Introduction to Monte Carlo Event Generators

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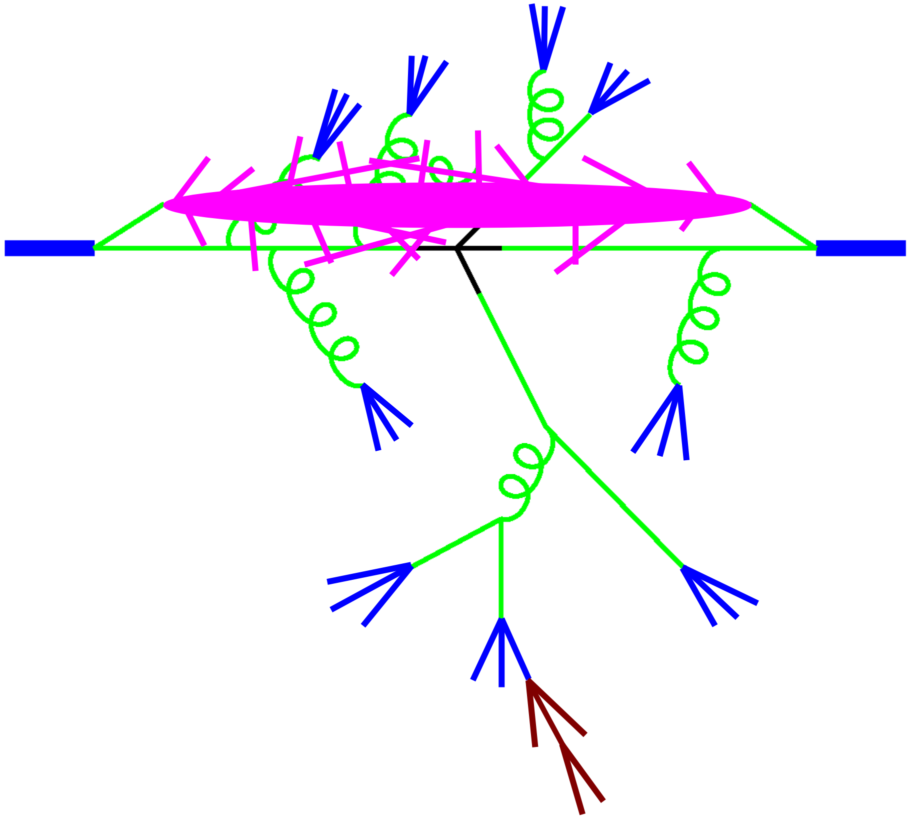
17th MCnet Summer School

CERN

June 10th – 14th 2024

Structure of LHC Events

1. Hard process
2. Parton shower
3. Hadronization
4. Underlying event
5. Unstable particle decays



Intro to Monte Carlo Event Generators

1. Monte Carlo technique / hard process
2. Parton showers
3. Hadronization
4. Underlying Event / Soft Inclusive Models

The Colour Dipole Model

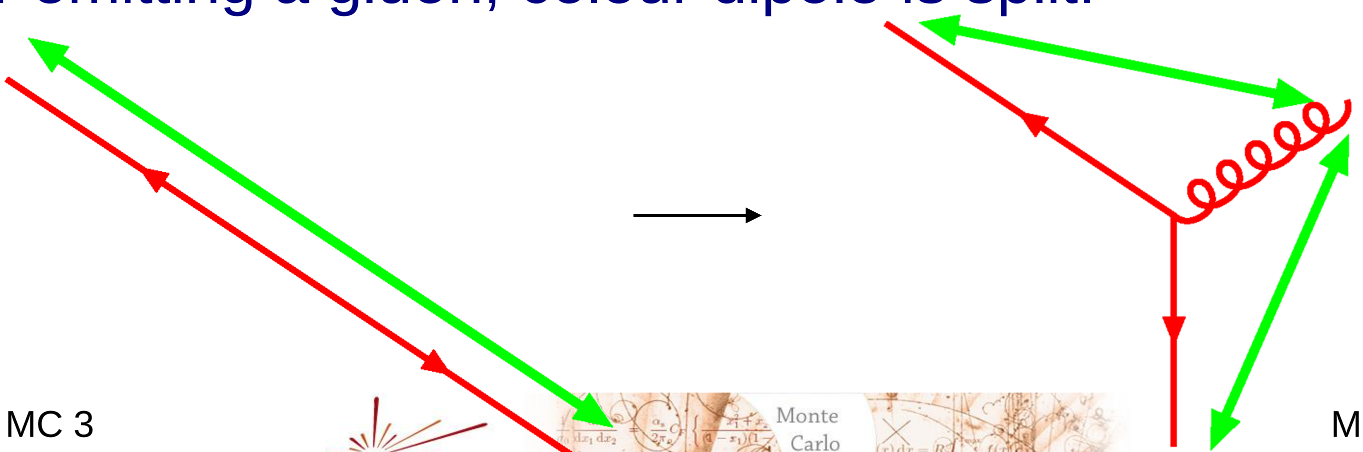
Conventional parton showers: start from collinear limit, modify to incorporate soft gluon coherence

Colour Dipole Model: start from soft limit

Emission of soft gluons from colour-anticolour dipole universal (and classical):

$$d\sigma \approx \sigma_0 \, \tfrac{1}{2} C_A \frac{\alpha_s(k_\perp)}{2\pi} \frac{dk_\perp^2}{k_\perp^2} \, dy, \qquad y = \text{rapidity} = \log\tan\theta/2$$

After emitting a gluon, colour dipole is split:

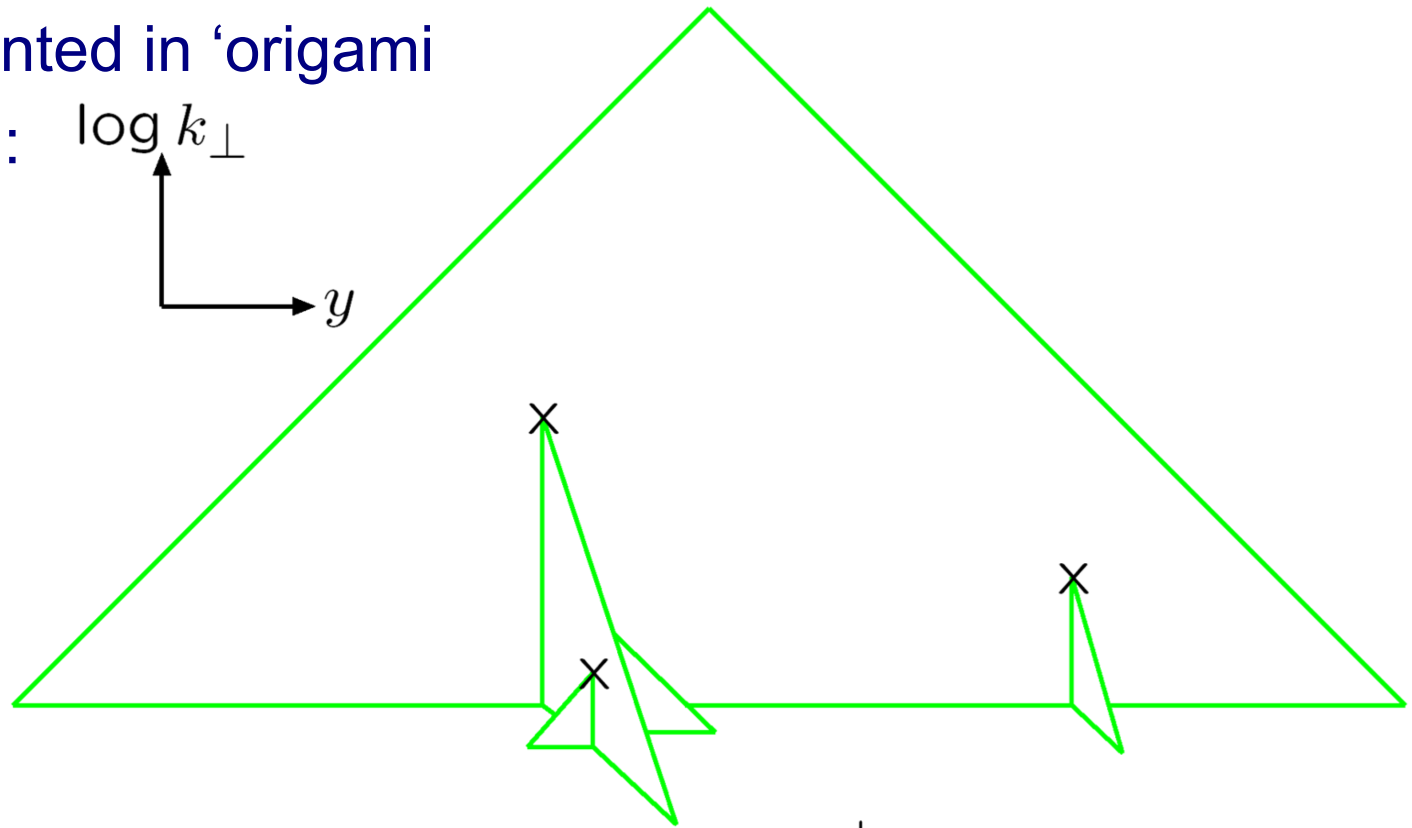


Subsequent dipoles continue to cascade

c.f. parton shower: one parton → two

CDM: one dipole → two = two partons → three

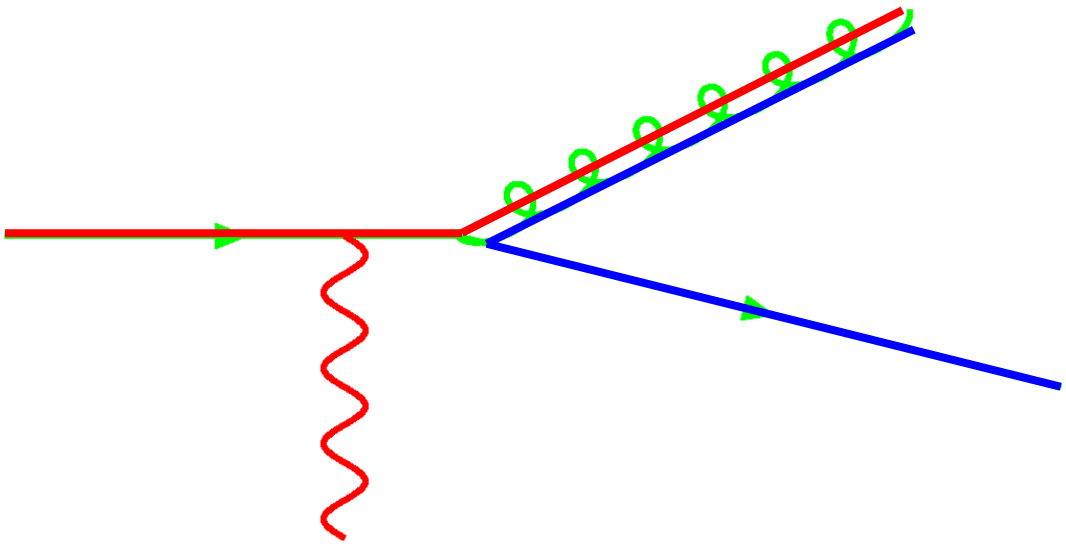
Represented in 'origami diagram':



Similar to angular-ordered parton shower for e^+e^- annihilation

Dipole cascades and colour coherence

Recall:



soft wide angle gluon sees the colour of the whole jet

⇒ emitted first in parton shower language

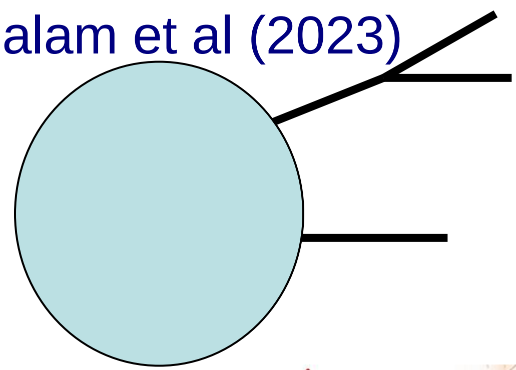
but colour of whole jet is carried by emitted gluon

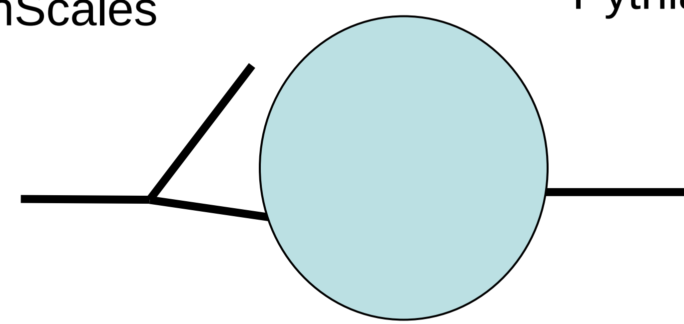
⇒ soft gluon emitted by hard gluon's dipole is emitted by the whole jet

Dipole Cascades

- Most new implementations based on dipole picture:
  - Catani & MHS (1997)
  - Kosower (1998)
  - Nagy & Soper (May 2007) DEDUCTOR
  - Giele, Kosower & Skands (July 2007) VINCIA
  - Dinsdale, Ternick & Weinzierl (Sept 2007)
  - Schumann & Krauss (Sept 2007) Sherpa
  - Winter & Krauss (Dec 2007) Sherpa
  - Plätzer & Gieseke (Sept 2009) Herwig / Matchbox
  - Salam et al (2023) PanScales

Dire
Pythia8

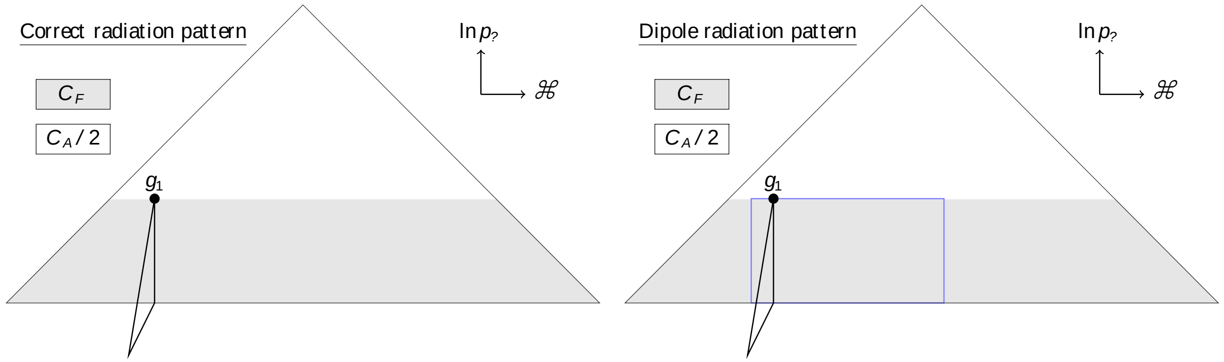




Dasgupta et al. *JHEP* 09 (2018) 033,

JHEP 03 (2020) 083 (erratum)

Matrix elements and phase space of dipole cascades are the same as in NLL and (almost) NNLL calculations → unless we get something wrong, can we reach that accuracy?



Partition of dipole into two single-singular pieces à la dipole subtraction method can easily be repartitioned into angular-ordered pieces

However…

Dasgupta et al. *JHEP* 09 (2018) 033,

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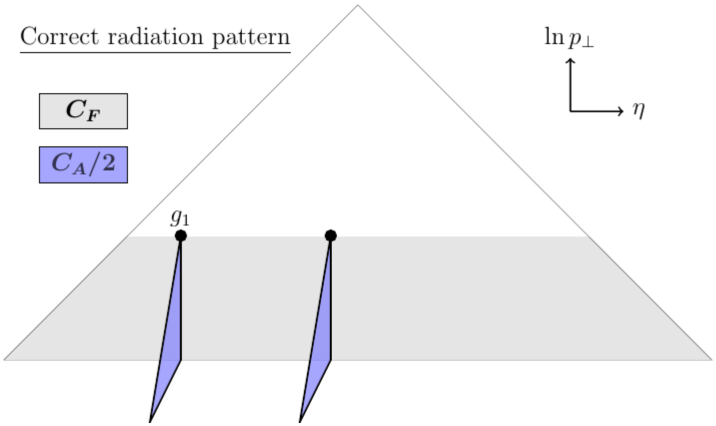
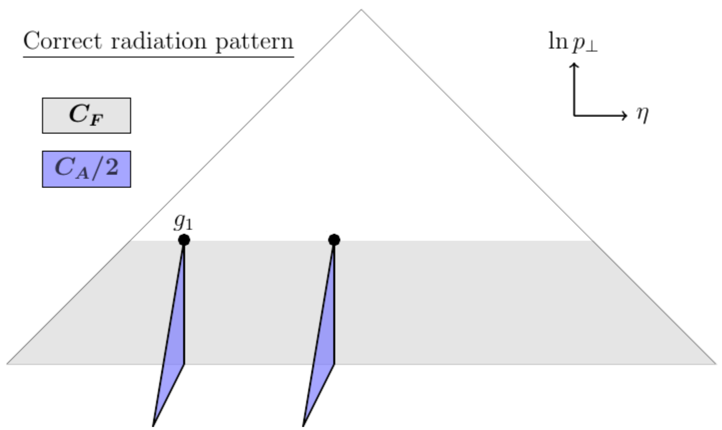
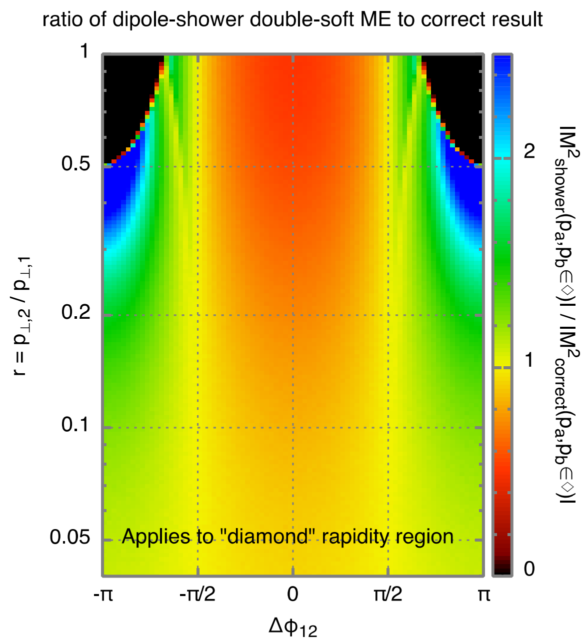


Dipole radiation between the gluons will always be given $C_A/2$
if they split to quarks they will always be given C_F

cf G. Gustafson Nucl. Phys. B392 (1993) 251-280

Dasgupta et al. *JHEP* 09 (2018) 033, *JHEP* 03 (2020) 083 (erratum)





Recoil from one gluon emission absorbed by the other requires global recoil scheme? à la angular-order parton showers

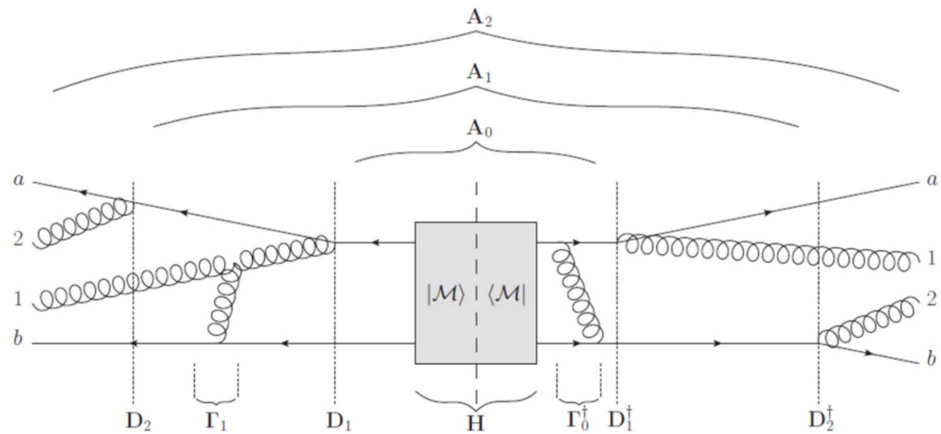
→ PanScales project

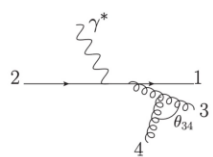
→ Forshaw, Holguin and Plätzer scheme → Herwig

Showers Beyond Leading Colour

Forshaw, Holguin and Plätzer, CVolver; Nagy and Soper, Deductor

$$\mathbf{A}_n(q_\perp;\{\tilde{p}\}_{n-1}\cup q_n) = \int \prod_{i=1}^{n_{\mathrm{H}}+n} \mathrm{d}^4 p_i \, \mathbf{V}_{q_\perp,q_{n\perp}} \mathbf{D}_n \mathbf{A}_{n-1}(q_{n\perp};\{p\}_{n-1}) \mathbf{D}_n^\dagger \mathbf{V}^\dagger_{q_\perp,q_{n\perp}} \Theta(q_\perp \le q_{n\perp}).$$





$$\mathbf{A}_4 = \frac{2\alpha_s}{\pi}\left({}^{(3)}\mathrm{S}_1^{2,3}[1\cdot 1] + {}^{(3)}\mathrm{S}_2^{1,3}[2\cdot 2] + {}^{(3)}\mathrm{S}_3^{1,2}[3\cdot 3]\right)$$

$$\mathbf{A}_4\big|_{n_1\to n_3} \approx \frac{2\alpha_s}{\pi}\Big[{}^{(3)}\mathrm{S}_1^{2,3}\Theta(\theta_{14}<\theta_{13})[1\cdot 1] + {}^{(3)}\mathrm{S}_2^{1,3}\Theta(\theta_{(1+3)4}>\theta_{13})[2\cdot 2] + {}^{(3)}\mathrm{S}_3^{1,2}\Theta(\theta_{34}<\theta_{13})[3\cdot 3]\Big]$$

$${}^{(n-1)}\mathrm{S}_i^{j,k} = \frac{1}{2}\left(\omega_{ij}+\omega_{ik}-\omega_{jk}\right)$$

$$[i\cdot j] = \mathbf{T}_i \,|M_{n-1}\rangle\langle M_{n-1}|\, \mathbf{T}_j^\dagger$$

$$\sum_j \mathbf{T}_j = 0 \qquad \omega_{ij}(q_n) = \frac{q_i\cdot q_j}{q_n\cdot q_i \; q_n\cdot q_j}$$

Up to terms of the order $(\theta_{13}^0, \theta_{i4}^0)$ and in any frame.

This and the usual coherent branching result precisely agree when $\theta_{12}=\pi$ but for $\theta_{12}\neq\pi$ the above result has greater accuracy.

Hadronization: Introduction

Partons are not physical particles: they cannot freely propagate.

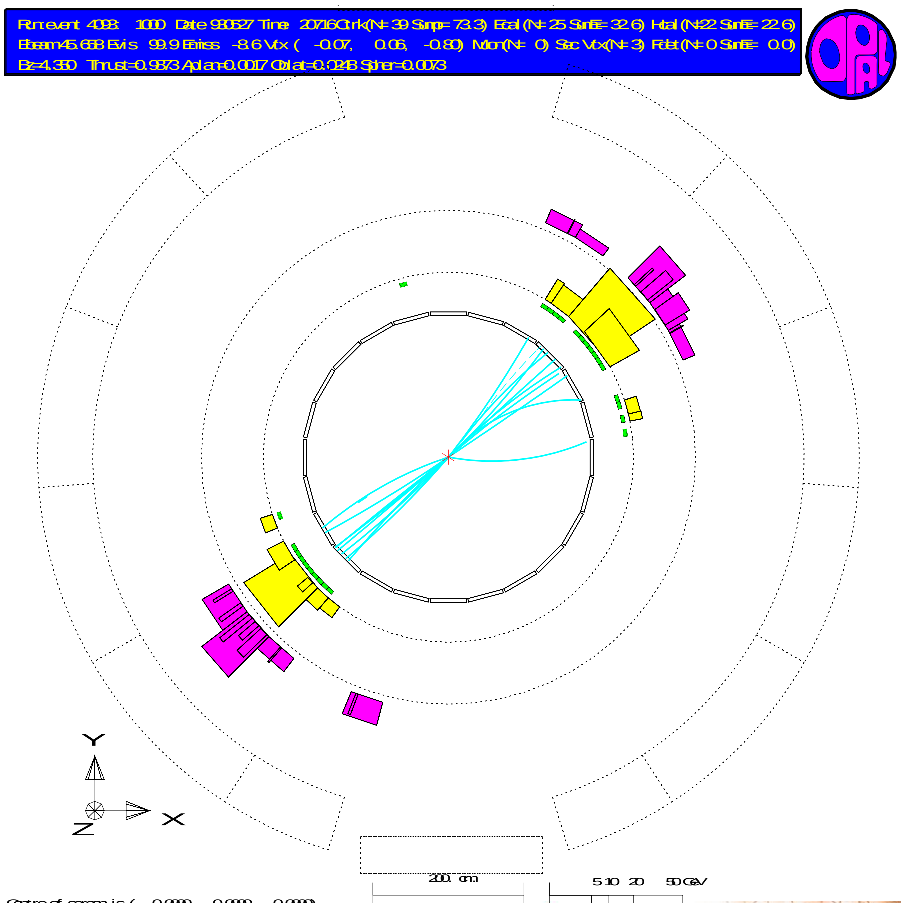
Hadrons are.

Need a model of partons' confinement into hadrons: hadronization.

1. Phenomenological models.
2. Confinement.
3. The string model.
4. Preconfinement.
5. The cluster model.
6. Secondary decays.
7. Underlying event models.

Jet production in $e^+e^- \rightarrow$ hadrons

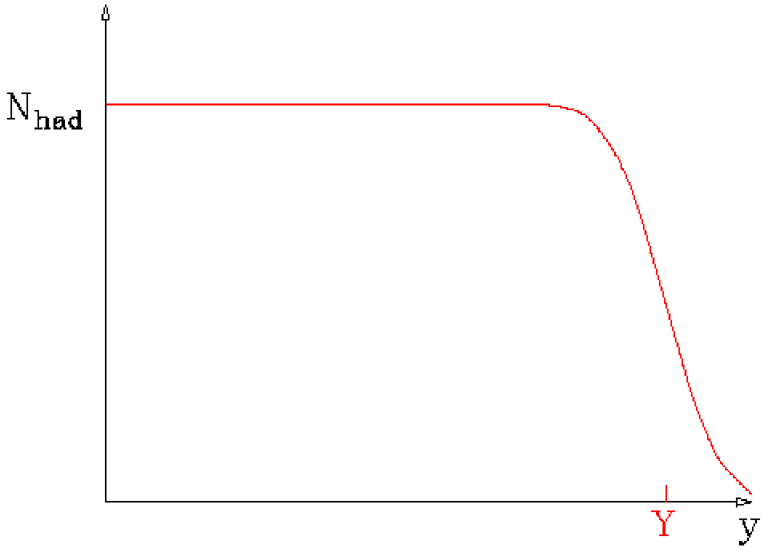
- Most e^+e^- events consist of two back-to-back jets

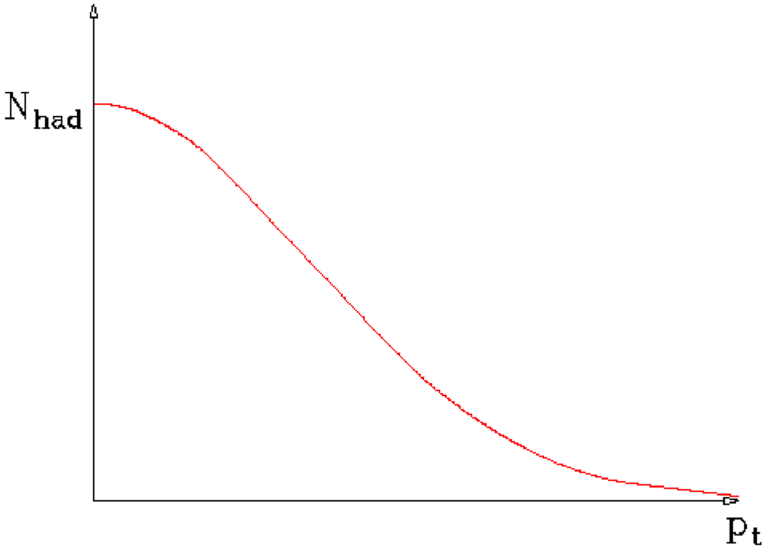


Phenomenological Models

Experimentally, $e^+e^- \rightarrow$ two jets:

Flat rapidity plateau and limited p_t, $\rho(p_t^2) \sim e^{-p_t^2/2p_0^2}$





Estimate of Hadronization Effects

Using this model, can estimate hadronization correction to perturbative quantities.

Jet energy and momentum:

$$E = \int_0^Y dy\, d^2p_t\, \rho(p_t^2)\, p_t\, \cosh y = \lambda \sinh Y$$

$$P = \int_0^Y dy\, d^2p_t\, \rho(p_t^2)\, p_t\, \sinh y = \lambda(\cosh Y - 1) \sim E - \lambda,$$

with $\lambda = \int d^2p_t\, \rho(p_t^2)\, p_t,$ mean transverse momentum.

Estimate from Fermi motion $\lambda \sim 1/R_{had} \sim m_{had}.$

Jet acquires non-perturbative mass: $M^2 = E^2 - P^2 \sim 2\lambda E$

Large: ~ 10 GeV for 100 GeV jets.

Independent Fragmentation Model ("Feynman—Field")

Direct implementation of the above.

Longitudinal momentum distribution = arbitrary fragmentation function: parameterization of data.

Transverse momentum distribution = Gaussian.

Recursively apply $q \to q' + \mathrm{had}.$

Hook up remaining soft q and $\bar{q}$.

Strongly frame dependent.

No obvious relation with perturbative emission.

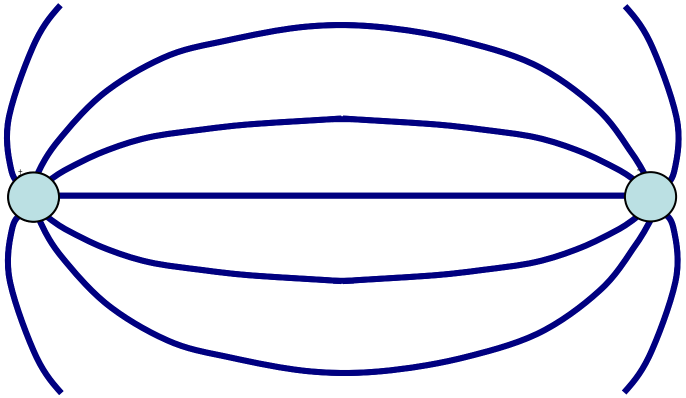
Not infrared safe.

Not a model of confinement.

Confinement

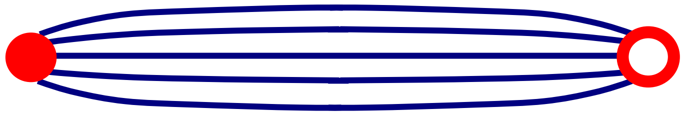
Asymptotic freedom: $Q\bar{Q}$ becomes increasingly QED-like at short distances.

QED:



but at long distances, gluon self-interaction makes field lines attract each other:

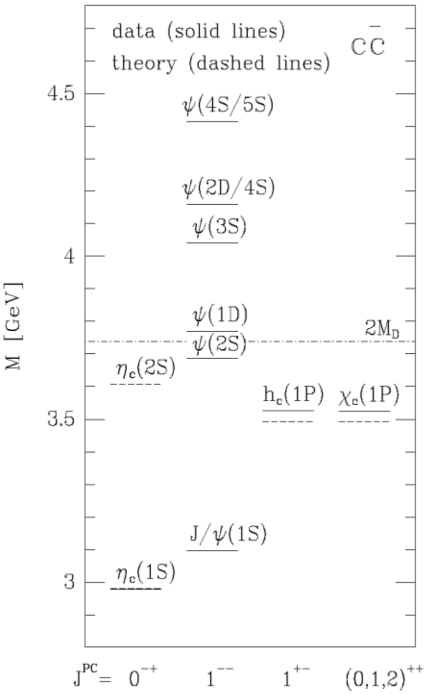
QCD:

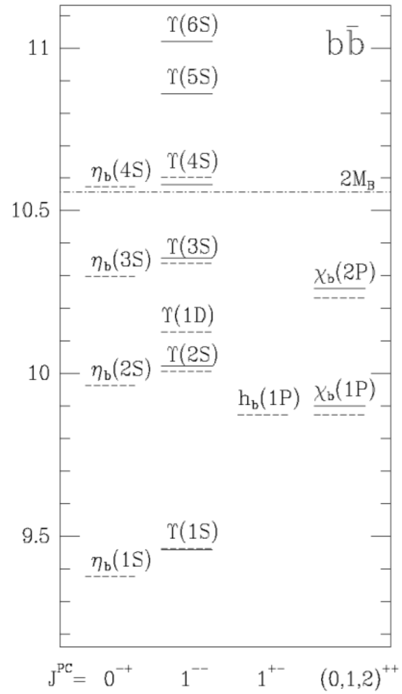


→linear potential → confinement

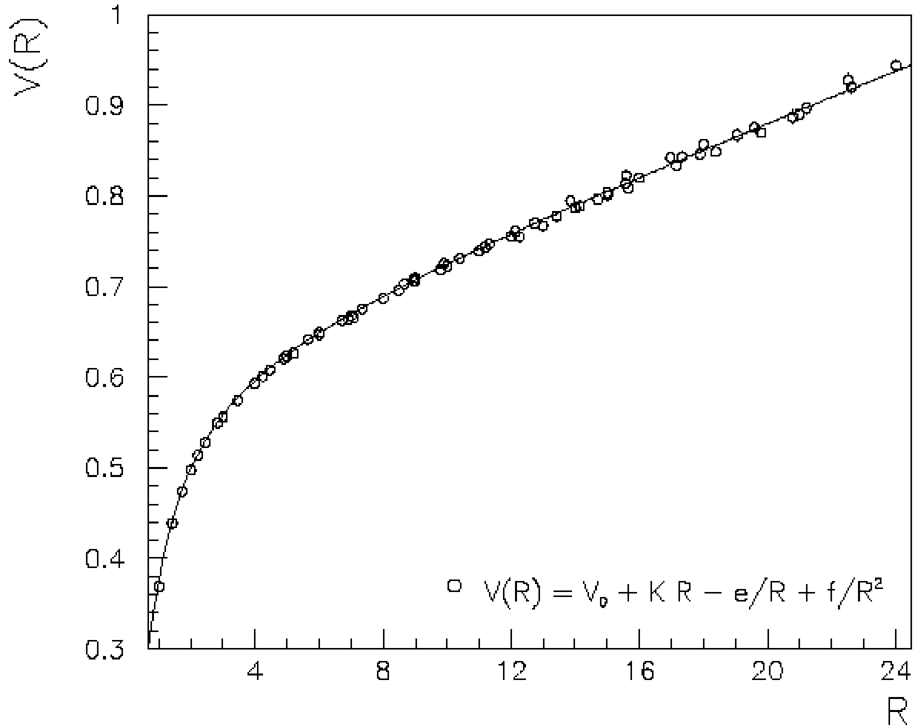
Interquark potential

Can measure from quarkonia spectra:





or from lattice QCD:



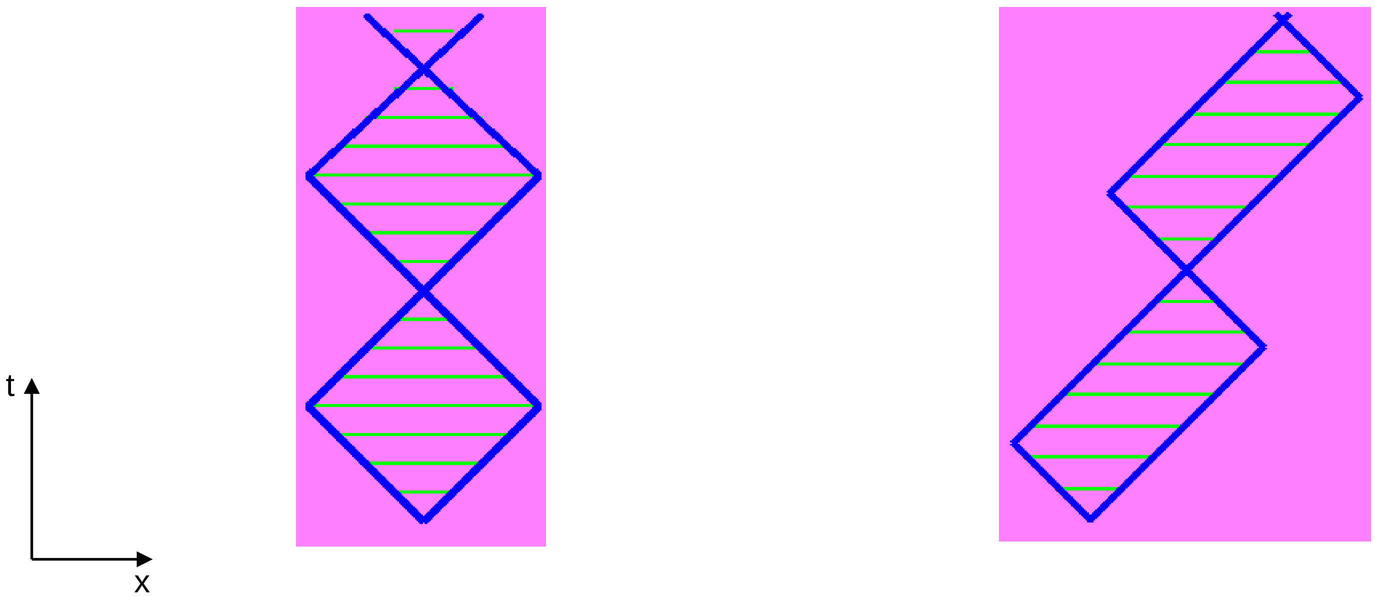
→ String tension

$$\kappa \approx 1\ \mathrm{GeV/fm}.$$

String Model of Mesons

Light quarks connected by string.

L=0 mesons only have ‘yo-yo’ modes:



Obeys area law: $m^2 = 2\kappa^2$ area

The Lund String Model

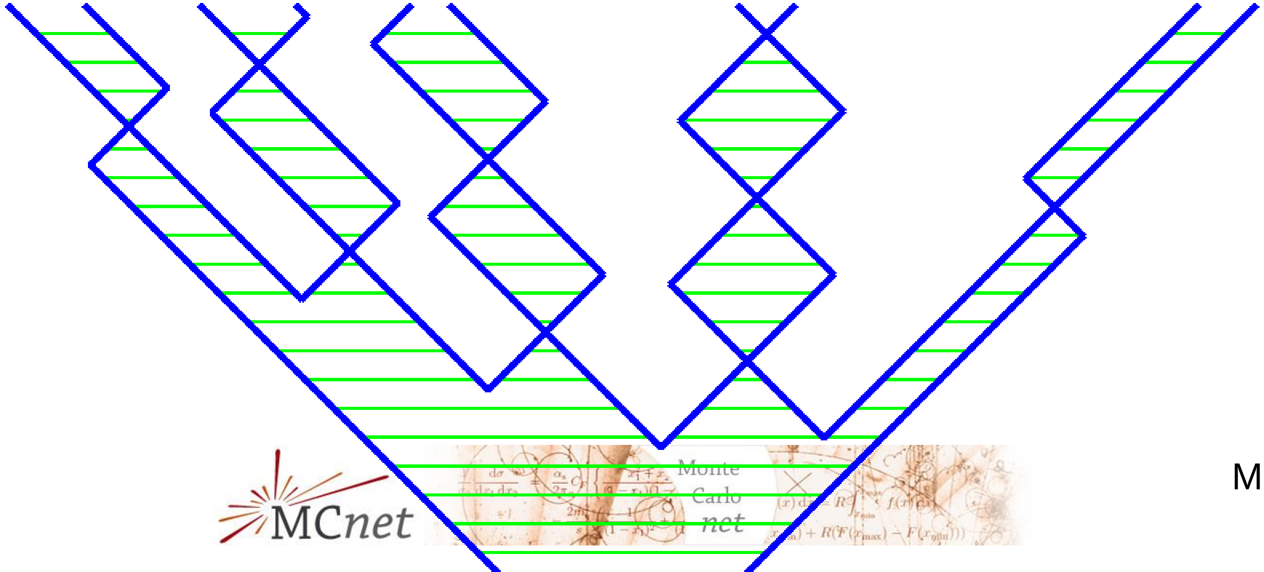
Start by ignoring gluon radiation:

e^+e^- annihilation = pointlike source of $q\bar{q}$ pairs

Intense chromomagnetic field within string → $q\bar{q}$ pairs created by tunnelling. Analogy with QED:

$$\frac{d(\text{Probability})}{dx\,dt} \propto \exp(-\pi m_q^2/\kappa)$$

Expanding string breaks into mesons long before yo-yo point.



Lund Symmetric Fragmentation Function

String picture → constraints on fragmentation function:

- Lorentz invariance
- Acausality
- Left—right symmetry

$$f(z) \propto z^{a_\alpha - a_\beta - 1}(1-z)^{a_\beta}$$

$a_{\alpha,\beta}$ adjustable parameters for quarks α and β.

Fermi motion → Gaussian transverse momentum.

Tunnelling probability becomes

$$\exp\left[-b(m_q^2 + p_t^2)\right]$$

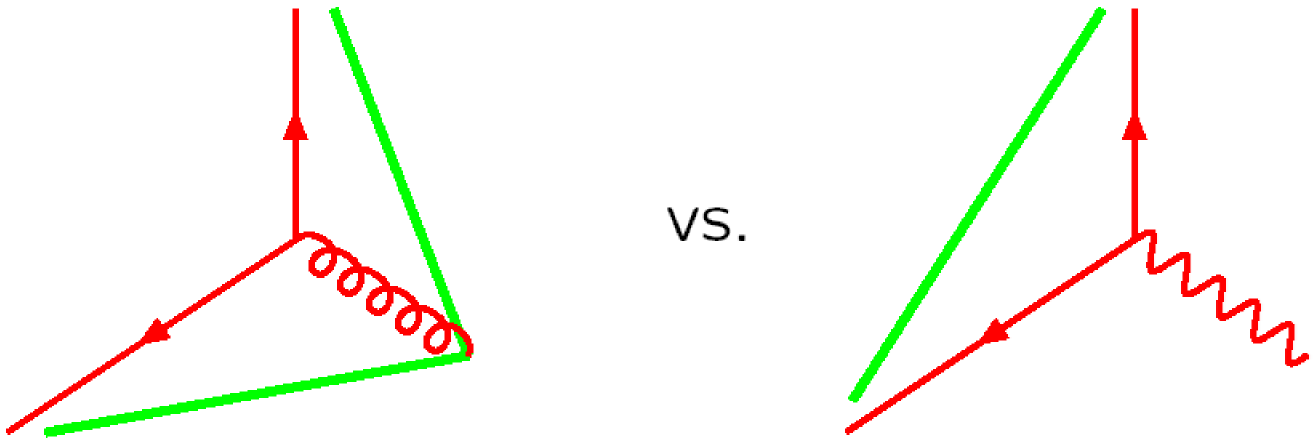
a, b and m_q^2 = main tuneable parameters of model

Three-jet Events

So far: string model = motivated, constrained independent fragmentation!

New feature: universal

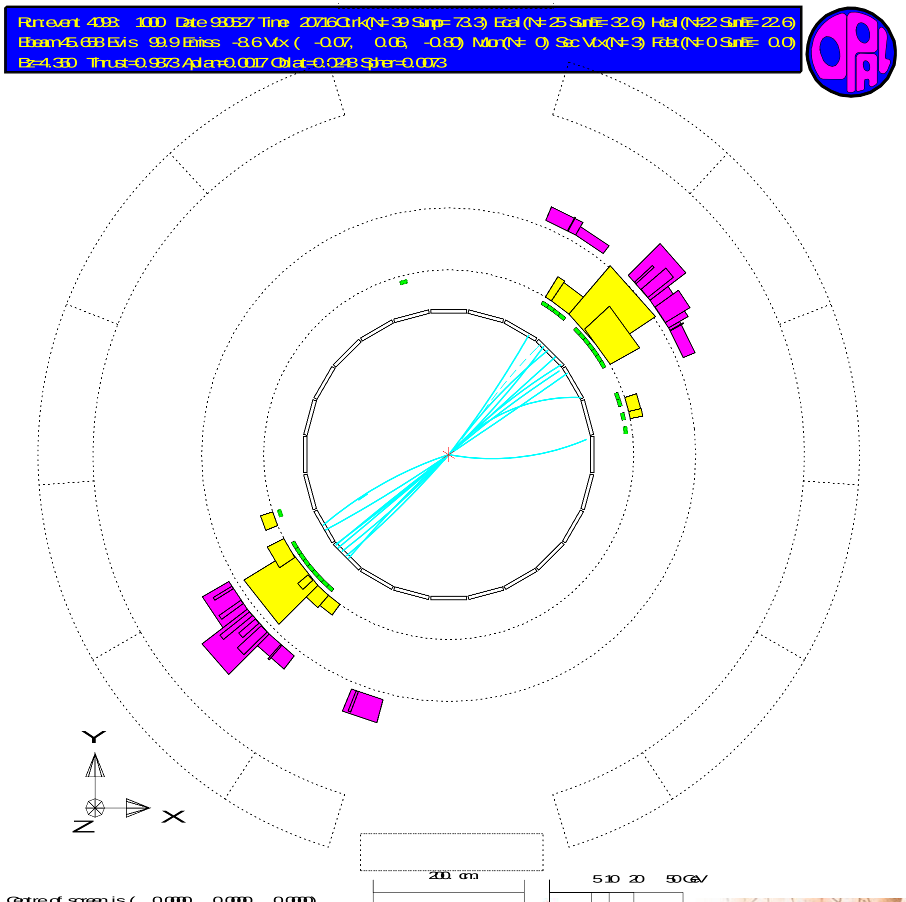
Gluon = kink on string → the string effect

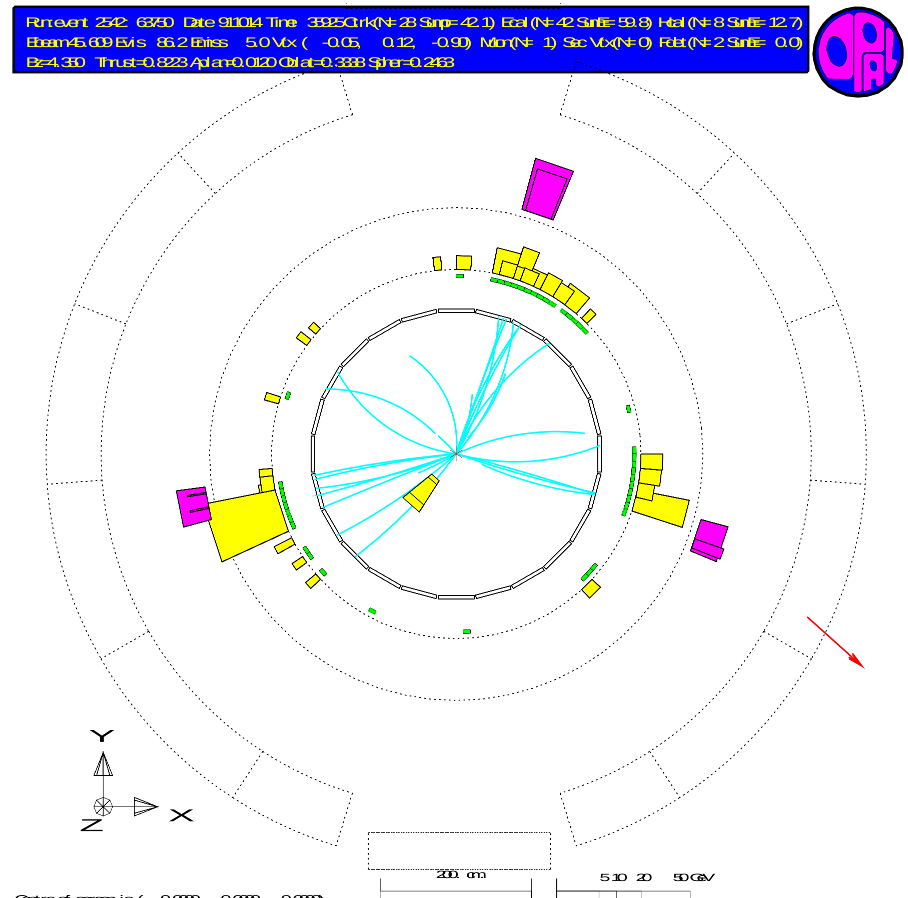


Infrared safe matching with parton shower: gluons with $k_\perp <$ inverse string width irrelevant.

Jet production in $e^+e^- \rightarrow$ hadrons

- Most e^+e^- events consist of two back-to-back jets
- But some consist of three (or more) jets → gluons





String Summary

- String model strongly physically motivated.
- Very successful fit to data.
- Universal: fitted to e^+e^- little freedom elsewhere.

- How does motivation translate to prediction?
  ~ one free parameter per hadron/effect!

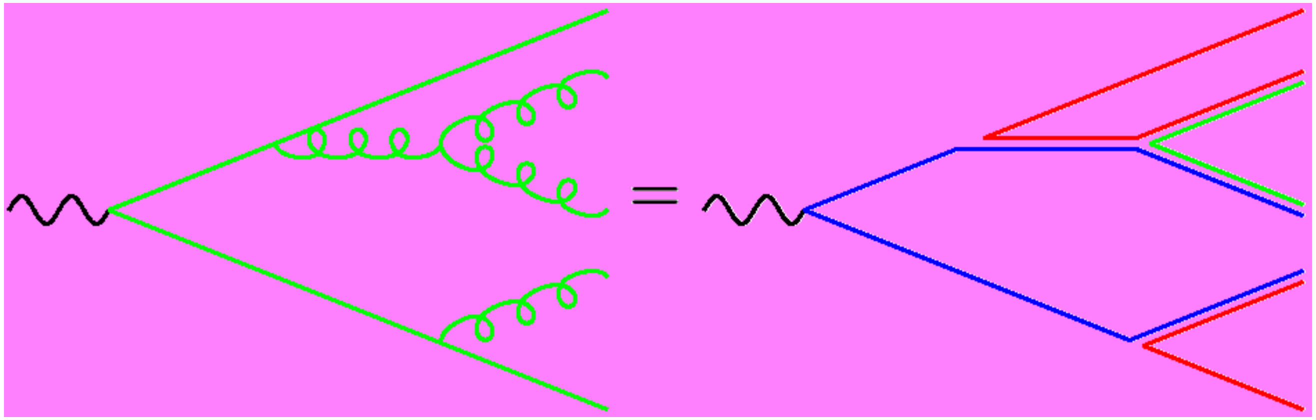
- Blankets too much perturbative information?

- Can we get by with a simpler model?

Preconfinement

Planar approximation: gluon = colour—anticolour pair.

Follow colour structure of parton shower: colour-singlet pairs end up close in phase space

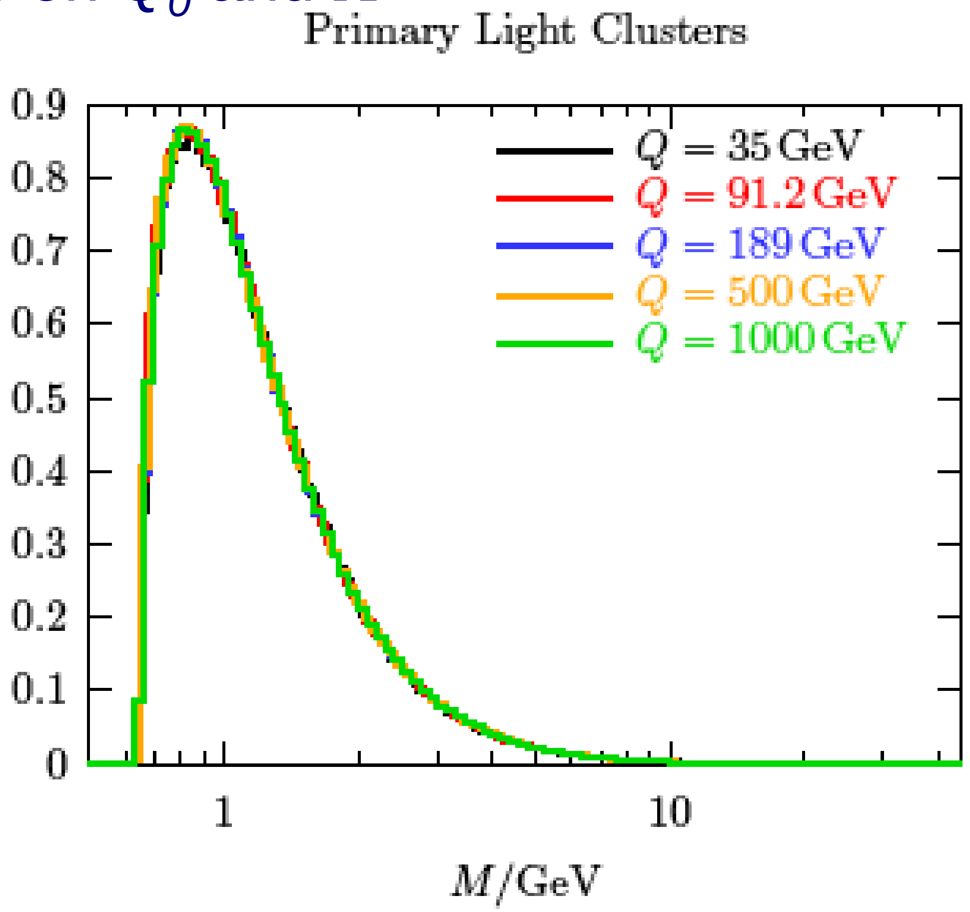


Mass spectrum of colour-singlet pairs asymptotically independent of energy, production mechanism, …

Peaked at low mass $\sim Q_0$.

Cluster mass distribution

- Independent of shower scale Q
  - depends on Q_0 and Λ



The Naïve Cluster Model

Project colour singlets onto continuum of high-mass mesonic resonances (=clusters). Decay to lighter well-known resonances and stable hadrons.

Assume spin information washed out:

decay = pure phase space.

→ heavier hadrons suppressed

→ baryon & strangeness suppression 'for free' (i.e. untuneable).

Hadron-level properties fully determined by cluster mass spectrum, i.e. by perturbative parameters.

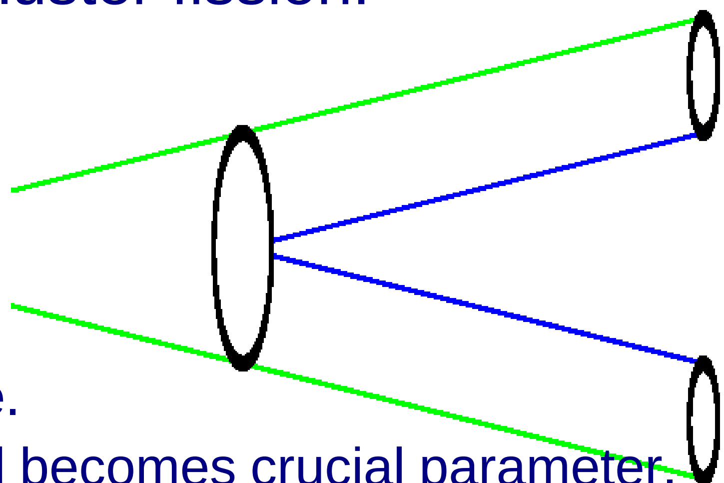
Q_0 crucial parameter of model.

The Cluster Model

Although cluster mass spectrum peaked at small m, broad tail at high m.

“Small fraction of clusters too heavy for isotropic two-body decay to be a good approximation”.

Longitudinal cluster fission:



Rather string-like.

Fission threshold becomes crucial parameter.

~15% of primary clusters get split but ~50% of hadrons come from them.

The Cluster Model

"Leading hadrons are too soft"

→ 'perturbative' quarks remember their direction somewhat

$$P(\theta^2) \sim \exp(-\theta^2/2\theta_0^2)$$

Rather string-like.

Extra adjustable parameter.

Strings

"Hadrons are produced by hadronization: you must get the non-perturbative dynamics right"

Improving data has meant successively refining perturbative phase of evolution…

Clusters

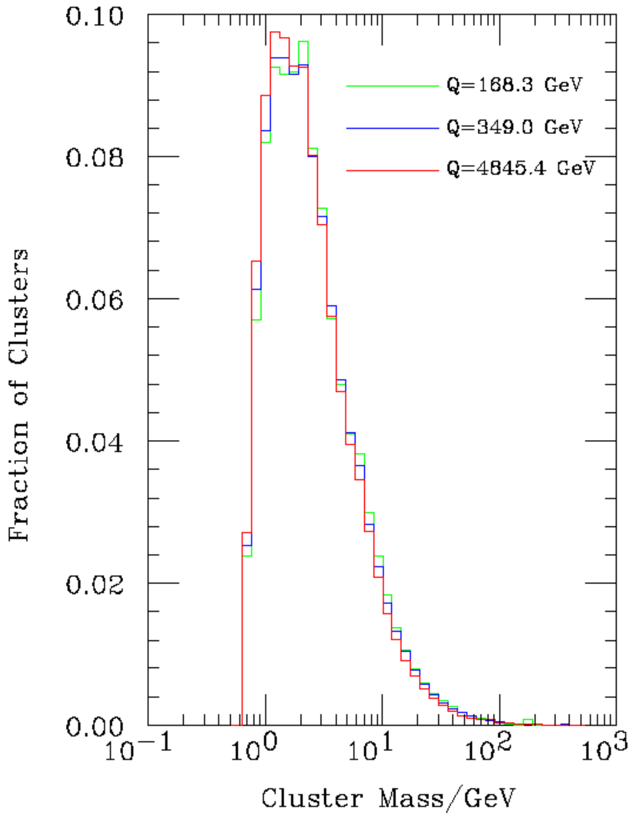
"Get the perturbative phase right and any old hadronization model will be good enough"

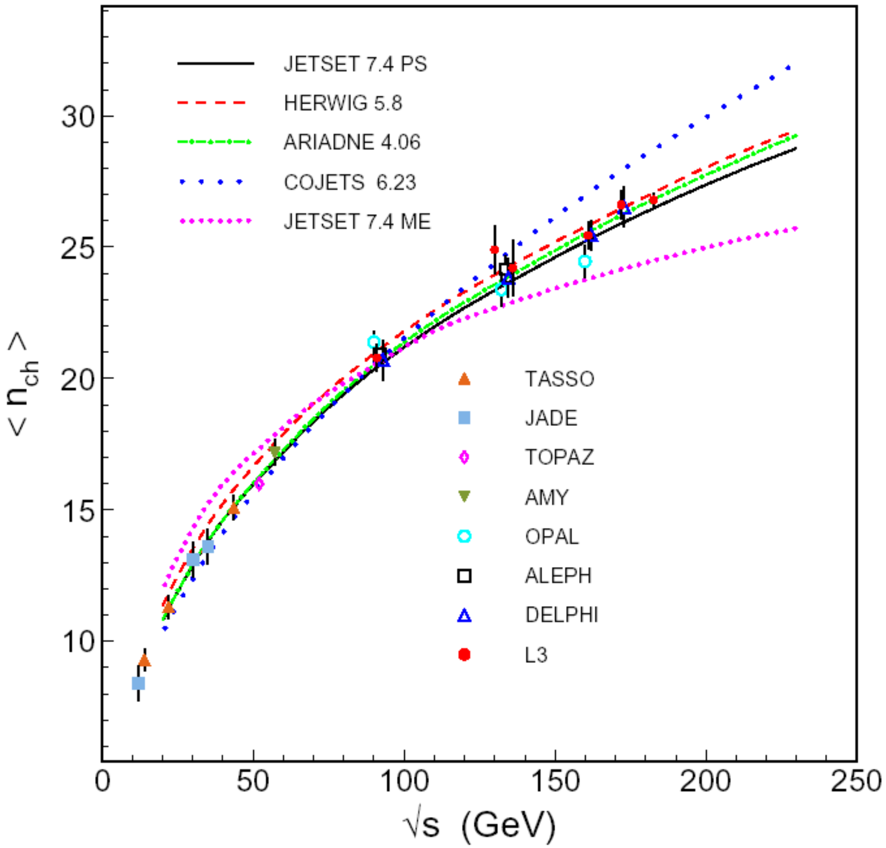
Improving data has meant successively making non-perturbative phase more string-like…

???

Universality of Hadronization Parameters

- Is guaranteed by preconfinement: do not need to retune at each energy





→ Only tune what's new in hadron—hadron collisions

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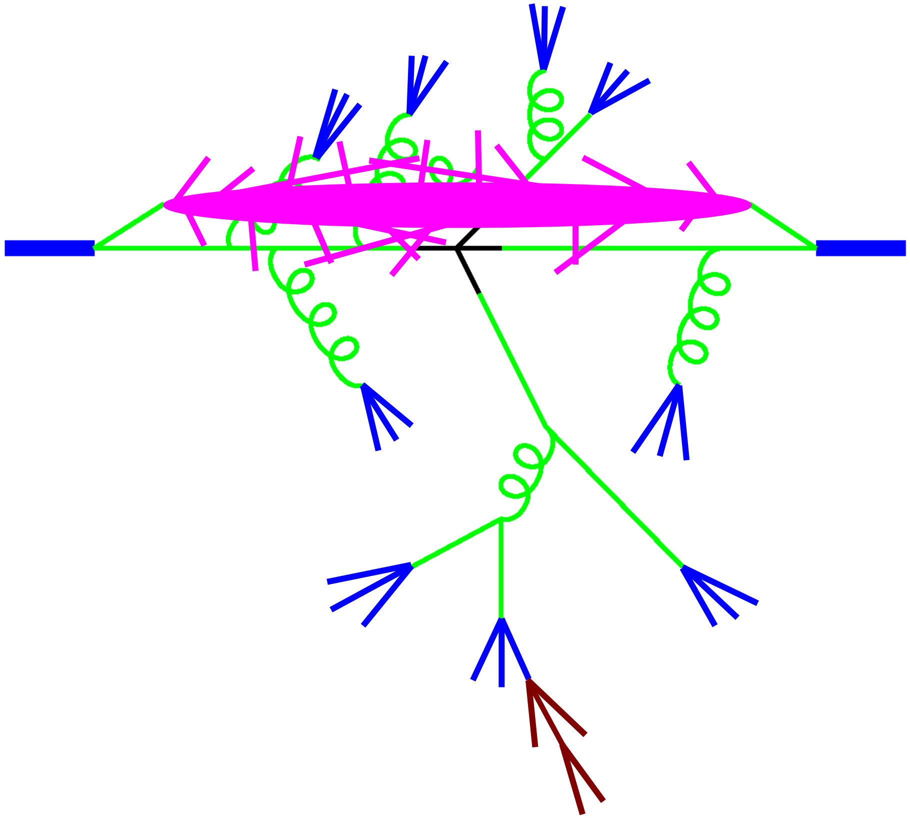


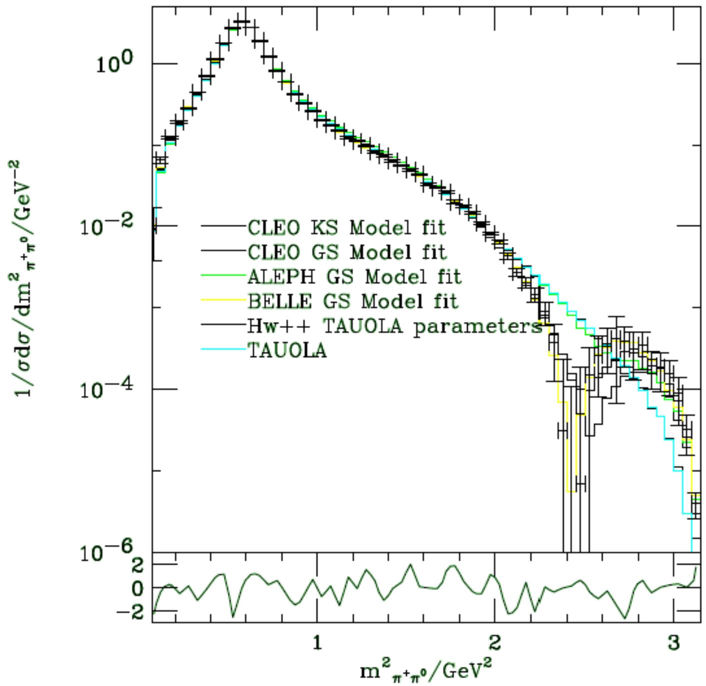
Secondary Decays and Decay Tables

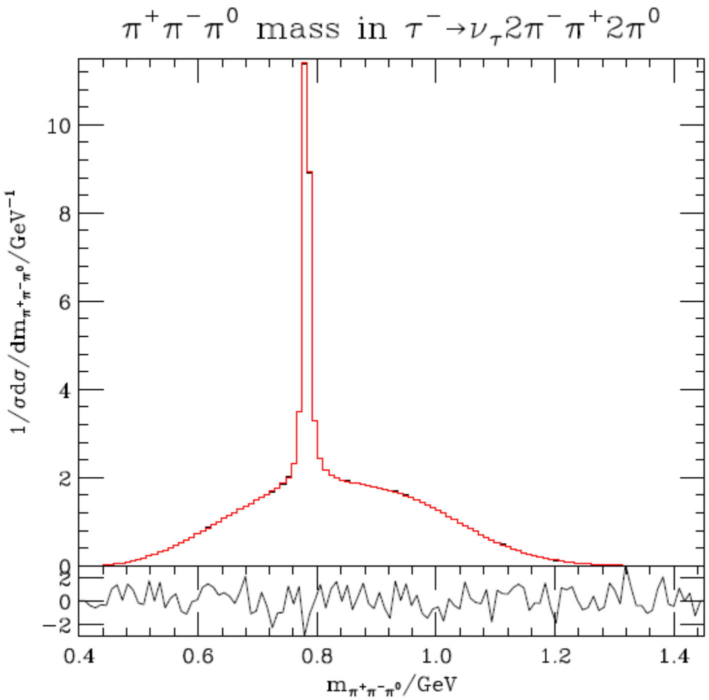
- Often forgotten ingredient of event generators:
  - String and cluster decay to some stable hadrons but mainly unstable resonances
  - These decay further "according to PDG data tables"
    - Matrix elements for n-body decays
  - But…
    - Not all resonances in a given multiplet have been measured
    - Measured branching fractions rarely add up to 100% exactly
    - Measured branching fractions rarely respect isospin exactly
  - So need to make a lot of choices
  - Has a significant effect on hadron yields, transverse momentum release, hadronization corrections to event shapes, …
  - Should consider the decay table choice part of the tuned set

Secondary particle decays

- Previous generations typically used external packages, e.g. TAUOLA, PHOTOS, EVTGEN
- Sherpa & Herwig++ contain at least as complete a description in all areas…
- without interfacing issues (c.f. τ spin)

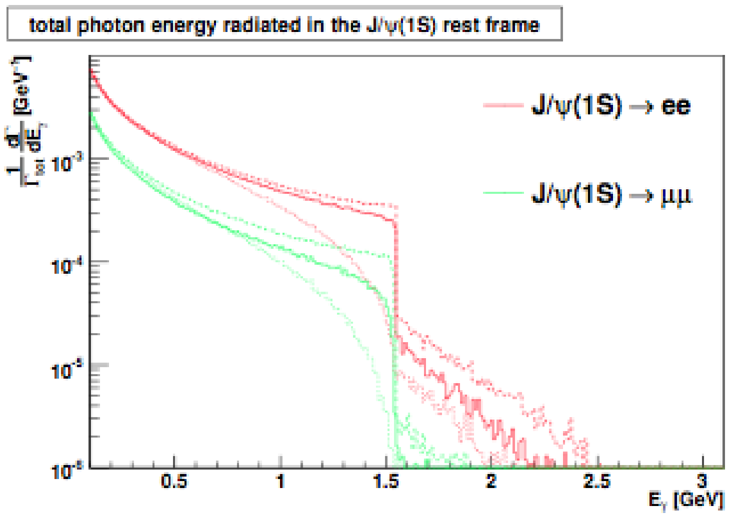
Tau Decays



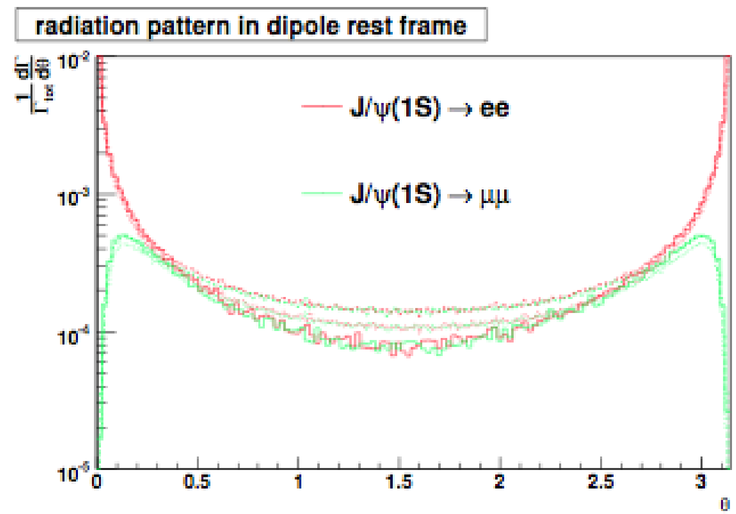


Mass spectrum of $\pi\pi$ in $\tau\to\pi\pi\nu_\tau$ for various models and example of mass distribution in $\tau\to 5\pi\nu_\tau$ comparing Herwig++ and TAUOLA.

Leptonic hadron decays: $J/\psi \to \ell\bar{\ell}$



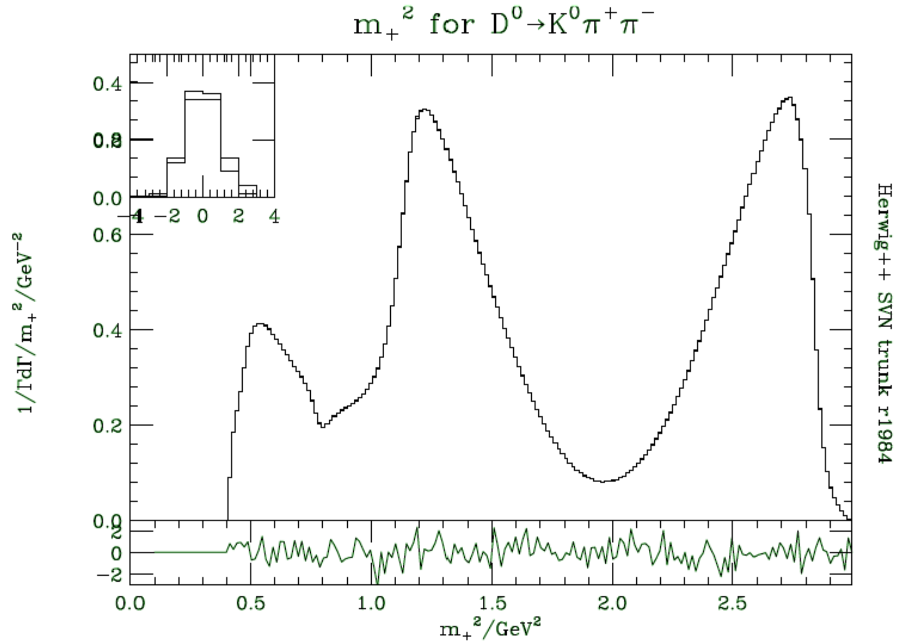
total radiated energy in the J/ψ rest frame

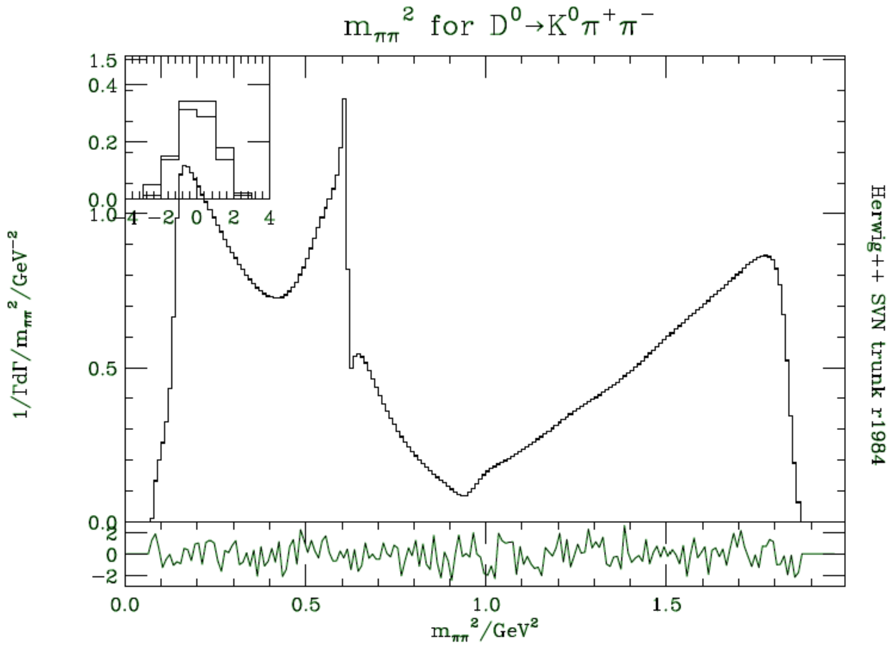


angular spectrum in the rest frame of the dipole

- soft only (dotted)
- collinear approximated ME (dashed)
- exact ME (solid)

D→Kππ

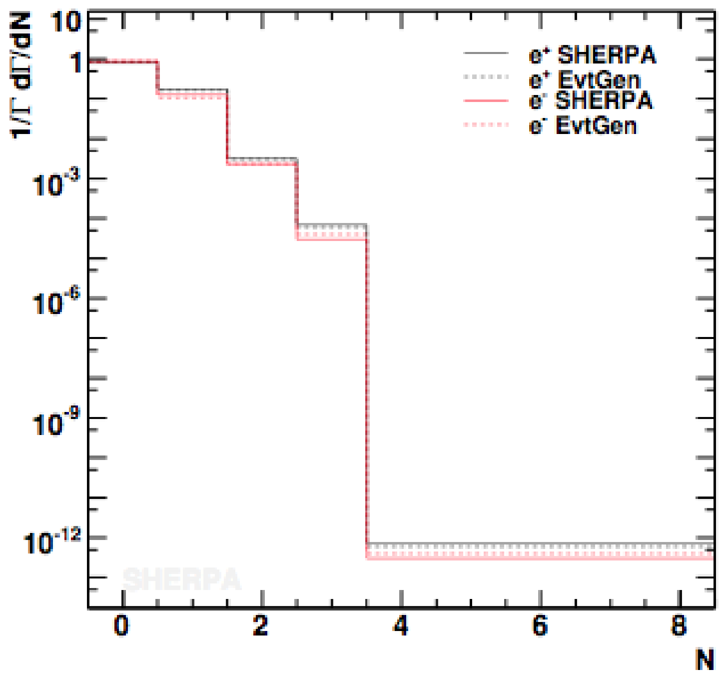




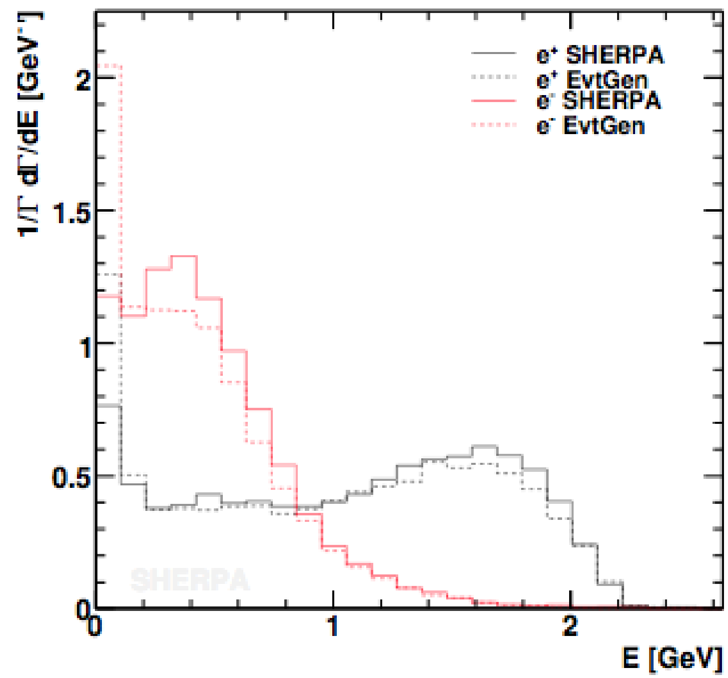
Comparison of Herwig++ and EvtGen implementations of the fit of Phys. Rev. D63 (2001) 092001 (CLEO).

Inclusive observables for B^+ decay

Electron multiplicity



Electron energy spectrum



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