Introduction to parton showers, matching and merging

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Overview of lectures

1) Introduction to parton showers
   - approximate higher-order corrections
   - building a parton shower

2) Improving parton showers
   - assessing the properties of a parton shower
   - NLL accuracy and beyond

3) Matching and merging
   - matching
   - merging

Introduction to parton showers – recap

- parton shower generate universal approximate higher-order corrections using the soft-collinear limit
- as the description of the branching process is probabilistic, it provides an event-like structure that can be iterated
  $\Rightarrow$ description of soft-collinear (intrajet) evolution

The question is now:

How accurate is this approximation.
How can we systematically improve it?

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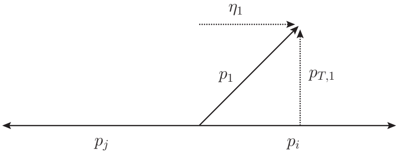
Improving parton showers

Tools to assess the properties of a parton shower
–
The Lund plane

The Lund plane – example: $e^+e^- \to q\bar{q} + g$

- Compute everything in centre-of-mass frame of the dipole ($q\bar{q}$)



using Sudakov decomposition $p_1 = p_1^+ + p_1^- + p_{\mathrm{T},1}$, with $p_1^2 = 2(p_1^+ p_1^- - p_{\mathrm{T},1}^2)$, $p_1^- = 2p_i p_1/\sqrt{2p_i p_j}$, and $p_1^+ = 2p_j p_1/\sqrt{2p_i p_j}$

- Simple expressions for transverse momentum and rapidity

$$p_{T,1}^2 = \frac{2(p_i p_1)(p_j p_1)}{p_i p_j} \qquad \text{and} \qquad \eta_1 = \frac{1}{2}\ln\frac{p_i p_1}{p_j p_1}$$

- squared matrix element divergent as $\propto 1/p_T^2$, encoding both soft and collinear divergences

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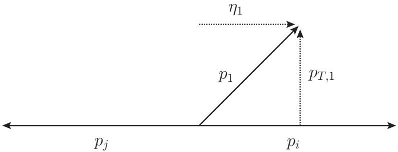

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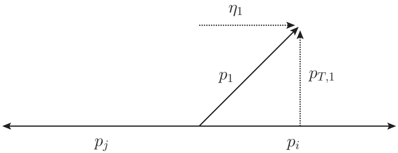

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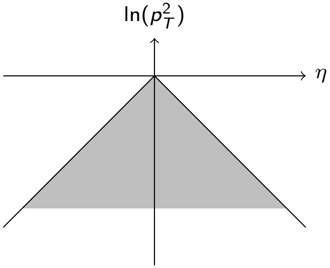
The **Lund plane**

- phase-space element $\propto \mathrm{d}p_T^2\,\mathrm{d}\eta$
  - $\left(\eta, \ln(p_T^2)\right)$ plane
  - phase space bounded by diagonals
  - emission classified by their $\left(\eta, \ln(p_T^2)\right)$ coordinates
  - emission opens new sheet with new η dimension



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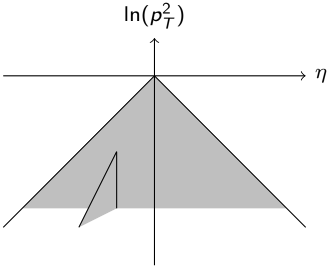

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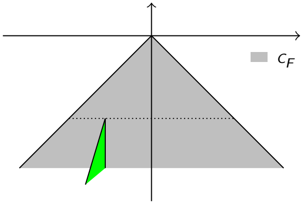
emission of a gluon off a $q\bar{q}$ dipole

- $N_c = 3$ (QCD):

$$\frac{C_F}{p_{T,1}^{2\,(i,j)}}$$



- $N_c \to \infty$, $C_A = \text{const}$ (large N_c limit):

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remember: in $N_c \to \infty$ limit $C_F \to C_A/2$
so this is consistent

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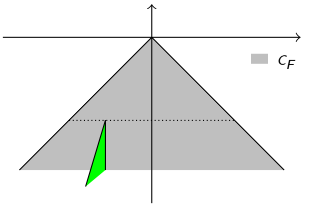

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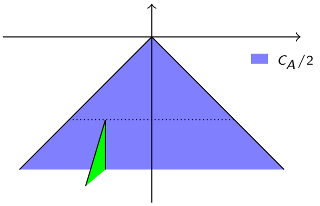

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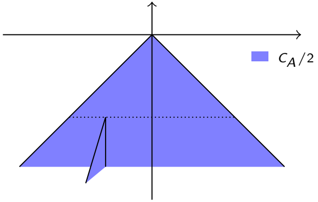
Emitting a second gluon



- $N_c \to \infty$, $C_A = \text{const}$ (large N_c limit):

$$\frac{(C_A/2)^2}{p_{T,1}^{2\,(i,j)}\,p_{T,2}^{2\,(i,1)}} + \left(i \leftrightarrow j\right)$$

  emission off gluon also with $C_A/2$

- $N_c = 3$ (QCD): in the soft-collinear limit, let's take a small detour

Angular radiation pattern

- matrix element can be written in terms of energies and angles

$$\frac{2p_i \cdot p_k}{(p_i \cdot q)(p_k \cdot q)} = \frac{W_{ik,j}}{E^2}$$

angular radiator function

$$W_{ik,j} = \frac{1-\cos\theta_{ik}}{(1-\cos\theta_{ij})(1-\cos\theta_{kj})}$$

- divergent as $\theta_{ij} \to 0$ and as $\theta_{kj} \to 0$
  $\to$ expose individual singularities using $W_{ik,j} = \tilde{W}^i_{ik,j} + \tilde{W}^k_{ik,j}$

$$\tilde{W}^i_{ik,j} = \frac{1}{2}\left[\frac{1-\cos\theta_{ik}}{(1-\cos\theta_{ij})(1-\cos\theta_{kj})} + \frac{1}{1-\cos\theta_{ij}} - \frac{1}{1-\cos\theta_{kj}}\right]$$

  - divergent as $\theta_{ij} \to 0$, but regular as $\theta_{kj} \to 0$

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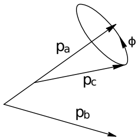


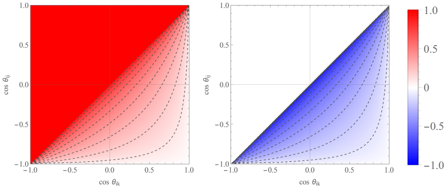
- work in a frame where direction of $\vec{p}_i$ aligned with z-axis

$$\cos\theta_{kj} = \cos\theta_k \cos\theta_j + \sin\theta_k \sin\theta_j \cos\phi_j$$

- integration over ϕ_j yields

$$\frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\phi_j\, \tilde{W}^i_{ik,j} = \frac{1}{1-\cos\theta_j} \times \begin{cases} 1 & \text{if} \quad \theta_j < \theta_k \\ 0 & \text{else} \end{cases}$$

  - on average, no radiation outside cone defined by parent dipole
  - differential radiation pattern more intricate: positive & negative contributions outside cone sum to zero



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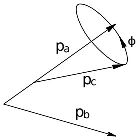

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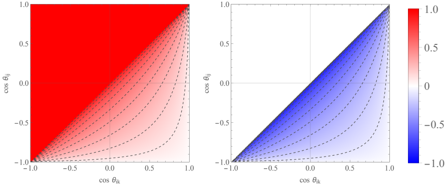
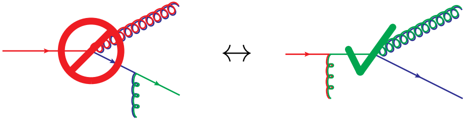


⇒ **angular ordering**

Colour coherence

Physical intepretation

- individual colour charges inside a colour dipole cannot be resolved if gluon wavelength larger than dipole size
  it only "sees" the combined colour charge of the "mother" parton



- net effect is destructive interference outside a cone
  with opening angle set by emitting colour dipole
- known in QED as the Chudakov effect

The Lund plane – example: $e^+e^- \to q\bar{q} + gg$

- Full 2-guon matrix element in the soft-collinear limit

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- $N_c \to \infty$ $(C_F \to C_A/2)$ ✓
  improved $N_c \to \infty$ $(C_F \neq C_A/2)$?
  - azimuthally integrated $\tilde{W}^i_{i1,2} = \tilde{W}^i_{ij,2}$ if $\theta_{i2} < \min(\theta_{i1}, \theta_{ij})$
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  - if both conditions simultaneously met (background sheet), all $C_A/2$ terms vanish $\to$ radiation from C_F term alone

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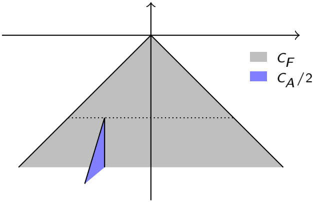

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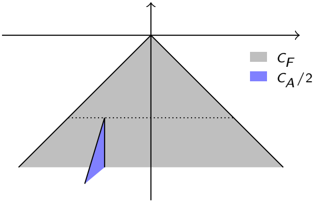


The simplest manifestation of angular ordering in QCD

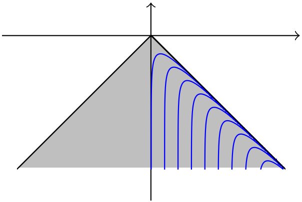
Properties of angular ordered and transverse-momentum ordered parton showers

Angular ordered parton showers

- differential radiation probability

$$\mathrm{d}V_a = \sum_b \frac{\mathrm{d}\tilde{q}^2}{\tilde{q}^2}\,\mathrm{d}z\,\frac{\alpha_s}{2\pi}\,P_{ab}(z)$$

  - dipole radiation becomes monopole radiation
    $\rightarrow$ parton (not *dipole*) shower
  - non-Abelian structure of QCD simplifies
    $\rightarrow$ radiation off mean charge C_F or C_A
- Lund plane filled from center to edges
  - random walk in p_T^2
  - colour factors correct for observables insensitive to azimuthal correlations

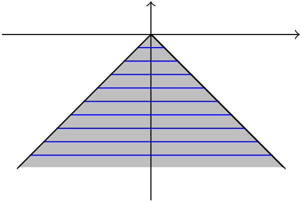


Transverse-momentum ordered dipole showers

- differential radiation probability for the dipole

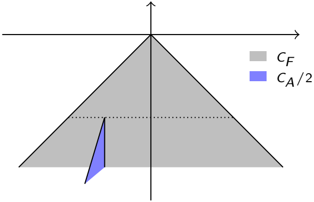
$$\mathrm{d}V_a = \sum_b \frac{\mathrm{d}p_{T,b}^2}{p_{T,b}^2}\,\mathrm{d}z\,\frac{\alpha_s}{2\pi}\,\tilde{P}_{ab}(z)$$

  - unified picture of parton and dipole evolution, inclusion if partial-fractioned soft limit in $\tilde{P}_{ab}(z)$
  - due to ordering in $p_{T,b}^2$ no natural way to recover correct colour factors as angular ordered shower does
- Lund plane filled from top to bottom
  - random walk in η
  - colour factors in improved leading color approximation



Radiation pattern of angular ordered and dipole showers

- **angular ordered parton showers**
  angles in centre-of-mass frame
  $\rightarrow$ coherence effects modeled by angular ordering variable agree on average with matrix element



- **p_T ordered dipole showers**
  angles in colour dipole frame
  $\rightarrow$ colour coherence not reflected by QCD charge of the dipole
  $\rightarrow$ emission off "back plane" in Lund plane should be associated with C_F, but is partly $C_A/2$
  (fine in strict large-N_c, wrong in improved large-N_c)
  $\rightarrow$ all-orders problem that appears first in 2-gluon emission case

Radiation pattern of angular ordered and dipole showers

- **angular ordered parton showers**
  angles in centre-of-mass frame
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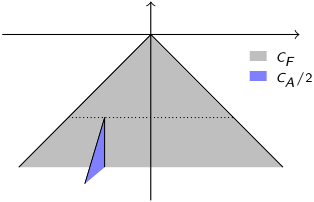

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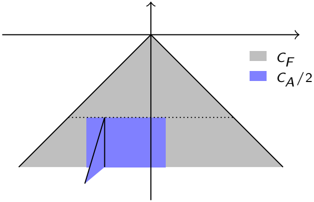
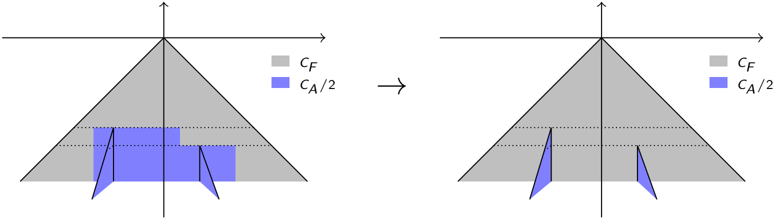


Correcting the radiation pattern of dipole showers

- analyse rapidity of gluon emission in event center-of-mass frame
- sectorise phase space and assign gluon to closest parton
  → choose corresponding color charge for evolution
- same technology for higher number of emissions



- starting with 4 emissions, there be "colour monsters"
  - quartic Casimir operators (easy)
  - non-factorisable contributions (hard)

Not captured in either angular ordered or corrected dipole evolution.

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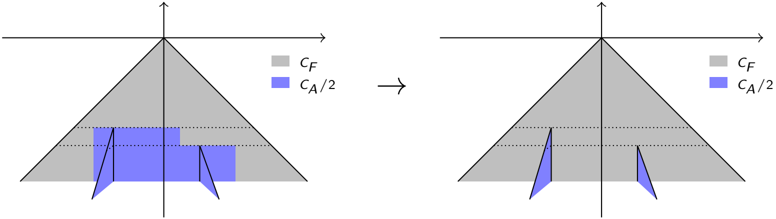

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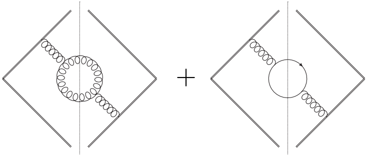
The CMW scheme



- approximate soft-gluon emission followed by collinear splitting gives rise to integrated NLO correction factor in dim. reg. and $\overline{\text{MS}}$

$$K = \left(\frac{67}{18} - \frac{\pi^2}{3}\right) C_A - \frac{10}{9}\, T_R\, n_f$$

local K-factor for soft-gluon emission

- K can be absorbed in an effective coupling

This is the so-called CMW scheme. Catani, Marchesini, Webber '91

Parton shower accuracy

Parton shower accuracy

How accurate is the parton shower's resummation
DGLAP resums leading single collinear logarithms.
Most parton showers reproduce this by construction.

How well does the parton shower resum related quantities?
Examples: jet rates, thrust, total broadening, ...

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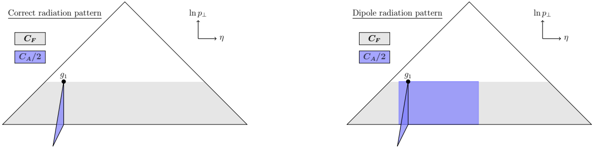
How to assess formal precision?

Prerequisites:

- parton shower must recover the soft-collinear structure of higher-order matrix elements
  a crucial aspect is the colour assignment in splitting function



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$$\frac{\Delta k_T^{ij}}{k_T^{ij}} \xrightarrow{k_T^j \to 0} 0$$

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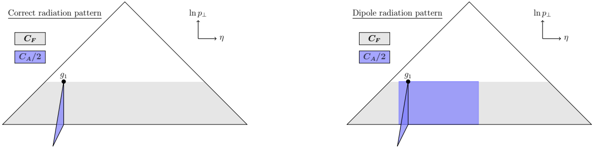

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$$\underbrace{\int_{z_{\min}}^{z_{\max}} \mathrm{d}z}_{\text{parton shower}} \longrightarrow \underbrace{\int_0^1 \mathrm{d}z}_{\textit{resummation}}$$

- evaluate

$$\frac{\Sigma^{\text{shower}}}{\Sigma^{\text{NLL}}} \propto \exp\left(f_{\text{shower}}^{\text{LL}} - L g_1(\alpha_s L)\right) \times \exp\left(f_{\text{shower}}^{\text{NLL}} - g_2(\alpha_s L)\right) \times \exp\left(\mathcal{O}(\alpha_s^{n+1} L^n)\right)$$

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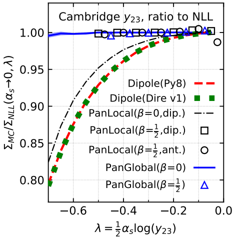
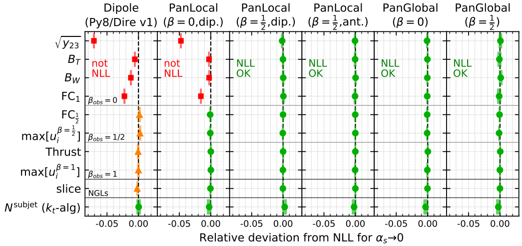
NLL accuracy in parton showers

Dasgupta, Dreyer, Hamilton, Monni, Salam, Soyez '20

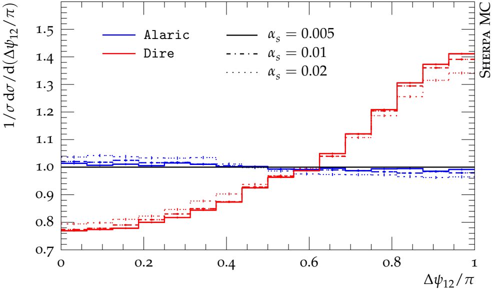




- the standard PYTHIA and DIRE showers are not NLL accurate
- newly formulated PANGLOBAL and PANLOCAL family of parton showers designed for NLL accuracy

NLL accuracy

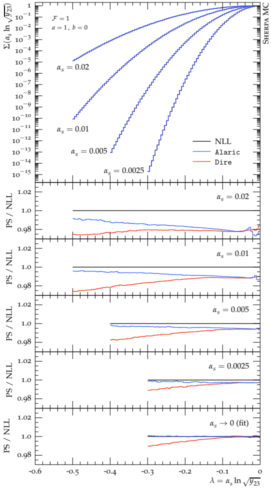
Höche, Krauss, Reichelt, MS '22



- subsequent emissions are uncorrelated to NLL accuracy
- DIRE introduces spurious correlation between jet planes

NLL accuracy

Höche, Krauss, Reichelt, MS '22



limit: $\alpha_s \to 0, \lambda = \alpha_s \log \mathcal{O} = \text{const.}$

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- Total jet broadening B_T $\beta = 0$
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- Thrust $1 - T$ $\beta = 1$

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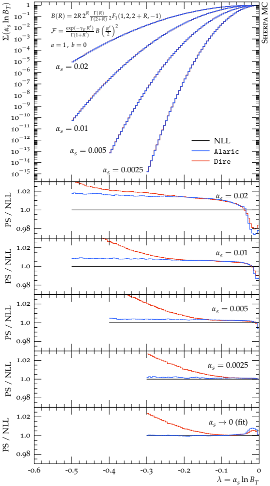

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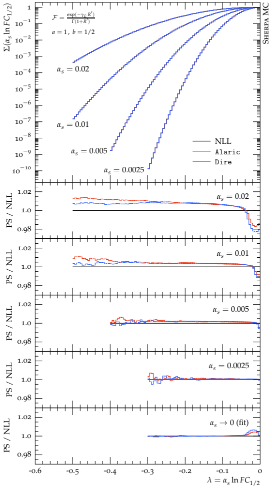

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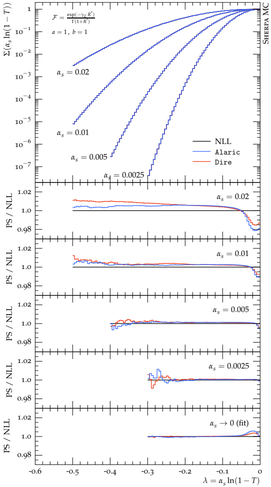

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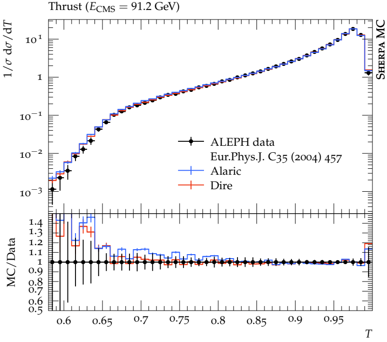
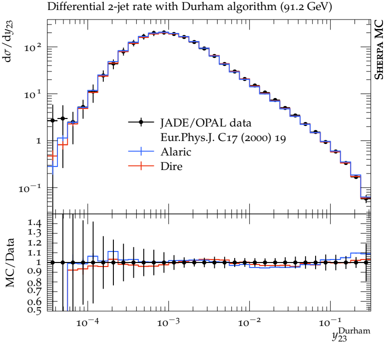
Effects of NLL accurate parton showers

LEP phenomenology

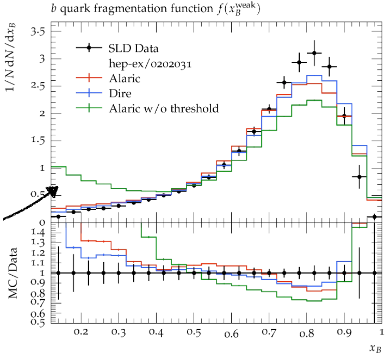
Höche, Krauss, Reichelt, MS '22





- ALARIC+PYTHIA string had., no matching or multijet merging
- hadronisation models are not infrared safe and depend on distribution of soft gluons

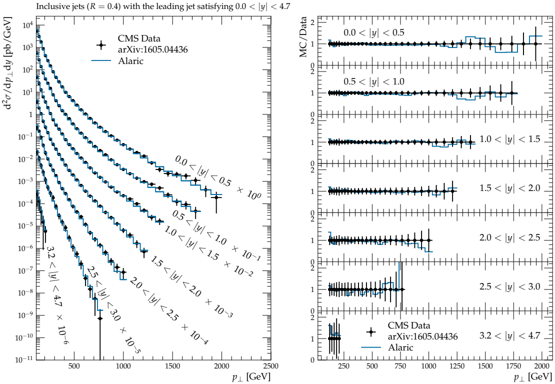
LEP phenomenology



- ALARIC is constructed with massless quarks so far
- quark masses are phenomenologically relevant
- quick fix: flavour thresholds for $g \to c\bar{c}$ and $g \to b\bar{b}$

LHC phenomenology

Höche, Krauss, Reichelt '24



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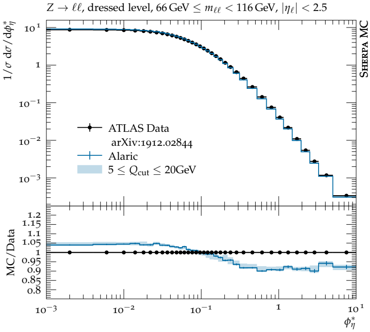
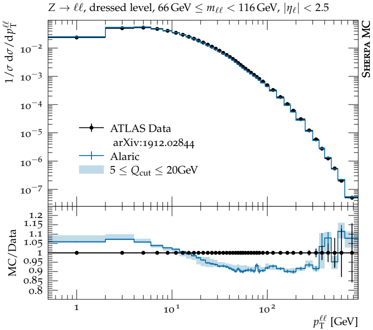




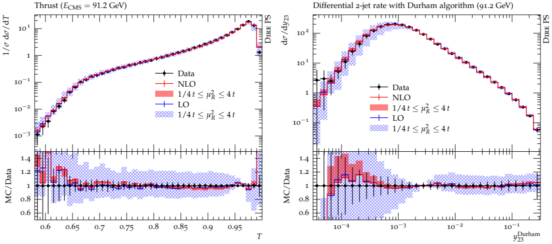
- ALARIC+PYTHIA string had., LO multijet merged

Developments towards higher (logarithmic) accuracy

To increase the logarithmic accuracy of the parton shower we need to

1) increase the perturbative order of the splitting functions
$\rightarrow$ NLO splitting functions
more involved singularity structure (double collinear, soft-collinear, double soft) $\rightarrow$ richer flavour structure ($P^{(1)}_{qq}$, $P^{(1)}_{qq'}$, ...)

2) ...



Evolution with NLO kernels $\neq$ NNLL accurate Höche, Krauss, Prestel '17

Other developments

- beyond leading colour approximation
  - many effects already included in improved large-N_c
  - $N_c = 3$ colour correlators, amplitude evolution
- beyond QCD and QED: EW showers
  - suffer from small relevance of resummed expressions at the LHC, more so at FCC, both -ee and -hh
    can often get by with fixed-order results
  - spin correlations a necessity
- spin correlations
  - mainstay of HERWIG parton shower, relevant for correlations of jet planes

Recap

This lecture:

- to achieve NLL accuracy parton showers must have
  - correct colour assignments
  - infrared-safe momentum maps
  - reproduce NLL coefficients (in the absence of momentum conservation)
- most standard showers are not NLL correct, with the exception of HERWIG's angular ordered shower $\rightarrow$ many new developments
- impact of formal NLL accuracy minute in standard observables

Next lectures:

- improving parton showers by process-dependent corrections $\rightarrow$ matching and merging