Bootstrapping Lagrangian PT for LSS

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New Physics from Galaxy Clustering III

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Bootstrap approach

$$\delta(\mathbf{x},\tau) = \delta^{(1)}(\mathbf{x},\tau) + \delta^{(2)}(\mathbf{x},\tau) + \cdots$$

$$\delta^{(2)}(\mathbf{k},\tau) = \int \frac{d^3p_1}{(2\pi^3)} \int \frac{d^3p_2}{(2\pi^3)} F_2(\mathbf{p}_1,\mathbf{p}_2,\tau)(2\pi^3)\delta_D(\mathbf{k}-\mathbf{p}_{12})\delta^{(1)}(\mathbf{p}_1,\tau)\delta^{(1)}(\mathbf{p}_2,\tau)$$

What is the most general form of F_2 that is allowed by the symmetries?

D'Amico, Marinucci, Pietroni, Vernizzi, 21
see also Fujita, Vlah 20

see Matteo's talk from Monday

EdS vs bootstrap, an example

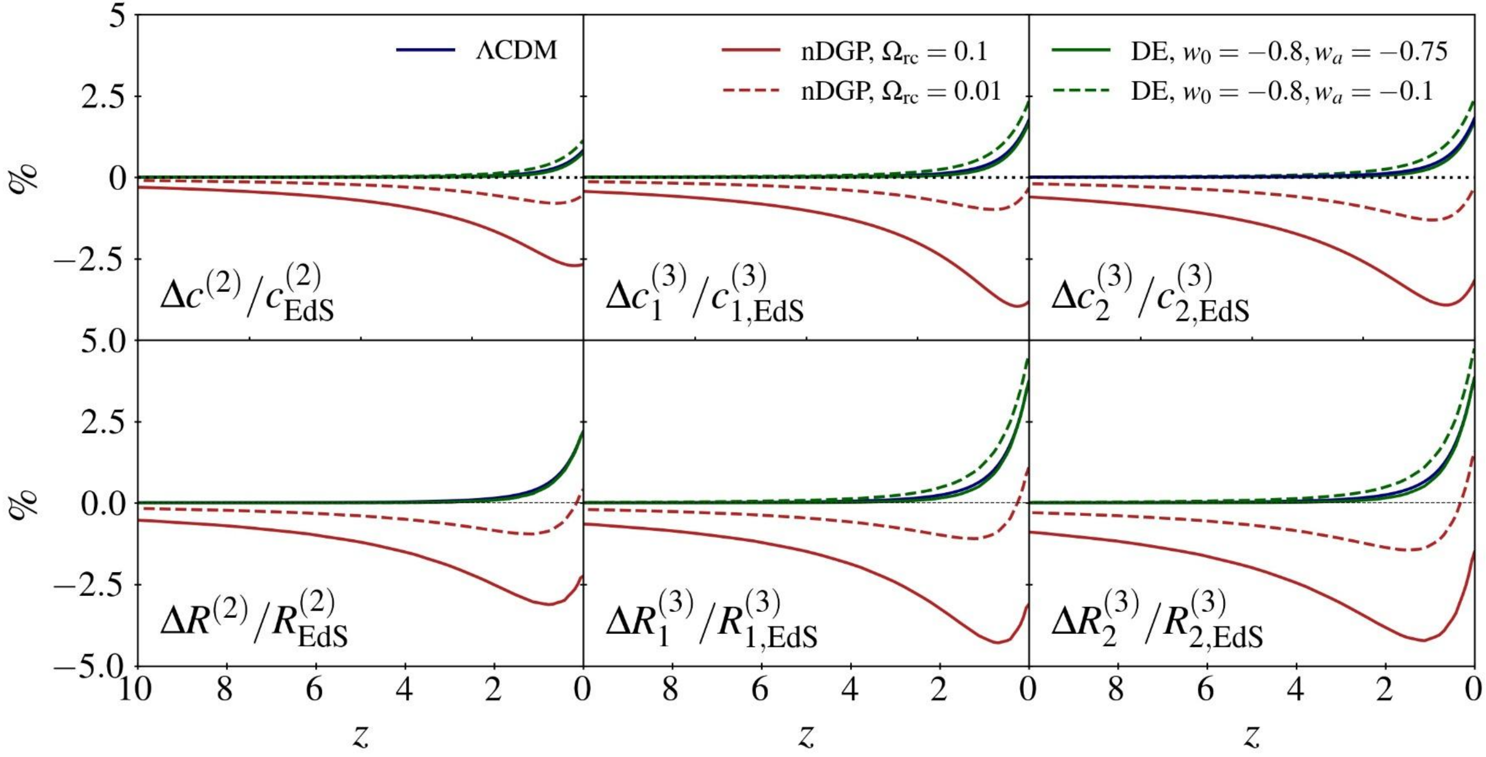
In EdS approximation:

$$F_2(\mathbf{p}_1, \mathbf{p}_2, \tau) = \frac{5}{7} + \frac{1}{2}\frac{\mathbf{p}_1 \cdot \mathbf{p}_2}{p_1 p_2}\left(\frac{p_1}{p_2} + \frac{p_2}{p_1}\right) + \frac{2}{7}\left(\frac{\mathbf{p}_1 \cdot \mathbf{p}_2}{p_1 p_2}\right)^2$$

In general D'Amico, Marinucci, Pietroni, Vernizzi, 21

$$F_2(\mathbf{p}_1, \mathbf{p}_2, \tau) = \frac{1 - c^{(2)}(\tau)}{2} + \frac{1}{2}\frac{\mathbf{p}_1 \cdot \mathbf{p}_2}{p_1 p_2}\left(\frac{p_1}{p_2} + \frac{p_2}{p_1}\right) + \frac{1 + c^{(2)}(\tau)}{2}\left(\frac{\mathbf{p}_1 \cdot \mathbf{p}_2}{p_1^2 p_2^2}\right)^2$$

vs EdS



can reach a difference of a few percent

Extended Galilean Invariance (EGI)

$$\tau \rightarrow \tilde{\tau}, \quad \mathbf{x} \rightarrow \tilde{\mathbf{x}} = \mathbf{x} + \mathbf{d}(\tau)$$

Eulerian vs Lagrangian

A *nonlinear* transf. of the overdensity field

$$\delta(\mathbf{x},\tau) \to \delta(\mathbf{x}-\mathbf{d}(\tau),\tau)$$

connect kernels at *different* order

$$\lim_{\mathbf{p}_1\to 0} F_2(\mathbf{p}_1,\mathbf{p}_2,\tau) = \frac{\mathbf{p}_1\cdot\mathbf{p}_2}{p_1^2} F_1(\mathbf{p_2},\tau) + \mathcal{O}(p_1^0)$$

Eulerian vs Lagrangian

In Lagrangian PT

$$\delta(\mathbf{x}) = \int d^3q\, \delta_D(\mathbf{x} - \mathbf{q} - \psi(\mathbf{q}, \tau)) - 1$$

Extended Galilean Invariance (EGI):

$$\psi(\mathbf{q}, \tau) \to \psi(\mathbf{q}, \tau) + \mathbf{d}(\tau)$$

A *linear* transformation of the displacement field

How to build higher-order displacement?

Extended Galilean Invariance (EGI):

$$\psi^{(1)}(\mathbf{q},\tau) \to \psi^{(1)}(\mathbf{q},\tau) + \mathbf{d}(\tau)$$

$$\psi^{(n)}(\mathbf{q},\tau) \to \psi^{(n)}(\mathbf{q},\tau),\ \ n \geq 2$$

Assuming no initial (transverse) vector $\psi^{(1)i} = \partial^i \varphi^{(1)}$& no new scale

<u>Basic building block</u> is linear scalar potential fields with 2 spatial derivatives

Displacement field at *n*-th order – 2nd order scalar

EGI + rotational invariant + no new scale

$$\varphi^{(2)}_{,ii} = a\,\varphi^{(1)}_{,ll}\,\varphi^{(1)}_{,jj} + b\,\varphi^{(1)}_{,lj}\,\varphi^{(1)}_{,lj}$$

For matter, impose mass conservation $\int d^3x\ \delta(\mathbf{x},\tau) = 0 \implies \int d^3q\ \varphi^{(2)}_{,ii}(\mathbf{q},\tau) = 0$

$$\varphi^{(2)}_{,ii} = \frac{1}{2}\left(\varphi^{(1)}_{,ll}\,\varphi^{(1)}_{,jj} - \varphi^{(1)}_{,lj}\,\varphi^{(1)}_{,lj}\right)$$

Therefore $\phi^{(2)}(\mathbf{q},\tau) = c^{(2)}(\tau)\varphi^{(2)}(\mathbf{q},\tau)$

Displacement field at n-th order: $n > 2$ scalar

3rd order

$$\varphi^{(3)}_{1,ii} = \frac{1}{3!}\epsilon^{i_1 i_2 i_3}\epsilon^{j_1 j_2 j_3}\varphi^{(1)}_{,i_1 j_1}\varphi^{(1)}_{,i_2 j_2}\varphi^{(1)}_{,i_3 j_3}$$

$$\varphi^{(3)}_{2,ii} = \frac{1}{2!}\epsilon^{i_1 i_2 k}\epsilon^{j_1 j_2 k}\varphi^{(2)}_{,i_1 j_1}\varphi^{(1)}_{,i_2 j_2}$$

structure of the Galileons …

4th order: there are 5 operators

$$\varphi^{(4)}_{5,ii} = \frac{1}{2}\varphi^{(1)}_{,lj}\left(\epsilon^{jmn}v^{(3)\,n}_{,lm} + \epsilon^{lmn}v^{(3)n}_{,jm}\right)$$

start to have contributions from the transverse vectors

Extension to biased tracers

In this formalism, the biased tracers move as

"additional mass non-conserving displacement"

$$\delta_t(\mathbf{x},,\tau) = \int d^3q\, \delta_D\left[\mathbf{x} - \mathbf{q} - \psi(\mathbf{q},,\tau) - \boxed{\psi_t(\mathbf{q},,\tau)}\right] - 1$$

"Resummed Lagrangian bias (in prep.)"

Example of mass-non conserving displacement

In this formalism, the biased tracers move as

"additional mass non-conserving displacement"

$$\delta_t(\mathbf{x}, , \tau) = \int d^3q\, \delta_D\left[\mathbf{x} - \mathbf{q} - \psi(\mathbf{q}, , \tau) - \boxed{\psi_t(\mathbf{q}, , \tau)}\right] - 1$$

Example: at 2nd order, both operators are allowed separately

$$\phi_t^{(2)}(\mathbf{q}, \tau) = b_1^{(2)}(\tau)\varphi_1^{(2)}(\mathbf{q}, \tau) + b_2^{(2)}(\tau)\varphi_2^{(2)}(\mathbf{q}, \tau)$$

where $\nabla^2\varphi_1^{(2)} = \varphi_{,ii}^{(1)}\varphi_{,jj}^{(1)}$ and $\nabla^2\varphi_2^{(2)} = \varphi_{,ij}^{(1)}\varphi_{,ij}^{(1)}$

"Resummed Lagrangian bias (in prep.)"

Conclusion

Lagrangian bootstrap approach, in the case of matter,
have been developed

- Requirements have been identified
- Generic structure of displacement fields up to 6th-order
  and an algorithm
- Beyond GR/LCDM
- Multispecies
- EPT vs LPT bootstrap relation
- Redshift space

NEXT: biased tracers, field level analysis, correlators, …

Thank you!

Additional slides

The (transverse) vectors contribution

Starts to be generated at 3rd-order

$$v^{(3)\,i}_{,kk} = \epsilon^{iln}\varphi^{(2)}_{,jl}\varphi^{(1)}_{,jn}$$

In general

$$v^{(n)\,i}_{,kk} = \epsilon^{iln}\psi^{(m_1)j}_{,l}\psi^{(m_2)j}_{,n}$$

We note that $\epsilon^{ilp}\psi^{(n_1)l}_{,k}\psi^{(n_1)k}_{,r}\psi^{(n_1)r}_{,p}$ is forbidden by momentum conservation

Multispecies

Two cold DM species 'α' and 'β',

with relative abundances ω_α , ω_β , such that $\omega_\alpha + \omega_\beta = 1$

$$\mathbf{x} = \mathbf{q}_\alpha + \psi_\alpha(\mathbf{q}_\alpha) = \mathbf{q}_\beta + \psi_\alpha(\mathbf{q}_\beta)$$

EGI constraint: potential with one spatial derivative terms are allowed

$$\varphi^{(2),rel}_{\beta\alpha,kk} \equiv \left[\varphi^{(1)}_{\beta,il}\left(\varphi^{(1)}_{\alpha,l} - \varphi^{(1)}_{\beta,l}\right)\right]_{,i}$$

being a total derivative, mass conservation is satisfied

Multispecies - 2

So, these new terms are generated

$$\psi_\alpha^{(2)k} = \cdots + c_{\beta\alpha}^{(2),\mathrm{rel}}(\tau)\varphi_{\beta\alpha,k}^{(2),\mathrm{rel}}$$

$$\psi_\beta^{(2)k} = \cdots + c_{\alpha\beta}^{(2),\mathrm{rel}}(\tau)\varphi_{\alpha\beta,k}^{(2),\mathrm{rel}}$$

Momentum conservation gives non trivial constraints

$$\int d^3q \left(\omega_\alpha \psi_\alpha^{(2)k} + \omega_\beta \psi_\beta^{(2)k}\right) = 0 \implies \omega_\alpha c_{\beta\alpha}^{(2),\mathrm{rel}} = \omega_\beta c_{\alpha\beta}^{(2),\mathrm{rel}}$$

Multispecies - 3

Transverse vectors are generated at second order

$$v^i_{,kk} = \epsilon^{iln}\varphi^{(1)}_{\alpha,jl}\varphi^{(1)}_{\beta,jn}$$

Modified gravity

In Horndeski theory, the Poisson equation is modified into

$$\nabla_x^2 \Phi(\mathbf{x},\tau) = \frac{3}{2}\alpha \mathcal{H}^2 \Omega_m \delta(\mathbf{x},\tau) + \frac{B'_{ab}}{4\mathcal{H}^2}\epsilon_{ikm}\epsilon_{jlm}\frac{\partial^2 \Phi_a(\mathbf{x},\tau)}{\partial x_i \partial x_j}\frac{\partial^2 \Phi_b(\mathbf{x},\tau)}{\partial x_k \partial x_l} + \frac{C'_{abc}}{12\mathcal{H}^2}\epsilon_{ikm}\epsilon_{jln}\frac{\partial^2 \Phi_a(\mathbf{x},\tau)}{\partial x_i \partial x_j}\frac{\partial^2 \Phi_b(\mathbf{x},\tau)}{\partial x_k \partial x_l}\frac{\partial^2 \Phi_c(\mathbf{x},\tau)}{\partial x_m \partial x_n},$$

The additional terms respect mass conservation

→ no new terms are being generated