Higher orders and resummation for diboson processes

CMS Topical Workshop on Off-shell Higgs Boson Production,
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John Campbell (Fermilab)

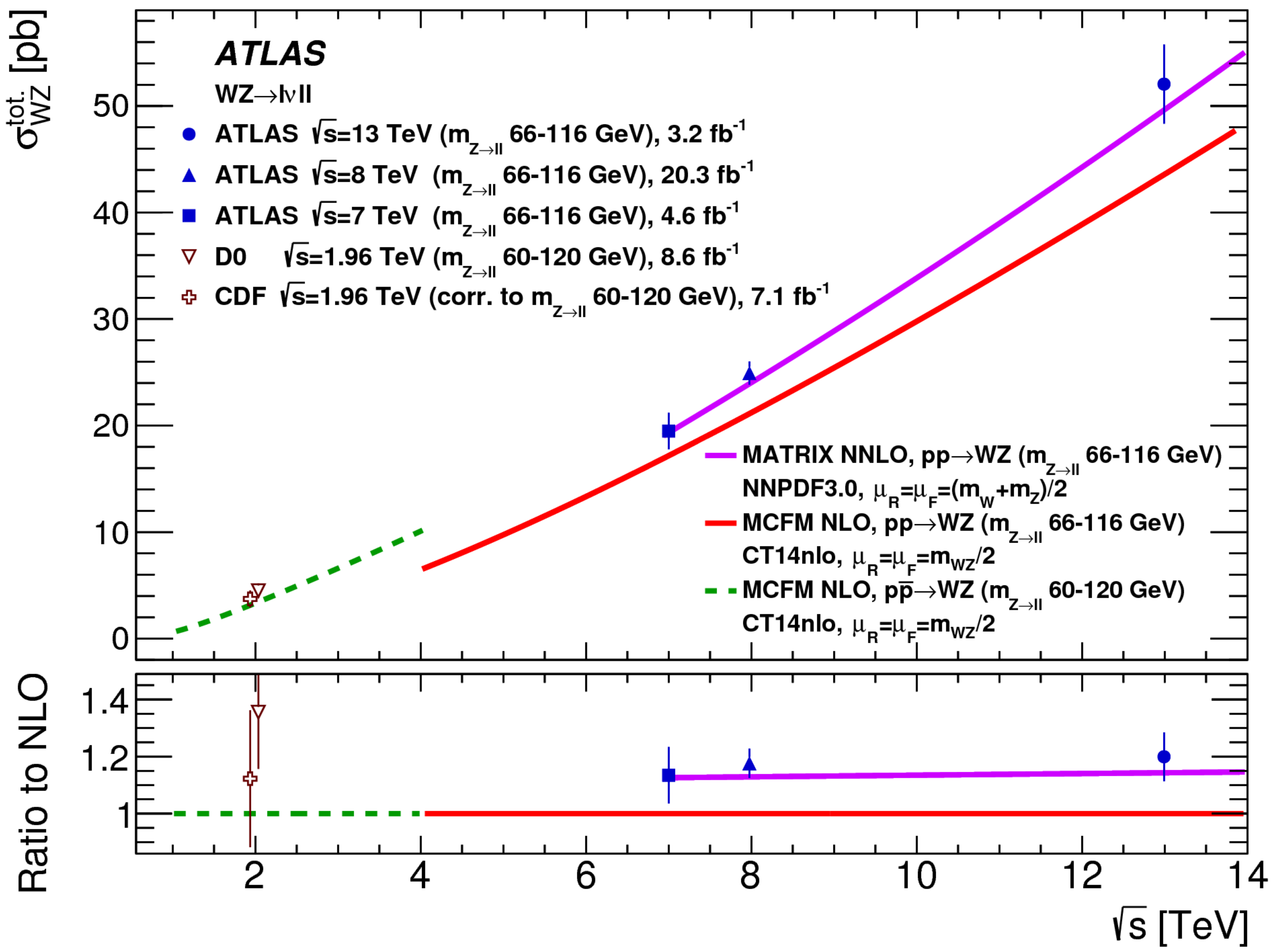
(work with Keith Ellis, Tobias Neumann, Satyajit Seth)

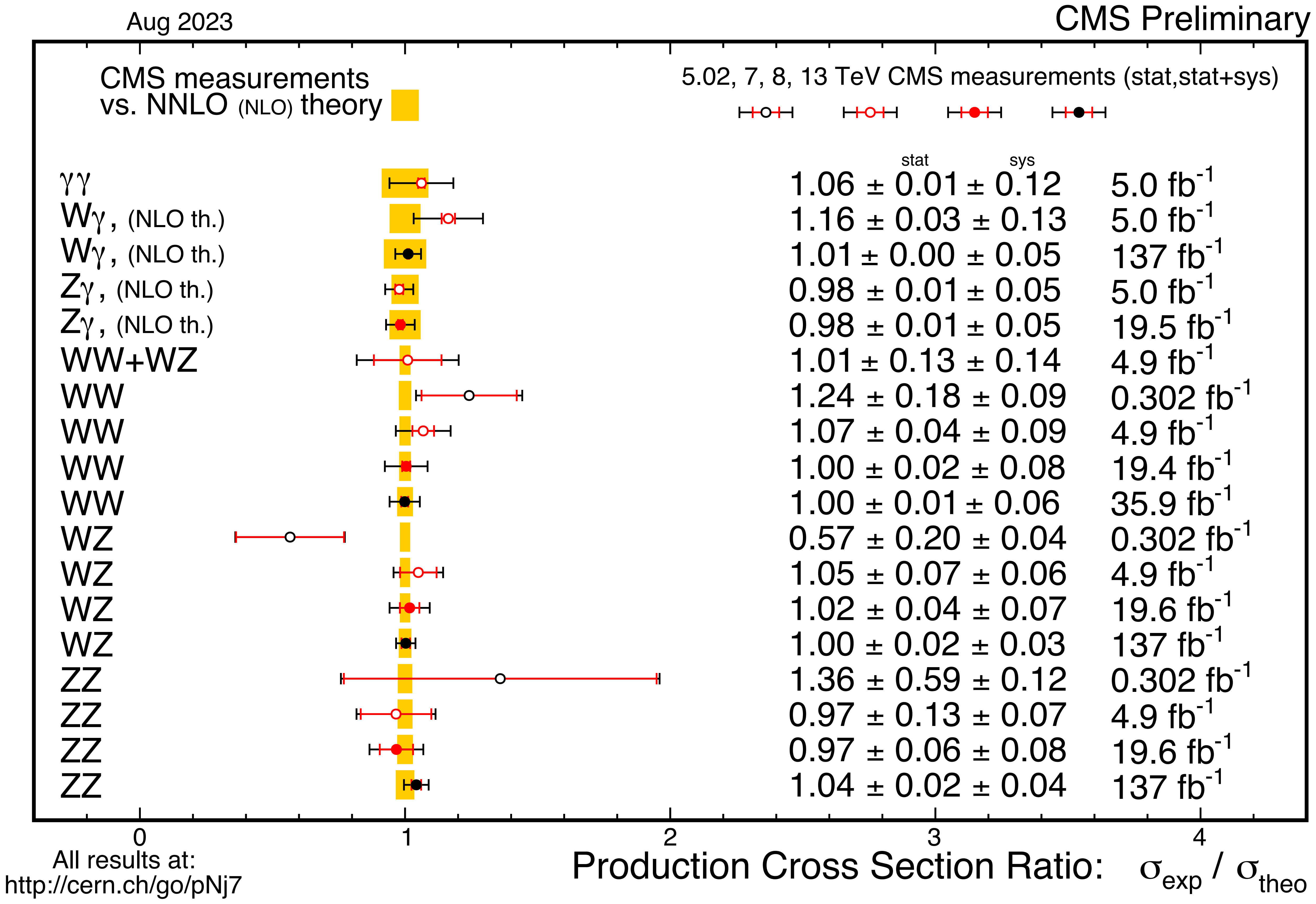
Motivation

- Will only talk about continuum contribution here, not interference, and only a little about pure Higgs production. **(see talks by Marco, Raoul)**
- Important to have a precise prediction for “boring” background in off shell analysis, plenty of other physics applications besides.
  - theory advances on many fronts in the last 5 years or so.
- As data continues to flow in, precision measurements will provide more opportunity to validate physics modeling and refine analyses:
  - differential measurements, q_T spectra
  - measurements with a jet veto / in jet bins

Importance of NNLO QCD

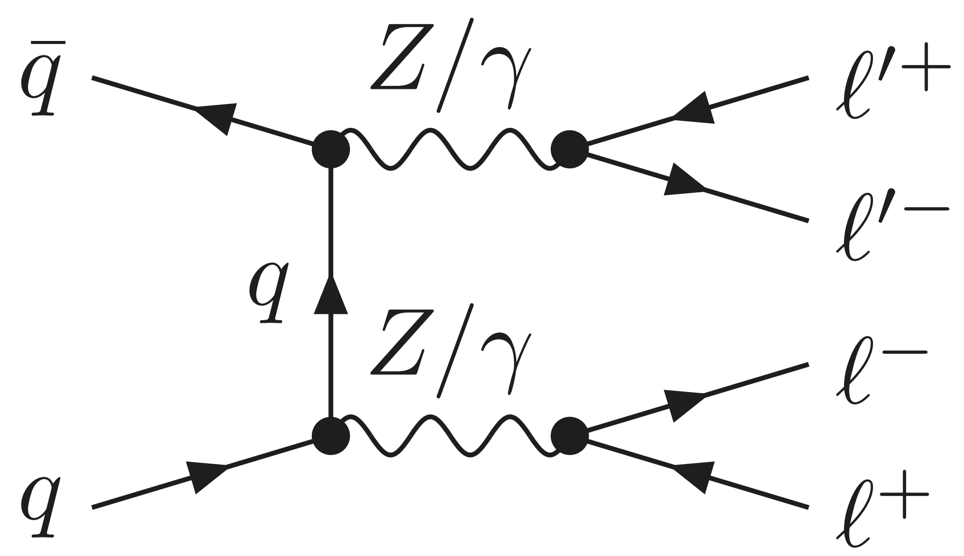
- Calculations at NNLO in QCD are essential to properly describe much of the current diboson data at the LHC and are mostly widespread.

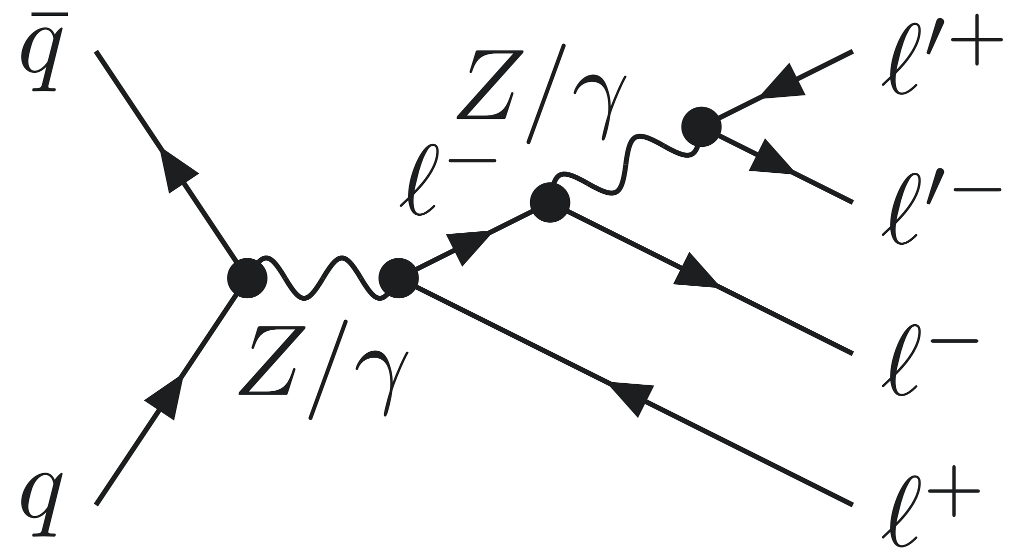




NNLO QCD and beyond

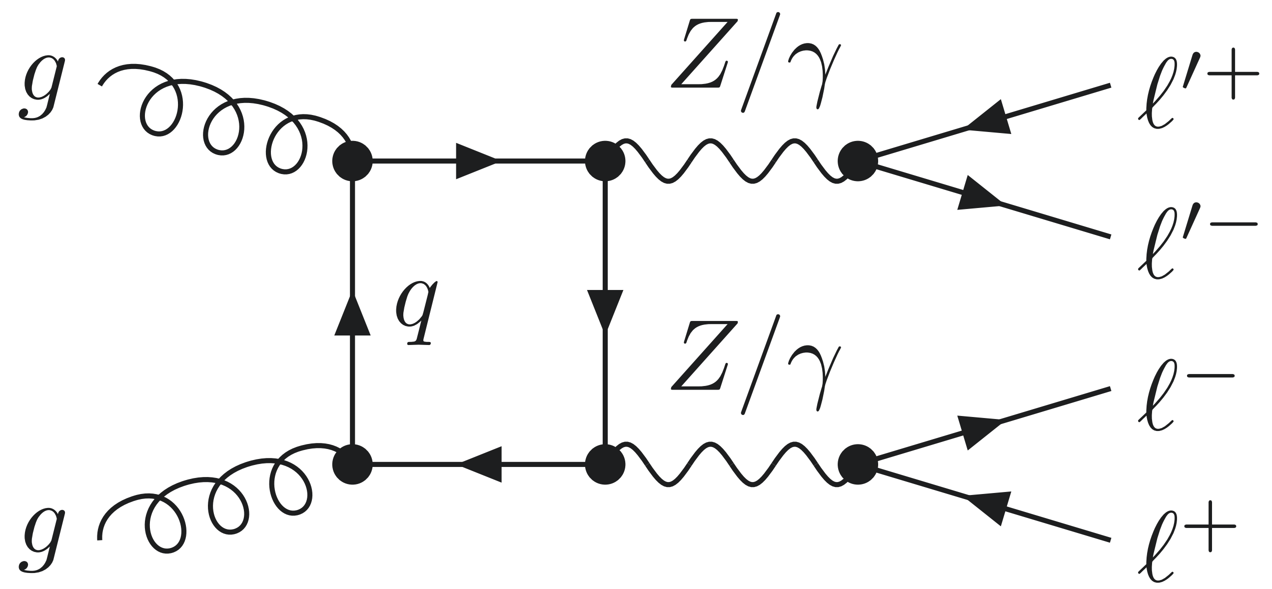
Up to $\mathcal{O}(\alpha_s^2)$ corrections to Born-level quark-antiquark channel





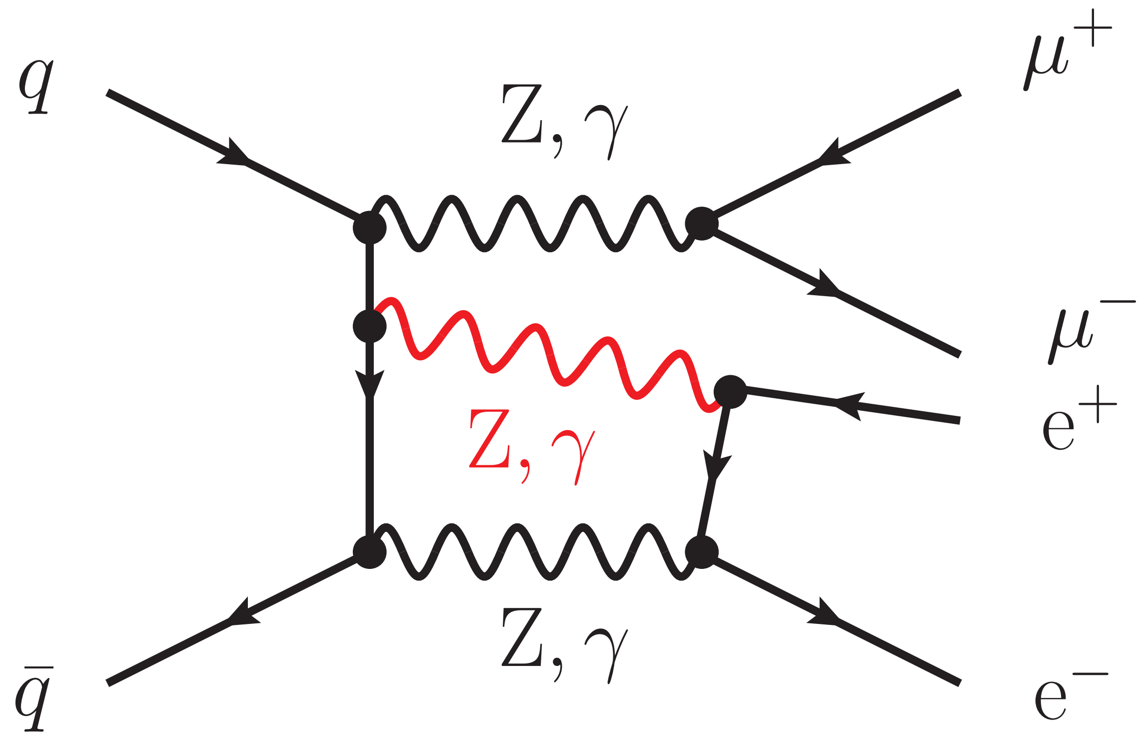
gg channel: first enters at NNLO QCD, corrections are leading part of $\mathcal{O}(\alpha_s^3)$ contribution ("nNNLO")

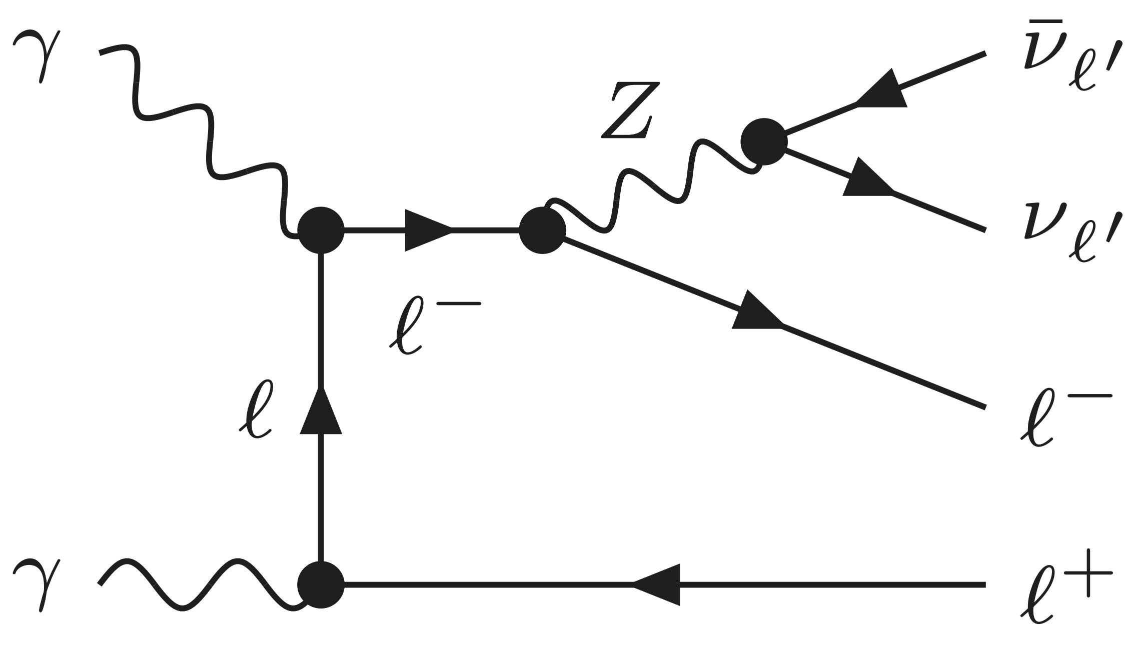
Grazzini et al., 1811.09593



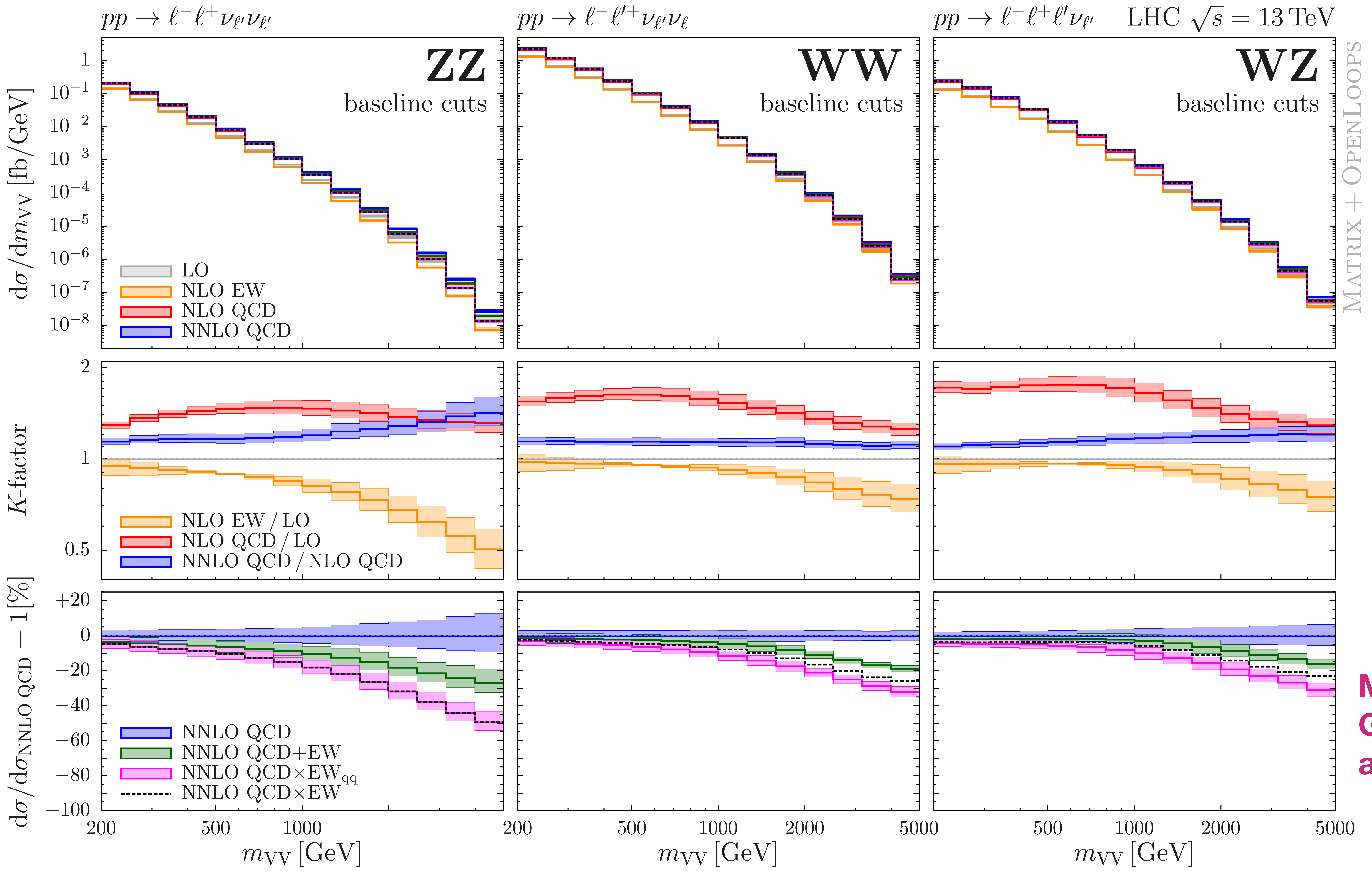
NLO electroweak corrections, including photon-initiated channels

Grazzini et al., 1912.00068





NNLO QCD & NLO EW



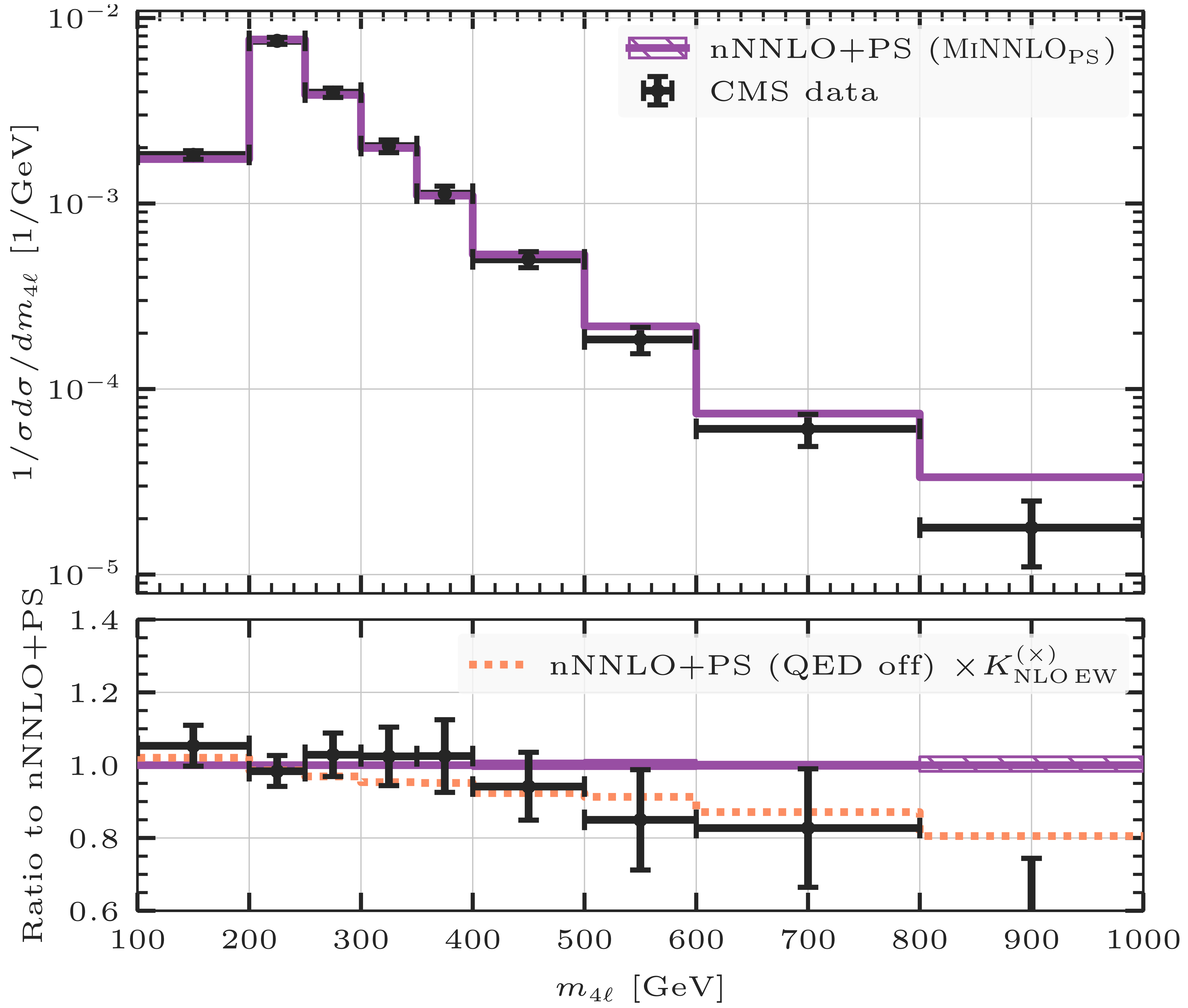
MATRIX
Grazzini et al.
arXiv: 1912.00068

Higher orders + parton shower

(see also talk by Simone)

- Quark-antiquark NNLO channels matched with MINNLOPS.
- Gluon-gluon NLO with POWHEG.
- No NLO EW effects, just estimated in figure to right.
  - expect significant impact in tail ~ $\alpha_w \log^2(M_{4l}/m_Z)$
  - agreement improved with their inclusion.

Buonocore et al. 2108.05337



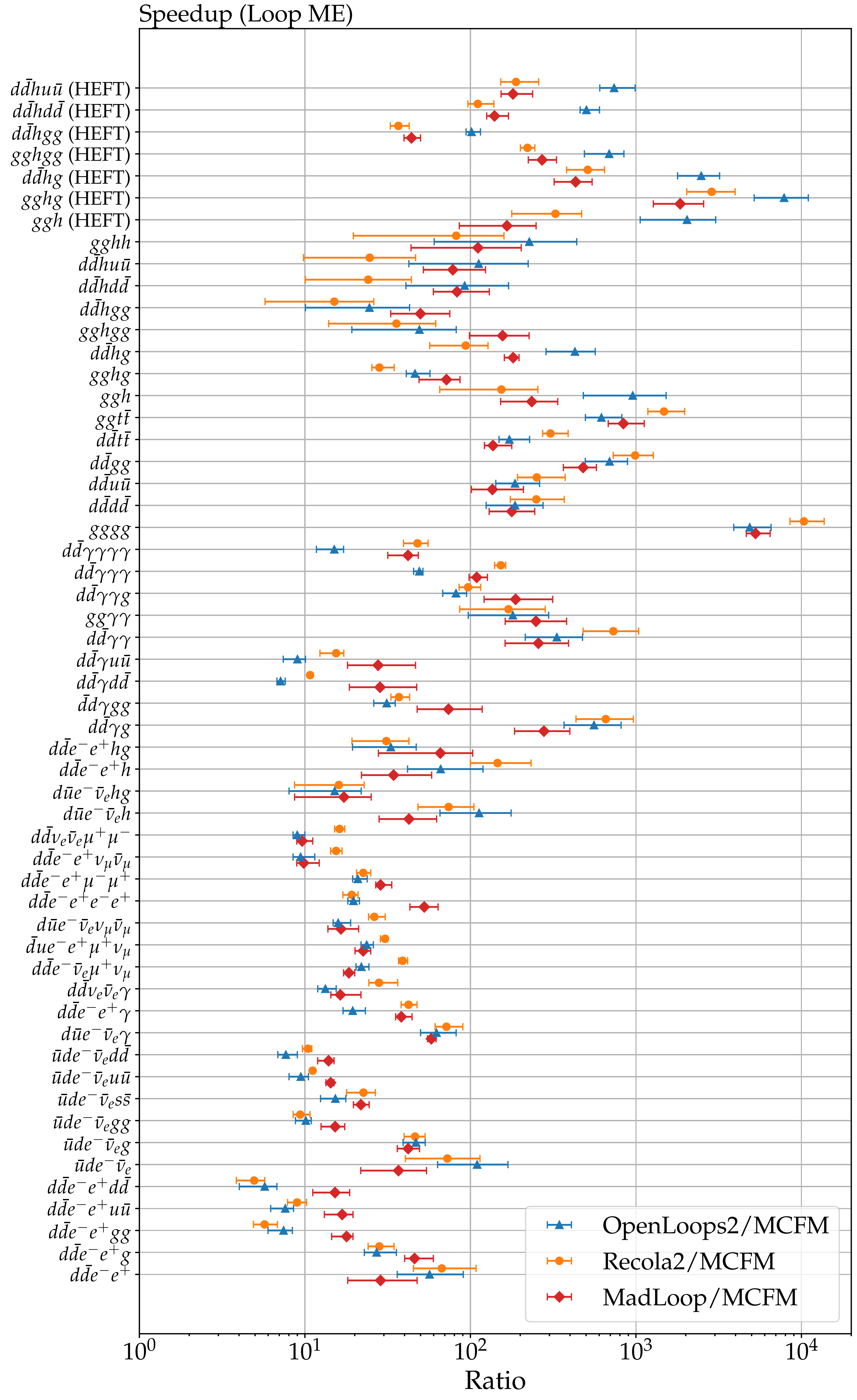
MCFM

Ellis, Neumann, Williams, JC + more: mcfm.fnal.gov

- MCFM contains about 350 processes at hadron-colliders evaluated at NLO.
- Since matrix elements are calculated using analytic formulae one can expect better performance, in terms of stability and computer speed, than fully numerical codes.
- Recent(ish) additions to virtual matrix elements:
  - H+4 partons with full mass effects at one-loop — Budge, De Laurentis, Ellis, Seth, JC, 2107.04472
  - Vector boson pair production at one loop: simplified analytic results for the process $q\bar{q}\ell\bar{\ell}\ell'\bar{\ell}'g$ — De Laurentis, Ellis, JC, 2203.17170
- Color-singlet and a handful of other processes now at NNLO, simplest at N^3LO.
- Most recently: resummation of large logarithms (as $q_T \to 0$ and when using jet veto) matched to NNLO calculations.

MCFM 1-loop library

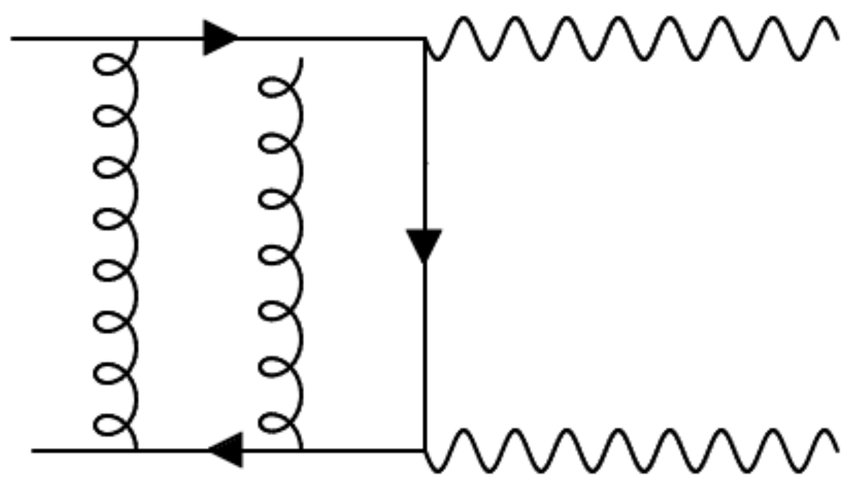
- Analytic 1-loop matrix elements from MCFM are also available in the form of a standalone library. **Hoeche, Preuss, JC, 2107.04472**
  - easily accessed in a similar way as, e.g. OpenLoops, through a C++ interface.
  - potential for significant speed gains vs. a numerical one-loop provider, either as component of higher-order calculation, parton shower, other tools. **(c.f. JHUGen-MELA)**
  - diboson (and + jet) amplitudes all available in this interface.



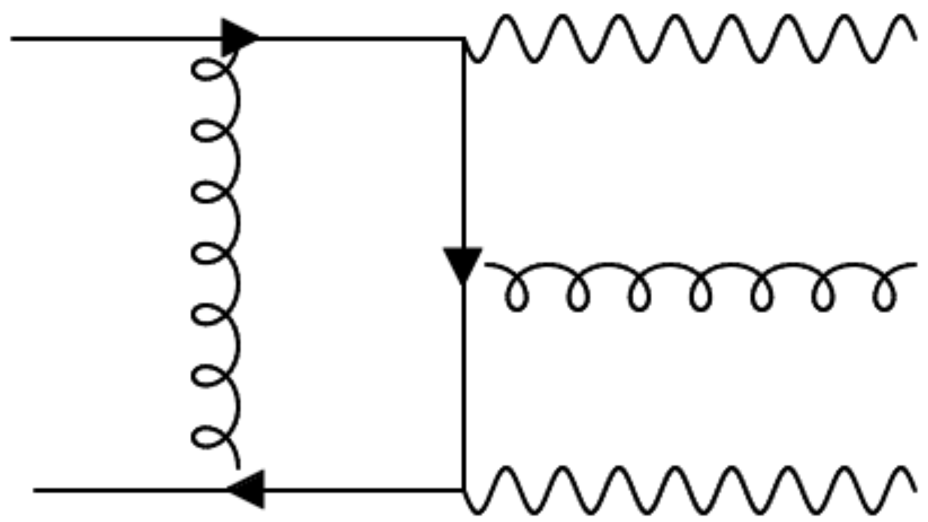
NNLO overview

- NNLO calculations not fully automated in the way that NLO calculations are now; very much by-hand and case-by-case.
  - multiple competing methods with different degrees of ease of calculation, technical challenges, applicability and availability.
- MCFM obtains NNLO predictions using both the jettiness and the q_T slicing schemes — methods for isolating and cancelling infrared (soft and collinear) divergences.
- NNLO results for pp → X require:
  - two loop matrix elements for pp → X
  - process pp → X + 1 parton at NLO
  - so mostly limited to color-singlet processes.

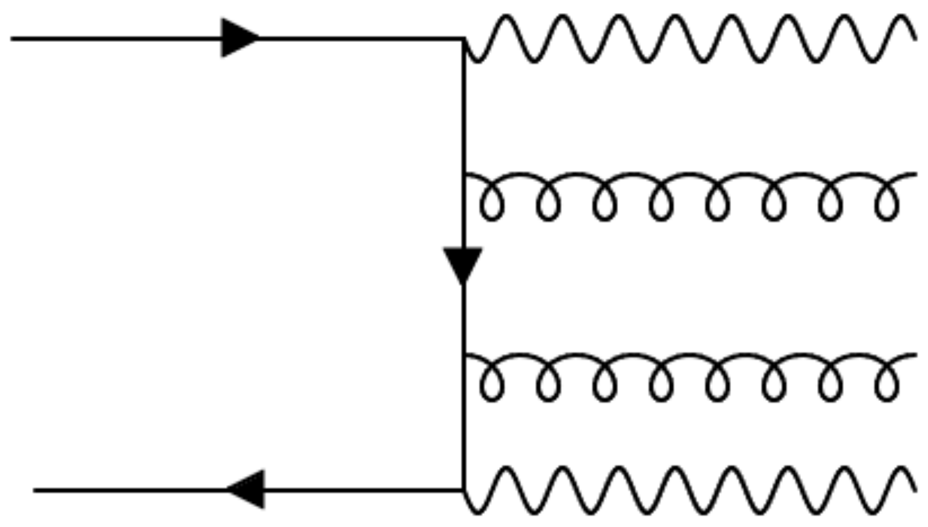
"Pure virtual", e.g. 2-loop diagrams (Born topology)



"Real-virtual", 1-loop with an additional parton



"Real-real", two additional partons

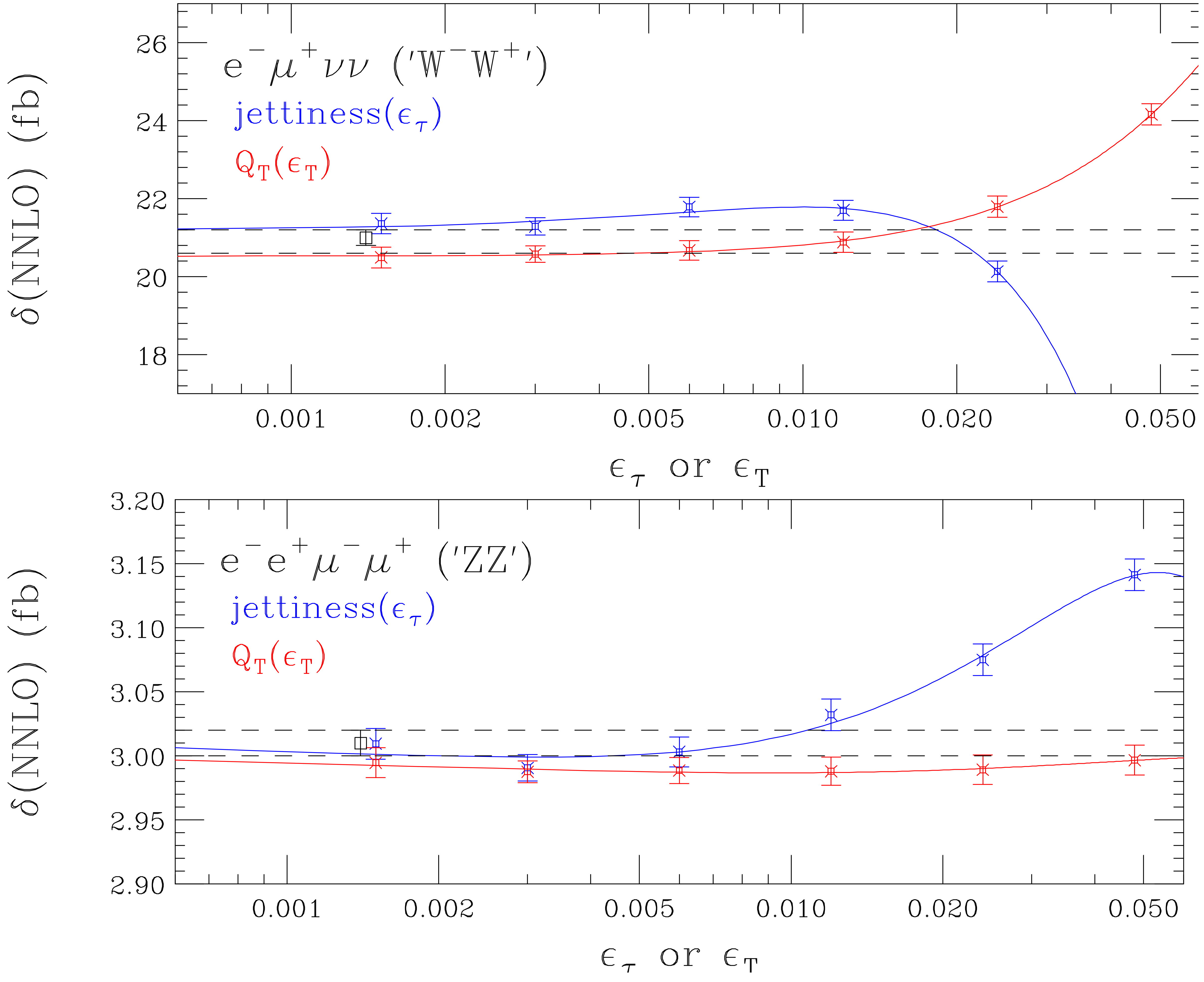


Dibosons @ NNLO: slicing methods

qT $\epsilon_T = q_T^{\rm cut}/Q$

jettiness $\epsilon_\tau = (\tau^{\rm cut}/Q)^{\frac{1}{\sqrt{2}}}$

- Slicing methods depend on a parameter (ε) that must be kept finite, but result only formally correct in limit ε → 0.
  - away from limit there are differences due to power corrections.
- q_T slicing method appears to have smaller power corrections in most cases for equal computational burden.
- However jettiness has the proven ability to deal with final states containing a jet.
  - c.f. attempt to develop formalism for new slicing variables ("k_T-ness"), so far only to NLO.



Buonocore et al, 2201.11519

q_T resummation in MCFM

- Use the SCET-based "collinear anomaly" q_T resummation formalism:
  Becher, Neubert, +Hager, Wilhelm, 1109.6027, 1212.2621, 1904.08325

$$\mathrm{d}\sigma_{ij}(p_1,p_2,\{\underline{q}\}) = \int_0^1 \mathrm{d}z_1 \int_0^1 \mathrm{d}z_2\, \mathrm{d}\sigma^0_{ij}(z_1p_1,z_2p_2,\{\underline{q}\})\,\mathcal{H}_{ij}(z_1p_1,z_2p_2,\{\underline{q}\},\mu)$$
$$\times\, \frac{1}{4\pi}\int \mathrm{d}^2x_\perp\, e^{-iq_\perp x_\perp} \left(\frac{x_T^2Q^2}{b_0^2}\right)^{-F_{ij}(x_\perp,\mu)} \times B_i(z_1,x_\perp,\mu)\cdot B_j(z_2,x_\perp,\mu)\,,$$

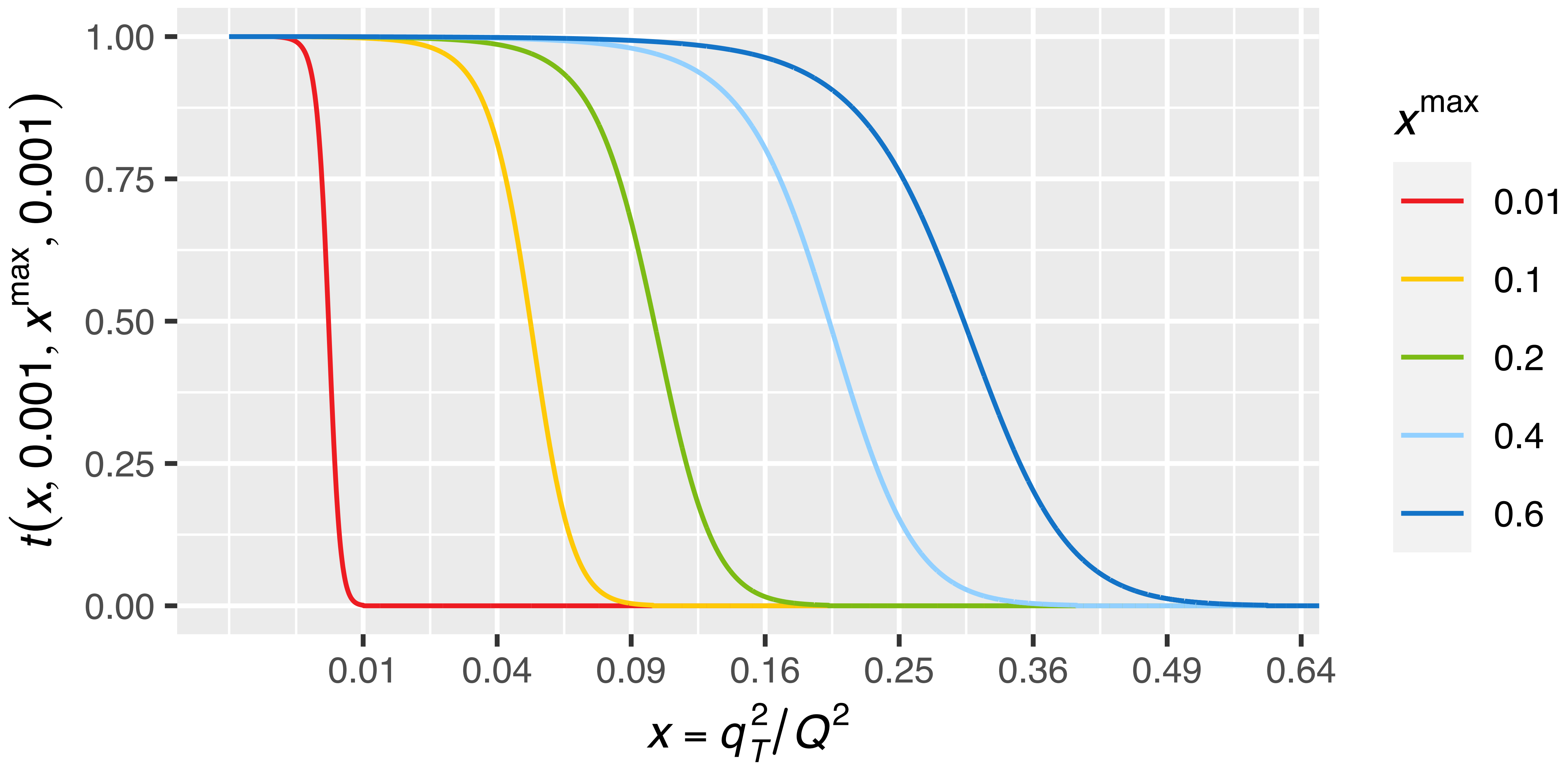
- All universal ingredients (beam functions, B_i , B_j and collinear anomaly exponent F_{ij}) known up to 3 loops.

- Resums large logarithms of the form $\log(q_T/Q)$, cures fixed-order divergence as $q_T \rightarrow 0$.
  - piggybacks existing machinery of NNLO calculations in MCFM to reach N^{3}LL+NNLO accuracy for important processes.
  - implemented as "CuTe-MCFM", first results for DY, Higgs, VH, $\gamma\gamma$, $Z\gamma$.
    Becher, Neumann 2009.11437

Matching to fixed order

- Fixed order result recovered up to higher order terms, which can induce unphysical behavior at large q_T.
- Match by expanding resummed result and replacing with fixed-order one — but computationally demanding at small q_T (introduce cutoff q_0).
- Implement a transition function to smoothly pass between resummed and fixed-order domains, choosing its parameters on a case-by-case basis.
- Sensitivity to transition function reduced order by order, parameters can be tuned to data.

$$\left.\frac{\mathrm{d}\sigma^{\mathrm{N^3LL}}}{\mathrm{d}q_T}\right|_{\text{naively matched to NNLO}} = \frac{\mathrm{d}\sigma^{\mathrm{N^3LL}}}{\mathrm{d}q_T} + \underbrace{\frac{\mathrm{d}\sigma^{\mathrm{NNLO}}}{\mathrm{d}q_T} - \left.\frac{\mathrm{d}\sigma^{\mathrm{N^3LL}}}{\mathrm{d}q_T}\right|_{\text{exp. to NNLO}}}_{\text{matching correction } \Delta\sigma}$$



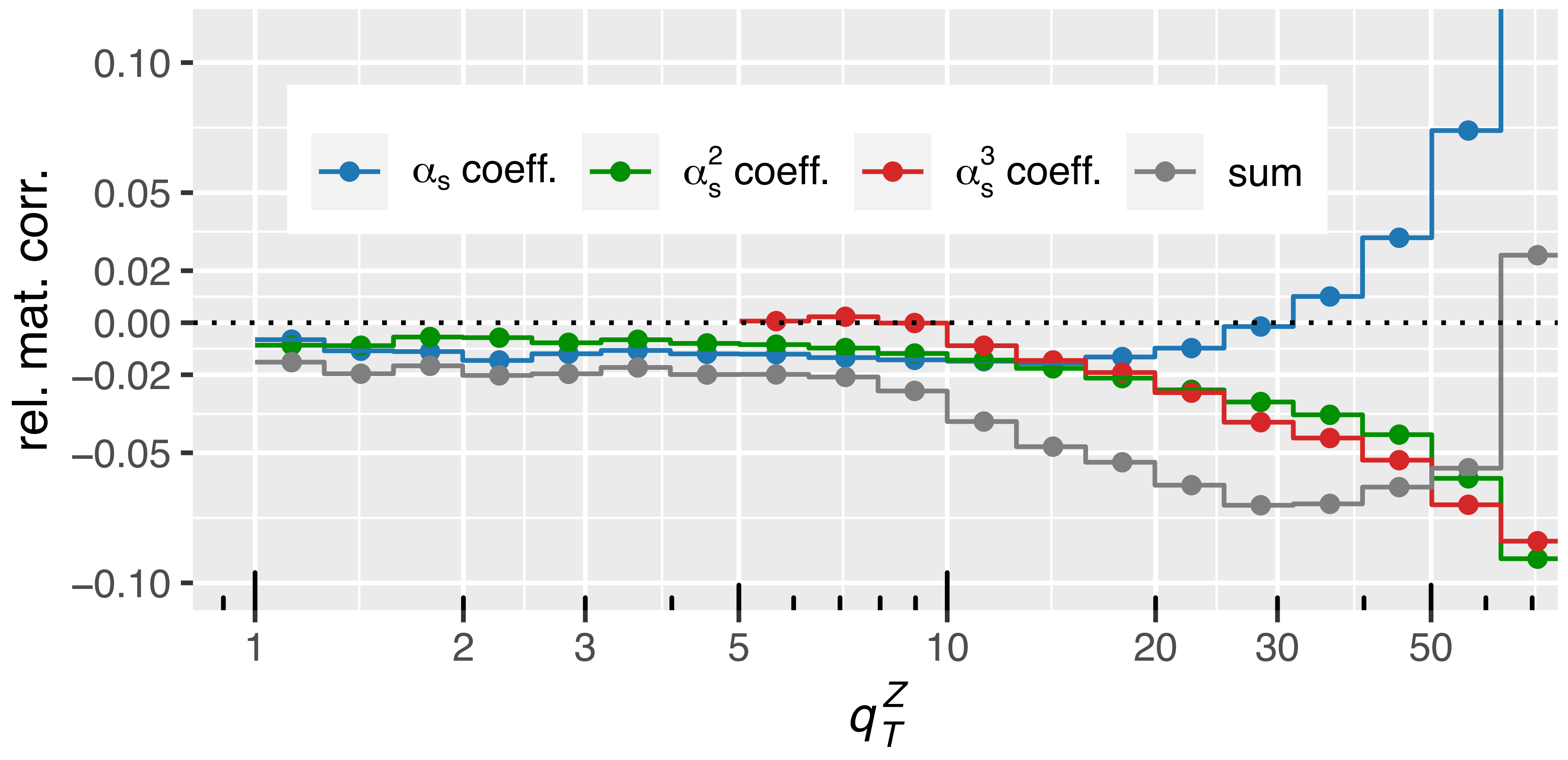
$$\left.\frac{\mathrm{d}\sigma^{\mathrm{N^3LL}}}{\mathrm{d}q_T}\right|_{\text{matched to NNLO}} = t(x)\left(\frac{\mathrm{d}\sigma^{\mathrm{N^3LL}}}{\mathrm{d}q_T} + \Delta\sigma|_{q_T>q_0}\right) + (1-t(x))\frac{\mathrm{d}\sigma^{\mathrm{NNLO}}}{\mathrm{d}q_T}$$

Becher, Neumann 2009.11437

Validation: Drell-Yan at N^4LL_p+N^3LO

Neumann, JC, 2207.07056

- Use recent calculations to push logarithmic accuracy to next order.
  - 3-loop beam functions 1912.05778, 2006.05329, 2012.03256, Luo et al. and Ebert et al.
  - 4-loop rapidity anomalous dimension Duhr et al., 2205.02242; Moult et al., 2205.02249
- "p": 5-loop cusp estimated (negligible) and missing unknown N^3LO PDFs.
- Combine with MCFM Z+jet calculation at NNLO to also reach N^3LO accuracy for this process.
- Performing pure fixed-order calculation tough at very low q_T but in practice only need to be convinced that matching corrections approach zero and are sufficiently small.



Comparison with CMS

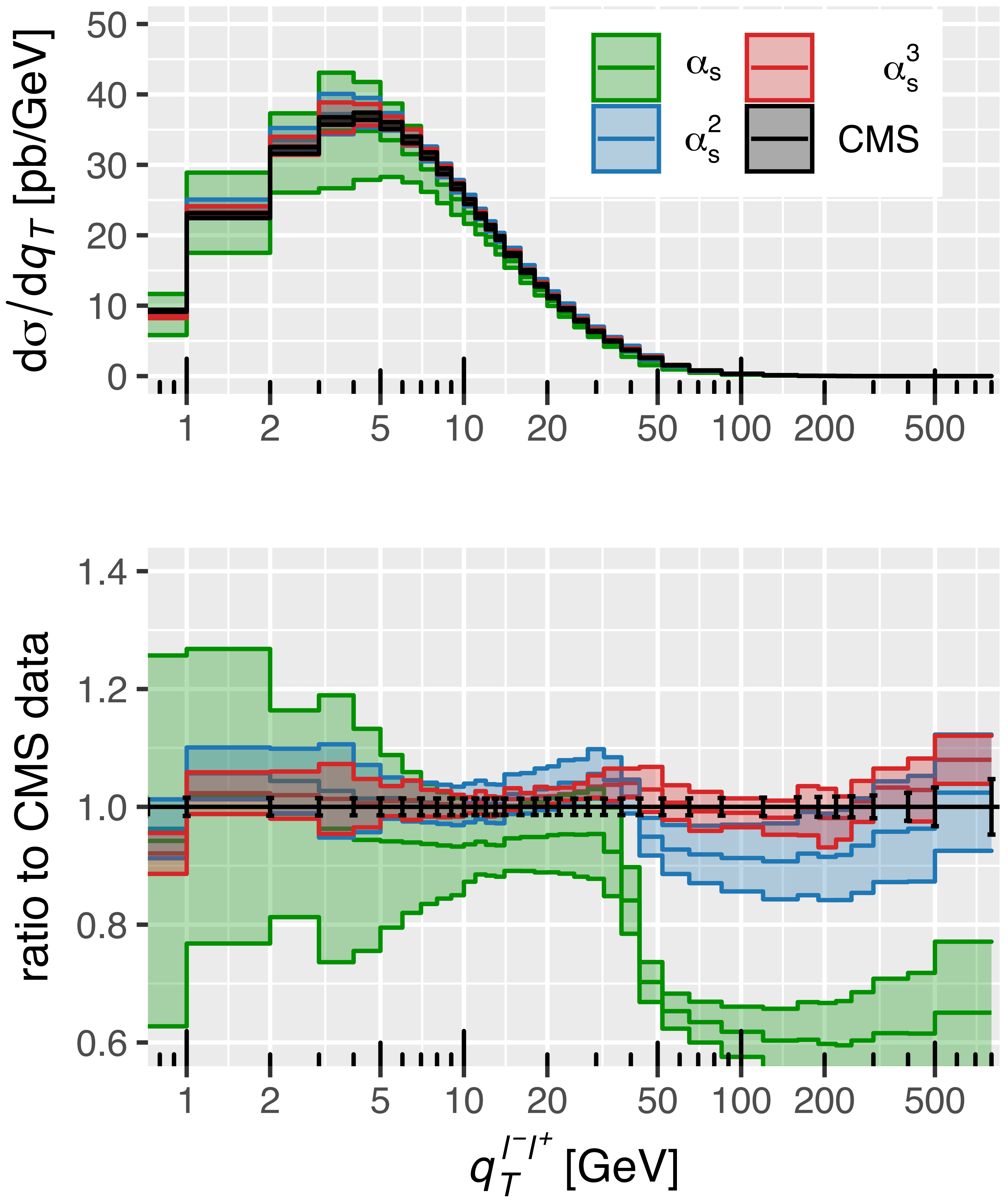
- Excellent agreement with CMS data at the highest order, noticeable improvement at both low and high q_T.
- Integrate over spectrum for a cross-section comparison.

Order k	fixed-order α_s^k	res. improved α_s^k
0	694^{+85}_{-92}	—
1	732^{+19}_{-30}	$637 \pm 8_{\text{mat.}} \pm 70_{\text{sc.}}$
2	720^{+4}_{-3}	$707 \pm 3_{\text{mat.}} \pm 29_{\text{sc.}}$
3	$700^{+4}_{-6} \pm 1_{\text{slicing}}$	$702 \pm 1_{\text{mat.}} \pm 1_{\text{m.c.}} \pm 17_{\text{sc.}}$

$699 \pm 5\,(\text{syst.}) \pm 17\,(\text{lumi.})$ (e, μ combined) [3]

- Total uncertainty larger by factor 2 than RadISH+NNLOJET.

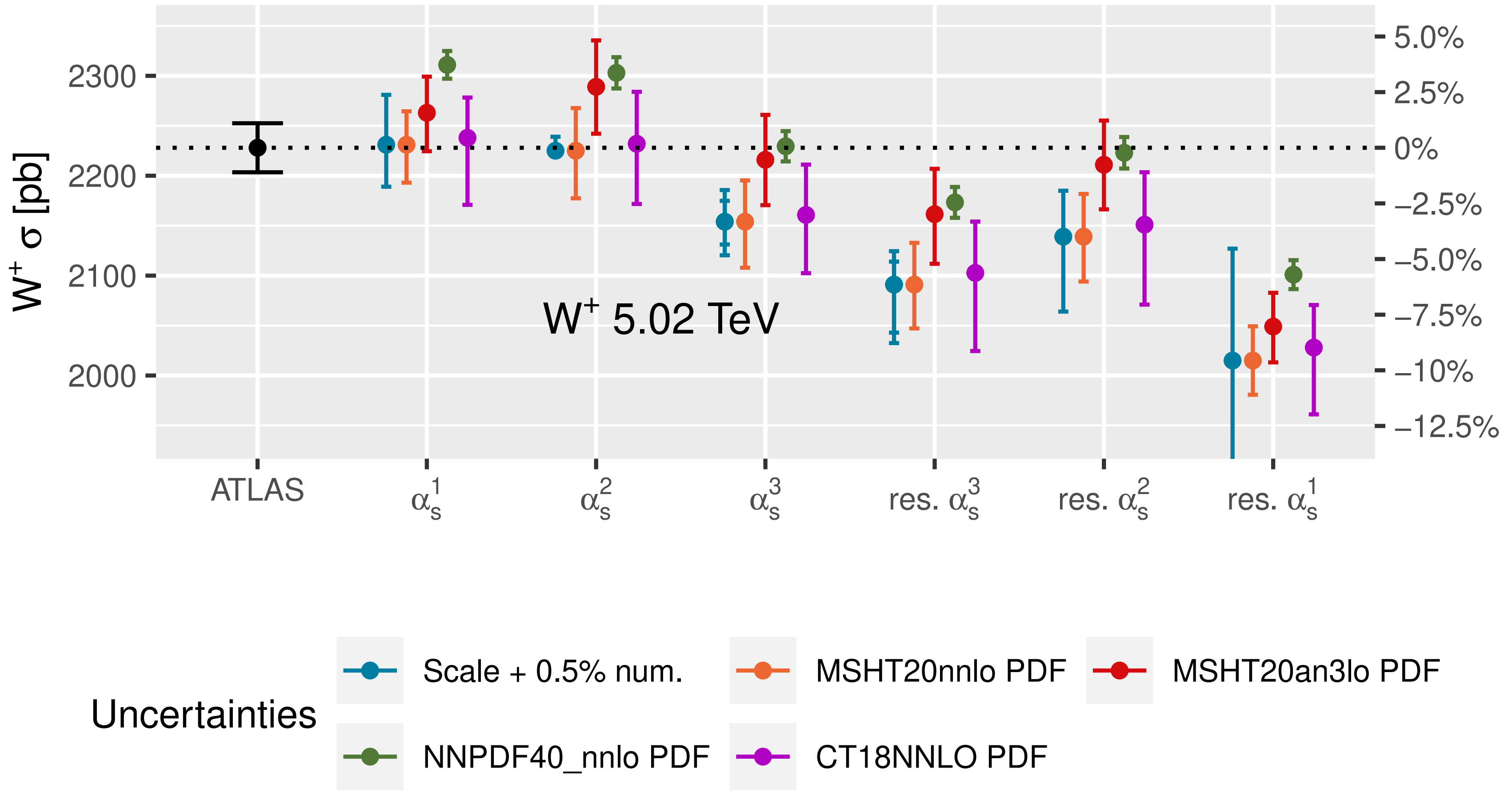
Chen et al., 2203.01565



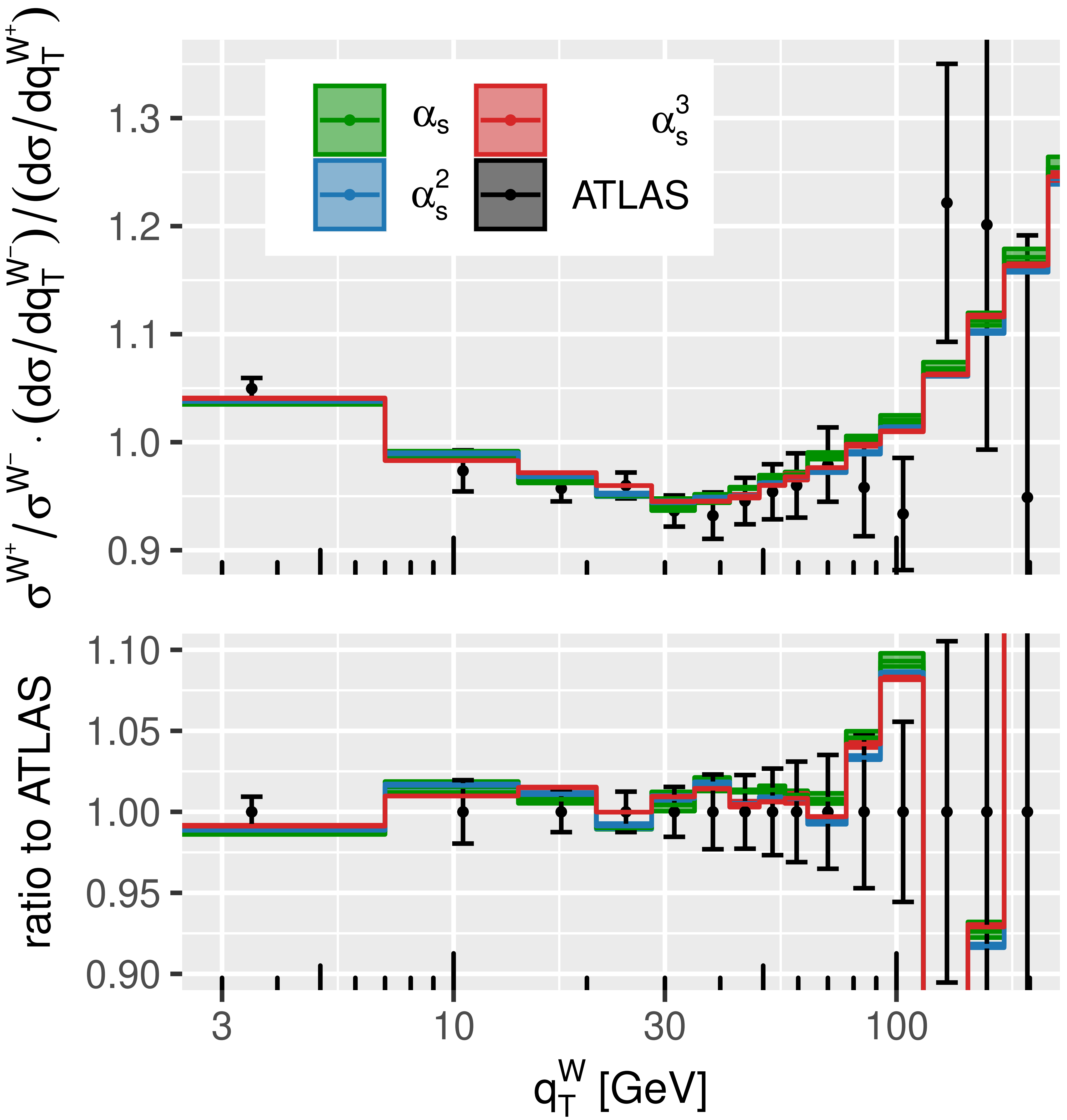
Advert: W production

Neumann, JC, 2308.15382

- W production using the same formalism.
- Surprisingly large N3LO corrections unless also using (approx.) N3LO PDFs.



- Comparison with low-pileup ATLAS data @ 5.02 TeV.
  - Lacking publication of detailed data to compare most predictions.



Resummation for dibosons

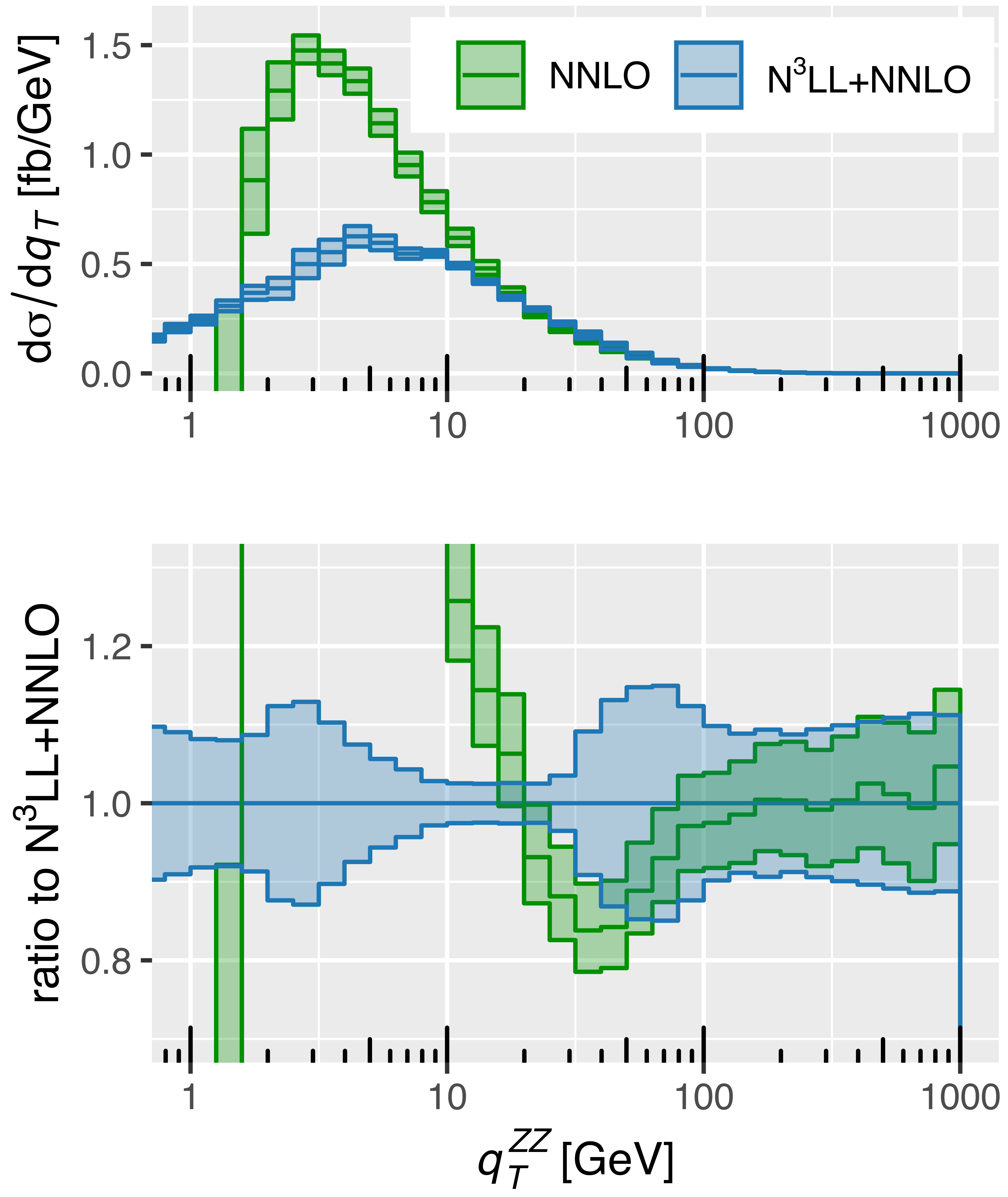
- Now turn to similar studies for diboson production.
- Much of formalism essentially the same; process-independent features such as scale dependence and non-perturbative effects should carry over.
  - in the future, exploit improved understanding gained from studies of Drell-Yan process, e.g. tuning of matching, non-perturbative input (not yet included here).
- Many different approaches for performing resummed calculations and matching, understanding uncertainty estimates.
  - good to have multiple approaches, c.f. MATRIX+RADISH and GENEVA for WW, see ongoing discussion in LHC EW WG for Drell-Yan case.

Kallweit et al., 2004.07720; Gavardi et al, 2308.11577

ZZ production at small q_T

Ellis, Neumann, Seth, JC, 2210.10724

- Resummation effects are potentially more important for vector boson pair production at the same q_T since Q is larger.
- Transition between about 50 and 100 GeV, $(q_T/Q)^2 \sim [0.05,\ 0.2]$, leading to total uncertainty up to 15% in that region.
- Resummation at N³LL+NNLO becomes important below those scales, small uncertainties until ~ 5 GeV.



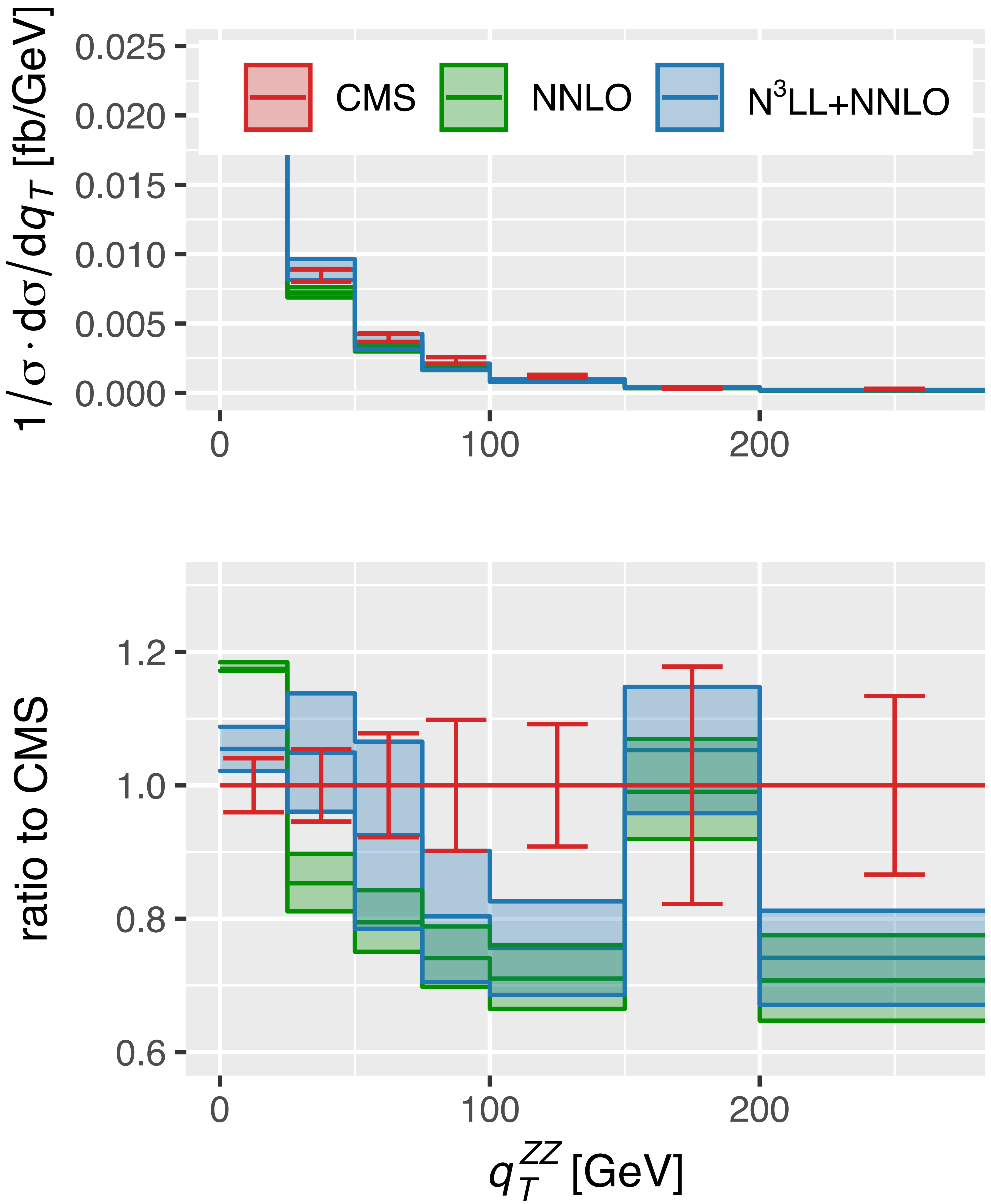
Transverse momentum distribution of the ZZ pair at NNLO and NNNLL+NNLO using CMS cuts at $\sqrt{s} = 13.6$ TeV

Comparison with CMS 13 TeV data

- We simplify the CMS analysis, by applying the same cuts to both electrons and muons and neglect (tiny) identical particle effects.

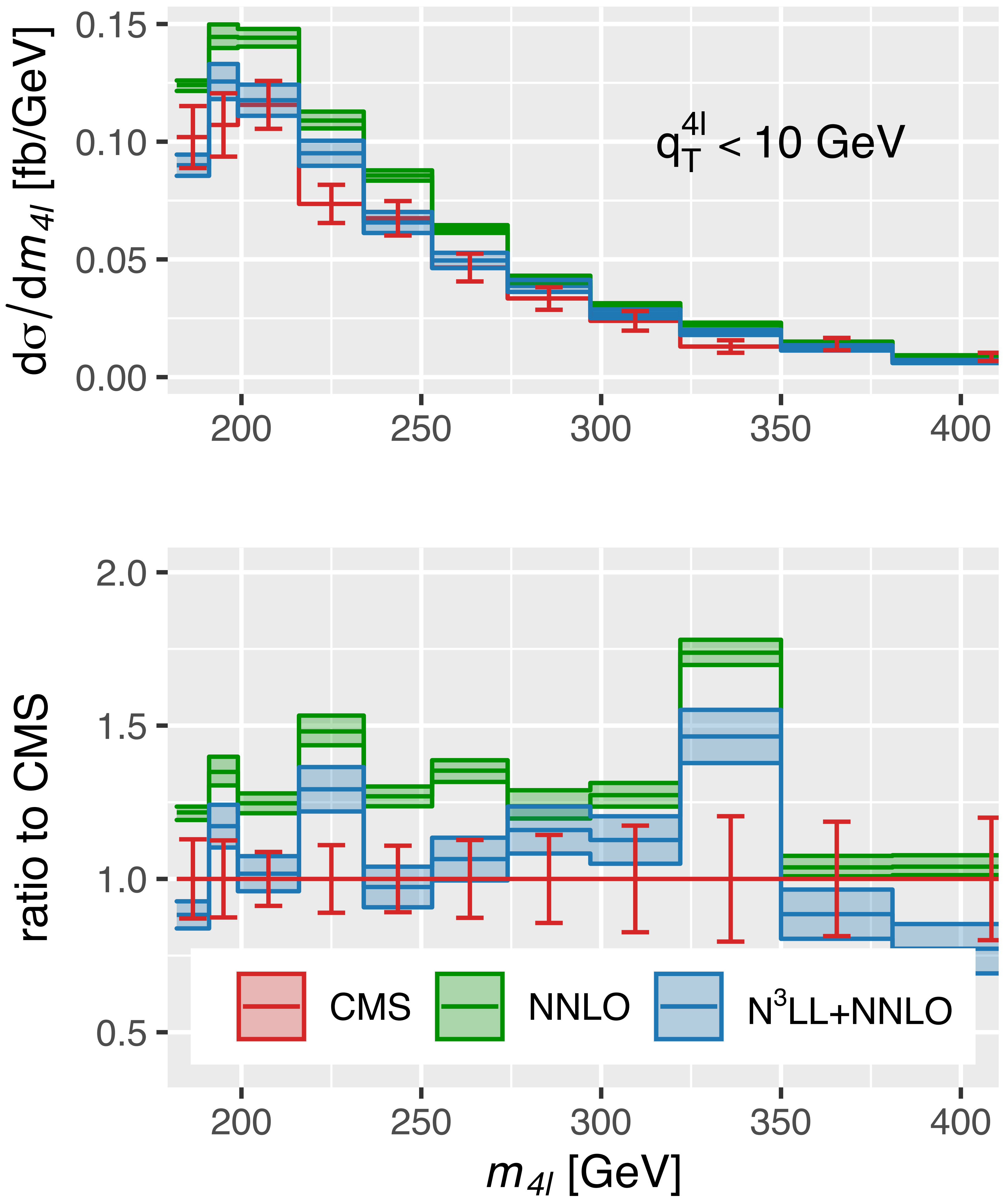
lepton cuts	$q_T^{l_1} > 20\,\mathrm{GeV},\ q_T^{l_2} > 10\,\mathrm{GeV},$ $q_T^{l_{3,4}} > 5\,\mathrm{GeV},\ \eta^l < 2.5$
lepton pair mass	$60\,\mathrm{GeV} < m_{l^-l^+} < 120\,\mathrm{GeV}$

- Resummation improves description below $q_T \sim 75$ GeV.
- More data will allow finer binning, so the resummation effects will be more prominent.



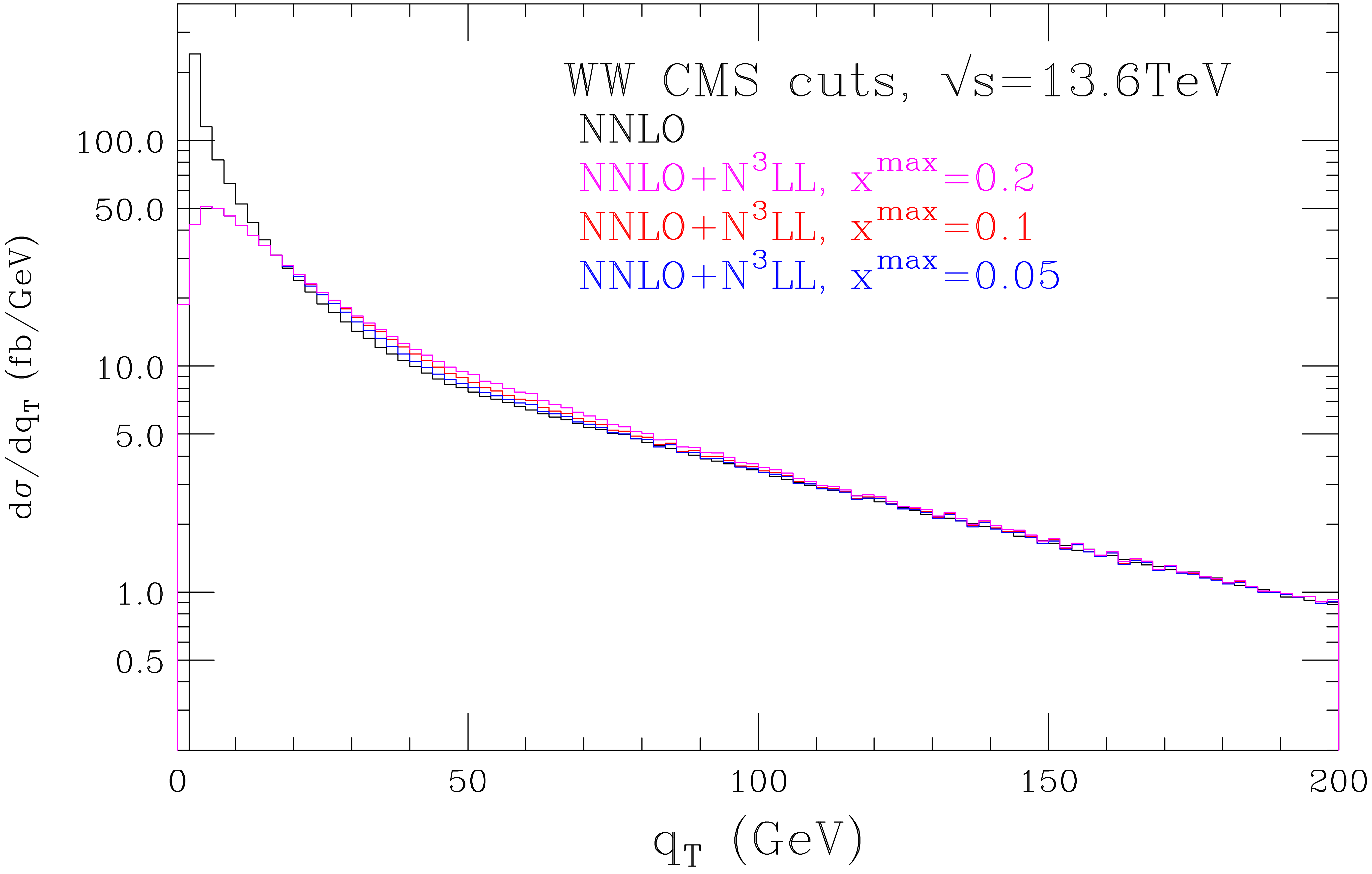
ZZ data: ATLAS

- The ATLAS collaboration (2103.01918) performed measurements of the m_{4l} distribution in five slices of q_T^{4l}.
- Expectation is that resummation should improve agreement with the data, as m_{4l} increases, as observed.
- Highly-correlated observables will show effects of resummation, e.g. leading-lepton p_T; not, for example, p_T of all leptons.



Other diboson processes

- WZ and WW q_T distributions show similar pattern but of course not directly measurable.
    - limited experimental interest.
- Much more important for WW is the cross section under the application of a jet veto, to reduce the $t\bar{t}$ background or to look at interference effects in jet bins.



Jet-veto results

- Since, to first approximation, diboson q_T balances jet p_T might think to obtain jet-veto results by integrating out diboson q_T distribution up to jet cut.
- A few subtleties to consider:
  1. This argument only applies for the first emission; more complicated beyond that (i.e. NNLO) and becomes sensitive to jet clustering (cone size, R).
  2. Would assume jet veto extends to all rapidities. Of course this is not what can be done in practice.
  3. How big are the logs anyway? We are not really directly probing small transverse momenta like when we examine q_T distribution.
- Effect of jet veto scales as $(\text{initial state color factor}) \times \log^k(Q/p_T(\text{veto}))$
  → enhanced for dibosons (larger Q) and for Higgs (color); also for off-shell studies.

Jet veto formalism

see, for example, Becher et al, 1307.0025 , Stewart et al, 1307.1808

- Well-developed formalism, primarily focussed on (important) Higgs case;
  - jets defined using sequential recombination jet algorithms.
- Jet vetos generate large logarithms, as codified in factorization formula.
- Beam and soft functions for leading jet p_T recently calculated at two-loop order using an exponential regulator by Abreu et al.
- Jet veto cross sections are simpler than the q_T resummed calculation (no b-space, directly in p_T).

$$d_{ij} = \min(p_{Ti}^n, p_{Tj}^n) \frac{\sqrt{\Delta y_{ij}^2 + \Delta\phi_{ij}^2}}{R}, \qquad d_{iB} = p_{Ti}^n$$

Rapidity regulator ν

$$\frac{d^2\sigma(p_T^{veto})}{dM^2 dy} = \sigma_0 \left| C_V(-M^2, \mu) \right|^2$$
$$\left[\mathcal{B}_c(\xi_1, M, p_T^{veto}, R^2, \mu, \nu)\, \mathcal{B}_{\bar{c}}(\xi_2, M, p_T^{veto}, R^2, \mu, \nu) \times \mathcal{S}(p_T^{veto}, R^2, \mu, \nu) \right]$$

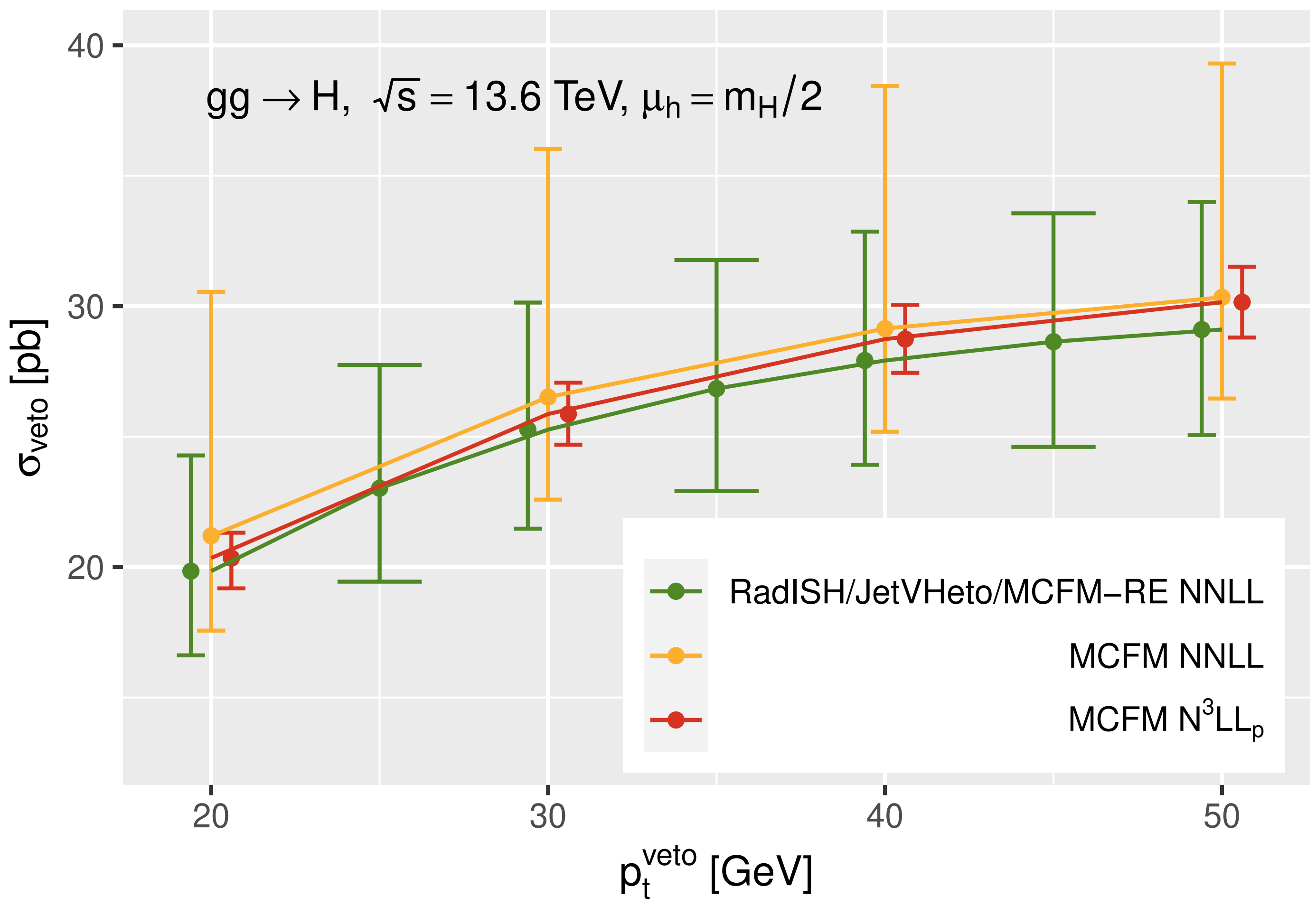
Beam functions
Abreu et al, 2207.07037

Soft function
Abreu et al, 2204.03987

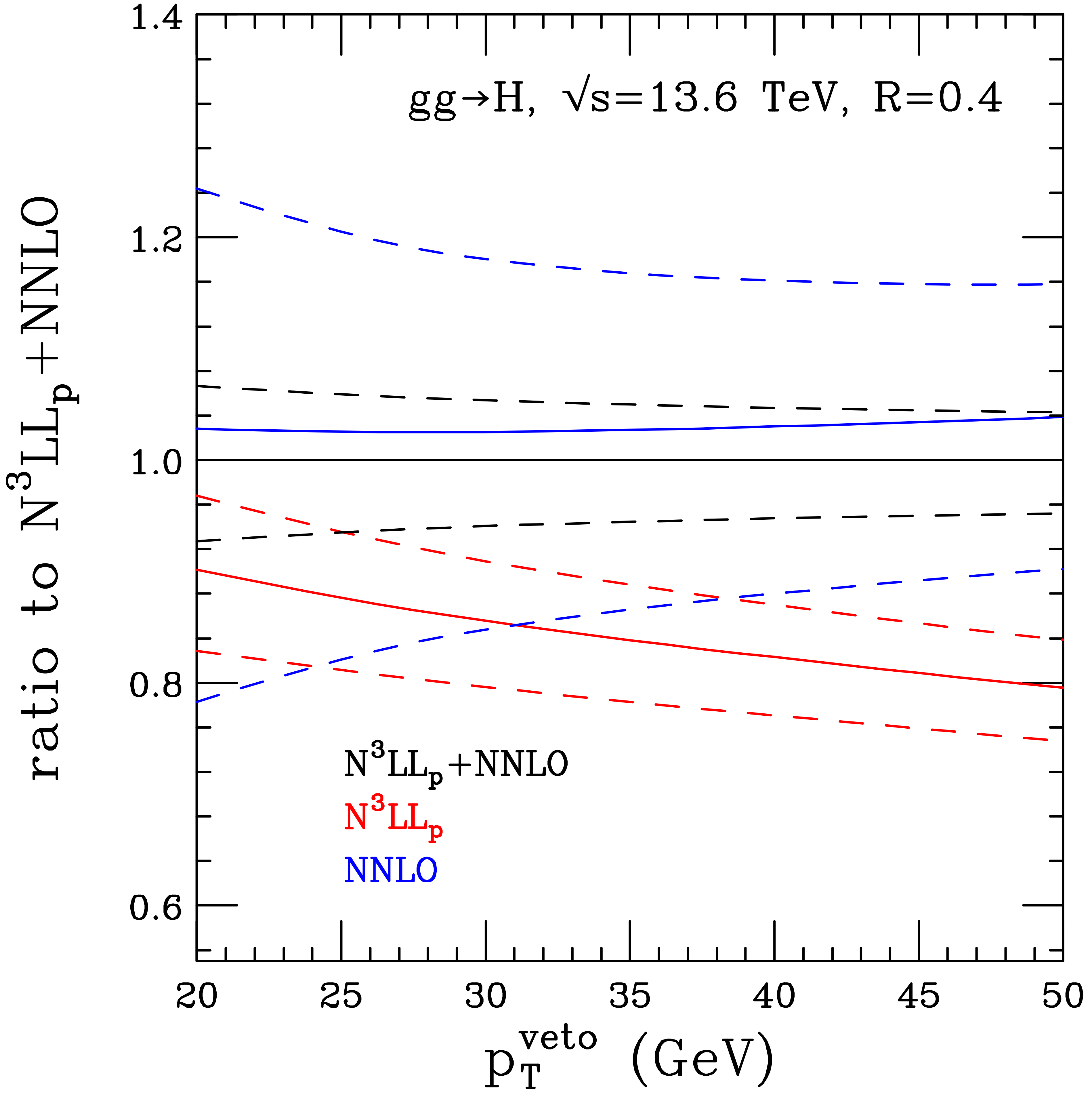
$$\sigma_0 = \frac{4\pi\alpha^2}{3N_c M^2 s} \qquad \xi_{1,2} = (M/\sqrt{s})\, e^{\pm y}$$

Higgs case

- Improves on previously available public implementations to NNLL in RadISH and JetVHeto.



- N3LLp not quite full N3LL, to be explained shortly.
- Right: comparison of fixed order and resummed calculations at highest orders.
    - smaller uncertainties in matched N3LLp+NNLO calculation than at NNLO.



Jet veto in a limited rapidity range

- Factorization formula earlier is valid for cross sections that are vetoed for all values of the jet rapidity.
- Unfortunately for theorists, analyses actually perform jet rapidity cuts, i.e. $\eta < \eta_{\mathrm{cut}}$.
- Can identify three theoretical regions:

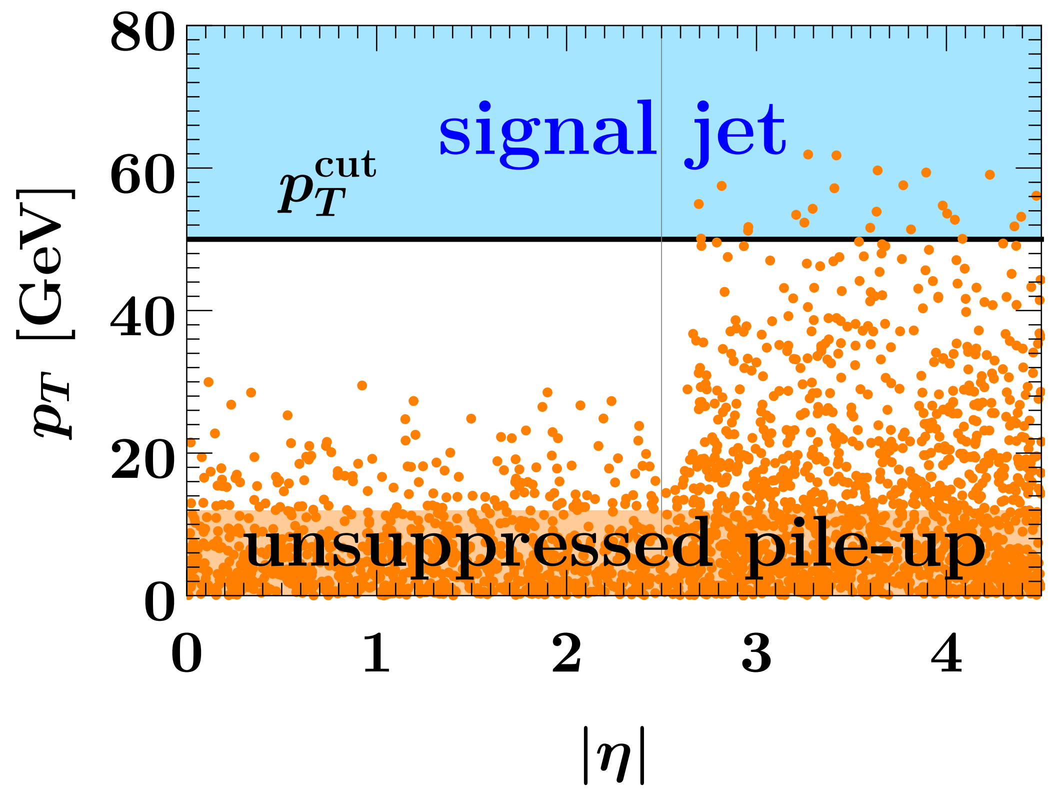
  Michel, Pietrulewicz, Tackmann, 1810.12911

  - $\eta_{\mathrm{cut}} \gg \ln(Q/p_T^{\mathrm{veto}})$

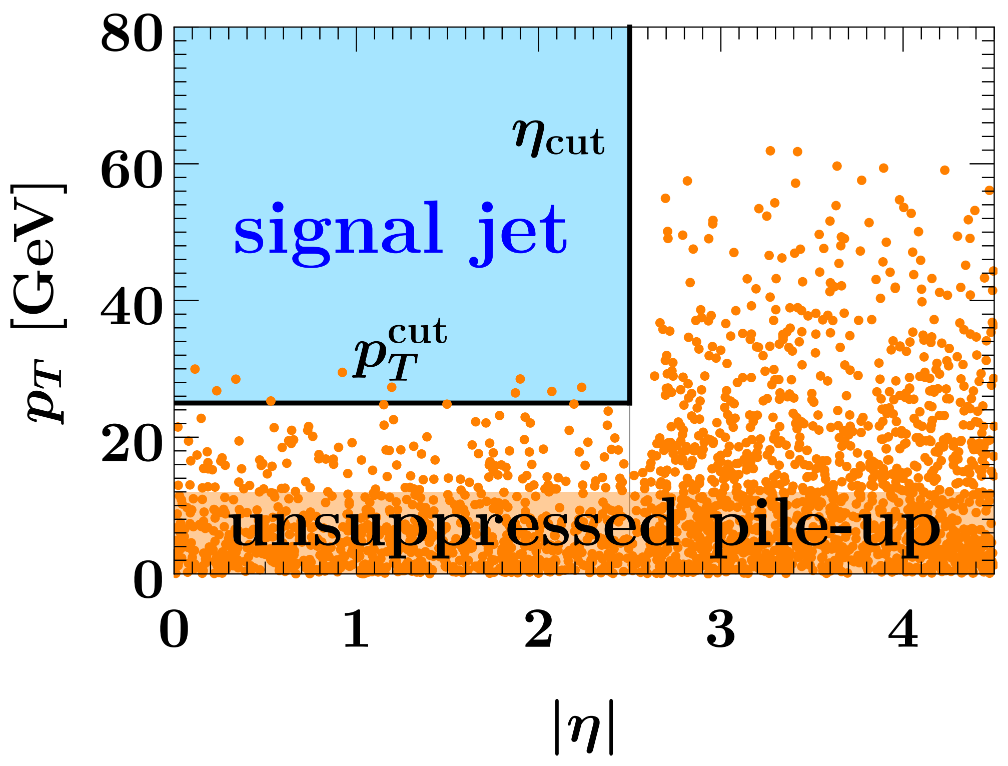
    standard jet veto resummation
  - $\eta_{\mathrm{cut}} \sim \ln(Q/p_T^{\mathrm{veto}})$

    η_{cut}-dependent beam functions
  - $\eta_{\mathrm{cut}} \ll \ln(Q/p_T^{\mathrm{veto}})$

    collinear non-global logs



Current theory calculation



Typical Experimental cuts

Strategy: determine where resummation is potentially important, before considering limited rapidity range resummation

Factorization and N^{3}LL$_p$

$$\left[\mathscr{B}_c(\xi_1, Q, p_T^{veto}, R^2, \mu, \nu)\, \mathscr{B}_{\bar{c}}(\xi_2, Q, p_T^{veto}, R^2, \mu, \nu\ )\mathscr{S}(p_T^{veto}, R, \mu, \nu)\right]_{q^2=Q^2}$$

$$= \left(\frac{Q}{p_T^{veto}}\right)^{-2F_{qq}(p_T^{veto}, R, \mu)} e^{2h^F(p_T^{veto},\mu)}\, \bar{B}_q(\xi_1, p_T^{veto}, R)\, \bar{B}_{\bar{q}}(\xi_2, p_T^{veto}, R)$$

"Collinear anomaly"

- Collinear anomaly expansion: $F_{qq}(p_T^{\text{veto}}, \mu) = a_S F_{qq}^{(0)} + a_S^2 F_{qq}^{(1)} + a_S^3 F_{qq}^{(2)} + \ldots\ , \quad a_S = \dfrac{\alpha_S}{4\pi}$

$$F_{qq}^{(0)} = \Gamma_0^F L_\perp + d_1^{\text{veto}}(R, F)$$

$$L_\perp = 2\ln\frac{\mu}{p_T^{\text{veto}}}$$

$$F_{qq}^{(1)} = \frac{1}{2}\Gamma_0^F \beta_0 L_\perp^2 + \Gamma_1^F L_\perp + d_2^{\text{veto}}(R, F)$$

$$F_{qq}^{(2)} = \frac{1}{3}\Gamma_0^F \beta_0^2 L_\perp^3 + \frac{1}{2}(\Gamma_0^F \beta_1 + 2\Gamma_1^F \beta_0)L_\perp^2 + (\Gamma_2^F + 2\beta_0 d_2^{\text{veto}}(R, F))L_\perp + d_3^{\text{veto}}(R, F)$$

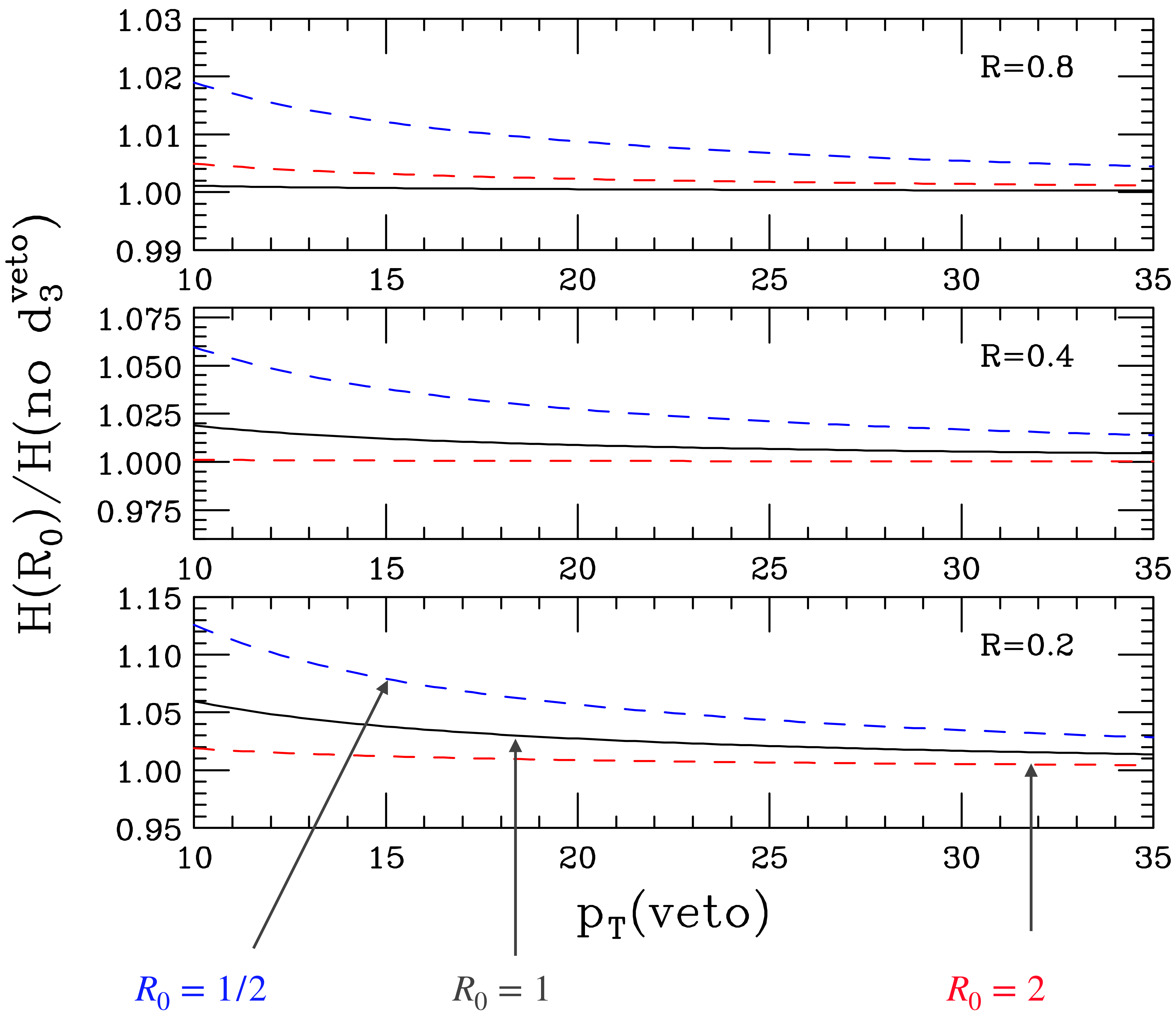
- Full N^{3}LL requires (R-dependent) coefficient d_3^{veto}, which is currently unknown.
  - Extracted in small-R limit — good to O(25%) in d_2^{veto} (for typical R) $\longrightarrow$ only claim N^{3}LL$_p$.
    Banfi et al, 1511.02886

Dependence on approximate d_3^{veto}

Ellis, Neumann, Seth, JC, 2301.11768

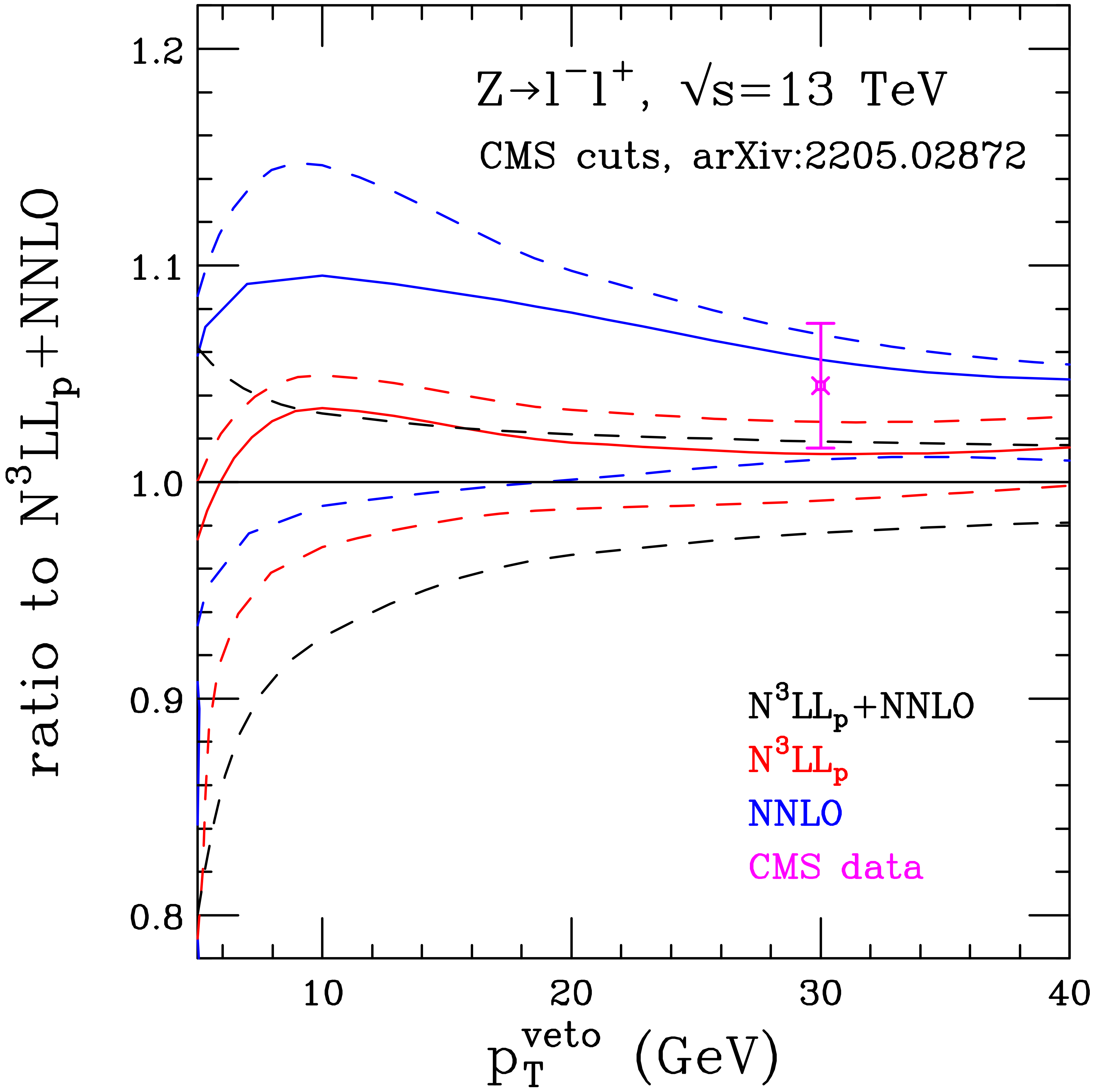
- $d_3^{\text{veto}} \sim -8.4 \times 64 C_B \ln^2(R/R_0)$
- R_0 varied as an uncertainty: for R=0.4, varying between 0.5 and 2 scales d_3^{veto} in the range [0.06,3].
- Contributes as $\left(\frac{m_H}{p_T^{\text{veto}}}\right)^{-2\left(\frac{\alpha_s(\mu)}{4\pi}\right)^3 d_3^{\text{veto}}}$

  so in this approximation ($d_3^{\text{veto}} < 0$) it increases the cross section.
- Estimate $\leq 2.5\ \%$ uncertainty at p_T^{veto}= 25 GeV and $R = 0.4$.



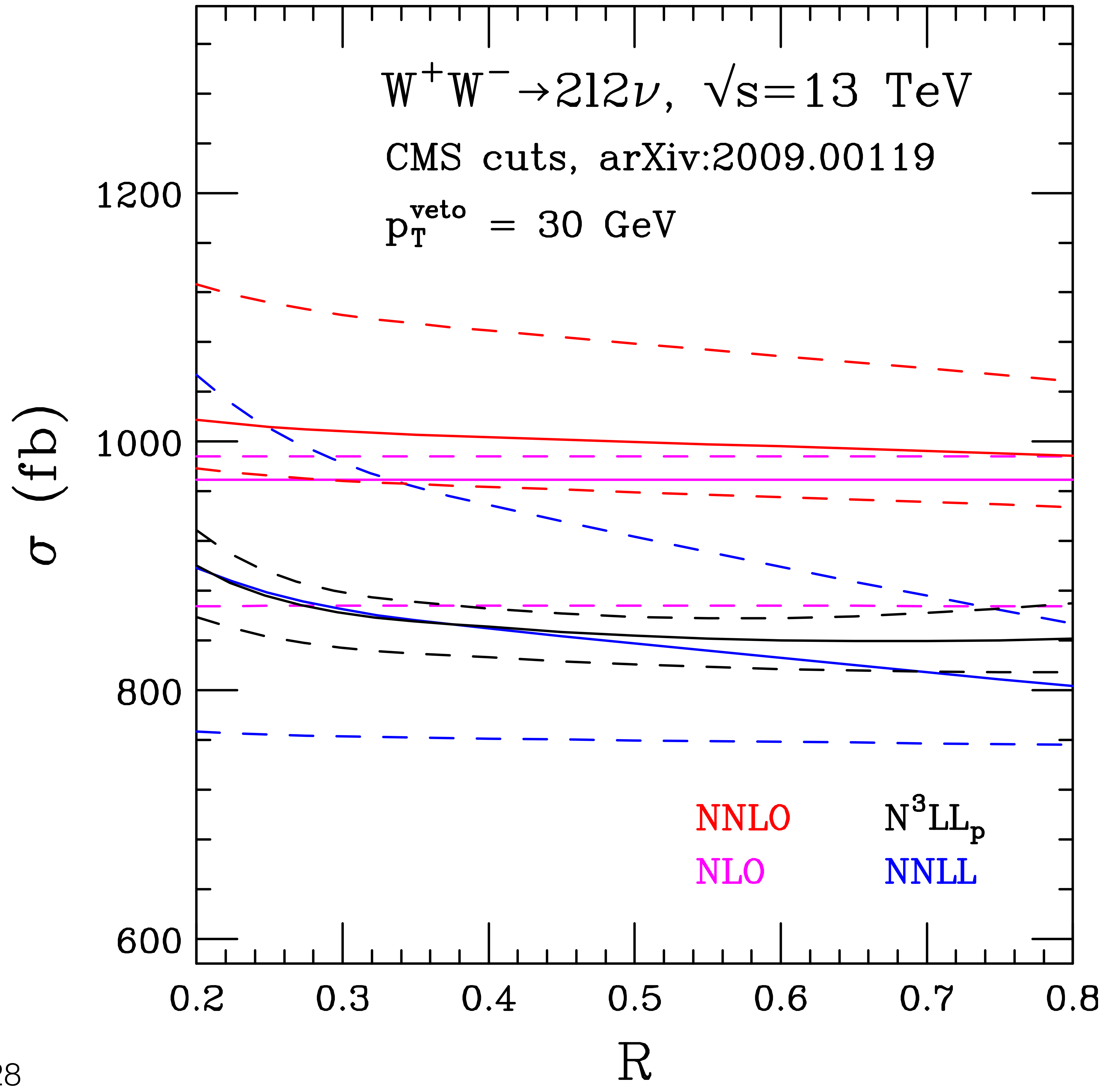
Warmup: Z production @ CMS

- For $p_T^{\rm veto} = 30$ GeV, $(\ln(Q/p_T^{\rm veto} = 1.1) \ll (\eta_{\rm cut} = 2.4)$
  - resummation formalism appropriate but expect that logs are not large enough to require it.
  - indeed, actual calculations differ only by about 5%, within errors.
- No large differences between NNLO and N^3LL_p +NNLO calculations across the range
  - but uncertainties are smaller in the resummed calculation, particularly (as expected) at small $p_T^{\rm veto}$.

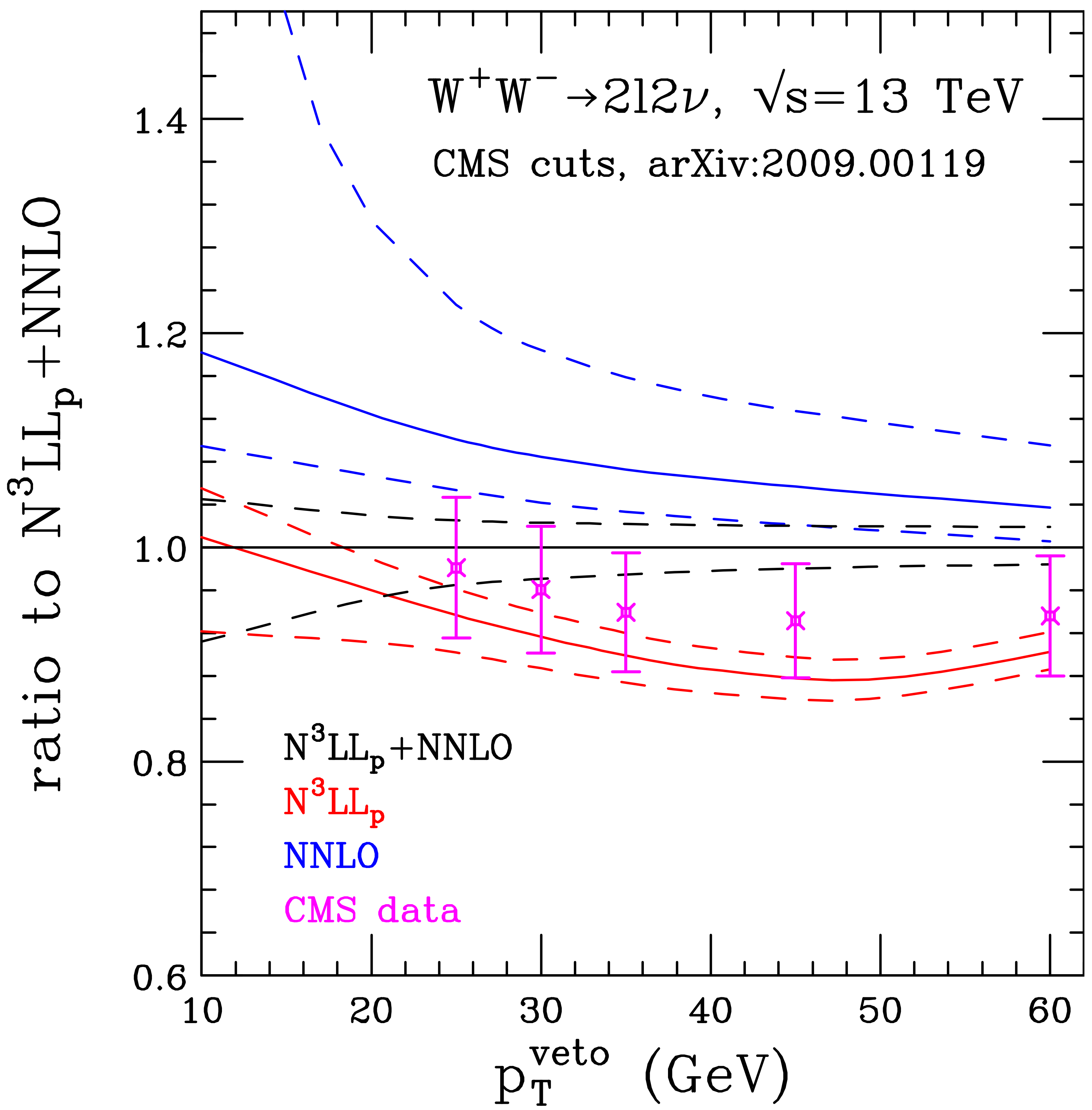


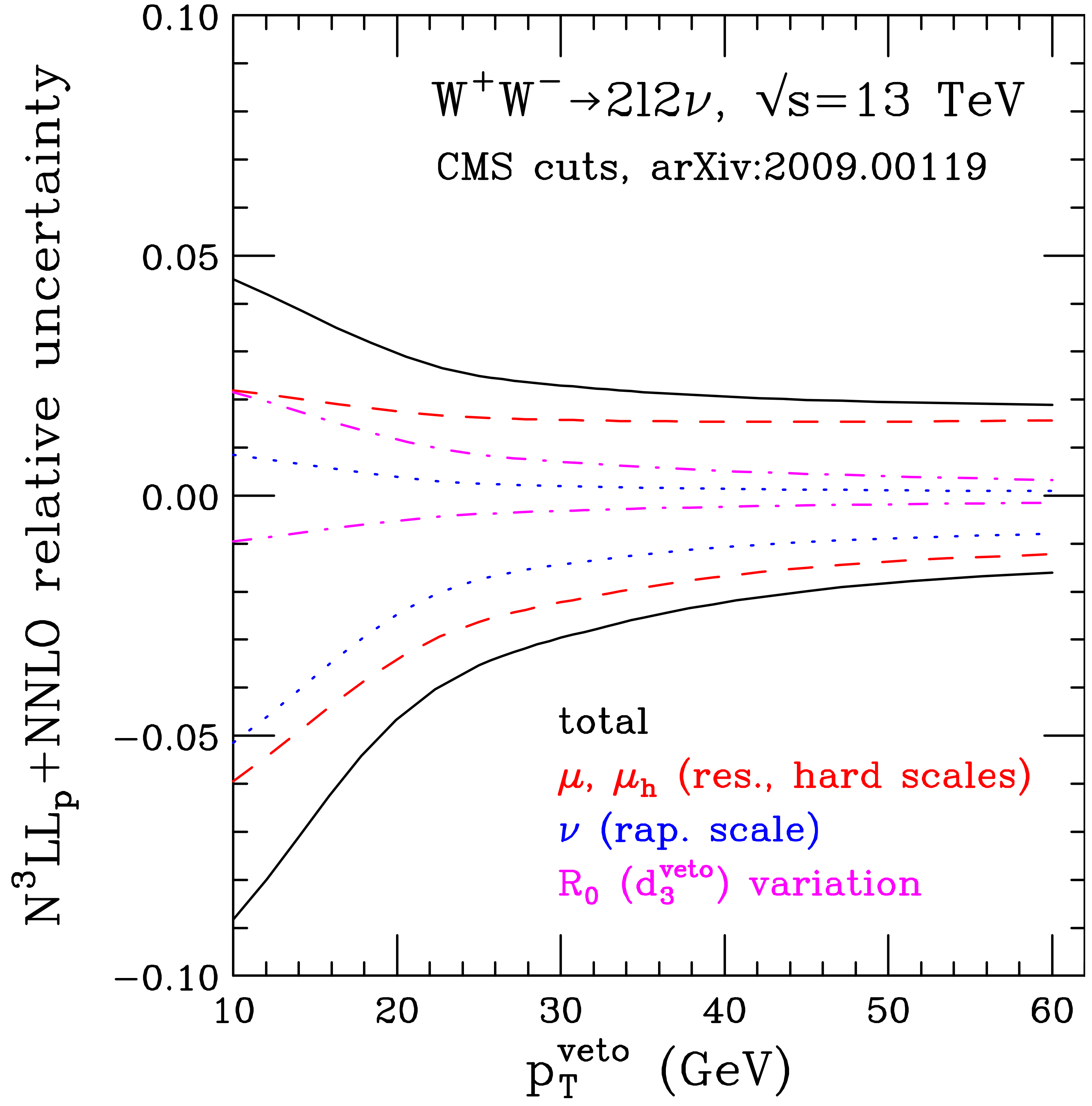
W^+W^- production @ CMS

- Loose jet cut $|\eta_{\text{cut}}| < 4.5$, $(\ln(Q/p_T^{\text{veto}}) = (1.3 - 3.1) \ll 4.5$ for jet vetoes in the range 10-60 GeV.
    - resummation formalism appropriate and expect impact from large logs.
    - this is borne out in actual calculation.
- Study suggests that neither pure NNLO nor N^3LL is sufficient, for $p_T^{\text{veto}} = 30$ GeV.
    - effect of matching will be substantial.
- R dependence is modest (zero at NLO!) and reduced from NNLL to $\text{N}^3\text{LL}_\text{p}$.



Matched prediction for W^+W^- @ CMS





- Effect of matching important; better agreement with resummed calculations than pure NNLO although experimental errors are still large. Will be interesting to see more data (only 36/fb).
- Contribution of d_3^{veto} uncertainty (N^{3}LL$_p$ vs. full N^{3}LL) to error budget small.

Summary

- For diboson final states, precision comparisons demand state-of-the-art theory. Depending on the analysis this may include NNLO QCD and NLO EW effects and/or resummation to N^3LL.
- MCFM provides NNLO QCD predictions for all diboson final states.
  - The small q_T resummation in CuTe-MCFM, accurate to N^3LL+NNLO, has been extended to all color singlet final states with pairs of massive vector bosons and is publicly available.
  - Extension to $N^4LL_p+N^3LO$ for Z and W production also available (CPU intensive). Relevant for precision studies of W mass and understanding resummation parameters.
  - We have also resummed cross sections at N^3LL_p+NNLO for all color singlet final state processes with a jet veto (at all rapidities). Necessary for Higgs production and for vector boson pair production, particularly WW → relevance for off-shell studies.
- The fine-grained experimental study of vector boson pair processes where resummation effects will be crucial is, in the main, still to come.
  - another part of the toolkit for precision studies in both the on- and off-shell regions.

Backup material

Uncertainty estimate

- Estimate the perturbative truncation uncertainty by varying the renormalization/hard scale and the factorization/resummation scale by the multipliers

$$(k_F; k_R) \in \{(2,2), (0.5,0.5), (2,1), (1,1), (0.5,1), (1,2), (1,0.5)\}.$$

- For fixed order $\mu_F = k_F\,\hat{Q}$, $\mu_R = k_R\,\hat{Q}$.
- Hard scale is $k_R\,\hat{Q}$. To set the resummation scale, first calculate characteristic scale $q^* = Q^2 \exp(-\pi/C_i/\alpha_s(q^*))$ and then set $\mu = \max\{k_F \times qT + q^* \exp(-q_T/q^*), 2\text{ GeV}\}$ so that for small qT, μ approaches q^* and it remains in the perturbative region.
- Additional important resummation uncertainties:
  - reintroduce rapidity scale dependence (fixed-order remnant of analytic regulator)
  - vary parameters in transition function.

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