Vertical Spin build-up due to Vertical Offset in Quadrupoles and Horizontal Betatron Oscillations

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- Introduction – Initial considerations on the effect
- Hybrid ring – the case of interest
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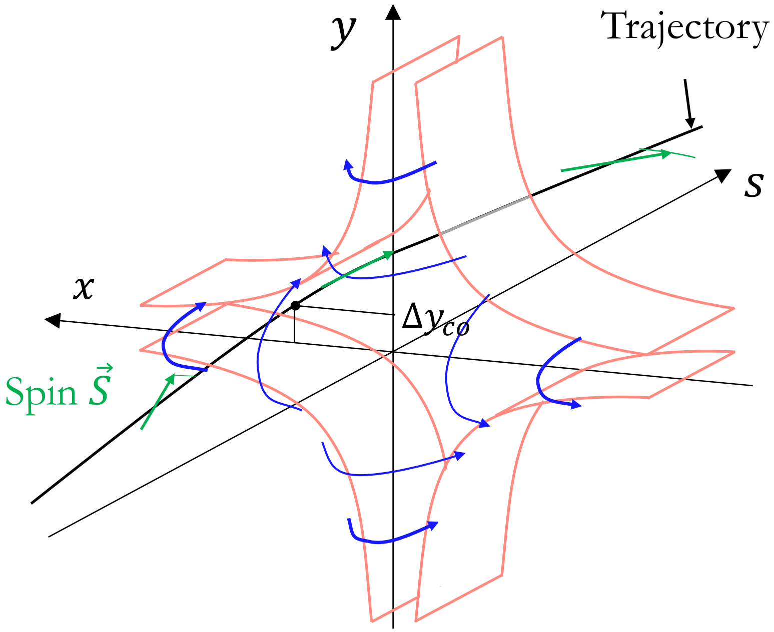
  - Simulation Results for a focusing quadrupole
    - Comparison between analytical estimates and simulation results
  - Simulation results for a defocusing quadrupole
- Lattice with electric focusing
  - Simulation Results for electric focusing
  - Analytical Estimates – electric quadrupoles
- Summary and Conclusions

Introduction

Initial considerations on the effect

- Combination of
  - Vertical offset Δy_{co} between beam and quadrupole
  - Horizontal betatron oscillations
- Lead to
  - Longitudinal magnetic field components in entrance and exit fringe field regions => Spin rotation around longitudinal axis
  - Horizontal deflection inside quadrupole magnet => Spin rotation around vertical axis
- Result: classical geometric phase effet

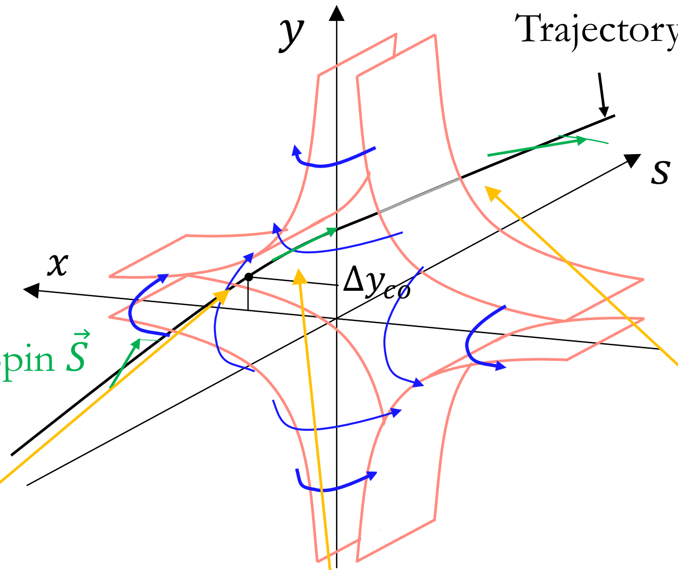


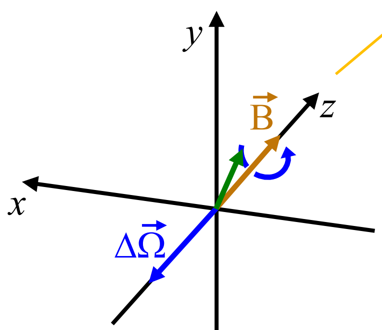
- Note:
  - Changing horizontal position of particle (e.g., from positive x to negative x corresponding to 180° change of betatron phase) inverts all rotations => Effect adds up
  - There may be other not yet understood spin rotations in addition (possibly explaining discrepancy between analytical estimates and simulation results)

Introduction

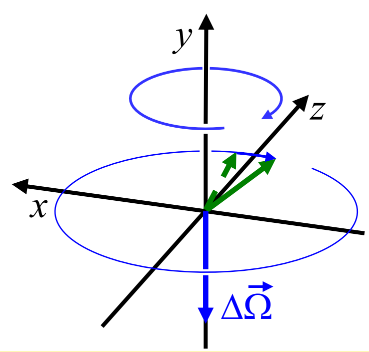
Initial considerations on the effect

A classical geometric phase effect!

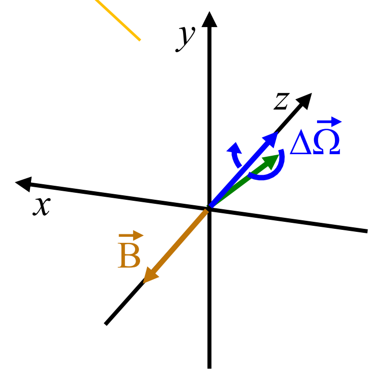




Entrance fringe field:
Positive longitudinal B
& positive radial spin
=> Negative vertical spin



Inside Quadrupole
Spin rotates faster than direction motion

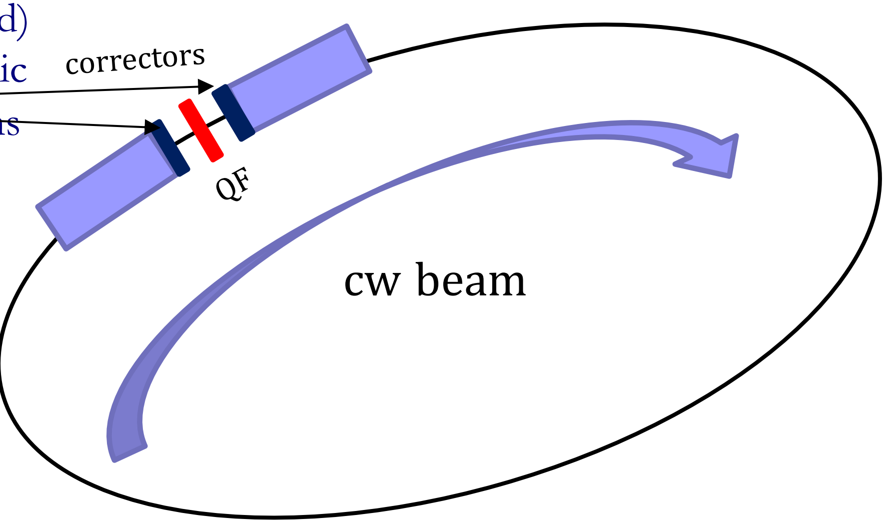


Exit fringe field:
Negative longitudinal B
& negative radial spin
=> Negative vertical spin

Simulation Results
Hybrid Ring - the case of interest

- The symmetric-hybrid ring lattice design has been used (https://journals.aps.org/prd/abstract/10.1103/PhysRevD.105.032001)

- A vertical offset of one QF quadrupole (focusing quad) in the middle of the machine has been added (magnetic correctors have been also added) – no orbit distortions outside:
  - Case 1: one quad offset of 0.1 mm
  - Case 2: one quad offset of 0.2 mm



- Spin tracking results are shown for a longitudinally polarized beam

- The results have been computed for particles executing different betatron oscillations

- The simulation results have been compared with analytical estimates

Analytical Estimates – Magnetic Quadrupoles

- Inside quadrupole and considering vertical component of angular frequencies for spin rotation $\omega_{s,y} = -\frac{q}{m}\left(G+\frac{1}{\gamma}\right)B_y$ rotation of particle direction $\omega_{p,y} = -\frac{q}{m\gamma}B_y$

=> Radial spin component: $S_x \approx (\omega_{s,y}/\omega_{p,y})x' = (\gamma G+1)x'$ and $S_x - x' \approx \gamma G\, x'$ (somewhat smaller for hybrid ring with part of focusing from electric bendings?)

Upper (lower) sign for quad entrance (exit)

- Integrated longitudinal magnetic field region of focusing magnet with strength k with x and Δy_{co} the transverse coordinates
  - Generates rotation around longitudinal axis

$$\int B_s\, ds = \pm \frac{m\gamma\beta c}{q} k\, x\, \Delta y_{co}$$

$$\Delta\alpha_s = -\frac{q}{m}\frac{G+1}{\gamma}\int B_s \frac{ds}{\beta c} = \mp(G+1)k\, x\, \Delta y_{co}$$

- Gives vertical spin component

$$\Delta S_y = \Delta\alpha_s(S_x - x') = \mp(G+1)k\Delta y_{co}\, x(S_x - x') \approx \mp\gamma\, G(G+1)k\, \Delta y_{co}\, x\, x'$$

- Averaging $x\,x'$ over betatron oscillations with β_x and α_x the Twiss parameters and J_x the action variable

$$\langle xx'\rangle = \frac{1}{2\pi}\int d\mu\, \sqrt{2J_x\beta_x}\, \cos\mu \sqrt{2J_x/\beta_x}\,(\sin\mu - \alpha_x\cos\mu) = -J_x\alpha_x$$

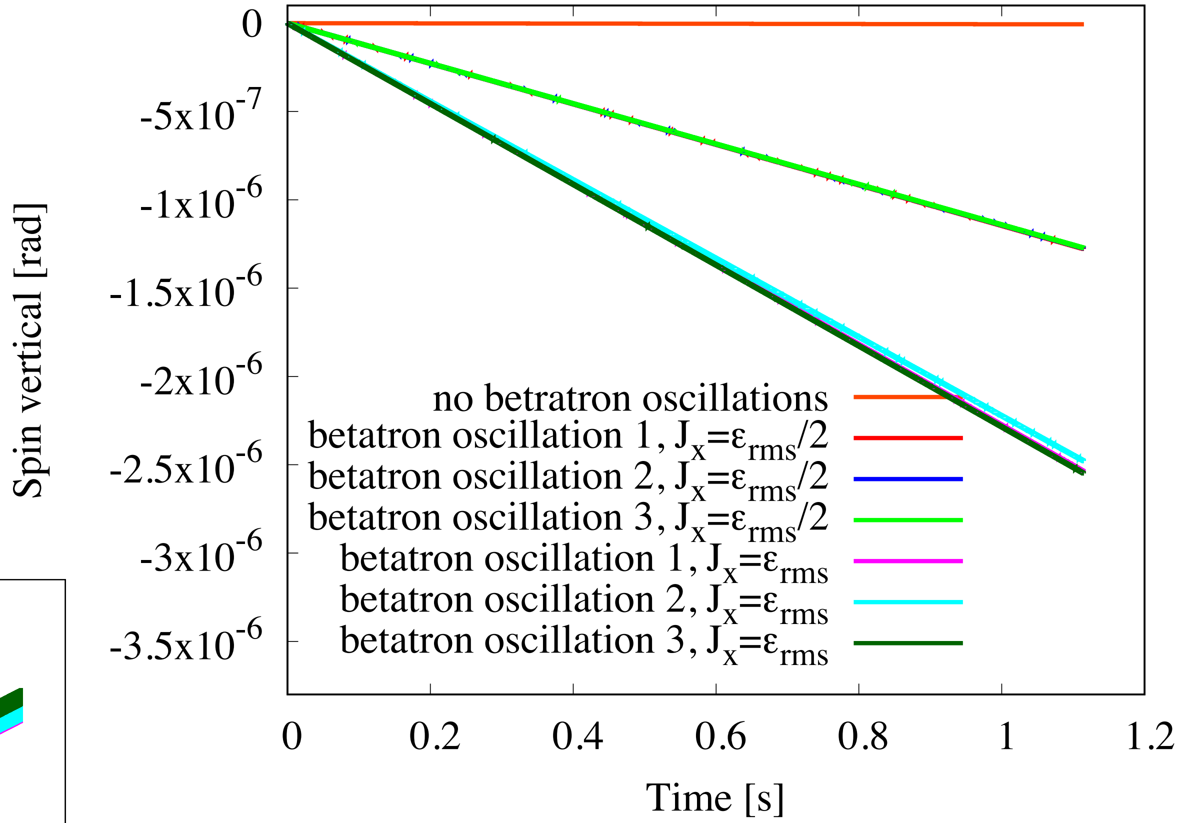
- Average spin buil-up rate with indices i and o for quadrupole entrance and exit

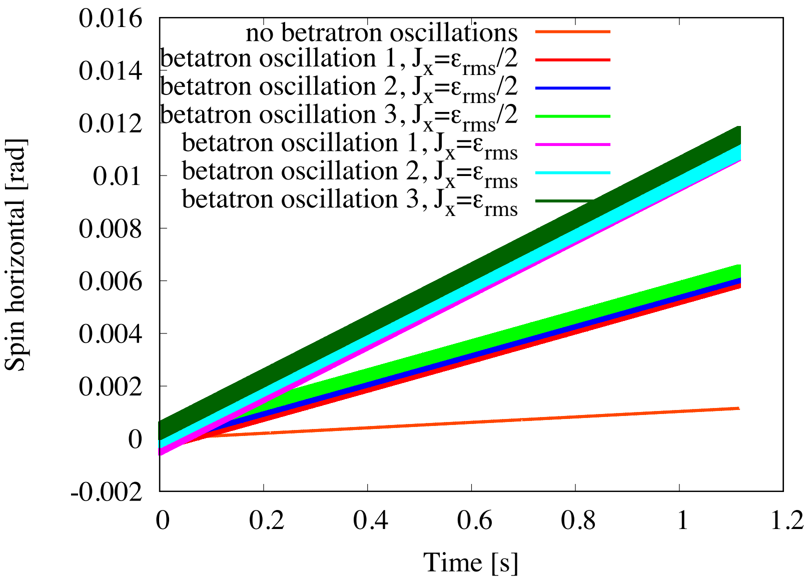
$$\dot{S}_y = \frac{\gamma\, G(G+1)k}{C/(\beta c)}\left(\Delta y_{co,i}\alpha_{x,i} - \Delta y_{co,o}\alpha_{x,o}\right) J_x$$

(rather upper limit for hybrid ring - should be exact for structure without bendings)

Simulation Results
for a focusing quadrupole

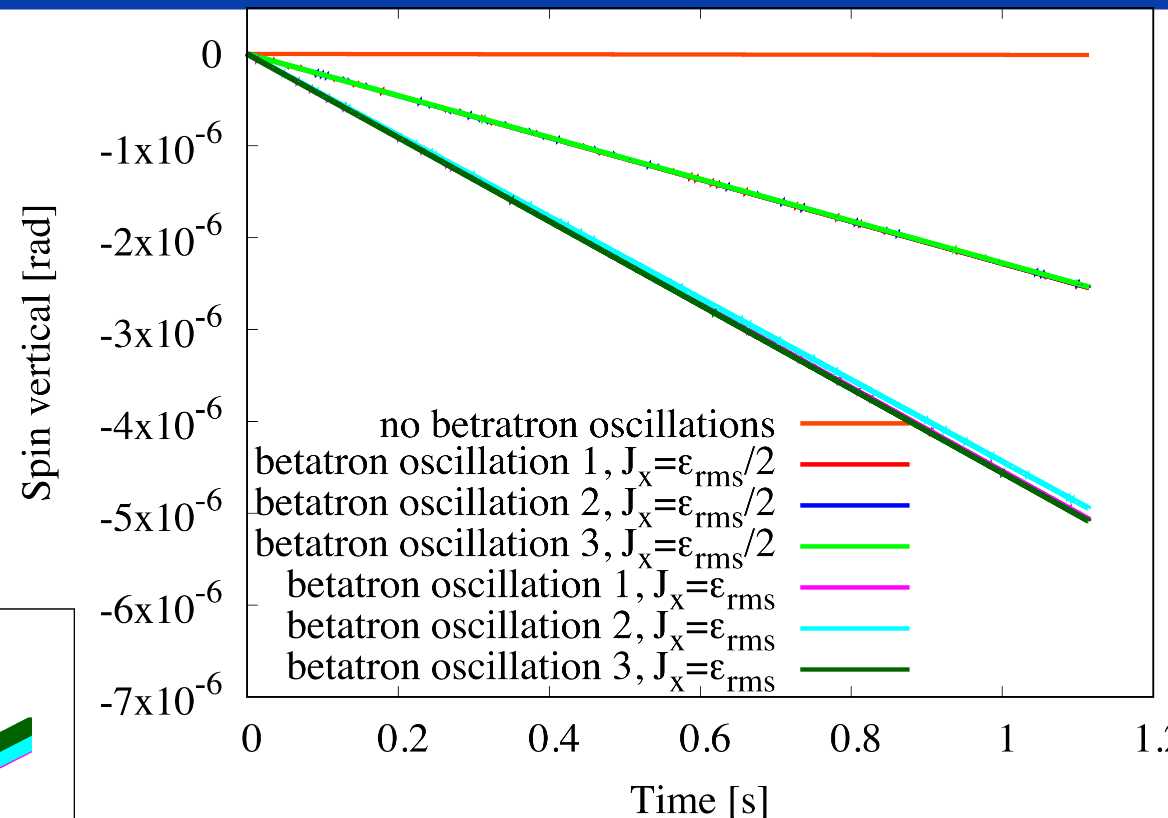
- Case of an offset of 0.1 mm

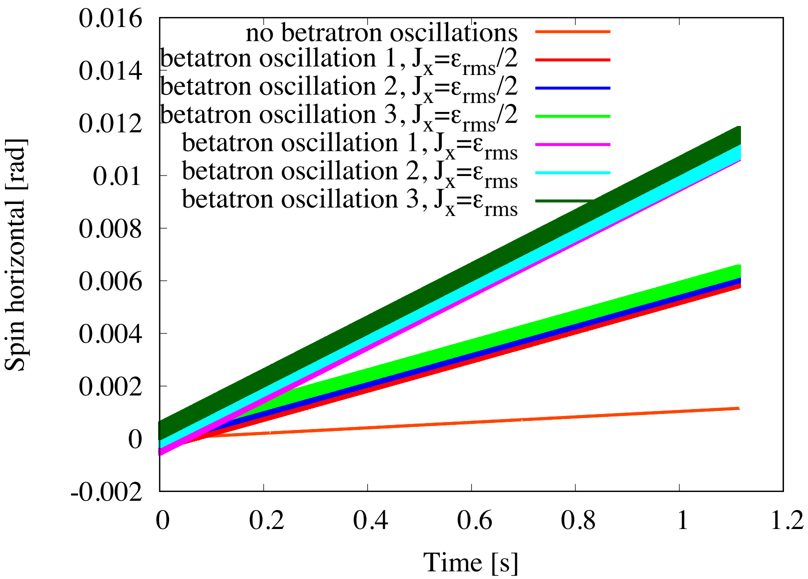




Simulation Results
for a focusing quadrupole

- Case of an offset of 0.2 mm





Comparison between Analytical estimates and Simulation Results

- From analytical estimates – case of 0.1 mm:
  - $J_x = \varepsilon_{rms} = 0.214\ \mu\text{m} \rightarrow \dot{S}_y = 5.9 \cdot 10^{-6}$ rad/s

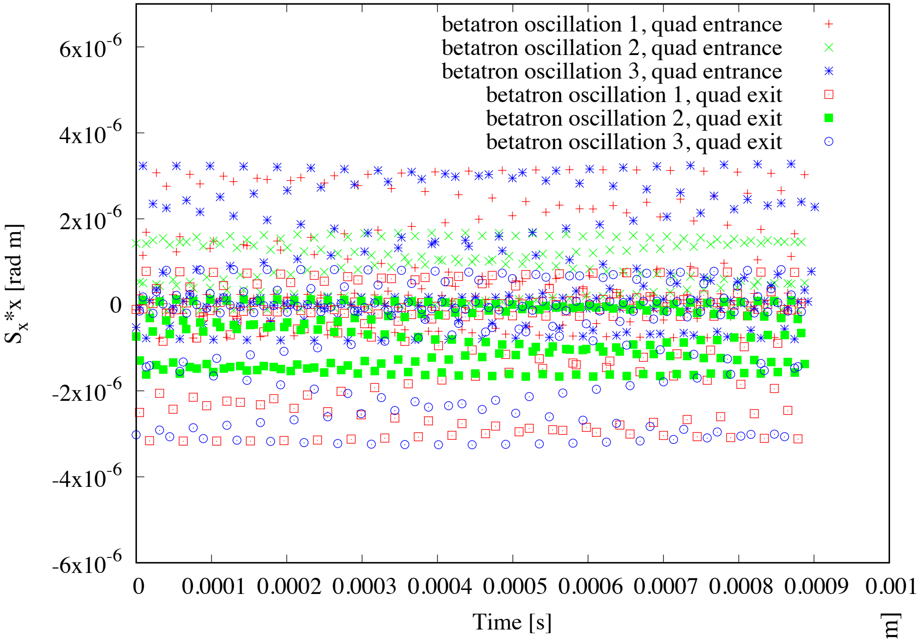
- Structure with bendings (from simulations) – case offset 0.1 mm:
  - $J_x = \varepsilon_{rms}/2 \rightarrow \dot{S}_y = -1.28 \cdot 10^{-6}$ rad/s
  - $J_x = \varepsilon_{rms} \rightarrow \dot{S}_y = -2.54 \cdot 10^{-6}$ rad/s

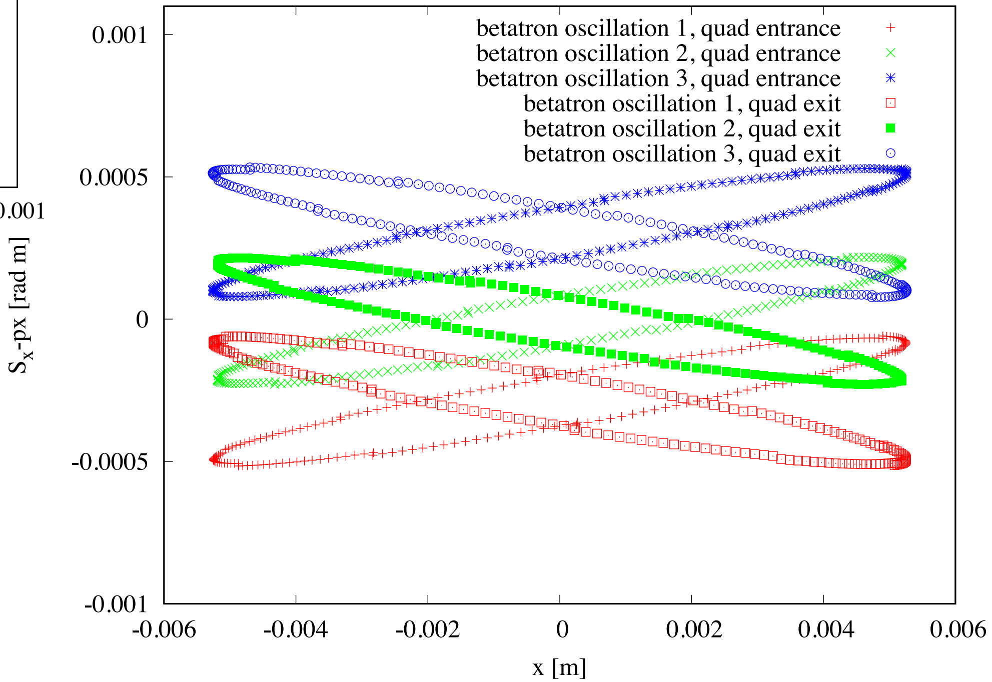
- Structure with bendings (from simulations) – case offset 0.2 mm:
  - $J_x = \varepsilon_{rms}/2 \rightarrow \dot{S}_y = -2.54 \cdot 10^{-6}$ rad/s
  - $J_x = \varepsilon_{rms} \rightarrow \dot{S}_y = -5.06 \cdot 10^{-6}$ rad/s

- Structure without bendings (from simulations) - analytical estimates should be exact - case of 0.1 mm:
  - $J_x = \varepsilon_{rms}/2 \rightarrow \dot{S}_y = -1.51 \cdot 10^{-6}$ rad/s
  - $J_x = \varepsilon_{rms} \rightarrow \dot{S}_y = -3.02 \cdot 10^{-6}$ rad/s

Simulation Results
for a focusing quadrupole

- Positions of the offset quadrupole at the entrance and the exit (case of $J_x = \varepsilon_{rms}$)

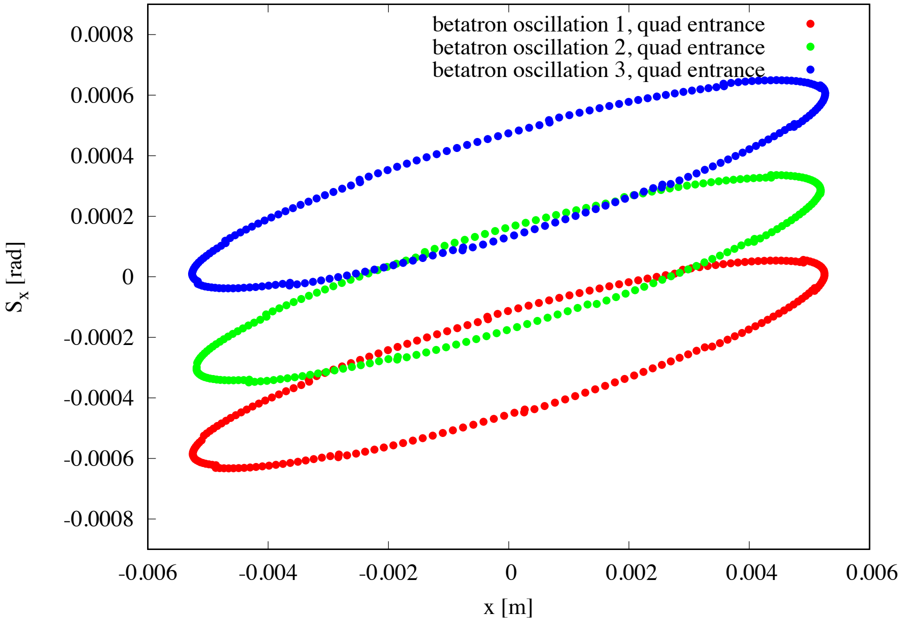




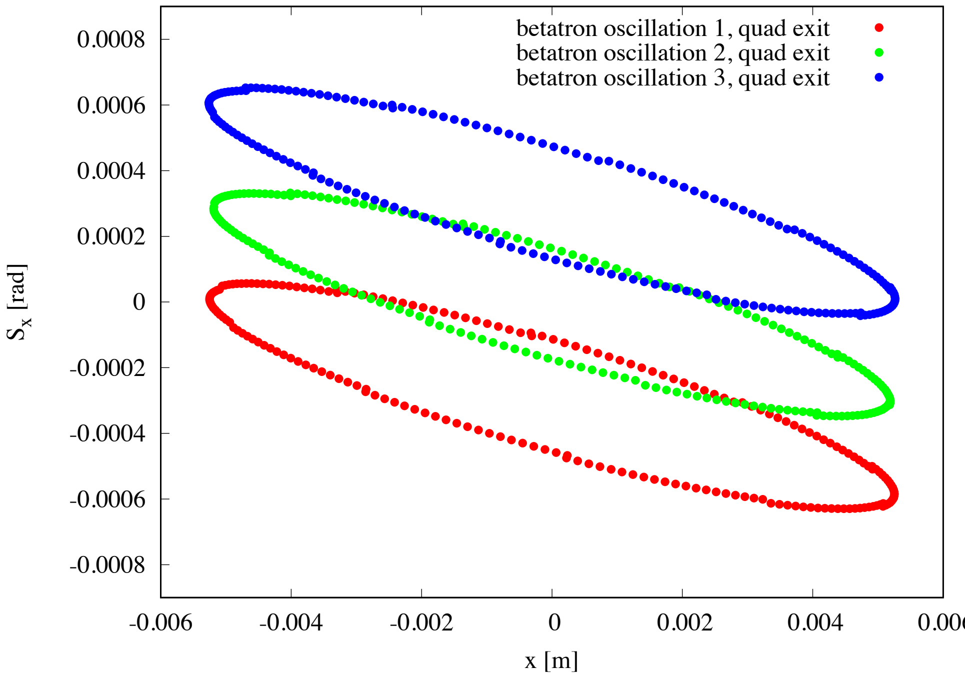
Simulation Results
for a focusing quadrupole

- Positions of the offset quadrupole at the entrance and the exit (case of $J_x = \varepsilon_{rms}$)

Quad entrance

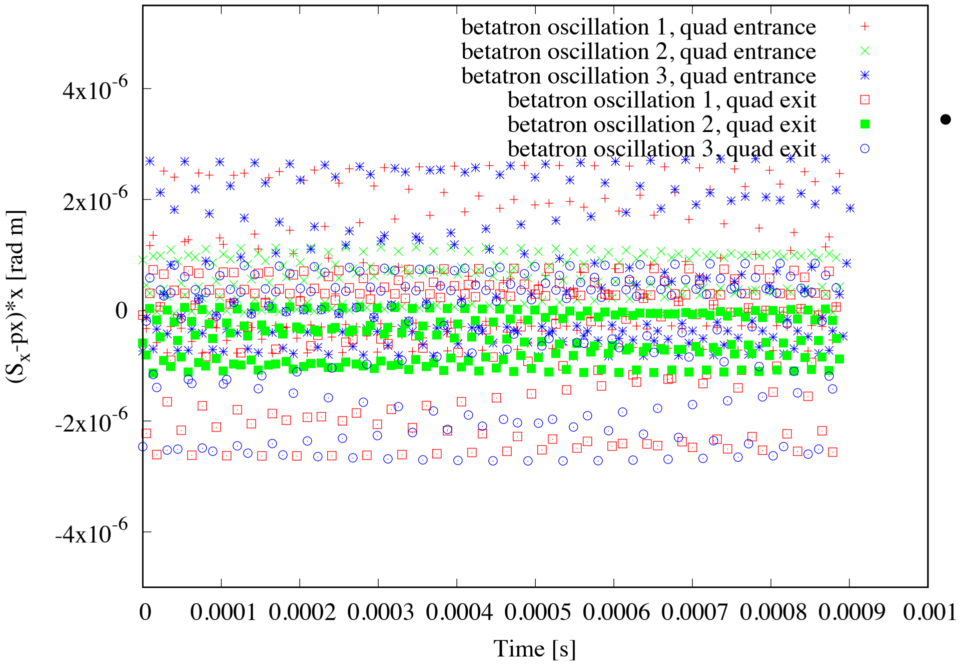


Quad exit



Simulation Results
for a focusing quadrupole

- Positions of the offset quadrupole at the entrance and the exit (case of $J_x = \varepsilon_{rms}$)



- From analytical estimates:
  $< (S_x - x') * x > \approx \gamma G < xx' > = -\gamma G J_x \alpha_x = \pm 5.39 \cdot 10^{-7}$ rad m

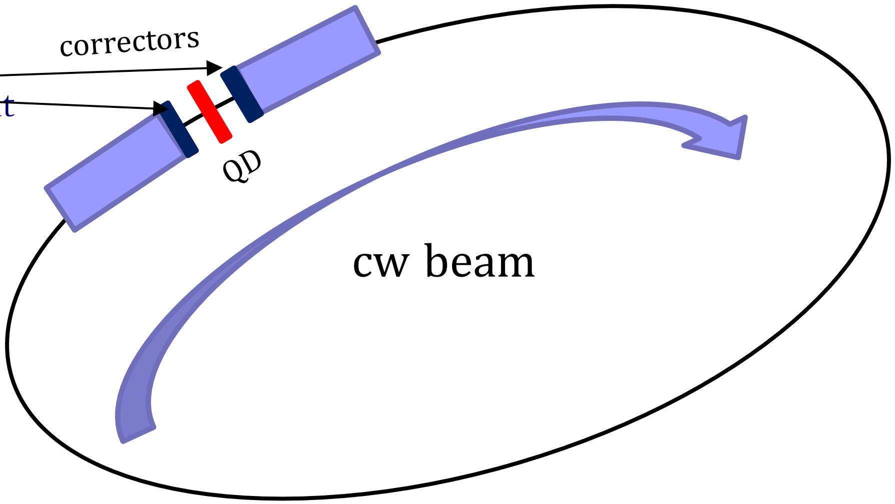
- From simulations:
  - Betatron 1:
    $< (S_x - x') * x > = 5.4097 \cdot 10^{-7}$ (entrance)
    $< (S_x - x') * x > = -5.4222 \cdot 10^{-7}$ (exit)
  - Betatron 2:
    $< (S_x - x') * x > = 5.2821 \cdot 10^{-7}$ (entrance)
    $< (S_x - x') * x > = -5.284 \cdot 10^{-7}$ (exit)
  - Betatron 3:
    $< (S_x - x') * x > = 5.4158 \cdot 10^{-7}$ (entrance)
    $< (S_x - x') * x > = -5.4678 \cdot 10^{-7}$ (exit)

showing good agreement with simulations…

Simulation Results
Hybrid Ring - case of the offset in the defocusing quad

- The symmetric-hybrid ring lattice design has been used (https://journals.aps.org/prd/abstract/10.1103/PhysRevD.105.032001)

- A vertical offset of one QD quadrupole (defocusing quad) in the middle of the machine has been added (magnetic correctors have been also added) – no orbit distortions outside:
  - Case 1: one quad offset of 0.1 mm

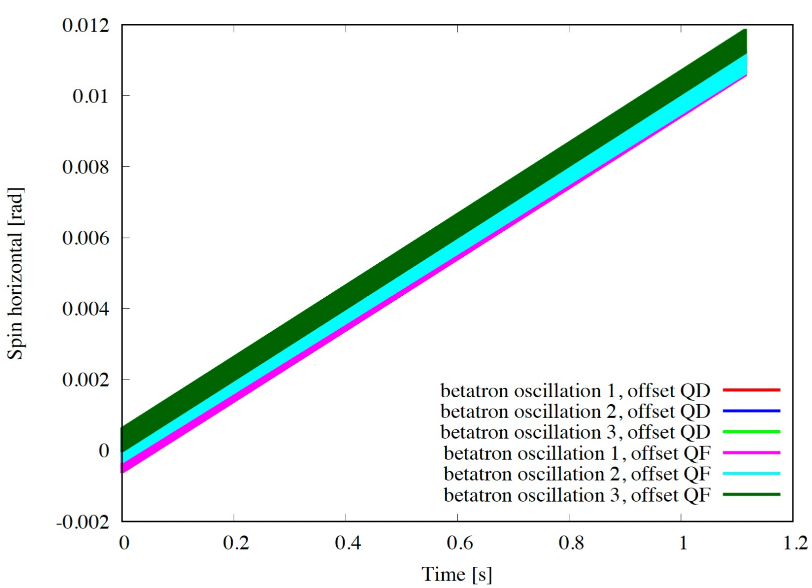


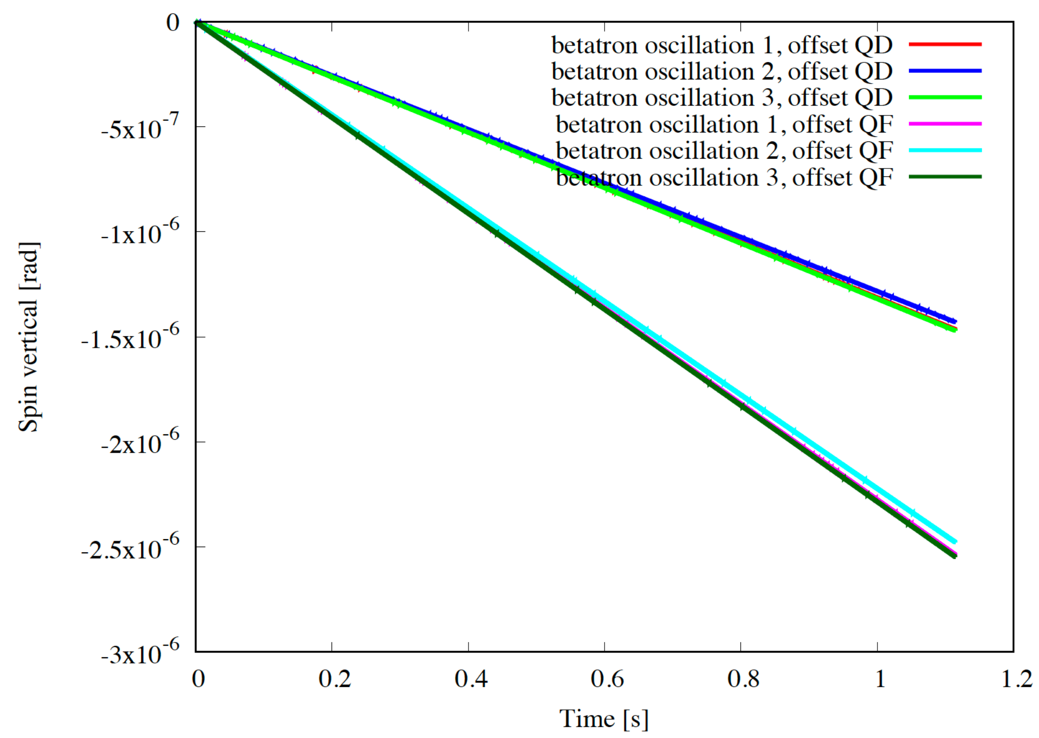
- Spin tracking results are shown for a longitudinally polarized beam

- The results have been computed for particles executing different betatron oscillations

Simulation Results
for a defocusing quadrupole

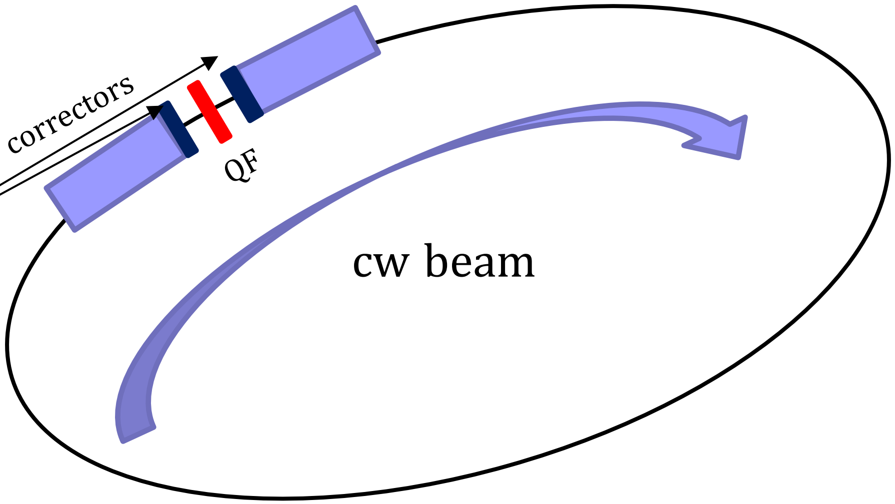
- Case of an offset of 0.1 mm - case of $J_x = \varepsilon_{rms} = 0.214\ \mu$m





Motivation and Simulation Set-up for the case of electric focusing

- Study triggered by a comment of Yannis during the meeting of 06/06/2023
  - What is the effect with electric focusing?
  - Similar effect with the same order of magnitude?

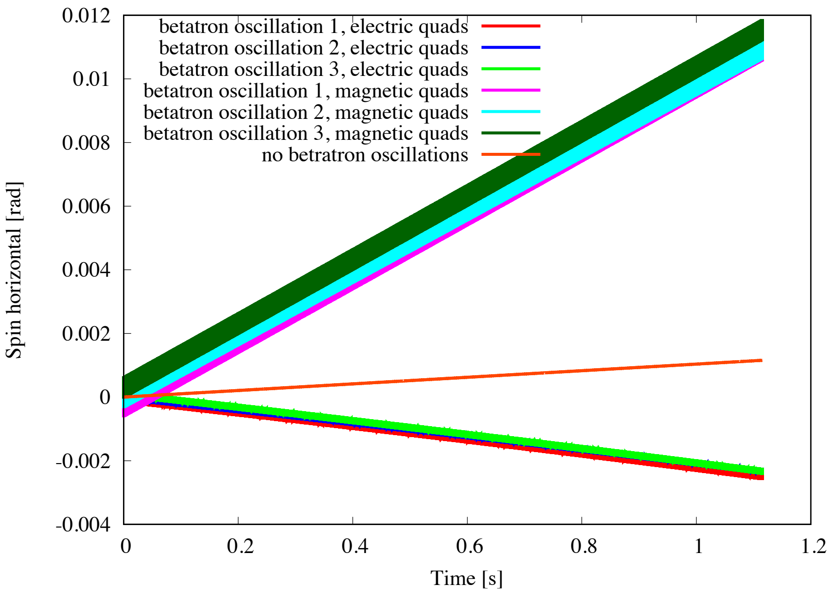


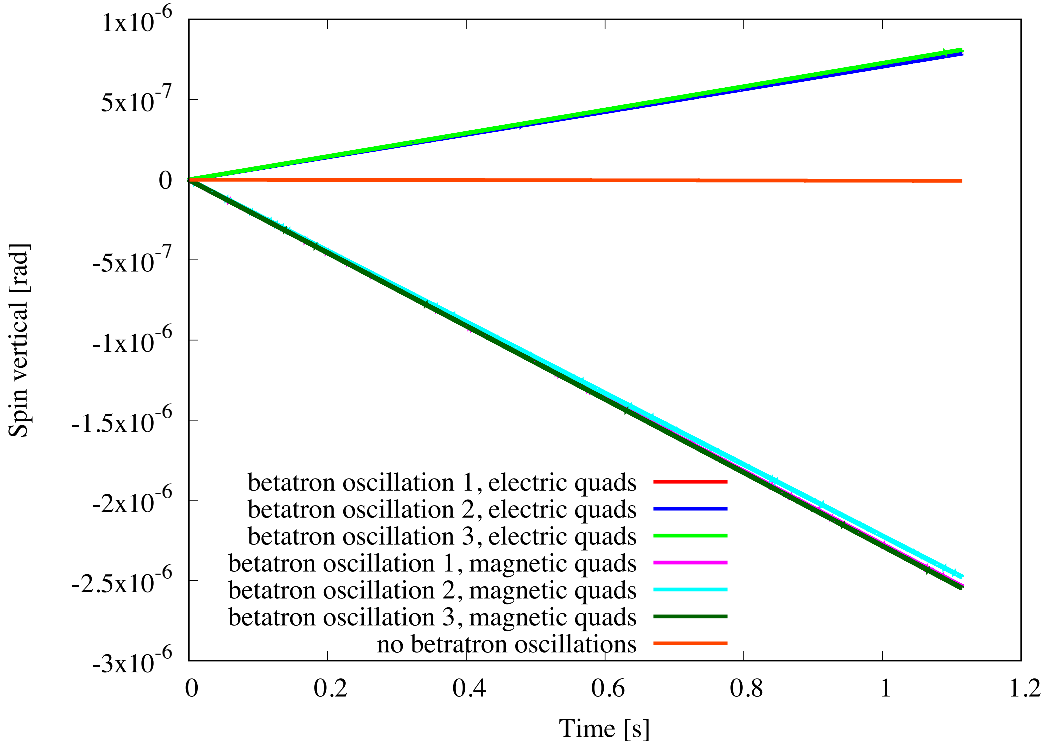
- The symmetric-hybrid ring lattice design has been used with electric quadrupoles
- A vertical offset of one QF quadrupole (focusing quad) in the middle of the machine has been added (electric correctors have been also added) – no orbit distortions outside:
  - ☐ Case 1: one quad offset of 0.1 mm
  - ☐ Case 2: one quad offset of 0.2 mm
- The results have been computed for particles executing different betatron oscillations
- Comparison of results between the lattice with magnetic and electric quadrupoles
- The simulation results have been compared with analytical estimates

Simulation Results

The case of electric focusing

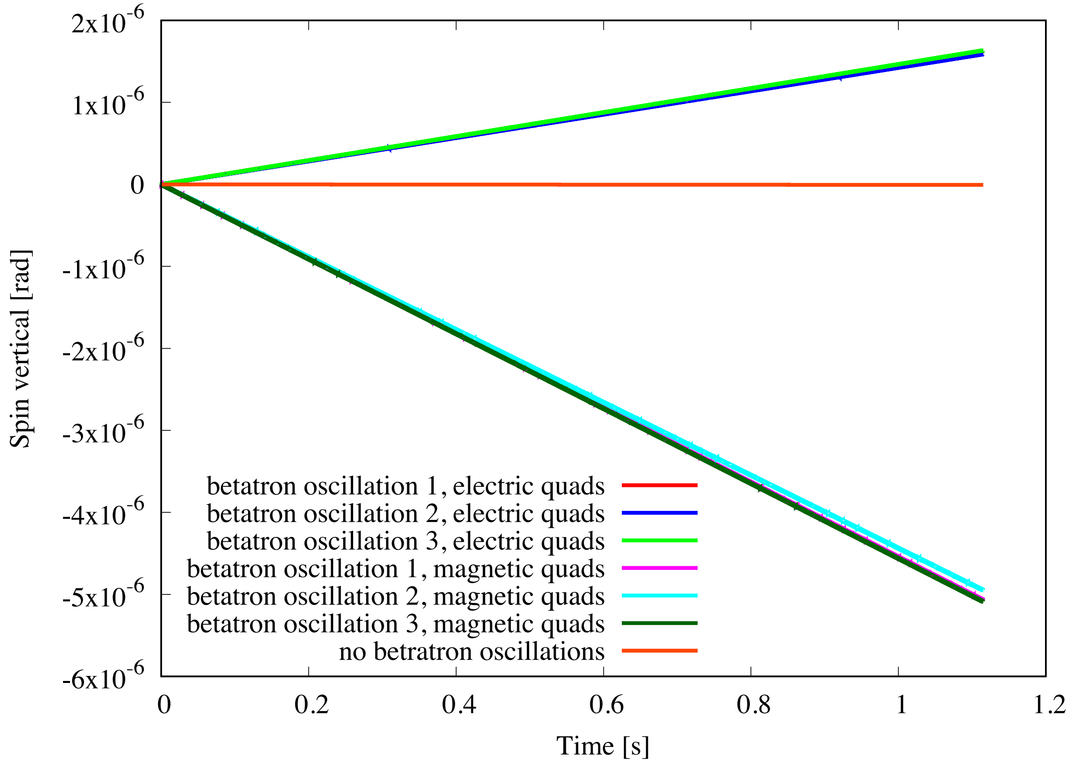
- Case of an offset of 0.1 mm - case of $J_x = \varepsilon_{rms} = 0.214\ \mu$m

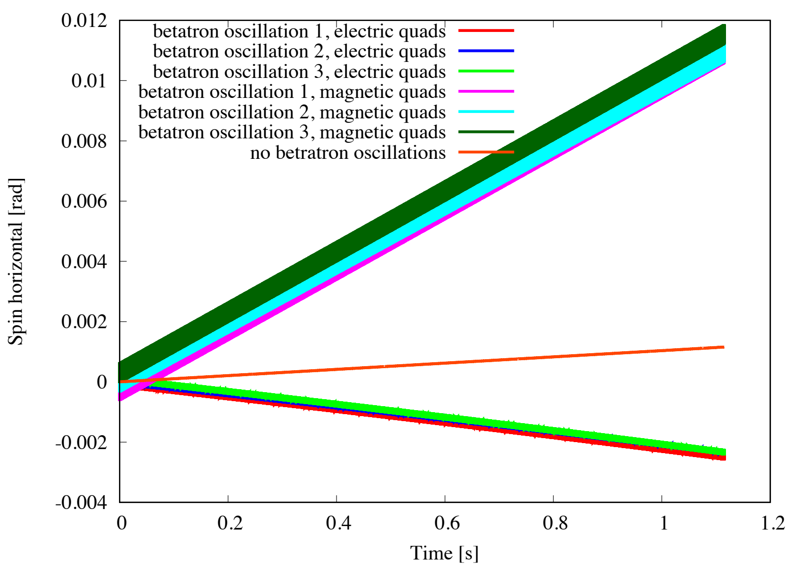




Simulation Results
The case of electric focusing

- Case of an offset of 0.2 mm - case of $J_x = \varepsilon_{rms} = 0.214\ \mu$m





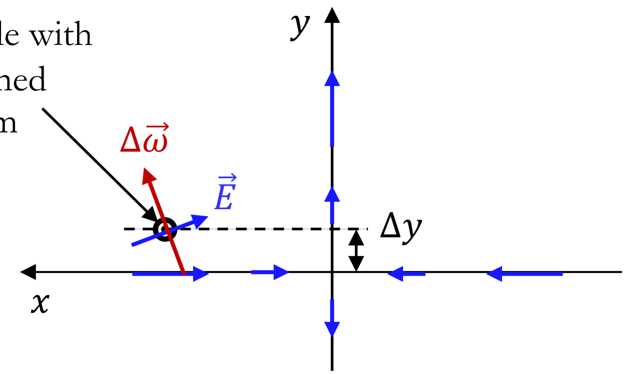
Analytical Estimates – Electric Quadrupoles

- Electric field $\vec{E} = k\frac{\gamma\beta^2 mc^2}{q}(-x, y)$
- Electric potential $U = k\frac{\gamma\beta^2 mc^2}{q}\frac{x^2-y^2}{2}$
- Change of Lorentz factor
  $\Delta\gamma = -\frac{qU}{mc^2} = k\,\gamma\beta^2\,\frac{y^2-x^2}{2}$
- Offset of beam w.r.t quad Δy results in (replace y by Δy)

$$\Delta\omega_x = -\frac{q}{m}\left(G - \frac{1}{\gamma^2-1}\right)\frac{\beta\,E_y}{c} = -\frac{\beta c}{\gamma}k^2(\Delta y^2 - x^2)\Delta y \approx \frac{\beta c}{\gamma}k^2 x^2 \Delta y$$

$$\Delta\left(\frac{-1}{\gamma^2-1}\right) = \frac{2\gamma\,\Delta\gamma}{(\gamma^2-1)^2}$$

Skipping higher orders $\propto \Delta y^3$



Direct spin rotation inside electric quad
From longitudinal into vertical direction

Neglected effect proportional to Δy^3 (quad and correctors) significant?

- Factors L_{quad}/C for averaging over circumference and replacing x^2 by average $\langle x^2\rangle = \beta_x J_x$ gives for initial polarization parallel to movement

$$\dot{S}_y = -\widehat{\Delta\omega}_x = -\frac{L_{quad}}{C}\frac{\beta c}{\gamma}k^2\beta_x J_x\,\Delta y$$

- For hybrid ring lattice after replacing magnetic quads by electric ones with $\Delta y = -0.1$ mm, $L_{quad} = 0.4$ m, $C = 800$ m, $k = 0.0877\ \text{m}^{-2}$, $\beta_x = 64$ m and $J_x = \varepsilon_{rms} = 0.214\ \mu$m $\quad \dot{S}_y = 0.757\ \mu$rad/s
- From simulations: $\dot{S}_y = 0.788\ \mu$rad/s

showing good agreement with simulations…

Summary and Conclusions

- Vertical spin build-up due to vertical offset in **magnetic quadrupole** and betatron oscillations
  - Classical geometric phase effect in fringes (plus possibly additional effects)
    - Spin rotations w.r.t. particle direction not supressed with magnetic fields
  - Order of magnitude agreement between analytical estimate and simulations
    - After changing settings of the simulation set-up
      - Initially about two orders of magnitude less vertical spin build-up with simulations
    - Different signs between estimates and simulations to be understood
    - Effect proportional to quad offset and horizontal action variable in simulations as expected
  - Operation with counter rotating beams and runs with different quadruploes polarities forseen to mitigate by hybrid ring EDM team
    - Residual effect from different beam emittances and imperfect magnetic field inversion (e.g., residual stray fields)

- Vertical spin build-up due to vertical offset in **electric quadrupole** and betatron oscillations
  - Study triggered by Yannis comment
  - Contrary to expectation vertical spin build up seen in simulation and understood in between
    - Decreased by a factor close to 3
    - Difference in sign with respect to magnetic quadrupoles
    - Good agreement with analytical estimates

Summary and Conclusions

- Observation on horizontal spin drifts due to betatron oscillations with magnetic and electric focusing
  - Different sign for electric and magnetic focusing
  - Hypothesis qualitative explanation
    - Electric focusing
      - Betratron oscillations inside bendings increase the path lengths, and for electric bends the deflection
      - Trajectory displaced in average towards the inside and the energy is increasing
      - Spin rotates faster than momentum generating negative radial spin components
    - Magnetic focusing
      - In addition, quadrupoles bend the beam towards the outside
      - Spin rotates faster than momentum generating larger positive radial spin components
  - Difference in Spin decoherence between the electric and the magnetic lattice

- Thorought studies still needed to understand and asses possible systematic effects
  - Simulation starting with reasonable assumptions on imperfections (alignment errors, unwanted field components, stray fields …) of initial machine
  - Implementation of beam based corrections (orbit differences, additional gradients and magnetic fields …)
  - Comparison with analytical estimates to ensure all effects are modelled correctly