A Blueprint for Understanding the MC Top Mass Parameter

This talk reports on new work Oliver Jin,
Simon Plätzer and Daniel Samitz

arXiv:1807.06617

arXiv:2404.09856

arXiv:2405.xxxxx

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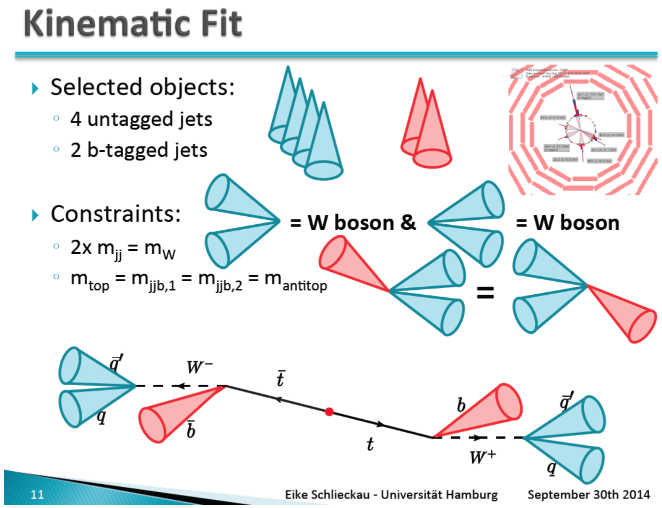
University of Vienna

LHC Top WG Meeting, April 24-26 2024

Most Precise Top Mass Measurements Method

LHC+Tevatron: Direct top mass measurements



Kinematic Fit

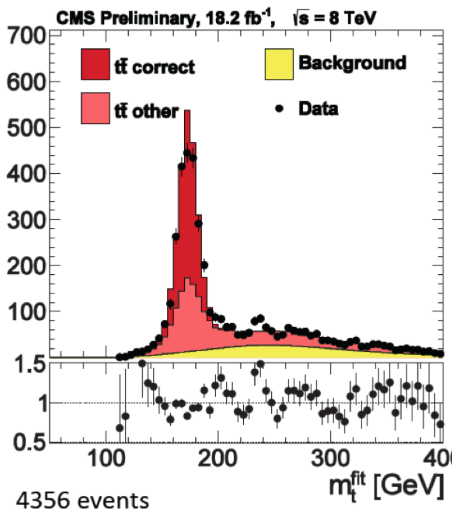
- Selected objects:
  - 4 untagged jets
  - 2 b-tagged jets
- Constraints:
  - 2x $m_{jj} = m_W$
  - $m_{top} = m_{jjb,1} = m_{jjb,2} = m_{antitop}$

= W boson & = W boson

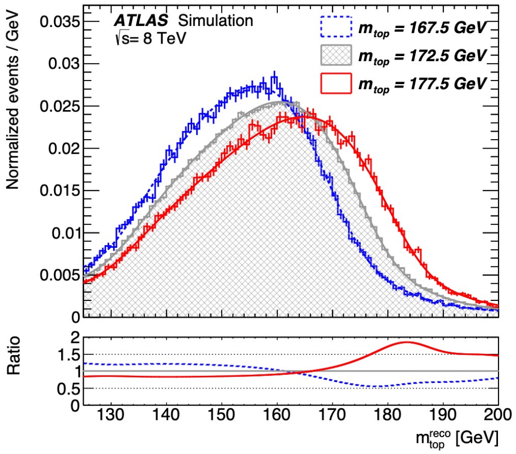
=

11 Eike Schlieckau - Universität Hamburg September 30th 2014

CMS-PAS-TOP-14-002



4356 events



$$m_t^{\mathrm{MC}} = 171.77 \pm 0.37\ \mathrm{GeV}$$

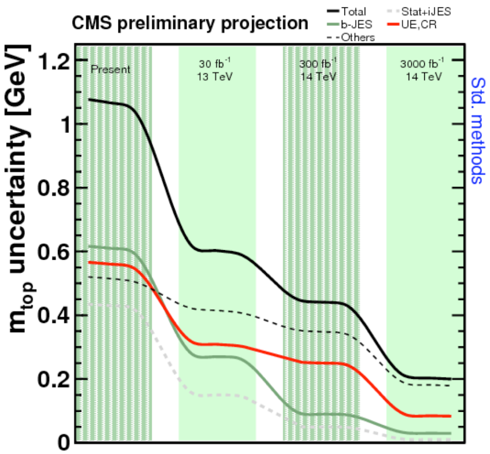
CMS collaboration. arXiv: 2302.01967

→ talk by Mark Owen

kinematic mass determination

based on the picture of a top quark particle

Determination of the best-fit value of the Monte-Carlo top quark mass parameter



⊕ High top mass sensitivity

⊖ Precision of MC ?

⊖ Meaning of m_t^{MC} ?

← Δ m_t ~ 200 MeV (projection)

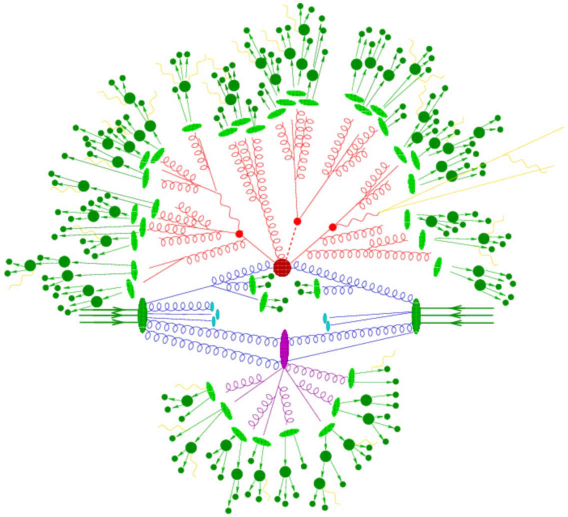
What is m_t^{MC} ?

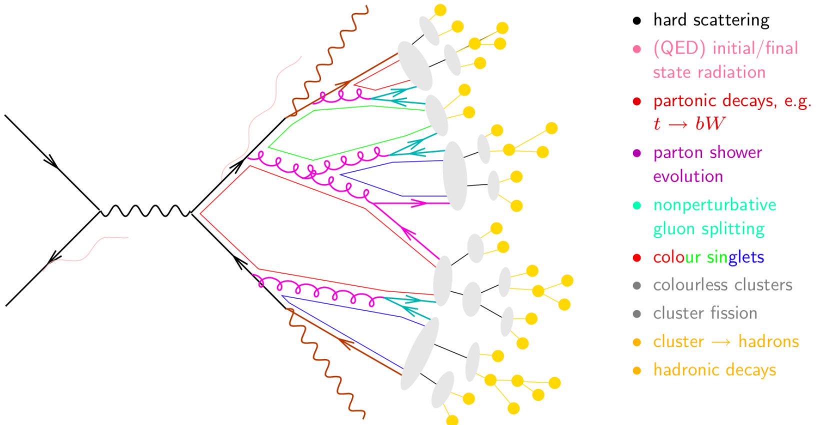
What does the question mean in the first place?

→ It means that we can provide the relation where δm^{scheme} can be **computed in pQCD**

$$m_t^{\text{MC}} = m_t^{\text{scheme}}(\mu) + \frac{\alpha_s(\mu)}{\pi}\delta m^{\text{scheme}} + \ldots$$

The issue is messy as we must understand and control the interplay of the components of MC event generators.





→ The “**MC top mass scheme**”

- Defined in pQCD
- Controlled at the observable hadron level.

Approaches to remedy the m_t^{MC} problem

- Indirect top quark mass measurements → ATLAS/CMS e.g. arXiv:2403.01313
  - Unfold data to parton level top-anti top on-shell particle distributions (e.g. m_{tt}) to be compared to N(N)LO fixed order calculations for on-shell top quarks → talk by Oleksandr Zanaiev
  - MC modelling aspects now contained in the hadron-to-parton unfolding carried out with the MC generator (no "theory of unfolding")
  - Uncertainties not yet as small as for direct determinations as observables are of more inclusive character

- 'Hadron' level analytic predictions for top mass determinations (ongoing work)
  - Fat top jets with soft drop grooming
    → MPI currently provides a practical limitation for LHC

    Mantry, Pathak, Stewart (2017)
    Mantry, Pathak, Stewart (2019)
  - Energy correlators
    → new type of top mass sensitivity related to decay opening angle

    Holguin, Moult, Pathak, Procura (2022)
    Holguin, Moult, Pathak, Procura, Schöfbeck (2023)

- This talk is about work to truly understand and control the MC top quark mass parameter m_t^{MC} → Improve MCs so that direct measurements can (eventually) be interpreted reliably.

What is m_t^{MC} ?

There are 4 essential ingredients to address the problem from first principles:

(B) Hadron level analytic QCD predictions

(A) Understanding of top as a quantum state

→ beyond the picture of a top particle

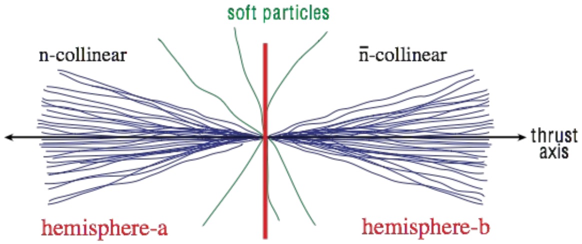
(C) NLL precise MC parton shower

(D) Factorization compatible MC hadronization model

Is there an observable where all 4 ingredients are known ?

Yes: Event-shape observables in e^+e^- collisions for boosted top pair production: 2-jettiness, thrust, …

(builds on sequence of work since 2007)



Aim of this talk:

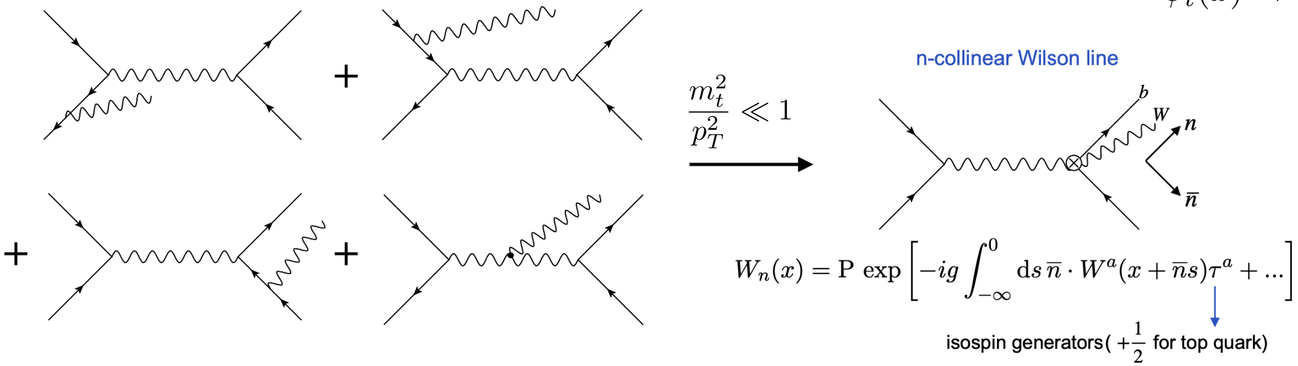
- Discuss interplay of (A) – (D) provide conceptual and practical basis to determine and control m_t^{MC} for MC event generators
- (A) – (C) from previous work. New development for (D)
- Explicit realization for e^+e^- event-shape top resonance distribution (e.g. 2-jettiness)

(A) Beyond the picture of a top particle

If we stick to the picture of a physical top particle the only mass that is ever relevant is the pole mass = pole of the top propagator.

For boosted top quarks the effects of QCD and electroweak radiation (incl. top decay) associated to the top direction can be described by a QCD+electroweak Wilson line, which generalizes the top particle picture in a process-independent way.

$$\psi_t(x) \to J_t(x) = W^\dagger(x)\psi_t(x)$$



$$\frac{m_t^2}{p_T^2} \ll 1 \longrightarrow$$

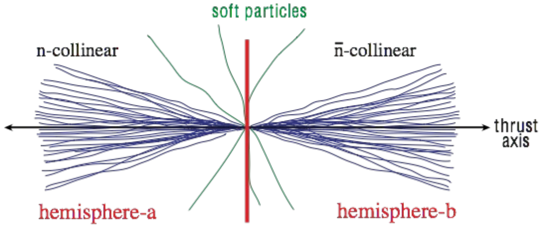
n-collinear Wilson line

$$W_n(x) = \mathrm{P}\ \exp\left[-ig\int_{-\infty}^{0} \mathrm{d}s\, \bar{n}\cdot W^a(x+\bar{n}s)\tau^a + \ldots\right]$$

isospin generators($+\frac{1}{2}$ for top quark)

- Wilson line contains ultra-collinear gluons, soft in top rest frame (+ top decay)
- Describes coherent (collinear) gauge-invariant coherent off-shell (top decay products)+gluon states → "top state" defined by measurement
- Allows for gauge-invariant treatment of IR cutoff dependence for off-shell top
  → IR cutoff of parton calculation
  = resolution scale defining the asymptotic "top state"

(B) Boosted top eventshapes



$$\tau \;=\; 1-\max_{\vec{n}}\frac{\sum_i |\vec{n}\cdot\vec{p}_i|}{Q} \;\overset{\tau\to 0}{\approx}\; \frac{M_1^2+M_2^2}{Q^2}$$

$$Q = E_{\mathrm{c.m.}}$$

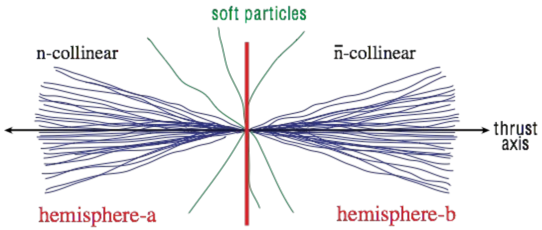
Insensitive to details of top decay

Hadron level:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau}(\tau,Q,m,\delta m) = \int_0^{Q\tau}\mathrm{d}\ell\;\underbrace{\frac{\mathrm{d}\hat{\sigma}_s}{\mathrm{d}\tau}\Big(\tau-\frac{\ell}{Q},Q,m,\delta m\Big)}_{\text{Parton cross section}}\;S_{\mathrm{mod}}(\ell)$$

Shape function

(B) Boosted top eventshapes



$$\tau = 1 - \max_{\vec n} \frac{\sum_i |\vec n \cdot \vec p_i|}{Q} \overset{\tau\to 0}{\approx} \frac{M_1^2 + M_2^2}{Q^2}$$

$$Q = E_{\mathrm{c.m.}}$$

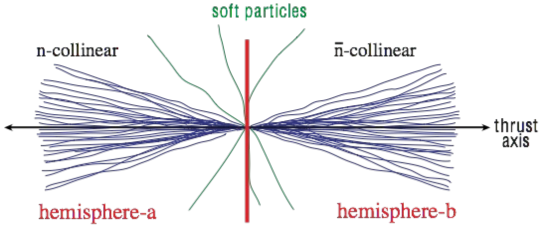
Insensitive to details of top decay

Hadron level: $$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau}(\tau, Q, m, \delta m) = \int_0^{Q\tau} \mathrm{d}\ell \, \underbrace{\frac{\mathrm{d}\hat\sigma_s}{\mathrm{d}\tau}\left(\tau - \frac{\ell}{Q}, Q, m, \delta m\right)}_{\text{Parton cross section}} \, S_{\mathrm{mod}}(\ell)$$

Shape function

- S_{mod} leading nonperturbative corrections <u>only from large-angle soft radiation</u>: linear sensitive to Λ_{QCD}
- Any top mass renormalization scheme can be implemented $m_t^{\mathrm{pole}} = m + \delta m$
- Can be calculated with a finite IR cutoff Q_0 for the parton cross section
- IR cutoff Q_0 = factorization scale for parton-level vs. hadronization corrections
    - Defines scheme for S_{mod} (large-angle soft radiation): $S_{\mathrm{mod}}(l) \to S_{\mathrm{mod}}(l, Q_0)$
    - Defines scheme for parton distribution: $\frac{d\hat\sigma}{d\tau}(\tau, Q, m) \to \frac{d\hat\sigma}{d\tau}(\tau, Q, m, Q_0)$

(B) Boosted top eventshapes



$$\tau = 1 - \max_{\vec{n}} \frac{\sum_i |\vec{n}\cdot\vec{p}_i|}{Q} \overset{\tau\to 0}{\approx} \frac{M_1^2 + M_2^2}{Q^2}$$

$$Q = E_{\mathrm{c.m.}}$$

Insensitive to details of top decay

Hadron level:
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau}(\tau, Q, m, \delta m) = \int_0^{Q\tau} \mathrm{d}\ell\, \underbrace{\frac{\mathrm{d}\hat{\sigma}_s}{\mathrm{d}\tau}\left(\tau - \frac{\ell}{Q}, Q, m, \delta m\right)}_{\text{Parton cross section}}\, S_{\mathrm{mod}}(\ell)$$

Shape function

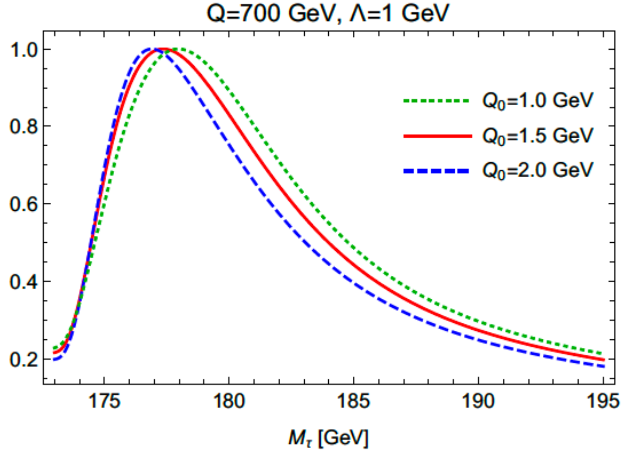
- IR cutoff Q_0 = factorization scale for parton-level vs. hadronization corrections
  - Modifies pole of top propagator away from m_t^{pole}: (ultra-collinear radiation)
    $m_t^{\mathrm{pole}} \to m_t(Q_0) = m_t^{\mathrm{pole}} - \delta m(Q_0)$, $\delta m(Q_0) \sim \alpha_s(Q_0)\, Q_0$
  - Peak position:

For $q_\perp^{\mathrm{gluon}} > Q_0$:

$$\frac{d}{d\ln Q_0}\tau_{\mathrm{peak}}^{\mathrm{parton}}(Q_0) = \frac{C_F\alpha_s(Q_0)}{4\pi}\frac{Q_0}{Q}\left[16 - 8\pi\frac{m_t}{Q}\right]$$

Large-angle soft → hadronization

→ top mass



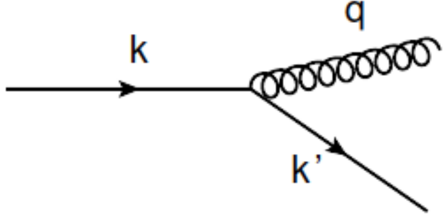
(C) Angular ordered parton shower (Herwig)

→ Coherent Branching algorithm (default Herwig shower):

Dokshitzer, Fadin, Khoze (1982)
Bassetto, Ciafaloni, Marchesini (1983)
Catani, Marchesini, Webber (1991)
Gieseke, Stephens, Webber (2003)

$$k'^\mu = z k^- \frac{n^\mu}{2} + \frac{k'^2 - q_\perp^2}{z k^-}\frac{\bar{n}^\mu}{2} - q_\perp^\mu$$

$$q^\mu = (1-z) k^- \frac{n^\mu}{2} + \frac{q^2 - q_\perp^2}{(1-z) k^-}\frac{\bar{n}^\mu}{2} + q_\perp^\mu$$



momentum conservation:

$$k^2 = \frac{k'^2}{z} + \frac{q^2}{1-z} + \frac{q_\perp^2}{z(1-z)}$$

evolution variables: $z\,,\ \tilde{q} = \frac{q_\perp^2}{z^2(1-z)^2}$

probabilities from splitting functions and Sudakov form factors

color coherence of soft gluon emissions → angular ordering: $z_i^2 \tilde{q}_i^2 > \tilde{q}_{i+1}^2$

→ jet mass distribution (inv. mass generated from CB from one boosted quark)

$k^2 \approx$ hemisphere mass (does not account for out of cone radiation)

$$\begin{aligned} J(Q^2, k^2 - m^2, m^2) &= \delta(k^2 - m^2) \\ &+ \int_0^{Q^2} \frac{\mathrm{d}\tilde{q}^2}{\tilde{q}^2} \int_0^1 \mathrm{d}z\, P_{QQ}\left[\alpha_s(z(1-z)\tilde{q}), z, m\right] \theta\Big(\tilde{q}^2 - \frac{Q_0^2 + m^2(1-z)^2}{z^2(1-z)^2}\Big) \\ &\times \Big[z J\big(z^2\tilde{q}^2, z(k^2 - m^2) - z^2(1-z)\tilde{q}^2\big) - J\big(\tilde{q}^2, k^2 - m^2\big)\Big] \end{aligned}$$

(C) Angular ordered parton shower (Herwig)

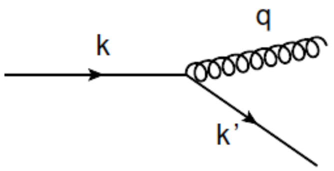
Partonic level cross section

Catani, Trentadue, Turnock Webber (1993)

$$\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\tau} = \int \mathrm{d}k^2\, \mathrm{d}k'^2\, \delta\Big(\tau - \frac{k^2 + k'^2}{Q^2}\Big)\, J(Q^2, k^2) J(Q^2, k'^2)$$

AHH, Plätzer, Samitz (2018)

- Agrees exactly with partonic cross section obtained from analytic factorized calculations at NLL!
- CB is NLL precise for inclusive event shapes.
- For vanishing IR cutoff: CB mass parameter $m_t^{\mathrm{CB}} = m_t^{\mathrm{pole}}$



BUT: Parton showers in MC generators have an IR cutoff to prevent infinite multiplicities

$$\boxed{q_\perp > Q_0}$$

- Linear dependence on Q_0 from large-angle soft and ultracollinear radiation
- Matches analogous calculations for analytic calculations
- Realized accurately by Herwig's shower

$$m_t^{\mathrm{CB}}(Q_0) = m_t^{\mathrm{pole}} - \frac{2}{3} Q_0 \alpha_s(Q_0) + \mathcal{O}(\alpha_s(Q_0)^2)$$

$$m_t^{\mathrm{CB}}(Q_0) = m_t^{\mathrm{MSR}}(Q_0) - \frac{2}{3}\left(1 - \frac{2}{\pi}\right) Q_0 \alpha_s(Q_0) + \mathcal{O}(\alpha_s^2(Q_0))$$

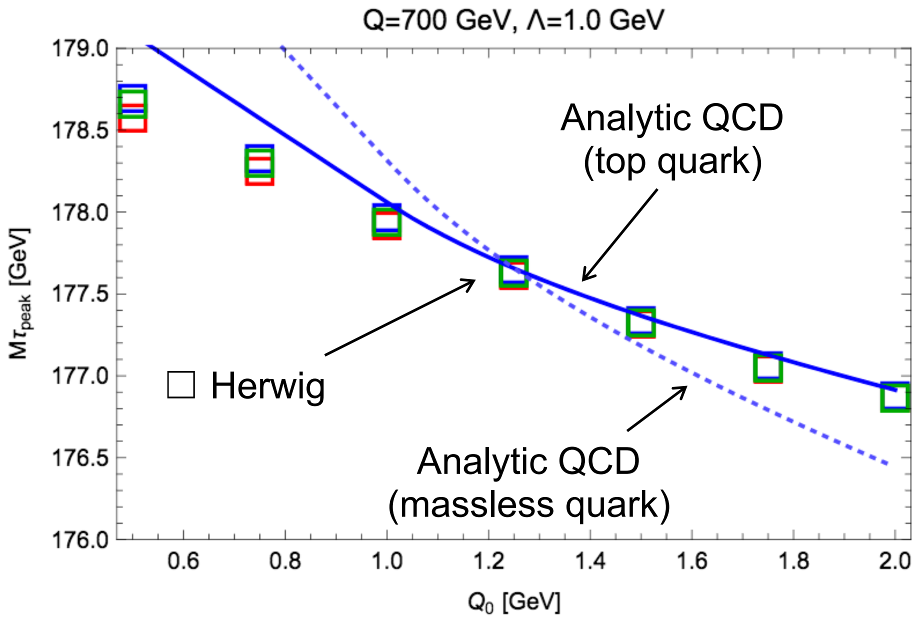
(C) Angular ordered parton shower (Herwig)

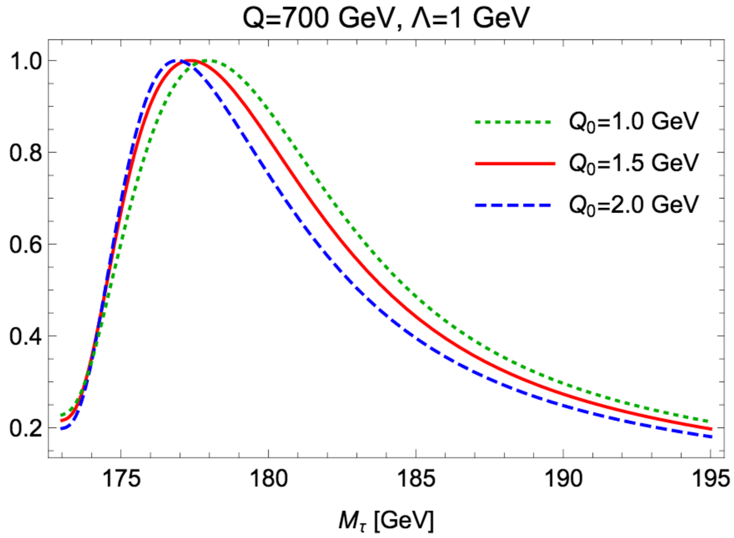
AHH, Plätzer, Samitz (2018)

Peak position of $M_\tau = \dfrac{Q^2\tau_2}{2m_t} \quad (Q = E_\mathrm{cm})$

Parton level analysis (no hadronization corrections
fixed value for m_t)

$$\frac{d}{dQ_0}\tau_\mathrm{peak}(Q_0) = \frac{C_F\alpha_s(Q_0)}{4\pi}\frac{Q_0}{Q}\left[16 - 8\pi\frac{m_t}{Q}\right]$$





- Herwig parton level simulations in full agreement with analytic calculations
- Change of Q_0:

  Physical predictions should remain unchanged: Q_0-invariance

How well does a hadronization model satisfy this criterium?

- hadronization model is retuned (Q_0 dependent tune)
- generator mass is interpreted as $m_t^{CB}(Q_0)$

(D) Factorization compatible hadronization model

AHH, Jin, Plätzer, Samitz
arXiv:2404.09856

Standard shower cut treatment for all MC generators:

- Shower-cutoff scale Q_0 = one of many hadronization model parameters

BUT: To gain control over the shower's top mass parameter:

Plätzer arXiv:2204.06956

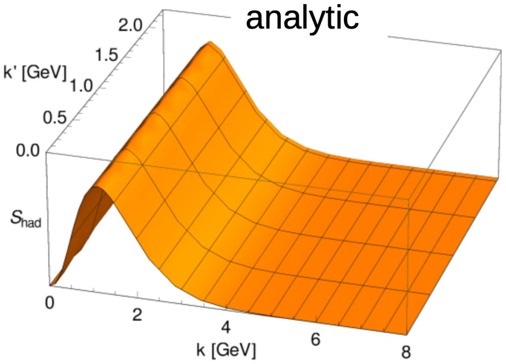
- The shower-cutoff scale Q_0 must be <u>promoted to a factorization scale</u>, such that hadron level descriptions are shower-cut independent.
- The parton-level to hadron-level migration matrix must behave like a shape function!

$$\frac{d\sigma}{d\tau}(\tau,Q) = \int d\hat{\tau}\, \frac{d\hat{\sigma}}{d\hat{\tau}}(\hat{\tau},Q)\, \underbrace{T(\tau,\hat{\tau},\{Q,Q_0\})}$$

Migration matrix should have the property $T(\tau,\hat{\tau},\{Q,Q_0\}) = T(\tau-\hat{\tau})$

$$T\left(\frac{k}{Q}=\tau-\hat{\tau}, \frac{k'}{Q}=\hat{\tau}, \{Q,Q_0\}\right)$$

Transfer matrix should have this form:



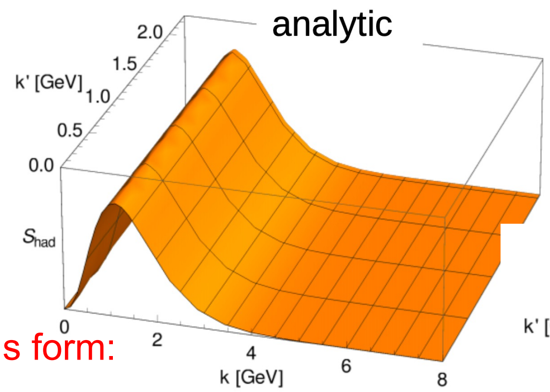
(D) Factorization compatible hadronization model

Tunes generated to LEP Z-pole data where the shower cutoff Q_0 has a fixed value

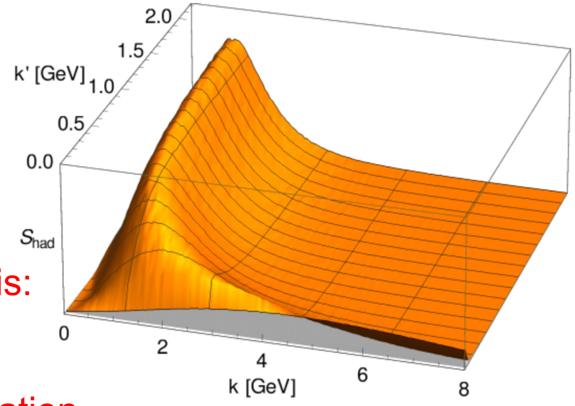
AHH, Jin, Plätzer, Samitz
arXiv:2404.09856

$$T\left(\frac{k}{Q}=\tau-\hat{\tau}, \frac{k'}{Q}=\hat{\tau}, \{Q, Q_0\}\right)$$

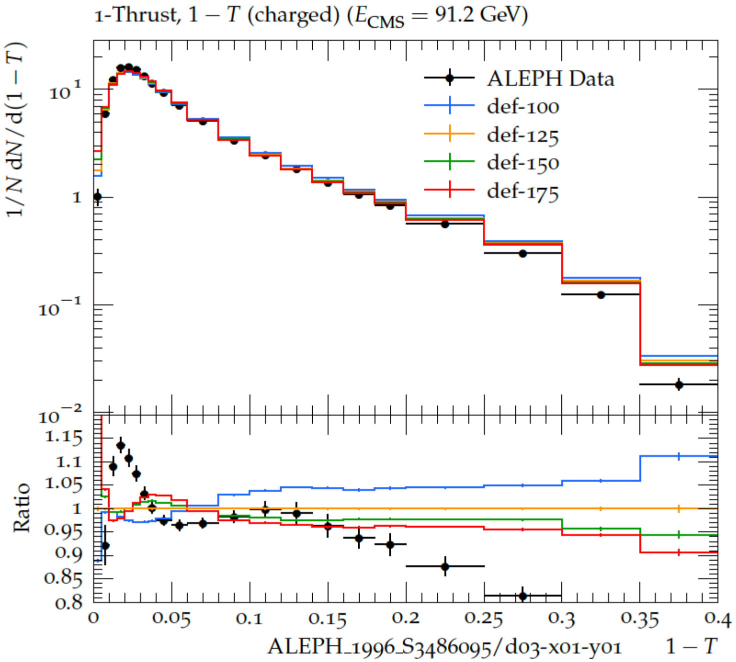
migration matrix should have this form:



But it actually looks like this:



Peak region hadronization inconsistent with QCD factorization!



Description of observables at hadron level not quite shower-cutoff independent (Thrust at $Q=M_z$)

Q_0 = (1.00, 1.25, 1.50, 1.75) GeV

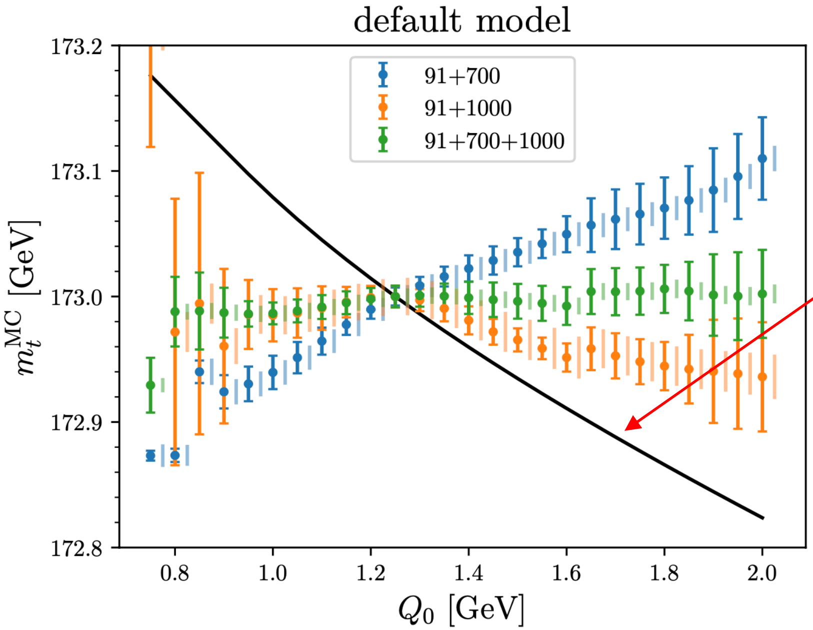
(Reference data for tune: Simulation for Q_0=1.25 GeV)

(D) Factorization compatible hadronization model

AHH, Jin. Plätzer, Samitz to appear

Include top "pseudo" 2-jettiness data into the standard tuning data set:

- Also tune the top mass parameter m_t^{MC} for different Q_0 values (to top pseudo data generated for Q_0=1.25 GeV



Q_0 dependence expected if hadronization model would be consistent with QCD factorization

preliminary

Default Herwig hadronization model modifies m_t^{MC} in an unphysical way incompatible with QCD factorization: uncertainty ~ 0.5 GeV

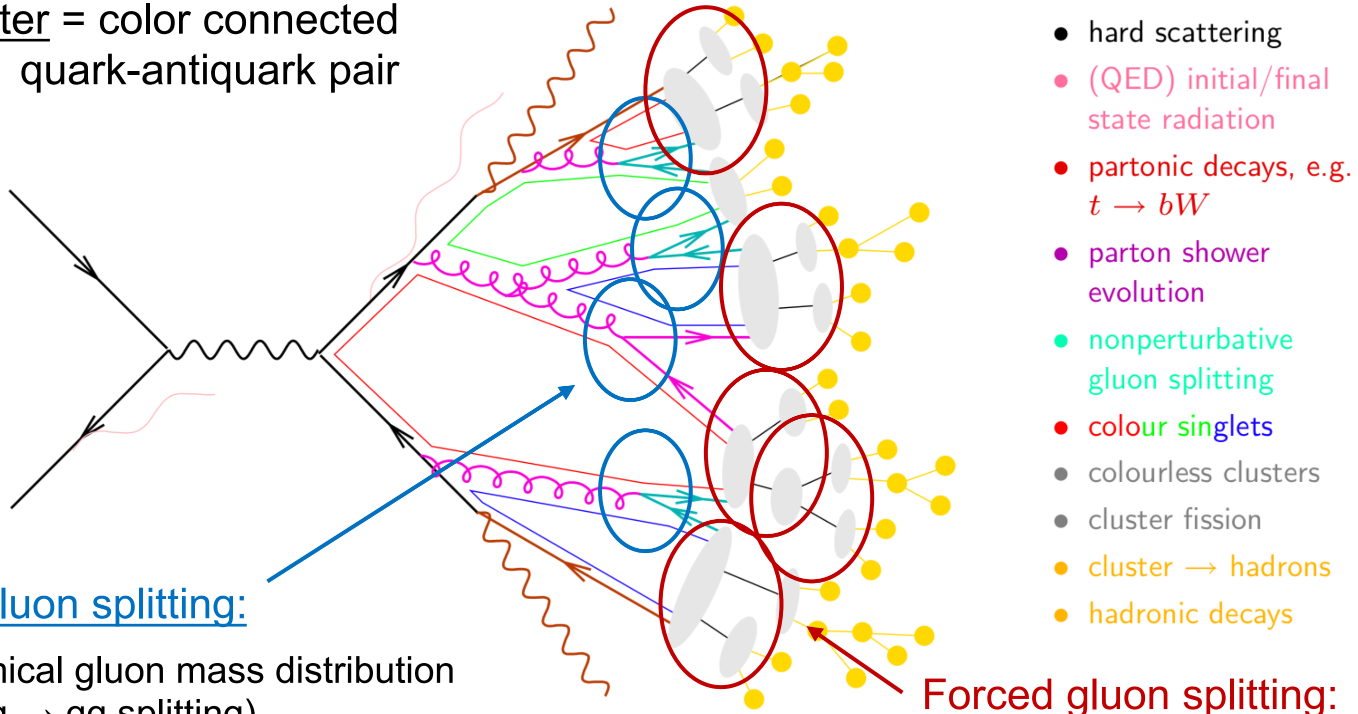
→ $m_t^{Herwig}(Q_0) \neq m_t^{CB}(Q_0)$ for the default hadronization model

(D) Factorization compatible hadronization model

AHH, Jin. Plätzer, Samitz to appear

Modified cluster hadronization that mimics aspects of parton shower dynamics:

Cluster = color connected quark-antiquark pair



Forced gluon splitting:

- Dynamical gluon mass distribution (from g → qq splitting)
- Kinematics in analogy to parton shower

Forced gluon splitting:

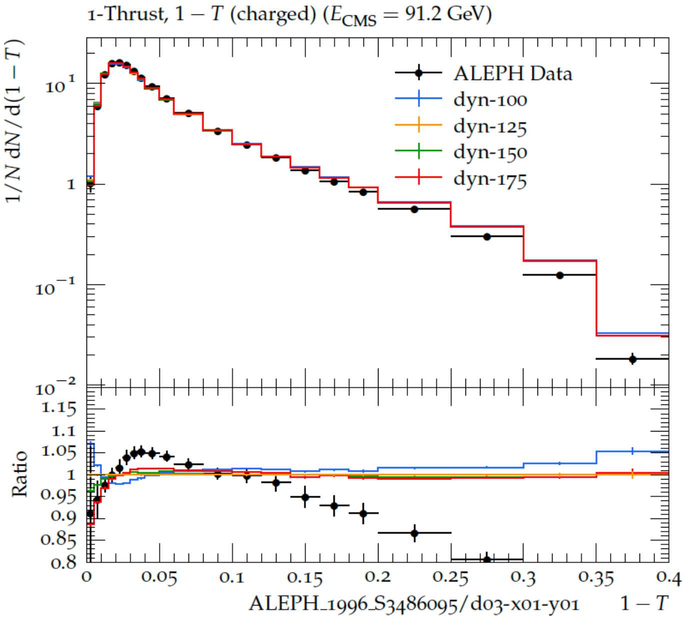
- Cluster splitting from branching q → q g and splitting g → qq
- Kinematics in analogy to the parton shower

➡ Model parameters can consistently adapt to changes of Q_0

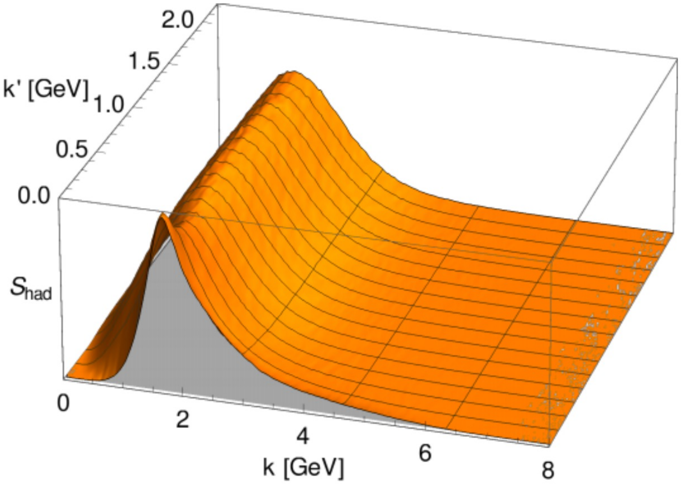
(D) Factorization compatible hadronization model

AHH, Jin. Plätzer, Samitz to appear

Migration function much better consistent with QCD factorization

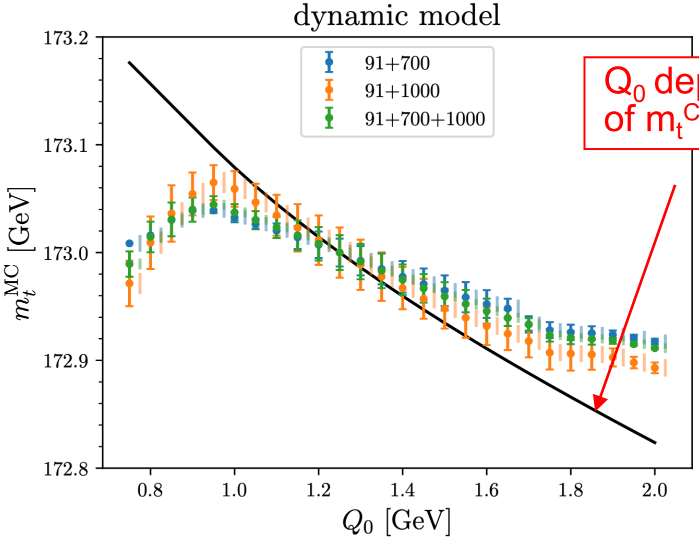


Observables much less dependent on Q_o



preliminary

Thrust



Q_0 dependence of $m_t^{CB}(Q_0)$

Tunes m_t^{MC} fully consistent with expectations from analytic QCD calculation

(“pseudo data” generated for Q_0=1.25 GeV)

⇒ $m_t^{Herwig}(Q_0) = m_t^{CB}(Q_0)$

within a precision of better than 50 MeV

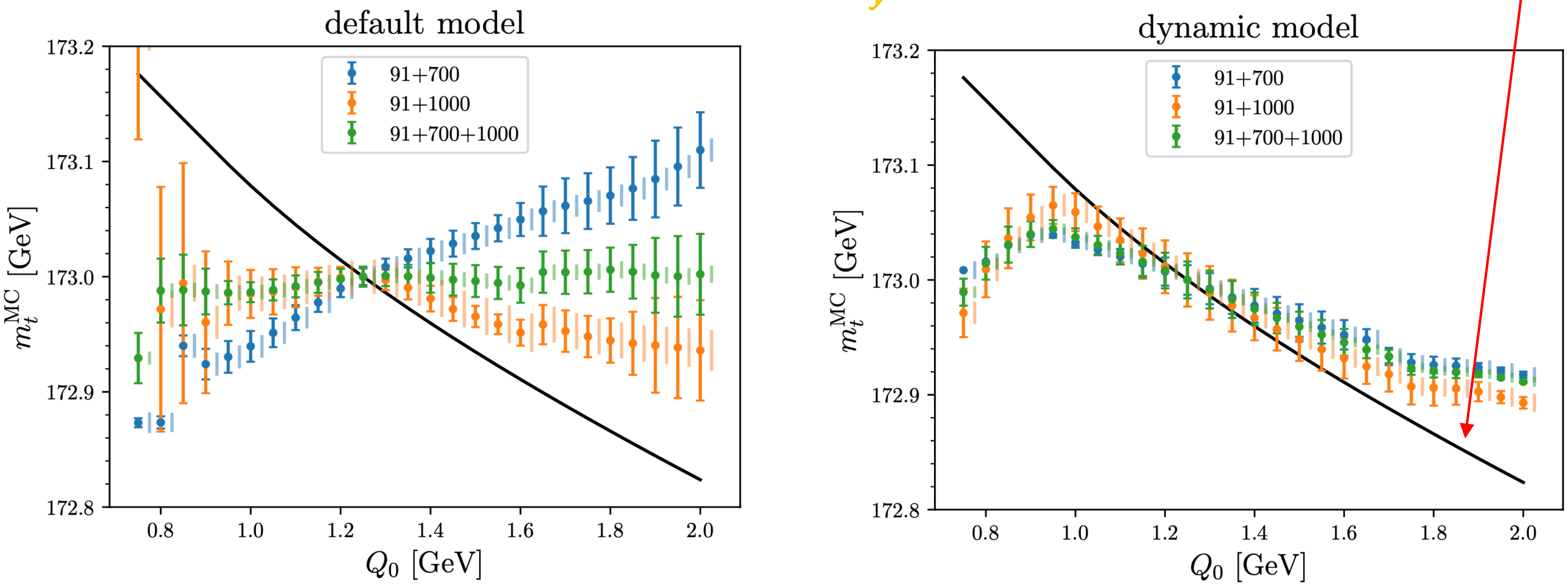
Old Default Model vs. New Dynamical Model

AHH, Jin. Plätzer, Samitz to appear

Shower cutoff dependence of tuned MC top quark mass to reference data including top quark 2-jettiness distributions at 700 and/or 1000 GeV

preliminary

Q_0 dependence expected from $m_t^{CB}(Q_0)$



Agreement of m_t^{MC} with $m_t^{CB}(Q_0)$

within 50 MeV !

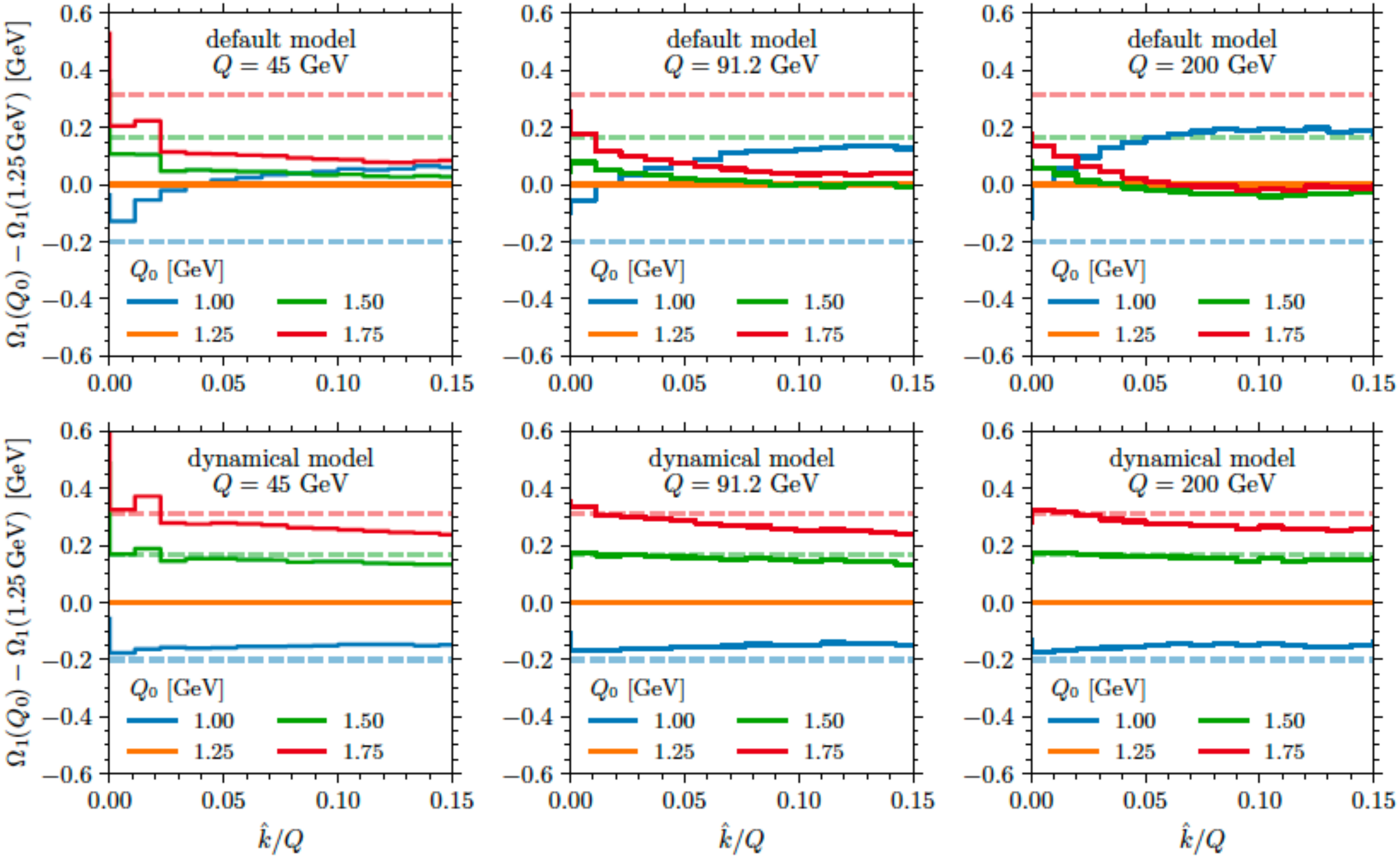
Old Default Model vs. New Dynamical Model

AHH, Jin. Plätzer, Samitz to appear

Shower cutoff dependence of first moment of migration matrix from simulations for 2-jettiness → ”MC scheme for hadronization correction“

preliminary

$$\Omega_1^{\mathrm{MC}}(\hat{k}, Q, Q_0) - \Omega_1^{\mathrm{MC}}(\hat{k}, Q, Q_{0,\mathrm{ref}} = 1.25\,\mathrm{GeV})$$

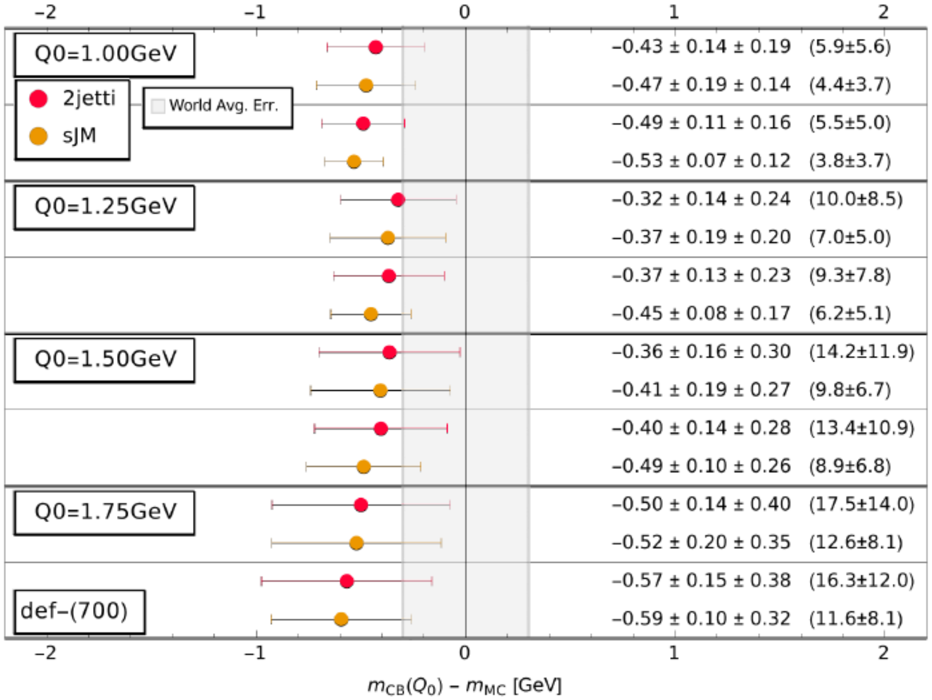


Old Default Model vs. New Dynamical Model

AHH, Jin. Plätzer, Samitz to appear

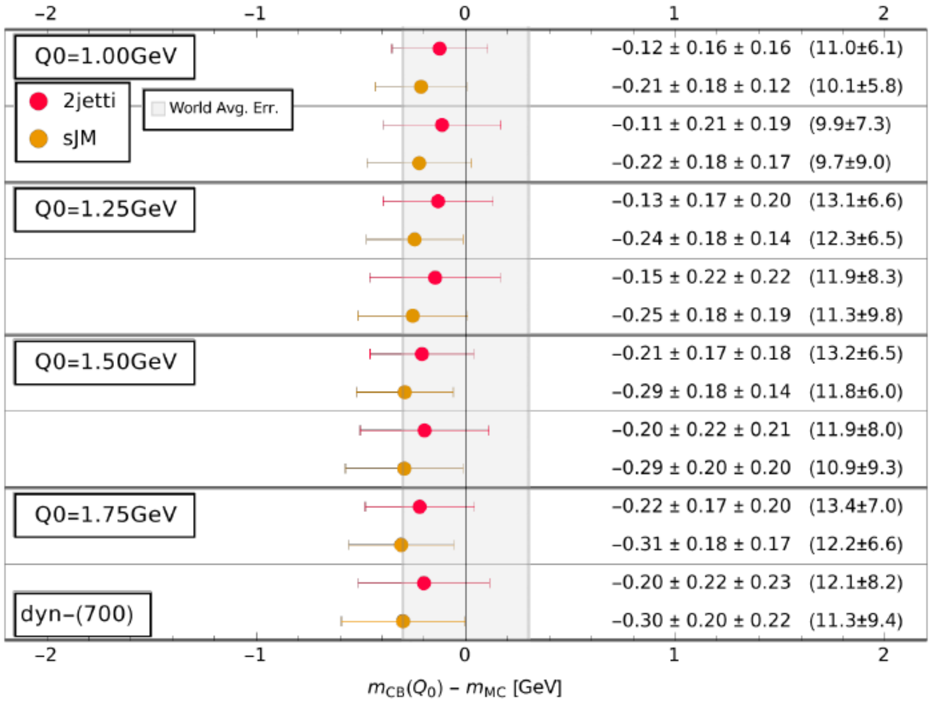
Cross check: apply top mass calibration to determine $m_t^{CB}(Q_0)$

preliminary



Default: m_t^{MC} incompatible with $m_t^{CB}(Q_0)$

hadronization model adds unphysical contributions to the relation $m_t^{MC} = m_t^{CB}(Q_0)$



Dynamical: m_t^{MC} compatible with $m_t^{CB}(Q_0)$

hadronization does not affect the relation $m_t^{MC} = m_t^{CB}(Q_0)$

Final remarks and Outlook

→ The MC top mass parameter m_t^{MC} can be promoted to a renormalization scheme and its NLO relation to any other top mass renormalization scheme can be calculated.

→ Key aspect: Parton shower cutoff Q_0 = Factorization scale separating pQCD and npQCD

- Universality of the current insights

→ $m_t^{MC} = m_t^{CB}(Q_0)$

  - So far we have all theoretical ingredients to interpret m_t^{MC} only for e^+e^- eventshape distributions and the Herwig MC generator (coherent branching + cluster hadronization)
  - MC generators do not have the same precision for all observables
- Progress to generalize the current results will take some more years of work because many new theory tools need to be developed (→ e.g. differential in top decay)
- Future plans:
  - investigate dipole shower, string hadronization (Pythia)
  - universality: observables differential in top decay (→ e.g. $M_{\text{b-jet lepton}}$)
  - final aim: b-jets with small jet radius
  - establish a m_t^{MC} verification tool box
  - theory of "unfolding" → relevant for indirect m_t measurements
- LHC: MPI and UE hadronization models still need to be better understood from the QCD perspective as long as b

Blueprint to elevate m_t^{MC} to a mass scheme

What does the question mean in the first place?

→ It means that we can provide the relation

$$m_t^{\mathrm{MC}} = m_t^{\mathrm{scheme}}(\mu) + \frac{\alpha_s(\mu)}{\pi}\delta m^{\mathrm{scheme}} + \ldots$$

There are 4 essential ingredients to address the problem from first principles:

Understanding of top as a quantum state

- Abandon top as on-shell asymptotic state (“top particle”)
- Beyond narrow-width limit (top = top decay final states)
- Top has color and needs additional gluons to become observable
- Observable top state = coherent top+gluon (color singlet) state

Hadron level analytic QCD predictions

- NLL precise analytic QCD perturbative prediction
- First principles treatment of hadronization corrections
- NLO control over IR factorization scale for perturb. and non-pert. effects
- Analytic control of shower cut Q_0 dependence

NLL precise MC parton shower

- NLL precision needed to achieve NLO accurate definition of m_t^{MC}
- Analytic NLO precision concerning the IR cutoff Q_0 of the parton shower evolution
- Shower cutoff Q_0 = an IR factorization scale
- Parton shower determines the meaning of $m_t^{MC}(Q_0)$

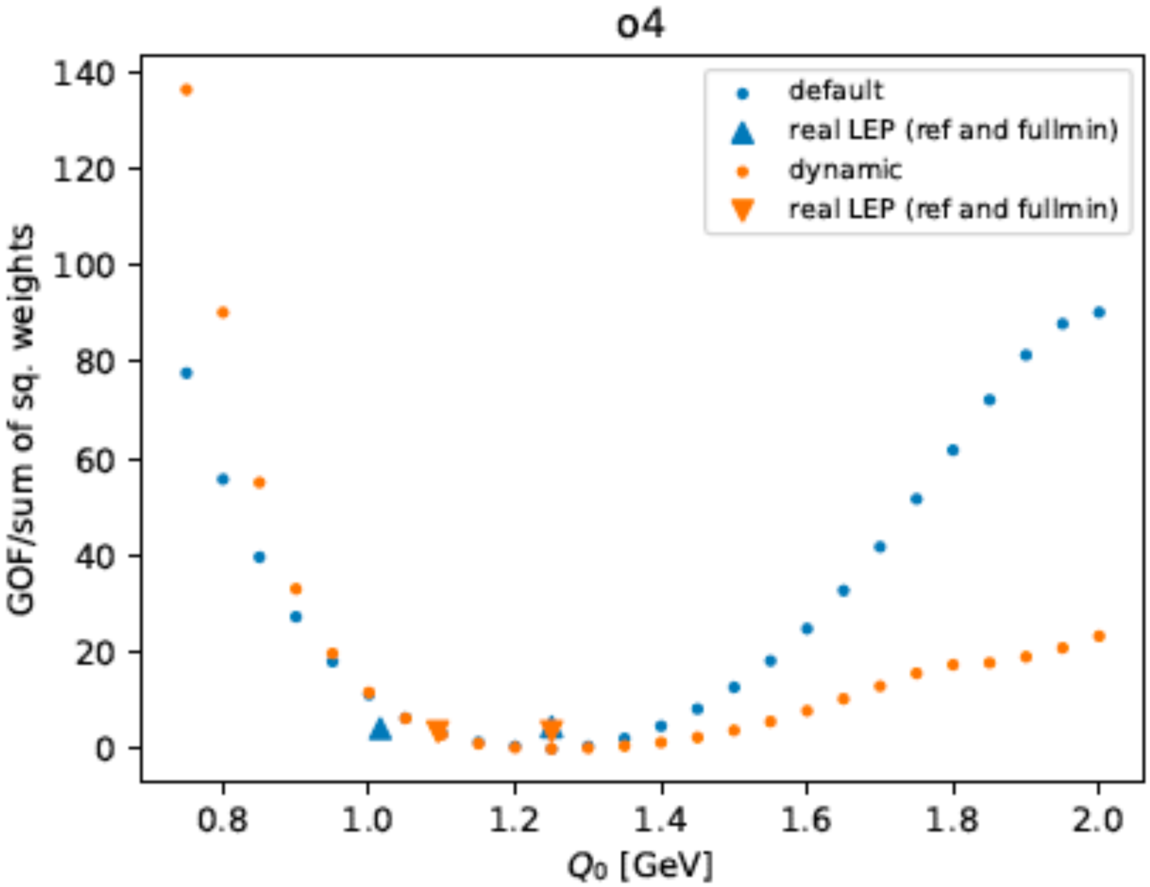
Factorization compatible MC hadronization model

- Shower cutoff Q_0 is not treated as a tuning variable, but as factorization scale
- Hadronization model has to respond to different choices of Q_0 to compensate for changes of Q_0 in the parton shower.
- Should not provide “nonperturbative corrections” to m_t^{MC}

Old Default Model vs. New Dynamical Model

AHH, Jin. Plätzer, Samitz to appear

Shower cutoff Q_0 minimal χ^2-values obtained in the tuning fits



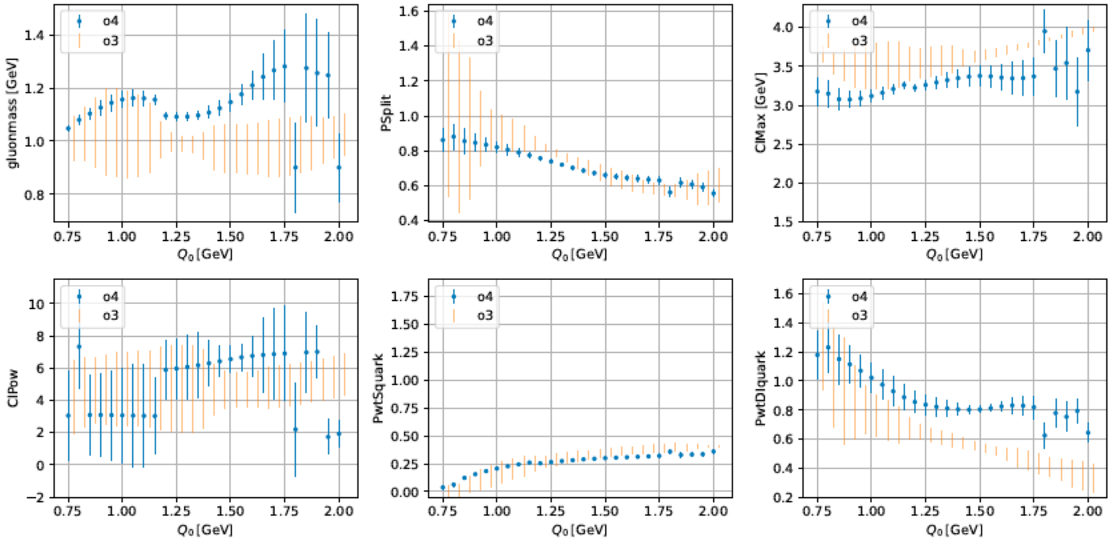
preliminary

Old Default Model vs. New Dynamical Model

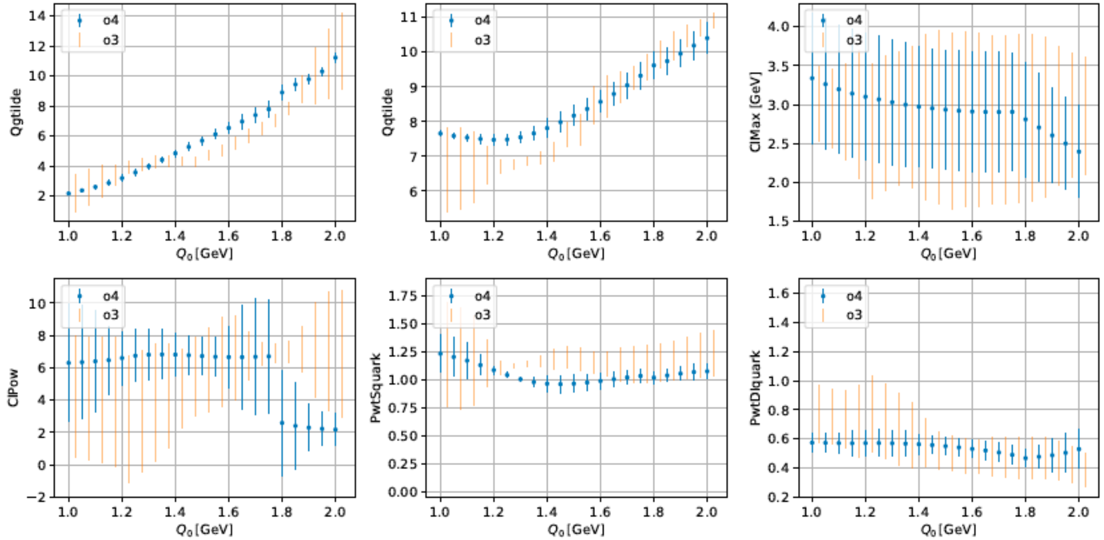
Tuned parameters for Q_0-dependent tuning analyses (apart from m_t^{MC})

AHH, Jin, Plätzer, Samitz
arXiv:2404.09856

Default model



Dynamical model



Factorization compatible hadronization model

AHH, Jin. Plätzer, Samitz to appear

Dynamic model: forced gluon splitting

- If the splitting had taken place in the parton shower it would have been generated from the splitting function

$$\mathrm{d}P(g \to q\bar{q}) \sim \frac{\mathrm{d}q^2}{q^2}\alpha_s(q^2)\left(1 - 2z(1-z) + \frac{2m_q^2}{q^2}\right)\Theta\left(q^2 z(1-z) - m_q^2\right)$$

  giving the gluon a virtuality q^2

- Use the probability distribution for this dynamically generated virtuality as „gluon constituent mass" m_g

- Set a highest possible scale for the non-pert. gluon splitting $\tilde{Q}_g$

  (new tuning parameter instead of fixed m_g)

- Need to IR regularize the splitting function (because evolve below cutoff): $\mathrm{d}P(g \to q\bar{q}) \to \mathrm{d}\tilde{P}(g \to q\bar{q})$

  freeze out strong coupling at some low scale to avoid Landau pole

  use constituent masses for quarks

Factorization compatible hadronization model

AHH, Jin. Plätzer, Samitz to appear

Dynamic model: cluster fission

- Want a mass distribution of the daughter clusters that resembles as much as possible the mass distribution dynamically generated by low scale emissions of the parton shower
- Radiate a gluon from one of the cluster's constituents according to $\mathrm{d}\tilde{P}(q \to qg)$ set a maximum scale $\tilde{Q}_q$ of the splitting (new tuning parameter for fission instead of P_{Split})
- Split the gluon according to $\mathrm{d}\tilde{P}(g \to q\bar{q})$
- Construct new clusters from the $q\bar{q}$ pairs