Matching and merging

Marek Schönherr

Institute for Particle Physics Phenomenology, Durham University

CERN, 12 Jun 2024

Matching and merging

Recap – Master equation

Expectation value of an observable at a hadron collider

$$\langle O \rangle = \sum_X \int_0^1 \mathrm{d}x_a \int_0^1 \mathrm{d}x_b \int \mathrm{d}\Phi_X \; f_a(x_1, \mu_\mathrm{F})\, f_b(x_b, \mu_\mathrm{F})\, \hat{\sigma}_{ab\to X}(\Phi_X; \mu_\mathrm{R}, \mu_\mathrm{F})\; O(\Phi_X)$$

The phase space element Φ_X is a set of initial and final state four momenta and corresponding flavours, obeying on-shell conditions and momentum conservation. X is an arbitrarily complex final state.

x_a, x_b are the Bjorken x momentum fractions of the initial state partons a and b. f_a and f_b are the associated parton distribution functions.

$\hat{\sigma}_{ab\to X}$ is the partonic transition matrix element for a and b reacting to produce X.
The observable O is our measurement function, it generally consist of a set of Θ-functions (cuts) etc., and takes the value 0 or 1.

Recap – Fixed-order calculations

At **leading order** approximation

$$\langle O\rangle^{\mathrm{LO}} = \int \mathrm{d}\Phi_B\, \mathrm{B}(\Phi_B)\, O(\Phi_B)$$

At **next-to-leading order**

$$\langle O\rangle^{\mathrm{NLO}} = \int \mathrm{d}\Phi_B \Big[\mathrm{B}(\Phi_B) + \mathrm{V}(\Phi_B) + \int \mathrm{d}\Phi_1 \mathrm{D}(\Phi_B \cdot \Phi_1)\Big]\, O(\Phi_B)$$
$$+ \int \mathrm{d}\Phi_R \Big[\mathrm{R}(\Phi_R)\, O(\Phi_R) - \mathrm{D}(\Phi_B \cdot \Phi_1)\, O(\Phi_B)\Big]$$

in any subtraction scheme

Recap – Fixed-order calculations

At **leading order** approximation

$$\langle O\rangle^{\text{LO}} = \int \mathrm{d}\Phi_B\, \mathrm{B}(\Phi_B)\; O(\Phi_B)$$

At **next-to-leading order**

$$\begin{aligned}\langle O\rangle^{\text{NLO}} = &\int \mathrm{d}\Phi_B \left[\mathrm{B}(\Phi_B) + \mathrm{V}(\Phi_B) + \int \mathrm{d}\Phi_1 \mathrm{D}(\Phi_B \cdot \Phi_1)\right] O(\Phi_B)\\ &+ \int \mathrm{d}\Phi_R \left[\mathrm{R}(\Phi_R)\; O(\Phi_R) - \mathrm{D}(\Phi_B \cdot \Phi_1)\; O(\Phi_B)\right]\end{aligned}$$

in any subtraction scheme

Recap – Fixed-order calculations

At **leading order** approximation

$$\langle O\rangle^{\mathsf{LO}} = \int \mathrm{d}\Phi_B\,\mathrm{B}(\Phi_B)\;O(\Phi_B)$$

At **next-to-leading order**

$$\begin{aligned}\langle O\rangle^{\mathsf{NLO}} =& \int \mathrm{d}\Phi_B\Big[\mathrm{B}(\Phi_B)+\mathrm{V}(\Phi_B)+\mathrm{I}_\mathrm{D}(\Phi_B)\Big]\;O(\Phi_B)\\ &+\int \mathrm{d}\Phi_R\,\Big[\mathrm{R}(\Phi_R)\;O(\Phi_R)-\mathrm{D}(\Phi_B\cdot\Phi_1)\;O(\Phi_B)\Big]\end{aligned}$$

in any subtraction scheme

Recap – Parton showers

LoPs

$$\begin{aligned}\langle O\rangle^{\text{LoPs}} &= \int \mathrm{d}\Phi_B\;\mathrm{B}(\Phi_B)\;\mathrm{PS}_B(t_0, O)\\ &= \int \mathrm{d}\Phi_B\;\mathrm{B}(\Phi_B)\;\Bigg[\Delta_B(t_c, t_0)\,O(\Phi_B)\\ &\qquad\qquad + \int_{t_c}^{t_0}\mathrm{d}\Phi_1\,\mathrm{K}_B(\Phi_1)\Delta_B(t, t_0)\,\mathrm{PS}_{B+1}(t, O)\Bigg]\end{aligned}$$

Parton showers give an estimate of higher order corrections in the soft-collinear limit.

Higher-order corrections in Monte-Carlo event generators II

1. Recap

2. Matching
   - Matching parton showers to fixed-order calculations

3. Merging
   - Multijet Merging at leading and next-to-leading order

4. Summary

Matching at LO

Let's start with the LO expression for the expectation value

$$\langle O\rangle^{\text{LoPs}} = \int \mathrm{d}\Phi_B\, \mathrm{B}(\Phi_B)\, O(\Phi_B)$$

Can we simply replace $O(\Phi_B)$ with the parton shower $\mathrm{PS}_n(t_B, O)$?

Yes.

There is no overlap in the terms included in the parton shower and the leading order matrix element.

The parton shower provides all higher order corrections to the given Born configuration in LOPS.

Matching at LO

Let's start with the LO expression for the expectation value

$$\langle O \rangle^{\text{LoPs}} = \int \mathrm{d}\Phi_B \, \mathrm{B}(\Phi_B) \, \mathrm{PS}_B(t_B, O)$$

Can we simply replace $O(\Phi_B)$ with the parton shower $\mathrm{PS}_n(t_B, O)$?

Yes.

There is no overlap in the terms included in the parton shower and the leading order matrix element.

The parton shower provides all higher order corrections to the given Born configuration in LoPs.

LoPs is trivial.

Matching at LO

Let's start with the LO expression for the expectation value

$$\langle O \rangle^{\text{LoPs}} = \int \mathrm{d}\Phi_B \, \mathrm{B}(\Phi_B) \, \mathrm{PS}_B(t_B, O)$$

Can we simply replace $O(\Phi_B)$ with the parton shower $\mathrm{PS}_n(t_B, O)$?

Yes.
There is no overlap in the terms included in the parton shower and the leading order matrix element.
The parton shower provides all higher order corrections to the given Born configuration in LoPs.

LoPs is trivial.
splitting kernels, starting conditions, NLL accuracy, etc.

Matching at LO

Let's start with the LO expression for the expectation value

$$\langle O\rangle^{\text{LoPs}} = \int \mathrm{d}\Phi_B\, \mathrm{B}(\Phi_B)\, \mathrm{PS}_B(t_B, O)$$

Can we simply replace $O(\Phi_B)$ with the parton shower $\mathrm{PS}_n(t_B, O)$?

Yes.
There is no overlap in the terms included in the parton shower and the leading order matrix element.
The parton shower provides all higher order corrections to the given Born configuration in LoPs.

LoPs is trivial.
splitting kernels, starting conditions, NLL accuracy, etc.

Matching at LO

Let's start with the LO expression for the expectation value

$$\langle O \rangle^{\text{LoPs}} = \int \mathrm{d}\Phi_B \, \mathrm{B}(\Phi_B) \, \mathrm{PS}_B(t_B, O)$$

Can we simply replace $O(\Phi_B)$ with the parton shower $\mathrm{PS}_n(t_B, O)$?

Yes.
There is no overlap in the terms included in the parton shower and the leading order matrix element.
The parton shower provides all higher order corrections to the given Born configuration in LoPs.

LoPs still contains many interesting problems:
splitting kernels, starting conditions, NLL accuracy, etc.

Matching at NLO

Let's start with the NLO expression for the expectation value

$$\langle O\rangle^{\text{NLO}} = \int \mathrm{d}\Phi_B \Big[\mathrm{B}+\mathrm{V}+\mathrm{I}_\mathrm{D}\Big](\Phi_B)\, O(\Phi_B) + \int \mathrm{d}\Phi_R \, \Big[\mathrm{R}(\Phi_R)\; O(t_R) - \mathrm{D}(\Phi_B\cdot\Phi_1)\; O(t_B)\Big]$$

Can we simply replace $O(\Phi_n)$ with the parton shower $\mathrm{PS}_n(t_n, O)$?

The first line seems fine, but in the second line there is a problem:

- R and D receive different parton shower corrections
- this spoils the subtraction in the IR limit

Additionally, as we have seen in the last lecture, $\mathrm{B}\cdot\mathrm{PS}_B(t_B,O)$ creates $\mathrm{V}_{\text{approx}}$ and $\mathrm{R}_{\text{approx}}$. They interfere with the proper V and R and spoil the NLO accuracy. Let's try again.

Matching at NLO

Let's start with the NLO expression for the expectation value

$$\begin{aligned}\langle O\rangle^{\text{NLO}} =&\int \mathrm{d}\Phi_B\Big[\mathrm{B}+\mathrm{V}+\mathrm{I_D}\Big](\Phi_B)\,\mathrm{PS}_B(t_B,O)\\ &+\int \mathrm{d}\Phi_R\,\Big[\mathrm{R}(\Phi_R)\,\mathrm{PS}_R(\Phi_R,O)-\mathrm{D}(\Phi_B\cdot\Phi_1)\,\mathrm{PS}_B(\Phi_B,O)\Big]\end{aligned}$$

Can we simply replace $O(\Phi_n)$ with the parton shower $\mathrm{PS}_n(t_n,O)$?

The first line seems fine, but in the second line there is a problem:

R and D receive different parton shower corrections, this spoils the subtraction in the IR limit

Additionally, as we have seen in the last lecture, $\mathrm{B}\cdot\mathrm{PS}_B(t_B,O)$ creates $\mathrm{V}_{\text{approx}}$ and $\mathrm{R}_{\text{approx}}$. They interfere with the proper V and R and spoil the NLO accuracy. **Let's try again.**

Matching at NLO

Let's start with the NLO expression for the expectation value

$$\langle O\rangle^{\text{NLO}} = \int \mathrm{d}\Phi_B \Big[\mathrm{B}+\mathrm{V}+\mathrm{I_D}\Big](\Phi_B)\,\mathrm{PS}_B(t_B,O) + \int \mathrm{d}\Phi_R\,\Big[\mathrm{R}(\Phi_R)\,\mathrm{PS}_R(\Phi_R,O) - \mathrm{D}(\Phi_B\cdot\Phi_1)\,\mathrm{PS}_B(\Phi_B,O)\Big]$$

Can we simply replace $O(\Phi_n)$ with the parton shower $\mathrm{PS}_n(t_n, O)$?

The first line seems fine, but in the second line there is a problem:

R and D receive different parton shower corrections,
this spoils the subtraction in the IR limit

Additionally, as we have seen in the last lecture, $\mathrm{B}\cdot\mathrm{PS}_B(t_B,O)$ creates $\mathrm{V}_{\text{approx}}$ and $\mathrm{R}_{\text{approx}}$. They interfere with the proper V and R and spoil the NLO accuracy. **Let's try again.**

Matching at NLO

Let's start with the NLO expression for the expectation value

$$
\begin{aligned}
\langle O\rangle^{\text{NLO}} =& \int \mathrm{d}\Phi_B \Big[\mathrm{B} + \mathrm{V} + \mathrm{I}_\mathrm{D}\Big](\Phi_B)\, \mathrm{PS}_B(t_B, O) \\
&+ \int \mathrm{d}\Phi_R \,\Big[\mathrm{R}(\Phi_R)\, \mathrm{PS}_R(\Phi_R, O) - \mathrm{D}(\Phi_B \cdot \Phi_1)\, \mathrm{PS}_B(\Phi_B, O)\Big]
\end{aligned}
$$

Can we simply replace $O(\Phi_n)$ with the parton shower $\mathrm{PS}_n(t_n, O)$?

The first line seems fine, but in the second line there is a problem:

R and D receive different parton shower corrections,
this spoils the subtraction in the IR limit

Additionally, as we have seen in the last lecture, $\mathrm{B} \cdot \mathrm{PS}_B(t_B, O)$ creates $\mathrm{V}_{\text{approx}}$ and $\mathrm{R}_{\text{approx}}$. They interfere with the proper V and R and spoil the NLO accuracy. **Let's try again.**

Matching at NLO

Let's start with the NLO expression for the expectation value

$$\langle O\rangle^{\text{NLO}} = \int \mathrm{d}\Phi_B \Big[\mathrm{B}+\mathrm{V}+\mathrm{I}_\mathrm{D}\Big](\Phi_B)\,\mathrm{PS}_B(t_B,O) + \int \mathrm{d}\Phi_R\,\Big[\mathrm{R}(\Phi_R)\,\mathrm{PS}_R(\Phi_R,O) - \mathrm{D}(\Phi_B\cdot\Phi_1)\,\mathrm{PS}_B(\Phi_B,O)\Big]$$

Can we simply replace $O(\Phi_n)$ with the parton shower $\mathrm{PS}_n(t_n,O)$?

The first line seems fine, but in the second line there is a problem:

R and D receive different parton shower corrections,
this spoils the subtraction in the IR limit

Additionally, as we have seen in the last lecture, $\mathrm{B}\cdot\mathrm{PS}_B(t_B,O)$ creates $\mathrm{V}_{\text{approx}}$ and $\mathrm{R}_{\text{approx}}$. They interfere with the proper V and R and spoil the NLO accuracy. **Let's try again.**

Matching at NLO

$$\langle O\rangle^{\mathsf{NLO}} = \int \mathrm{d}\Phi_B \Big[\mathrm{B} + \mathrm{V}\Big](\Phi_B)\, O(\Phi_B)$$
$$+ \int \mathrm{d}\Phi_R\, \mathrm{R}(\Phi_R)\; O(\Phi_R)$$

1. Let's examine what happens when we effect the PS on Φ_B to $\mathcal{O}(\alpha_s)$.

2. Still, NLO accuracy is spoiled, however, except for the sign, this looks very much like a subtraction at NLO.

Matching at NLO

$$\langle O\rangle^{\text{NLOPS}} = \int \mathrm{d}\Phi_B \Big[\mathrm{B}+\mathrm{V}\Big](\Phi_B)\,\mathrm{PS}(t_B, O) + \int \mathrm{d}\Phi_R\;\mathrm{R}(\Phi_R)\;O(\Phi_R)$$

1. Let's examine what happens when we effect the PS on Φ_B to $\mathcal{O}(\alpha_s)$.
   $\rightarrow$ we generate approximate real and virtual corrections

$$\mathrm{R}_{\text{approx}}(\Phi_B\Phi_1) = \mathrm{B}(\Phi_B)\cdot \mathrm{K}_B(\Phi_1) = \mathrm{D}_{\mathrm{K}}(\Phi_B\Phi_1)$$

$$\mathrm{V}_{\text{approx}}(\Phi_B) = -\,\mathrm{B}(\Phi_B)\int_{t_c}^{t_0}\mathrm{d}\Phi_1\,\mathrm{K}_B(\Phi_1) = -\mathrm{I}_{\mathrm{K}}(\Phi_B)$$

$$\mathrm{B}(\Phi_B)\mathrm{PS}_B(t_B,O)\big|_{\mathcal{O}(\alpha_s)} = \left[\mathrm{B}-\mathrm{I}_{\mathrm{K}}\right](\Phi_B)\,O(\Phi_B) + \mathrm{D}_{\mathrm{K}}(\Phi_B\Phi_1)\,O(\Phi_R)$$

2. Still, NLO accuracy is spoiled, however, except for the sign, this looks very much like a subtraction at NLO.

Matching at NLO

$$\langle O\rangle^{\text{NLOPS}} = \int \mathrm{d}\Phi_B \Big[\mathrm{B}+\mathrm{V}\Big](\Phi_B)\,\mathrm{PS}(t_B,O) + \int \mathrm{d}\Phi_R\;\mathrm{R}(\Phi_R)\;O(\Phi_R)$$

1. Let's examine what happens when we effect the PS on Φ_B to $\mathcal{O}(\alpha_s)$.
$\rightarrow$ we generate approximate real and virtual corrections

$$\begin{aligned}
\mathrm{R}_{\text{approx}}(\Phi_B\Phi_1) &= \mathrm{B}(\Phi_B)\cdot\mathrm{K}_B(\Phi_1) = \mathrm{D}_{\mathrm{K}}(\Phi_B\Phi_1)\\
\mathrm{V}_{\text{approx}}(\Phi_B) &= -\,\mathrm{B}(\Phi_B)\int_{t_c}^{t_0}\mathrm{d}\Phi_1\,\mathrm{K}_B(\Phi_1) = -\mathrm{I}_{\mathrm{K}}(\Phi_B)\\
\mathrm{B}(\Phi_B)\mathrm{PS}_B(t_B,O)\big|_{\mathcal{O}(\alpha_s)} &= \left[\mathrm{B}-\mathrm{I}_{\mathrm{K}}\right](\Phi_B)\,O(\Phi_B)+\mathrm{D}_{\mathrm{K}}(\Phi_B\Phi_1)\,O(\Phi_R)
\end{aligned}$$

2. Still, NLO accuracy is spoiled, however, except for the sign, this looks very much like a subtraction at NLO.

Matching at NLO

$$\langle O\rangle^{\text{NLOPS}} = \int \mathrm{d}\Phi_B \Big[\mathrm{B}+\mathrm{V}-\mathrm{I_K}\Big](\Phi_B)\, O(\Phi_B) + \int \mathrm{d}\Phi_R\, \Big[\mathrm{R}+\mathrm{D_K}\Big](\Phi_R)\, O(\Phi_R)$$

1. Let's examine what happens when we effect the PS on Φ_B to $\mathcal{O}(\alpha_s)$.
   $\rightarrow$ we generate approximate real and virtual corrections

$$\mathrm{R}_{\text{approx}}(\Phi_B\Phi_1) = \mathrm{B}(\Phi_B)\cdot \mathrm{K}_B(\Phi_1) = \mathrm{D_K}(\Phi_B\Phi_1)$$

$$\mathrm{V}_{\text{approx}}(\Phi_B) = -\,\mathrm{B}(\Phi_B)\int_{t_c}^{t_0}\mathrm{d}\Phi_1\,\mathrm{K}_B(\Phi_1) = -\mathrm{I_K}(\Phi_B)$$

$$\mathrm{B}(\Phi_B)\mathrm{PS}_B(t_B,O)\big|_{\mathcal{O}(\alpha_s)} = [\mathrm{B}-\mathrm{I_K}]\,(\Phi_B)\,O(\Phi_B)+\mathrm{D_K}(\Phi_B\Phi_1)\,O(\Phi_R)$$

2. Still, NLO accuracy is spoiled, however, except for the sign, this looks very much like a subtraction at NLO.

Matching at NLO

$$\langle O\rangle^{\text{NLOPS}} = \int \mathrm{d}\Phi_B \Big[\mathrm{B}+\mathrm{V}-\mathrm{I}_\mathrm{K}\Big](\Phi_B)\, O(\Phi_B) + \int \mathrm{d}\Phi_R \,\Big[\mathrm{R}+\mathrm{D}_\mathrm{K}\Big](\Phi_R)\; O(\Phi_R)$$

1. Let's examine what happens when we effect the PS on Φ_B to $\mathcal{O}(\alpha_s)$.
   $\rightarrow$ we generate approximate real and virtual corrections

$$\begin{aligned}
\mathrm{R}_\text{approx}(\Phi_B\Phi_1) &= \mathrm{B}(\Phi_B)\cdot\mathrm{K}_B(\Phi_1) \;=\; \mathrm{D}_\mathrm{K}(\Phi_B\Phi_1)\\
\mathrm{V}_\text{approx}(\Phi_B) &= \;-\,\mathrm{B}(\Phi_B)\int_{t_c}^{t_0}\mathrm{d}\Phi_1\,\mathrm{K}_B(\Phi_1) \;=\; -\mathrm{I}_\mathrm{K}(\Phi_B)\\
\mathrm{B}(\Phi_B)\mathrm{PS}_B(t_B,O)\big|_{\mathcal{O}(\alpha_s)} &= \left[\mathrm{B}-\mathrm{I}_\mathrm{K}\right](\Phi_B)\,O(\Phi_B)+\mathrm{D}_\mathrm{K}(\Phi_B\Phi_1)\,O(\Phi_R)
\end{aligned}$$

2. Still, NLO accuracy is spoiled, however, except for the sign, this looks very much like a subtraction at NLO.

Matching at NLO

$$\langle O\rangle^{\text{NLOPS}} = \int \mathrm{d}\Phi_B \Big[\mathrm{B}+\mathrm{V}+\mathrm{I}_\mathrm{K}\Big](\Phi_B)\,\mathrm{PS}_B(t_B,O) + \int \mathrm{d}\Phi_R\,\Big[\mathrm{R}-\mathrm{D}_\mathrm{K}\Big](\Phi_R)\;O(\Phi_R)$$

3. Let's examine what happens when we effect the PS on Φ_B to $\mathcal{O}(\alpha_s)$.

4. To $\mathcal{O}(\alpha_s)$ the exact NLO expression is recovered

In order for both $\Big[\mathrm{B}+\mathrm{V}+\mathrm{I}_\mathrm{K}\Big](\Phi_B)$ and $\Big[\mathrm{R}-\mathrm{D}_\mathrm{K}\Big]$ to be finite in the soft-collinear limit and, thus, integrable in 4 dimensions puts requirements on the accuracy of the parton shower $\to \widetilde{\mathrm{PS}}_B$

Matching at NLO

$$\langle O\rangle^{\text{NLOPS}} = \int \mathrm{d}\Phi_B \Big[\mathrm{B}+\mathrm{V}+\mathrm{I}_\mathrm{K}\Big](\Phi_B)\,\mathrm{PS}_B(t_B,O) + \int \mathrm{d}\Phi_R \,\Big[\mathrm{R}-\mathrm{D}_\mathrm{K}\Big](\Phi_R)\; O(\Phi_R)$$

3. Let's examine what happens when we effect the PS on Φ_B to $\mathcal{O}(\alpha_s)$.

$$\mathrm{B}(\Phi_B)\mathrm{PS}_B(t_B,O)\big|_{\mathcal{O}(\alpha_s)} = \left[\mathrm{B}-\mathrm{I}_\mathrm{K}\right](\Phi_B)\,O(\Phi_B) + \mathrm{D}_\mathrm{K}(\Phi_B\Phi_1)\,O(\Phi_R)$$

4. To $\mathcal{O}(\alpha_s)$ the exact NLO expression is recovered

In order for both $\Big[\mathrm{B}+\mathrm{V}+\mathrm{I}_\mathrm{K}\Big](\Phi_B)$ and $\Big[\mathrm{R}-\mathrm{D}_\mathrm{K}\Big]$ to be finite in the soft-collinear limit and, thus, integrable in 4 dimensions puts requirements on the accuracy of the parton shower $\to \widetilde{\mathrm{PS}}_B$

Matching at NLO

$$\langle O\rangle^{\text{NLOPS}} = \int \mathrm{d}\Phi_B \Big[\mathrm{B}+\mathrm{V}+\mathrm{I_K}\Big](\Phi_B)\,\mathrm{PS}_B(t_B,O) + \int \mathrm{d}\Phi_R\,\Big[\mathrm{R}-\mathrm{D_K}\Big](\Phi_R)\;O(\Phi_R)$$

3. Let's examine what happens when we effect the PS on Φ_B to $\mathcal{O}(\alpha_s)$.

$$\mathrm{B}(\Phi_B)\mathrm{PS}_B(t_B,O)\big|_{\mathcal{O}(\alpha_s)} = \left[\mathrm{B}-\mathrm{I_K}\right](\Phi_B)\,O(\Phi_B)+\mathrm{D_K}(\Phi_B\Phi_1)\,O(\Phi_R)$$

4. To $\mathcal{O}(\alpha_s)$ the exact NLO expression is recovered

In order for both $\Big[\mathrm{B}+\mathrm{V}+\mathrm{I_K}\Big](\Phi_B)$ and $\Big[\mathrm{R}-\mathrm{D_K}\Big]$ to be finite in the soft-collinear limit and, thus, integrable in 4 dimensions puts requirements on the accuracy of the parton shower $\to \widetilde{\mathrm{PS}}_B$

Matching at NLO

$$\langle O\rangle^{\text{NLOPS}} = \int \mathrm{d}\Phi_B \Big[\mathrm{B}+\mathrm{V}\Big](\Phi_B)\, O(\Phi_B) + \int \mathrm{d}\Phi_R\, \mathrm{R}(\Phi_R)\; O(\Phi_R)$$

3. Let's examine what happens when we effect the PS on Φ_B to $\mathcal{O}(\alpha_s)$.

$$\mathrm{B}(\Phi_B)\mathrm{PS}_B(t_B, O)\big|_{\mathcal{O}(\alpha_s)} = [\mathrm{B}-\mathrm{I_K}]\,(\Phi_B)\; O(\Phi_B) + \mathrm{D_K}(\Phi_B\Phi_1)\; O(\Phi_R)$$

4. To $\mathcal{O}(\alpha_s)$ the exact NLO expression is recovered

In order for both $\Big[\mathrm{B}+\mathrm{V}+\mathrm{I_K}\Big](\Phi_B)$ and $\Big[\mathrm{R}-\mathrm{D_K}\Big]$ to be finite in the soft-collinear limit and, thus, integrable in 4 dimensions puts requirements on the accuracy of the parton shower $\to \widetilde{\mathrm{PS}}_B$

Matching at NLO

$$\langle O\rangle^{\text{NLOPS}} = \int \mathrm{d}\Phi_B \Big[\mathrm{B} + \mathrm{V}\Big](\Phi_B)\, O(\Phi_B) + \int \mathrm{d}\Phi_R\, \mathrm{R}(\Phi_R)\; O(\Phi_R)$$

3. Let's examine what happens when we effect the PS on Φ_B to $\mathcal{O}(\alpha_s)$.

$$\mathrm{B}(\Phi_B)\mathrm{PS}_B(t_B, O)\big|_{\mathcal{O}(\alpha_s)} = \left[\mathrm{B} - \mathrm{I_K}\right](\Phi_B)\, O(\Phi_B) + \mathrm{D_K}(\Phi_B\Phi_1)\, O(\Phi_R)$$

4. To $\mathcal{O}(\alpha_s)$ the exact NLO expression is recovered

In order for both $\Big[\mathrm{B} + \mathrm{V} + \mathrm{I_K}\Big](\Phi_B)$ and $\Big[\mathrm{R} - \mathrm{D_K}\Big]$ to be finite in the soft-collinear limit and, thus, integrable in 4 dimensions puts requirements on the accuracy of the parton shower $\to \widetilde{\mathrm{PS}}_B$

Matching at NLO

$$\langle O\rangle^{\text{NLOPS}} = \int \mathrm{d}\Phi_B \Big[\mathrm{B}+\mathrm{V}+\mathrm{I}_\mathrm{K}\Big](\Phi_B)\,\widetilde{\mathrm{PS}}_B(t_B,O) + \int \mathrm{d}\Phi_R \,\Big[\mathrm{R}-\mathrm{D}_\mathrm{K}\Big](\Phi_R)\; O(\Phi_R)$$

3. Let's examine what happens when we effect the PS on Φ_B to $\mathcal{O}(\alpha_s)$.

$$\mathrm{B}(\Phi_B)\widetilde{\mathrm{PS}}_B(t_B,O)\Big|_{\mathcal{O}(\alpha_s)} = [\mathrm{B}-\mathrm{I}_\mathrm{K}]\,(\Phi_B)\,O(\Phi_B) + \mathrm{D}_\mathrm{K}(\Phi_B\Phi_1)\,O(\Phi_R)$$

4. To $\mathcal{O}(\alpha_s)$ the exact NLO expression is recovered

In order for both $\Big[\mathrm{B}+\mathrm{V}+\mathrm{I}_\mathrm{K}\Big](\Phi_B)$ and $\Big[\mathrm{R}-\mathrm{D}_\mathrm{K}\Big]$ to be finite in the soft-collinear limit and, thus, integrable in 4 dimensions puts requirements on the accuracy of the parton shower $\to \widetilde{\mathrm{PS}}_B$

POWHEG/MC@NLO

$$\langle O\rangle^{\text{NLOPS}} = \int \mathrm{d}\Phi_B \Big[\mathrm{B}+\mathrm{V}+\mathrm{I}_\mathrm{K}\Big](\Phi_B)\,\widetilde{\mathrm{PS}}_B(t_0,O) \qquad \mathbb{S}\text{-event}$$

$$+\int \mathrm{d}\Phi_R\,\Big[\mathrm{R}-\mathrm{D}_\mathrm{K}\Big](\Phi_R)\,\mathrm{PS}_R(t_R,O) \qquad \mathbb{H}\text{-event}$$

There are still a few choices possible

1) choice of kernel D_K in the matched emission in $\widetilde{\mathrm{PS}}$
2) choice of parton shower starting scale t_0

	MC@NLO	POWHEG
D_K	$\mathrm{B}\cdot\widetilde{\mathrm{K}}_\mathrm{PS}$	R
t_0	μ_F^2	S_had

- MC@NLO preserves parton shower resummation, corrected to $\mathcal{O}(\alpha_s)$
- POWHEG eliminates the second line, the source of negative weights

POWHEG/MC@NLO

$$\langle O \rangle^{\text{NLOPS}} = \int \mathrm{d}\Phi_B \, \overline{\mathrm{B}}\,(\Phi_B)\, \widetilde{\mathrm{PS}}_B(t_0, O) \qquad \mathbb{S}\text{-event}$$

$$+ \int \mathrm{d}\Phi_R \, \mathrm{H}\,(\Phi_R)\, \mathrm{PS}_R(t_R, O) \qquad \mathbb{H}\text{-event}$$

There are still a few choices possible

1) choice of kernel $\mathrm{D_K}$ in the matched emission in $\widetilde{\mathrm{PS}}$
2) choice of parton shower starting scale t_0

	MC@NLO	POWHEG
$\mathrm{D_K}$	$\mathrm{B} \cdot \widetilde{\mathrm{K}}_{\mathrm{PS}}$	R
t_0	μ_F^2	S_{had}

- MC@NLO preserves parton shower resummation, corrected to $\mathcal{O}(\alpha_s)$
- POWHEG eliminates the second line, the source of negative weights

POWHEG/MC@NLO

Description of emission spectrum:

$$\langle O_{\text{em}} \rangle^{\text{NLOPS}} = \int \mathrm{d}\Phi_B \mathrm{d}\Phi_1 \, \overline{\mathrm{B}}(\Phi_B) \, \frac{\mathrm{D_K}(\Phi_B \cdot \Phi_1)}{\mathrm{B}(\Phi_B)} \, O(\Phi_R) + \int \mathrm{d}\Phi_R \left[\mathrm{R} - \mathrm{D_K}\right](\Phi_R) \, O(\Phi_R)$$

In the resummation region $\mathrm{D_K} \neq 0$, spectrum enhanced with $\overline{\mathrm{B}}/\mathrm{B}$

	MC@NLO	POWHEG
$\mathrm{D_K}$	$\mathrm{B} \cdot \widetilde{\mathrm{K}}_{\mathrm{PS}}$	R
t_0	μ_F^2	S_{had}

Powheg:

split $\quad \mathrm{R} = \mathrm{R}_{\text{soft}} + \mathrm{R}_{\text{hard}}$

with

$$\mathrm{R}_{\text{soft}} = \frac{h^2}{p_\perp^2 + h^2} \, \mathrm{R}$$

and $\quad \mathrm{D_K} = \mathrm{R}_{\text{soft}}$

POWHEG/MC@NLO

Description of emission spectrum:

$$\langle O_{\text{em}}\rangle^{\text{NLOPS}} = \int \mathrm{d}\Phi_B \mathrm{d}\Phi_1 \, \frac{\overline{\mathrm{B}}(\Phi_B)}{\mathrm{B}(\Phi_B)} \, \mathrm{D_K}(\Phi_B \cdot \Phi_1) \, O(\Phi_R)$$
$$+ \int \mathrm{d}\Phi_R \, \Big[\mathrm{R} - \mathrm{D_K}\Big](\Phi_R) \, O(\Phi_R)$$

In the resummation region $\mathrm{D_K} \neq 0$, spectrum enhanced with $\overline{\mathrm{B}}/\mathrm{B}$

	MC@NLO	POWHEG
$\mathrm{D_K}$	$\mathrm{B} \cdot \widetilde{\mathrm{K}}_{\mathrm{PS}}$	R
t_0	μ_F^2	S_{had}

Powheg:

split $\quad \mathrm{R} = \mathrm{R}_{\text{soft}} + \mathrm{R}_{\text{hard}}$

with

$$\mathrm{R}_{\text{soft}} = \frac{h^2}{p_\perp^2 + h^2}\,\mathrm{R}$$

and $\quad \mathrm{D_K} = \mathrm{R}_{\text{soft}}$

POWHEG/MC@NLO

Description of emission spectrum:

$$\langle O_{\text{em}}\rangle^{\text{NLOPS}} = \int \mathrm{d}\Phi_R \left[\mathrm{R} + \left(\frac{\overline{\mathrm{B}}(\Phi_B)}{\mathrm{B}(\Phi_B)} - 1\right)\mathrm{D_K}\right](\Phi_R)\; O(\Phi_R)$$

In the resummation region $\mathrm{D_K} \neq 0$, spectrum enhanced with $\overline{\mathrm{B}}/\mathrm{B}$

	MC@NLO	POWHEG
$\mathrm{D_K}$	$\mathrm{B}\cdot\tilde{\mathrm{K}}_{\mathrm{PS}}$	R
t_0	μ_F^2	S_{had}

Powheg:

split $\quad \mathrm{R} = \mathrm{R}_{\text{soft}} + \mathrm{R}_{\text{hard}}$

with

$$\mathrm{R}_{\text{soft}} = \frac{h^2}{p_\perp^2 + h^2}\,\mathrm{R}$$

and $\quad \mathrm{D_K} = \mathrm{R}_{\text{soft}}$

Powheg/Mc@Nlo

Description of emission spectrum:

$$\langle O_{\text{em}} \rangle^{\text{NLOPS}} = \int \mathrm{d}\Phi_R \left[\mathrm{R} + \left(\frac{\overline{\mathrm{B}}(\Phi_B)}{\mathrm{B}(\Phi_B)} - 1 \right) \mathrm{D}_\mathrm{K} \right] (\Phi_R)\; O(\Phi_R)$$

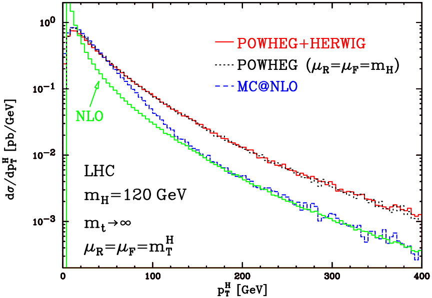
In the resummation region $\mathrm{D}_\mathrm{K} \neq 0$, spectrum enhanced with $\overline{\mathrm{B}}/\mathrm{B}$

	Mc@Nlo	Powheg
D_K	$\mathrm{B} \cdot \widetilde{\mathrm{K}}_\mathrm{PS}$	R
t_0	μ_F^2	S_had

Powheg:
split $\quad \mathrm{R} = \mathrm{R}_\text{soft} + \mathrm{R}_\text{hard}$
with

$$\mathrm{R}_\text{soft} = \frac{h^2}{p_\perp^2 + h^2}\,\mathrm{R}$$

and $\quad \mathrm{D}_\mathrm{K} = \mathrm{R}_\text{soft}$



POWHEG/MC@NLO

Description of emission spectrum:

$$\langle O_{\text{em}} \rangle^{\text{NLOPS}} = \int \mathrm{d}\Phi_R \left[\mathrm{R} + \left(\frac{\overline{\mathrm{B}}(\Phi_B)}{\mathrm{B}(\Phi_B)} - 1\right)\mathrm{D_K}\right](\Phi_R)\; O(\Phi_R)$$

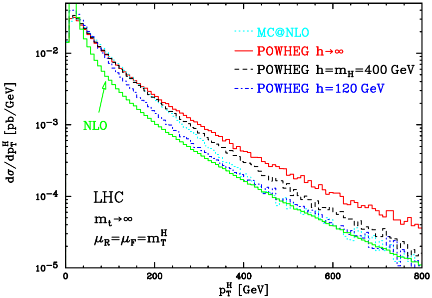
In the resummation region $\mathrm{D_K} \neq 0$, spectrum enhanced with $\overline{\mathrm{B}}/\mathrm{B}$

	MC@NLO	POWHEG
$\mathrm{D_K}$	$\mathrm{B} \cdot \widetilde{\mathrm{K}}_{\mathrm{PS}}$	R
t_0	μ_F^2	S_{had}

Powheg:
split $\mathrm{R} = \mathrm{R}_{\text{soft}} + \mathrm{R}_{\text{hard}}$
with

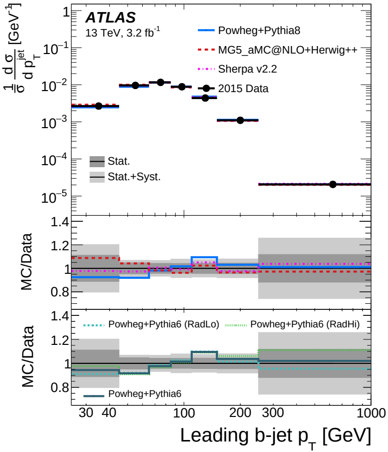
$$\mathrm{R}_{\text{soft}} = \frac{h^2}{p_\perp^2 + h^2}\,\mathrm{R}$$

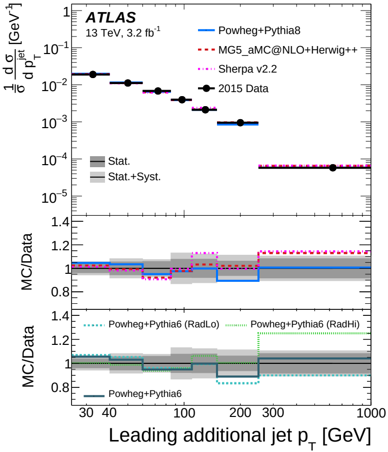
and $\mathrm{D_K} = \mathrm{R}_{\text{soft}}$



POWHEG/MC@NLO

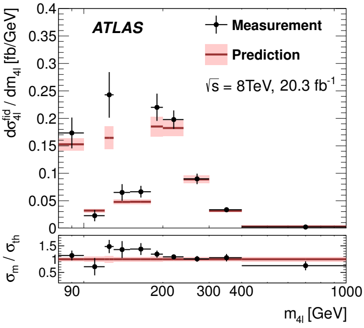
$t\bar{t}$ production arXiv:1610.09978

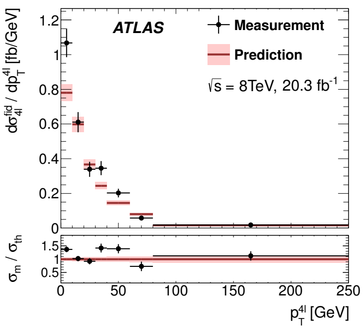




POWHEG/MC@NLO

4ℓ production   arXiv:1509.07844





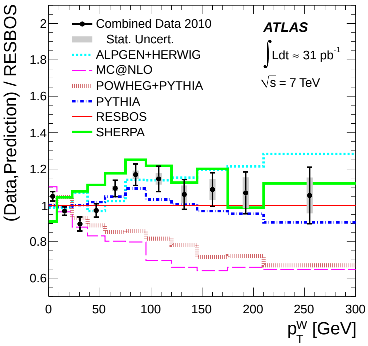
POWHEG/MC@NLO

arXiv:1108.6308

W production

The W transverse momentum in different regions is dominated by contributions from different final states:
W, Wj, Wjj, ...

NLOPS for incl. W production describe Wj at LO, Wjj with PS only



POWHEG/MC@NLO

POWHEG

- \+ almost only positive weights
- − modifies resummation
- − may need modification of resummation region

implemented in
- POWHEG-BOX
- HERWIG

MC@NLO

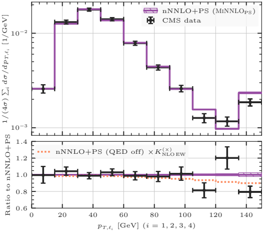
- \+ retains exact PS resummation
- − possibility of negative weights

implemented in
- MC@NLO
- SHERPA
- aMC@NLO
- HERWIG

Matching beyond NLO

MiNLO/MiNNLO

Buoncuore et.al. '21



- key idea:
  - start with a NLOPS simulation of $pp \to X + j$
  - superimpose a Sudakov factor on the first jet (present at Born level, no regular PS Sudakov)
  - now take p_T of first jet to zero, creating an inclusive sample $\to$ MiNLO
  - reweight in all Born phase space variables to fixed-order NNLO calculation $\to$ MiNNLO
- X (Born) observables NNLO correct through reweighting
  $X + j$ observables NLO correct through construction,
  reweighted only by $\mathcal{O}(\alpha_s^2)$
  $X + jj$ observables LO correct likewise

Higher-order corrections in Monte-Carlo event generators II

Multijet Merging

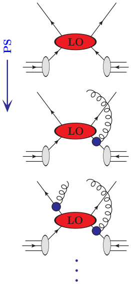
LoPs, NloPs and NnloPs describe observables dominated by topologies of a single multiplicity very well.

However, many observables receive contributions from many final state multiplicities. Examples: H_T, $p_\perp$, etc.
NloPs, for example, will describe the low end at NLO accuracy, an intermediate region at LO accuracy, and the high end at PS accuracy only

We want to describe these observables as uniformly as possible
$\Rightarrow$ **multijet merging**

At the same time, multijet merged samples provide the LHC experiments with largest freedom of projecting these samples onto observables without the loss of accuracy.

Multijet merging at LO

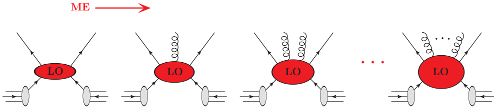


Parton showers

resummation of (soft-)coll. limit
$\rightarrow$ intrajet evolution

- matrix elements (ME) and parton showers (PS) are approximations in different regions of phase space
- MEPs combines multiple LOPs – keeping either accuracy
- NLOPs elevate LOPs to NLO accuracy
- MENLOPs supplements core NLOPs with higher multiplicities LOPs

Multijet merging at LO

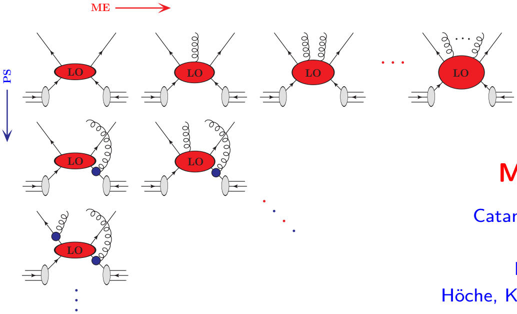


Matrix elements

fixed-order in α_s
$\rightarrow$ hard wide-angle emissions
$\rightarrow$ interference terms

- matrix elements (ME) and parton showers (PS) are approximations in different regions of phase space
- MEPS combines multiple LOPS – keeping either accuracy
- NLOPS elevate LOPS to NLO accuracy
- MENLOPS supplements core NLOPS with higher multiplicities LOPS

Multijet merging at LO



MEPS (CKKW,MLM)

Catani, Krauss, Kuhn, Webber JHEP11(2001)063
Lönnblad JHEP05(2002)046
Mangano, Moretti, Pittau NPB632(2002)343
Höche, Krauss, Schumann, Siegert JHEP05(2009)053
Lönnblad, Prestel JHEP03(2012)019

- matrix elements (ME) and parton showers (PS) are approximations in different regions of phase space
- MEPS combines multiple LOPS – keeping either accuracy
- NLOPS elevate LOPS to NLO accuracy
- MENLOPS supplements core NLOPS with higher multiplicities LOPS

Multijet merging at LO

$$\langle O\rangle^{\text{MEPS}} = \int \mathrm{d}\Phi_n\,\mathrm{B}_n\,\mathrm{PS}_n(O)\,\Theta(Q_{\text{cut}}-Q_{n+1})$$
$$+\int \mathrm{d}\Phi_{n+1}\,\mathrm{B}_{n+1}\,\Theta(Q_{n+1}-Q_{\text{cut}})\,\Delta_n(t_{n+1},t_n)$$
$$\mathrm{PS}_{n+1}(O)\,\Theta(Q_{\text{cut}}-Q_{n+2})$$
$$+\int \mathrm{d}\Phi_{n+2}\;,\mathrm{B}_{n+2}\,\Theta(Q_{n+2}-Q_{\text{cut}})\,\Delta_n(t_{n+1},t_n)\,\Delta_{n+1}(t_{n+2},t_{n+1})$$
$$\mathrm{PS}_{n+2}(O)$$

- restrict the parton shower on $2\to n$ to emit only below Q_{cut}
- arbitrary jet measure $Q_n=Q_n(\Phi_n)$
- add the $n+1$ ME and its parton shower
- multiply by Sudakov wrt. $2\to n$ process to restore resummation
- iterate

Multijet merging at LO

$$
\begin{aligned}
\langle O\rangle^{\text{MEPS}} = &\int \mathrm{d}\Phi_n\,\mathrm{B}_n\,\mathrm{PS}_n(O)\,\Theta(Q_{\text{cut}}-Q_{n+1})\\
&+\int \mathrm{d}\Phi_{n+1}\,\mathrm{B}_{n+1}\,\Theta(Q_{n+1}-Q_{\text{cut}})\,\Delta_n(t_{n+1},t_n)\\
&\qquad \mathrm{PS}_{n+1}(O)\,\Theta(Q_{\text{cut}}-Q_{n+2})\\
&+\int \mathrm{d}\Phi_{n+2}\,,\mathrm{B}_{n+2}\,\Theta(Q_{n+2}-Q_{\text{cut}})\,\Delta_n(t_{n+1},t_n)\,\Delta_{n+1}(t_{n+2},t_{n+1})\\
&\qquad \mathrm{PS}_{n+2}(O)
\end{aligned}
$$

- restrict the parton shower on $2\to n$ to emit only below Q_{cut}
- arbitrary jet measure $Q_n = Q_n(\Phi_n)$
- add the $n+1$ ME and its parton shower
- multiply by Sudakov wrt. $2\to n$ process to restore resummation
- iterate

Multijet merging at LO

$$\begin{aligned}\langle O\rangle^{\text{MEPS}} = &\int \mathrm{d}\Phi_n \, \mathrm{B}_n \, \mathrm{PS}_n(O) \, \Theta(Q_{\text{cut}} - Q_{n+1}) \\ &+ \int \mathrm{d}\Phi_{n+1} \, \mathrm{B}_{n+1} \, \Theta(Q_{n+1} - Q_{\text{cut}}) \, \Delta_n(t_{n+1}, t_n) \\ &\quad \mathrm{PS}_{n+1}(O) \, \Theta(Q_{\text{cut}} - Q_{n+2}) \\ &+ \int \mathrm{d}\Phi_{n+2} \, , \mathrm{B}_{n+2} \, \Theta(Q_{n+2} - Q_{\text{cut}}) \, \Delta_n(t_{n+1}, t_n) \, \Delta_{n+1}(t_{n+2}, t_{n+1}) \\ &\quad \mathrm{PS}_{n+2}(O)\end{aligned}$$

- restrict the parton shower on $2 \to n$ to emit only below Q_{cut}
- arbitrary jet measure $Q_n = Q_n(\Phi_n)$
- add the $n + 1$ ME and its parton shower
- multiply by Sudakov wrt. $2 \to n$ process to restore resummation
- iterate

Multijet merging at LO

$$
\begin{aligned}
\langle O\rangle^{\text{MEPS}} = & \int \mathrm{d}\Phi_n \, \mathrm{B}_n \, \mathrm{PS}_n(O) \, \Theta(Q_\text{cut} - Q_{n+1}) \\
& + \int \mathrm{d}\Phi_{n+1} \, \mathrm{B}_{n+1} \, \Theta(Q_{n+1} - Q_\text{cut}) \, \Delta_n(t_{n+1}, t_n) \\
& \quad \mathrm{PS}_{n+1}(O) \, \Theta(Q_\text{cut} - Q_{n+2}) \\
& + \int \mathrm{d}\Phi_{n+2} \,, \mathrm{B}_{n+2} \, \Theta(Q_{n+2} - Q_\text{cut}) \, \Delta_n(t_{n+1}, t_n) \, \Delta_{n+1}(t_{n+2}, t_{n+1}) \\
& \quad \mathrm{PS}_{n+2}(O)
\end{aligned}
$$

- restrict the parton shower on $2 \to n$ to emit only below Q_cut
- arbitrary jet measure $Q_n = Q_n(\Phi_n)$
- add the $n+1$ ME and its parton shower
- multiply by Sudakov wrt. $2 \to n$ process to restore resummation
- iterate

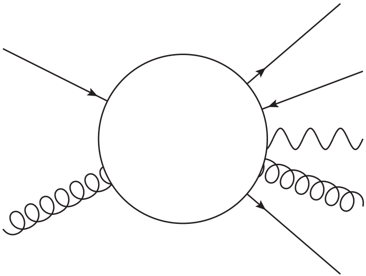
Multijet merging at LO

$$\begin{aligned}\langle O\rangle^{\text{MEPs}} = &\int \mathrm{d}\Phi_n\,\mathrm{B}_n\,\mathrm{PS}_n(O)\,\Theta(Q_\text{cut}-Q_{n+1})\\ &+\int \mathrm{d}\Phi_{n+1}\,\mathrm{B}_{n+1}\,\Theta(Q_{n+1}-Q_\text{cut})\,\Delta_n(t_{n+1},t_n)\\ &\qquad \mathrm{PS}_{n+1}(O)\,\Theta(Q_\text{cut}-Q_{n+2})\\ &+\int \mathrm{d}\Phi_{n+2}\,,\mathrm{B}_{n+2}\,\Theta(Q_{n+2}-Q_\text{cut})\,\Delta_n(t_{n+1},t_n)\,\Delta_{n+1}(t_{n+2},t_{n+1})\\ &\qquad \mathrm{PS}_{n+2}(O)\end{aligned}$$

- restrict the parton shower on $2\to n$ to emit only below Q_cut
- arbitrary jet measure $Q_n = Q_n(\Phi_n)$
- add the $n+1$ ME and its parton shower
- multiply by Sudakov wrt. $2\to n$ process to restore resummation
- iterate

Determination of scales t_i

Example: Drell-Yan production in association with jets

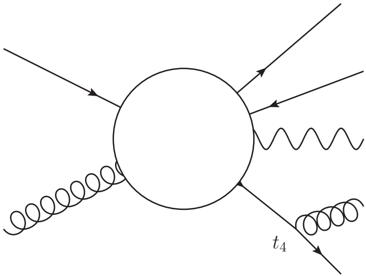


- cluster external particles using inverse parton shower $\rightarrow$ flavour conscious, initial state aware, probability determined through splitting kernels
- identify a shower history (probabilistically), determine scale t_i up to predefined t_I
- choose

$$\alpha_s^{n+k}(\mu_R^2) = \alpha_s^k(\mu_{\text{core}}^2)\prod_{i=1}^{n}\alpha_s(t_i)$$

Determination of scales t_i

Example: Drell-Yan production in association with jets

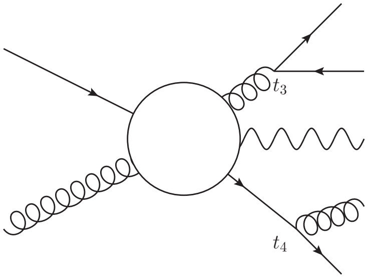


- cluster external particles using inverse parton shower $\rightarrow$ flavour conscious, initial state aware, probability determined through splitting kernels
- identify a shower history (probabilistically), determine scale t_i up to predefined t_I
- choose

$$\alpha_s^{n+k}(\mu_R^2) = \alpha_s^k(\mu_{\text{core}}^2) \prod_{i=1}^{n} \alpha_s(t_i)$$

Determination of scales t_i

Example: Drell-Yan production in association with jets

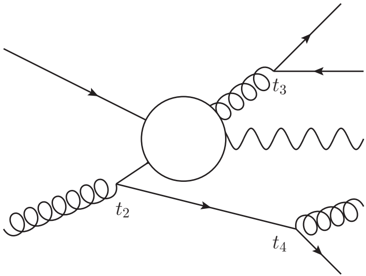


- cluster external particles using inverse parton shower $\rightarrow$ flavour conscious, initial state aware, probability determined through splitting kernels
- identify a shower history (probabilistically), determine scale t_i up to predefined t_I
- choose

$$\alpha_s^{n+k}(\mu_R^2) = \alpha_s^k(\mu_{\text{core}}^2) \prod_{i=1}^{n} \alpha_s(t_i)$$

Determination of scales t_i

Example: Drell-Yan production in association with jets

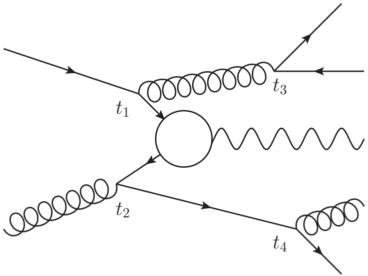


- cluster external particles using inverse parton shower $\rightarrow$ flavour conscious, initial state aware, probability determined through splitting kernels
- identify a shower history (probabilistically), determine scale t_i up to predefined t_I
- choose

$$\alpha_s^{n+k}(\mu_R^2) = \alpha_s^k(\mu_{\text{core}}^2) \prod_{i=1}^{n} \alpha_s(t_i)$$

Determination of scales t_i

Example: Drell-Yan production in association with jets

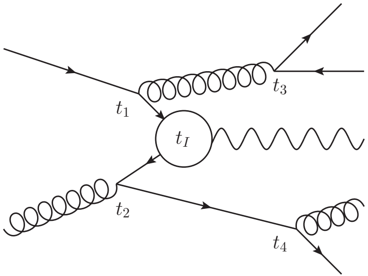


- cluster external particles using inverse parton shower $\rightarrow$ flavour conscious, initial state aware, probability determined through splitting kernels
- identify a shower history (probabilistically), determine scale t_i up to predefined t_I
- choose

$$\alpha_s^{n+k}(\mu_R^2) = \alpha_s^k(\mu_{\text{core}}^2)\prod_{i=1}^{n}\alpha_s(t_i)$$

Determination of scales t_i

Example: Drell-Yan production in association with jets

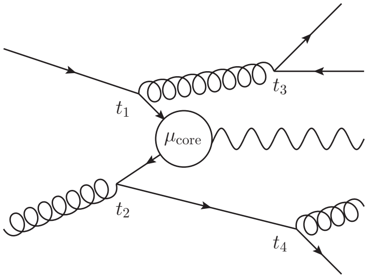


- cluster external particles using inverse parton shower $\rightarrow$ flavour conscious, initial state aware, probability determined through splitting kernels
- identify a shower history (probabilistically), determine scale t_i up to predefined t_I
- choose

$$\alpha_s^{n+k}(\mu_R^2) = \alpha_s^k(\mu_{\text{core}}^2)\prod_{i=1}^{n}\alpha_s(t_i)$$

Determination of scales t_i

Example: Drell-Yan production in association with jets



- cluster external particles using inverse parton shower $\rightarrow$ flavour conscious, initial state aware, probability determined through splitting kernels
- identify a shower history (probabilistically), determine scale t_i up to predefined t_I
- choose

$$\alpha_s^{n+k}(\mu_R^2) = \alpha_s^k(\mu_{\text{core}}^2) \prod_{i=1}^{n} \alpha_s(t_i)$$

Multijet merging at LO

- PS: higher order real emission corrections in (soft-)collinear limit
  $\rightarrow$ identify hard region, replace kernel with LO matrix element

$$\langle O\rangle^{\text{MEPS}} = \int \mathrm{d}\Phi_n\,\mathrm{B}\left[\Delta_n(t_c,t_{\max}) + \int_{t_c}^{t_{\max}} \mathrm{d}\Phi_1\;\mathrm{K}_n\;\Delta_n(t,t_{\max})\;\Theta(Q_{\text{cut}}-Q_{n+1})\right]$$
$$+\int \mathrm{d}\Phi_{n+1}\,\mathrm{B}_{n+1}\;\Delta_n(t',t_{\max})\;\Theta(Q_{n+1}-Q_{\text{cut}})$$

$$= \int \mathrm{d}\Phi_n\,\mathrm{B}\left[\Delta_n(t_c,t_{\max}) + \int_{t_c}^{t_{\max}} \mathrm{d}\Phi_1\;\mathrm{K}_n\;\Delta_n(t',t_{\max})\;\Theta(Q_{\text{cut}}-Q_{n+1}) + \int_{t_c}^{t_{\max}} \mathrm{d}\Phi_1\;\frac{\mathrm{B}_{n+1}}{\mathrm{B}_n}\;\Delta_n(t',t_{\max})\;\Theta(Q_{n+1}-Q_{\text{cut}})\right]$$
$$+\int \mathrm{d}\Phi_{n+1}\,\mathrm{B}_{n+1}\;\Theta(t-t_{\max})$$

- replace shower kernels in hard regeion by ratio of matrix elements
  $\rightarrow$ contains correct description of hard emissions & interference
  $\rightarrow$ spoils unitarity, beyond LOPS accuracy

Multijet merging at LO

- PS: higher order real emission corrections in (soft-)collinear limit
  $\rightarrow$ identify hard region, replace kernel with LO matrix element

$$
\begin{aligned}
\langle O\rangle^{\mathrm{MEPS}} &= \int \mathrm{d}\Phi_n\, \mathrm{B}\left[\Delta_n(t_c,t_{\max}) + \int_{t_c}^{t_{\max}} \mathrm{d}\Phi_1\, \mathrm{K}_n\, \Delta_n(t,t_{\max})\, \Theta(Q_{\mathrm{cut}}-Q_{n+1})\right] \\
&\quad + \int \mathrm{d}\Phi_{n+1}\, \mathrm{B}_{n+1}\, \Delta_n(t',t_{\max})\, \Theta(Q_{n+1}-Q_{\mathrm{cut}}) \\
&= \int \mathrm{d}\Phi_n\, \mathrm{B}\left[\Delta_n(t_c,t_{\max}) + \int_{t_c}^{t_{\max}} \mathrm{d}\Phi_1\, \mathrm{K}_n\, \Delta_n(t',t_{\max})\, \Theta(Q_{\mathrm{cut}}-Q_{n+1})\right. \\
&\qquad\qquad \left. + \int_{t_c}^{t_{\max}} \mathrm{d}\Phi_1\, \frac{\mathrm{B}_{n+1}}{\mathrm{B}_n}\, \Delta_n(t',t_{\max})\, \Theta(Q_{n+1}-Q_{\mathrm{cut}})\right] \\
&\quad + \int \mathrm{d}\Phi_{n+1}\, \mathrm{B}_{n+1}\, \Theta(t-t_{\max})
\end{aligned}
$$

- replace shower kernels in hard regeion by ratio of matrix elements
  $\rightarrow$ contains correct description of hard emissions & interference
  $\rightarrow$ spoils unitarity, beyond LOPS accuracy

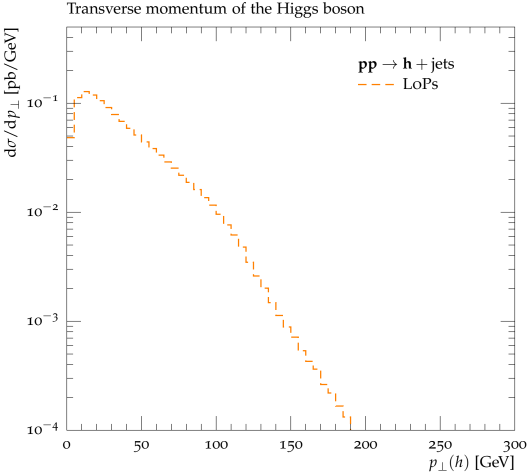
Multijet merging at LO

- PS: higher order real emission corrections in (soft-)collinear limit
  $\rightarrow$ identify hard region, replace kernel with LO matrix element

$$\begin{aligned}
\langle O\rangle^{\text{MEPS}} &= \int \mathrm{d}\Phi_n\,\mathrm{B}\left[\Delta_n(t_c,t_{\max}) + \int_{t_c}^{t_{\max}} \mathrm{d}\Phi_1\,\mathrm{K}_n\,\Delta_n(t,t_{\max})\,\Theta(Q_{\text{cut}}-Q_{n+1})\right] \\
&\quad + \int \mathrm{d}\Phi_{n+1}\,\mathrm{B}_{n+1}\,\Delta_n(t',t_{\max})\,\Theta(Q_{n+1}-Q_{\text{cut}}) \\
&= \int \mathrm{d}\Phi_n\,\mathrm{B}\left[\Delta_n(t_c,t_{\max}) + \int_{t_c}^{t_{\max}} \mathrm{d}\Phi_1\,\mathrm{K}_n\,\Delta_n(t',t_{\max})\,\Theta(Q_{\text{cut}}-Q_{n+1})\right. \\
&\qquad\qquad \left. + \int_{t_c}^{t_{\max}} \mathrm{d}\Phi_1\,\frac{\mathrm{B}_{n+1}}{\mathrm{B}_n}\,\Delta_n(t',t_{\max})\,\Theta(Q_{n+1}-Q_{\text{cut}})\right] \\
&\quad + \int \mathrm{d}\Phi_{n+1}\,\mathrm{B}_{n+1}\,\Theta(t-t_{\max})
\end{aligned}$$

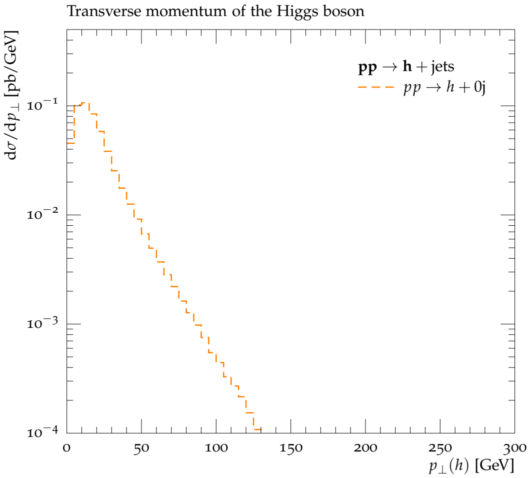
- replace shower kernels in hard regeion by ratio of matrix elements
  $\rightarrow$ contains correct description of hard emissions & interference
  $\rightarrow$ spoils unitarity, beyond LoPs accuracy

Multijet merging at LO



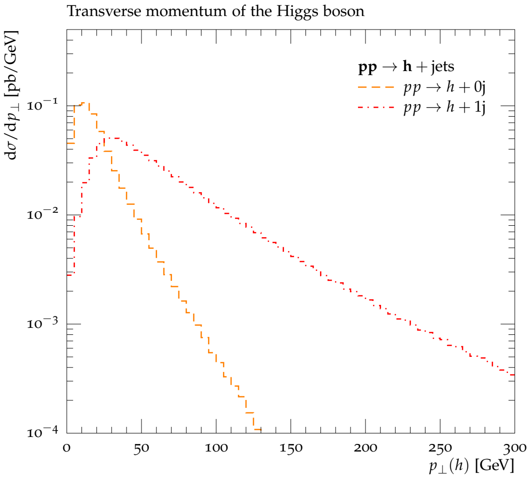
- first emission by PS, restrict to $Q_{n+1} < Q_{\text{cut}}$
- LoPs $pp \to h + \text{jet}$ for $Q_{n+1} > Q_{\text{cut}}$
- restrict emission off $pp \to h + \text{jet}$ to $Q_{n+2} < Q_{\text{cut}}$
- LoPs $pp \to h + 2\text{jets}$ for $Q_{n+2} > Q_{\text{cut}}$
- iterate
- sum all contributions

Multijet merging at LO



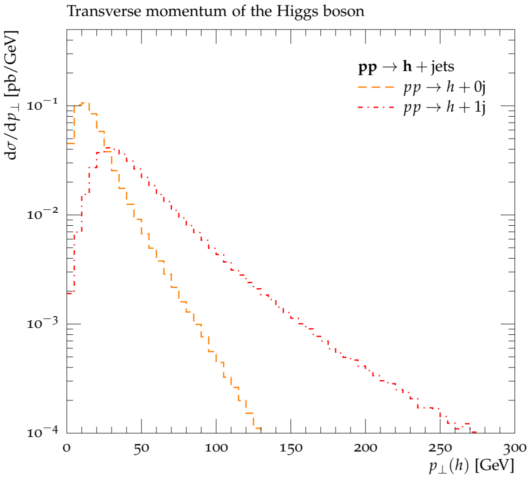
- first emission by PS, restrict to $Q_{n+1} < Q_{\text{cut}}$
- LoPs $pp \to h + \text{jet}$ for $Q_{n+1} > Q_{\text{cut}}$
- restrict emission off $pp \to h + \text{jet}$ to $Q_{n+2} < Q_{\text{cut}}$
- LoPs $pp \to h + 2\text{jets}$ for $Q_{n+2} > Q_{\text{cut}}$
- iterate
- sum all contributions

Multijet merging at LO



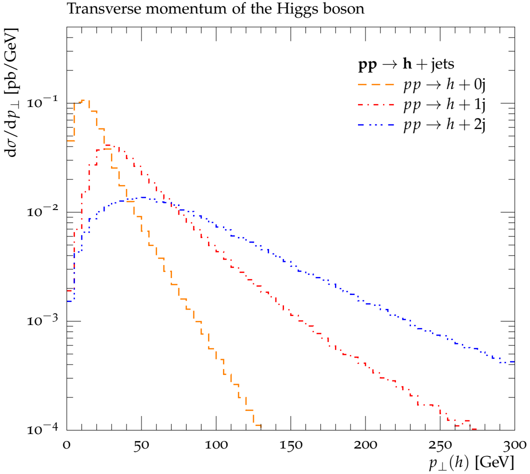
- first emission by PS, restrict to $Q_{n+1} < Q_{\text{cut}}$
- LoPs $pp \to h + \text{jet}$ for $Q_{n+1} > Q_{\text{cut}}$
- restrict emission off $pp \to h + \text{jet}$ to $Q_{n+2} < Q_{\text{cut}}$
- LoPs $pp \to h + 2\text{jets}$ for $Q_{n+2} > Q_{\text{cut}}$
- iterate
- sum all contributions

Multijet merging at LO



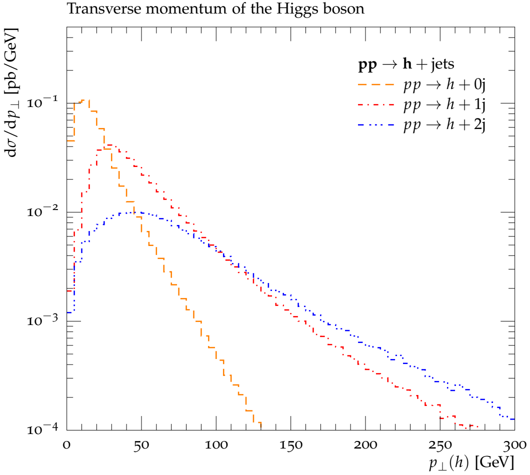
- first emission by PS, restrict to $Q_{n+1} < Q_{\text{cut}}$
- LoPs $pp \to h + \text{jet}$ for $Q_{n+1} > Q_{\text{cut}}$
- restrict emission off $pp \to h + \text{jet}$ to $Q_{n+2} < Q_{\text{cut}}$
- LoPs $pp \to h + 2\text{jets}$ for $Q_{n+2} > Q_{\text{cut}}$
- iterate
- sum all contributions

Multijet merging at LO



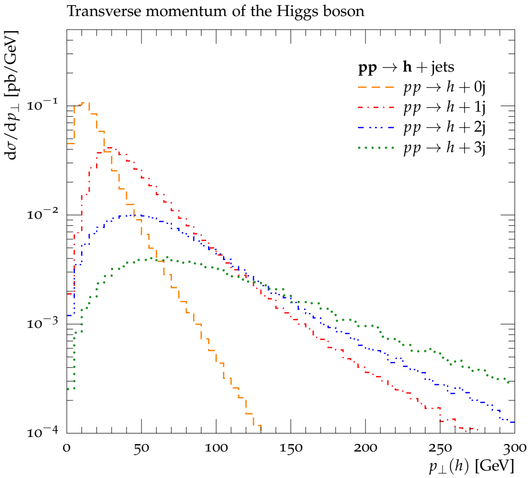
- first emission by PS, restrict to $Q_{n+1} < Q_{\text{cut}}$
- LoPs $pp \to h + \text{jet}$ for $Q_{n+1} > Q_{\text{cut}}$
- restrict emission off $pp \to h + \text{jet}$ to $Q_{n+2} < Q_{\text{cut}}$
- LoPs $pp \to h + 2\text{jets}$ for $Q_{n+2} > Q_{\text{cut}}$
- iterate
- sum all contributions

Multijet merging at LO



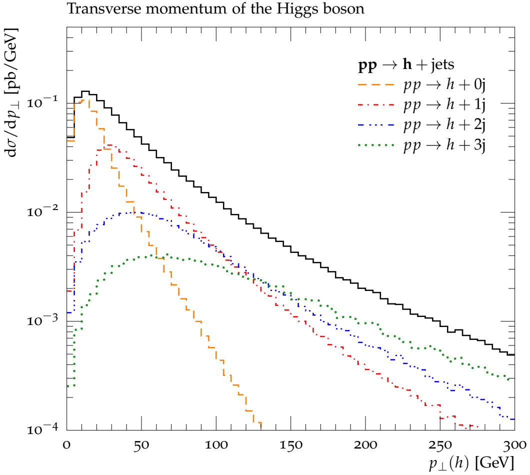
- first emission by PS, restrict to $Q_{n+1} < Q_\mathrm{cut}$
- LoPs $pp \to h + \mathrm{jet}$ for $Q_{n+1} > Q_\mathrm{cut}$
- restrict emission off $pp \to h + \mathrm{jet}$ to $Q_{n+2} < Q_\mathrm{cut}$
- LoPs $pp \to h + 2\mathrm{jets}$ for $Q_{n+2} > Q_\mathrm{cut}$
- iterate
- sum all contributions

Multijet merging at LO



- first emission by PS, restrict to $Q_{n+1} < Q_{\mathrm{cut}}$
- LoPs $pp \to h + \mathrm{jet}$ for $Q_{n+1} > Q_{\mathrm{cut}}$
- restrict emission off $pp \to h + \mathrm{jet}$ to $Q_{n+2} < Q_{\mathrm{cut}}$
- LoPs $pp \to h + 2\mathrm{jets}$ for $Q_{n+2} > Q_{\mathrm{cut}}$
- iterate
- sum all contributions

Multijet merging at LO



- first emission by PS, restrict to $Q_{n+1} < Q_{\text{cut}}$
- LoPs $pp \to h + \text{jet}$ for $Q_{n+1} > Q_{\text{cut}}$
- restrict emission off $pp \to h + \text{jet}$ to $Q_{n+2} < Q_{\text{cut}}$
- LoPs $pp \to h + 2\text{jets}$ for $Q_{n+2} > Q_{\text{cut}}$
- iterate
- sum all contributions

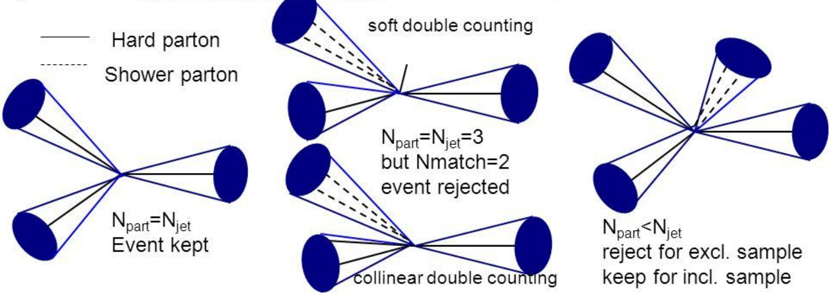
MLM

Mangano (2002)
Used in Alpgen

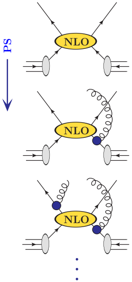
Prevent parton shower harder than any emission by ME using cone algo:

1) Generate hard parton configuration for given n=N_{part} with ME, imposing $|\eta^i| < \eta^{max}, E_T^i > E_T^{min}, \Delta R_{ij} > R^{min}$
2) Define tree branching structure using K_T-algo allowing only pairing consistent with color flow
3) Compute α_s at the nodal values, but do not apply Sudakov factors
4) Shower the hard event without any veto using Herwig/Pythia
   when done, find N_{jet} jet with cone algorithm with E_T^{min}, R^{min}
   if $N_{part}<N_{jet}$ reject event
5) Matched jets to hard partons using $\min \Delta R_{i,jets}$
   Only keep events, if each hard parton is uniquely contained in jets
   Events with $N_{part}<N_{jet}$ are rejected except for highest multiplicities
6) Define exclusive N-jet sample by requiring N_{part} =N_{jet}
7) After matching, combine exclusive samples to one inclusive sample

This is equivalent to tSudakov reweighting in CKKW (external lines)



Multijet merging at NLO

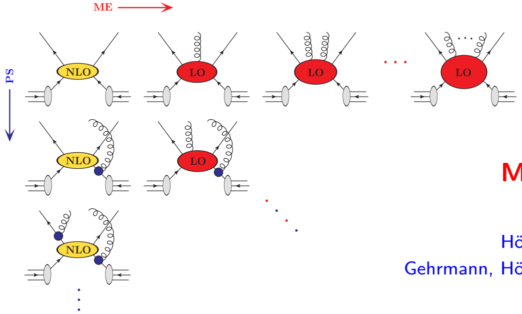


NLOPS (MC@NLO,POWHEG)

Frixione, Webber JHEP06(2002)029
Nason JHEP11(2004)040, Frixione et.al. JHEP11(2007)070
Höche, Krauss, MS, Siegert JHEP09(2012)049

- matrix elements (ME) and parton showers (PS) are approximations in different regions of phase space
- Multijet merging at LO combines multiple LOPS
- NLOPS elevate LOPS to NLO accuracy
- First steo supplements core NLOPS with higher multiplicities LOPS
- **Multijet merging at NLO combines multiple NLOPS**

Multijet merging at NLO

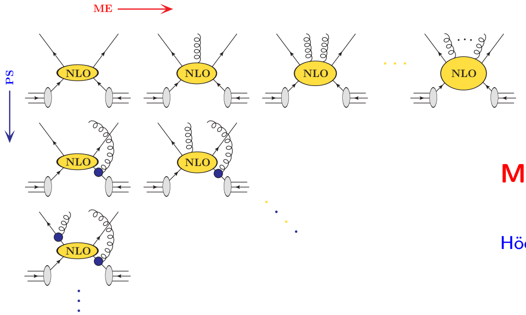


Multijet merging

Hamilton, Nason JHEP06(2010)039
Höche, Krauss, MS, Siegert JHEP08(2011)123
Gehrmann, Höche, Krauss, MS, Siegert JHEP01(2013)144

- matrix elements (ME) and parton showers (PS) are approximations in different regions of phase space
- Multijet merging at LO combines multiple LoPs
- NLoPs elevate LoPs to NLO accuracy
- First steo supplements core NLoPs with higher multiplicities LoPs
- Multijet merging at NLO combines multiple NLoPs

Multijet merging at NLO



Multijet merging at NLO

Lavesson, Lönnblad JHEP12(2008)070
Höche, Krauss, MS, Siegert JHEP04(2013)027
Fredrerix, Frixione JHEP12(2012)061
Lönnblad, Prestel JHEP03(2013)166
Plätzer JHEP08(2013)114

- matrix elements (ME) and parton showers (PS) are approximations in different regions of phase space
- Multijet merging at LO combines multiple LOPS
- NLOPS elevate LOPS to NLO accuracy
- First steo supplements core NLOPS with higher multiplicities LOPS
- **Multijet merging at NLO combines multiple NLOPS**

Multijet merging at NLO

$$
\begin{aligned}
\langle O\rangle^{\text{MePs@Nlo}} = &\int \mathrm{d}\Phi_n\,\overline{\mathrm{B}}_n\,\widetilde{\mathrm{PS}}_n(O)\,\Theta(Q_\text{cut}-Q_{n+1})\\
&+\int \mathrm{d}\Phi_{n+1}\,\mathrm{H}_n\,\Theta(Q_\text{cut}-Q_{n+1})\,\mathrm{PS}_{n+1}(O)\\
&+\int \mathrm{d}\Phi_{n+1}\,\overline{\mathrm{B}}_{n+1}\,\Theta(Q_{n+1}-Q_\text{cut})\left(\Delta_n(t_{n+1},t_n)-\Delta_n^{(1)}(t_{n+1},t_n)\right)\\
&\qquad\widetilde{\mathrm{PS}}_{n+1}(O)\,\Theta(Q_\text{cut}-Q_{n+2})\\
&+\int \mathrm{d}\Phi_{n+2}\,\mathrm{H}_{n+1}\,\Theta(Q_{n+1}-Q_\text{cut})\\
&\qquad\mathrm{PS}_{n+1}(O)\,\Theta(Q_\text{cut}-Q_{n+2})\\
&+\ldots
\end{aligned}
$$

- NloPs for $2\to n$, restricted to emit only below Q_cut
- add the NloPs for $2\to n+1$
- multiply by Sudakov wrt. $2\to n$ process to restore resummation
- remove overlap of Δ_n and $\mathrm{d}\sigma_{n+1}^{\text{NLO}}$, iterate

Multijet merging at NLO

$$
\begin{aligned}
\langle O\rangle^{\text{MEPS@NLO}} = & \int \mathrm{d}\Phi_n\, \overline{\mathrm{B}}_n\, \widetilde{\mathrm{PS}}_n(O)\, \Theta(Q_{\text{cut}} - Q_{n+1}) \\
& + \int \mathrm{d}\Phi_{n+1}\, \mathrm{H}_n\, \Theta(Q_{\text{cut}} - Q_{n+1})\, \mathrm{PS}_{n+1}(O) \\
& + \int \mathrm{d}\Phi_{n+1}\, \overline{\mathrm{B}}_{n+1}\, \Theta(Q_{n+1} - Q_{\text{cut}}) \left(\Delta_n(t_{n+1}, t_n) - \Delta_n^{(1)}(t_{n+1}, t_n)\right) \\
& \qquad \widetilde{\mathrm{PS}}_{n+1}(O)\, \Theta(Q_{\text{cut}} - Q_{n+2}) \\
& + \int \mathrm{d}\Phi_{n+2}\, \mathrm{H}_{n+1}\, \Theta(Q_{n+1} - Q_{\text{cut}}) \\
& \qquad \mathrm{PS}_{n+1}(O)\, \Theta(Q_{\text{cut}} - Q_{n+2}) \\
& + \ldots
\end{aligned}
$$

- NLOPS for $2 \to n$, restricted to emit only below Q_{cut}
- add the NLOPS for $2 \to n+1$
- multiply by Sudakov wrt. $2 \to n$ process to restore resummation
- remove overlap of Δ_n and $\mathrm{d}\sigma_{n+1}^{\text{NLO}}$, iterate

Multijet merging at NLO

$$
\begin{aligned}
\langle O\rangle^{\textsc{MePs@Nlo}} = & \int \mathrm{d}\Phi_n\,\overline{\mathrm{B}}_n\,\widetilde{\mathrm{PS}}_n(O)\,\Theta(Q_{\text{cut}}-Q_{n+1}) \\
& + \int \mathrm{d}\Phi_{n+1}\,\mathrm{H}_n\,\Theta(Q_{\text{cut}}-Q_{n+1})\,\mathrm{PS}_{n+1}(O) \\
& + \int \mathrm{d}\Phi_{n+1}\,\overline{\mathrm{B}}_{n+1}\,\Theta(Q_{n+1}-Q_{\text{cut}})\left(\Delta_n(t_{n+1},t_n)-\Delta_n^{(1)}(t_{n+1},t_n)\right) \\
& \qquad\quad \widetilde{\mathrm{PS}}_{n+1}(O)\,\Theta(Q_{\text{cut}}-Q_{n+2}) \\
& + \int \mathrm{d}\Phi_{n+2}\,\mathrm{H}_{n+1}\,\Theta(Q_{n+1}-Q_{\text{cut}}) \\
& \qquad\quad \mathrm{PS}_{n+1}(O)\,\Theta(Q_{\text{cut}}-Q_{n+2}) \\
& + \ldots
\end{aligned}
$$

- NLOPS for $2 \to n$, restricted to emit only below Q_{cut}
- add the NLOPS for $2 \to n+1$
- multiply by Sudakov wrt. $2 \to n$ process to restore resummation
- remove overlap of Δ_n and $\mathrm{d}\sigma_{n+1}^{\text{NLO}}$, iterate

Multijet merging at NLO

$$
\begin{aligned}
\langle O\rangle^{\text{MePs@Nlo}} = &\int \mathrm{d}\Phi_n\, \overline{\mathrm{B}}_n\, \widetilde{\mathrm{PS}}_n(O)\, \Theta(Q_{\text{cut}} - Q_{n+1}) \\
&+ \int \mathrm{d}\Phi_{n+1}\, \mathrm{H}_n\, \Theta(Q_{\text{cut}} - Q_{n+1})\, \mathrm{PS}_{n+1}(O) \\
&+ \int \mathrm{d}\Phi_{n+1}\, \overline{\mathrm{B}}_{n+1}\, \Theta(Q_{n+1} - Q_{\text{cut}}) \left(\Delta_n(t_{n+1}, t_n) - \Delta_n^{(1)}(t_{n+1}, t_n) \right) \\
&\qquad \widetilde{\mathrm{PS}}_{n+1}(O)\, \Theta(Q_{\text{cut}} - Q_{n+2}) \\
&+ \int \mathrm{d}\Phi_{n+2}\, \mathrm{H}_{n+1}\, \Theta(Q_{n+1} - Q_{\text{cut}}) \\
&\qquad \mathrm{PS}_{n+1}(O)\, \Theta(Q_{\text{cut}} - Q_{n+2}) \\
&+ \ldots
\end{aligned}
$$

- NLOPS for $2 \to n$, restricted to emit only below Q_{cut}
- add the NLOPS for $2 \to n+1$
- multiply by Sudakov wrt. $2 \to n$ process to restore resummation
- remove overlap of Δ_n and $\mathrm{d}\sigma_{n+1}^{\text{NLO}}$, iterate

Multijet merging at NLO

$$
\begin{aligned}
\langle O\rangle^{\text{MEPS@NLO}} = &\int \mathrm{d}\Phi_n\, \overline{\mathrm{B}}_n\, \widetilde{\mathrm{PS}}_n(O)\, \Theta(Q_{\text{cut}} - Q_{n+1}) \\
&+ \int \mathrm{d}\Phi_{n+1}\, \mathrm{H}_n\, \Theta(Q_{\text{cut}} - Q_{n+1})\, \mathrm{PS}_{n+1}(O) \\
&+ \int \mathrm{d}\Phi_{n+1}\, \overline{\mathrm{B}}_{n+1}\, \Theta(Q_{n+1} - Q_{\text{cut}}) \left(\Delta_n(t_{n+1}, t_n) - \Delta_n^{(1)}(t_{n+1}, t_n)\right) \\
&\qquad \widetilde{\mathrm{PS}}_{n+1}(O)\, \Theta(Q_{\text{cut}} - Q_{n+2}) \\
&+ \int \mathrm{d}\Phi_{n+2}\, \mathrm{H}_{n+1}\, \Theta(Q_{n+1} - Q_{\text{cut}}) \\
&\qquad \mathrm{PS}_{n+1}(O)\, \Theta(Q_{\text{cut}} - Q_{n+2}) \\
&+ \ldots
\end{aligned}
$$

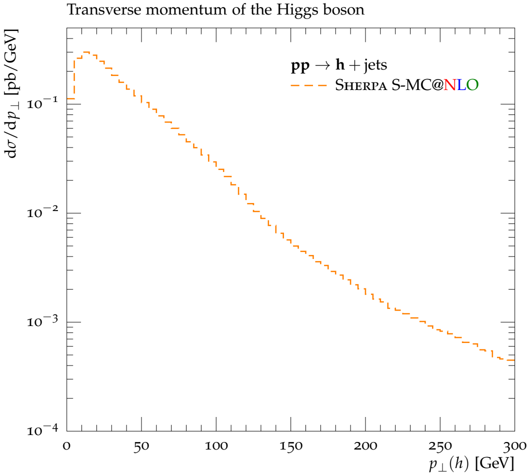
- NLOPS for $2 \to n$, restricted to emit only below Q_{cut}
- add the NLOPS for $2 \to n+1$
- multiply by Sudakov wrt. $2 \to n$ process to restore resummation
- remove overlap of Δ_n and $\mathrm{d}\sigma_{n+1}^{\text{NLO}}$, iterate

Multijet merging at NLO

$$
\begin{aligned}
\langle O\rangle^{\text{MEPS@NLO}} = &\int \mathrm{d}\Phi_n\,\overline{\mathrm{B}}_n\,\widetilde{\mathrm{PS}}_n(O)\,\Theta(Q_{\text{cut}}-Q_{n+1})\\
&+\int \mathrm{d}\Phi_{n+1}\,\mathrm{H}_n\,\Theta(Q_{\text{cut}}-Q_{n+1})\,\mathrm{PS}_{n+1}(O)\\
&+\int \mathrm{d}\Phi_{n+1}\,\overline{\mathrm{B}}_{n+1}\,\Theta(Q_{n+1}-Q_{\text{cut}})\left(\Delta_n(t_{n+1},t_n)-\Delta_n^{(1)}(t_{n+1},t_n)\right)\\
&\qquad\quad \widetilde{\mathrm{PS}}_{n+1}(O)\,\Theta(Q_{\text{cut}}-Q_{n+2})\\
&+\int \mathrm{d}\Phi_{n+2}\,\mathrm{H}_{n+1}\,\Theta(Q_{n+1}-Q_{\text{cut}})\\
&\qquad\quad \mathrm{PS}_{n+1}(O)\,\Theta(Q_{\text{cut}}-Q_{n+2})\\
&+\ldots
\end{aligned}
$$

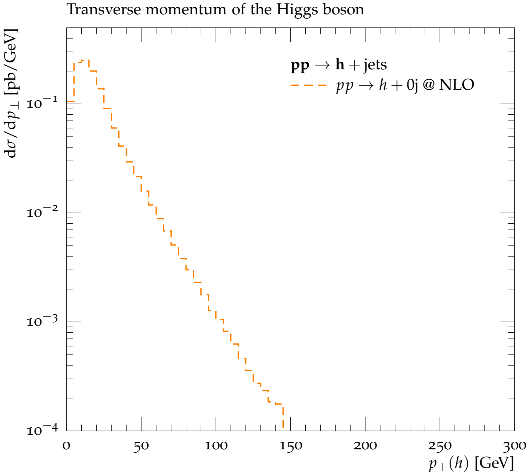
- NLOPS for $2\to n$, restricted to emit only below Q_{cut}
- add the NLOPS for $2\to n+1$
- multiply by Sudakov wrt. $2\to n$ process to restore resummation
- remove overlap of Δ_n and $\mathrm{d}\sigma_{n+1}^{\text{NLO}}$, iterate

Multijet merging at NLO



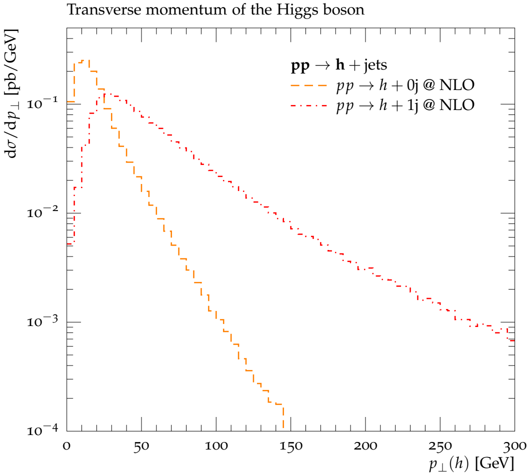
- first emission by NLOPS , restrict to $Q_{n+1} < Q_{\text{cut}}$
- NLOPS $pp \to h +$ jet for $Q_{n+1} > Q_{\text{cut}}$
- restrict emission off $pp \to h +$ jet to $Q_{n+2} < Q_{\text{cut}}$
- NLOPS $pp \to h +$ 2jets for $Q_{n+2} > Q_{\text{cut}}$
- iterate
- sum all contributions

Multijet merging at NLO



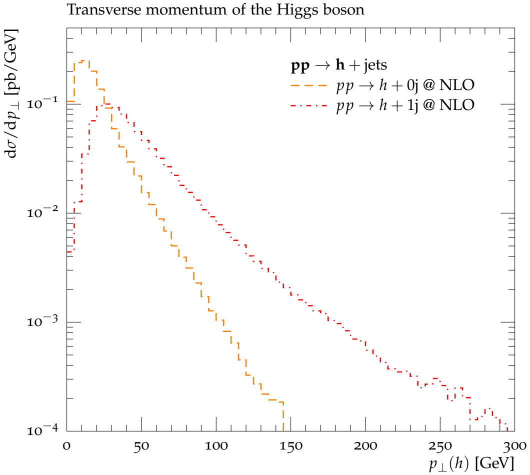
- first emission by NLOPS , restrict to $Q_{n+1} < Q_{\text{cut}}$
- NLOPS $pp \rightarrow h + \text{jet}$ for $Q_{n+1} > Q_{\text{cut}}$
- restrict emission off $pp \rightarrow h + \text{jet}$ to $Q_{n+2} < Q_{\text{cut}}$
- NLOPS $pp \rightarrow h + 2\text{jets}$ for $Q_{n+2} > Q_{\text{cut}}$
- iterate
- sum all contributions

Multijet merging at NLO



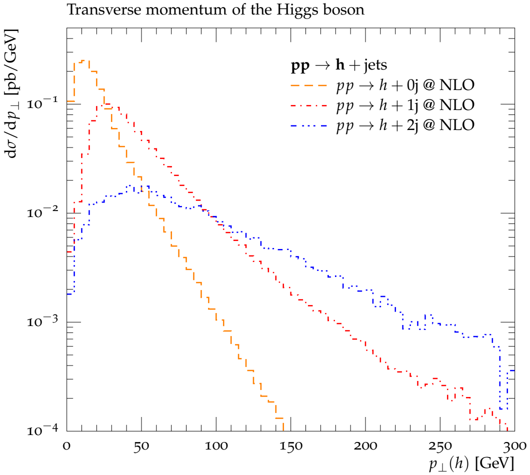
- first emission by NLOPS , restrict to $Q_{n+1} < Q_{\mathrm{cut}}$
- NLOPS $pp \to h + \mathrm{jet}$ for $Q_{n+1} > Q_{\mathrm{cut}}$
- restrict emission off $pp \to h + \mathrm{jet}$ to $Q_{n+2} < Q_{\mathrm{cut}}$
- NLOPS $pp \to h + 2\mathrm{jets}$ for $Q_{n+2} > Q_{\mathrm{cut}}$
- iterate
- sum all contributions

Multijet merging at NLO



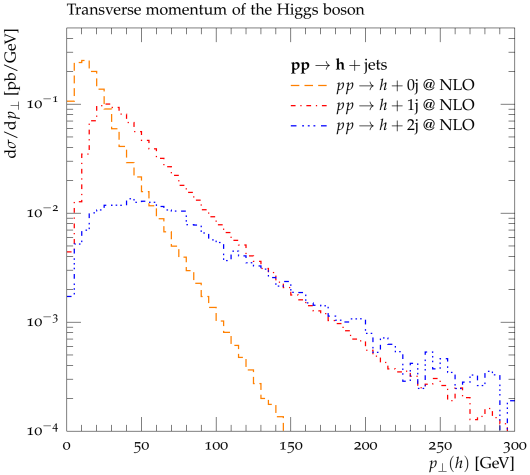
- first emission by NLOPS , restrict to $Q_{n+1} < Q_\text{cut}$
- NLOPS $pp \to h + \text{jet}$ for $Q_{n+1} > Q_\text{cut}$
- restrict emission off $pp \to h + \text{jet}$ to $Q_{n+2} < Q_\text{cut}$
- NLOPS $pp \to h + 2\text{jets}$ for $Q_{n+2} > Q_\text{cut}$
- iterate
- sum all contributions

Multijet merging at NLO



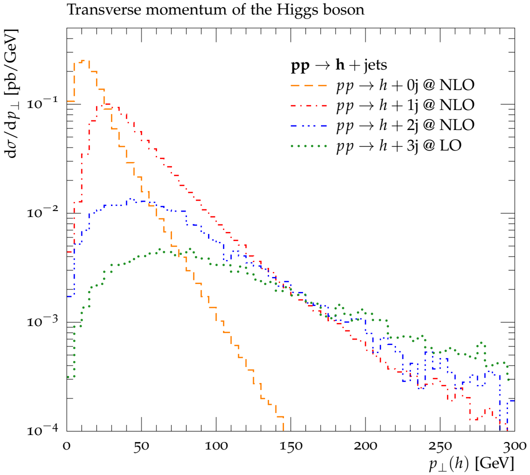
- first emission by NLOPS, restrict to $Q_{n+1} < Q_{\text{cut}}$
- NLOPS $pp \to h +$ jet for $Q_{n+1} > Q_{\text{cut}}$
- restrict emission off $pp \to h +$ jet to $Q_{n+2} < Q_{\text{cut}}$
- NLOPS $pp \to h +$ 2jets for $Q_{n+2} > Q_{\text{cut}}$
- iterate
- sum all contributions

Multijet merging at NLO



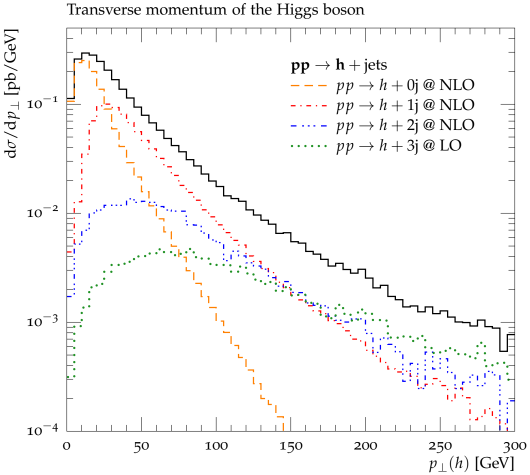
- first emission by NLOPS , restrict to $Q_{n+1} < Q_{\mathrm{cut}}$
- NLOPS $pp \to h + \mathrm{jet}$ for $Q_{n+1} > Q_{\mathrm{cut}}$
- restrict emission off $pp \to h + \mathrm{jet}$ to $Q_{n+2} < Q_{\mathrm{cut}}$
- NLOPS $pp \to h + 2\mathrm{jets}$ for $Q_{n+2} > Q_{\mathrm{cut}}$
- iterate
- sum all contributions

Multijet merging at NLO



- first emission by NLOPS , restrict to $Q_{n+1} < Q_{\text{cut}}$
- NLOPS $pp \to h + \text{jet}$ for $Q_{n+1} > Q_{\text{cut}}$
- restrict emission off $pp \to h + \text{jet}$ to $Q_{n+2} < Q_{\text{cut}}$
- NLOPS $pp \to h + 2\text{jets}$ for $Q_{n+2} > Q_{\text{cut}}$
- iterate
- sum all contributions

Multijet merging at NLO



- first emission by NLOPS , restrict to $Q_{n+1} < Q_{\text{cut}}$
- NLOPS $pp \to h +$ jet for $Q_{n+1} > Q_{\text{cut}}$
- restrict emission off $pp \to h +$ jet to $Q_{n+2} < Q_{\text{cut}}$
- NLOPS $pp \to h +$ 2jets for $Q_{n+2} > Q_{\text{cut}}$
- iterate
- sum all contributions

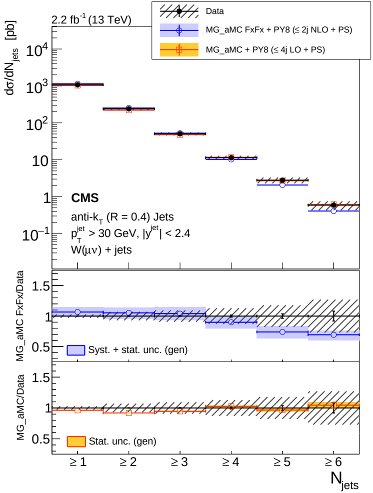
Multijet merging at NLO

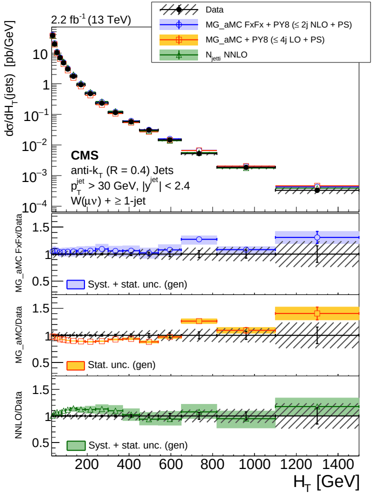
Available implementations

- ALPGEN+HERWIG/PYTHIA
  MLM (LO)
- MADGRAPH/LOOPPROVIDER+HERWIG
  UNLOPS (NLO), UMEPS (LO)
- MADGRAPH+PYTHIA
  FXFX (NLO), UNLOPS (NLO), MLM (LO), UMEPS (LO)
- SHERPA+LOOPPROVIDER
  MEPS@NLO (NLO), MEPS (LO)

Multijet merging at NLO

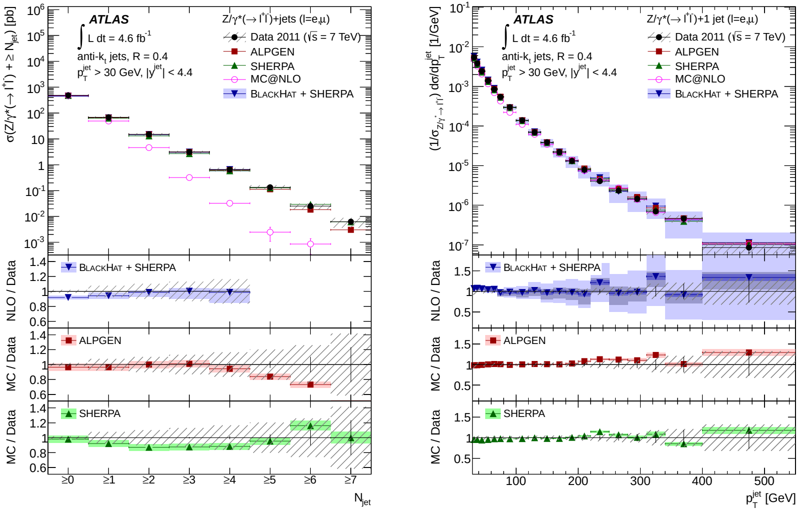
lepton + MET production





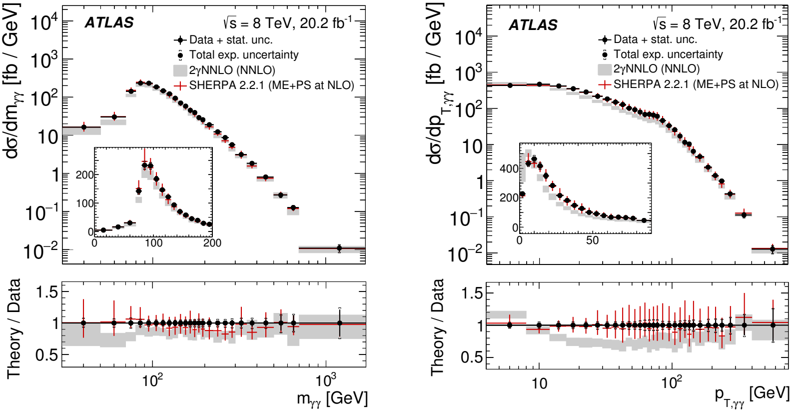
Multijet merging at NLO

lepton pair production



Multijet merging at NLO

diphoton production



Summary

- NLoPs matching is automated and is a standard input for any LHC analysis
  - standard schemes (Powheg/Mc@Nlo) closely related, but predictions can differ substantially
  - accuracy of the prediction restricted to observables which are described by a single multiplicity
  - extensions to NNLOPs exist for the simplest cases
- multijet merging improves the accuracy for the emission of additional jets
  - #emissions limited by CPU resources
  - NLO accuracy can be reached for the lowest few multiplicities
- computational complexity
  LO $<$ LoPs $<$ Multijet merging at LO
  NLO $<$ NLoPs $<$ Multijet merging at NLO