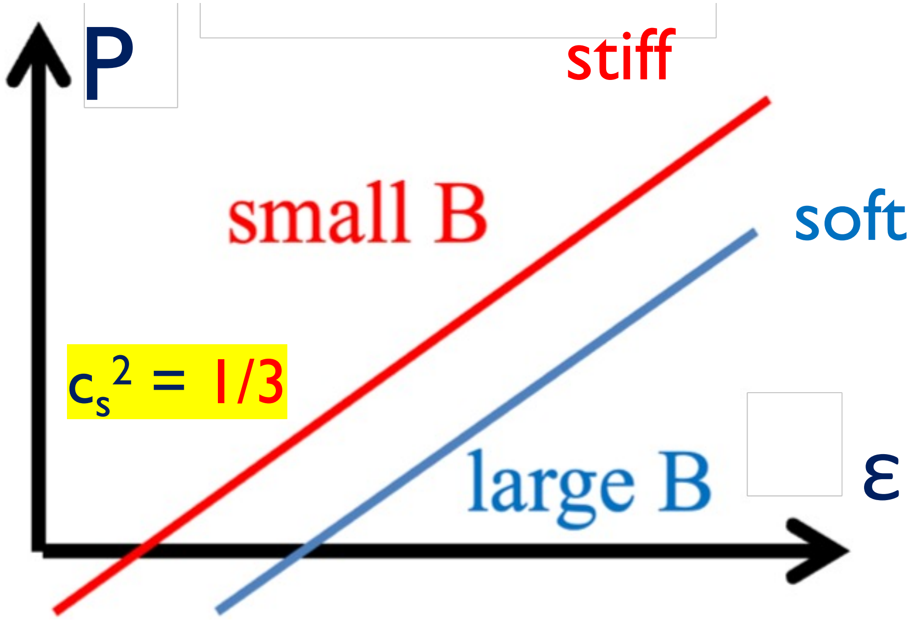
e.g.) free massless quarks

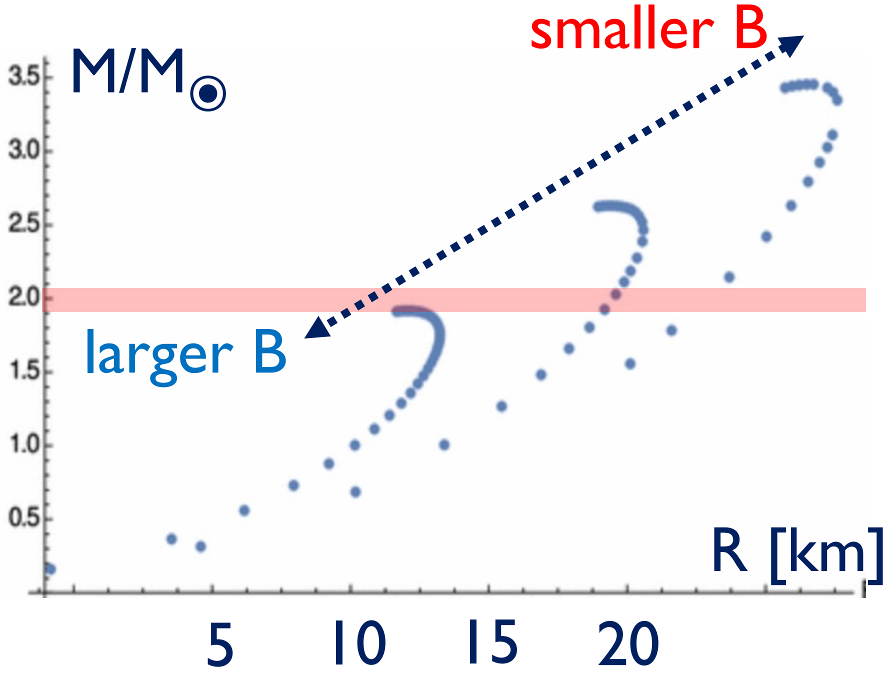
$$P = \frac{\varepsilon}{3} - B'$$

normalization

quark kin. pressure >> baryon kin. pressure

O(N_c) O($1/N_c$)





relativistic pressure → stiff EOS ?

*depends on **where** to start...*

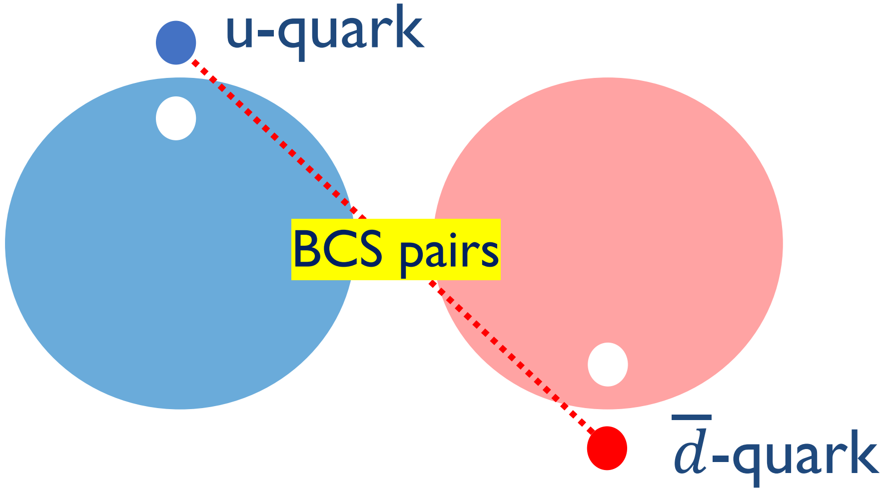
Isospin QCD

isospin chemical pot. $\mu_I = \mu_u = -\mu_d$

$n_0 = 0.16\ \mathrm{fm}^{-3}$

- onset of finite density → begin with pions (instead of nucleons)

high density

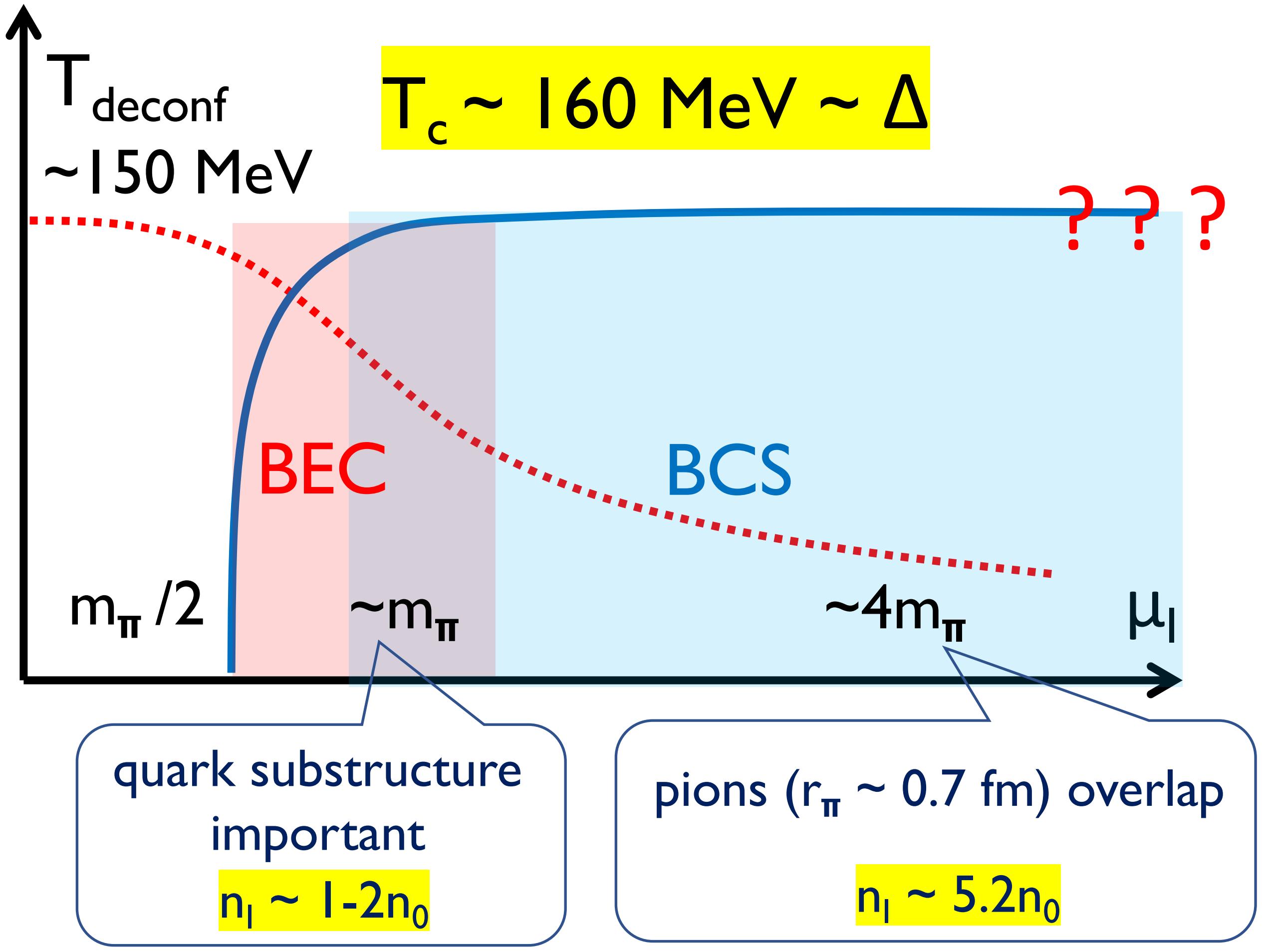


pairing → pion cond. π^+ (p-a) & π^- (h-ah)

→ quark mass gap Δ

~ T_c/0.57 ~ 280 MeV

[see also Fujimoto '24]



Hints for new scale & saturation

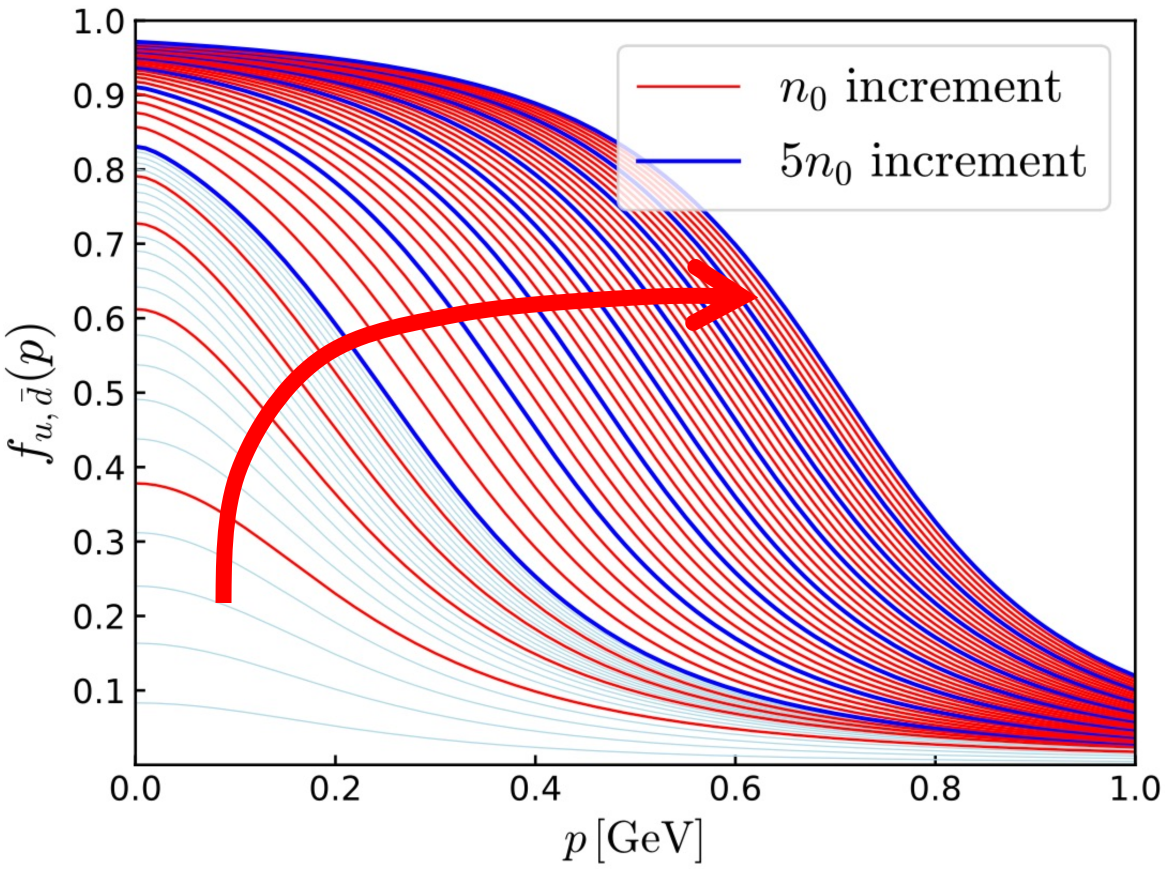
[quark-meson model study, Chiba+ '23; TK+ ‘24; ...]

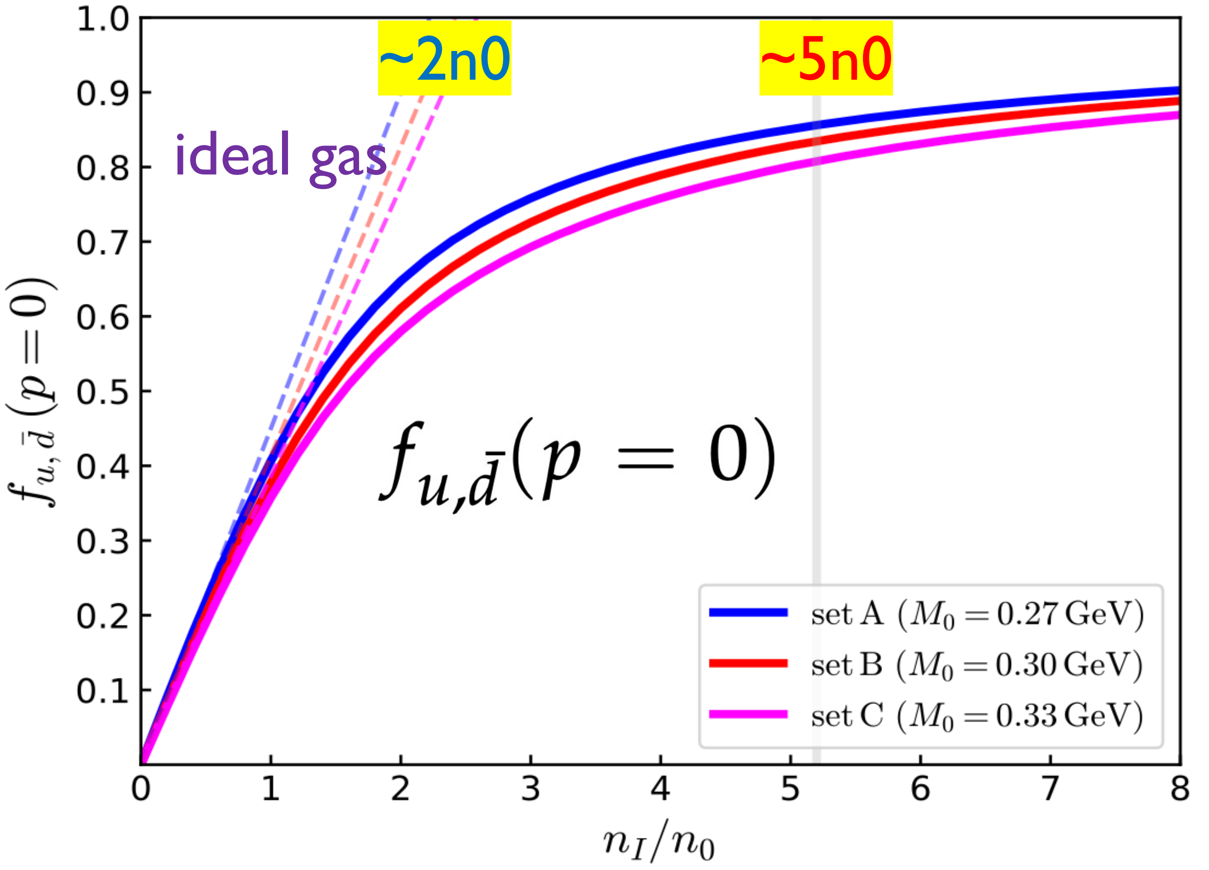
- BCS occ. probability:

$$f_{u,\bar{d}}(p;n_I) = \frac{1}{2}\left(1 - \frac{E_l - \mu_I}{\sqrt{(E_l-\mu_I)^2+\Delta^2}}\right)$$

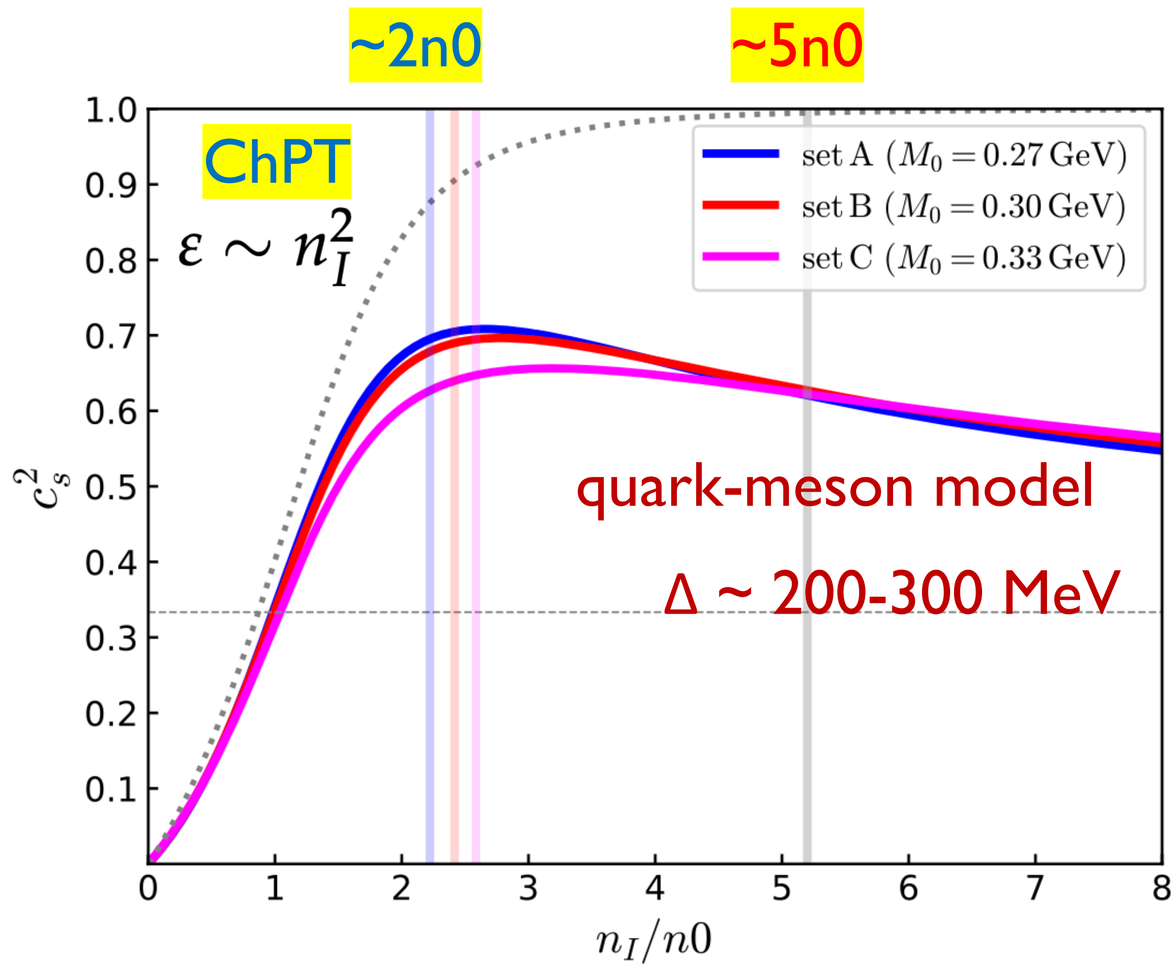
ideal pion gas pic. definitely violated

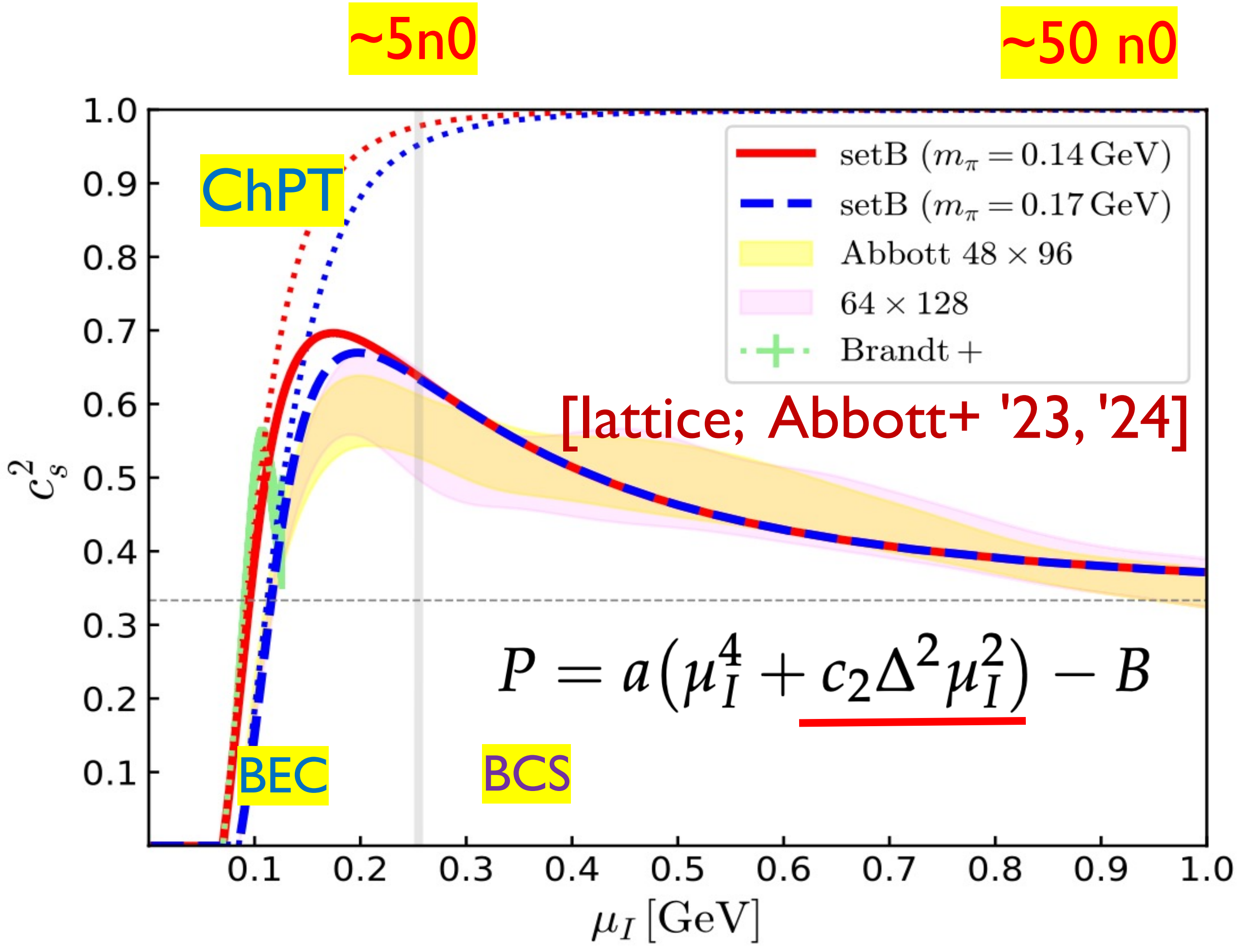
pions with r ~ 0.66 fm overlap





Sound speed: quark-meson model, ChPT, and Lattice



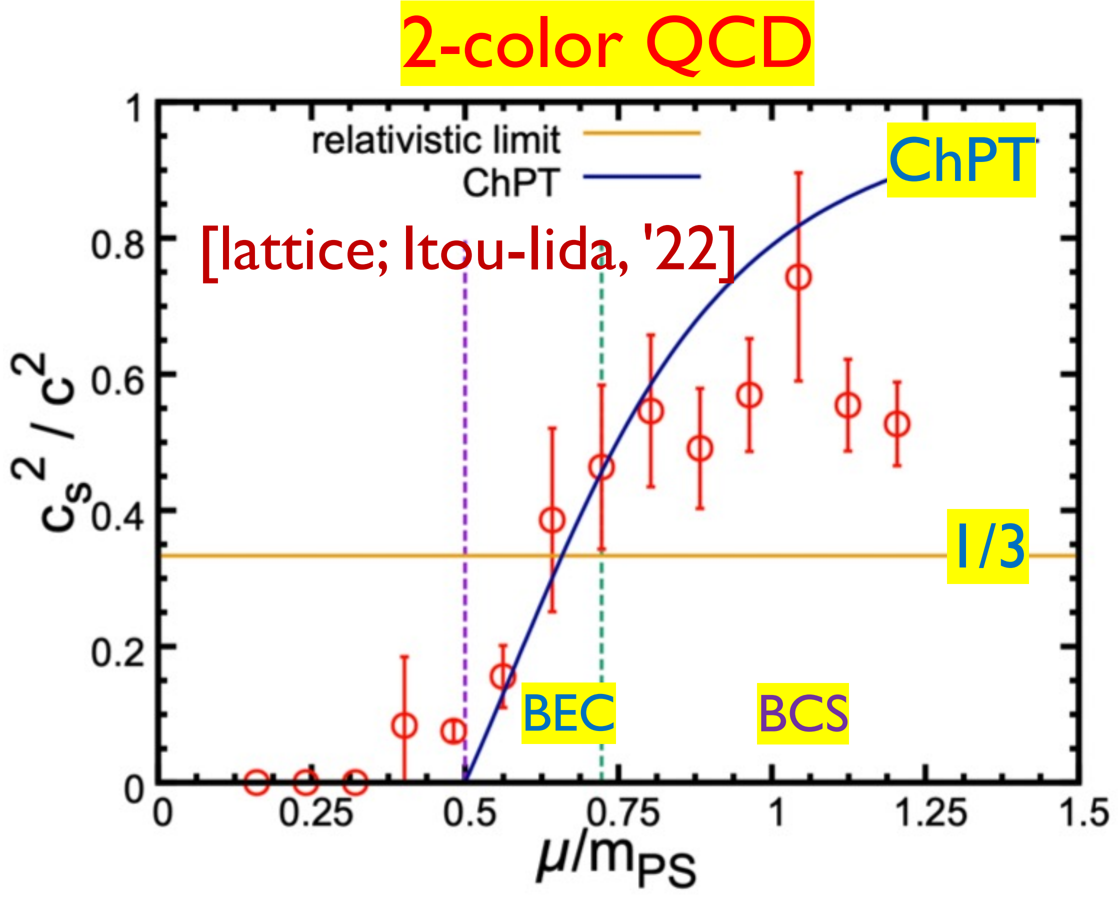


see also two-color QCD, Iida-Itou-Murakami-... ('22, '23, '24)

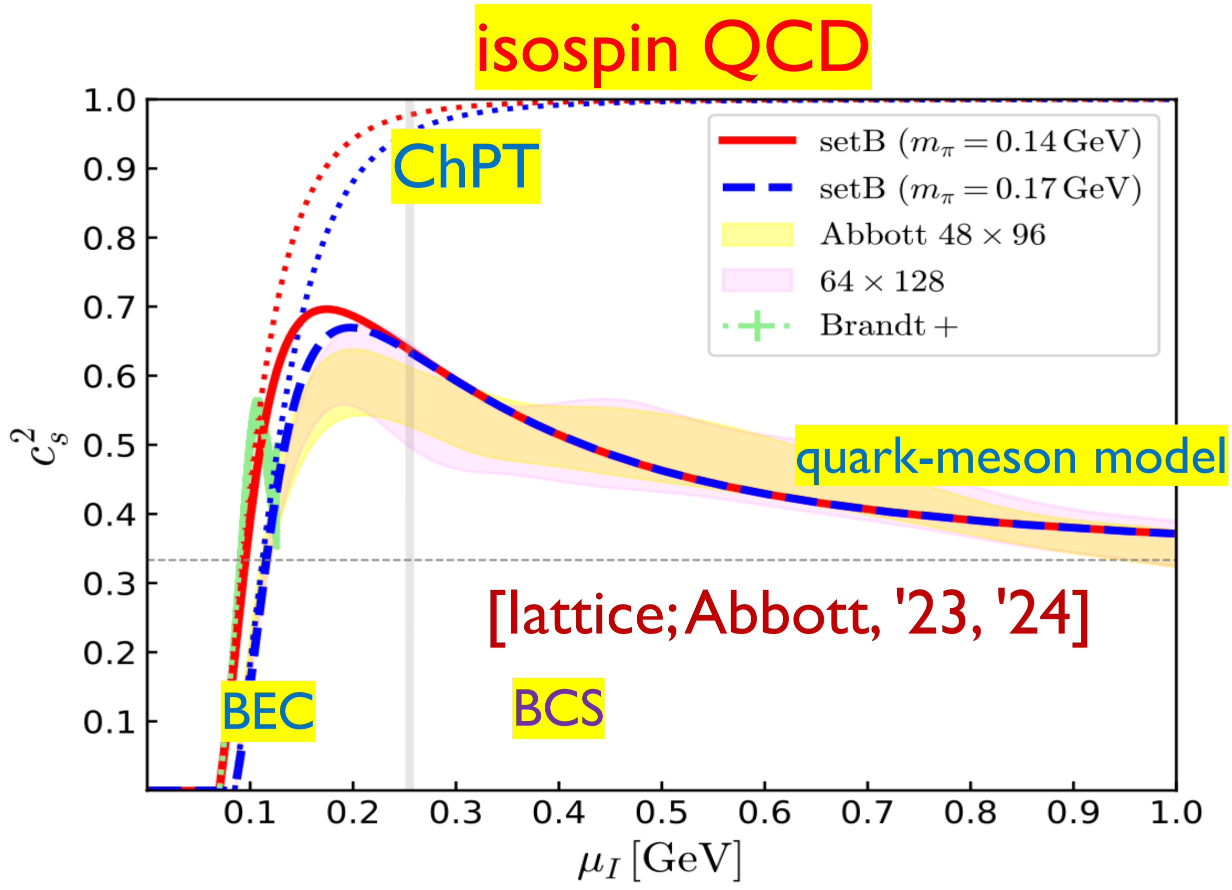
[model study; TK-Suenaga-Chiba '24]

Examples from QCD-like theories

tests on the lattice



[model study, TK-Suenaga '21]



[model study, Chiba-TK '23; TK-Suenaga-Chiba '24]

Peak in the BEC-BCS type crossover

Stiff quark matter

The appearance of c_s^2 peak is characteristic in the QHC scenarios:

good baseline, but NOT necessarily sufficient for ~ 2.1-2.3$M_\odot$ NS.

(just after the crossover, quarks are not fully relativistic.)

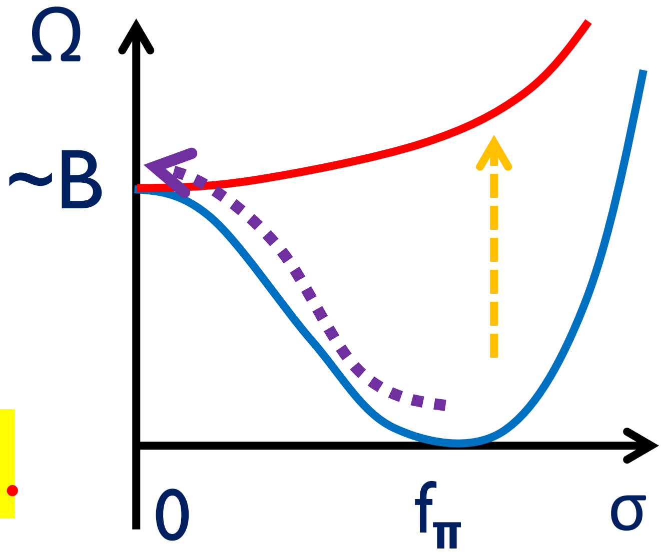
Can the chiral restoration stiffens EOS by making quarks relativistic?

Unlikely: "*the bag constant*" from the Dirac sea

$$\varepsilon \rightarrow \varepsilon + B$$

$$P \rightarrow P - B$$

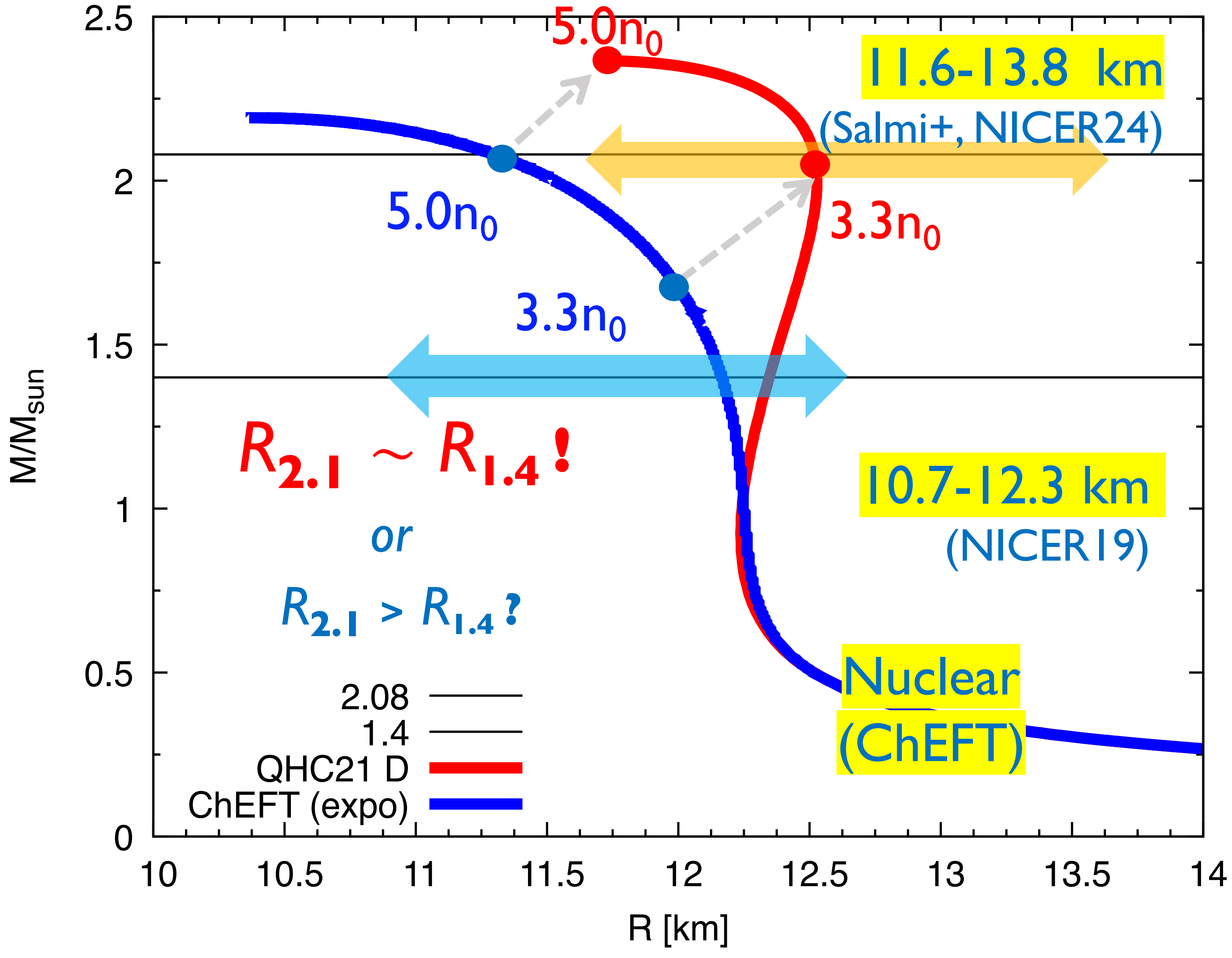
significant softening!

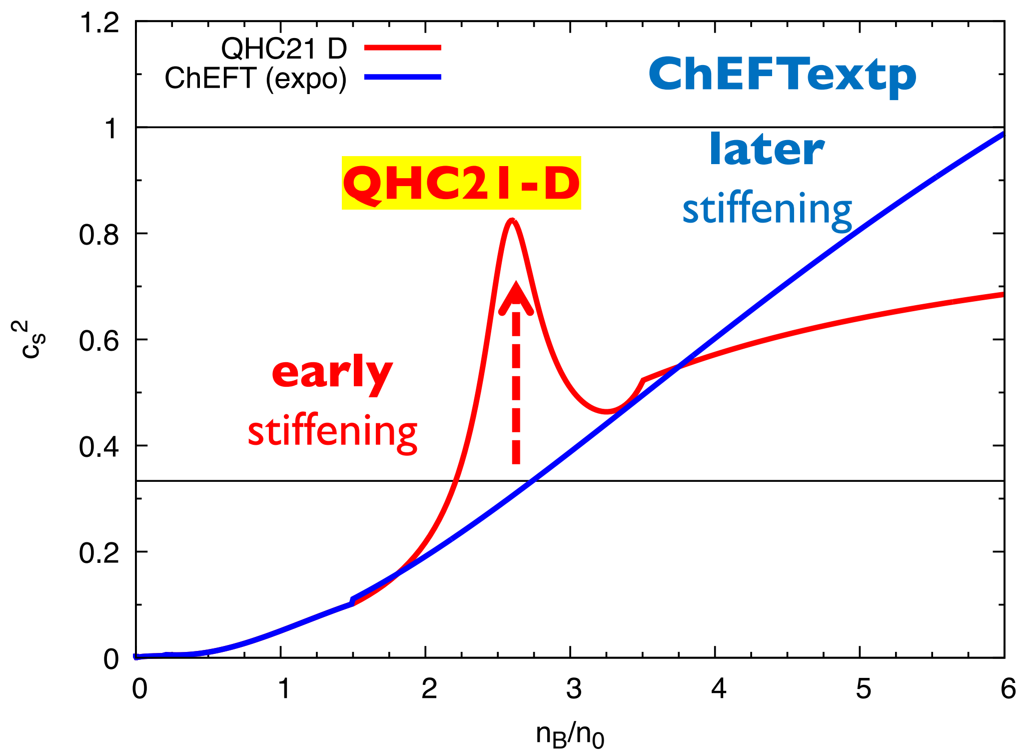


At this stage, we begin to discuss interactions...

Early vs later stiffening: QHC21 vs ChEFTextp

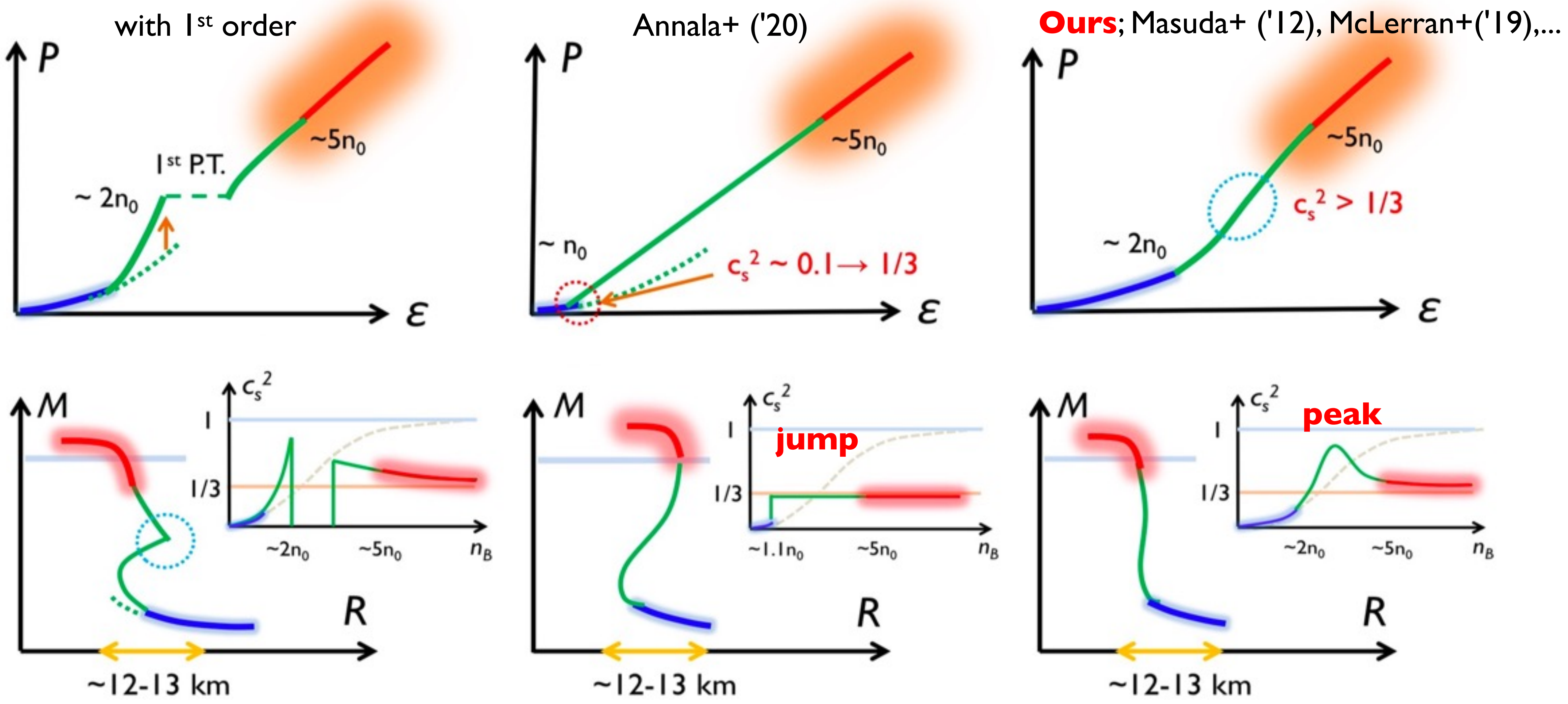
[TK-Hatsuda-Baym '21]





$2\text{-}3n_0$: already beyond purely nucleonic regime?

Three possible scenarios



→ $R_{1.4}$ and $R_{2.1}$?

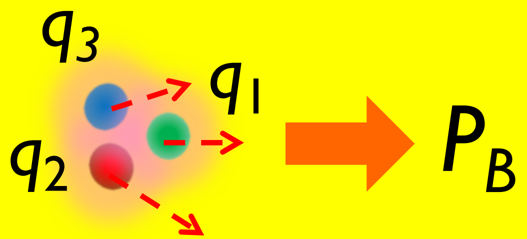
→ *nuclear physics ?*

→ *my favorite*

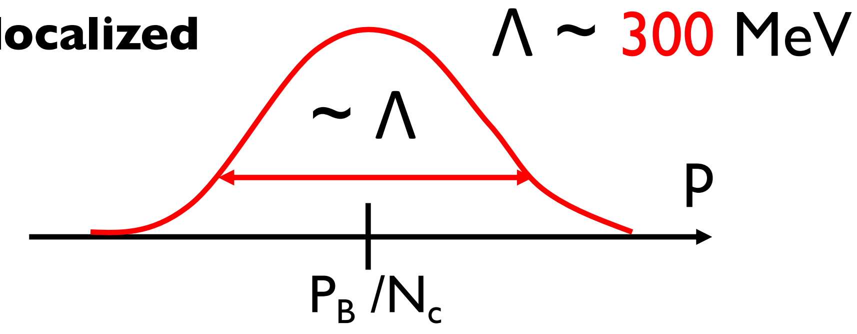
Quarks in a baryon

N_c (=3): number of colors

probability density: $$\varphi(\boldsymbol{q};\boldsymbol{P}_\mathrm{B}) = \mathcal{N} \mathrm{e}^{-\frac{1}{\Lambda^2}\left(\boldsymbol{q}-\frac{\boldsymbol{P}_\mathrm{B}}{N_\mathrm{c}}\right)^2}$$



localized



variance: $$\left\langle \left(\boldsymbol{p} - \frac{\boldsymbol{P}_B}{N_c}\right)^2 \right\rangle \sim \Lambda^2$$ *energetic !*

→ large "*mechanical*" pressure

$$\langle E_q(\boldsymbol{p})\rangle_{\boldsymbol{P}_B} = \mathcal{N}\int_{\boldsymbol{p}} E_q(\boldsymbol{p}) \mathrm{e}^{-\frac{1}{\Lambda^2}\left(\boldsymbol{p}-\frac{\boldsymbol{P}_B}{N_c}\right)^2} \simeq \langle E_q(\boldsymbol{p})\rangle_{\boldsymbol{P}_B=0} + \frac{1}{6}\left\langle \frac{\partial^2 E_q}{\partial p_i \partial p_i}\right\rangle_{\boldsymbol{P}_B=0}\left(\frac{\boldsymbol{P}_B}{N_c}\right)^2 + \cdots$$

average energy (quark)

x N_c x N_c

~ N_c (M_q + E_{kin}) ≫ ~ P_B^2 /($N_c E_q$)

baryon mass baryon kin. energy

Quantum numbers ?

quark quantum numbers; N_c, N_f, 2-spins (for a given spatial w.f.)

how many baryon species are needed to saturate quark states?

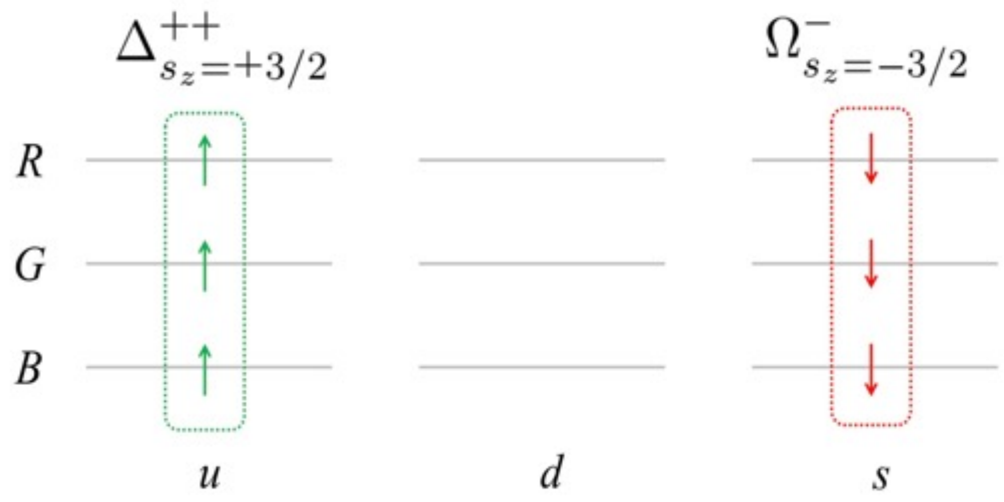
→ need only **$2N_f$ = 6** species for $N_f = 3$

(full members of singlet, octet, decuplet are NOT necessary)

convenient **color-flavor-spin** bases

[neglect N-Δ splitting etc. for simplicity]

$$\Delta^{++}_{s_z=\pm 3/2} = [u_R\uparrow u_G\uparrow u_B\uparrow],\quad [u_R\downarrow u_G\downarrow u_B\downarrow],$$
$$\Delta^{-}_{s_z=\pm 3/2} = [d_R\uparrow d_G\uparrow d_B\uparrow],\quad [d_R\downarrow d_G\downarrow d_B\downarrow],$$
$$\Omega^{-}_{s_z=\pm 3/2} = [s_R\uparrow s_G\uparrow s_B\uparrow],\quad [s_R\downarrow s_G\downarrow s_B\downarrow],$$



Color-magnetic interaction play **many** roles

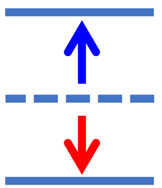
1) **Coupling** $\propto$ **velocity** ~ **p/E**

become important in relativistic regime & high density

cf) Δ (1232)

3M$_q$ + ...

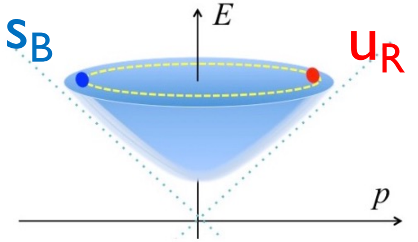
N (938)



2) **Pairing** : strongly channel dependent

hadron mass ordering: N-Δ, etc. [DeRujula+ (1975), Isgur-Karl (1978), ...]

color-super-conductivity [Alford, Wilczek, Rajagopal, Schafer,... 1998-]



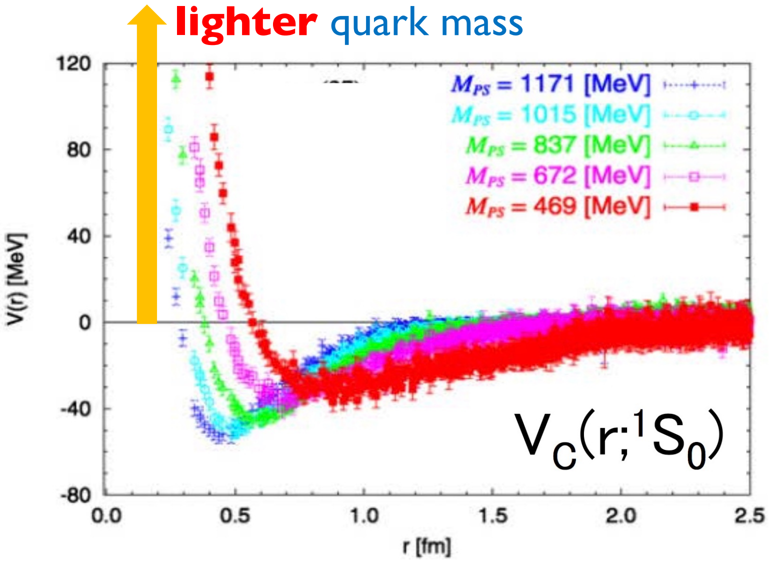
3) **Baryon-Baryon int.** : **short-range** correlation

(**Pauli** + **color-mag.**) [Oka-Yazaki (1980),...]

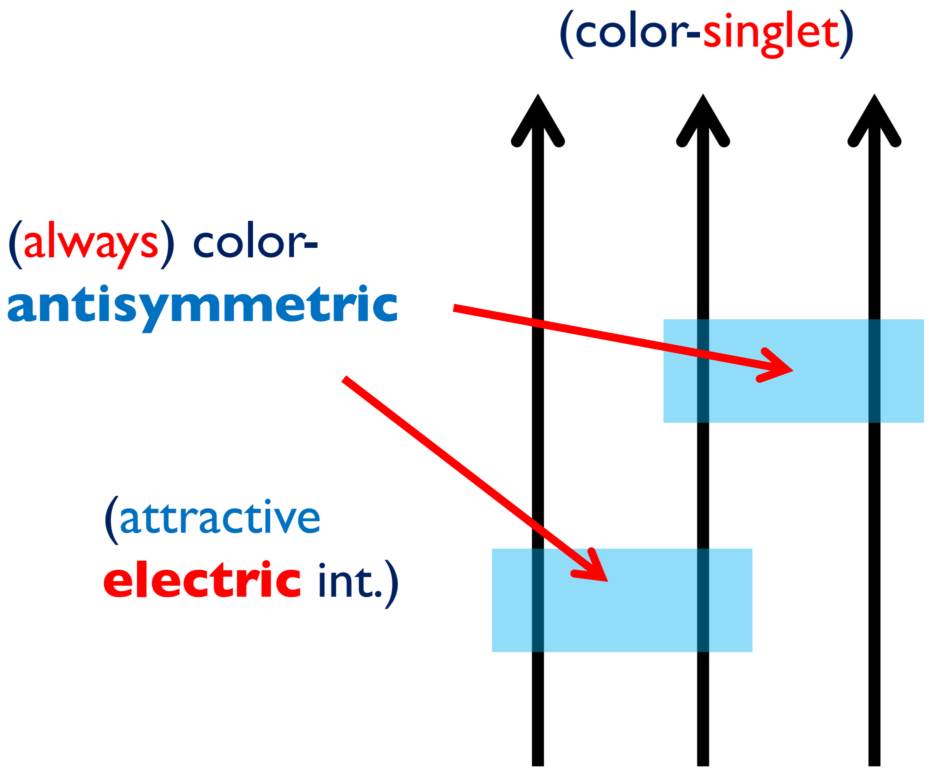
channel dep. → non-universal hard core (some are attractive!)

mass dep. → stronger hard core in relativistic quarks

→ **consistent with the lattice QCD** [HAL-collaboration]



a baryon in dilute regime

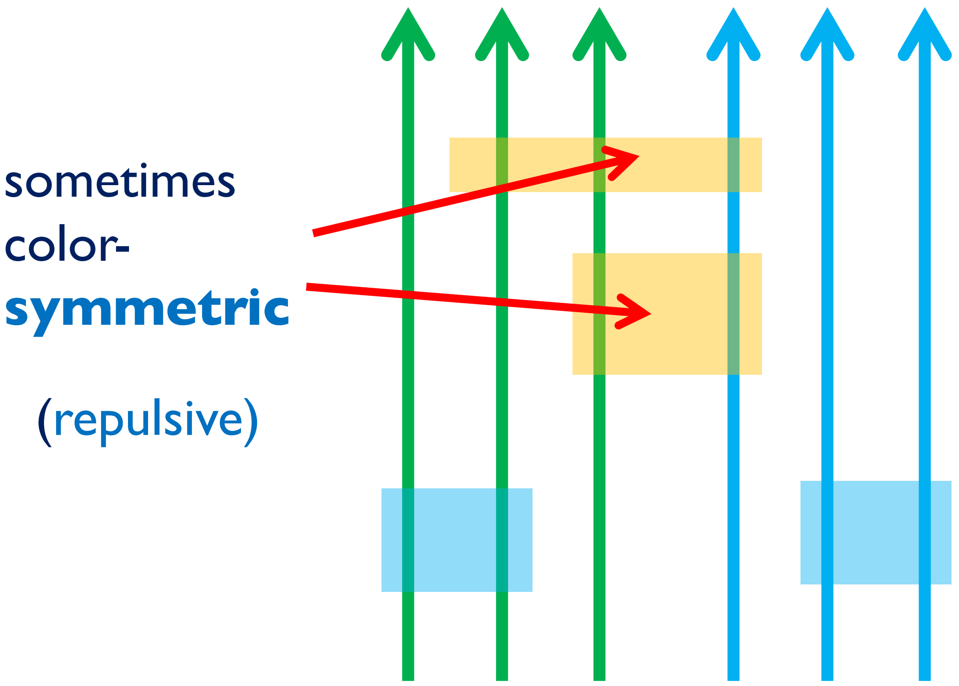


e.g., **nucleons**

$M_N \sim 3M_{q.} + \text{kin.} + \text{color-EM}$

~ 940MeV ~ 1100MeV ~ −150-200MeV

in dense regime



more chances to feel repulsion

EoS with interactions

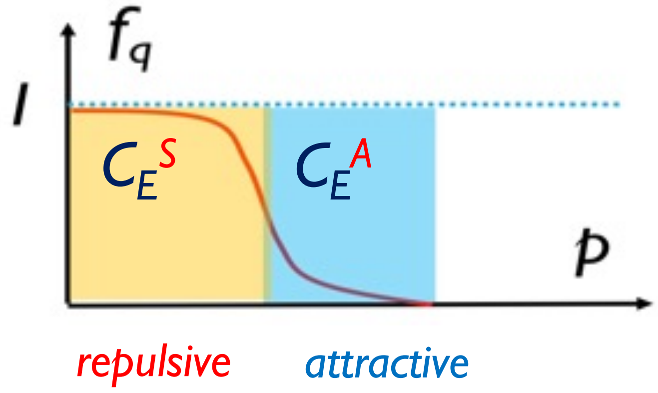
cf) [TK '21, TK-Suenaga '21]

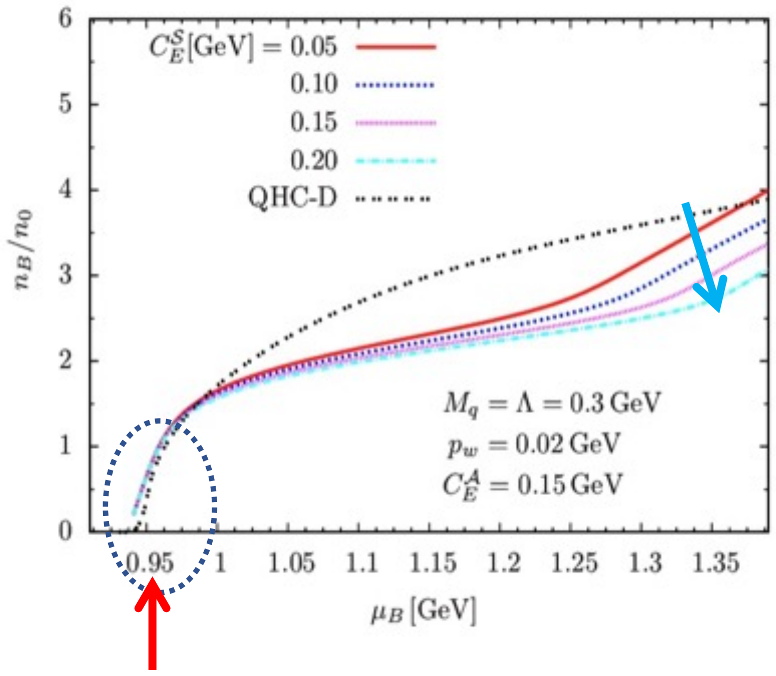
e.g.,

$$\mathcal{V}[f_Q] = -C_E^{\mathcal{A}}\left[1-(f_Q)^n\right] + C_E^{\mathcal{S}}(f_Q)^n$$

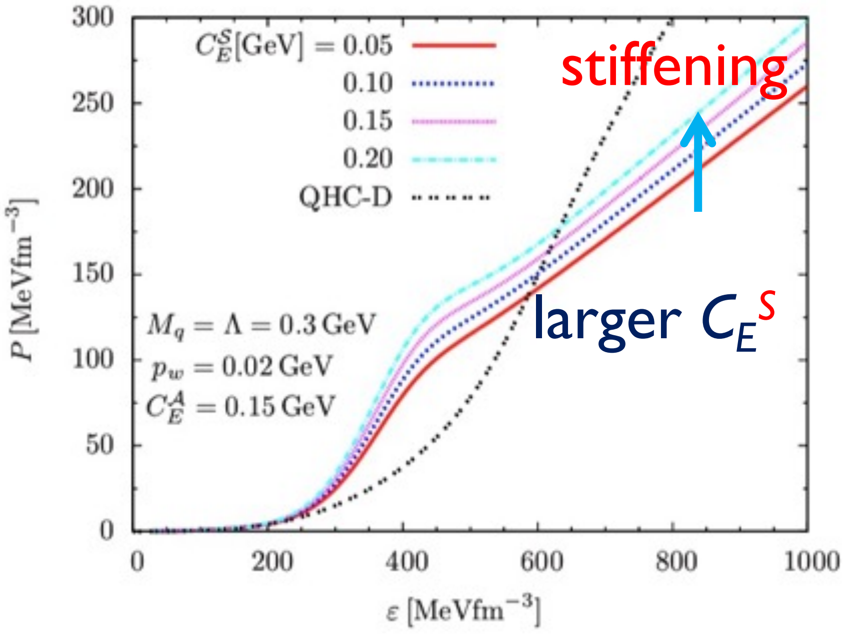
$[1-(f_Q)^n]$ → 1 (dilute), → 0 (dense)

$(f_Q)^n$ → 0 (dilute), → 1 (dense)

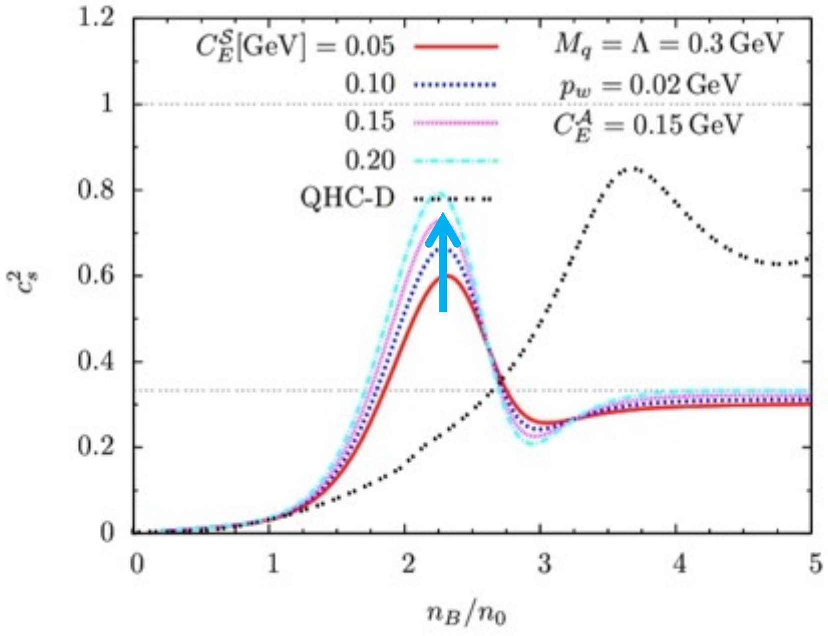




adjust C_E^A (fit M_B = 939 MeV)



high density stiffening



stronger peak in c_s

Important relations

sum rule

single baryon contain single R- or G- or B- quark

$$n_q^{R,G,B} = \int_{\mathbf{p}} f_q(p) = \int_{\mathbf{p}} \left(\int_{\mathbf{P_B}} \mathcal{B}(P_B) Q_{\rm in}(\mathbf{p}; \mathbf{P_B}) \right) = \int_{\mathbf{P_B}} \mathcal{B}(P_B) = n_B$$

energy density

$$E_B(P_B) \equiv N_c \int_{\mathbf{p}} E_q(\mathbf{p}) Q_{\rm in}(\mathbf{p}; \mathbf{P_B})$$

$$\varepsilon = \int_{\mathbf{P_B}} E_B(P_B) \mathcal{B}(P_B) = N_c \int_{\mathbf{P_B}} \left(\int_{\mathbf{p}} E_q(\mathbf{p}) Q_{\rm in}(\mathbf{p}; \mathbf{P_B}) \right) \mathcal{B}(P_B) = N_c \int_{\mathbf{p}} E_q(\mathbf{p}) f_q(p)$$

Dual expression: one can freely switch descriptions

No double counting

Finite-T model

Hadron Resonance Gas model for quark distribution

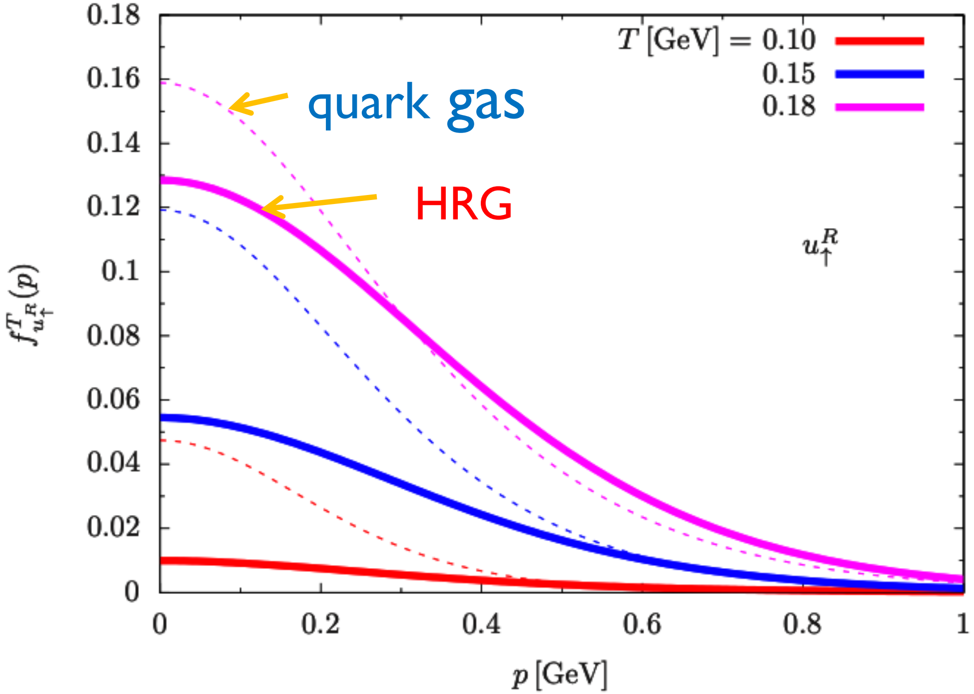
see [TK-Suenaga, '22]

$$f_{\mathbf{q}}^T(\boldsymbol{p}) = \sum_h \int_{\boldsymbol{P}_h} n_h^T(\boldsymbol{P}_h) Q_{\mathrm{in}}^{h\mathbf{q}}(\boldsymbol{p};\boldsymbol{P}_h)$$

$$n_h^T(\boldsymbol{P}_h) = [\mathrm{e}^{E_h(\boldsymbol{P}_h)/T} - 1]^{-1}$$

- calculate quark w.f. for mesons up to $L = 3$, $n_r = 4$; $E <\sim 2.5$ GeV

	$n_r{}^{2S+1}L_J$	$M_{\rm exp}$	$M_{\rm cal}$	$\bar{\boldsymbol{P}}^2$	$\sqrt{\langle \boldsymbol{r}^2\rangle}$	f_S	α_s
π	1^1S_0	0.14	0.16	0.47	0.50	0.70	0.80
	2^1S_0	1.30	1.28	0.43	0.98		
	3^1S_0	1.81	1.82	0.55	1.38		
	4^1S_0	2.07**	2.22	0.67	1.66		
ρ	1^3S_1	0.78	0.76	0.21	0.66	0.74	0.80
	2^3S_1	1.47	1.44	0.35	1.17		
	3^3S_1	1.91*	1.87	0.48	1.55		
	4^1S_1	2.27**	2.22	0.61	1.83		
K	1^1S_0	0.49	0.49	0.42	0.49	0.72	0.77
	2^1S_0	1.46*	1.46	0.45	0.98		
K^*	1^3S_1	0.89	0.91	0.24	0.63	0.75	0.77
	2^3S_1	1.41	1.54	0.39	1.10		



quark gas ~ HRG

at ~ 0.15-0.18 GeV