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Electroweak corrections to double Higgs production at the LHC

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The Higgs WG4 meeting

Based on: arvix: 2311.16963

In cooperation with:
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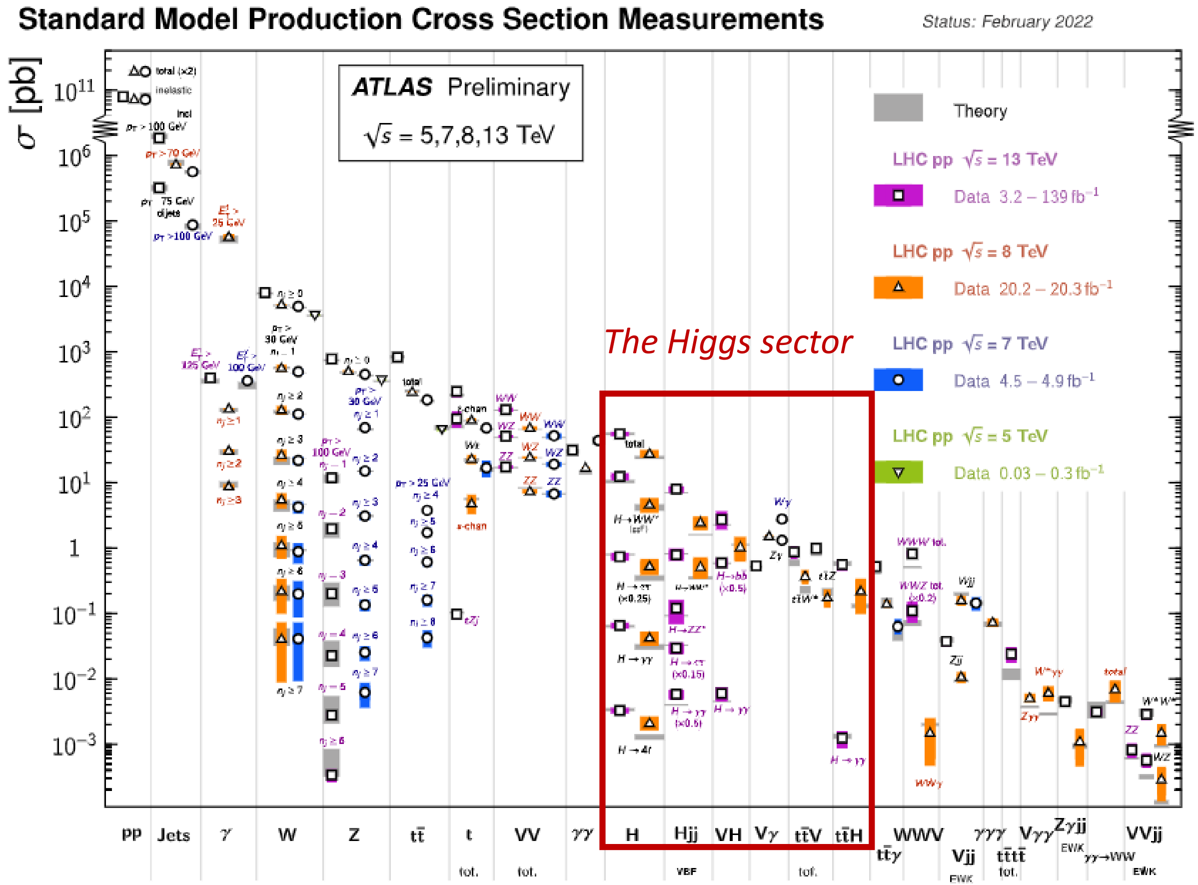
Introduction to Higgs

- Discovery of Higgs boson(2012,LHC): the last found elementary particle in SM
- Experiments at the ALTAS and CMS: consistent with the results predicted by SM

Azzurri: *Int.J.Mod.Phys.A* 38 (2023) 09n10, 23300077



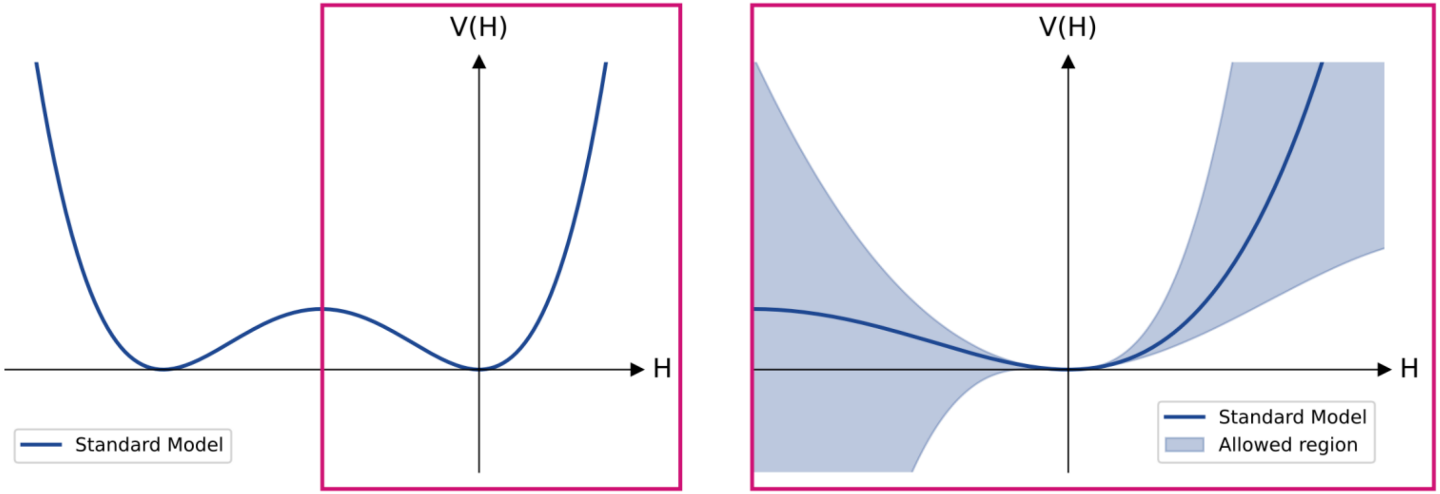
Standard Model Production Cross Section Measurements

Status: February 2022

Higgs Potential

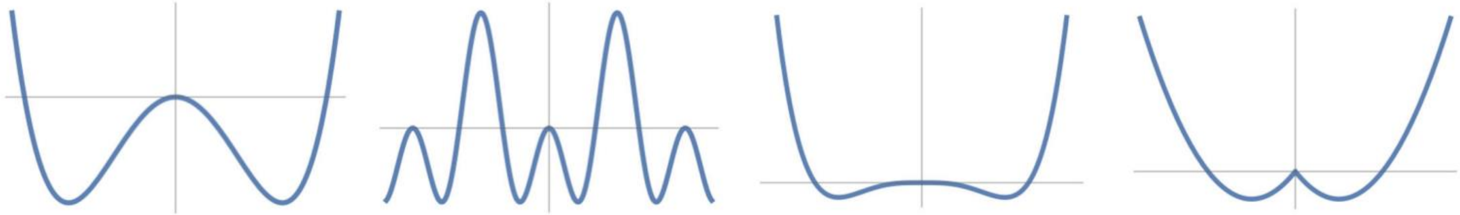
- Problems not clear: shape of Higgs potential, new physics beyond SM…

Plot taken from Moser: Higgs 2023



[using current ATLAS limits @ 95% CL]

Agrawal et al: 1907.02078



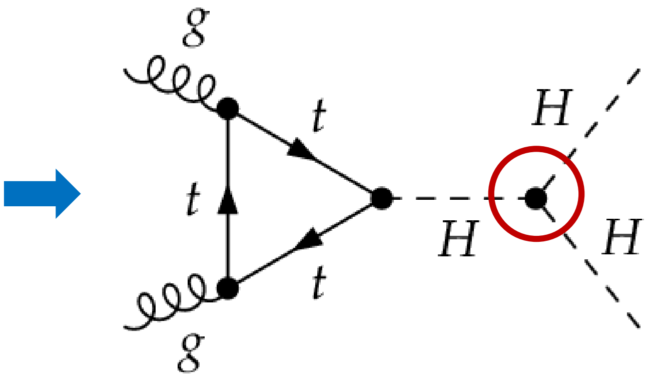
Landau-Ginzburg Higgs **Nambu-Goldstone Higgs** **Coleman-Weinberg Higgs** **Tadpole-Induced Higgs**

Higgs potential predicted by other BSM theories.

Higgs trilinear coupling

- Higgs potential is probed through determining the strength of Higgs boson self-interactions in searches for HH production. $(\lambda^{SM} \approx 1/8)$

$$V(h) = \frac{m_h^2}{2} h^2 + \lambda^{SM} v h^3 + \frac{1}{4} \lambda^{SM} h^4$$



- Experiment constraints on Higgs boson self-interactions:

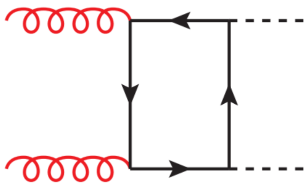
ATLAS: 2007.02873

CMS: 2202. 09617

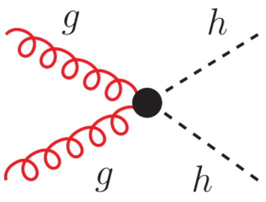
Jones: LHEP 2023 (2023) 442

  - Current: $-1.5 < \lambda_{hhh}^{EX}/\lambda^{SM} < 6.7$ for ATLAS, $-2.3 < \lambda_{hhh}^{EX}/\lambda^{SM} < 9.4$ for CMS.
  - Future: a limit of $-0.5 < \lambda_{hhh}^{EX}/\lambda^{SM} < 1.5$ will be achieved.

QCD corrections status



- corrections with full top quark mass dependence
  - NLO corrections keeping top quark mass, Borowka et al:1604.06447
  - NLO corrections matched to parton shower, Heinrich et al:1703.09252
  - NLO corrections with soft-gluon resummation, Ferrera et al: 1609.01691



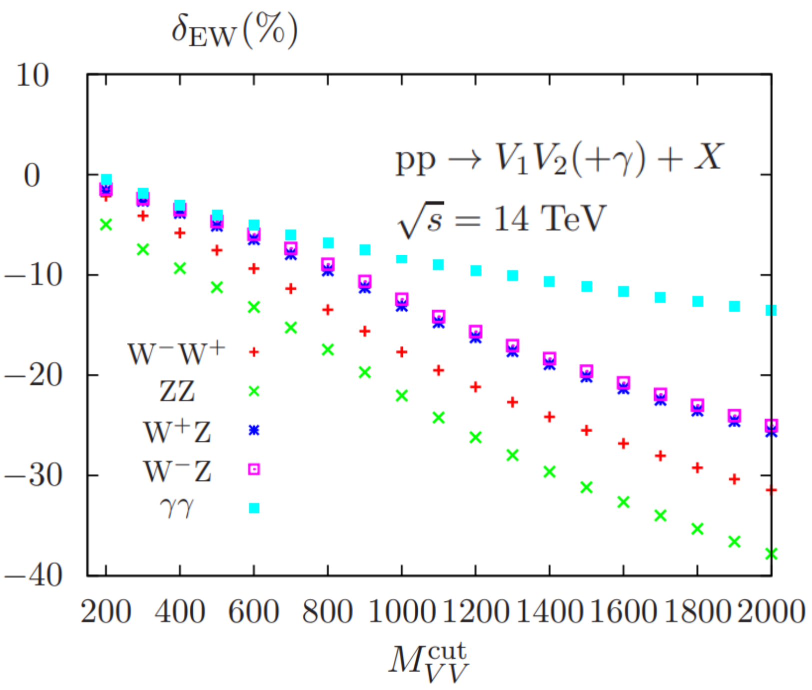
- corrections in heavy top limit (HTL) approximation
  - $\mathrm{N^2LO}$ in HTL, Florian et al:1305.5206
  - $\mathrm{N^3LO}$ in HTL, Chen et al:1909.06808
  - $\mathrm{N^2LO}$ in HTL+ NLO with full top-quark mass dependence, Florian et al:2106.14050
  - $\mathrm{N^3LO}$ in HTL include the top-quark mass effects, Chen et al:1912.13001
  - $\mathrm{N^2LO}$ in HTL matched to parton shower, Alioli et al: 2212.10489
  - $\mathrm{N^3LO}$ in HTL+ NLO with full top-quark mass dependence + soft-gluon resummation, Ajjath et al:2209.03914
- Current QCD corrections uncertainties: $\mathcal{O}(1\%)$ Jones: LHEP 2023 (2023) 442

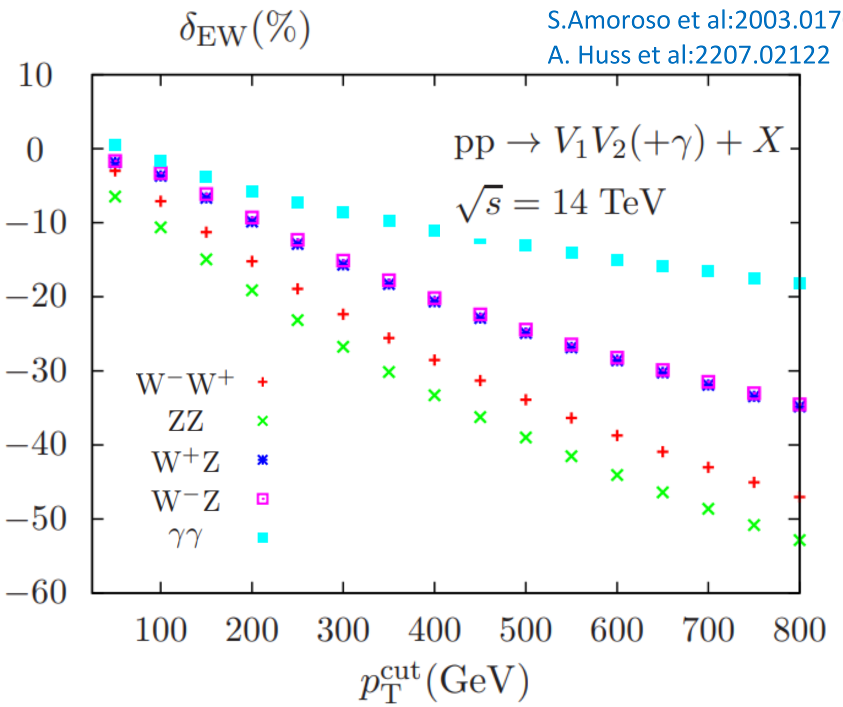
Process	QCD	$\sigma_{th}[pb]$	$\delta_{th}[\%]$
HH production via gg fusion	$\mathrm{N^3LO_{HTL}}$ $\mathrm{NLO_{QCD}}$	0.03105	+2.2 -5.0

Why EW corrections

- EW corrections
  - $\alpha \sim \mathcal{O}(1\%)$, the biggest uncertainty from theoretical side!
  - Sudakov enhancement, $\mathcal{O}(10\% \sim 30\%)$ corrections in high energy region. A Bierweiler et al:1305.5402
  - NLO EW corrections are crucial, a focal point in 2015, 2017, 2019 and 2021 Les Houches precision wish lists

J.R. Anderson et al:1605. 04692
Les Houches 2017:1803.07977
S.Amoroso et al:2003.01700
A. Huss et al:2207.02122





EW corrections status

- Partial results
  - Higgs self-coupling corrections, Borowka et al: 1811.12366
  - two-loop box diagrams, Davies et al:2207.02587
  - top-quark Yukawa corrections, Muhlleitner et al:2207.02524

https://indico.ihep.ac.cn/event/18025/

- Recent developments on this topic: Higgs 2023, Nov 27- Dec 2, Beijing
  - Full NLO EW corrections in large-m_t limit

    See talk by KIT group
  - Higgs-Higgs and Higgs-Yukawa parts corrections

    See talk by Shandong University group
  - Yukawa and Self-Coupling Corrections

    See talk by Durham University group

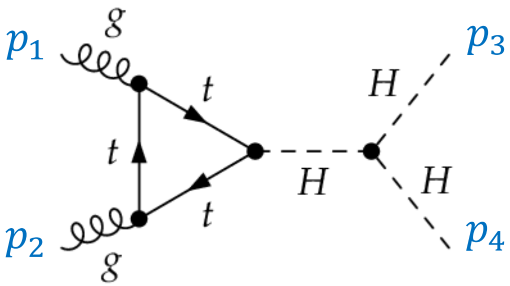
Production Rate

- Di-Higgs production cross section:

gluon PDF

$$\sigma(\mathrm{pp} \to \mathrm{HH}) = \int dx_1\, dx_2 f_g(x_1) f_g(x_2) \hat{\sigma}_{gg\to HH}(\hat{s}, m^2)$$

  - Kinematics invariants and mass scales include $m_h, m_t, m_W, m_Z, \hat{s}, \hat{t}$,



$$\Rightarrow \begin{cases} \hat{s} = (p_1 + p_1)^2 \\ \hat{t} = (p_1 - p_3)^2 \end{cases}$$

  - Multiple mass scales, analytic result for $\hat{\sigma}$ is challenging. ✗
  - Monte Carlo integration method can be adopted. ✓

$$\int dx_1\, dx_2 f_g(x_1) f_g(x_2) d\hat{t} \frac{d\hat{\sigma}}{d\hat{t}}(\hat{s}, \hat{t}) = \sum_{i,j} \Delta_{i,j} \times \frac{d\hat{\sigma}}{dt}(\hat{s}_i, \hat{t}_j)$$

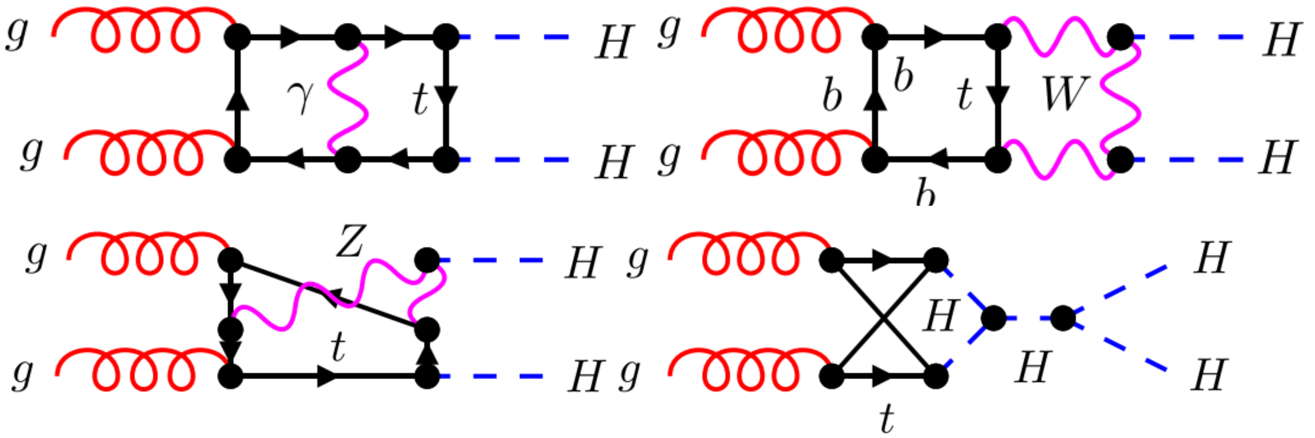
  - Lots of numerical results for $d\hat{\sigma}/d\hat{t}$ at different phase space points are required.

Calculation procedure

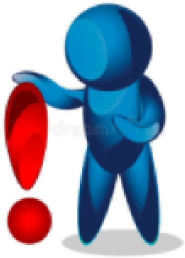
- Numerical results for $d\hat{\sigma}/d\hat{t}$ at a phase space point $(\hat{s}_i, \hat{t}_j)$ can be evaluated using following steps:

T. Hahn:0012260

  - Generate Feynman amplitudes: 8 LO diagrams, 2020 NLO virtual correction diagrams. NLO real corrections are forbidden due to Furry theorem.



  - Manipulate amplitudes to obtain scalar integrals.
  - Reduce scalar integrals to master integrals.
  - Calculate master integrals.
  - Remove divergence via renormalization



Manipulate amplitudes

- Amplitudes for $g(p_1)g(p_2) \rightarrow H(p_3)H(p_4)$:

$$M_{ab} = \delta_{ab}\epsilon_1^{\mu}\epsilon_2^{\nu}M_{\mu\nu}$$

  - Decomposition to form factor

$$M_{\mu\nu} = F_1(\hat{s},\hat{t},m^2)T_1^{\mu\nu} + F_2(\hat{s},\hat{t},m^2)T_2^{\mu\nu}$$

  - Decomposition to scalar integrals

To be reduced

$$F_i(\hat{s},\hat{t},m^2) = \sum_j C_{i,j}(\hat{s},\hat{t},m^2) \times \boxed{I_{i,j}(\hat{s},\hat{t},m^2)}$$

  - Reduction to master integrals with Blade program package https://gitlab.com/multiloop-pku

$$I_{i,j,k}(\hat{s},\hat{t},m^2) = \sum_k P_{i,j,k}(\hat{s},\hat{t},m^2) \times \boxed{M_{i,j,k}(\hat{s},\hat{t},m^2)}$$

To be calculated, results for $\mathcal{O}(10^4)$ $(\hat{s}_\mathrm{i},\hat{t}_\mathrm{i})$ are required

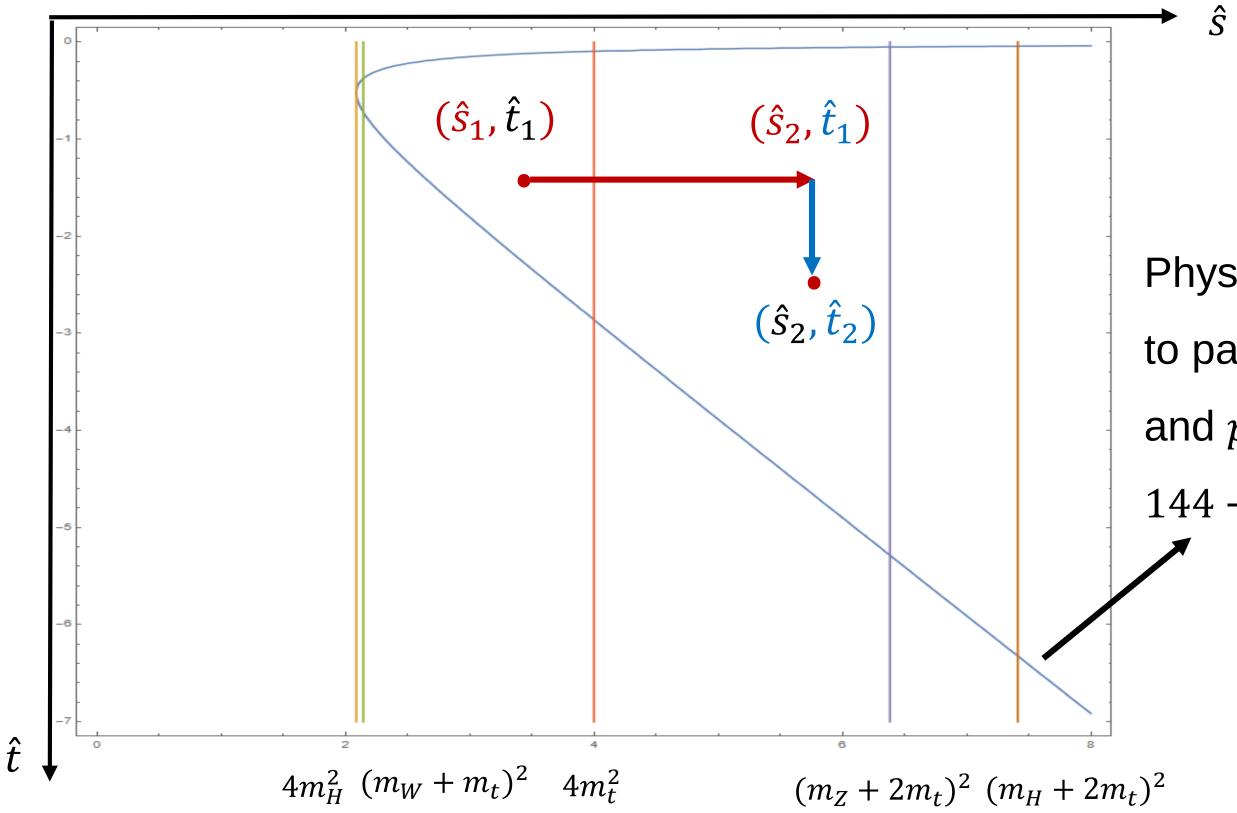
Calculate integrals

- Calculate integrals for a phase space point
  - Only two dimensional regulator ϵ points ($\epsilon = \pm\frac{1}{1000}$) are required
  - 3000 cpu.h run time using AMFlow program package https://gitlab.com/multiloop-pku
- Calculate integrals for 30000 phase space points
  - $\mathcal{O}(10^8)$ cpu.h run time with all points calculated by AMFlow
  - Differential equation running method, $\mathcal{O}(10^5)$ cpu.h run time
- Solution of differential equation
  - Differential equations provided by Blade, $\frac{\partial}{\partial\hat{s}}\vec{I} = A_{\hat{s}}\vec{I}$, $\frac{\partial}{\partial\hat{t}}\vec{I} = A_{\hat{t}}\vec{I}$
  - Numerical boundary provided by AMFlow, $\vec{I}(\hat{s}_0, \hat{t}_0)$
  - General solution for DE of single variable:

$$I_i = \sum_{\mu\in S}(x-x_0)^{\mu}\sum_{k=0}^{k_\mu}\log(x-x_0)^k\sum_{n=0}^{m}c_{i,\mu,k,n}(x-x_0)^n$$

DE running

- Only physical singularities in $\hat{s}$ direction when intermediate particles go on-shell
- Continuation direction for $\hat{s}$ is $+i0^-$
- Asymptotic expansion at singularities is required when crossing them for DE running in the $\hat{s}$ direction
- Taylor expansion for DE running in the $\hat{t}$ direction



Physical boundary corresponds to parallelization between p_1 and p_3.

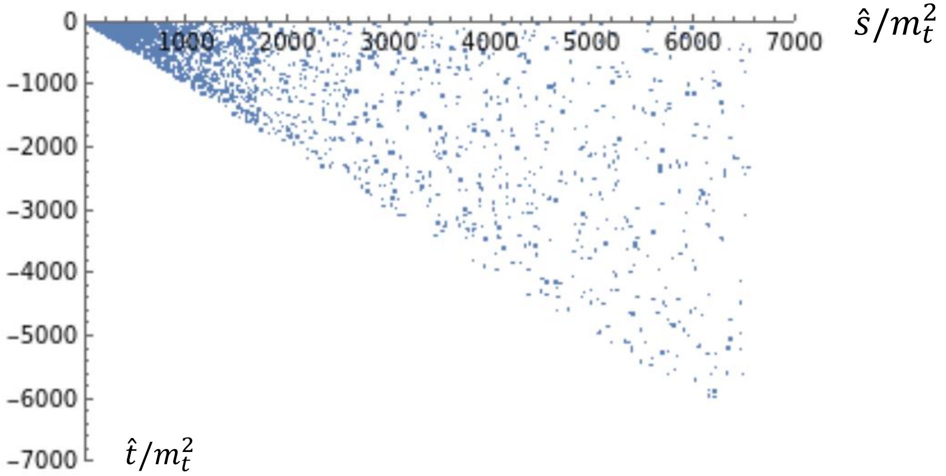
$$144 - 552\hat{t} + 529\hat{s}\hat{t} + 529\hat{t}^2 = 0$$

MC samples

- LO production cross section:

$$\sigma_{\text{LO}} = \sum_{i=1}^{N} \frac{1}{\text{flux}_i} f_g(x_1^i,\mu) f_g\left(x_2^i,\mu\right) |\overline{M_{\text{LO}}^i}(\hat{s},\hat{t})|^2 \, \Delta x_1^i \, \Delta x_2^i \Delta\Phi_2^i$$

  - μ is factorization scale and $\Delta\Phi_2$ corresponds to the element of phase space integral, $\Delta\Phi_2 \sim \Delta\hat{t}$
  - Importance sampling based on LO cross section



  - Samples are sparse at high energy region due to the suppression of gluon PDFs

Total cross sections

- Input parameters
  - $\alpha = \frac{\sqrt{2}}{\pi} G_\mu m_W^2 \left(1 - \frac{m_W^2}{m_Z^2}\right)$ $m_t = 172.69 \text{ GeV}$ $\alpha = 1/133.12$

    $G_\mu = 1.166378 \times 10^{-5} \text{ GeV}^{-2}$
  - NNPDF3.1 PDF set
- Renormalization
  - On-shell renormalization for masses and fields
  - G_μ-scheme renormalization for electromagnetic coupling
- Results: 1.8×10^4 events
  - K factor is stable. Its statistical error is smaller than cross sections
  - -4% NLO EW corrections

μ	$M_{HH}/2$	$\sqrt{p_T^2 + m_H^2}$	m_H
LO	19.96(6)	21.11(7)	25.09(8)
NLO	19.12(6)	20.21(6)	23.94(8)
$\mathcal{K}$-factor	0.958(1)	0.957(1)	0.954(1)

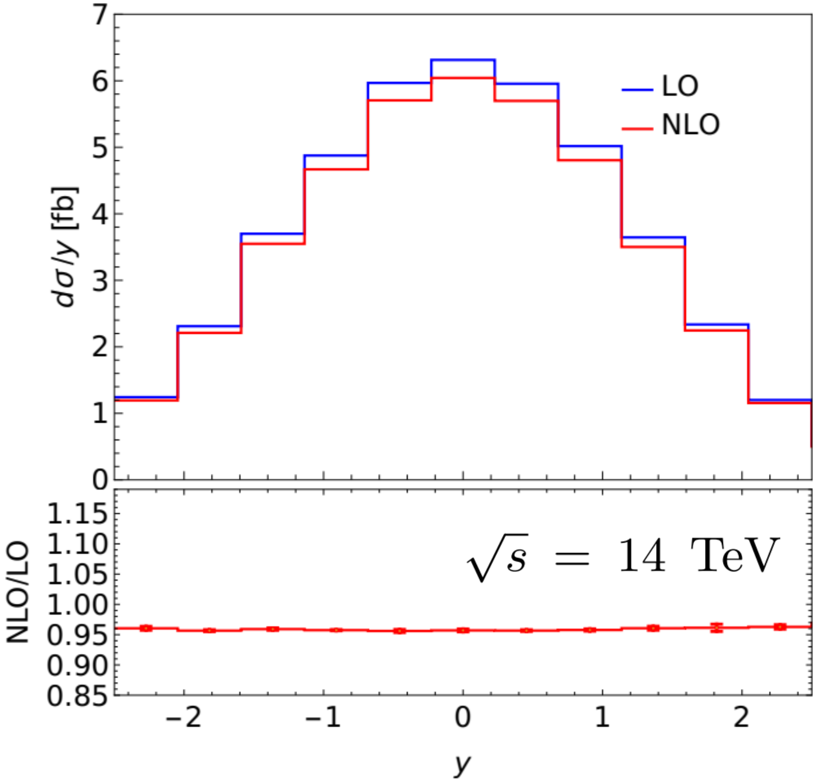
y differntial distribution

- The differential K factor can get a controllable error with far fewer events

$$\Delta\sigma^{\mathrm{NLO}} = \Delta\mathrm{K} \times \Delta\sigma^{\mathrm{LO}}$$

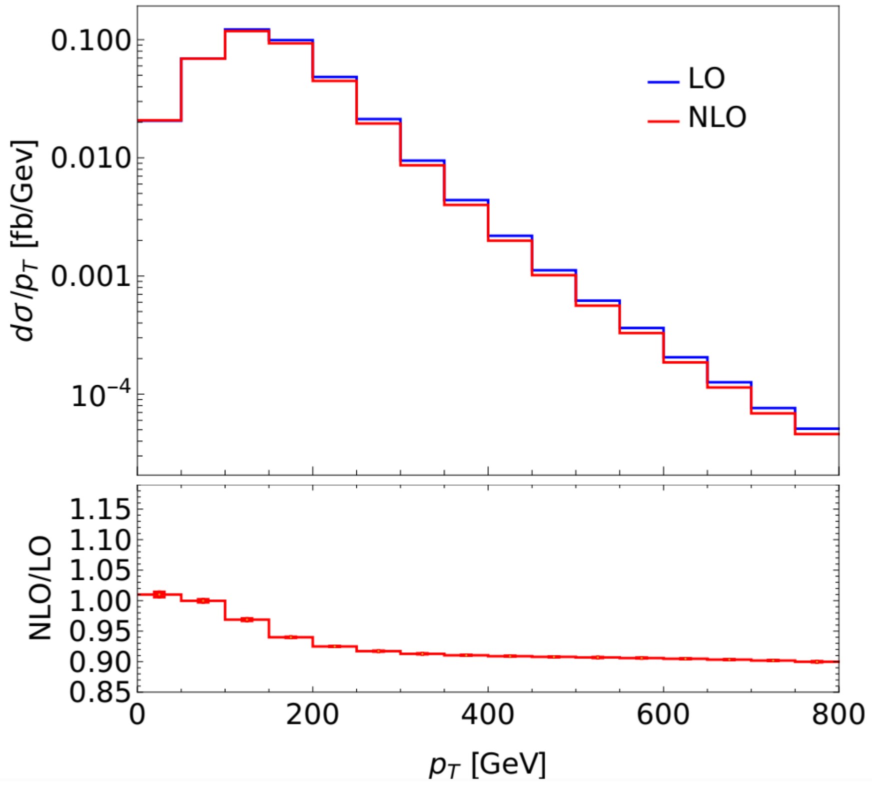
- ΔK uses 1.8×10^4 events for σ and additional

  400 events for each bin
- $\Delta\sigma^{\mathrm{LO}}$ uses 3×10^5 events
- Up to NLO, K ≈ 0.96

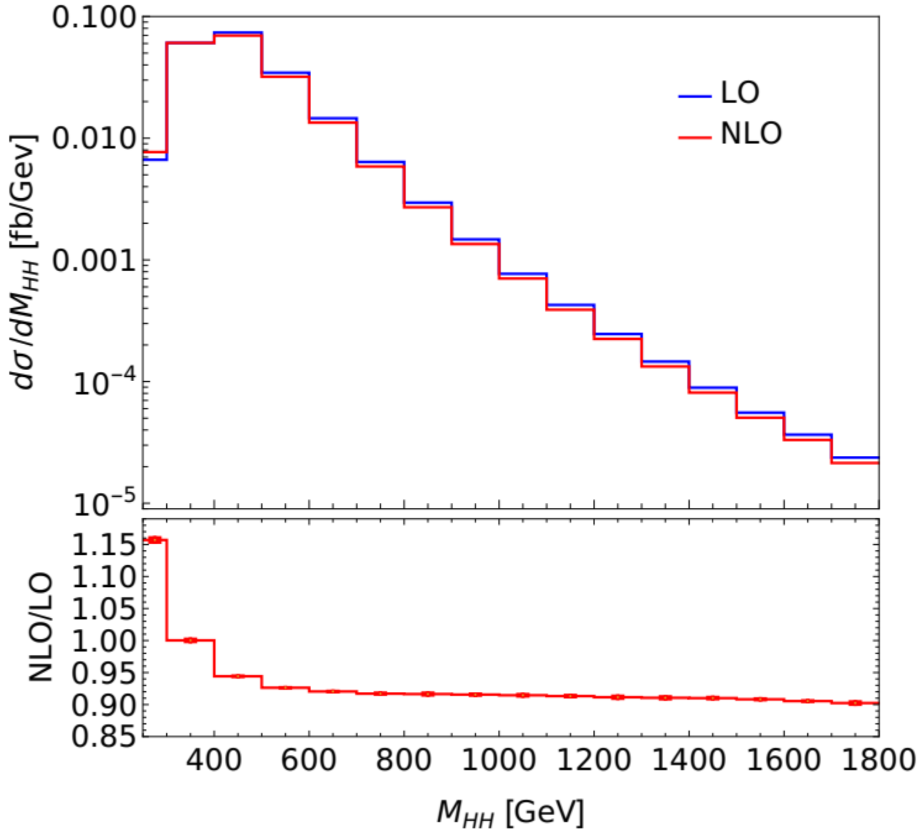


p_T differntial distribution

- -10% NLO corrections at the tail



M_{HH} differntial distribution



- $+15\%$ NLO corrections at the beginning of spectrum
- Substantial increase near threshold arises, considering heavy top-quark mass expansion, due to the leading term in the expansion at NLO is larger than that at LO by m_t^4
- -10% corrections in the tail, similar to p_T differential distribution
- Sudakov enhancement

Summary

- Higgs trilinear coupling is important
- NLO EW corrections to total cross sections is about -4%
- -4% NLO EW corrections to rapidity distribution
- $+15\%$ NLO corrections at the beginning of spectrum for the $\mathrm{M_{HH}}$, Sudakov effect was observed for both p_T and $\mathrm{M_{HH}}$ distribution
- Sufficient precision from current QCD corrections and NLO EW corrections for measurements at the HL-LHC

Thanks for your attention!