Università degli Studi di Milano

INFN Istituto Nazionale di Fisica Nucleare

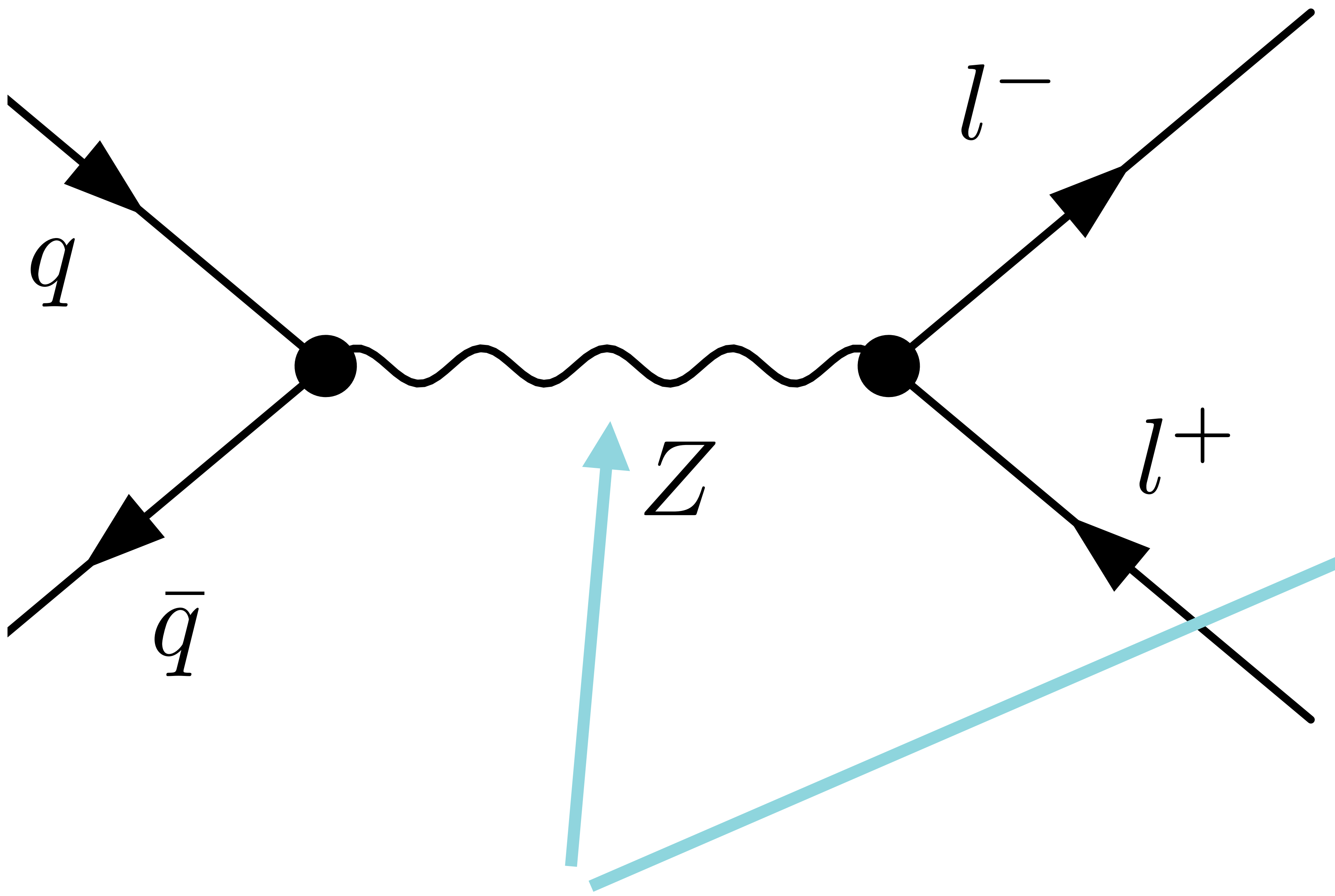
Lecture 1: A few comments about the electroweak Standard Model and its renormalisation

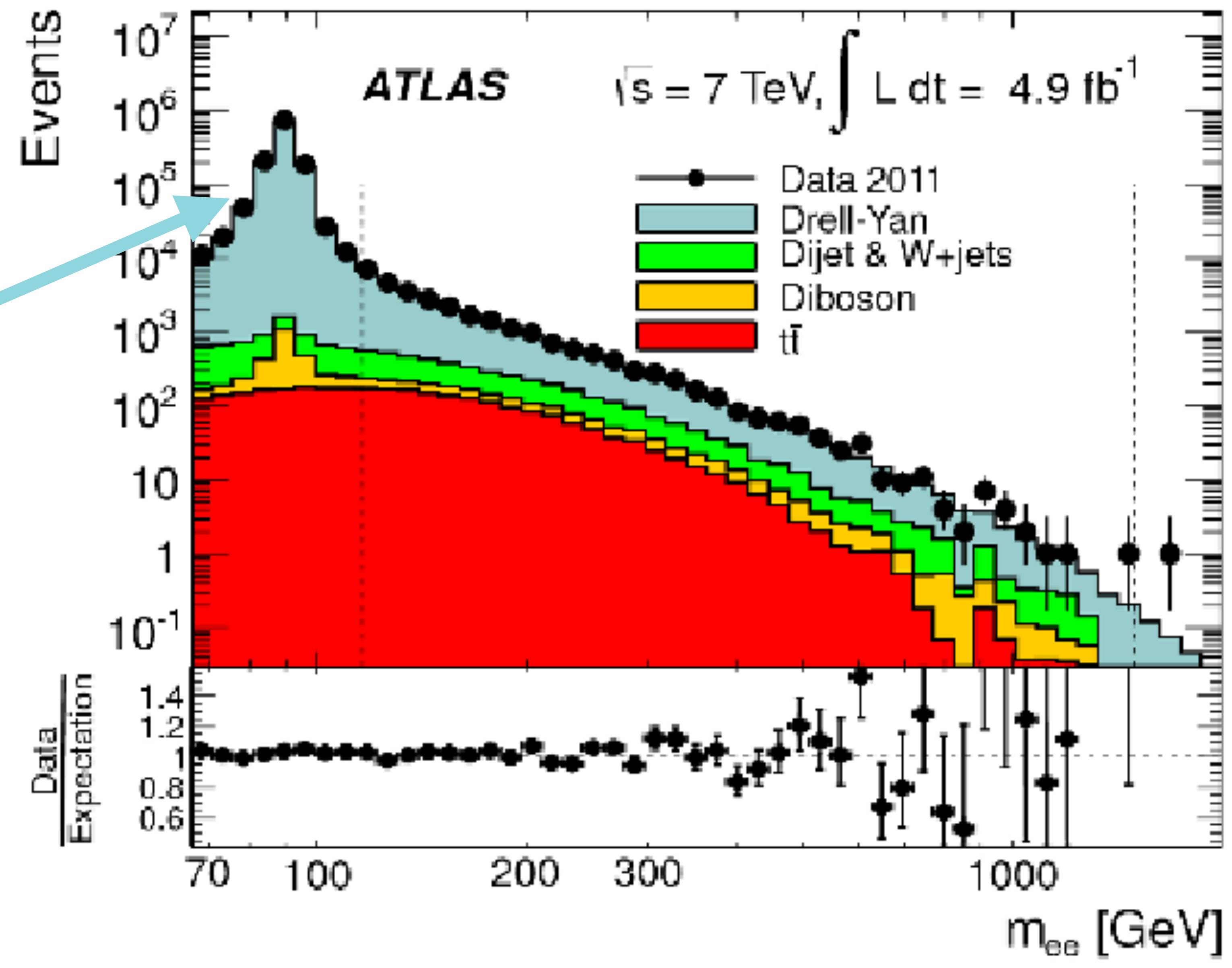
Alessandro Vicini

University of Milano, INFN Milano

CERN, MCnet school, June 2024

What can we learn from Elementary Particles ?

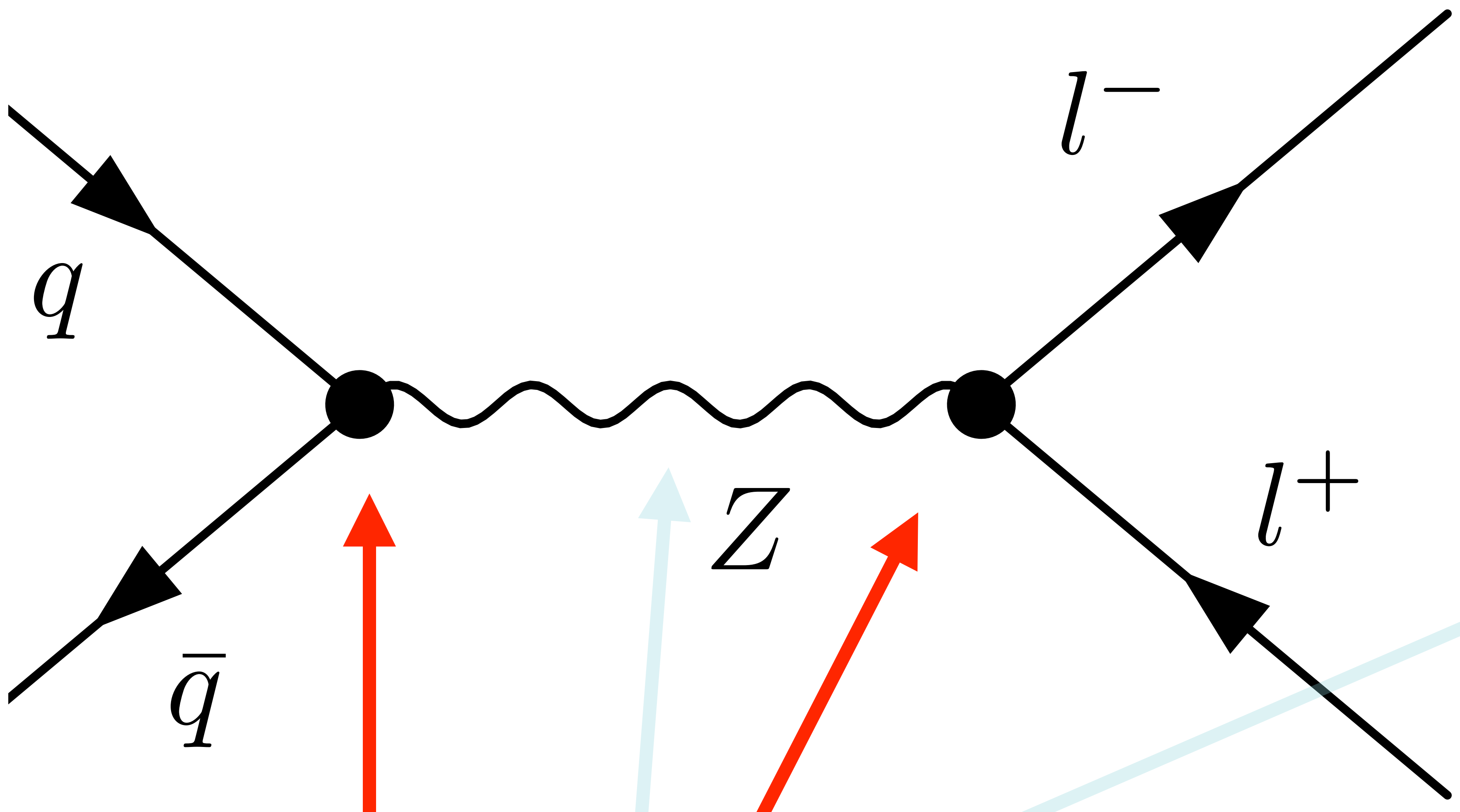


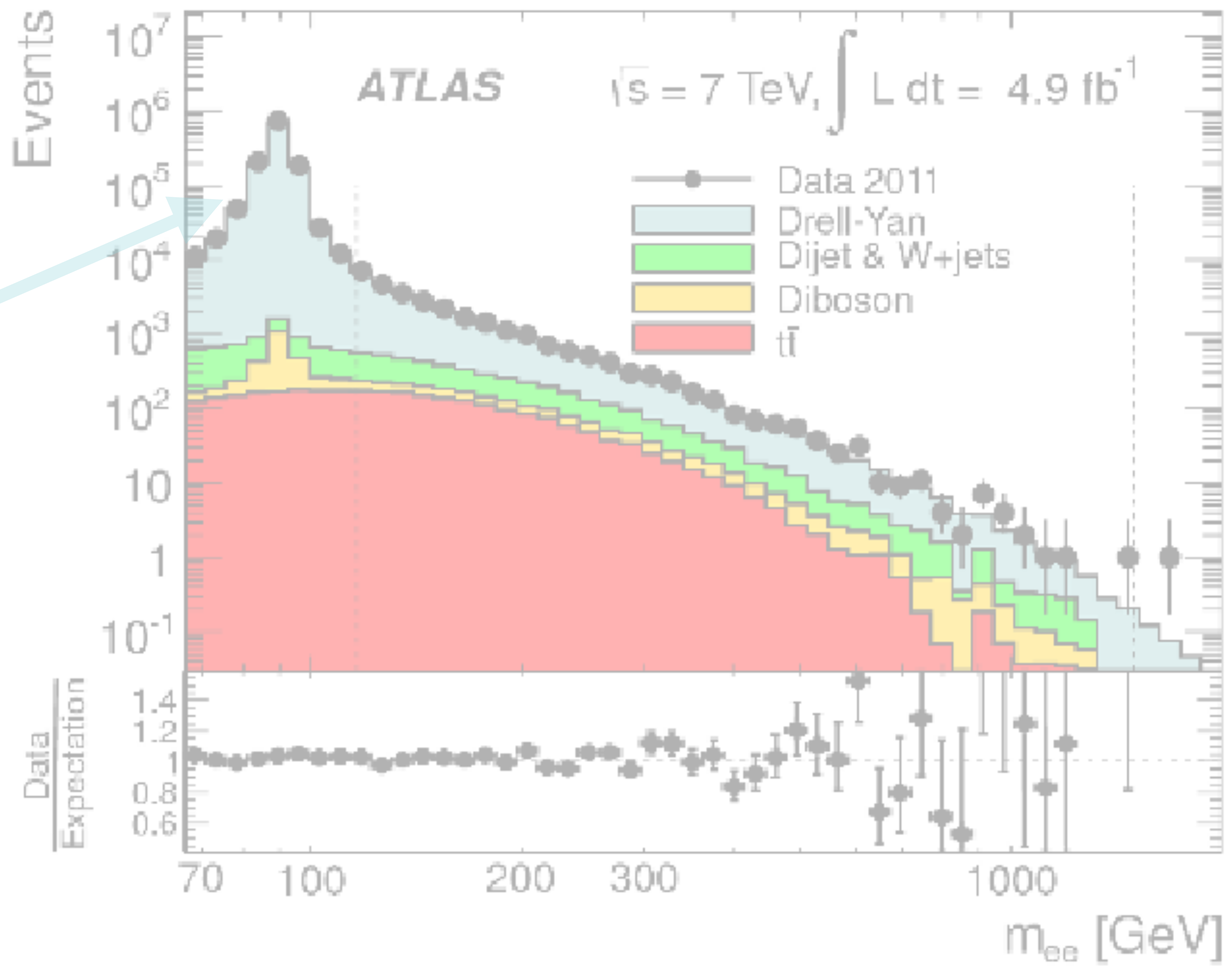


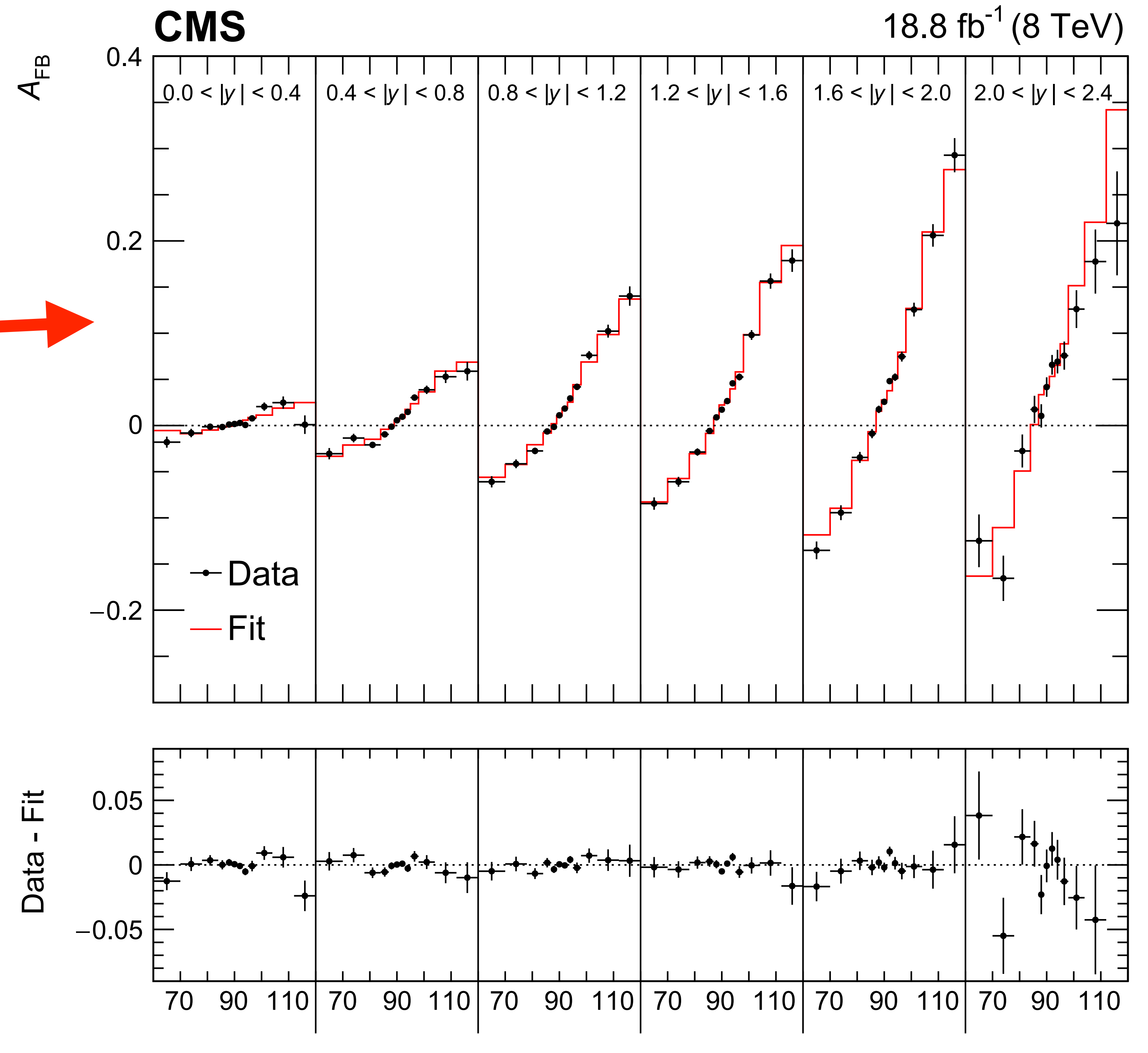
- the value of the masses of the intermediate particles

  (from the resonances, when measurable)

What can we learn from Elementary Particles ?



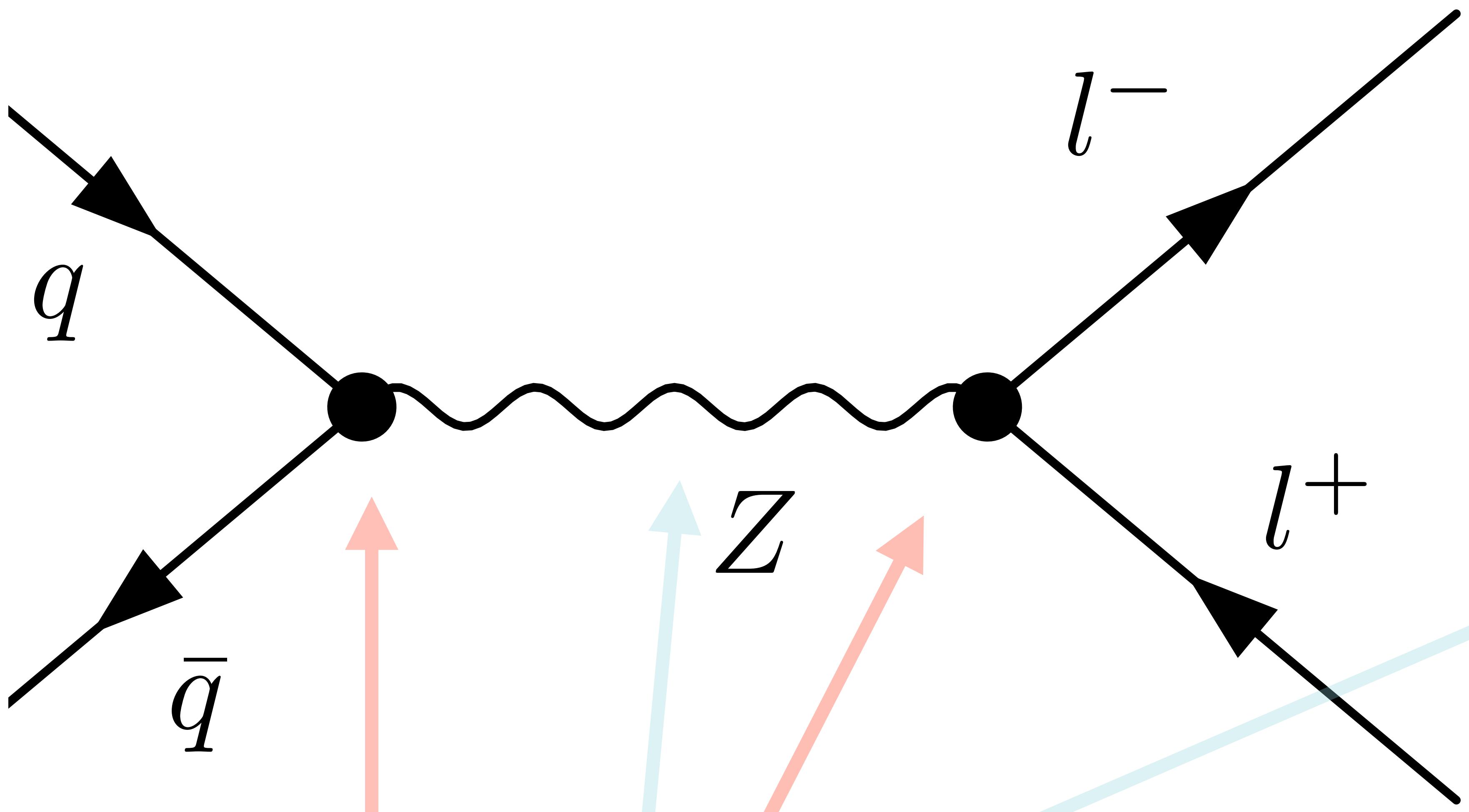


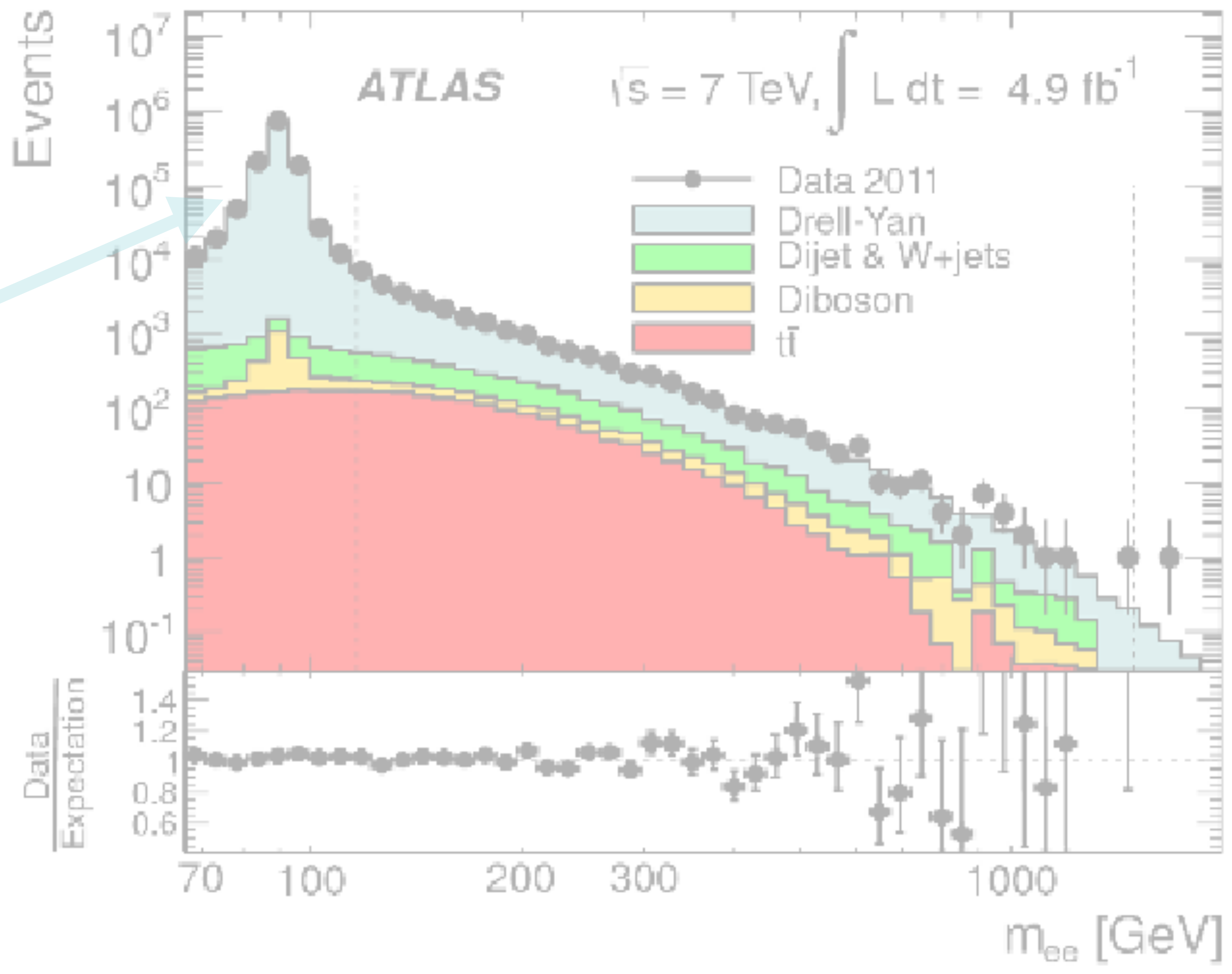


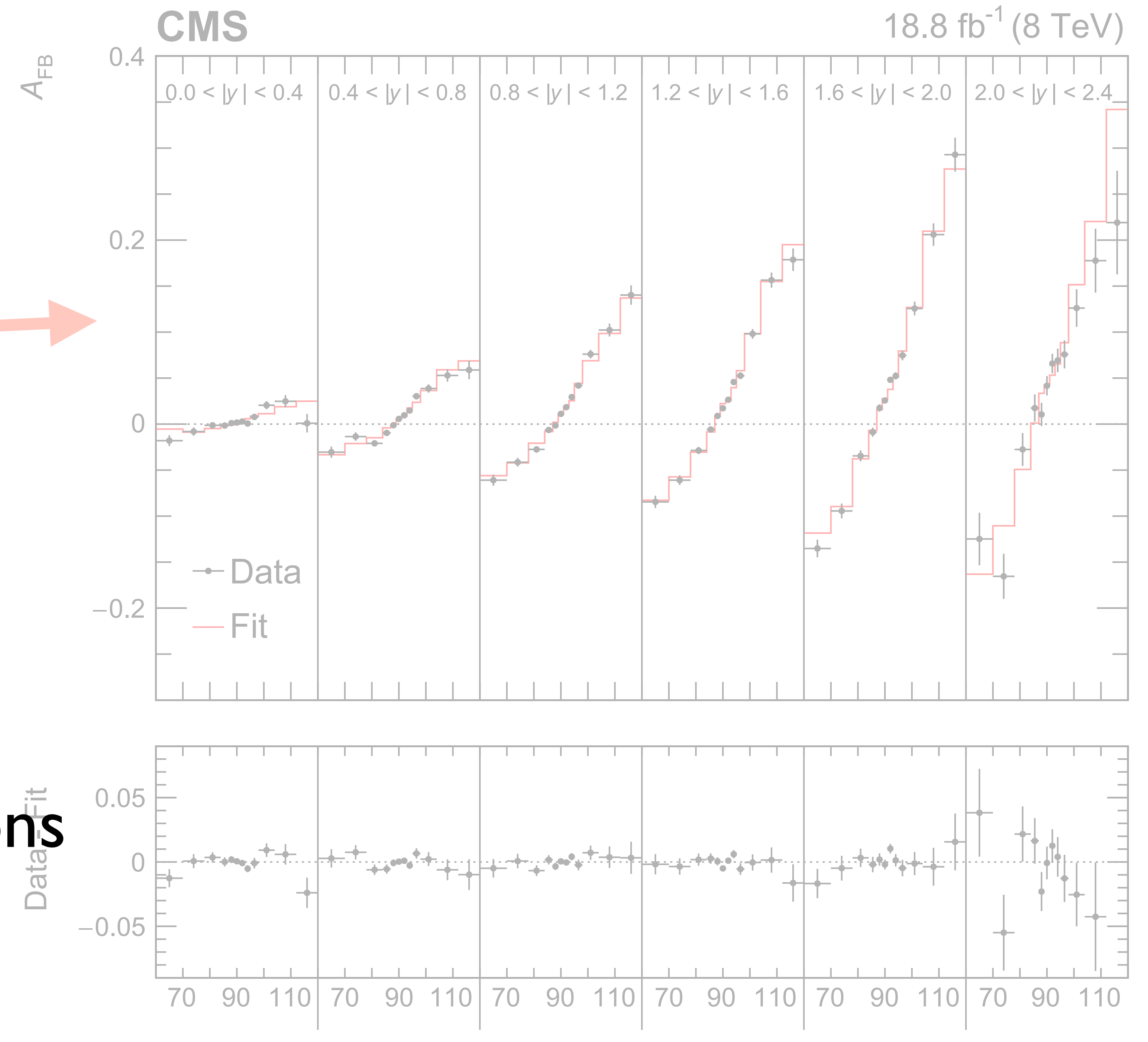
- the value of the masses of the intermediate particles
  (from the resonances, when measurable)

- the nature and strength of the interaction between gauge bosons and matter fields:
  scalar, pseudo-scalar, vector, axial-vector, …
  (observables with defined properties under Lorentz and discrete symmetries)

What can we learn from Elementary Particles ?







- the value of the masses of the intermediate particles
  (from the resonances, when measurable)

- the nature and strength of the interaction between gauge bosons and matter fields:
  scalar, pseudo-scalar, vector, axial-vector, …
  (observables with defined properties under Lorentz and discrete symmetries)

- these experimental results are compared against the best Standard Model predictions
  sensitivity to quantum corrections and New Physics

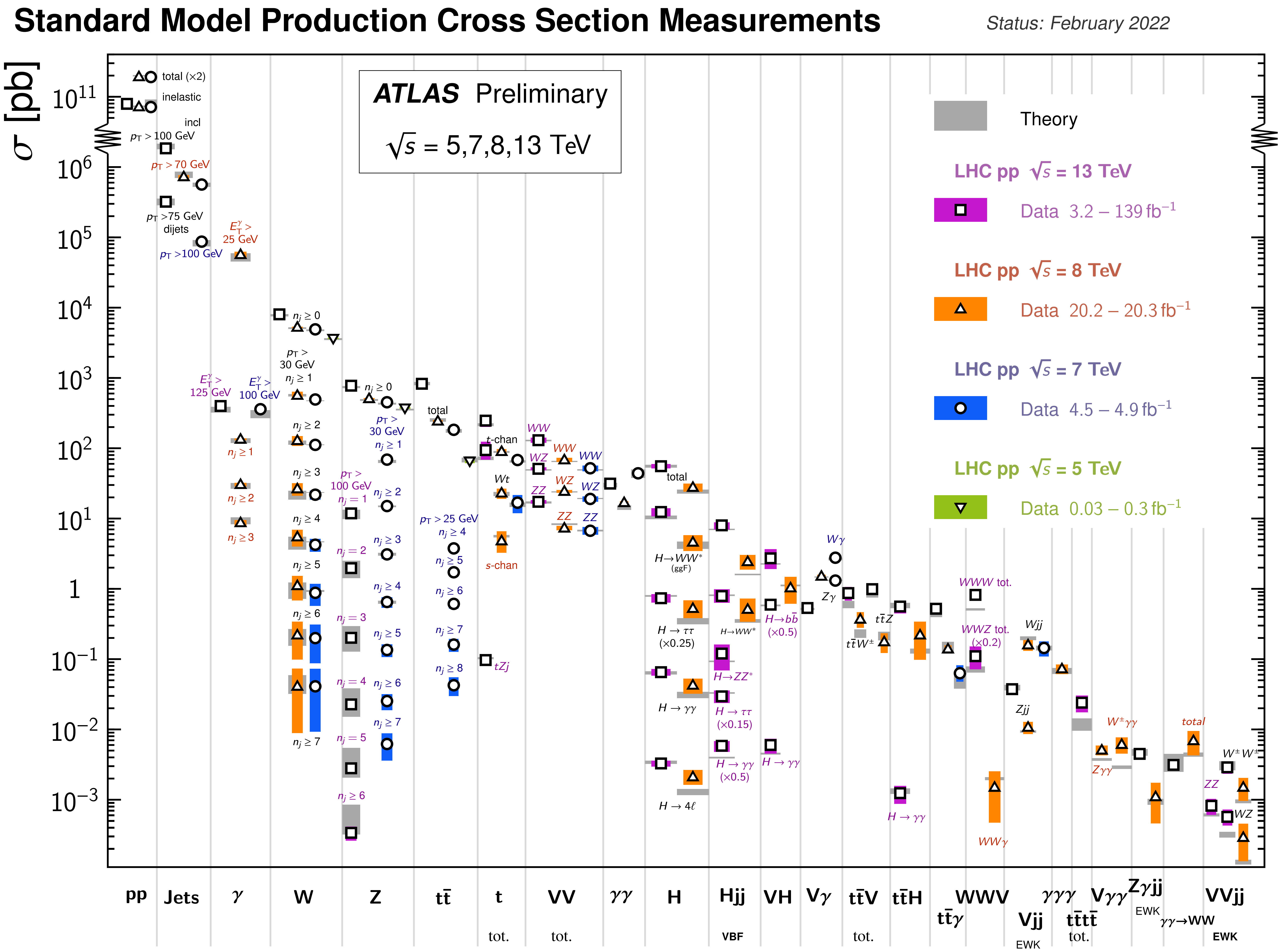
Exploring the strong and electroweak interactions at colliders

Rich amount of high-precision data
relative to different interactions
spanning many orders of magnitude

remarkable level of agreement
with the theoretical predictions
(grey bands)

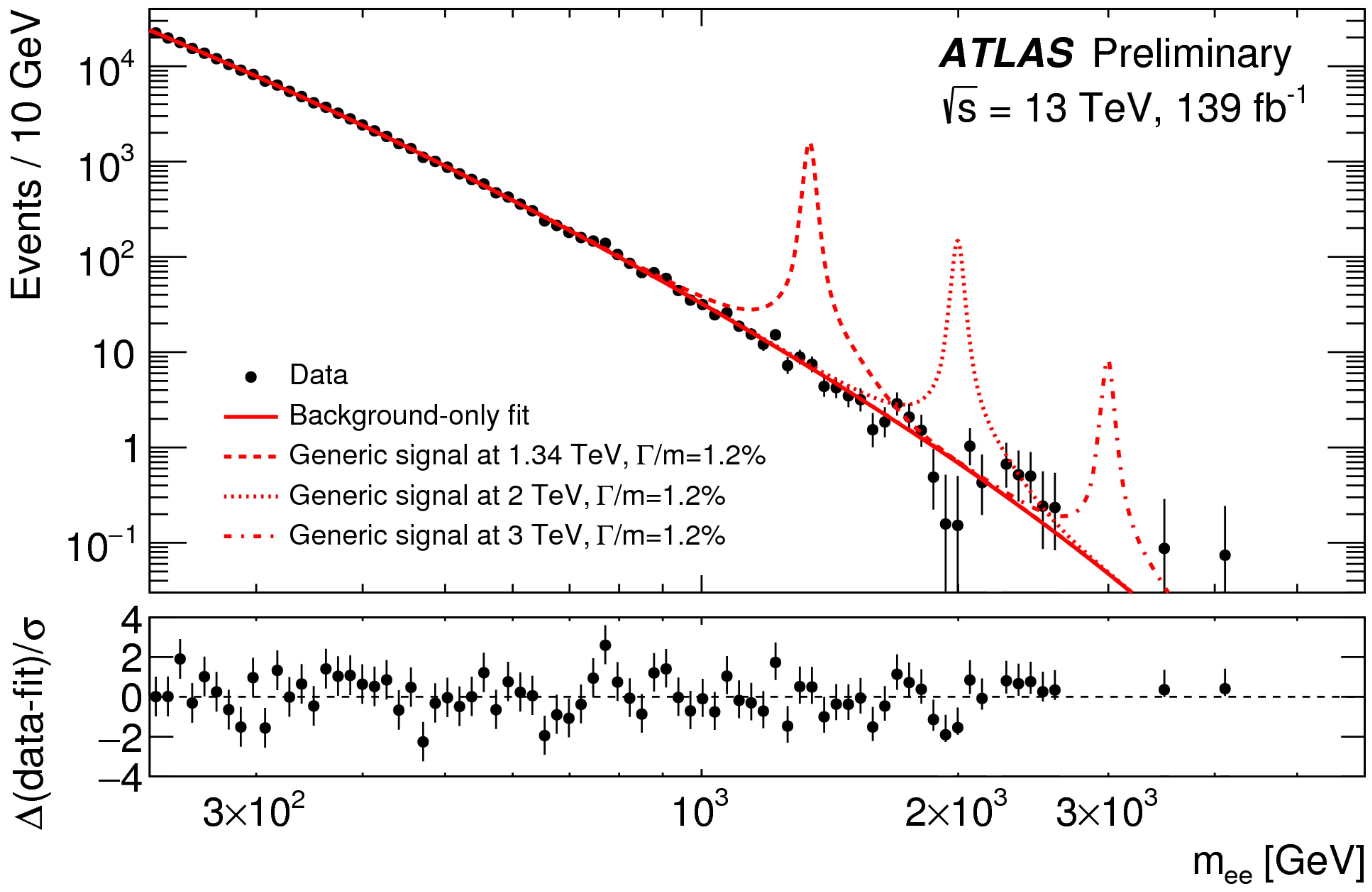
but

in several cases we need to push
the comparison
to an even higher level of precision



When Precision is a crucial tool

a new pronounced resonance would be a discovery

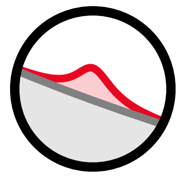


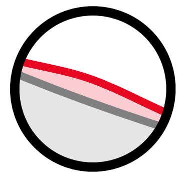
When Precision is a crucial tool

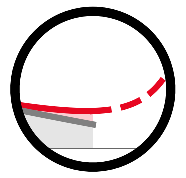
a new pronounced resonance would be a discovery

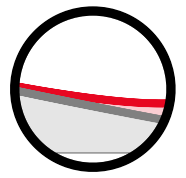
in the absence of a strong signal

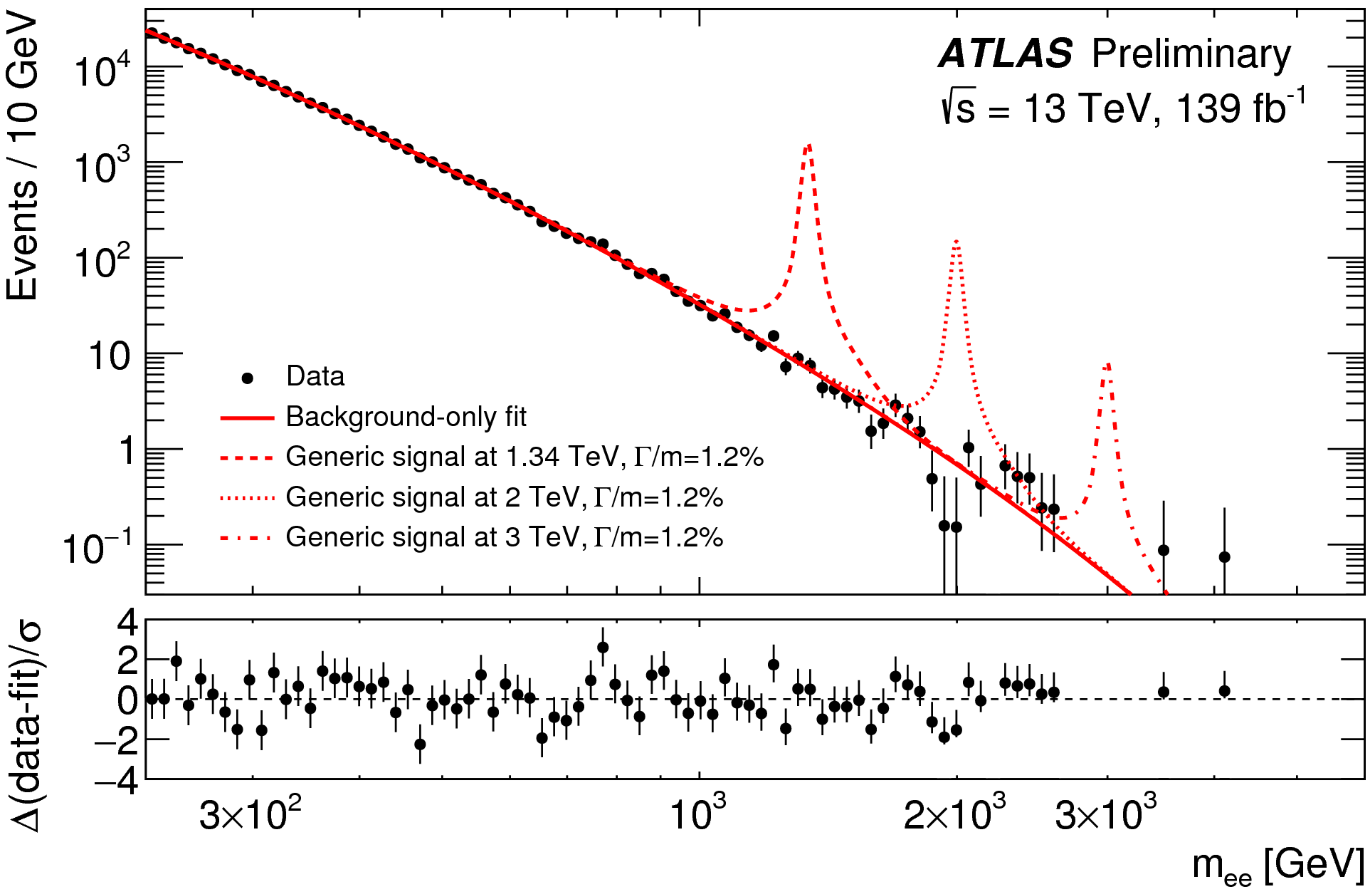
a discrepancy with the Standard Model (SM) predictions

 feeble interactions (low peak)

 broad resonance

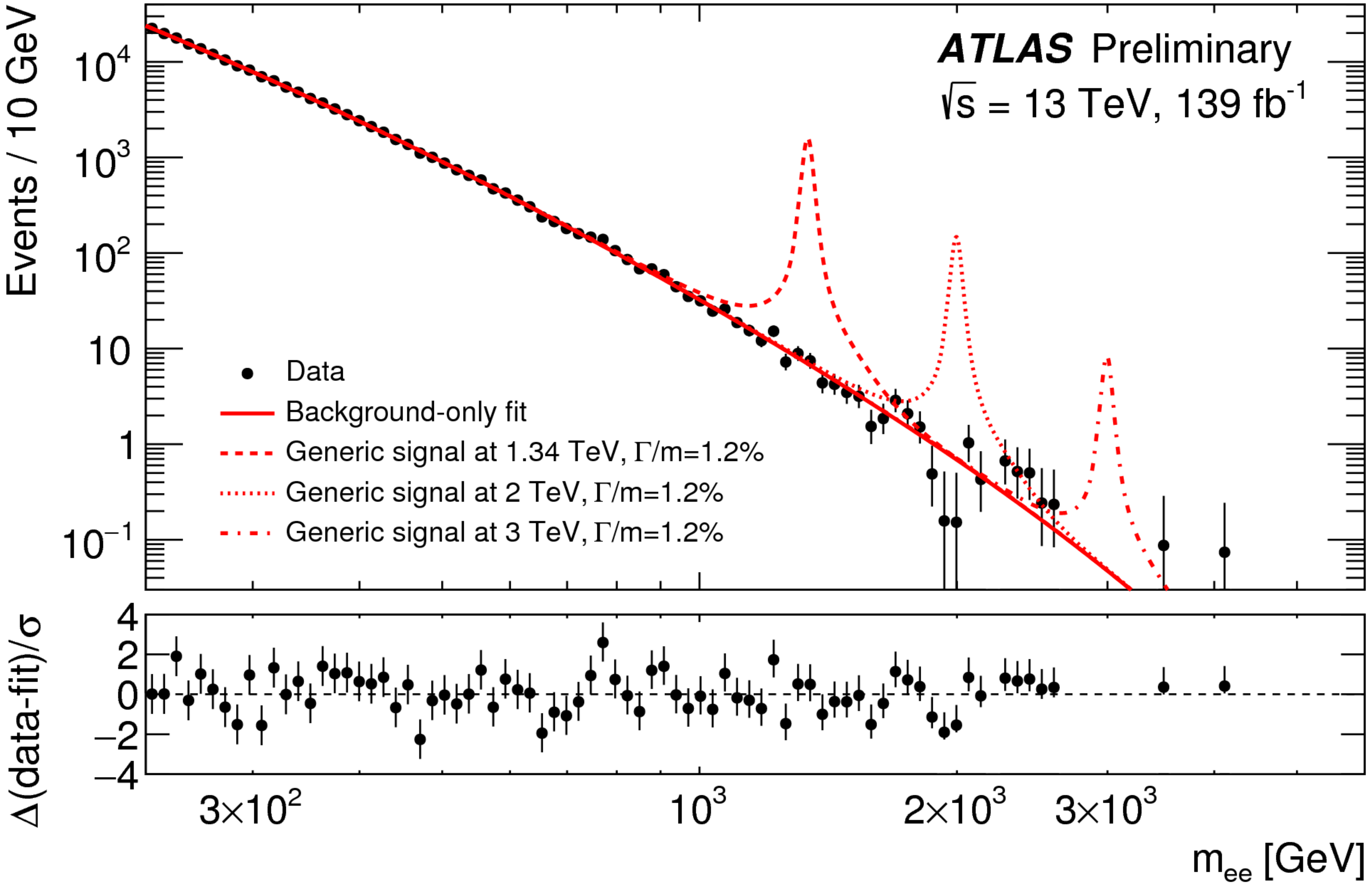
 a heavy new particle (out of reach)

 a generic deviation



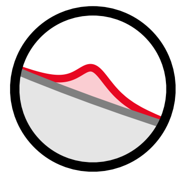
When Precision is a crucial tool

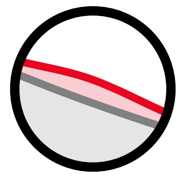
a new pronounced resonance would be a discovery

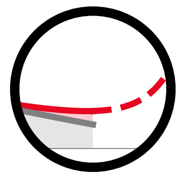


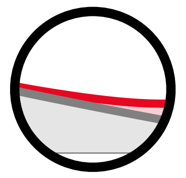
in the absence of a strong signal

a discrepancy with the Standard Model (SM) predictions

 feeble interactions (low peak)

 broad resonance

 a heavy new particle (out of reach)

 a generic deviation

1) What is the Standard Model (SM) ?

2) general gauge invariant parameterisation of New Physics

Standard Model Effective Field Theory (SMEFT):

$$\mathcal{L}_{\rm Eff} = \sum_{d=4}^{\infty} \frac{1}{\Lambda^{d-4}} \mathcal{L}_d = \mathcal{L}_{\rm SM} + \frac{1}{\Lambda}\mathcal{L}_5 + \frac{1}{\Lambda^2}\mathcal{L}_6 + \cdots$$

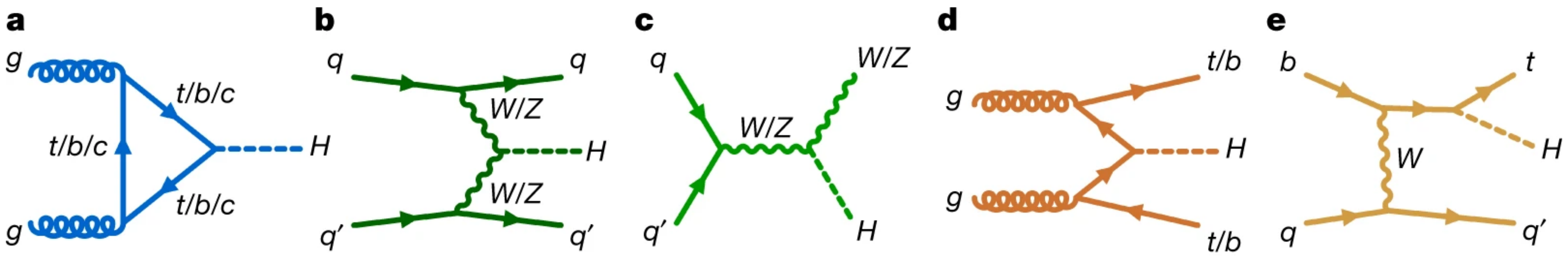
$$\mathcal{L}_d = \sum_i C_i^d \mathcal{O}_i \qquad [\mathcal{O}_i] = d \longrightarrow \left(\frac{q}{\Lambda}\right)^{d-4}$$

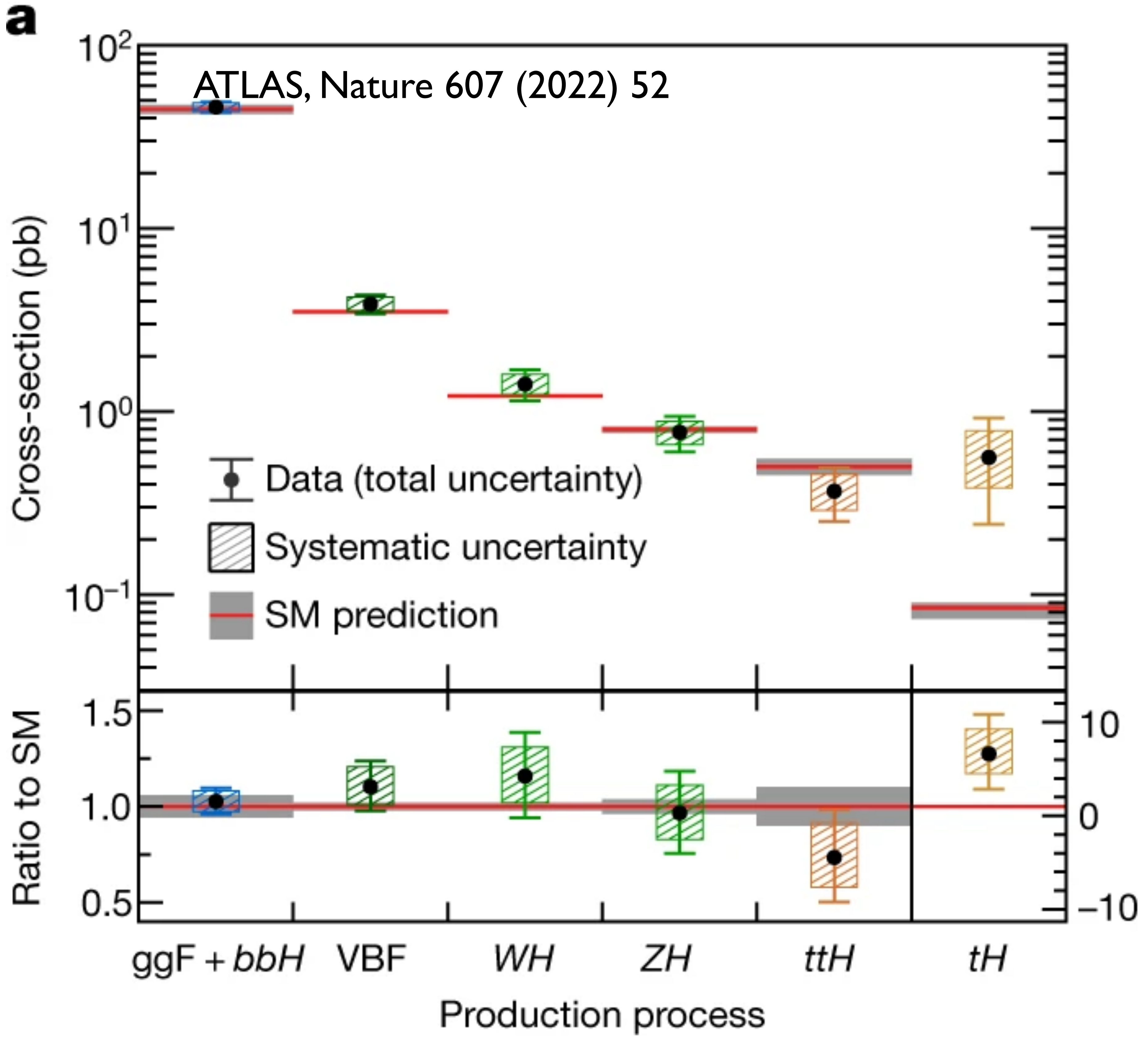
Effects suppressed by $q = v, E < \Lambda$

Λ: Cut-off of the EFT

When Precision is a crucial tool: deciphering the nature of the Higgs boson

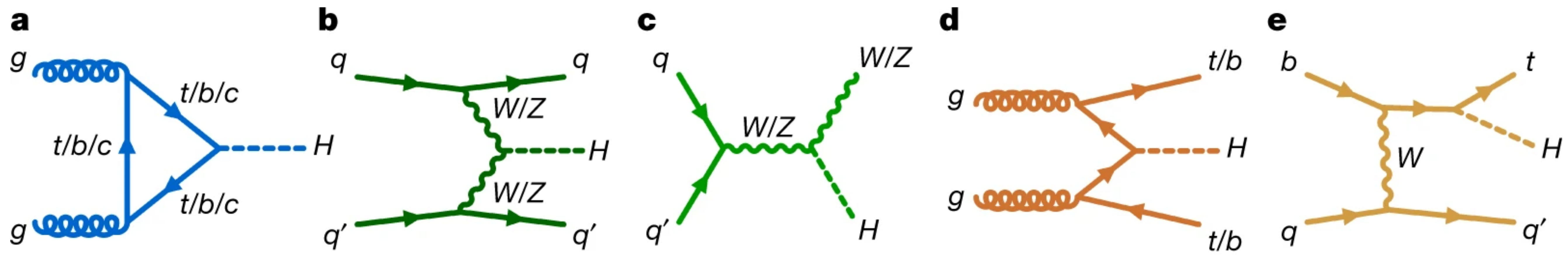
Is the observed scalar particle the Standard Model Higgs boson ?

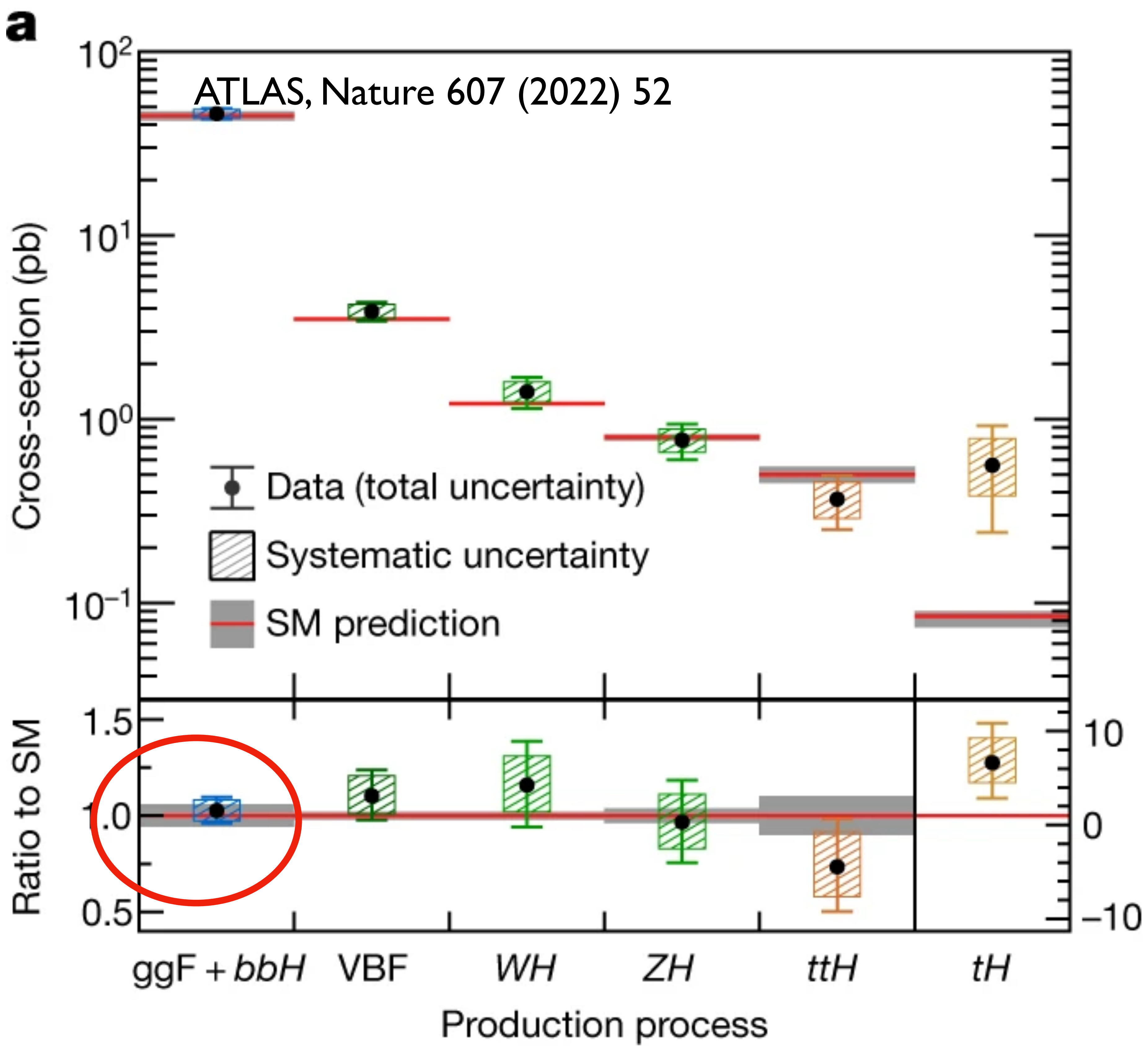




When Precision is a crucial tool: deciphering the nature of the Higgs boson

Is the observed scalar particle the Standard Model Higgs boson ?

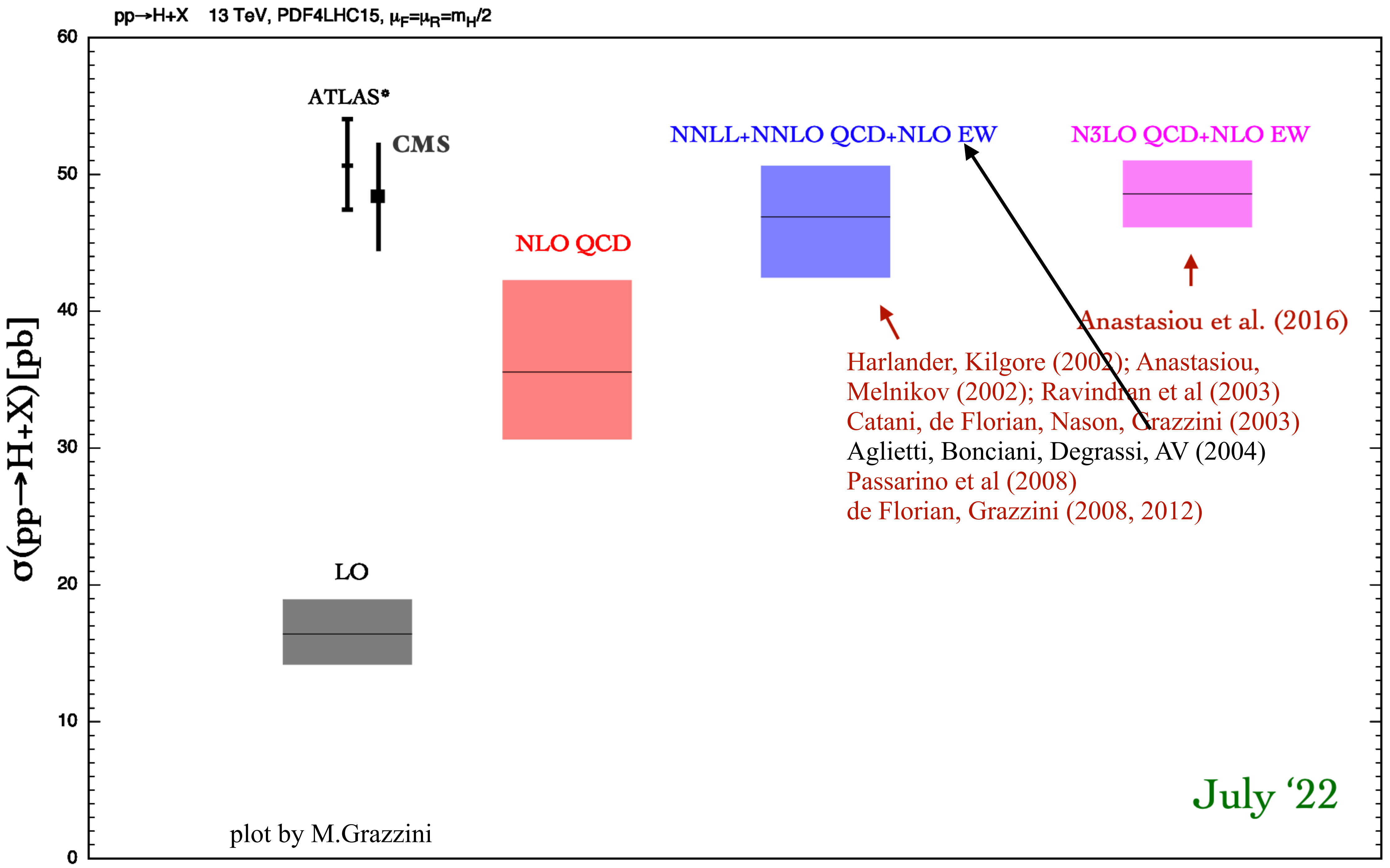




Quantum corrections up to third order needed

for a significant comparison

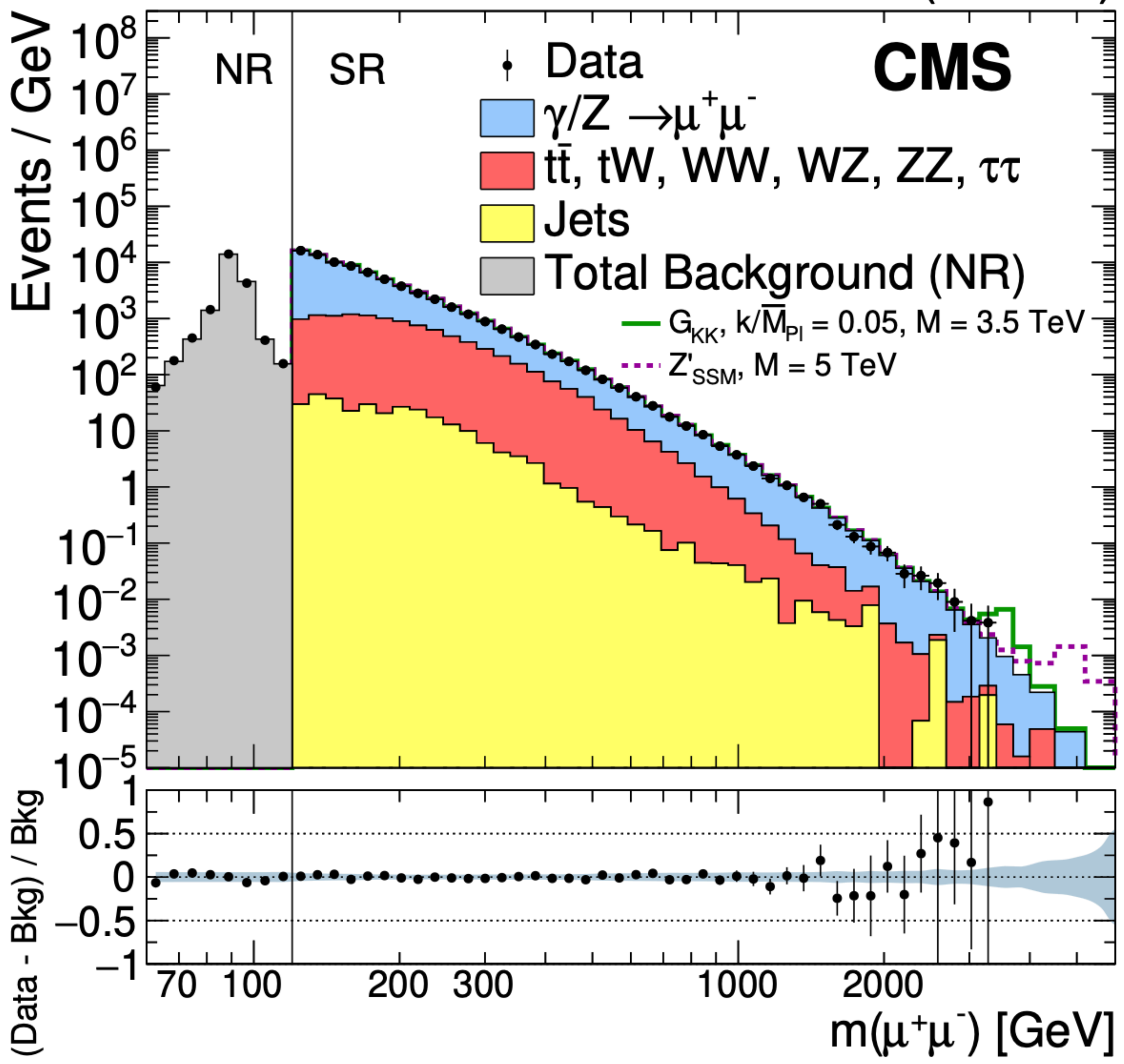
with the Higgs production cross sections



Motivations

- Global test of the data description offered by the SM → search for tensions
  - → possible explanation in terms of higher-dimension operators (effective description of New Physics)

arXiv:2103.02708



mass window [GeV]	stat. unc. 140fb^{-1}	stat. unc. 3ab^{-1}
$600<m_{\mu\mu}<900$	1.4%	0.2%
$900<m_{\mu\mu}<1300$	3.2%	0.6%

are our predictions / simulations adequate ? Can we identify BSM deviations in a significant way ?

Motivations

- Precision determination of SM parameters (often with precision better than 0.1%)

$$\delta m_W^{exp} \sim 16 \text{ MeV} \quad \text{i.e.} \quad 2 \cdot 10^{-4} \quad \text{precision}$$

$$\delta \sin^2 \theta_W^{exp} \sim 0.0003 \quad \text{i.e.} \quad 1.3 \cdot 10^{-3} \text{ precision}$$

How much do these results depend on the details of our simulations ?

- Electroweak Physics is a promising portal towards New Physics

Quantum corrections are not necessarily small

Outline of lecture 1

1) QED as a gauge theory. Conserved currents and SM gauge group choice. Prediction of the Z neutral current.
2) Issues with gauge invariance. The Higgs mechanism. Prediction of the existence of the Higgs boson.
3) The Higgs mechanism. Proportionality of the coupling of the Higgs boson to another field F with the mass of the field F.
4) Gauge boson polarisations. Tests of EW SSB. Cancellations avoiding unitarity violations
5) Renormalization. Basic concepts for mass and charge renormalisation.
6) Issues with the description of unstable particles. Complex mass scheme.
7) The couplings of the Z boson to fermions. Determination of the weak mixing angle(s)

Outline of lecture 2

1) Electroweak corrections classification
2) Mixed QCD-EW corrections at hadron colliders (partonic results, PDFs,)
3) W mass determination at hadron colliders
4) Pole expansion, on-shell boson production, off-shell effects
5) photons, leptons, isolation

SM formulation

Why gauge theories?

QED, QCD and the weak interactions are described by QFT invariant under a group of gauge transformations.
Why this request of invariance is so important

Why gauge theories?

QED, QCD and the weak interactions are described by QFT invariant under a group of gauge transformations.
Why this request of invariance is so important

Precision

Anomalous magnetic moment of the electron

$$(g-2)/2 = (1159.65218073 \pm 0.00000028) \times 10^{-6}$$

successfully tested against the prediction including fifth-order corrections in QED

Why gauge theories?

QED, QCD and the weak interactions are described by QFT invariant under a group of gauge transformations.
Why this request of invariance is so important

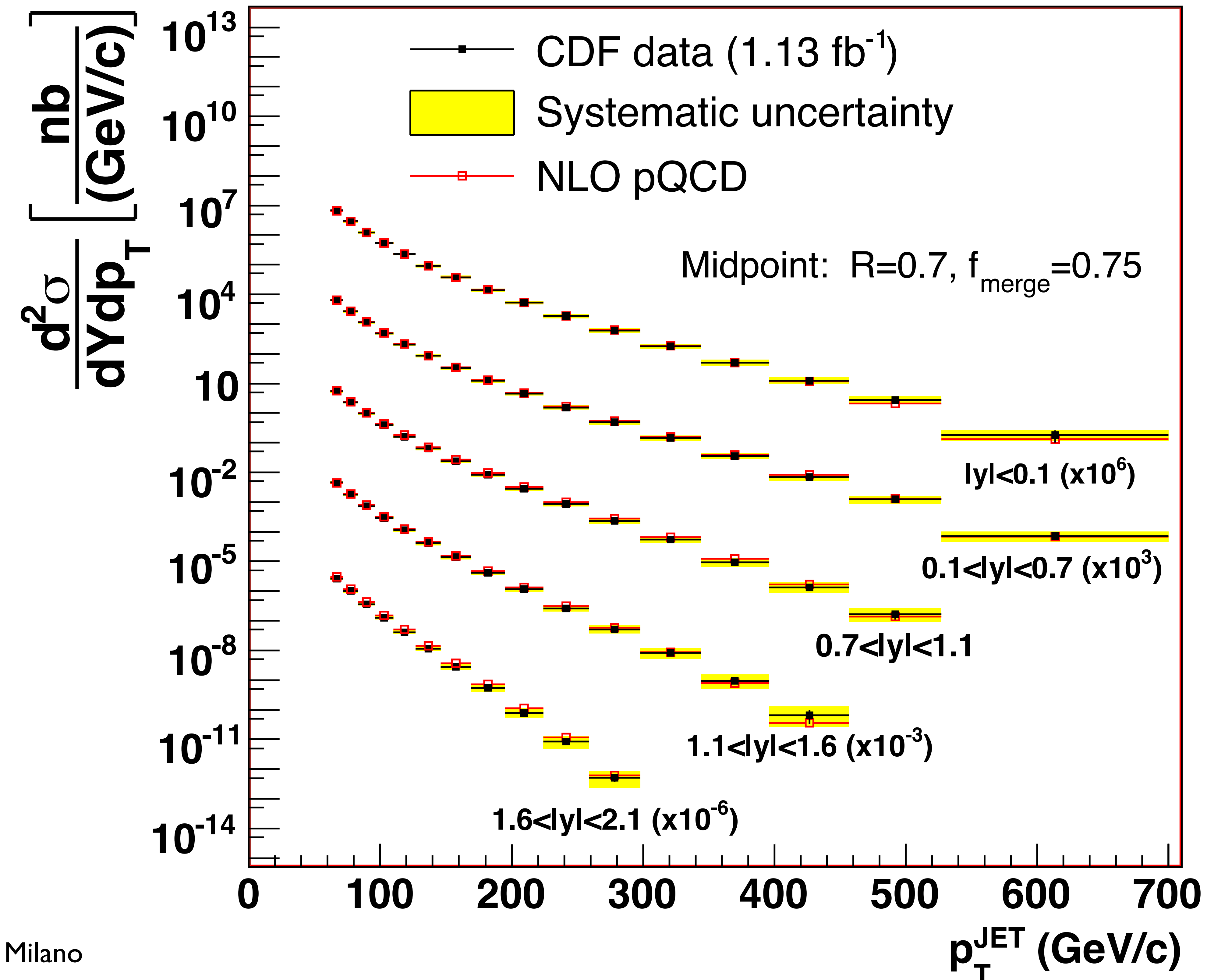
Precision

Anomalous magnetic moment of the electron

$$(g-2)/2 = (1159.65218073 \pm 0.00000028) \times 10^{-6}$$

successfully tested against the prediction including fifth-order corrections in QED

Accuracy



single jet production

QCD accurately describes the data at percent level

tested over more than 1 order of magnitude in energy
the rate varies by 7 orders of magnitude

The EW Standard Model: QED as a gauge theory

- In QED, the invariance of the fermionic lagrangian (electron field ψ) under a $U(1)$ group of global phase transformations implies (Noether's theorem) the existence of a conserved (electric) charge and of a conserved (fermion) vector current

$\mathscr{L}_{Dirac} = \overline{\psi}\left(i\gamma_\mu \partial^\mu - m\right)\psi$ is invariant when $\psi \to e^{ie\alpha}\,\psi$ with α a real constant ($U(1)$ global symmetry)

$J_\mu = \overline{\psi}\,\gamma_\mu\,\psi \qquad Q = \int d^3\vec{x}\,J_0(x)$ are the conserved e.m. current and electric charge

The EW Standard Model: QED as a gauge theory

- In QED, the invariance of the fermionic lagrangian (electron field ψ) under a $U(1)$ group of global phase transformations implies (Noether's theorem) the existence of a conserved (electric) charge and of a conserved (fermion) vector current

$\mathscr{L}_{Dirac} = \overline{\psi}\left(i\gamma_\mu \partial^\mu - m\right)\psi$ is invariant when $\psi \to e^{ie\alpha}\,\psi$ with α a real constant $(U(1)$ global symmetry)

$J_\mu = \overline{\psi}\,\gamma_\mu\,\psi \qquad Q = \int d^3\vec{x}\; J_0(x)$ are the conserved e.m. current and electric charge

- In QED, the request of invariance of the fermionic lagrangian (electron field ψ) under a group of local phase transformations can be satisfied by the introduction of a gauge field A_μ: $\qquad \psi \to e^{ie\alpha(x)}\,\psi, \quad A^\mu \to A^\mu \,+\, \partial^\mu\alpha(x)$

$\mathscr{L}_{QED} = \overline{\psi}\left[i\gamma_\mu(\partial^\mu - ieA^\mu) - m\right]\psi$ is invariant under the local trasnformation

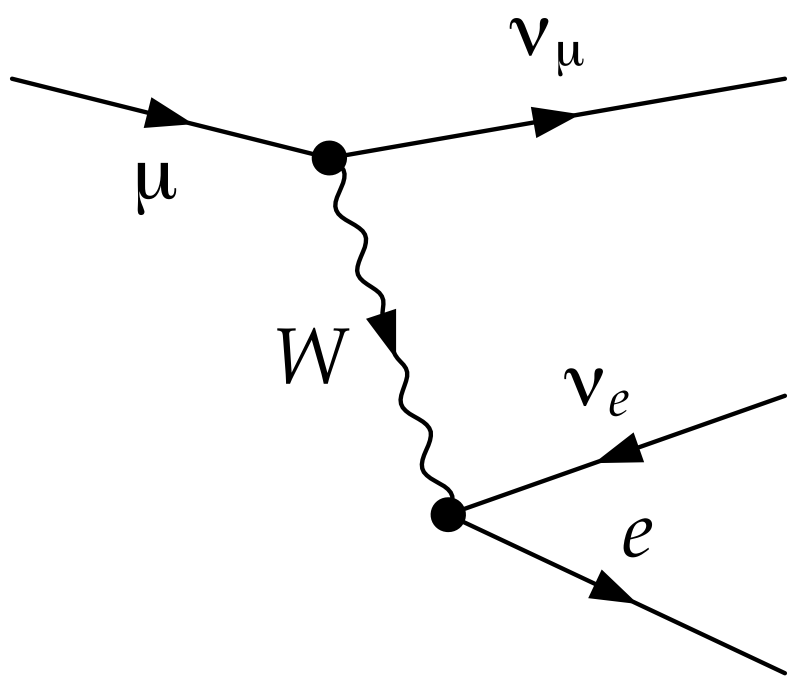
- The gauge field A_μ describes a massless spin-1 particle with two transverse degrees of freedom (the photon field) which interacts with the conserved fermionic current (e.m. interaction)

The EW Standard Model: the currents and the choice of the $SU(2)_L \times U(1)_Y$ gauge group

- The EW SM is built according to the same idea / pattern

Fermi has identified, in the study of neutron beta decay (or also muon decay) two fermionic electrically charged currents

$$\mathscr{L}_{Fermi} = \frac{G_\mu}{\sqrt{2}} J^\mu_{ud,\,L}\, J^{\ell\nu,\,L}_\mu \qquad \text{with} \qquad J^\mu_{ud,\,L} = \bar{u}\,\gamma^\mu \frac{1-\gamma_5}{2} d \quad \text{and } J^{\ell\nu,\,L}_\mu = \bar{\ell}\,\gamma_\mu \frac{1-\gamma_5}{2}\nu$$



We ask these currents to be the conserved currents of a global symmetry.

- The $SU(2)$ group has three generators $\tau_j = \sigma_j/2$ → two of its three conserved currents will be the Fermi currents. The Fermi currents are purely left-handed (parity violation in neutron beta decay) → $SU(2)_L$

$$\Psi_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \qquad \mathscr{L}_{doublet} = \overline{\Psi}_L \left(i\gamma_\mu \partial^\mu \right) \Psi_L \quad \text{is invariant when} \quad \Psi_L \to e^{i\frac{g}{2}\sigma_j\alpha_j}\,\Psi_L \ \text{ with } \alpha_j \text{ real}$$

- the third (neutral) conserved current can not be the e.m. current, because it is purely left-handed

The EW Standard Model: the currents and the choice of the $SU(2)_L \times U(1)_Y$ gauge group

- Enlarge the transformation group to $SU(2)_L \times U(1)_Y$ → 4 generators: 2 charged and 2 neutral → 4 conserved currents

- We promote the global $SU(2)_L \times U(1)_Y$ group to a group of local (gauge) phase transformations
  and then we introduce 4 gauge fields (W_1, W_2, W_3, B) which mediate the EW interactions

$$\mathscr{L}_{NC}^{int} = \overline{\Psi}\, \gamma_\mu \left[g\, \tau_3 W_3^\mu \;+\; g' \frac{Y}{2} B^\mu \right] \Psi$$

We consider a linear combination of the 2 neutral generators

$$B^\mu = A^\mu \cos\theta_W - Z^\mu \sin\theta_W$$
$$W_3^\mu = A^\mu \sin\theta_W + Z^\mu \cos\theta_W$$

obtaining

$$\mathscr{L}_{NC}^{int} = \overline{\Psi}\, \gamma_\mu \left(g\, \sin\theta_W\, \tau_3 \;+\; g' \cos\theta_W \frac{Y}{2} \right) \Psi\, A^\mu \;+\; \overline{\Psi}\, \gamma_\mu \left(g\, \cos\theta_W\, \tau_3 \;-\; g' \sin\theta_W \frac{Y}{2} \right) \Psi\, Z^\mu$$

The EW Standard Model: the currents and the choice of the $SU(2)_L \times U(1)_Y$ gauge group

- Enlarge the transformation group to $SU(2)_L \times U(1)_Y$ → 4 generators: 2 charged and 2 neutral → 4 conserved currents

- We promote the global $SU(2)_L \times U(1)_Y$ group to a group of local (gauge) phase transformations
  and then we introduce 4 gauge fields (W_1, W_2, W_3, B) which mediate the EW interactions

$$\mathscr{L}_{NC}^{int} = \overline{\Psi}\,\gamma_\mu \left[g\,\tau_3 W_3^\mu \;+\; g' \frac{Y}{2} B^\mu \right] \Psi$$

We consider a linear combination of the 2 neutral generators

$$B^\mu = A^\mu \cos\theta_W - Z^\mu \sin\theta_W$$
$$W_3^\mu = A^\mu \sin\theta_W + Z^\mu \cos\theta_W$$

obtaining

$$\mathscr{L}_{NC}^{int} = \overline{\Psi}\,\gamma_\mu \left(g\,\sin\theta_W\,\tau_3 \;+\; g'\,\cos\theta_W \frac{Y}{2} \right) \Psi\, A^\mu \;+\; \overline{\Psi}\,\gamma_\mu \left(g\,\cos\theta_W\,\tau_3 \;-\; g'\,\sin\theta_W \frac{Y}{2} \right) \Psi\, Z^\mu$$

- We require that the first combination is the electromagnetic current (assignment of the Y value for each L and R field)

The second neutral current is a prediction of this construction. It has been discovered by Gargamelle in 1973

The EW Standard Model: the currents and the choice of the $SU(2)_L \times U(1)_Y$ gauge group

- Enlarge the transformation group to $SU(2)_L \times U(1)_Y$ → 4 generators: 2 charged and 2 neutral → 4 conserved currents

- We promote the global $SU(2)_L \times U(1)_Y$ group to a group of local (gauge) phase transformations
  and then we introduce 4 gauge fields (W_1, W_2, W_3, B) which mediate the EW interactions

$$\mathscr{L}_{NC}^{int} = \overline{\Psi}\,\gamma_\mu \left[g\,\tau_3 W_3^\mu \;+\; g' \frac{Y}{2} B^\mu \right] \Psi$$

We consider a linear combination of the 2 neutral generators

$$B^\mu = A^\mu \cos\theta_W - Z^\mu \sin\theta_W$$
$$W_3^\mu = A^\mu \sin\theta_W + Z^\mu \cos\theta_W$$

obtaining

$$\mathscr{L}_{NC}^{int} = \overline{\Psi}\,\gamma_\mu \left(g\,\sin\theta_W\,\tau_3 \;+\; g'\,\cos\theta_W \frac{Y}{2} \right) \Psi\, A^\mu \;+\; \overline{\Psi}\,\gamma_\mu \left(g\,\cos\theta_W\,\tau_3 \;-\; g'\,\sin\theta_W \frac{Y}{2} \right) \Psi\, Z^\mu$$

- We require that the first combination is the electromagnetic current (assignment of the Y value for each L and R field)

The second neutral current is a prediction of this construction. It has been discovered by Gargamelle in 1973

The weak mixing angle

In the construction of the SM,
identification of the electromagnetic current and electric charge $e = g \sin\theta_W$
→ prediction of the second neutral current, coupling the Z boson to fermions

$$Zf\bar{f} \propto i \frac{g}{\cos\theta_W} \gamma^\mu \left(T_3 \frac{1-\gamma_5}{2} - \sin^2\theta_W Q_f \right) \qquad \sin^2\theta_W = \frac{(g')^2}{g^2 + (g')^2}$$

It is interesting to test both: the strength of the neutral current interaction
the mixing of the $SU(2)_L$ and $U(1)_Y$ gauge groups

Issues with gauge invariance

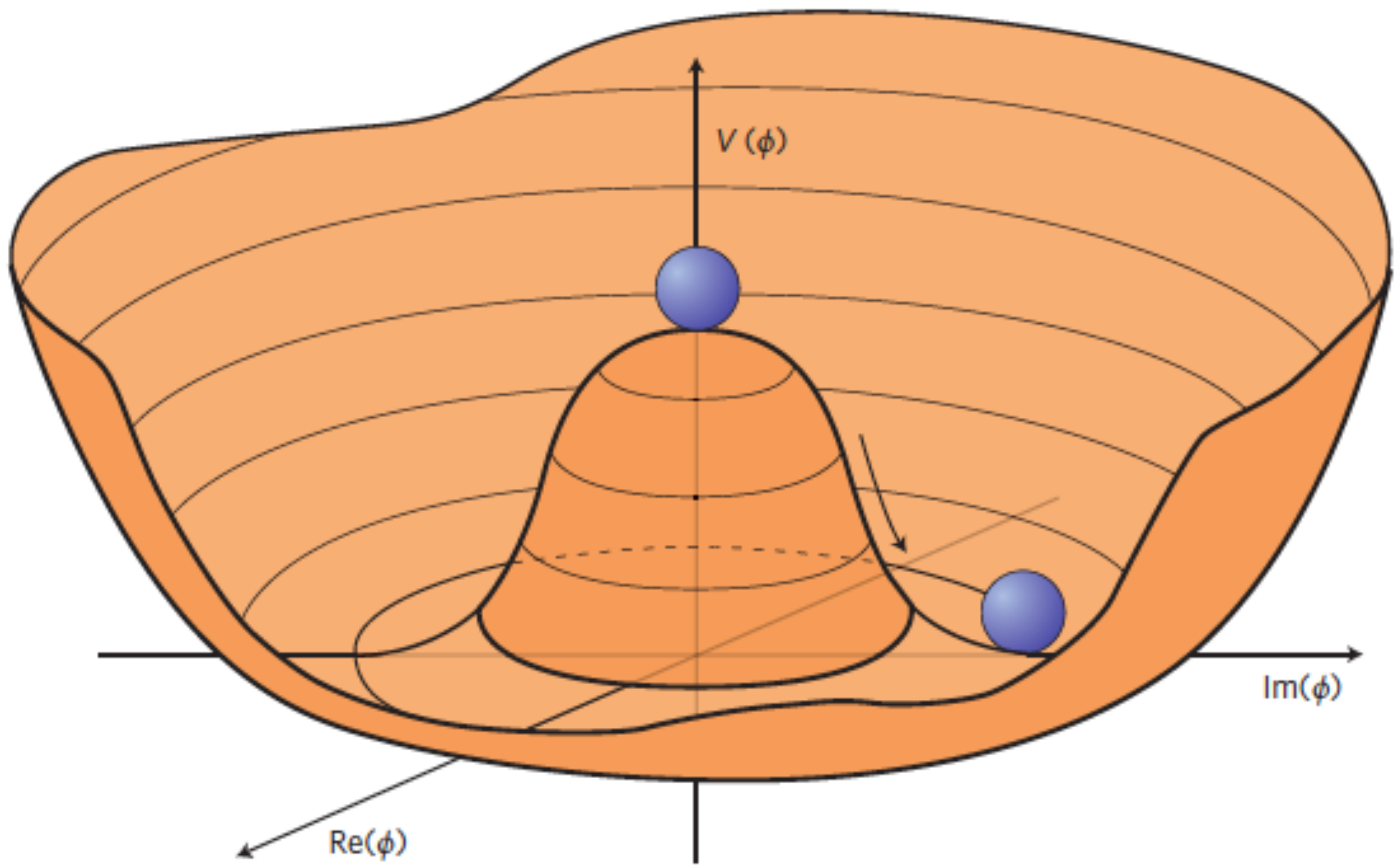
1) The gauge fields are by construction massless, but we know that $W^{\pm}$ are very heavy (80 times the proton mass)…
A massive spin-1 particle has 3 polarizations , while the gauge lagrangian describes only 2 polarizations per boson

An explicit mass term in the lagrangian breaks gauge invariance.

2) in the QED Dirac lagrangian, in the mass term, $\mathscr{L}_{mass} = -m\left(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L\right)$
but now ψ_L and ψ_R have different transformation properties under $SU(2)_L \times U(1)_Y$ breaking gauge invariance

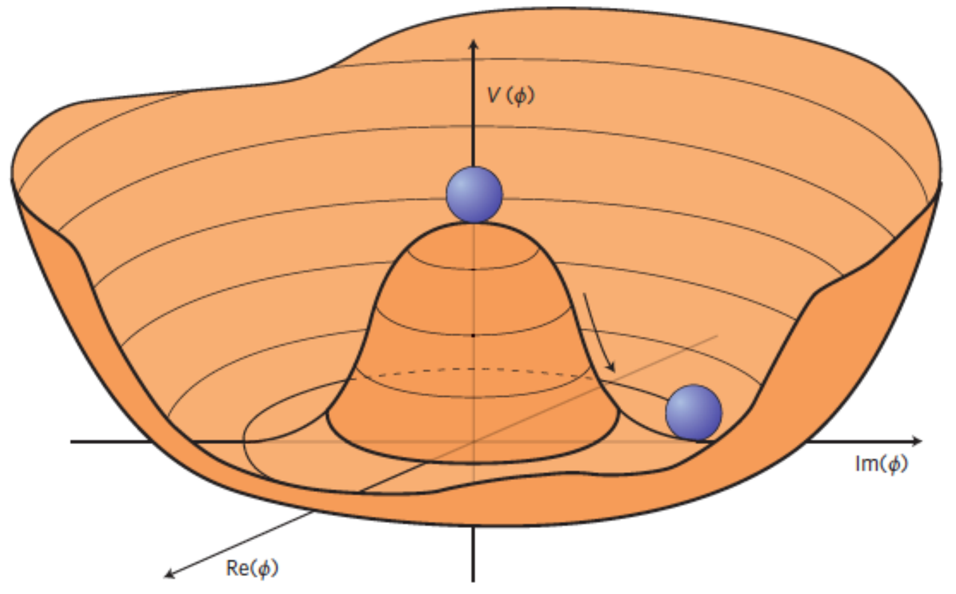
The Higgs mechanism

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ v + \sigma + i\chi \end{pmatrix} \qquad \langle 0 | \Phi | 0 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$



Issues with gauge invariance: the Higgs mechanism

Given $\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ v + \sigma + i\chi \end{pmatrix}$ and $D^\mu = \partial^\mu - i\frac{g}{2}\sigma_j W_j^\mu - i\frac{g'}{2} Y B^\mu$



$$\mathscr{L}_{higgs} = \left(D_\mu \Phi\right)^\dagger (D^\mu \Phi) + \mu^2 \left(\Phi^\dagger \Phi\right) - \lambda \left(\Phi^\dagger \Phi\right)^2$$

3 real fields (ϕ_1, ϕ_2, χ) describe the longitudinal pol. of $W^\pm$, Z,

1 real field σ is a neutral singlet → the Higgs boson

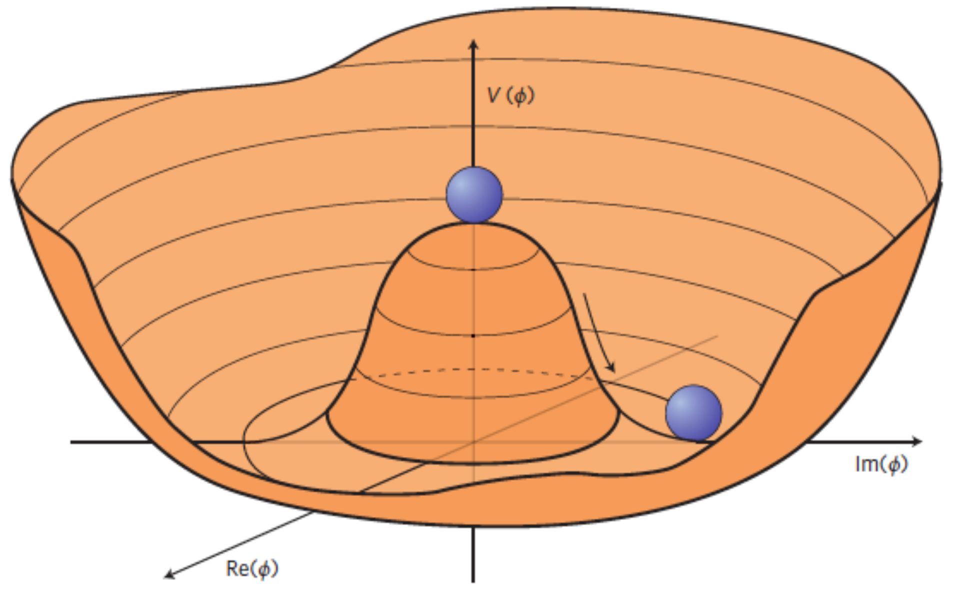
$$\mathscr{L}_{fm} = -d_{ij}\Phi^\dagger \Psi_{iL}\, d_{jR} - u_{ij}\tilde{\Phi}^\dagger \Psi_{iL}\, u_{jR} + h.c.$$

d_{ij} and u_{ij} are Yukawa matrices
with $i, j = 1,2,3$ over the fermion families
$\tilde{\Phi} = (\sigma_2 \Phi^*)^T$ has hypercharge opposite to Φ

$$\mathscr{L} = \mathscr{L}_{gauge} + \mathscr{L}_{higgs} + \mathscr{L}_{fm}$$

All these lagrangian terms are gauge invariant
After the spontaneous breaking of the EW symmetry, $v \neq 0$, we generate mass terms for fermions and gauge bosons

Issues with gauge invariance: the Higgs mechanism



Given $\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ v + \sigma + i\chi \end{pmatrix}$ and $D^\mu = \partial^\mu - i\frac{g}{2}\sigma_j W_j^\mu - i\frac{g'}{2} Y B^\mu$

$$\mathscr{L}_{higgs} = \left(D_\mu \Phi\right)^\dagger (D^\mu \Phi) + \mu^2 \left(\Phi^\dagger \Phi\right) - \lambda \left(\Phi^\dagger \Phi\right)^2$$

3 real fields (ϕ_1, ϕ_2, χ) describe the longitudinal pol. of $W^\pm$, Z,

1 real field σ is a neutral singlet → the Higgs boson

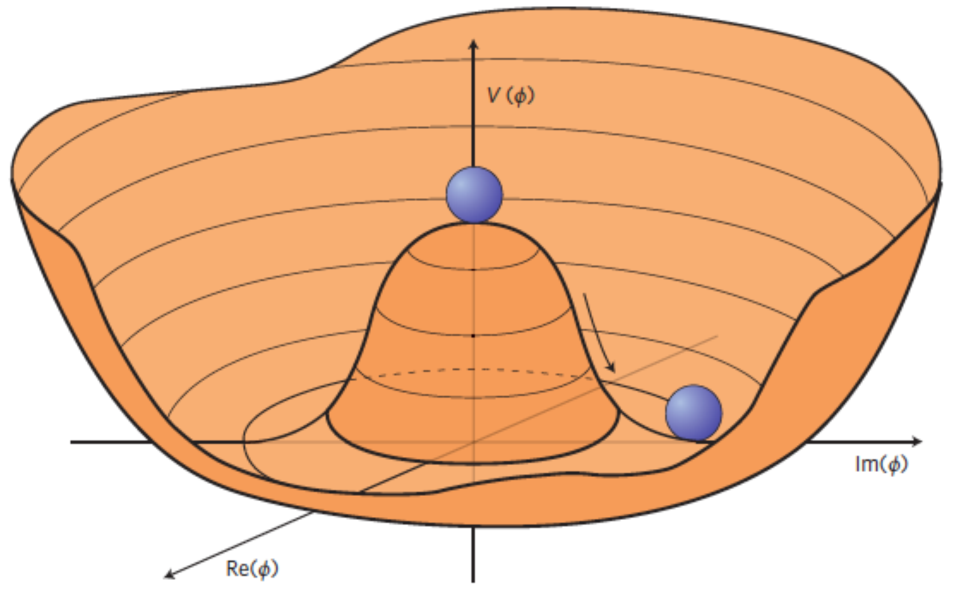
$$\mathscr{L}_{fm} = -\, d_{ij} \Phi^\dagger \Psi_{iL}\, d_{jR} - u_{ij} \tilde{\Phi}^\dagger \Psi_{iL}\, u_{jR} + h.c.$$

d_{ij} and u_{ij} are Yukawa matrices
with $i, j = 1,2,3$ over the fermion families
$\tilde{\Phi} = (\sigma_2 \Phi^*)^T$ has hypercharge opposite to Φ

$$\mathscr{L} = \mathscr{L}_{gauge} + \mathscr{L}_{higgs} + \mathscr{L}_{fm}$$

All these lagrangian terms are gauge invariant
After the spontaneous breaking of the EW symmetry, $v \neq 0$, we generate mass terms for fermions and gauge bosons

Issues with gauge invariance: the Higgs mechanism



Given $\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ v + \sigma + i\chi \end{pmatrix}$ and $D^\mu = \partial^\mu - i\frac{g}{2}\sigma_j W_j^\mu - i\frac{g'}{2} Y B^\mu$

$$\mathscr{L}_{higgs} = \left(D_\mu \Phi\right)^\dagger (D^\mu \Phi) - \mu^2 \left(\Phi^\dagger \Phi\right) - \lambda \left(\Phi^\dagger \Phi\right)^2$$

3 real fields (ϕ_1, ϕ_2, χ) describe the longitudinal pol. of $W^\pm$, Z,

1 real field σ is a neutral singlet → the Higgs boson

$$\mathscr{L}_{fm} = - d_{ij} \Phi^\dagger \Psi_{iL}\, d_{jR} - u_{ij} \tilde{\Phi}^\dagger \Psi_{iL}\, u_{jR} + h.c.$$

d_{ij} and u_{ij} are Yukawa matrices

with $i, j = 1,2,3$ over the fermion families

$\tilde{\Phi} = (\sigma_2 \Phi^*)^T$ has hypercharge opposite to Φ

$$\mathscr{L} = \mathscr{L}_{gauge} + \mathscr{L}_{higgs} + \mathscr{L}_{fm}$$

All these lagrangian terms are gauge invariant

After the spontaneous breaking of the EW symmetry, $v \neq 0$, we generate mass terms for fermions and gauge bosons

The EW Standard Model: masses and couplings

- Assuming that the vacuum expectation value (VEV) of the complex Higgs doublet is not vanishing we observe that whenever the Higgs doublet couples to other fields, we obtain two contributions:
  - one mass term for the other field
  - one interaction term between the Higgs and the other field, proportional to the mass of the latter

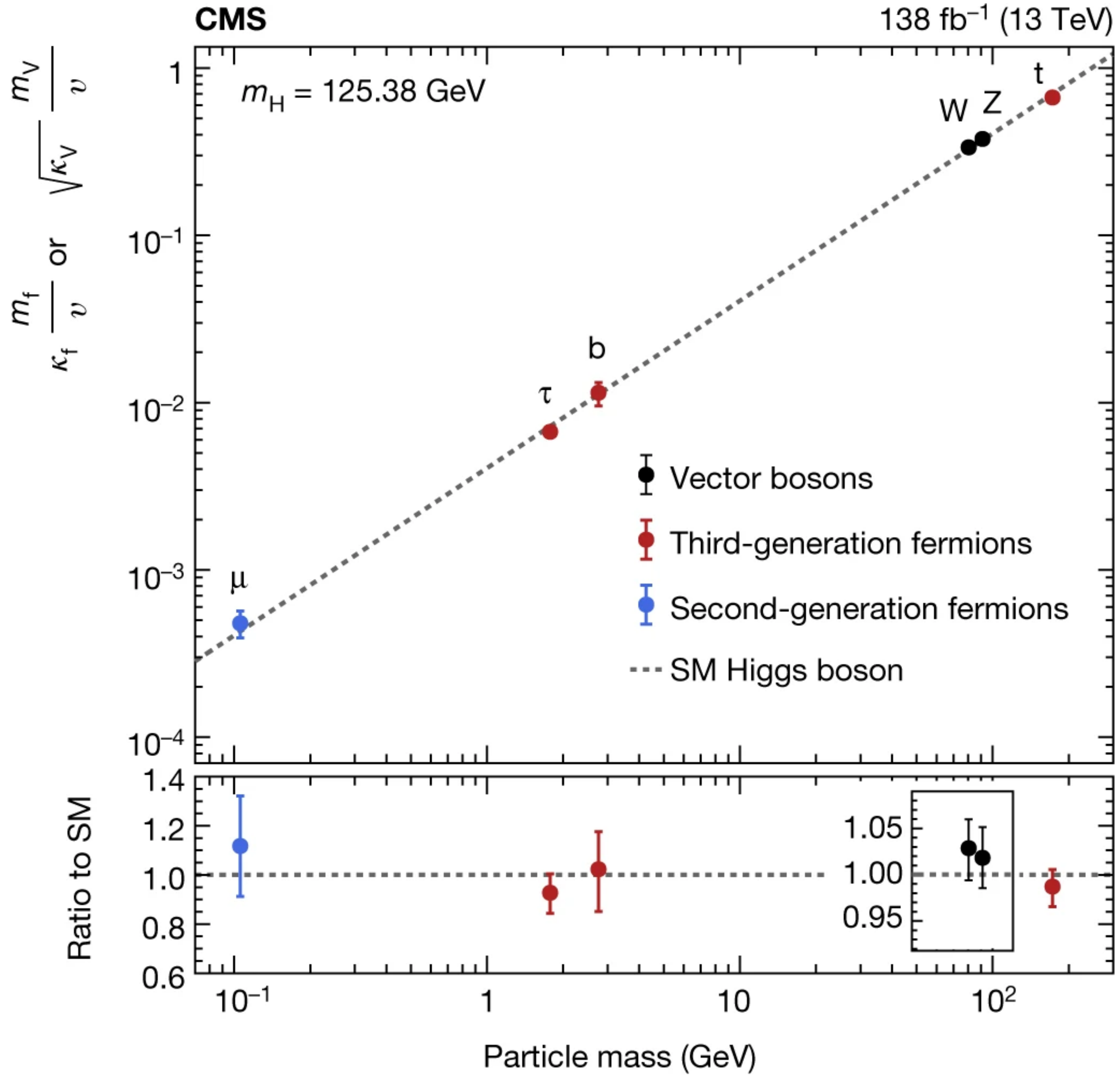
abelian example

$$\left(\partial_\mu + ieA_\mu\right) \frac{1}{\sqrt{2}}(v+\sigma-i\chi) \; (\partial^\mu - ieA^\mu) \frac{1}{\sqrt{2}}(v+\sigma+i\chi)$$

$$\left[\cdots + e^2 v^2 A_\mu A^\mu + \quad \cdots \quad + e^2 v\,\sigma A_\mu A^\mu + \cdots\right] \rightarrow \left[\cdots + M^2 A_\mu A^\mu + \quad \cdots \quad + e\,M\,\sigma A_\mu A^\mu + \cdots\right]$$

The EW Standard Model: masses and couplings

- the interaction strength between the Higgs and any other field is proportional to the mass of the latter



The EW Standard Model: weak gauge boson polarisations

A massive gauge boson has 3 polarisation states. In the laboratory frame they read

$$\varepsilon^{\mu}_{T1} = (0,1,0,0), \quad \varepsilon^{\mu}_{T2} = (0,0,1,0), \quad \varepsilon^{\mu}_{L} = \left(\frac{|\vec{k}|}{m},0,0,\frac{E_k}{m}\right)$$

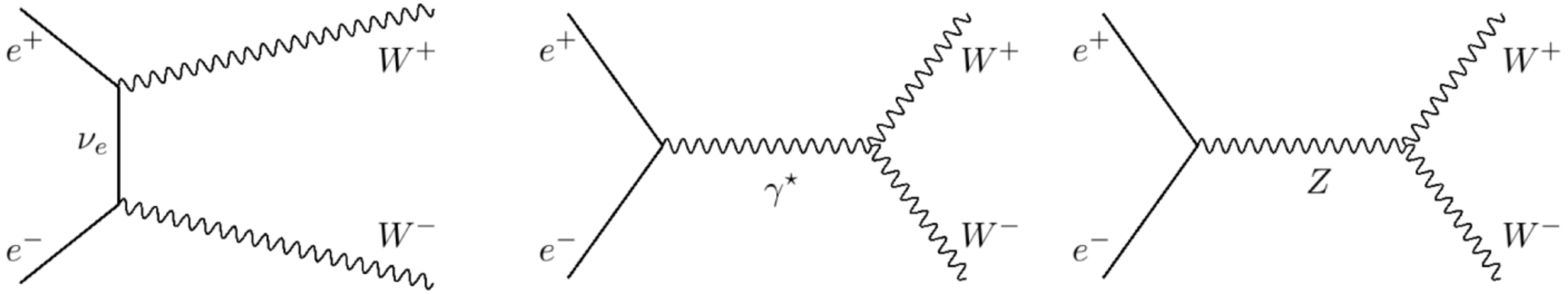
The ε^{μ}_{L} vector describes the longitudinal polarisation. In the high-energy limit it grows like $\sqrt{s}$, i.e. $\varepsilon^{\mu}_{L} \sim \frac{k^{\mu}}{m}$

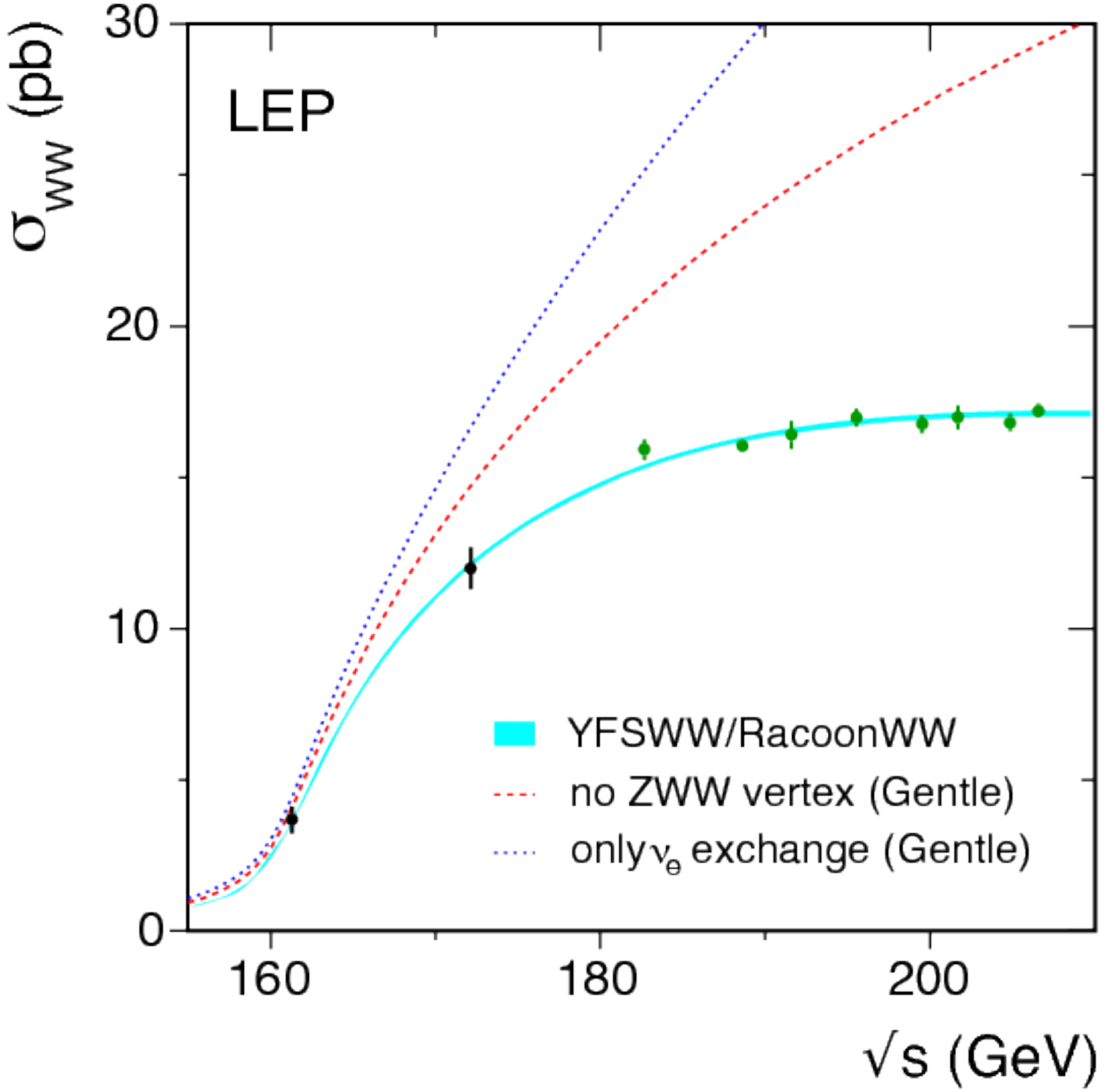
1) In the production of longitudinal gauge boson pairs, individual Feynman diagrams may diverge like s, violating the unitarity of the S matrix.
The gauge invariance constraints lead to specific cancellations in the full amplitude, which remove the "dangerous" terms

2) In $|\mathscr{M}|^2 = |\sum_i \mathscr{M}_i|^2$, only the combination of all the interference terms between all the Feynman diagrams (not only the squared of the individual diagrams) yields the correct physical prediction

The EW Standard Model: weak gauge boson polarisations

Illustration of "problem 2)"





The EW Standard Model: weak gauge boson polarisations

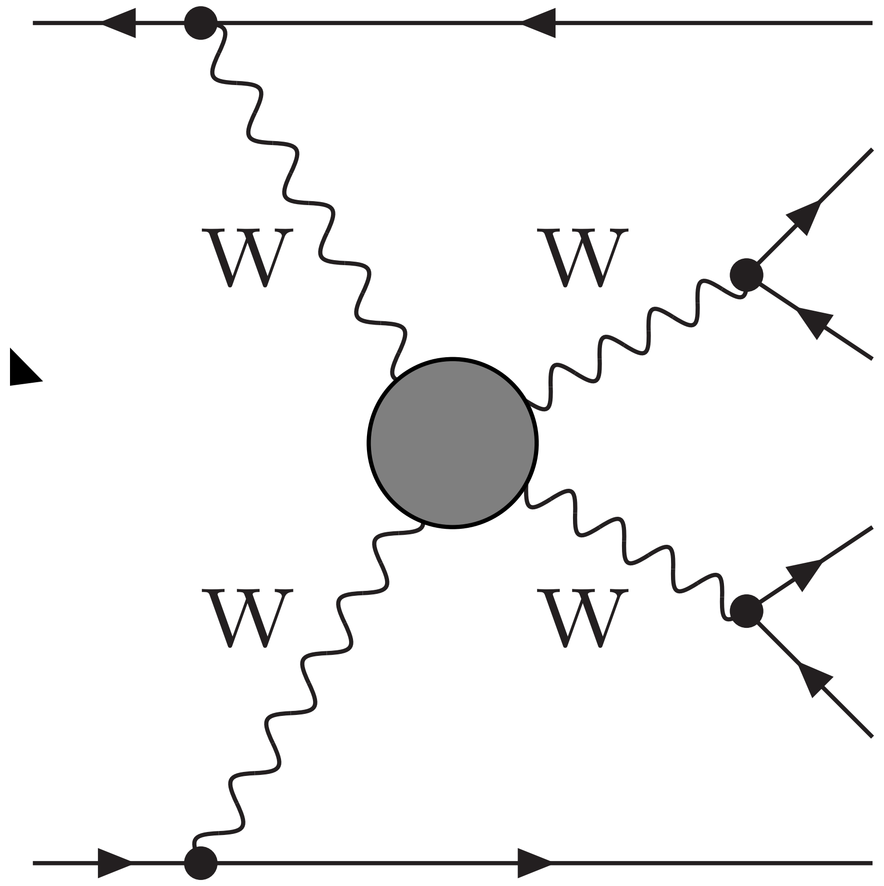
$$\mathscr{L}_{SM} = \mathscr{L}_{Dirac-gauge} + \mathscr{L}_{YM} + \mathscr{L}_{Higgs} + \mathscr{L}_{fm}$$

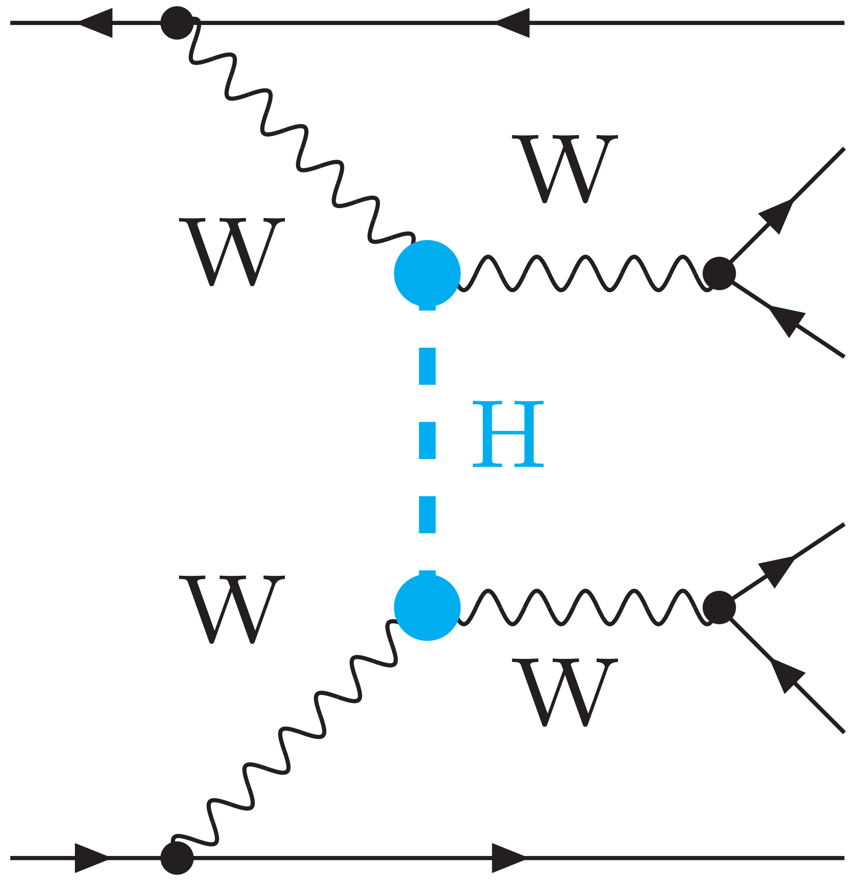
- The gauge-invariance request put strong constraints on the terms allowed in the lagrangian and their coefficients
  → unitarity cancellations

  The presence of BSM physics could modify the cancellation mechanism
  leading to e.g. a "delayed" unitarity, compared to the SM prediction

- After spontaneous breaking of the EW gauge symmetry (SSB), also longitudinal gauge boson may interact

  A careful study of Higgs and vector-boson scattering may reveal details of the EW SSB





Only the inclusion of all possible diagrams guarantees the unitarity cancellations

The EW Standard Model: the flavor sector, minimal *vs* non-minimal formulations

- the Higgs mechanism yields masses for the fermions without explicit breaking of gauge invariance

$$\mathscr{L}_{fm} = -d_{ij}\Phi^\dagger\Psi_{iL}\, d_{jR} - u_{ij}\tilde{\Phi}^\dagger\Psi_{iL}\, u_{jR} + h.c.$$

- since up-type and down-type fermions carry different hypercharge values,
  the gauge invariance of the fermion mass sector is achieved by using
  the Higgs doublet for the down-type fermions and the charged conjugate Higgs doublet $\tilde{\Phi}$ for the up-type quarks
  → minimal solution

- In full generality it is possible to assume that two distinct scalar doublets, with opposite hypercharge values, are present

$$\Phi \to \Phi_d \qquad \tilde{\Phi} \to \Phi_u \qquad\qquad Y(\Phi_d) = +1 \qquad Y(\Phi_u) = -1$$

→ two Higgs doublet models (2HDM) are a large class of models (including SUSY models)
predicting the existence of 4 additional fields (2 charged scalars, one neutral scalar, one neutral pseudoscalar)

The EW Standard Model: the custodial symmetry

- In addition to the gauge symmetry, explicitly enforced, we have accidental global symmetries of the lagrangian

$$\mathscr{L}_{scalar} = \left(\partial_\mu \Phi\right)^\dagger (\partial^\mu \Phi) + \mu^2 \left(\Phi^\dagger \Phi\right) - \lambda \left(\Phi^\dagger \Phi\right)^2$$

The scalar sector (before gauging: ∂^μ instead of D^μ) is invariant under a $SU(2)_L \times SU(2)_R$ global symmetry

After EW SSB, a residual global $SU(2)_C$ symmetry, called custodial, is still present in the scalar sector

- At tree level $\rho = \dfrac{m_W^2}{m_Z^2 \cos^2\theta_W} = 1$ is a consequence of $SU(2)_C$

Including the radiative corrections,

hypercharge effects and fermion mass splittings break $SU(2)_C$

$$\rho = 1 + \Delta\rho$$

The EW Standard Model: the custodial symmetry

- In addition to the gauge symmetry, explicitly enforced, we have accidental global symmetries of the lagrangian

$$\mathscr{L}_{scalar} = \left(\partial_\mu \Phi\right)^\dagger (\partial^\mu \Phi) + \mu^2 \left(\Phi^\dagger \Phi\right) - \lambda \left(\Phi^\dagger \Phi\right)^2$$

The scalar sector (before gauging: ∂^μ instead of D^μ) is invariant under a $SU(2)_L \times SU(2)_R$ global symmetry
After EW SSB, a residual global $SU(2)_C$ symmetry, called custodial, is still present in the scalar sector

- At tree level $\rho = \dfrac{m_W^2}{m_Z^2 \cos^2\theta_W} = 1$ is a consequence of $SU(2)_C$

Including the radiative corrections,
hypercharge effects and fermion mass splittings break $SU(2)_C$

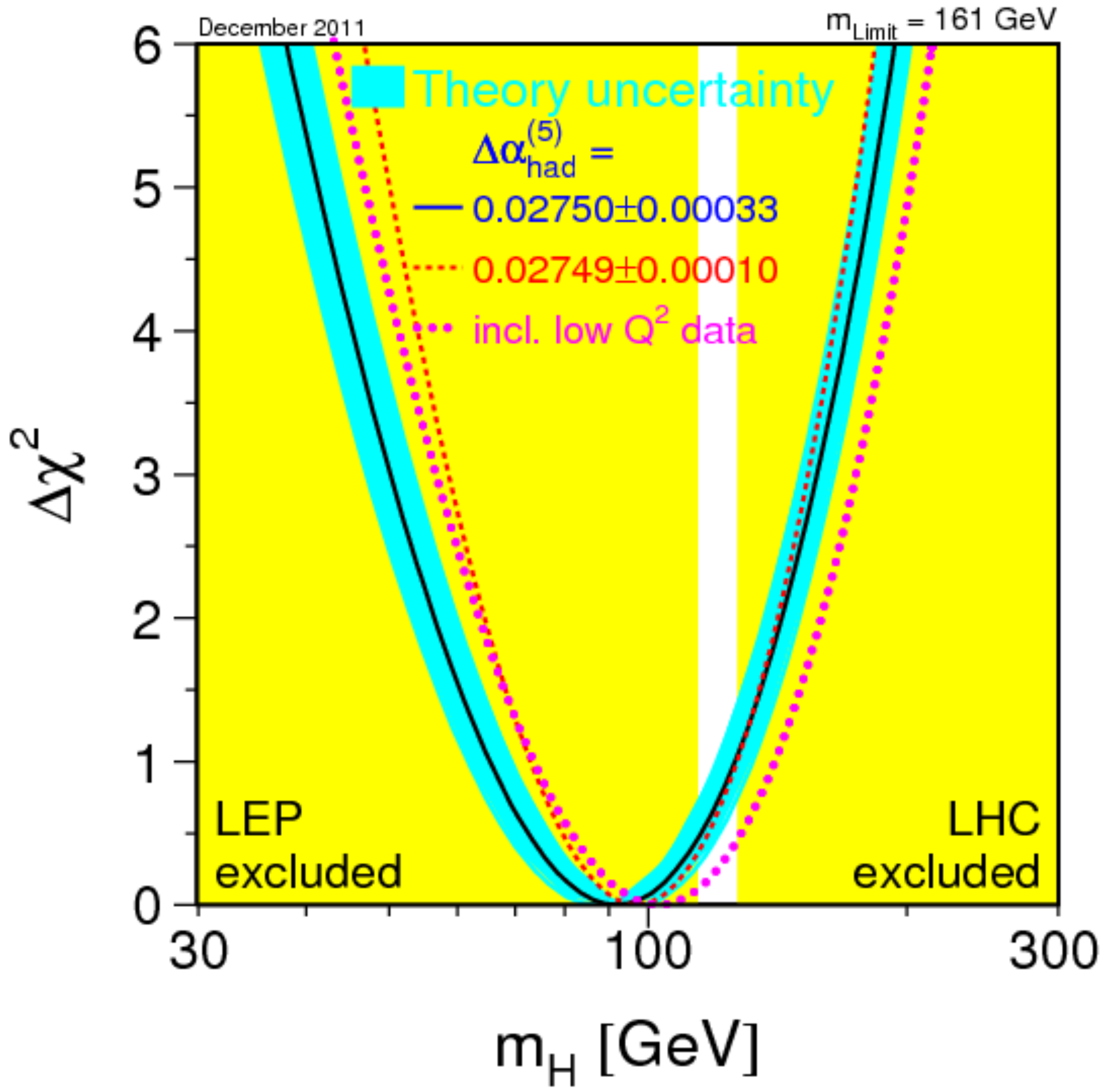
$$\rho = 1 + \Delta\rho$$

- The presence in the classical lagrangian of $SU(2)_C$,
implies that the evaluation of $\Delta\rho$ does not require renormalisation
→ the $\Delta\rho$ value is a prediction of the SM

$$\Delta\rho \sim \frac{3G_\mu m_t^2}{8\sqrt{2}\pi} \quad , \quad \Delta\rho \propto \log\left(\frac{m_H^2}{m_W^2}\right)$$

→ bounds on the top quark and Higgs masses

→ constraint on the HWW and HZZ couplings



SM renormalisation
or
what is a mass? what is a coupling?

The bare Lagrangian density

- The bare Lagrangian is a mathematical quantity,
  from which we derive the equations of motion of the fields and the scattering amplitudes

  It describes a k-fold infinite set of possible theories, parametric in the k masses and couplings
  (changing the value of the electric charge affects the chemistry but not the physics of our world!)

  e.g. in QED k=2

$$\mathscr{L}_{QED} = \overline{\psi}\left[i\gamma_\mu(\partial^\mu - ie_0 A^\mu) - m_0 \right]\psi$$

- A precise, physical, meaning of the Lagrangian is achieved imposing the renormalisation conditions

  The renormalisation program is not specifically related to the UV divergences,
  but it rather solves the k-fold infinite degeneracy of the bare lagrangian,
  choosing a specific value and meaning for the couplings

- The renormalisation conditions are imposed at a given energy scale, the renormalisation scale.
  The dependence of the theory on this choice is controlled by the Renormalization Group Equations

The couplings in the Lagrangian and their relation to physical quantities

- The EW SM is invariant under the gauge group $SU(2)_L \times U(1)_Y$, with gauge couplings g and g'
  The scalar sector depends on the VEV v of the Higgs field and on the quartic scalar coupling λ
  Neglecting the fermion masses and the fermion-scalar interaction, the theory is fully specified by 4 couplings

- In the construction of the SM there are two neutral currents
  we impose that one is the electromagnetic current, coupling to the photon field A^μ
  the second neutral current is in turn a prediction, coupling to the Z-boson field Z^μ
  The fields A^μ and Z^μ are a linear combination of the $SU(2)_L \times U(1)_Y$ gauge fields, with a rotation angle $\tan\theta_W = \dfrac{g'}{g}$
  and the electric charge is $e = g\sin\theta_W$

- After spontaneous symmetry breaking, the gauge bosons acquire mass, via the Higgs mechanism

$$m_W = \frac{1}{2} g v, \quad m_Z = \frac{1}{2} v\sqrt{g^2 + g'^2}, \; m_H = v\sqrt{2\lambda}$$

- The Lagrangian couplings are in simple direct relation with four physical parameters

$$(g, g', v, \lambda) \quad \leftrightarrow \quad (e, m_W, m_Z, m_H)$$

(the weak mixing angle is a derived parameter)

A simple renormalisation scheme (Sirlin 1980)

- The tree level relations between the Lagrangian couplings and the chosen physical parameters hold for the bare quantities

$$e_0 = \frac{g_0\, g_0'}{\sqrt{g_0^2 + g_0'^2}}, \quad m_{W,0} = \frac{1}{2} g_0\, v_0, \quad m_{Z,0} = \frac{1}{2} v_0 \sqrt{g_0^2 + g_0'^2}, \quad m_{H,0} = v_0 \sqrt{2\lambda_0}$$

- We express the bare physical parameters in terms of renormalised ones and counterterms

$$e_0 = e + \delta e, \quad m_{W,0}^2 = m_W^2 + \delta m_W^2, \quad m_{Z,0}^2 = m_Z^2 + \delta m_Z^2, \quad m_{H,0}^2 = m_H^2 + \delta m_H^2$$

and also the same replacement for the Lagrangian couplings

$$g_0 = g + \delta g, \; g_0' = g' + \delta g', \; v_0 = v + \delta v, \; \lambda_0 = \lambda + \delta\lambda$$

- We expand in powers of $\hbar$ the relations and identify the coefficients, order by order

$$\delta a = (\hbar)^1 \delta a^{(1)} + (\hbar)^2 \delta a^{(2)} + (\hbar)^3 \delta a^{(3)} + \ldots$$

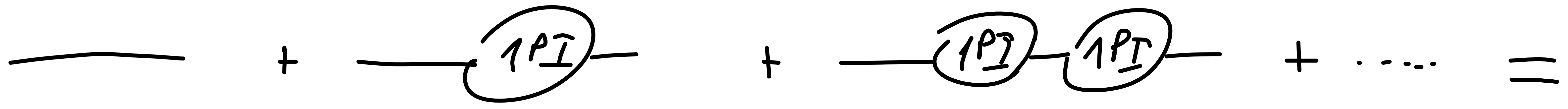
The Lagrangian coupling counterterms $(\delta g, \delta g', \delta v, \delta\lambda)$ can be expressed as a linear combination of $(\delta e, \delta m_W^2, \delta m_Z^2, \delta m_H^2,)$

- How can we compute $(\delta e, \delta m_W^2, \delta m_Z^2, \delta m_H^2)$?

Renormalisation conditions I

- The counterterm defines the renormalised parameter
  The relation between the renormalised parameter and the experimental input must then be specified

- In the on-shell renormalisation scheme, the renormalised mass coincides with the pole of the propagator
  (particle interpretation = simple pole of the propagator)



$$\frac{1}{p^2 - m_0^2 + \Sigma(p^2)} = \frac{1}{p^2 - m^2 - \delta m^2 + \Sigma(p^2)} = \frac{1}{p^2 - m^2 - \delta m^2 + \Sigma(m^2) + (p^2 - m^2)\Sigma'(m^2) + \ldots} = \frac{1}{(p^2 - m^2)\ \left(1 + \Sigma'(m^2)\right)}$$

with $\delta m^2 \ = \ \Sigma(m^2)$

- In the on-shell renormalisation scheme, the request of probabilistic interpretation of the fields, leads to a condition on the residue of the propagator, which must be 1
  This leads to the definition of the renormalised fields $\phi_0 = \phi\, Z_{wf}^{\frac{1}{2}}$ with $Z_{wf} = 1 + \Sigma'(m^2)$

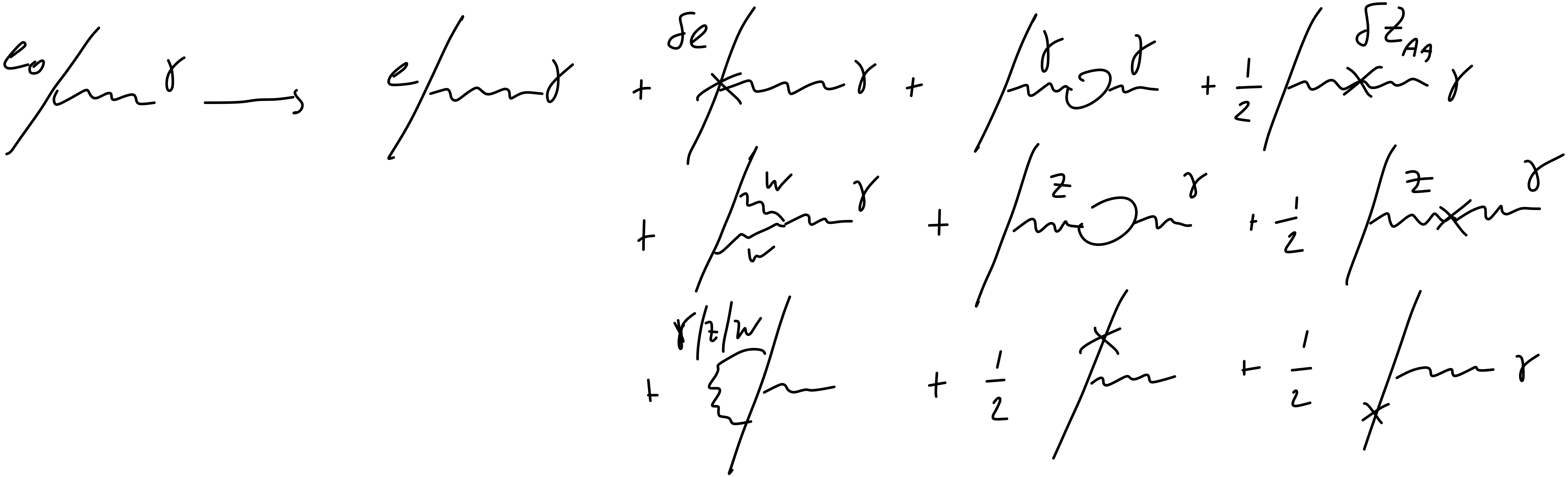
- These definitions stem from the study of the propagator and are completely general, for each field

Renormalisation conditions II

- The electric charge counterterm is defined via the study of the Thomson scattering i.e. the emission of a photon off a fermion, at vanishing momentum transfer

$q^\mu = 0$

- In the on-shell renormalisation scheme, the renormalised charge coincides with the experimental charge, at all orders in perturbation theory



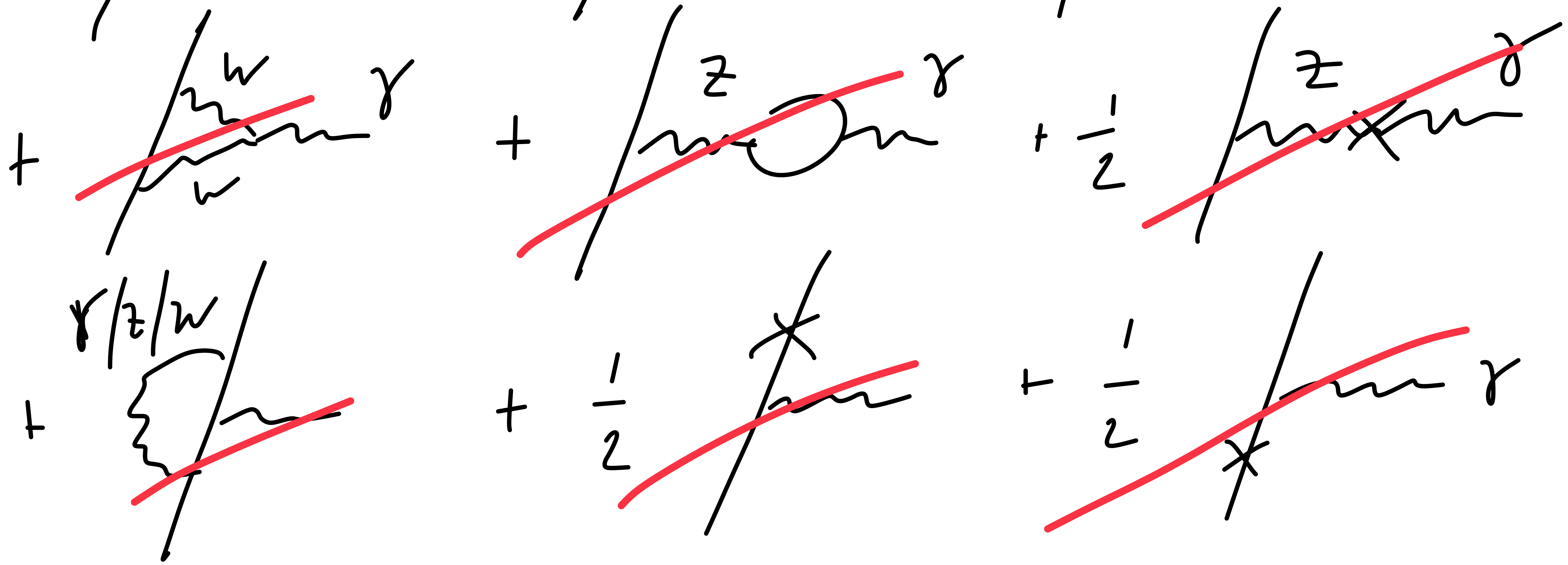
→ the electric charge counterterm cancels, order by order, the radiative corrections

Renormalisation conditions II

$q^\mu = 0$

- The electric charge counterterm is defined via the study of the Thomson scattering
  i.e. the emission of a photon off a fermion, at vanishing momentum transfer

- In the on-shell renormalisation scheme,
  the renormalised charge coincides with the experimental charge, at all orders in perturbation theory



Ward Identities

$U(1)_{e.m.}$ unbroken

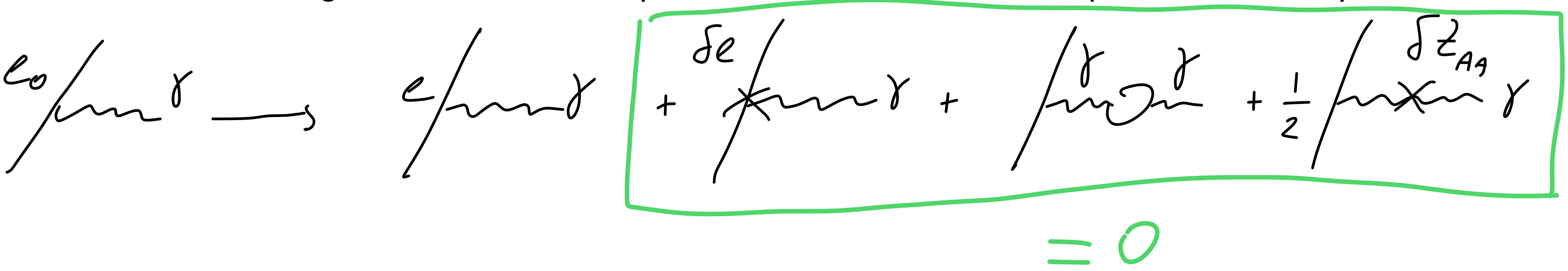
→ the electric charge counterterm cancels, order by order, the radiative corrections

Renormalisation conditions II

- The electric charge counterterm is defined via the study of the Thomson scattering
  i.e. the emission of a photon off a fermion, at vanishing momentum transfer

$q^\mu = 0$

- In the on-shell renormalisation scheme,
  the renormalised charge coincides with the experimental charge, at all orders in perturbation theory



→ the electric charge counterterm δe cancels, order by order, the radiative corrections

$$\alpha = \frac{e^2}{4\pi^2} = \frac{1}{137.035999}$$ from measurement of atomic transitions

The renormalised Lagrangian and the choice of the input parameters

- Once $(\delta e, \delta m_W^2, \delta m_Z^2, \delta m_H^2)$ have been computed from the relevant self-energies, the Lagrangian is completely assigned and expressed in terms of $\mathscr{L} = \mathscr{L}(e, m_W, m_Z, m_H)$
  → predictivity = any observable can computed and written in terms of these 4 inputs and compared with data → test

The renormalised Lagrangian and the choice of the input parameters

- Once $(\delta e, \delta m_W^2, \delta m_Z^2, \delta m_H^2)$ have been computed from the relevant self-energies, the Lagrangian is completely assigned and expressed in terms of $\mathscr{L} = \mathscr{L}(e, m_W, m_Z, m_H)$
  → predictivity = any observable can computed and written in terms of these 4 inputs and compared with data → test

- An orthogonal question:

  Given some experimental kinematical distributions, which parameters can be determined from them ?

The renormalised Lagrangian and the choice of the input parameters

- Once $(\delta e, \delta m_W^2, \delta m_Z^2, \delta m_H^2)$ have been computed from the relevant self-energies, the Lagrangian is completely assigned and expressed in terms of $\mathscr{L} = \mathscr{L}(e, m_W, m_Z, m_H)$
  → predictivity = any observable can computed and written in terms of these 4 inputs and compared with data → test

- An orthogonal question:

  Given some experimental kinematical distributions, which parameters can be determined from them ?

  → only (e, m_W, m_Z, m_H) can be determined by fitting the theoretical distributions (model dependent) to the data

The renormalised Lagrangian and the choice of the input parameters

- Once $(\delta e, \delta m_W^2, \delta m_Z^2, \delta m_H^2)$ have been computed from the relevant self-energies, the Lagrangian is completely assigned and expressed in terms of (e, m_W, m_Z, m_H)
  → predictivity = any observable can computed and written in terms of these 4 inputs

The renormalised Lagrangian and the choice of the input parameters

- Once $(\delta e, \delta m_W^2, \delta m_Z^2, \delta m_H^2)$ have been computed from the relevant self-energies, the Lagrangian is completely assigned and expressed in terms of (e, m_W, m_Z, m_H)
  → predictivity = any observable can computed and written in terms of these 4 inputs

- Is this choice of input parameters unique? no: we could use also G_μ or $\sin^2\theta_{eff}^{\ell}$ replacing e or m_W

The renormalised Lagrangian and the choice of the input parameters

- Once $(\delta e, \delta m_W^2, \delta m_Z^2, \delta m_H^2)$ have been computed from the relevant self-energies, the Lagrangian is completely assigned and expressed in terms of (e, m_W, m_Z, m_H)
  → predictivity = any observable can computed and written in terms of these 4 inputs

- Is this choice of input parameters unique? no: we could use also G_μ or $\sin^2\theta_{eff}^\ell$ replacing e or m_W

- How do we choose the input parameters?

1) minimize the parametric uncertainty of the final results → $(\alpha, G_\mu, m_Z, m_H)$ are the best known quantities

$$\alpha(0) = 1/137.035999$$

2) avoid the dependence on non-perturbative QCD uncertainties → (G_μ, m_W, m_Z, m_H)

$$\alpha_{G_\mu} = 1/132 \qquad \alpha_{G_\mu}/\alpha(0) \simeq 1.035$$

3) reabsorb in the definition of the input parameters large radiative corrections → (G_μ, m_W, m_Z, m_H) makes it

4) extract from the data the value of one input parameter via a fitting procedure

→ (G_μ, m_W, m_Z, m_H) allows to fit m_W, instead $(G_\mu, \sin^2\theta_{eff}^\ell, m_Z, m_H)$ is needed to fit $\sin^2\theta_{eff}^\ell$

The Fermi constant and the parameterisation of the charged-current weak interaction

Fermi theory of β decay

muon decay $\mu^- \to \nu_\mu e^- \bar{\nu}_e$

$$\frac{1}{\tau_\mu} \to \Gamma_\mu \to G_\mu$$

QED corrections to Γ_μ necessary for precise determination of G_μ
computable in the Fermi theory (Kinoshita, Sirlin, 1959)

The independence of the QED corrections of the underlying model (Fermi theory vs SM) allows

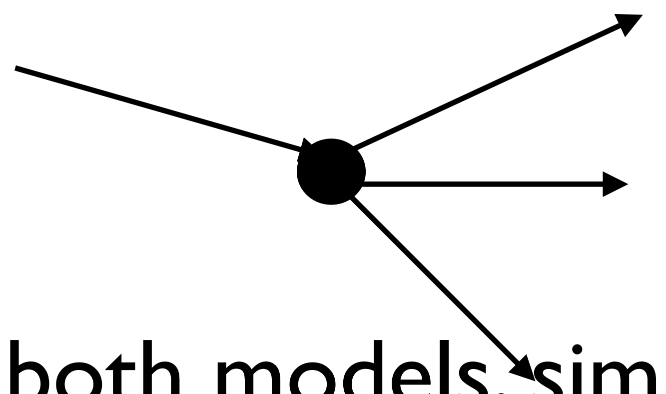
- to define G_μ and to measure its value with high precision G_μ = 1.1663787(6) 10^{-5} GeV^{-2}

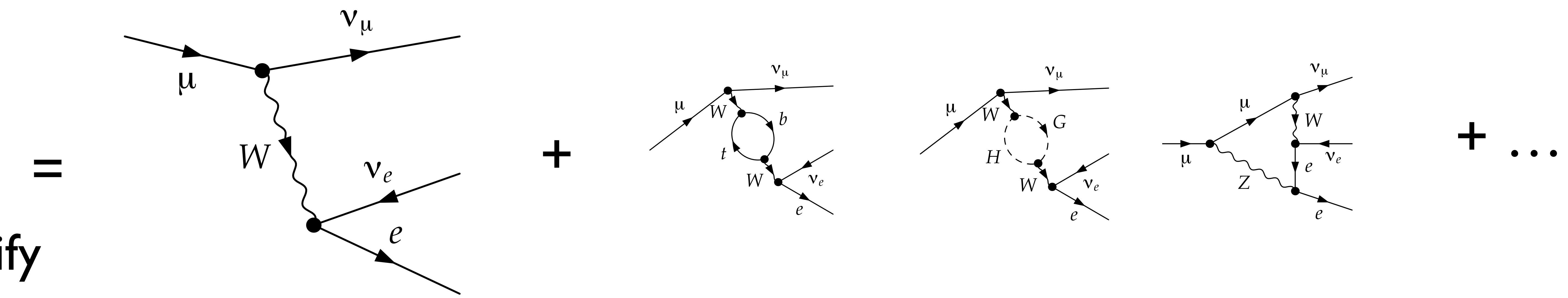
- to "reabsorb" in the G_μ definition the large logarithmic QED effects

The Fermi constant and the parameterisation of the charged-current weak interaction

- The Fermi theory and the SM can be identified, i.e. matched
  imposing that the muon decay amplitude at zero momentum transfer, in the two theories, coincide

$$\frac{G_\mu}{\sqrt{2}} = \frac{g^2}{8m_W^2}(1+\Delta r)$$





- the QED corrections, identical in both models, simplify
- the non-QED corrections contribute to the definition of the matching at a given perturbative order, via Δr

- It is possible to compute Δr using (e, m_W, m_Z, m_H) as inputs in the on-shell scheme

$$\Delta r = \Delta\alpha(m_Z^2) - \frac{\cos^2\theta_W}{\sin^2\theta_W}\rho \; + \; \Delta r_{rem} \qquad \text{and approximately} \qquad \Delta r \sim 0.07 - 3\cdot 0.01 + \mathcal{O}(0.001) \sim 0.035$$

- Δr is a finite physical correction.
  Its inclusion allows to use G_μ as input to express the strength of the weak interaction
  Δr is sensitive to BSM physics via the virtual corrections

Predictivity of the Standard Model

$$\mathcal{L}_{SM} = \mathcal{L}_{SM}(\alpha, G_\mu, m_Z; m_H; m_f; CKM)$$

We trade m_W for G_μ among the inputs, and solve the matching condition for m_W

$$\frac{G_\mu}{\sqrt{2}} = \frac{g^2}{8m_W^2}(1+\Delta r) \quad \Rightarrow \quad m_W^2 = \frac{m_Z^2}{2}\left(1+\sqrt{1-\frac{4\pi\alpha}{G_\mu\sqrt{2}m_Z^2}(1+\Delta r)}\right)$$

The W boson mass: theoretical prediction

Identification of the Fermi theory (effective theory) and Standard Model amplitudes for the muon decay process

$$\frac{G_\mu}{\sqrt{2}} = \frac{g^2}{8m_W^2}\left(1+\Delta r\right)$$

The W boson mass: theoretical prediction

Identification of the Fermi theory (effective theory) and Standard Model amplitudes for the muon decay process

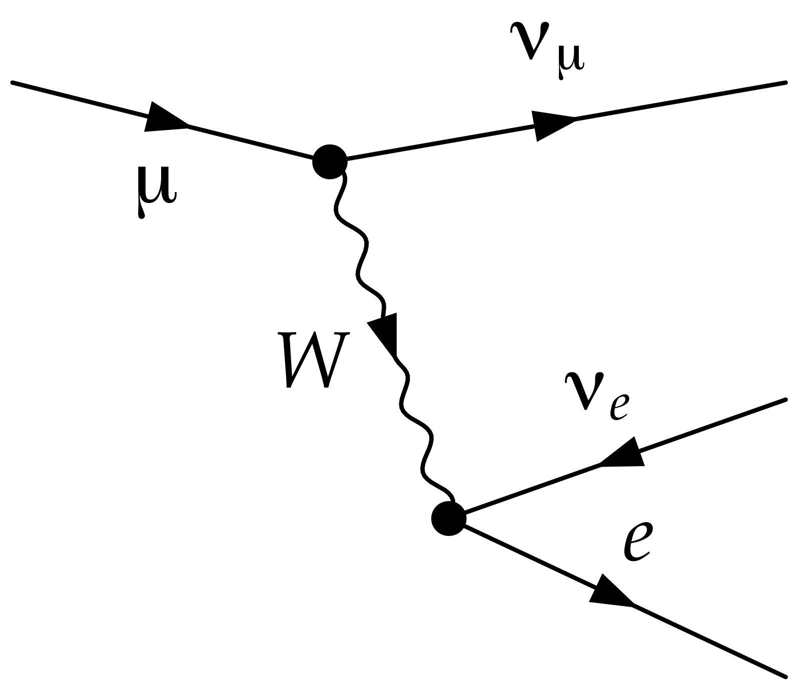
$$\frac{G_\mu}{\sqrt{2}} = \frac{g^2}{8 m_W^2}(1+\Delta r) \quad \Rightarrow \quad m_W^2 = \frac{m_Z^2}{2}\left(1+\sqrt{1-\frac{4\pi\alpha}{G_\mu\sqrt{2}m_Z^2}(1+\Delta r)}\right)$$

The W boson mass: theoretical prediction

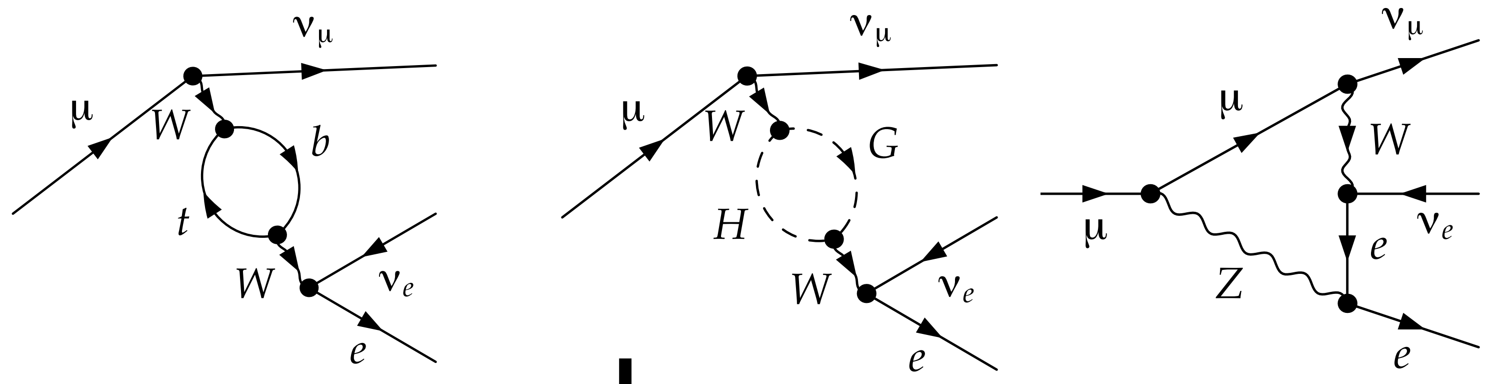
Identification of the Fermi theory (effective theory) and Standard Model amplitudes for the muon decay process

$$\frac{G_\mu}{\sqrt{2}} = \frac{g^2}{8m_W^2}(1+\Delta r) \quad \Rightarrow \quad m_W^2 = \frac{m_Z^2}{2}\left(1+\sqrt{1-\frac{4\pi\alpha}{G_\mu\sqrt{2}m_Z^2}(1+\Delta r)}\right)$$

In the Standard Model



+



+ …

+ full 2-loop + partial 3- and 4-loop

$m_W = 80.934$ GeV

$\Delta m_W = \mathcal{O}(0.5\ \mathrm{GeV})$

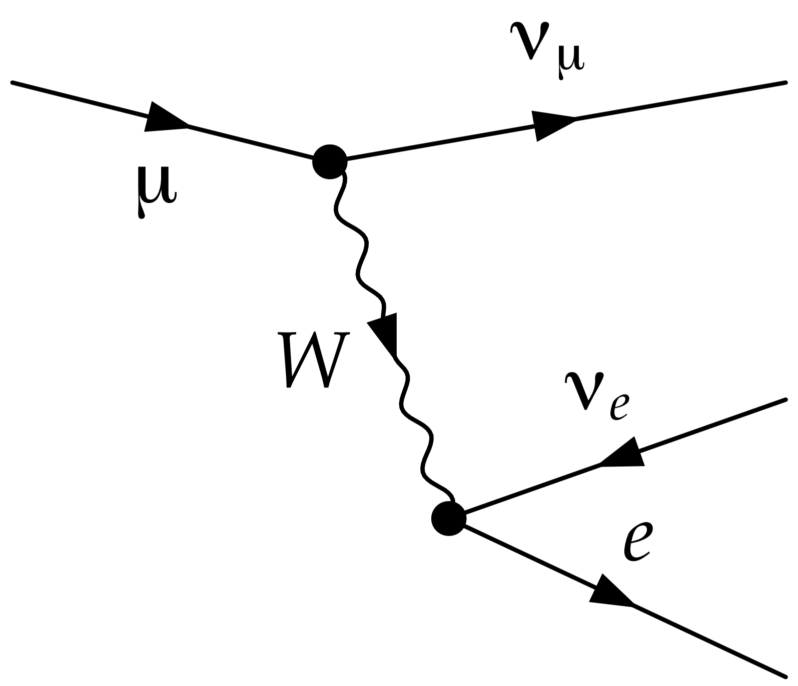
$\Delta m_W = \mathcal{O}(50\ \mathrm{MeV})$

The W boson mass: theoretical prediction

Identification of the Fermi theory (effective theory) and Standard Model amplitudes for the muon decay process

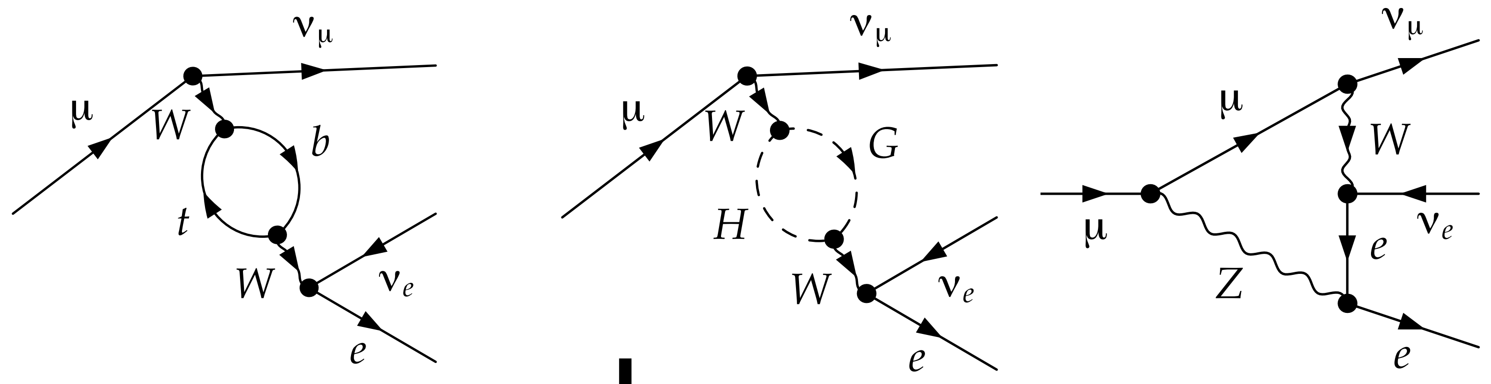
$$\frac{G_\mu}{\sqrt{2}} = \frac{g^2}{8 m_W^2}\,(1+\Delta r) \quad \Rightarrow \quad m_W^2 = \frac{m_Z^2}{2}\left(1+\sqrt{1-\frac{4\pi\alpha}{G_\mu\sqrt{2}m_Z^2}(1+\Delta r)}\right)$$

In the Standard Model



$m_W = 80.934$ GeV

+



+ ...

$\Delta m_W = \mathcal{O}(0.5\ \text{GeV})$

+ full 2-loop + partial 3- and 4-loop

$\Delta m_W = \mathcal{O}(50\ \text{MeV})$

on-shell scheme $m_W^{os} = 80.353 \pm 0.004$ GeV (Freitas, Hollik, Walter, Weiglein)

MSbar scheme. $m_W^{\overline{MS}} = 80.351 \pm 0.003$ GeV (Degrassi, Gambino, Giardino)

parametric uncertainties $\delta m_W^{par} = \pm 0.005$ GeV due to the $(\alpha, G_\mu, m_Z, m_H, m_t)$ values

$$\frac{\Delta m_W^{th}}{m_W} \sim 1 \cdot 10^{-4}$$

Experimental tests of the MW prediction

- See tomorrow's lecture for the theoretical details of the experimental determination

- A few comments now to discuss "what is the mass of an unstable particle ?"

The complex-mass scheme

- The quantisation of the gauge theory requires a gauge fixing term → gauge dependent Green's functions

  The BRS symmetry, including the Faddeev-Popov ghosts → the S-matrix elements (the xsecs) are gauge invariant

  What about the parameters, like masses and couplings ? Are they gauge invariant ?

- The position of the pole of the propagator, that we interpret as mass of the particle, depends on the mass ct definition

  If $\delta m_Z^2 = \mathrm{Re}(\Sigma_{ZZ}(p^2 = m_Z^2))$, then, starting from 2-loop EW, gauge dependent terms in the renormalised mass

  With $\delta\mu_Z^2 = \Sigma_{ZZ}(p^2 = \mu_Z^2)$ we solve the gauge invariance problem to all orders

  Since the bare mass is real valued, $m_0^2 = \mu^2 + \delta\mu^2$, then the renormalised mass $\mu = m - \frac{i}{2}\Gamma$ is complex valued

  The position of the complex mass is gauge invariant (as it should be if we want to fit it !!!) → complex mass scheme

The complex-mass scheme

- Given $\mu = m - \frac{i}{2}\Gamma$, the mass m is a free parameter,

  the decay width Γ, has to be computed

  at N[K]LO, we need the imaginary part of a self-energy a (k+1) loops

  → important for a smooth description of the resonance

- Given an input scheme like $(G_\mu, \mu_W, \mu_Z, \mu_H)$, then $s_w^2 = 1 - \frac{\mu_W^2}{\mu_Z^2}$ is a shortcut for that mass combination

Green's functions are in general complex valued (a factor like s_w^2 is not special)

The weak mixing angle

In the construction of the SM,
identification of the electromagnetic current and electric charge $e = g \sin\theta_W$
→ prediction of the second neutral current, coupling the Z boson to fermions

$$Zf\bar{f} \propto i \frac{g}{\cos\theta_W} \gamma^\mu \left(T_3 \frac{1-\gamma_5}{2} - \sin^2\theta_W Q_f \right)$$

$$\sin^2\theta_W = \frac{(g')^2}{g^2 + (g')^2}$$

At tree-level (more in general at LO-EW) , all definitions of the weak mixing angle are equivalent

Only after EW renormalisation (at NLO-EW or higher), the meaning of this coupling, at the quantum level, becomes unique

The weak mixing angle(s)

the on-shell weak mixing angle has been proposed by Sirlin in 1980
within the framework of the on-shell renormalisation scheme

$$\sin^2\theta_{OS} = 1 - \frac{m_W^2}{m_Z^2}$$

this definition is valid to all orders in perturbation theory, as it is related to a combination of physical parameters

when using (α, m_W, m_Z) as input parameters of the SM lagrangian, then $\sin^2\theta_{OS}$ is a "shortcut" for that mass combination

The weak mixing angle(s)

- the effective weak mixing angle for a fermion f enters in the definition of the effective $Zf\bar{f}$ vertex exactly at the Z resonance ($q^2 = m_Z^2$),

$$\mathscr{M}^{eff}_{Zf\bar{f}} = \bar{u}_l \gamma_\alpha \left[\mathscr{G}_v^f(m_Z^2) - \mathscr{G}_a^f(m_Z^2)\gamma_5 \right] v_l \varepsilon_Z^\alpha \qquad 4\,|\,Q_f|\sin^2\theta^f_{eff} \;=\; 1 \;-\; \frac{\mathscr{G}_v^f}{\mathscr{G}_a^f}$$

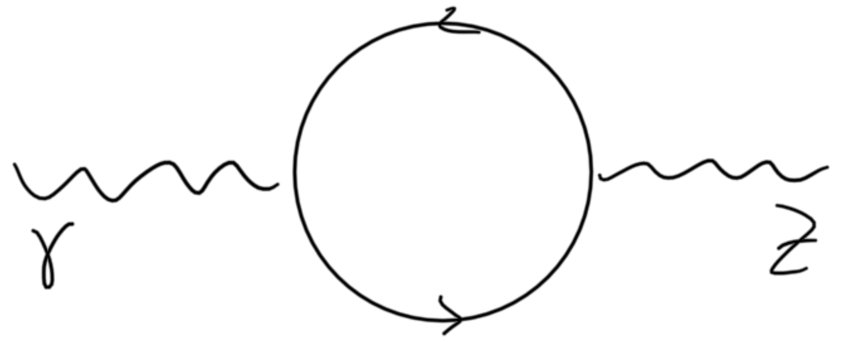
The weak mixing angle(s)

- the effective weak mixing angle for a fermion f enters in the definition of the effective $Zf\bar{f}$ vertex exactly at the Z resonance ($q^2 = m_Z^2$),

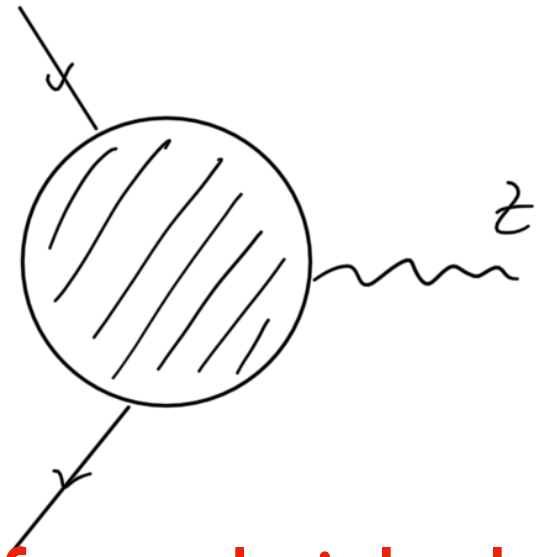
$$\mathscr{M}^{eff}_{Zf\bar{f}} = \bar{u}_l \gamma_\alpha \left[\mathscr{G}^f_v(m_Z^2) - \mathscr{G}^f_a(m_Z^2)\gamma_5 \right] v_l \varepsilon^\alpha_Z \qquad 4|Q_f| \sin^2\theta^f_{eff} = 1 - \frac{\mathscr{G}^f_v}{\mathscr{G}^f_a}$$

the effective weak mixing angle receives quantum corrections through

- the universal self-energy corrections



- the flavour-dependent vertex corrections

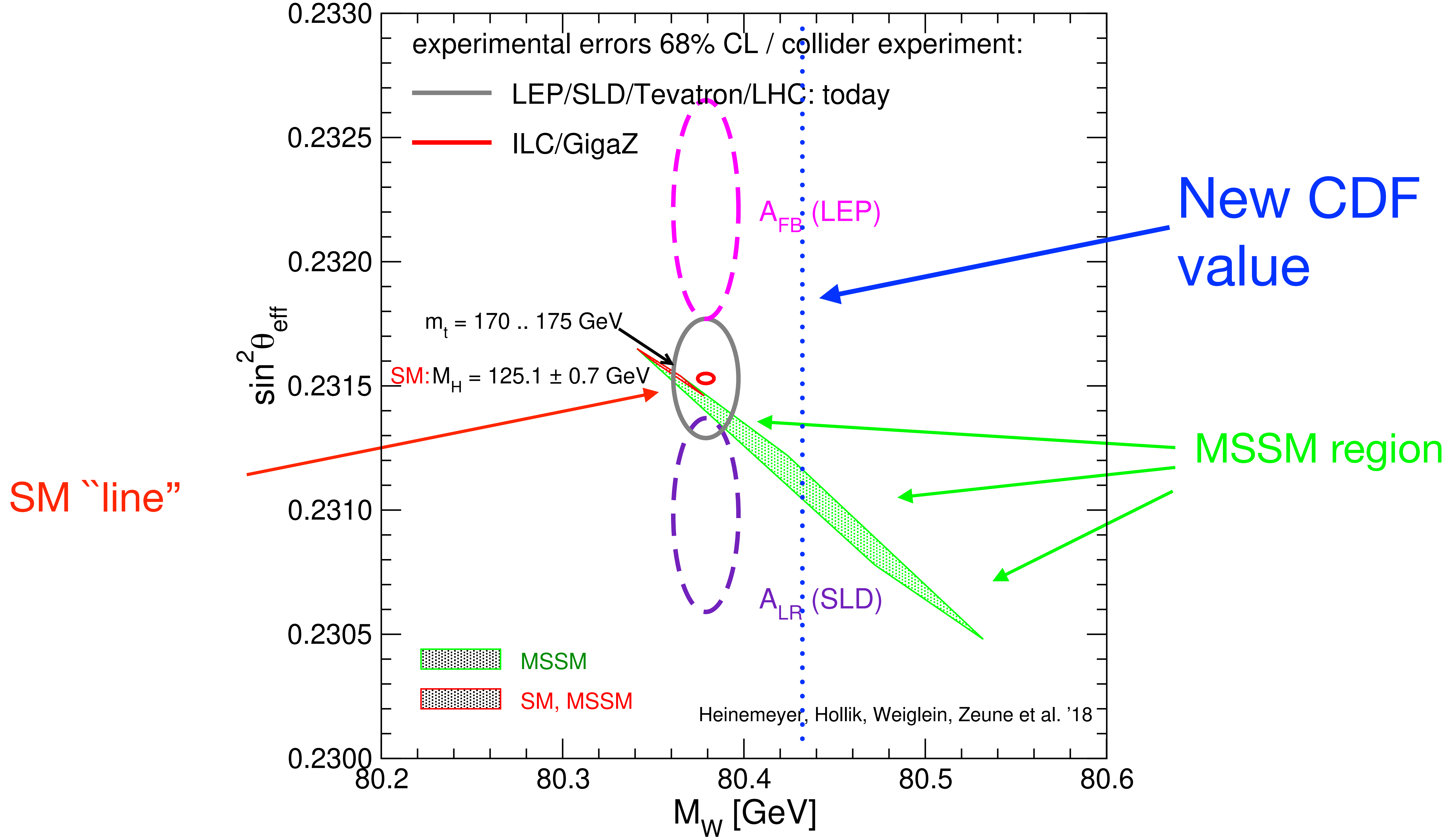


from all those diagram which yield a different corrections to left- and right-handed currents

sensitivity to BSM physics active at $q^2 = m_Z^2$, different than the BSM probed by m_W via Δr

Relevance of a simultaneous study of m_W and of the effective weak mixing angle

[S. Heinemeyer, W. Hollik, G. W., L. Zeune '18]



sensitivity to different sets of oblique corrections, i.e. to different combinations of gauge boson self-energies

independent determinations of these two parameters crucial for testing different New Physics alternatives

The weak mixing angle(s)

- the MSbar weak mixing angle stems from the renormalisation of the weak coupling in the MSbar renormalisation scheme

$$\frac{G_\mu}{\sqrt{2}} = \frac{g_0^2}{8 m_{W,0}^2} \longrightarrow \hat{s}^2 \hat{c}^2 = \frac{\pi\alpha}{\sqrt{2} G_\mu m_Z^2 (1-\Delta\hat{r})} \qquad \hat{s}^2 \equiv \sin^2\hat{\theta}(\mu_R = m_Z)$$

it is flavour independent

it has a weak dependence on the top-quark corrections → precise theoretical prediction (fast convergence)

The running of the MSbar weak mixing angle

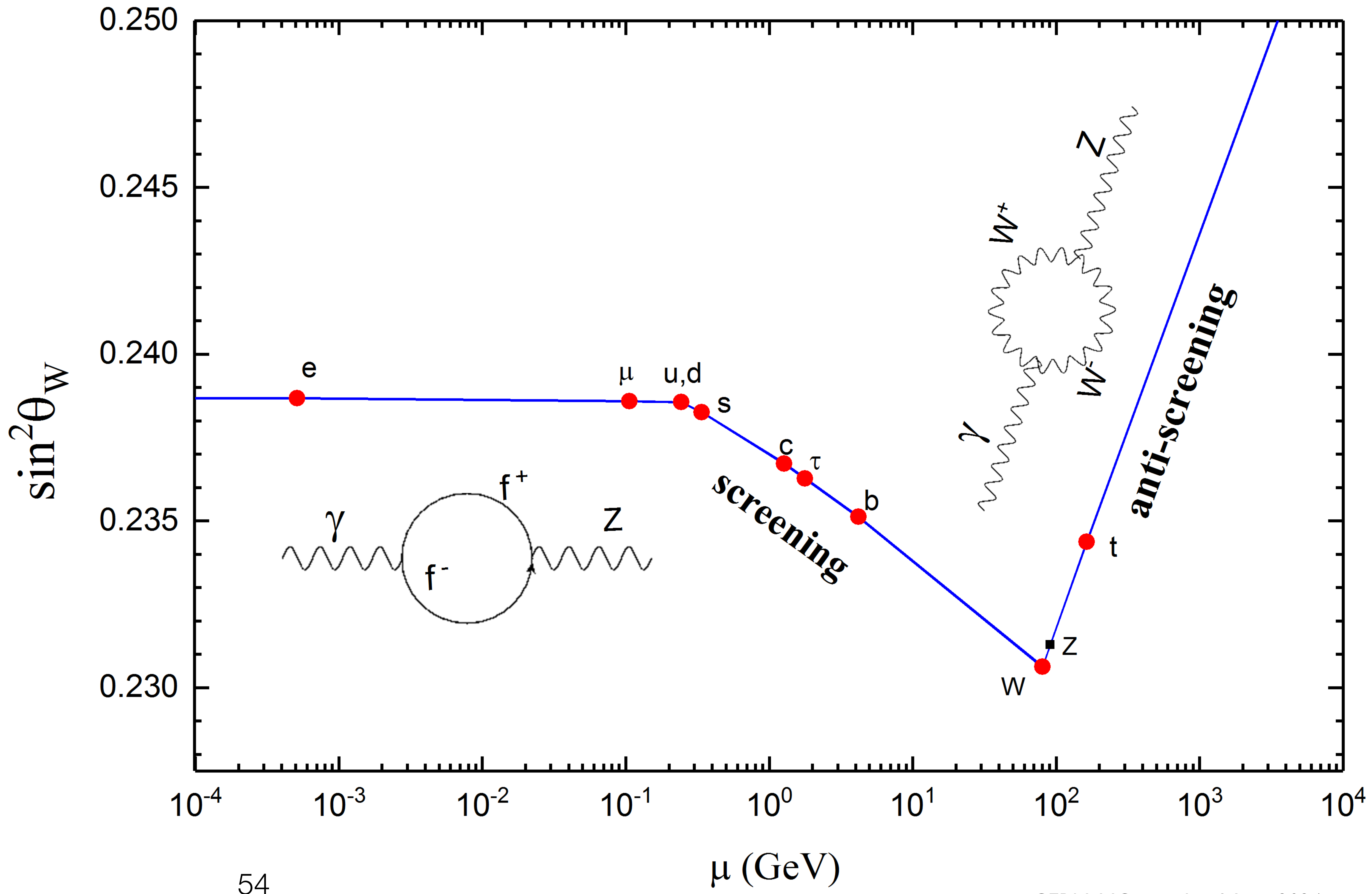
J.Erler, M. Ramsey Musolf, hep-ph/0409169, J.Erler, R.Ferro-Hernandez, arXiv:1712.09146,

The electric charge $\hat{\alpha}$ and the vector coupling $\hat{v}_f$ of a Z boson to a fermion f satisfy two Renormalization Group Equations, with solution

$$\hat{s}^2(\mu) = \hat{s}^2(\mu_0)\frac{\hat{\alpha}(\mu)}{\hat{\alpha}(\mu_0)} + \lambda_1\left[1 - \frac{\hat{\alpha}(\mu)}{\hat{\alpha}(\mu_0)}\right] + \frac{\hat{\alpha}(\mu)}{\pi}\left[\frac{\lambda_2}{3}\ln\frac{\mu^2}{\mu_0^2} + \frac{3\lambda_3}{4}\ln\frac{\hat{\alpha}(\mu)}{\hat{\alpha}(\mu_0)} + \tilde{\sigma}(\mu_0) - \tilde{\sigma}(\mu)\right]$$

expresses the dependence of $\hat{s}^2(\mu) \equiv \sin^2\hat{\theta}_W(\mu)$
- on the renormalisation scale μ
- on the coupled running of $\hat{\alpha}(\mu)$

Interesting test of the SM
from the MeV to the TeV energy range



Summary

- the SM is a renormalizable predictive theory: using $(\alpha, G_\mu, m_Z, m_H)$ as inputs, we predict m_W and $\sin^2\theta^\ell_{eff}$

  the comparison of the theoretical m_W and $\sin^2\theta^\ell_{eff}$ with their experimental determinations allow

  - to test the SM
  - to put constraints on the SMEFT extension

- the scalar sector in the SM is minimal

  we can understand the details of the EW SSB via

  - an experimental determination of the Higgs self-interaction couplings
  - the study of vector boson scattering

  → unitarity is a fundamental constraint

  → HL-LHC

Thank you

From the Fermi theory of weak interactions to the discovery of W and Z

The SM predicts the existence of a new neutral current, different than the electromagnetic one
(Glashow 1961, Weinberg 1967, Salam 1968)

The observation of weak neutral current immediately allowed the estimate of the value of the weak mixing angle in the correct range
GARGAMELLE, Phys.Lett. 46B (1973) 138-140

From the basic relation among the EW parameters it was immediately possible to estimate the order of magnitude of the mass of the weak bosons, in the 80 GeV range
(Antonelli, Maiani, 1981)

The discovery at the CERN SPPS of the W and Z bosons and the first determination of their masses allowed the planning of a new phase of precision studies accomplished with the construction of two e^+e^- colliders (SLC and LEP) running at the Z resonance

The precise determination of MZ and of the couplings of the Z boson to fermions
and in particular the value of the effective weak mixing angle
allowed to establish a framework for a test of the SM at the level of its quantum corrections

There is evidence of EW corrections beyond QED with 26 σ significance!
Full 1-loop and leading 2-loop radiative corrections are needed to describe the data
(indirect evidence of bosonic quantum effects, hints on the m_t and m_H values)

The W boson mass: theoretical prediction

Sirlin, 1980, 1984; Marciano, Sirlin, 1980, 1981;
van der Bij, Veltman, 1984; Barbieri, Ciafaloni, Strumia 1993;
Djouadi, Verzegnassi 1987; Consoli, Hollik, Jegerlehner, 1989;
Chetyrkin, Kühn, Steinhauser, 1995;
Barbieri, Beccaria, Ciafaloni, Curci, Viceré, 1992, 1993; Fleischer, Tarasov, Jegerlehner, 1993;
Degrassi, Gambino, AV, 1996; Degrassi, Gambino, Sirlin, 1997;
Freitas, Hollik, Walter, Weiglein, 2000, 2003;
Awramik, Czakon, 2002; Awramik, Czakon, Onishchenko, Veretin, 2003; Onishchenko, Veretin, 2003

The best available prediction includes
the full 2-loop EW result, leading higher-order EW and QCD corrections,
resummation of reducible terms
Missing 3-loop and 4-loop terms needed to reduce the uncertainties.

$$m_W = w_0 + w_1 dH + w_2 dH^2 + w_3 dh + w_4 dt + w_5 dH dt + w_6 da_s + w_7 da^{(5)}$$

$$dt = [(M_t/173.34\,\text{GeV})^2 - 1]$$

$$da^{(5)} = [\Delta\alpha^{(5)}_{\text{had}}(m_Z^2)/0.02750 - 1]$$

$$dH = \ln\left(\frac{m_H}{125.15\,\text{GeV}}\right)$$

$$dh = [(m_H/125.15\ \text{GeV})^2 - 1]$$

$$da_s = \left(\frac{\alpha_s(m_Z)}{0.1184} - 1\right)$$

	$124.42 \le m_H \le 125.87$ GeV	$50 \le m_H \le 450$ GeV
w_0	80.35712	80.35714
w_1	-0.06017	-0.06094
w_2	0.0	-0.00971
w_3	0.0	0.00028
w_4	0.52749	0.52655
w_5	-0.00613	-0.00646
w_6	-0.08178	-0.08199
w_7	-0.50530	-0.50259

on-shell scheme $m_W^{os} = 80.353 \pm 0.004$ GeV (Freitas, Hollik, Walter, Weiglein)
MSbar scheme. $m_W^{\overline{MS}} = 80.351 \pm 0.003$ GeV (Degrassi, Gambino, Giardino)

parametric uncertainties $\delta m_W^{par} = \pm 0.005$ GeV due to the $(\alpha, G_\mu, m_Z, m_H, m_t)$ values

The W boson mass: theoretical prediction

<u>on-shell scheme:</u> dominant contributions to Δr

$$\Delta r = \Delta\alpha - \frac{c_{\rm w}^2}{s_{\rm w}^2}\Delta\rho + \Delta r_{\rm rem}$$

$$\Delta\alpha = \Pi^{\gamma}_{\rm ferm}(M_Z^2) - \Pi^{\gamma}_{\rm ferm}(0) \quad \rightarrow \quad \alpha(M_Z) = \frac{\alpha}{1-\Delta\alpha}$$

$$\Delta\rho = \frac{\Sigma_Z(0)}{M_Z^2} - \frac{\Sigma_W(0)}{M_W^2} = 3\frac{G_F m_t^2}{8\pi^2\sqrt{2}} \quad \textit{[one-loop]} \quad \sim \frac{m_t^2}{v^2} \sim \alpha_t$$

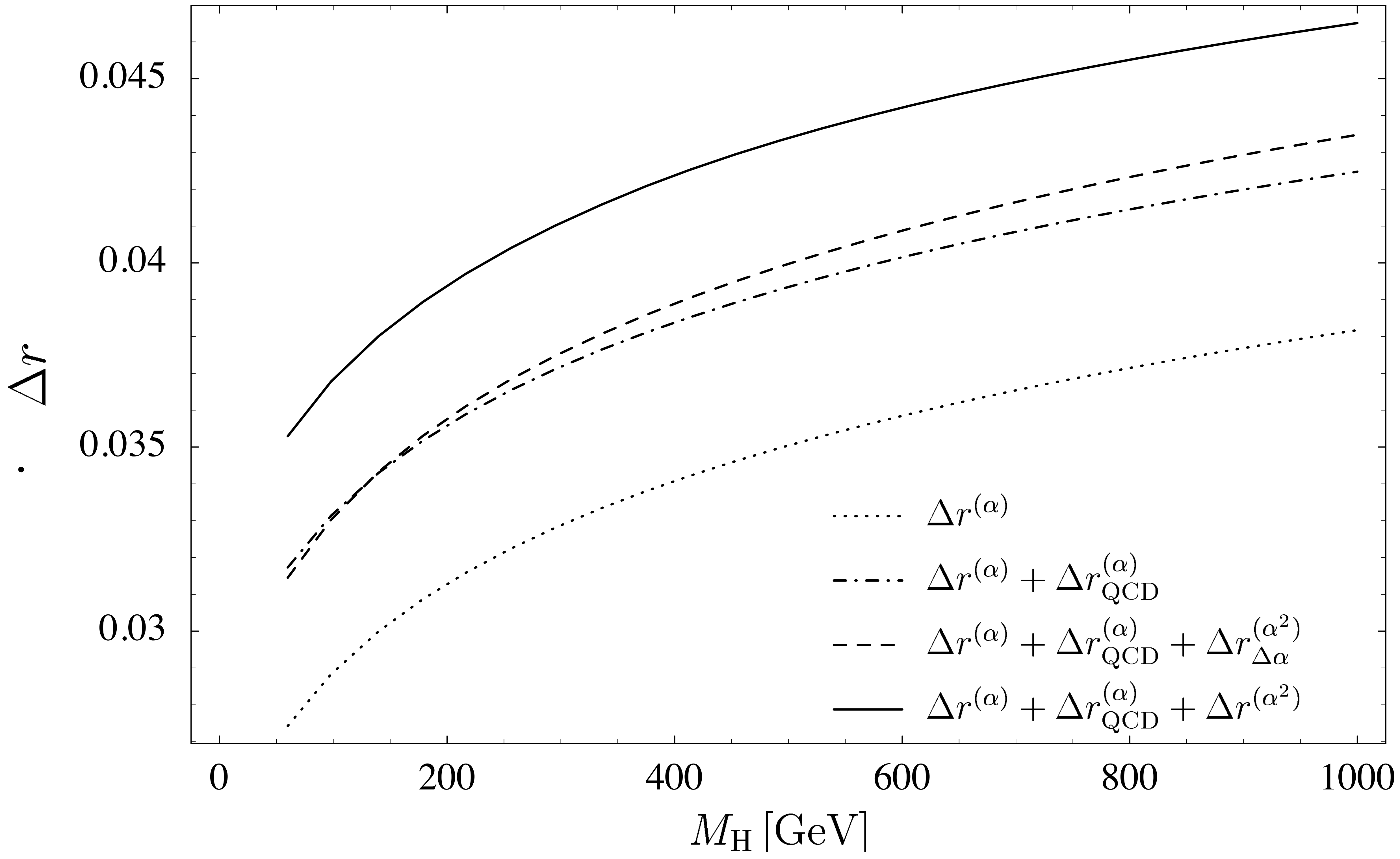
beyond one-loop order: $\sim \alpha^2,\, \alpha\alpha_t,\, \alpha_t^2,\, \alpha^2\alpha_t,\, \alpha\alpha_t^2,\, \alpha_t^3, \ldots$

reducible higher order terms from $\Delta\alpha$ and $\Delta\rho$ via

$$1 + \Delta r \rightarrow \frac{1}{(1-\Delta\alpha)\left(1+\frac{c_{\rm w}^2}{s_{\rm w}^2}\Delta\rho\right) + \cdots}$$

$$\rho = 1 + \Delta\rho \rightarrow \frac{1}{1-\Delta\rho}$$

effects of higher-order terms on Δr



(Consoli, Hollik, Jegerlehner)

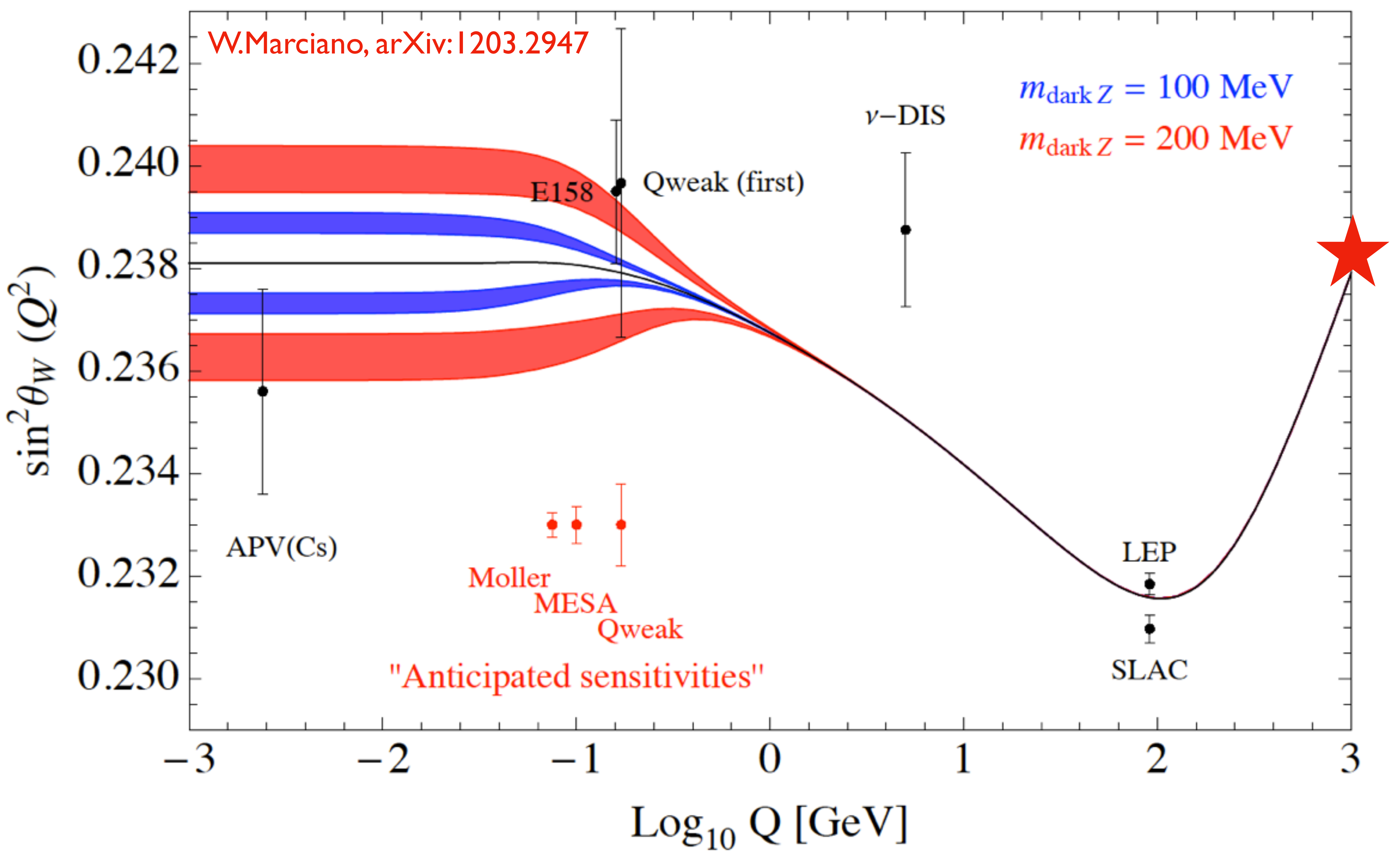
The running of the MSbar weak mixing angle and sensitivity to New Physics

Additional (BSM) virtual contributions modify the β-function changing the slope of the running (or even the sign)

At low energies,
there is sensitivity to the effects due to light new particles otherwise swamped at the Z resonance or at higher scales

At high energies, we might hope to have indirect hints of new heavy particles

The reference experimental precision is still set by the LEP value
$\sin^2\hat\theta_W(m_Z^2) = 0.23121(4)$ (PDG) or $\Delta\sin^2\theta^\ell_{eff} = 16\cdot 10^{-5}$



Given the size of the running effects, a SM test achievable with $\mathcal{O}(1\%)$ determinations

Given the rich literature on the possible studies at low-energy facilities
it is natural to investigate the possibility of a determination in the TeV region
exploiting the sub-percent precision expected at the end of HL-LHC