Introduction to Monte Carlo Event Generators

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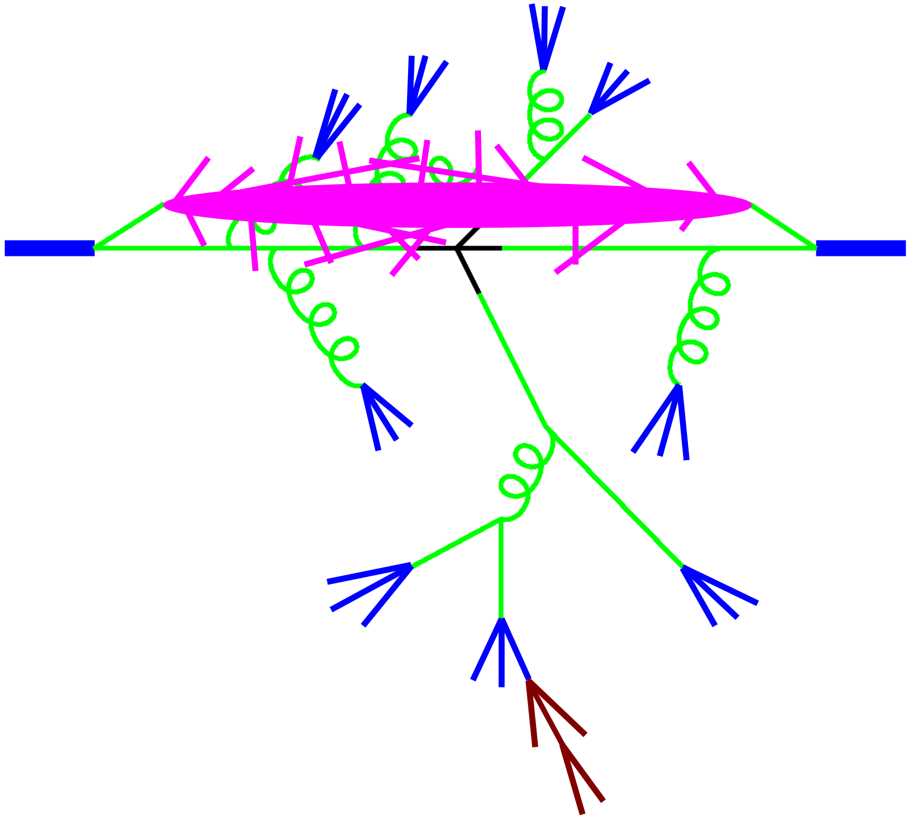
Structure of LHC Events

1. Hard process
2. Parton shower
3. Hadronization
4. Underlying event
5. Unstable particle decays



Parton Showers: Introduction

QED: accelerated charges radiate.

QCD identical: accelerated colours radiate.

gluons also charged.

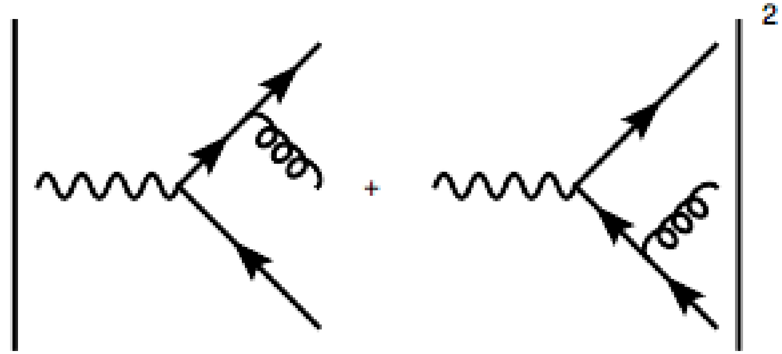
→ cascade of partons.

= parton shower.

1. e^+e^-annihilation to jets.
2. Universality of collinear emission.
3. Sudakov form factors.
4. Universality of soft emission.
5. Angular ordering.
6. Initial-state radiation.
7. Hard scattering.
8. Heavy quarks.
9. Dipole cascades.
10. Matching and merging

QCD emission matrix elements diverge

e.g. $e^+e^- \to 3$ partons:



$$\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta\,\mathrm{d}z_g} \sim \sigma_0\, C_F\, \frac{\alpha_s}{2\pi}\, \frac{2}{\sin^2\theta}\, \frac{1+(1-z_g)^2}{z_g}$$

$E_g/E_{g,max}$

$e^+e^- \to 2$ partons

"quark charge squared"

QCD running coupling ~ 0.1

Divergent in collinear limit $\theta \to 0,\pi$ (for massless quarks)

and soft limit $z_g \to 0$

can separate into two independent jets:

$$\begin{aligned}\frac{2\, d\cos\theta}{\sin^2\theta} &= \frac{d\cos\theta}{1-\cos\theta} + \frac{d\cos\theta}{1+\cos\theta} \\ &= \frac{d\cos\theta}{1-\cos\theta} + \frac{d\cos\bar\theta}{1-\cos\bar\theta} \\ &\approx \frac{d\theta^2}{\theta^2} + \frac{d\bar\theta^2}{\bar\theta^2}\end{aligned}$$

jets evolve independently

$$d\sigma = \sigma_0 \sum_{\text{jets}} C_F \frac{\alpha_s}{2\pi} \frac{d\theta^2}{\theta^2}\, dz \frac{1+(1-z)^2}{z}$$

Exactly same form for anything $\propto \theta^2$

eg transverse momentum: $k_\perp^2 = z^2(1-z)^2\,\theta^2\,E^2$

invariant mass: $q^2 = z(1-z)\,\theta^2\,E^2$

$$\frac{d\theta^2}{\theta^2} = \frac{dk_\perp^2}{k_\perp^2} = \frac{dq^2}{q^2}$$

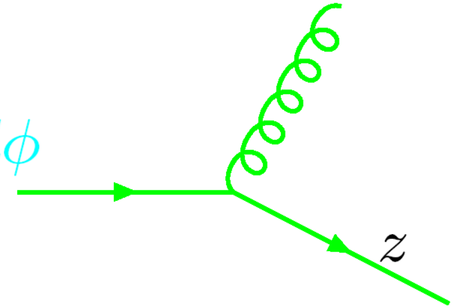
Collinear Limit

Universal:

$$d\sigma = \sigma_0 \, \frac{\alpha_s}{2\pi} \, \frac{d\theta^2}{\theta^2} \, dz \, P(z, \phi) d\phi$$

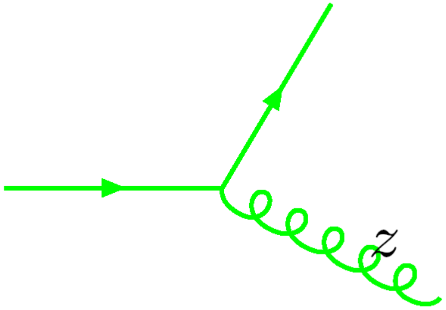
$P(z, \phi) =$

Dokshitzer-Gribov-Lipatov-Altarelli-Parisi splitting kernel: dependent on flavour and spin



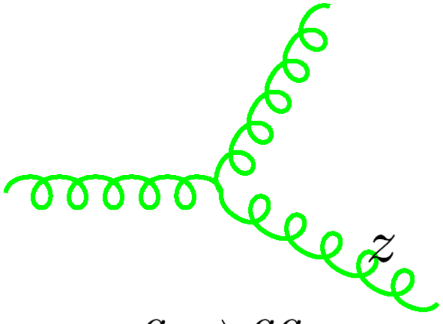
$q \to qg$

$C_F \frac{1+z^2}{1-z}$



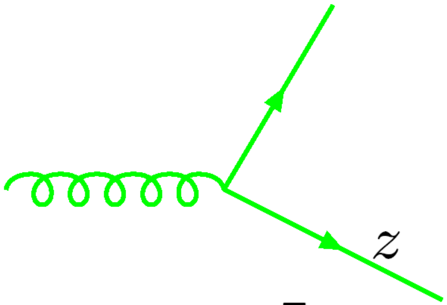
$q \to gq$

$C_F \frac{1+(1-z)^2}{z}$



$g \to gg$

$C_A \frac{z^4+1+(1-z)^4}{z(1-z)}$



$g \to q\bar{q}$

$T_R \left(z^2 + (1-z)^2\right)$

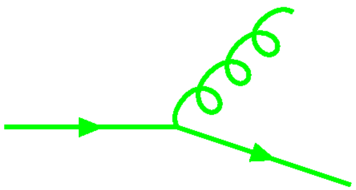
Resolvable partons

What is a parton?

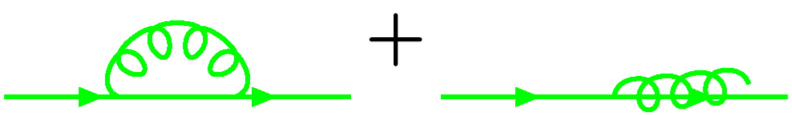
Collinear parton pair $\longleftrightarrow$ single parton

Introduce resolution criterion, eg $k_\perp > Q_0$.

Virtual corrections must be combined with unresolvable real emission



Resolvable emission
Finite



Virtual + Unresolvable emission
Finite

Unitarity: P(resolved) + P(unresolved) = 1

Sudakov form factor

Probability(emission between q^2 and $q^2 + dq^2$)

$$d\mathcal{P} = \frac{\alpha_s}{2\pi}\frac{dq^2}{q^2}\int_{Q_0^2/q^2}^{1-Q_0^2/q^2} dz\ P(z) \equiv \frac{dq^2}{q^2}\bar{P}(q^2).$$

Define probability(no emission between Q^2 and q^2) to be $\Delta(Q^2, q^2)$. Gives evolution equation

$$\frac{d\Delta(Q^2,q^2)}{dq^2} = \Delta(Q^2,q^2)\frac{d\mathcal{P}}{dq^2}$$

$$\Rightarrow \Delta(Q^2,q^2) = \exp - \int_{q^2}^{Q^2} \frac{dk^2}{k^2}\bar{P}(k^2).$$

c.f. radioactive decay

atom has probability λ per unit time to decay.

Probability(no decay after time T) = $\exp - \int^T dt\, \lambda$

Sudakov form factor

Probability(emission between q^2 and $q^2 + dq^2$)

$$d\mathcal{P} = \frac{\alpha_s}{2\pi}\frac{dq^2}{q^2}\int_{Q_0^2/q^2}^{1-Q_0^2/q^2} dz\ P(z) \equiv \frac{dq^2}{q^2}\bar{P}(q^2).$$

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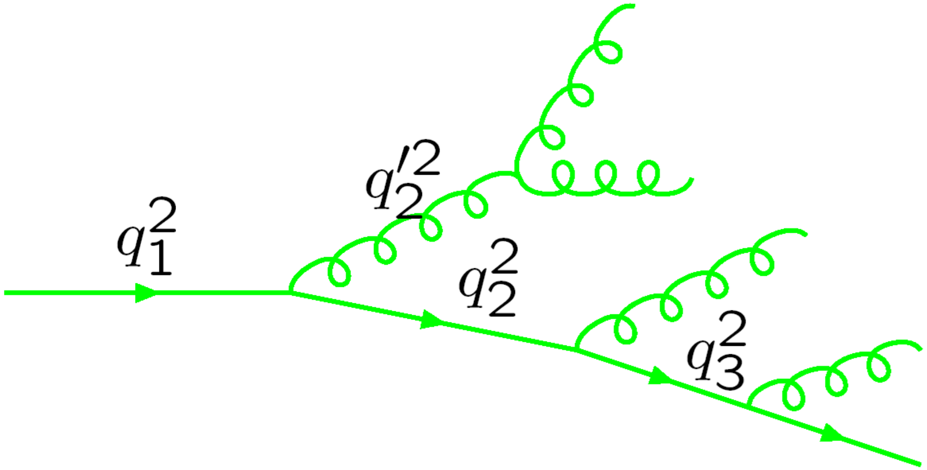
$$\Rightarrow \Delta(Q^2,q^2) = \exp - \int_{q^2}^{Q^2}\frac{dk^2}{k^2}\bar{P}(k^2).$$

$\Delta(Q^2, Q_0^2) \equiv \Delta(Q^2)$ Sudakov form factor

=Probability(emitting no resolvable radiation)

$$\Delta_q(Q^2) \sim \exp - C_F\frac{\alpha_s}{2\pi}\log^2\frac{Q^2}{Q_0^2}$$

Multiple emission



$$q_1^2 > q_2^2 > q_3^2 > \ldots$$
$$q_1^2 > q_2'^2 \ldots$$

But initial condition? $q_1^2 < ???$

Process dependent

Monte Carlo implementation

Can generate branching according to

$$d\mathcal{P} = \frac{dq^2}{q^2}\bar{P}(q^2)\,\Delta(Q^2,q^2)$$

By choosing $0 < \rho < 1$ uniformly:

If $\rho < \Delta(Q^2)$ no resolvable radiation, evolution stops.

Otherwise, solve $\rho = \Delta(Q^2,q^2)$

for q^2=emission scale

Considerable freedom:

Evolution scale: $q^2/k_\perp^2/\theta^2$?

z: Energy? Light-cone momentum?

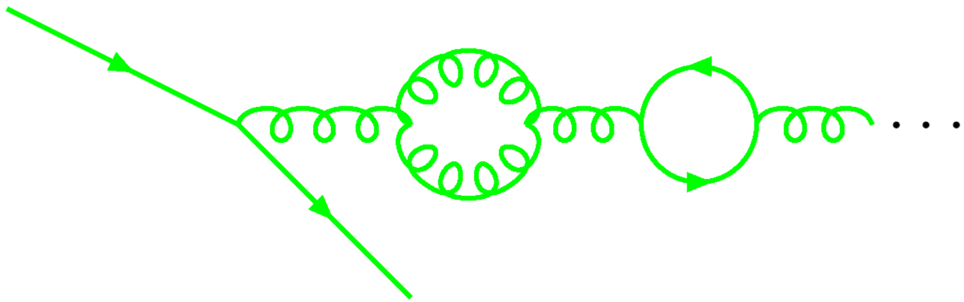
Massless partons become massive. How?

Upper limit for q^2?

All formally free choices, but can be very important numerically

Running coupling

Effect of summing up higher orders:



absorbed by replacing α_s by $\alpha_s(k_\perp^2)$.

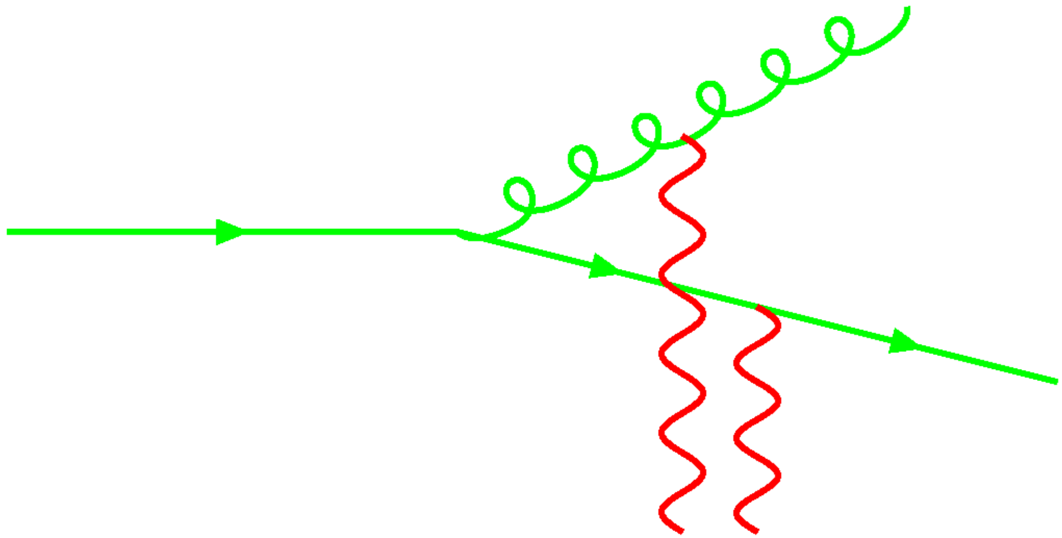
Much faster parton multiplication – phase space fills with soft gluons.

Must then avoid Landau pole: $k_\perp^2 \gg \Lambda^2$.

Q_0 now becomes physical parameter!

Soft limit

Also universal. But at amplitude level…



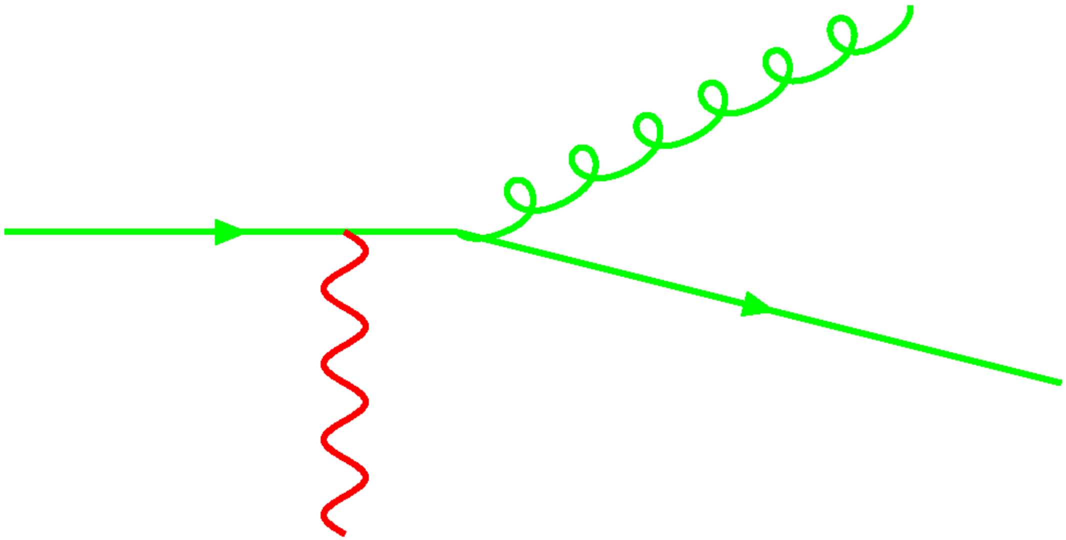
soft gluon comes from everywhere in event.

→ Quantum interference.

Spoils independent evolution picture?

Angular ordering

NO:



outside angular ordered cones, soft gluons sum coherently: only see colour charge of whole jet.

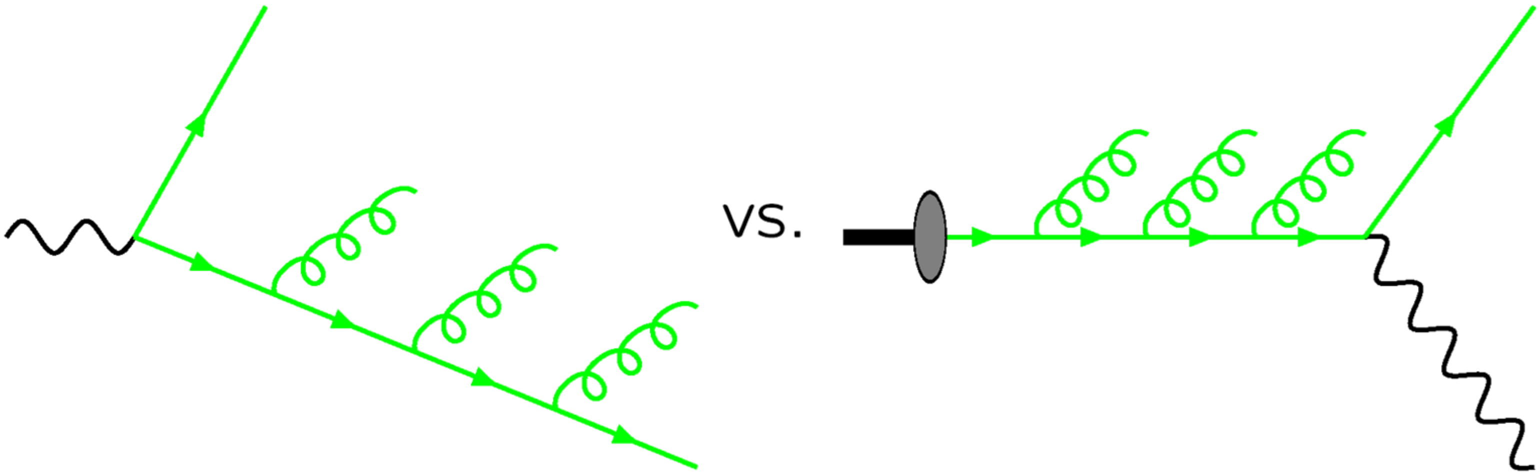
Soft gluon effects fully incorporated by using θ^2 as evolution variable: angular ordering

First gluon not necessarily hardest!

Initial state radiation

In principle identical to final state (for not too small x)

In practice different because both ends of evolution fixed:



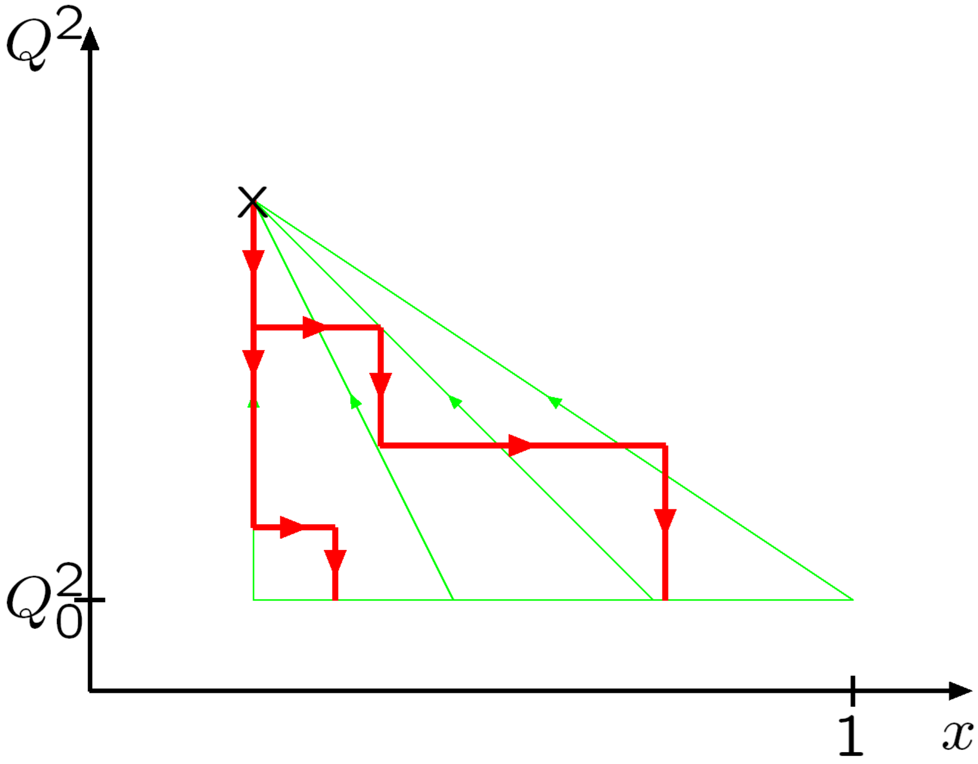
Use approach based on evolution equations…

Backward evolution

DGLAP evolution: pdfs at (x, Q^2) as function of pdfs at $(> x, Q_0^2)$:

Evolution paths sum over all possible events.

Formulate as backward evolution: start from hard scattering and work down in q^2, up in x towards incoming hadron.

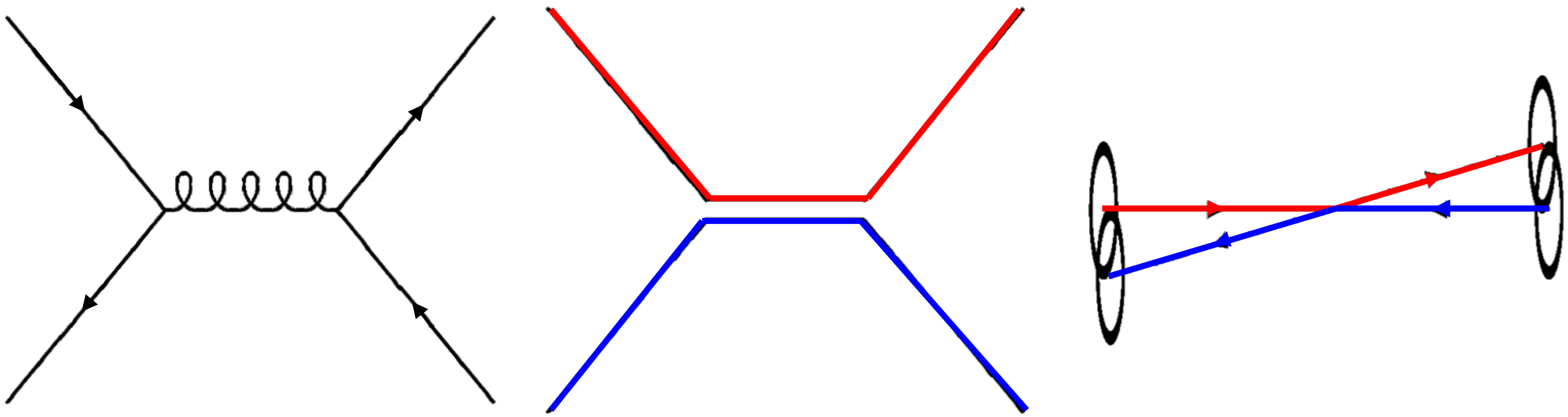


Algorithm identical to final state with $\Delta_i(Q^2, q^2)$ replaced by $\Delta_i(Q^2, q^2)/f_i(x, q^2)$.

Hard Scattering

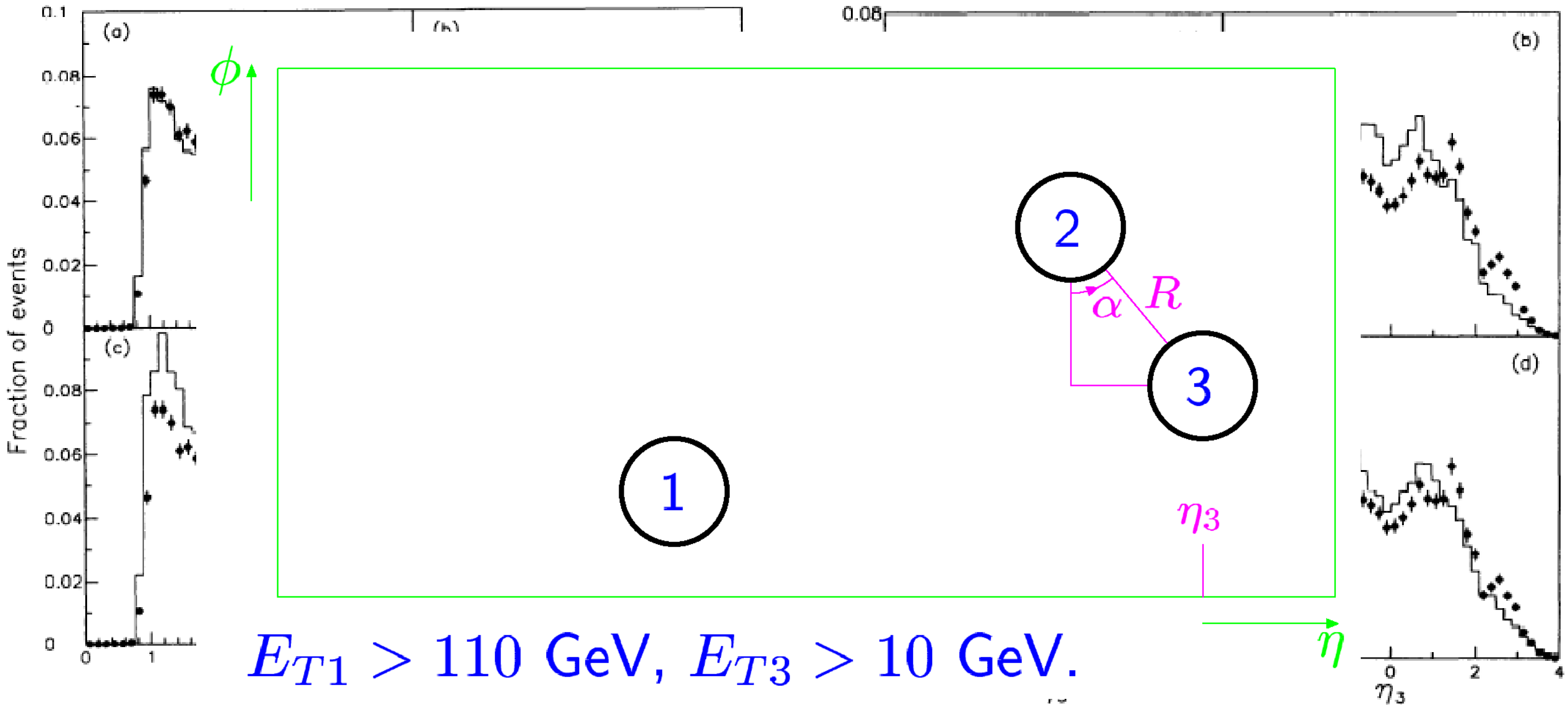
Sets up initial conditions for parton showers.

Colour coherence important here too.



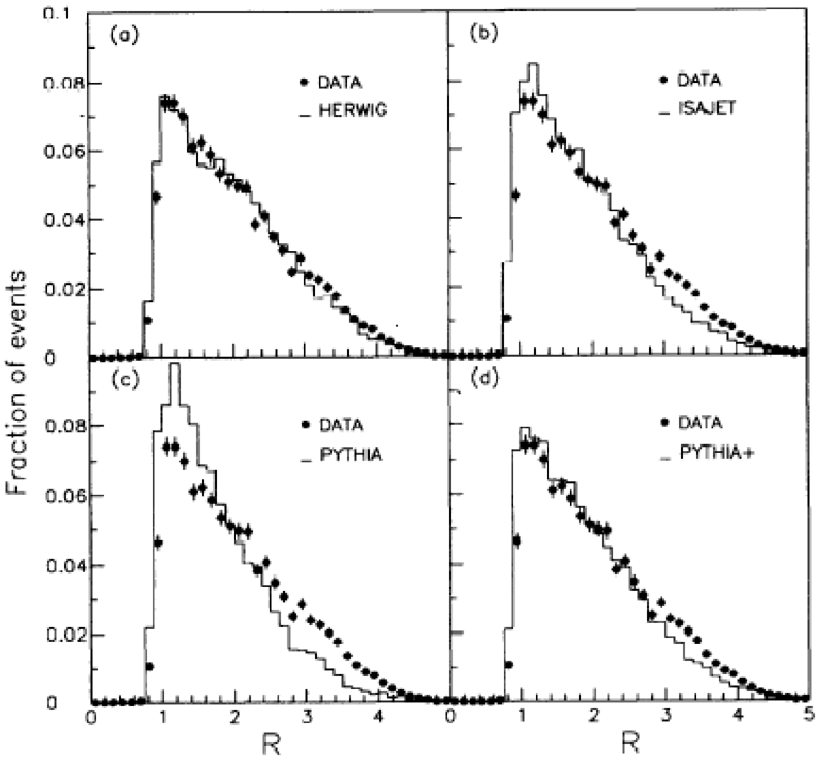
Emission from each parton confined to cone stretching to its colour partner

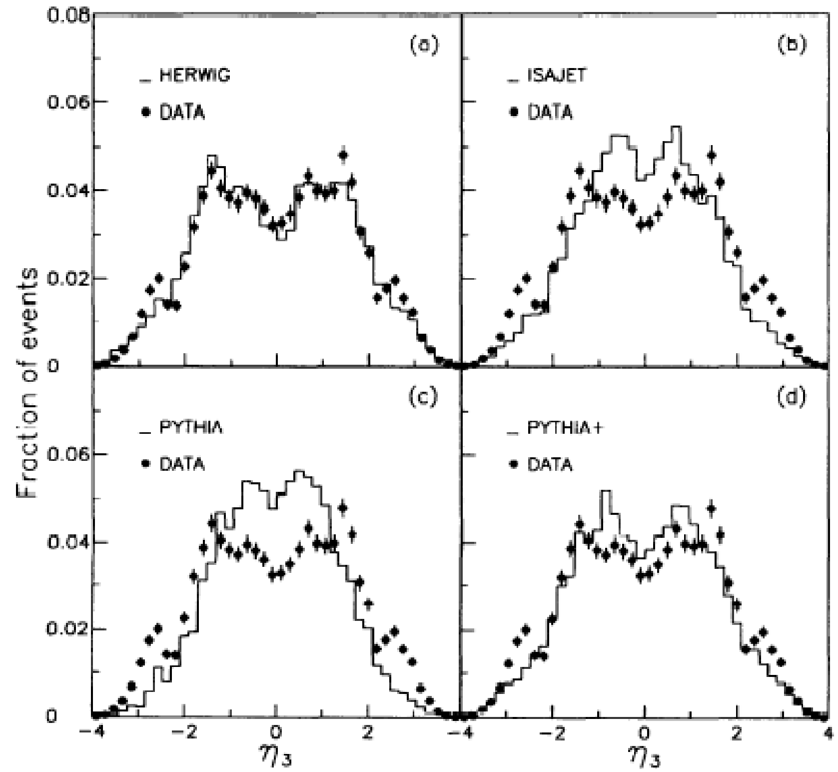
Essential to fit Tevatron data…



$E_{T1} > 110$ GeV, $E_{T3} > 10$ GeV.

Distributions of third-hardest jet in multi-jet events



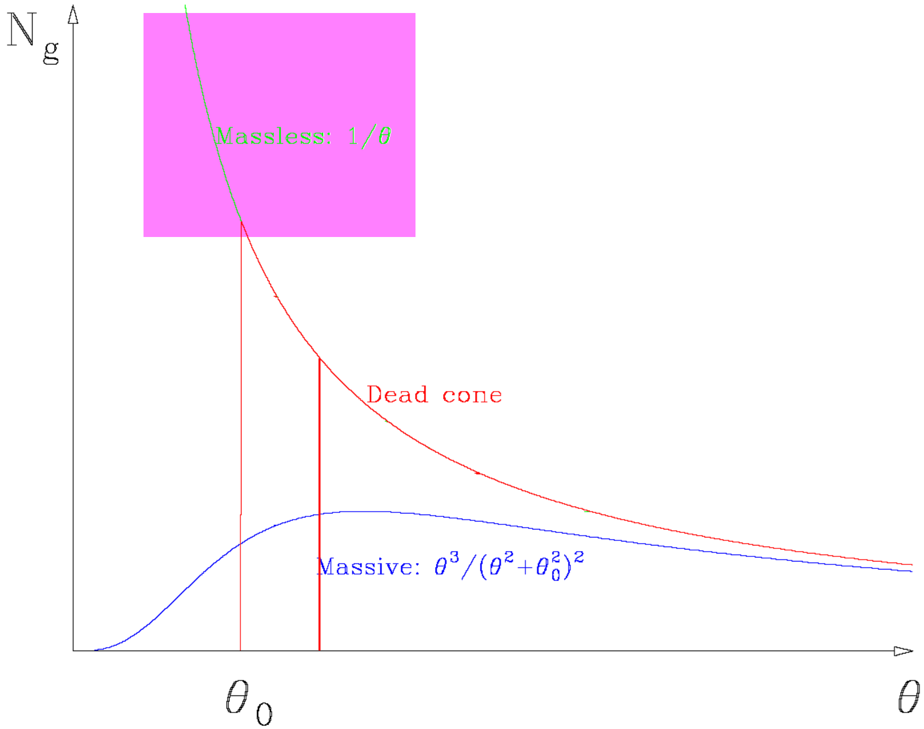


Distributions of third-hardest jet in multi-jet events

HERWIG has complete treatment of colour coherence, PYTHIA+ has partial

Heavy Quarks/Spartons

look like light quarks at large angles, sterile at small angles:



approximated as energy-dependent cutoff: $\theta > \theta_0 = \frac{m_q}{E_q}$.

The 'dead cone'. Too extreme?

Heavy Quarks/Spartons

More properly treated using quasi-collinear splitting:

$$\mathrm{d}\mathcal{P}_{\widetilde{ij}\to ij} = \frac{\alpha_S}{2\pi}\frac{\mathrm{d}\tilde{q}^2}{\tilde{q}^2}\,\mathrm{d}z\,P_{\widetilde{ij}\to ij}\left(z,\tilde{q}\right),$$

$$P_{q\to qg} = \frac{C_F}{1-z}\left[1+z^2-\frac{2m_q^2}{z\tilde{q}^2}\right],$$

$$P_{g\to gg} = C_A\left[\frac{z}{1-z}+\frac{1-z}{z}+z\left(1-z\right)\right],$$

$$P_{g\to q\bar{q}} = T_R\left[1-2z\left(1-z\right)+\frac{2m_q^2}{z\left(1-z\right)\tilde{q}^2}\right],$$

$$P_{\tilde{g}\to\tilde{g}g} = \frac{C_A}{1-z}\left[1+z^2-\frac{2m_{\tilde{g}}^2}{z\tilde{q}^2}\right],$$

$$P_{\tilde{q}\to\tilde{q}g} = \frac{2C_F}{1-z}\left[z-\frac{m_{\tilde{q}}}{z\tilde{q}^2}\right],$$

$\to$ smooth suppression in forward region

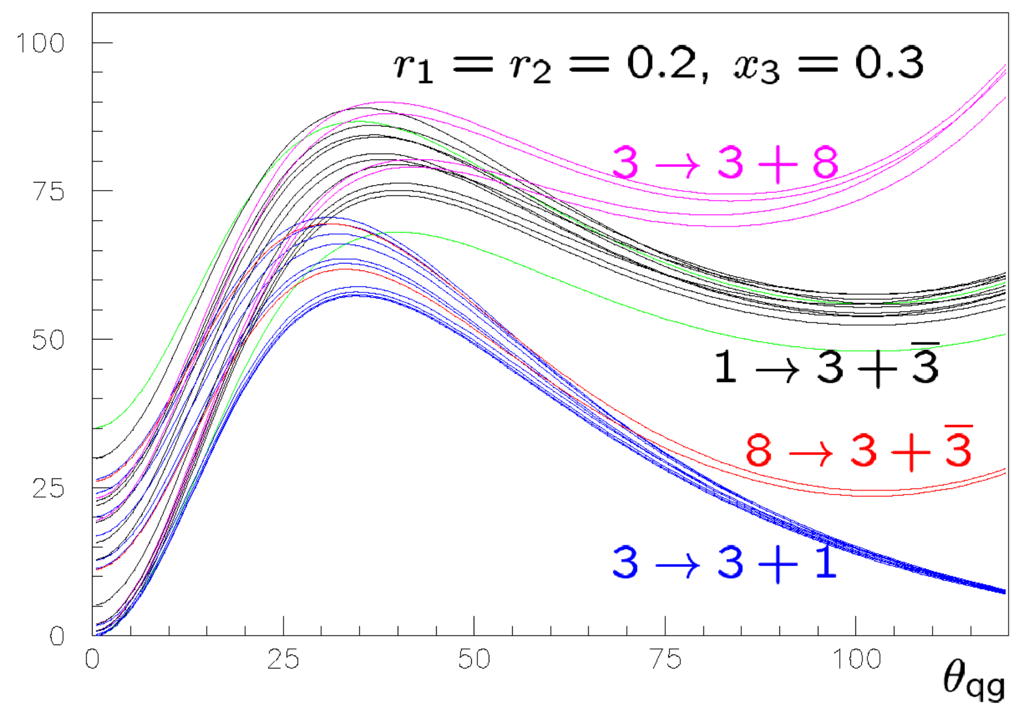
Heavy Quarks/Spartons

- Dead cone only exact for
  - emission from spin-0 particle, or
  - infinitely soft emitted gluon

- In general, depends on
  - energy of gluon
  - colours and spins of emitting particle and colour partner

→ process-dependent mass corrections

colour	spin	γ_5	example
$1 \to 3 + \bar{3}$	—	—	(eikonal)
$1 \to 3 + \bar{3}$	$1 \to \frac{1}{2} + \frac{1}{2}$	$1, \gamma_5, 1 \pm \gamma_5$	$Z^0 \to q\bar{q}$
$3 \to 3 + 1$	$\frac{1}{2} \to \frac{1}{2} + 1$	$1, \gamma_5, 1 \pm \gamma_5$	$t \to bW^+$
$1 \to 3 + \bar{3}$	$0 \to \frac{1}{2} + \frac{1}{2}$	$1, \gamma_5, 1 \pm \gamma_5$	$H^0 \to q\bar{q}$
$3 \to 3 + 1$	$\frac{1}{2} \to \frac{1}{2} + 0$	$1, \gamma_5, 1 \pm \gamma_5$	$t \to bH^+$
$1 \to 3 + \bar{3}$	$1 \to 0 + 0$	1	$Z^0 \to \tilde{q}\bar{\tilde{q}}$
$3 \to 3 + 1$	$0 \to 0 + 1$	1	$\tilde{q} \to \tilde{q}'W^+$
$1 \to 3 + \bar{3}$	$0 \to 0 + 0$	1	$H^0 \to \tilde{q}\bar{\tilde{q}}$
$3 \to 3 + 1$	$0 \to 0 + 0$	1	$\tilde{q} \to \tilde{q}'H^+$
$1 \to 3 + \bar{3}$	$\frac{1}{2} \to \frac{1}{2} + 0$	$1, \gamma_5, 1 \pm \gamma_5$	$\chi \to q\bar{\tilde{q}}$
$3 \to 3 + 1$	$0 \to \frac{1}{2} + \frac{1}{2}$	$1, \gamma_5, 1 \pm \gamma_5$	$\tilde{q} \to q\chi$
$3 \to 3 + 1$	$\frac{1}{2} \to 0 + \frac{1}{2}$	$1, \gamma_5, 1 \pm \gamma_5$	$t \to \tilde{t}\chi$
$8 \to 3 + \bar{3}$	$\frac{1}{2} \to \frac{1}{2} + 0$	$1, \gamma_5, 1 \pm \gamma_5$	$\tilde{g} \to q\bar{\tilde{q}}$
$3 \to 3 + 8$	$0 \to \frac{1}{2} + \frac{1}{2}$	$1, \gamma_5, 1 \pm \gamma_5$	$\tilde{q} \to q\tilde{g}$
$3 \to 3 + 8$	$\frac{1}{2} \to 0 + \frac{1}{2}$	$1, \gamma_5, 1 \pm \gamma_5$	$t \to \tilde{t}\tilde{g}$



The Colour Dipole Model

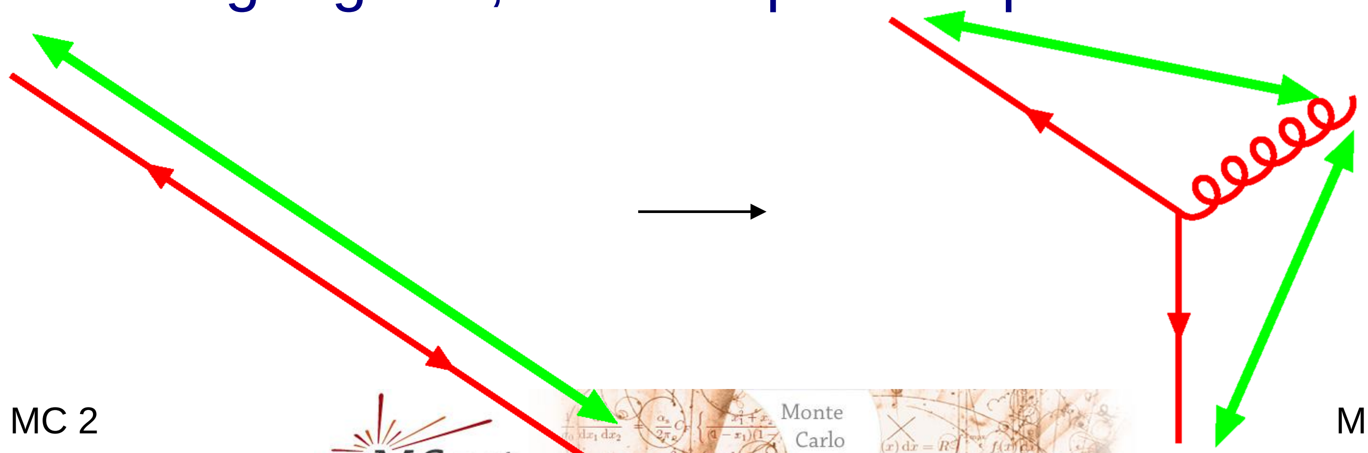
Conventional parton showers: start from collinear limit, modify to incorporate soft gluon coherence

Colour Dipole Model: start from soft limit

Emission of soft gluons from colour-anticolour dipole universal (and classical):

$$d\sigma \approx \sigma_0 \, \tfrac{1}{2} C_A \frac{\alpha_s(k_\perp)}{2\pi} \frac{dk_\perp^2}{k_\perp^2} \, dy, \qquad y = \text{rapidity} = \log \tan \theta/2$$

After emitting a gluon, colour dipole is split:

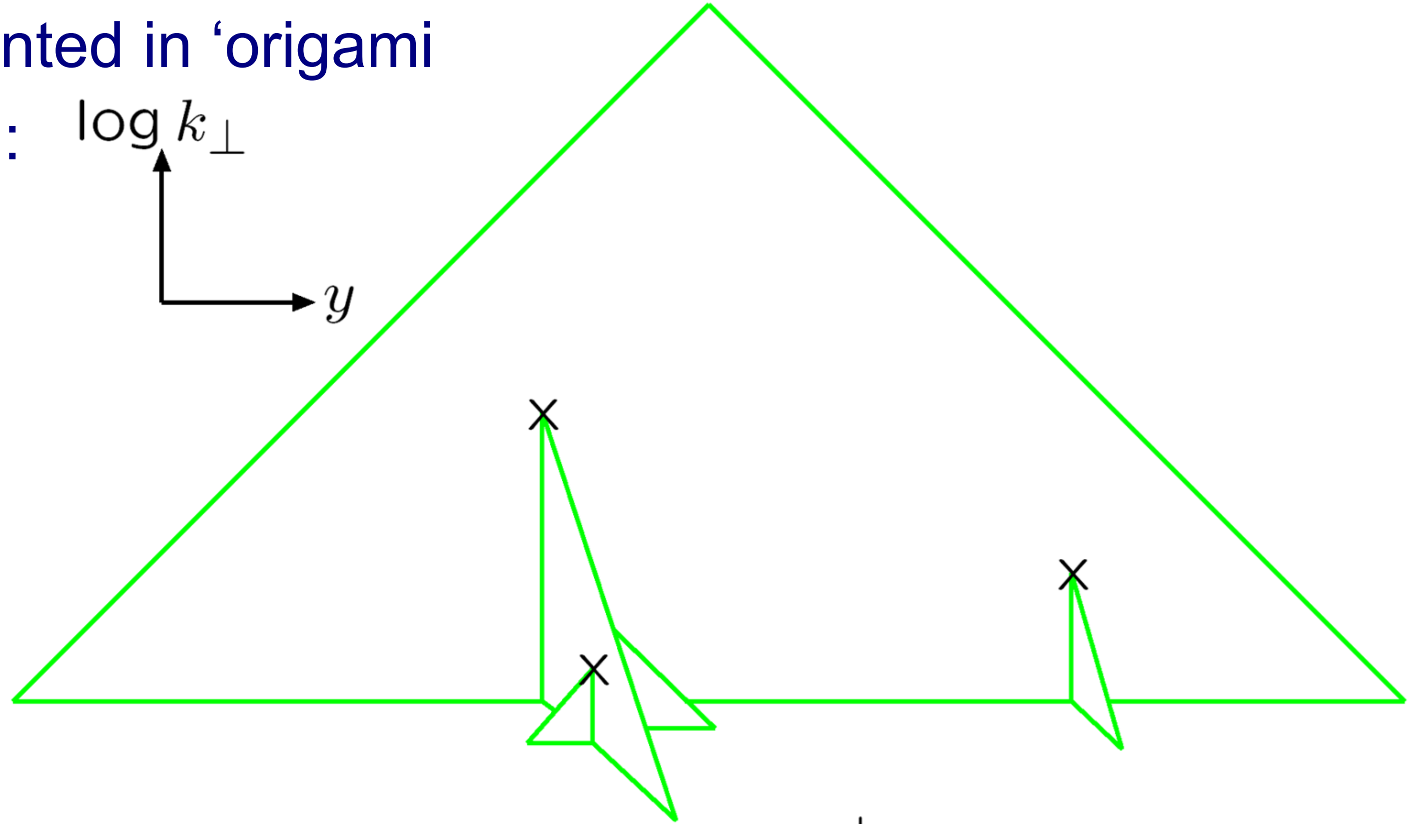


Subsequent dipoles continue to cascade

c.f. parton shower: one parton → two

CDM: one dipole → two = two partons → three

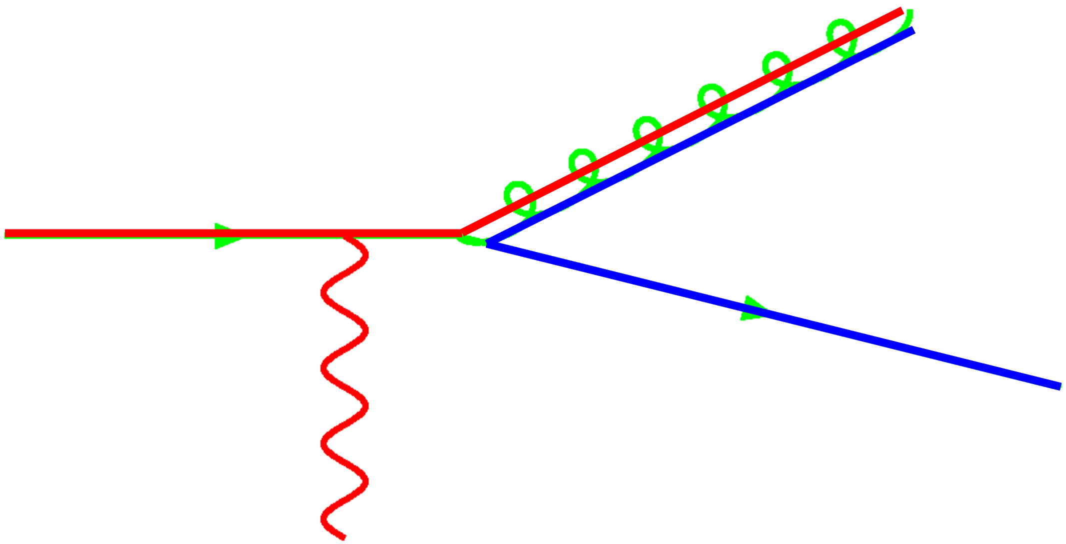
Represented in 'origami diagram':



Similar to angular-ordered parton shower for e^+e^- annihilation

Dipole cascades and colour coherence

Recall:



soft wide angle gluon sees the colour of the whole jet

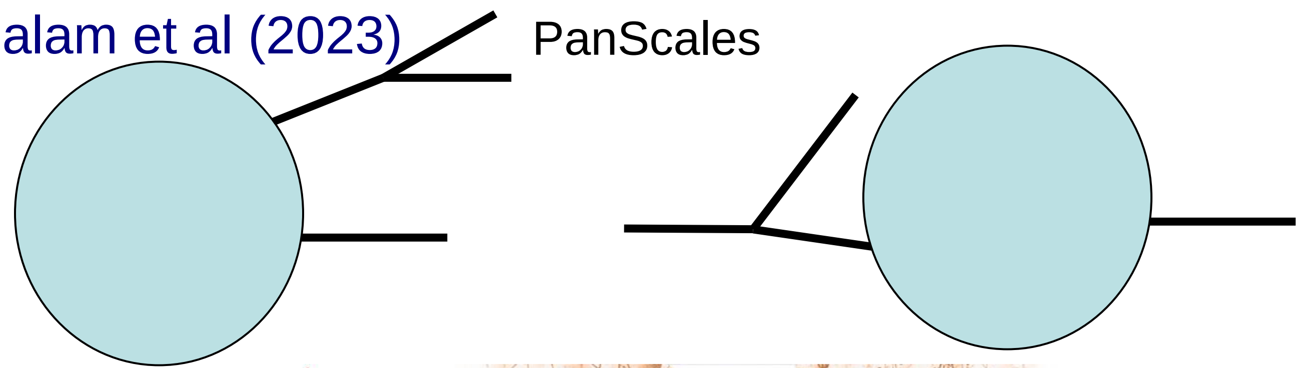
⇒ emitted first in parton shower language

but colour of whole jet is carried by emitted gluon

⇒ soft gluon emitted by hard gluon’s dipole is emitted by the whole jet

Dipole Cascades

- Most new implementations based on dipole picture:
  - Catani & MHS (1997)
  - Kosower (1998)
  - Nagy & Soper (May 2007) DEDUCTOR
  - Giele, Kosower & Skands (July 2007) VINCIA
  - Dinsdale, Ternick & Weinzierl (Sept 2007)
  - Schumann & Krauss (Sept 2007) Sherpa
  - Winter & Krauss (Dec 2007) Sherpa
  - Plätzer & Gieseke (Sept 2009) Herwig / Matchbox
  - Salam et al (2023) PanScales



Matrix Element Matching

Parton shower built on approximations to QCD matrix elements valid in **collinear** and **soft** approximations

$\rightarrow$ describe bulk of radiation well $\rightarrow$ hadronic final state

$\rightarrow$ but …

- searches for new physics
- top mass measurement
- *n* jet cross sections
- …

$\rightarrow$ hard, well-separated jets

- described better by fixed ("leading") order matrix element
- would also like next-to-leading order normalization (or better)

$\rightarrow$ need matrix element matching $\rightarrow$ Marek Schoenherr

Summary

- Accelerated colour charges radiate gluons.

  Gluons are also charged → cascade.

- Probabilistic language derived from factorization theorems of full gauge theory.

  Colour coherence is central to both angular-ordered parton showers and dipole showers

- Modern parton shower models are very sophisticated implementations of perturbative QCD, but would be useless without hadronization models…