The Renormalization Group for Large-Scale Structure (RGforLSS)

Henrique Rubira

(Cambridge/LMU fellow)

With Fabian Schmidt and Charalampos Nikolis

2307.15031,
2404.16929,
2405.21002

Parma, November 2024

Message to take home

Henrique Rubira

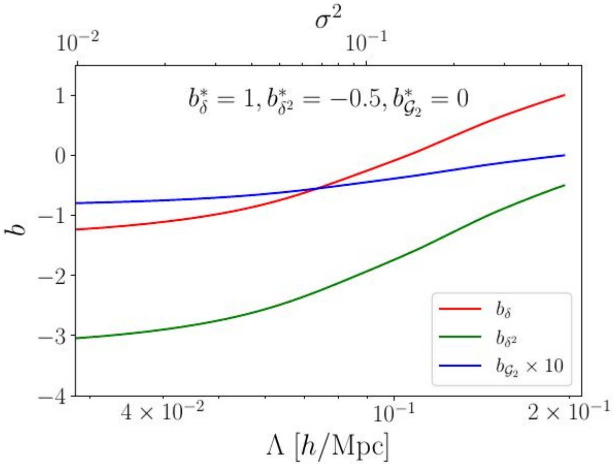
We derive the Callan-Symanzik equation for the galaxy bias+stochastic+PNG parameters

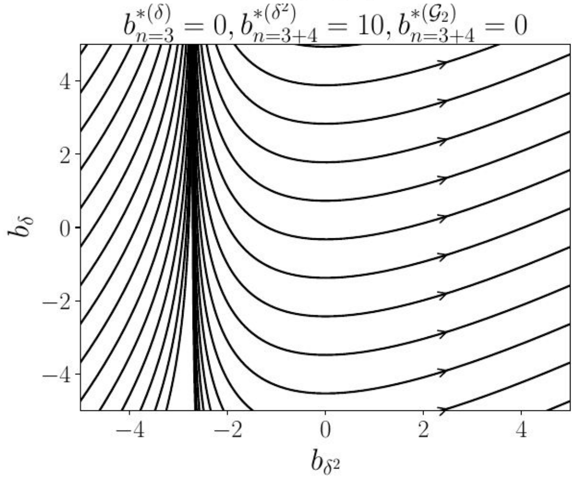
$$\frac{db_\delta}{d\Lambda} = -\left[\frac{68}{21} b_{\delta^2} + 3 b^*_{\delta^3} - \frac{4}{3} b^*_{\mathcal{G}_2\delta}\right] \frac{d\sigma^2_\Lambda}{d\Lambda} \,,$$

$$\frac{db_{\delta^2}}{d\Lambda} = -\left[\frac{8126}{2205} b_{\delta^2} + \frac{17}{7} b^*_{\delta^3} - \frac{376}{105} b^*_{\mathcal{G}_2\delta} + b^{*(\delta^2)}_{n=4}\right] \frac{d\sigma^2_\Lambda}{d\Lambda} \,,$$

$$\frac{db_{\mathcal{G}_2}}{d\Lambda} = -\left[\frac{254}{2205} b_{\delta^2} + \frac{116}{105} b^*_{\mathcal{G}_2\delta} + b^{*(\mathcal{G}_2)}_{n=4}\right] \frac{d\sigma^2_\Lambda}{d\Lambda} \,.$$

Many things to explore:
- Systematic construction of operator basis,
- Systematic renormalization,
- Cross-checks,
- More information from galaxy clustering (to be investigated)





The future

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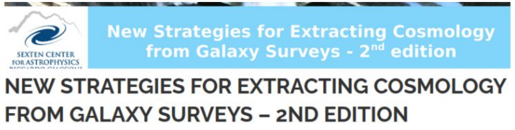
1) **Higher n-point functions and higher-loops**
   - We need: quick loop computations (e.g. COBRA from Bakx, Chisari, Vlah)
   - Why? Inflation, neutrinos, DE, mediators, smaller scales, …

2) **Field-level** (
   - all n-pt function tower
   - Difficulties: many degrees of freedom (convergence), hard noise-modelling

3) **Other statistics and Multi-tracing**



4) **Priors**

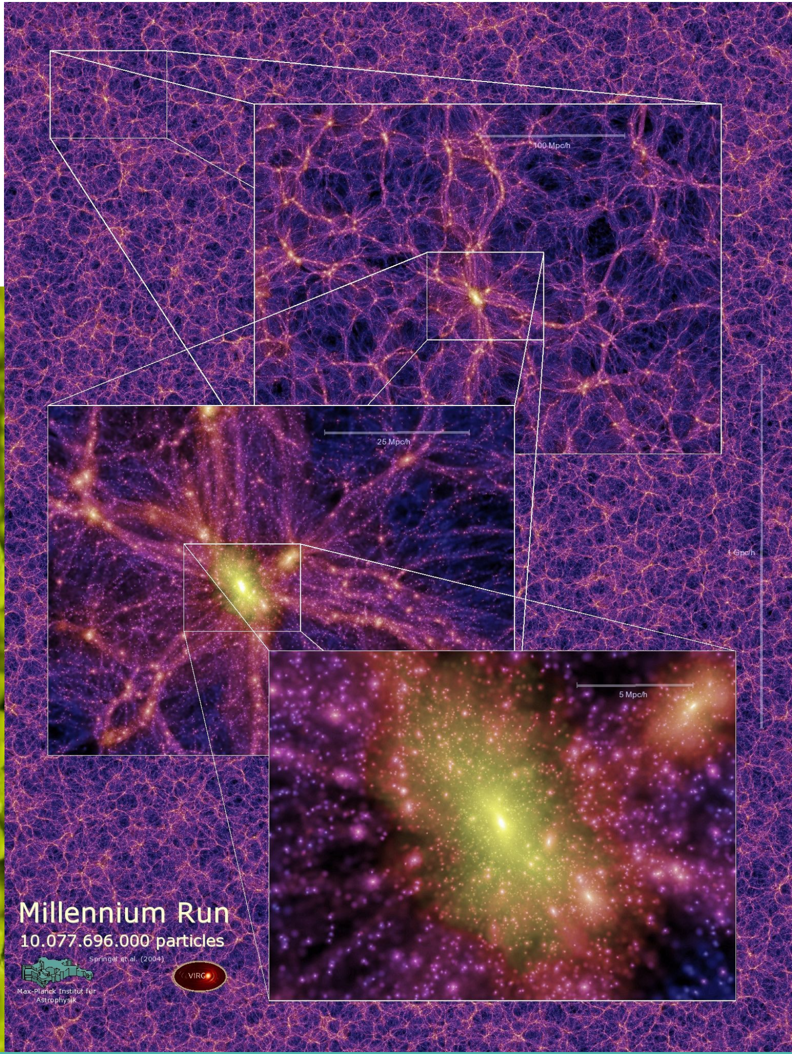
5) **Beyond-LCDM**: new interaction vertex

6) **Other observables**: Lyman-alpha, intensity maps, lensing

7) **Theoretical pathway**: the **RGforLSS**

Part I - Preamble(s)

How things change with scale?
(from food to galaxies)



QFT101

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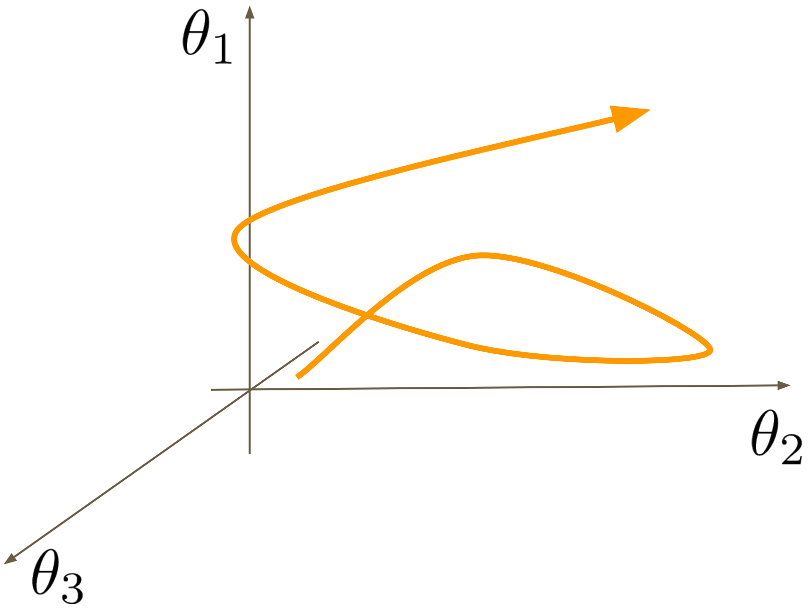
Renormalization group: coupling constants evolve with the cutoff ("flow").

Observables don't depend on the cutoff!



Callan-Symanzik equation:

$$\frac{\partial g}{\partial \ln \mu} = \beta(g)$$

QED: $$\beta(e) = \frac{e^3}{12\pi^2}$$

QCD: $$\beta(g) = -\left(11 - \frac{n_s}{6} - \frac{2n_f}{3}\right)\frac{g^3}{16\pi^2}$$

The galaxy bias expansion

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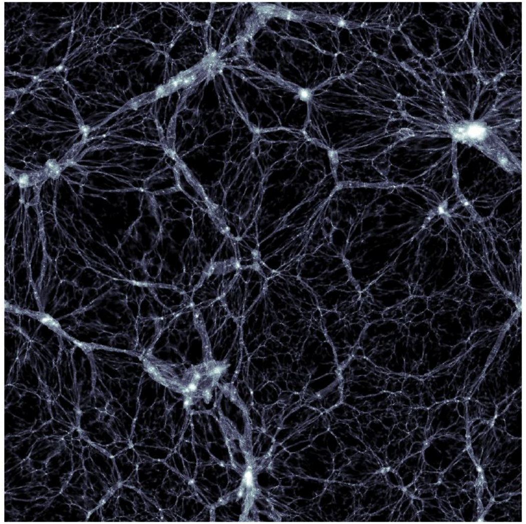
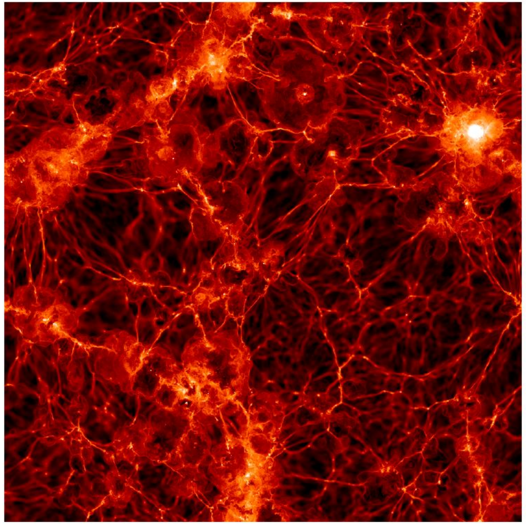


(a) dark matter



(b) baryons

From Illustris simulation, Haiden, Steinhauser, Vogelsberger, Genel, Springel, Torrey, Hernquist, 15

Stochastic field

$$\delta_g(\boldsymbol{x},\tau) \equiv \frac{n_g(\boldsymbol{x},\tau)}{\bar{n}_g(\tau)} - 1 = \sum_O \left[b_O(\tau) + c_{\epsilon,O}(\tau)\epsilon(\boldsymbol{x},\tau)\right] O(\boldsymbol{x},\tau) + \epsilon(\boldsymbol{x},\tau)$$

Bias

Bias review: Desjacques, Jeong, Schmidt

Part II - Renormalization in LSS

Renormalizing the bias parameters

Henrique Rubira

$$\delta_g(\boldsymbol{x},\tau) \equiv \frac{n_g(\boldsymbol{x},\tau)}{\bar{n}_g(\tau)} - 1 = \sum_O \left[b_O(\tau) + c_{\epsilon,O}(\tau)\epsilon(\boldsymbol{x},\tau)\right] O(\boldsymbol{x},\tau) + \epsilon(\boldsymbol{x},\tau)$$

$$O[\delta](\boldsymbol{k}) = \int_{\boldsymbol{p}_1,\ldots,\boldsymbol{p}_n} \delta_{\rm D}(\boldsymbol{k} - \boldsymbol{p}_{1\ldots n}) S_O(\boldsymbol{p}_1,\ldots \boldsymbol{p}_n)\delta(\boldsymbol{p}_1)\cdots\delta(\boldsymbol{p}_n)$$

First order:	$\delta\,;$
Second order:	$\delta^2,\ \mathcal{G}_2\,;$
Third order:	$\delta^3,\ \delta\,\mathcal{G}_2,\ \Gamma_3,\ \mathcal{G}_3\,;$

Contribution from arbitrarily small scales!

Renormalizing the bias parameters

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$$\delta_g(\boldsymbol{x},\tau) \equiv \frac{n_g(\boldsymbol{x},\tau)}{\bar{n}_g(\tau)} - 1 = \sum_O \left[b_O^{\Lambda}(\tau) + c_{\epsilon,O}^{\Lambda}(\tau)\epsilon^{\Lambda}(\boldsymbol{x},\tau)\right] O^{\Lambda}(\boldsymbol{x},\tau) + \epsilon^{\Lambda}(\boldsymbol{x},\tau)$$

\+ counter-terms (Λ)

$$O[\delta](\boldsymbol{k}) = \int_{\boldsymbol{p}_1,\ldots,\boldsymbol{p}_n}^{\Lambda} \delta_{\rm D}(\boldsymbol{k} - \boldsymbol{p}_{1\ldots n}) S_O(\boldsymbol{p}_1,\ldots \boldsymbol{p}_n)\delta(\boldsymbol{p}_1)\cdots\delta(\boldsymbol{p}_n)$$

Notation:

$$[\![O]\!] = O^{\Lambda} + \text{counter-terms} \, (\Lambda)$$

How to determine the renormalization condition?

First order:	δ ;
Second order:	$\delta^2, \ \mathcal{G}_2$;
Third order:	$\delta^3, \ \delta\,\mathcal{G}_2, \ \Gamma_3, \ \mathcal{G}_3$;

Contribution from arbitrarily small scales!

Motivation

RENORMALIZATION AND EFFECTIVE LAGRANGIANS

Joseph POLCHINSKI*

Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138, USA

Received 27 April 1983

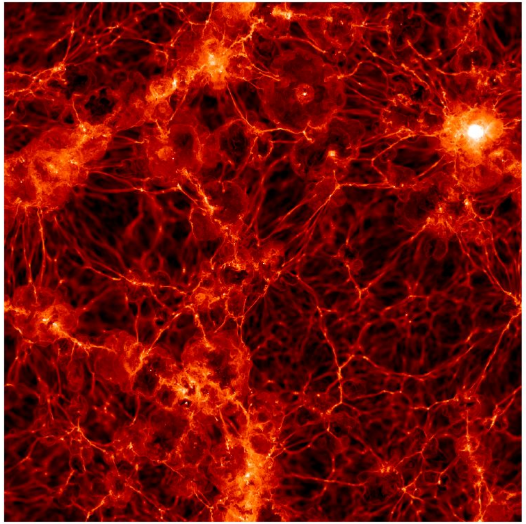
1. Introduction

The understanding of renormalization has advanced greatly in the past two decades. Originally it was just a means of removing infinities from perturbative calculations. The question of why nature should be described by a renormalizable theory was not addressed. These were simply the only theories in which calculations could be done.

A great improvement comes when one takes seriously the idea of a physical cutoff at a very large energy scale Λ. The theory at energies above Λ could be another field

Intuition time

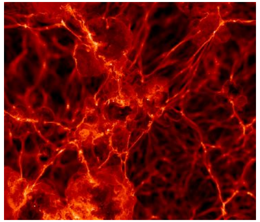
Take a box, smoothed on some scale Λ and measure b_O



*Technically, we are talking about operators constructed from differently the smoothed initial condition. But this picture should work as an intuition

Intuition time

In the same box, smooth on a new scale Λ and measure b_O



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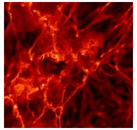
Intuition time

You can measure $b_O(\Lambda)$

Extra cross-check:
If the running does not match the theoretical prediction, something is missing



Does this scaling of the bias parameters carry information?
(Open question, I don't know the answer)

Part III - The Wilson-Polchinski path integral approach

Warning (and apologies in advance):
next 2 slides will be technical, they are just there to trigger interest

The bias partition function (based on Carroll, Leichenauer, Pollack, 13)

$$\mathcal{Z}[J_\Lambda] = \int \mathcal{D}\delta_\Lambda^{(1)} \mathcal{P}[\delta_\Lambda^{(1)}] \exp\left(\int_{\boldsymbol{k}} J_\Lambda(\boldsymbol{k}) \left[\sum_O b_O^\Lambda O[\delta_\Lambda^{(1)}](-\boldsymbol{k}) \right] + \frac{1}{2} P_\epsilon^\Lambda \int_{\boldsymbol{k}} J_\Lambda(\boldsymbol{k}) J_\Lambda(-\boldsymbol{k}) + \mathcal{O}[J_\Lambda^2 \delta_\Lambda^{(1)},\ J_\Lambda^3] \right)$$

Path-integral over linear-smoothed density, normalized

Single-current term

Double-current term captures stochasticity source

N-point correlators evaluated as:

$$\left. \frac{\partial \mathcal{Z}}{\partial J_\Lambda \ldots \partial J_\Lambda} \right|_{J_\Lambda = 0}$$

See Cabass, Schmidt 19

The shell expansion (Wilson formalism)

Henrique Rubira

Consider a very thin shell with width: $\Lambda = \Lambda' - \lambda$

$$\delta^{(1)}_{\Lambda'}(\boldsymbol{k}) = \delta^{(1)}_{\Lambda}(\boldsymbol{k}) + \delta^{(1)}_{\text{shell}}(\boldsymbol{k})$$

Idea: Integrate out the shell!

$$\mathcal{Z}[J_\Lambda] = \int \mathcal{D}\delta^{(1)}_\Lambda \mathcal{P}[\delta^{(1)}_\Lambda] \exp\left(\int_{\boldsymbol{k}} J_\Lambda(\boldsymbol{k}) \left[\sum_O b_O^{\Lambda'} O[\delta^{(1)}_\Lambda](-\boldsymbol{k}) \right] + \frac{1}{2} P_\epsilon^{\Lambda'} \int_{\boldsymbol{k}} J_\Lambda(\boldsymbol{k}) J_\Lambda(-\boldsymbol{k}) + \mathcal{O}[J_\Lambda^2 \delta^{(1)}_\Lambda,\ J_\Lambda^3] \right)$$

The running of the bias/stochastic operators is done connecting both cutoff

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$$\times \left(1 + \int_{\boldsymbol{k}} J_\Lambda(\boldsymbol{k}) \left[\sum_O b_O^{\Lambda'} \left(\mathcal{S}^1_O[\delta^{(1)}_\Lambda](-\boldsymbol{k}) + \mathcal{S}^2_O[\delta^{(1)}_\Lambda](-\boldsymbol{k}) + \dots \right) \right] + \frac{1}{2} \int_{\boldsymbol{k},\boldsymbol{k}'} J_\Lambda(\boldsymbol{k}) J_\Lambda(\boldsymbol{k}') \sum_{O,O'} b_O^{\Lambda'} b_{O'}^{\Lambda'} \left[\mathcal{S}^{11}_{OO'}[\delta^{(1)}_\Lambda](\boldsymbol{k},\boldsymbol{k}') + \dots \right] + \mathcal{O}[J_\Lambda^2 \delta^{(1)}_\Lambda,\ J_\Lambda^3] \right)$$

The running of the bias/stochastic operators is done connecting both cutoff

What appears after integrating out the shell

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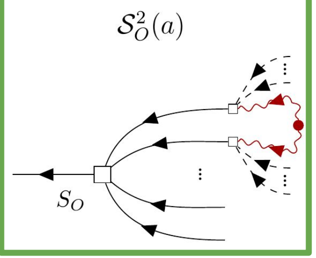
$$\times \left(1 + \int_{\boldsymbol{k}} J_\Lambda(\boldsymbol{k}) \left[\sum_O b_O^{\Lambda'} \left(\mathcal{S}^1_O[\delta^{(1)}_\Lambda](-\boldsymbol{k}) + \mathcal{S}^2_O[\delta^{(1)}_\Lambda](-\boldsymbol{k}) + \ldots \right) \right] + \frac{1}{2} \int_{\boldsymbol{k},\boldsymbol{k}'} J_\Lambda(\boldsymbol{k}) J_\Lambda(\boldsymbol{k}') \sum_{O,O'} b_O^{\Lambda'} b_{O'}^{\Lambda'} \left[\mathcal{S}^{11}_{OO'}[\delta^{(1)}_\Lambda](\boldsymbol{k},\boldsymbol{k}') + \ldots \right] \right)$$

The running of the bias/stochastic operators is done connecting both cutoff

What appears after integrating out the shell

Bias corrections



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Idea: Integrate out the shell!

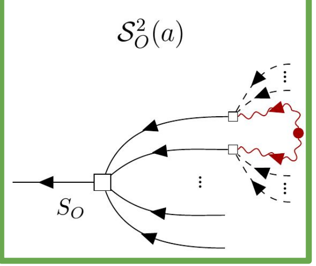
$$\mathcal{Z}[J_\Lambda] = \int \mathcal{D}\delta^{(1)}_\Lambda \mathcal{P}[\delta^{(1)}_\Lambda] \exp\Bigg(\int_{\boldsymbol{k}} J_\Lambda(\boldsymbol{k}) \Big[\sum_O b^{\Lambda'}_O O[\delta^{(1)}_\Lambda](-\boldsymbol{k}) \Big] + \frac{1}{2} P^{\Lambda'}_\epsilon \int_{\boldsymbol{k}} J_\Lambda(\boldsymbol{k}) J_\Lambda(-\boldsymbol{k}) + \mathcal{O}[J^2_\Lambda \delta^{(1)}_\Lambda,\ J^3_\Lambda] \Bigg)$$

$$\times \Bigg(1 + \int_{\boldsymbol{k}} J_\Lambda(\boldsymbol{k}) \Big[\sum_O b^{\Lambda'}_O \Big(\mathcal{S}^1_O[\delta^{(1)}_\Lambda](-\boldsymbol{k}) + \mathcal{S}^2_O[\delta^{(1)}_\Lambda](-\boldsymbol{k}) + \ldots \Big) \Big] + \frac{1}{2} \int_{\boldsymbol{k},\boldsymbol{k}'} J_\Lambda(\boldsymbol{k}) J_\Lambda(\boldsymbol{k}') \sum_{O,O'} b^{\Lambda'}_O b^{\Lambda'}_{O'} \Big[\mathcal{S}^{11}_{OO'}[\delta^{(1)}_\Lambda](\boldsymbol{k},\boldsymbol{k}') + \ldots$$

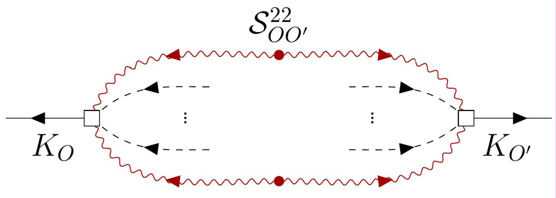
The running of the bias/stochastic operators is done connecting both cutoff

What appears after integrating out the shell

Bias corrections



Stochastic corrections



Example...

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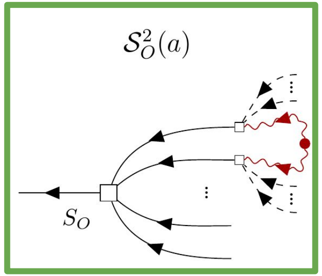
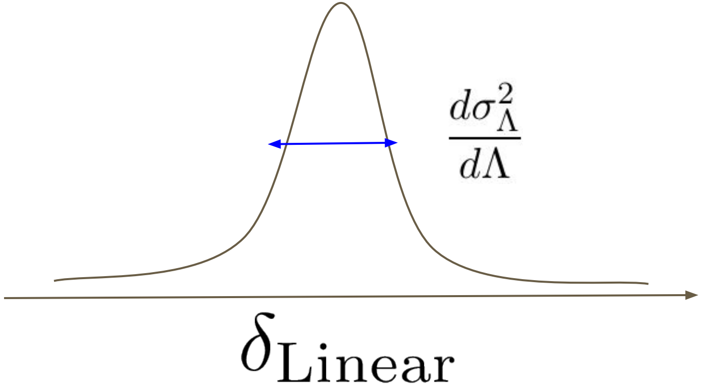


Correction to those operators!

$$\mathcal{S}^2_{\delta^2}[\delta^{(1)}_\Lambda](\boldsymbol{k}) = \left[\frac{68}{21}\delta^{(1+2)}(\boldsymbol{k}) + \frac{8126}{2205}\left[\delta^2(\boldsymbol{k})\right]^{(2)} + \frac{254}{2205}\mathcal{G}_2^{(2)}(\boldsymbol{k})\right]\int \frac{p^2dp}{2\pi^2}P_{\rm shell}(p)$$
$$+ \text{higher derivative (h.d.)} + \mathcal{O}\left[\left(\delta^{(1)}_\Lambda\right)^3\right],$$

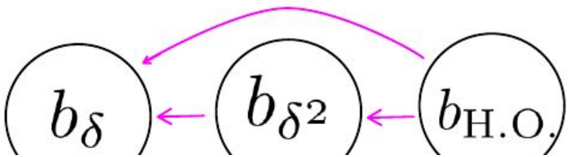
$$\int_{\boldsymbol{p}} P_{\rm shell}(p) = \int_\Lambda^{\Lambda+\lambda}\frac{p^2dp}{2\pi^2}P_{\rm L}(p) = \frac{d\sigma^2_\Lambda}{d\Lambda}\Big|_\Lambda \lambda + \mathcal{O}(\lambda^2),$$



Results

$$\frac{d}{d\Lambda} b_O(\Lambda) = -\frac{d\sigma_\Lambda^2}{d\Lambda} \sum_{O'} s^O_{O'}\, b_{O'}(\Lambda)\,,$$

J^1



$s^O_{O'}$	δ	δ^2	$\mathcal{G}_2$	δ^3	$\mathcal{G}_3$	Γ_3	$\delta\mathcal{G}_2$
$\mathbb{1}$	-	-	-	-	-	-	-
δ	-	$68/21$	-	3	-	-	$-4/3$
δ^2	-	$8126/2205$	-	$68/7$	-	-	$-376/105$
$\mathcal{G}_2$	-	$254/2205$	-	-	-	-	$116/105$

Solutions

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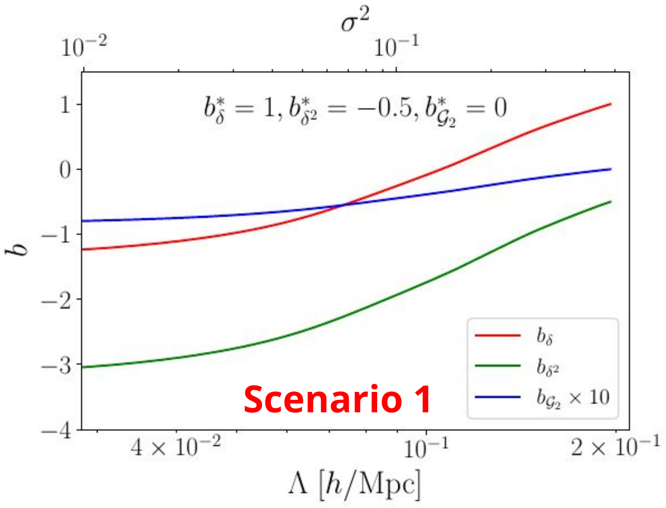
Wilson-Polchinski RG-equations

$$\frac{db_\delta}{d\Lambda} = -\left[\frac{68}{21}b_{\delta^2} + 3b^*_{\delta^3} - \frac{4}{3}b^*_{\mathcal{G}_2\delta}\right]\frac{d\sigma^2_\Lambda}{d\Lambda}\,,$$

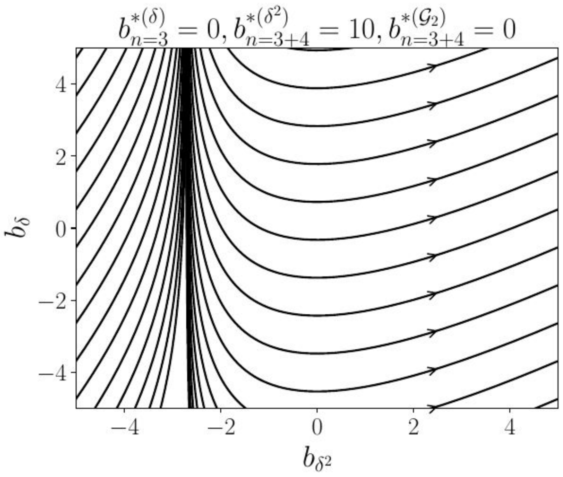
$$\frac{db_{\delta^2}}{d\Lambda} = -\left[\frac{8126}{2205}b_{\delta^2} + \frac{17}{7}b^*_{\delta^3} - \frac{376}{105}b^*_{\mathcal{G}_2\delta} + b^{*(\delta^2)}_{n=4}\right]\frac{d\sigma^2_\Lambda}{d\Lambda}\,,$$

$$\frac{db_{\mathcal{G}_2}}{d\Lambda} = -\left[\frac{254}{2205}b_{\delta^2} + \frac{116}{105}b^*_{\mathcal{G}_2\delta} + b^{*(\mathcal{G}_2)}_{n=4}\right]\frac{d\sigma^2_\Lambda}{d\Lambda}\,.$$



Notice that:

- **Bias parameter that are zero, may be sourced;**
- **Bias parameters may change sign!**



PNGs (2405.21002)

w/ Charalampos Nikolis

PNGs

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Spin-0

$$\text{First order:} \quad \delta,\ \Psi;$$
$$\text{Second order:} \quad \delta^2,\ \mathcal{G}_2,\ \delta\Psi;$$
$$\text{Third order:} \quad \delta^3,\ \delta^2\Psi,\ \delta\mathcal{G}_2,\ \Psi\mathcal{G}_2,\ \Gamma_3,\ \mathcal{G}_3$$

Spin-2

$$\text{First order:} \quad \delta;$$
$$\text{Second order:} \quad \delta^2,\ \mathcal{G}_2,\ \mathrm{Tr}\left[\Psi\Pi^{[1]}\right];$$
$$\text{Third order:} \quad \delta^3,\ \delta\mathcal{G}_2,\ \delta\ \mathrm{Tr}\left[\Psi\Pi^{[1]}\right],\ \Gamma_3,\ \mathcal{G}_3,\ \mathrm{Tr}\left[\Psi\Pi^{[2]}\right]$$

PNGs

Free term

New interaction

$$\frac{db_\delta}{d\Lambda} = -\left[\frac{68}{21} b_{\delta^2}(\Lambda) + b_{n=3}^{*\{\delta\}\mathrm{G}}\right] \frac{d\sigma_\Lambda^2}{d\Lambda} - a_0 f_{\mathrm{NL}} \left[-\frac{13}{21} b_\Psi + \frac{13}{21} b_{\Psi\delta} + b_{n=3}^{*\{\delta\}\mathrm{NG}}\right] \left(\frac{H_0}{\Lambda}\right)^2 \frac{3\,\Omega_m}{2\,T(\Lambda)} \frac{d\sigma_\Lambda^2}{d\Lambda};$$

Rederivation of Dalal+ 07 (in an elegant way)

Now a coupled set of ODEs

$$\frac{db_\Psi}{d\Lambda} = -a_0 f_{\mathrm{NL}} b_{n=3}^{*\{\Psi\}\mathrm{NG}} \frac{d\sigma_\Lambda^2}{d\Lambda} - 4 a_0 f_{\mathrm{NL}} b_{\delta^2} \frac{d\sigma_\Lambda^2}{d\Lambda},$$

$$\frac{db_{\Psi\delta}}{d\Lambda} = -a_0 f_{\mathrm{NL}} \left[\frac{272}{21} b_{\delta^2} + b_{n=3+4}^{*\{\Psi\delta\}\mathrm{G}} + b_{n=3+4}^{*\{\Psi\delta\}\mathrm{NG}}\right] \frac{d\sigma_\Lambda^2}{d\Lambda},$$

PNGs

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Free term

New interaction

$$\frac{db_\delta}{d\Lambda} = -\left[\frac{68}{21} b_{\delta^2}(\Lambda) + b^{*\{\delta\}\mathrm{G}}_{n=3}\right]\frac{d\sigma^2_\Lambda}{d\Lambda} - a_0 f_{\mathrm{NL}}\left[-\frac{13}{21} b_\Psi + \frac{13}{21} b_{\Psi\delta} + b^{*\{\delta\}\mathrm{NG}}_{n=3}\right]\left(\frac{H_0}{\Lambda}\right)^2 \frac{3\,\Omega_m}{2\,T(\Lambda)}\frac{d\sigma^2_\Lambda}{d\Lambda};$$

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$$\frac{db_\Psi}{d\Lambda} = -a_0 f_{\mathrm{NL}} b^{*\{\Psi\}\mathrm{NG}}_{n=3}\frac{d\sigma^2_\Lambda}{d\Lambda} - 4 a_0 f_{\mathrm{NL}} b_{\delta^2}\frac{d\sigma^2_\Lambda}{d\Lambda},$$

$$\frac{db_{\Psi\delta}}{d\Lambda} = -a_0 f_{\mathrm{NL}}\left[\frac{272}{21} b_{\delta^2} + b^{*\{\Psi\delta\}\mathrm{G}}_{n=3+4} + b^{*\{\Psi\delta\}\mathrm{NG}}_{n=3+4}\right]\frac{d\sigma^2_\Lambda}{d\Lambda},$$

Rederivation of Dalal+ 07 (in an elegant way)

$s^O_{O'}$	δ^2	δ^3	$\delta\mathcal{G}_2$	Ψ	$\Psi\delta$	$\Psi\delta^2$	$\Psi\mathcal{G}_2$	$\mathrm{Tr}\,\Psi\Pi^{[1]}$	$\delta\,\mathrm{Tr}\,\Psi\Pi^{[1]}$	$\mathrm{Tr}\,\Psi\Pi^{[2]}$
δ	68/21	3	−4/3	−13/21	13/21	2	−4/3	34/21	1	34/21
δ^2	8126/2205	68/7	−376/105	43/135	478/135	47/21	−31/21	124/315	178/105	14347/6027
$\mathcal{G}_2$	254/2205	-	116/105	−1699/13230	79/2205	-	−1/21	−661/4410	4/35	−241/735
Ψ	4	-	-	-	-	1	-	-	-	-
$\delta\Psi$	272/21	12	−8/3	-	-	68/21	-	-	-	-
$\mathrm{Tr}\,\Psi\Pi^{[1]}$	64/105	-	16/15	-	-	-	-	-	8/105	58/305

PNGs

Starting with no PNG in a cutoff, you dynamically generate them!!!
(extending the picture of Assassi, Baumann, Schmidt)

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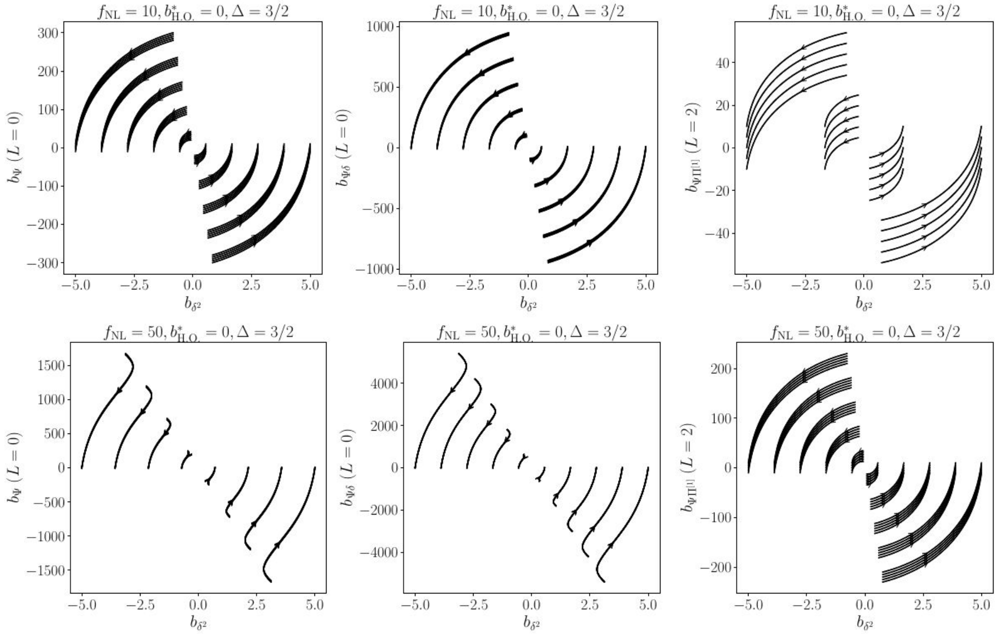

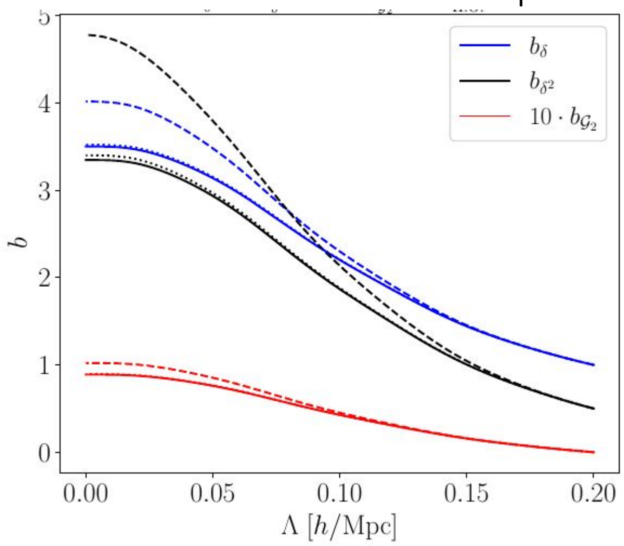

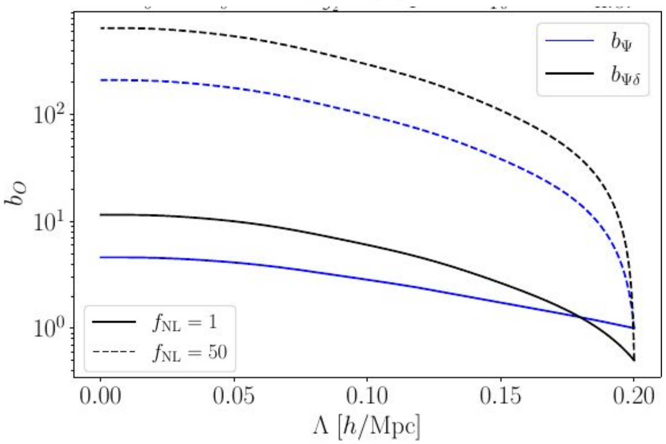

Stochasticity in LSS (2404.16929)

Stochasticity

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$$\delta_g(\boldsymbol{x},\tau) \equiv \frac{n_g(\boldsymbol{x},\tau)}{\bar{n}_g(\tau)} - 1 = \sum_O \left[b_O(\tau) + c_{\epsilon,O}(\tau)\epsilon(\boldsymbol{x},\tau)\right] O(\boldsymbol{x},\tau) + \epsilon(\boldsymbol{x},\tau)$$

Noise is a central part in modelling galaxy distribution

Properties of the noise:

How those C coefficients evolve?

$$\langle \epsilon(\boldsymbol{k}_1)\ldots\epsilon(\boldsymbol{k}_m) O(\boldsymbol{k}_{m+1})\rangle = \hat{\delta}_{\rm D}(\boldsymbol{k}_{1\ldots m}) C^{(m)}_{\epsilon,O} O(\boldsymbol{k}_{m+1})$$

$$\langle \epsilon(\boldsymbol{k}_1) O(\boldsymbol{k}_2) O'(\boldsymbol{k}_3)\ldots\rangle = 0$$ (linearly does not correlate with O's)

Example:
The shot-noise terms

$$\langle \epsilon(\boldsymbol{k}_1)\ldots\epsilon(\boldsymbol{k}_m)\rangle = \hat{\delta}_{\rm D}(\boldsymbol{k}_{1\ldots m}) C^{(m)}_{\epsilon,\mathbb{1}} \,.$$

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Stochasticity

Coupled to higher powers of J

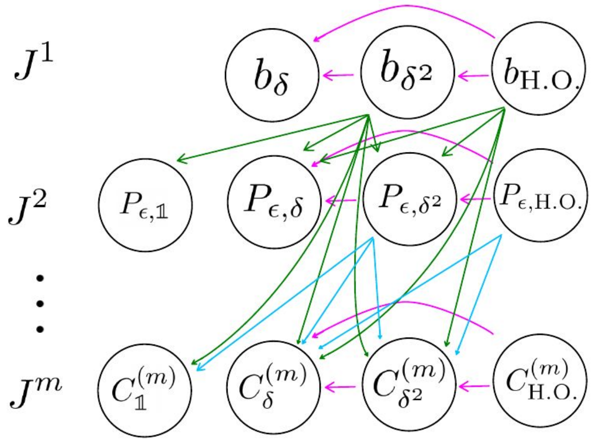
$$\mathcal{Z}[J_\Lambda] = \int \mathcal{D}\delta_\Lambda^{(1)} \mathcal{P}[\delta_\Lambda^{(1)}] \exp\left(\sum_m \left\{\frac{1}{m!}\int_{\boldsymbol{x}} \left[(J_\Lambda(\boldsymbol{x}))^m \sum_O C_O^{(m)}(\Lambda') O[\delta_\Lambda^{(1)}](\boldsymbol{x})\right] + \zeta^{(m)}[J_\Lambda, \delta_\Lambda^{(1)}]\right\}\right)$$

Shell corrections

$$\frac{d}{d\Lambda} b_O(\Lambda) = -\frac{d\sigma_\Lambda^2}{d\Lambda} \sum_{O'} s_{O'}^O \, b_{O'}(\Lambda)\,,$$

↓

$$\frac{d}{d\Lambda} C_O^{(m)}(\Lambda) \propto -\left[P_{\rm L}(\Lambda)\right]^{p-1} \frac{d\sigma_\Lambda^2}{d\Lambda} \sum_{O_1, O_2, \ldots O_m} s^O_{O_1 O_2 \ldots O_m} C_{O_1}^{(i_1)}(\Lambda) \ldots C_{O_p}^{(i_p)}(\Lambda)$$



Stochasticity

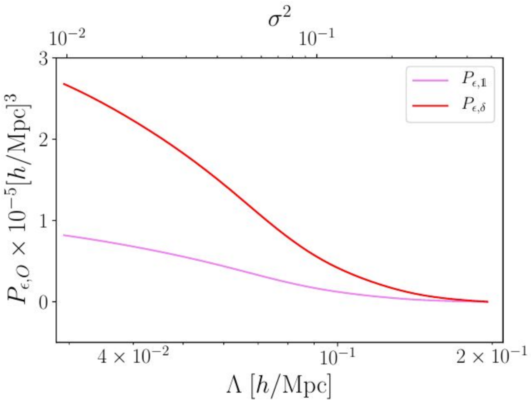
Henrique Rubira

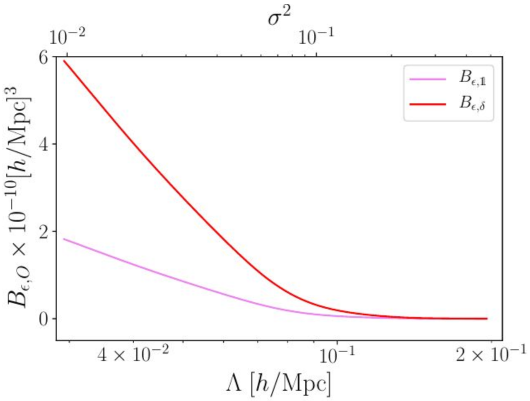
The delta^2 bias generates the whole tower of stochastic parameters!!!

Examples (shot-noise terms):

$$\frac{dP_{\epsilon,\mathbb{1}}}{d\Lambda} = -2[b_{\delta^2}(\Lambda)]^2 [2P_{\rm L}(\Lambda)] \frac{d\sigma^2_\Lambda}{d\Lambda}$$

$$\frac{dB_{\epsilon,\mathbb{1}}}{d\Lambda} = -2P^*_{\epsilon,\delta^2} b_{\delta^2}(\Lambda) [2P_{\rm L}(\Lambda)] \frac{d\sigma^2_\Lambda}{d\Lambda} - 8[b_{\delta^2}(\Lambda)]^3 3 [P_{\rm L}(\Lambda)]^2 \frac{d\sigma^2_\Lambda}{d\Lambda}$$





Again: starting with no stochasticity, we generate them

Stochasticity

Henrique Rubira

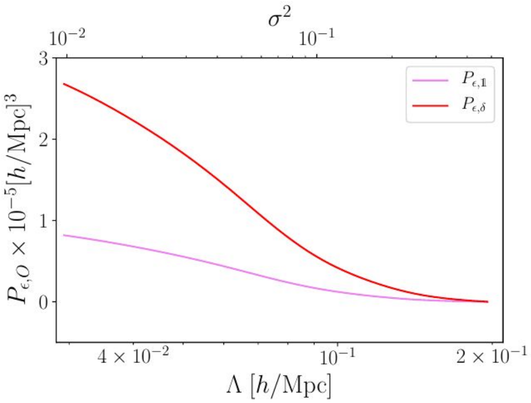
The delta^2 bias generates the whole tower of stochastic parameters!!!

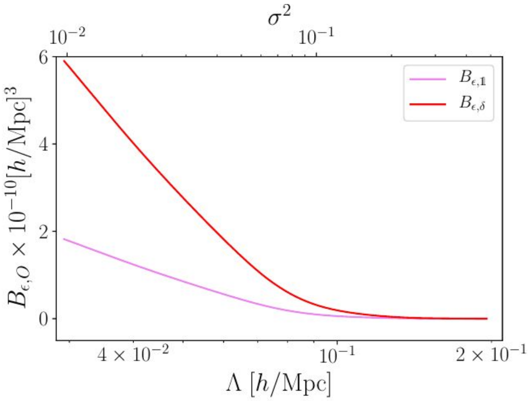
Examples (shot-noise terms):

Chain reaction

$$\frac{dP_{\epsilon,\mathbb{1}}}{d\Lambda} = -2[b_{\delta^2}(\Lambda)]^2 [2P_{\rm L}(\Lambda)] \frac{d\sigma^2_\Lambda}{d\Lambda}$$

$$\frac{dB_{\epsilon,\mathbb{1}}}{d\Lambda} = -2P^*_{\epsilon,\delta^2} b_{\delta^2}(\Lambda) [2P_{\rm L}(\Lambda)] \frac{d\sigma^2_\Lambda}{d\Lambda} - 8[b_{\delta^2}(\Lambda)]^3 3 [P_{\rm L}(\Lambda)]^2 \frac{d\sigma^2_\Lambda}{d\Lambda}$$





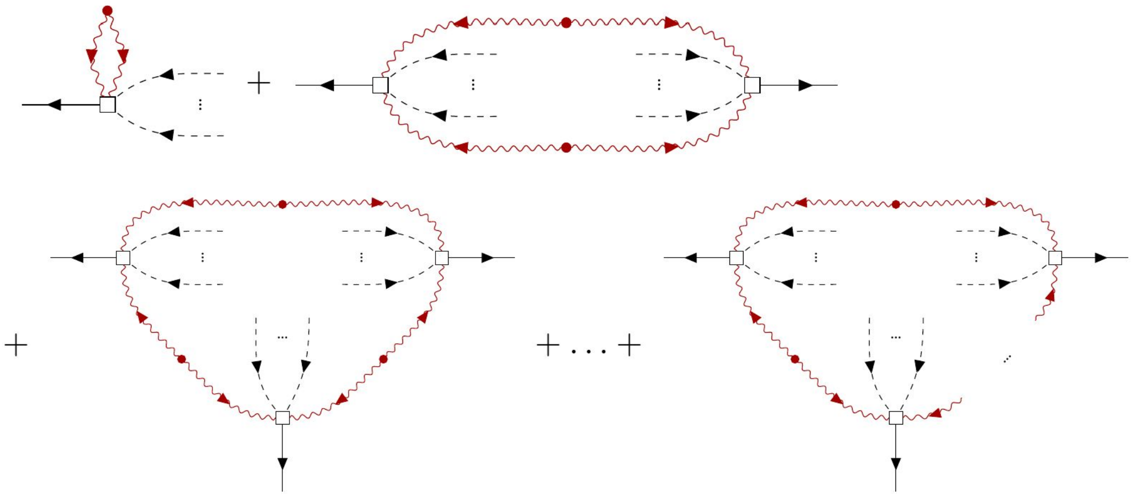
Again: starting with no stochasticity, we generate them

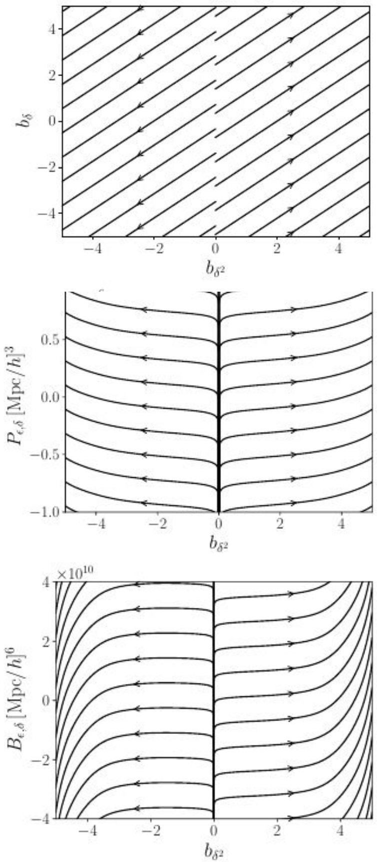
Stochasticity

Conclusion: very simple expression for how general terms in the partition function couple to each other

$$\frac{d}{d\Lambda} C_O^{(m)}(\Lambda) \propto -\left[P_{\rm L}(\Lambda)\right]^{p-1} \frac{d\sigma_\Lambda^2}{d\Lambda} \sum_{O_1, O_2, \ldots O_m} s^{O}_{O_1 O_2 \ldots O_m} C_{O_1}^{(i_1)}(\Lambda) \ldots C_{O_p}^{(i_p)}(\Lambda)$$

Simple diagrammatic interpretation





Part IV - Final remarks

How to relate the renormalization schemes?

Henrique Rubira

N-point function renormalized bias
(Assassi, Baumann, Green, Zaldarriaga)

Finite cutoff bias
(This work)

How to connect both?

$$[\![O']\!](\boldsymbol{k}') \longleftrightarrow O'[\delta_\Lambda^{(1)}](\boldsymbol{k}')$$

Henrique Rubira

How to relate the renormalization schemes?

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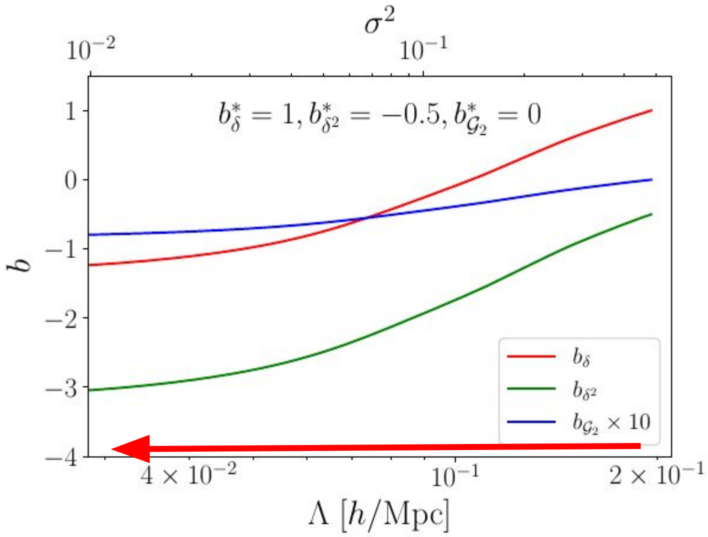


Solution: Run the bias towards

$$\Lambda \to 0$$

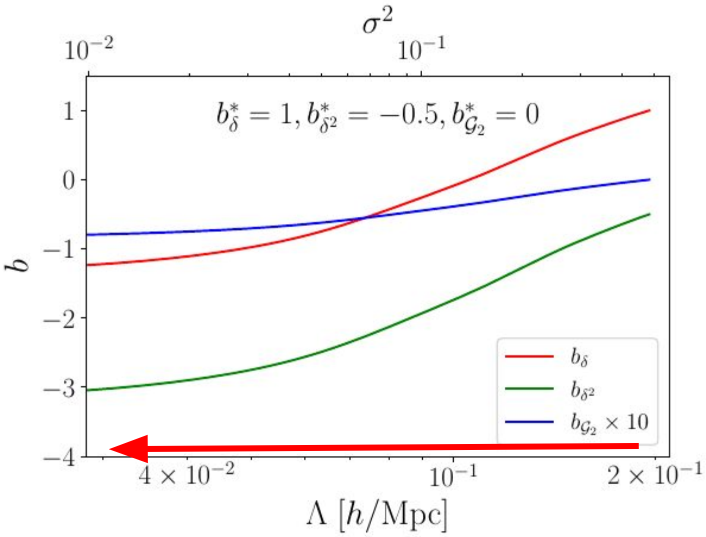
Henrique Rubira

How to relate the renormalization schemes?

N-point function renormalized bias
(Assassi, Baumann, Green, Zaldarriaga)

Finite cutoff bias
(This work)

$$\lim_{\Lambda\to 0;\ k/\Lambda\ \text{fixed}} \langle O[\delta_\Lambda^{(1)}](\boldsymbol{k})[\![O']\!](\boldsymbol{k}')\rangle = \lim_{\Lambda\to 0;\ k/\Lambda\ \text{fixed}} \langle O[\delta_\Lambda^{(1)}](\boldsymbol{k})O'[\delta_\Lambda^{(1)}](\boldsymbol{k}')\rangle$$



Solution: Run the bias towards

$$\Lambda \to 0$$

Conclusion: Why you should care

- Additional cross-check for EFT inference;
- Systematic renormalization of bias/stochastic parameters (including PNG);
- Self-consistent renormalization for P(k), B(k1,k2,k3), ... Also field level
- (Unambiguously) Define Priors for EFT analysis in $\Lambda \to 0$
- Absorb cutoff dependence in the counter-terms keeping also sub-leading contributions;
- More information? Connection to other fields is manifest, like phase transitions, critical exponents, etc (TBD)


Thanks a lot!

Why just not taking $\Lambda \to \infty$?

$$\delta^{(1)}_{\Lambda'}(\boldsymbol{k}) = \delta^{(1)}_{\Lambda}(\boldsymbol{k}) + \delta^{(1)}_{\rm shell}(\boldsymbol{k}) \qquad \Lambda = \Lambda' - \lambda$$

$$\mathcal{Z}[J_\Lambda] = \int \mathcal{D}\delta^{(1)}_\Lambda \mathcal{P}[\delta^{(1)}_\Lambda] \exp\left(\int_{\boldsymbol{k}} J_\Lambda(\boldsymbol{k}) \left[\sum_O b_O^{\Lambda'} O[\delta^{(1)}_\Lambda](-\boldsymbol{k})\right]\right)$$

Continuum of the theory is determined by taking $\Lambda' \to \infty$

This determines local terms to be added to the action that will cancel out UV dependence (the counter-terms)

In Wilson-Polchinski we integrate modes up to the cutoff of the theory $k_{\rm NL}$

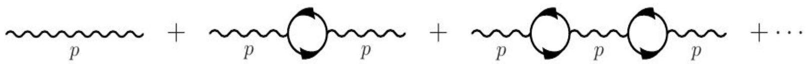
Renormalization scale $\Lambda_* < k_{\rm NL}$

Henrique Rubira

Logs in QFT

Logs in QFT: Arise when we have a hierarchy of scales $\lim_{E\to\infty} \Gamma(E,m) = E^d \Gamma(1, \frac{m}{E}) \times O\left[\ln\left(\frac{E}{m}\right)\right]$

Approach 1) Resum diagrams by hand (if you can)



$$\tilde{V}(p^2) = \frac{e_R^2}{p^2}\left[1 + \frac{e_R^2}{12\pi^2}\ln\frac{p^2}{p_0^2} + \left(\frac{e_R^2}{12\pi^2}\ln\frac{p^2}{p_0^2}\right)^2 + \cdots\right] = \frac{1}{p^2}\left[\frac{e_R^2}{1 - \frac{e_R^2}{12\pi^2}\ln\frac{p^2}{p_0^2}}\right]$$

$$e_{\text{eff}}^2(p^2) = \frac{e_R^2}{1 - \frac{e_R^2}{12\pi^2}\ln\frac{p^2}{p_0^2}}$$

Extracted from Schwartz's and Weinberg's books

Approach 2) Direct from the RGE

$$p_0^2 \frac{d}{dp_0^2}\tilde{V}(p^2) = 0$$

$$p_0^2 \frac{de_{\text{eff}}}{dp_0^2} = \frac{e_{\text{eff}}^3}{24\pi^2}$$

$$e_{\text{eff}}^2(p^2) = \frac{e_R^2}{1 - \frac{e_R^2}{12\pi^2}\ln\frac{p^2}{p_0^2}}$$

The n-point function renormalized bias
(Assassi, Baumann, Green, Zaldarriaga, 2014)

Henrique Rubira

Intuition: Define the bias parameter of order "n" as the large-scale limit of "n+1"-point functions

Example 1: Define the linear bias in the large-scale limit of P(k):

$$b_\delta = \lim_{k \to 0} \frac{\langle \delta_g \delta \rangle}{\langle \delta \delta \rangle}$$

Example 2: Define the 2nd-order bias parameters in the large-scale limit of B(k1,k2,k3)

More formally:

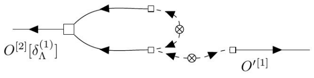
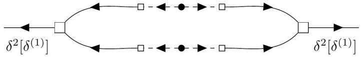
$$\langle \delta^{(1)}(\boldsymbol{k}_1) \cdots \delta^{(1)}(\boldsymbol{k}_m) [\![O]\!](\boldsymbol{k}) \rangle \xrightarrow{k_i \to 0} \langle \delta^{(1)}(\boldsymbol{k}_1) \cdots \delta^{(1)}(\boldsymbol{k}_m) O[\delta](\boldsymbol{k}) \rangle_{\mathrm{LO}}$$

Example:

$$[\![\delta^2]\!] = \delta^2 - \sigma_\infty^2 \left(1 + \frac{68}{21}\delta + \frac{8126}{2205}\delta^2 + \frac{254}{2205}\mathcal{G}_2 \right)$$

Henrique Rubira

Differences between renormalization schemes

	N-point renormalization	Finite Λ
In practice, one has to:	Subtract all UV-dep part	Nothing to subtract. Bias params run... and we know how
Potential to:	Error-prone, missing finite contributions	Extra sanity-checks
$\langle(\delta)^{(1)}(\delta^2)^{(3)}\rangle$ 	Completely removed by c.t., but missing sub-leading $k^2 P_{\rm L}(k)\int_{\boldsymbol{p}} p^{-2}P_{\rm L}(p)$	Finite: $4b_\delta^\Lambda b_{\delta^2}^\Lambda \int_p F_2(\boldsymbol{p},\boldsymbol{k}-\boldsymbol{p})P_{\rm L}^\Lambda(p)P_{\rm L}^\Lambda(k)$
$\langle(\delta^2)^{(2)}(\delta^2)^{(2)}\rangle$ 	Subtracted to the stochastic term, but missing sub-leading $k^2\int_{\boldsymbol{p}} p^{-2}[P_{\rm L}(p)]^2$	Finite and contributes to the stochastic running: $2\,(b_{\delta^2}^\Lambda)^2\int_{\boldsymbol{p}} P_{\rm L}^\Lambda(p)P_{\rm L}^\Lambda(\lvert\boldsymbol{k}-\boldsymbol{p}\rvert)$

Logs in LSS

$$\Delta^2_{1-loop} = \left(\frac{k}{k_{NL}}\right)^{3+n} + \left(\frac{k}{k_{NL}}\right)^{2(3+n)} \left[\alpha(n) + \tilde{\alpha}(n) \ln\left(\frac{k}{k_{NL}}\right)\right]$$

n	-2	-3/2	-1	-1/2	0	1/2	1	3/2	2	5/2	3
α_{13}	$\frac{5\pi^2}{112}$	$\frac{992\pi}{6{,}615}$	...	$-\frac{416\pi}{8{,}085}$	$-\frac{\pi^2}{336}$	...	...	...	$-\frac{\pi^2}{168}$	...	...
α_{22}	$\frac{75\pi^2}{784}$	-0.232	...	$.698$	$\frac{29\pi^2}{784}$	...	...	...	$\frac{\pi^2}{392}$	...	...
$\tilde{\alpha}_{13}$	0	0	$\frac{61}{315}$	0	0	0	$-\frac{4}{105}$	0	0	0	$\frac{20}{1{,}323}$
$\tilde{\alpha}_{22}$	0	0	0	0	0	$-\frac{9}{98}$	0	$\frac{31}{16{,}464}$	0	$-\frac{359}{26{,}880}$	0
α	1.38	.239	...	.537	.336	...	...	...	$-.0336$	...	...
$\tilde{\alpha}$	0	0	.194	0	0	$-.0918$	.0381	$-.00188$	0	$-.0134$	.0151

Pajer+Zaldarriaga, 2013

The shell expansion (Wilson formalism)

Henrique Rubira

Consider a very thin shell with width: $\Lambda = \Lambda' - \lambda$

$$\delta^{(1)}_{\Lambda'}(\boldsymbol{k}) = \delta^{(1)}_{\Lambda}(\boldsymbol{k}) + \delta^{(1)}_{\text{shell}}(\boldsymbol{k})$$

Idea: Integrate out the shell!

$$\mathcal{Z}[J_\Lambda] = \int \mathcal{D}\delta^{(1)}_\Lambda \mathcal{P}[\delta^{(1)}_\Lambda] \int \mathcal{D}\delta^{(1)}_{\text{shell}} \mathcal{P}[\delta^{(1)}_{\text{shell}}] \tag{2.7}$$
$$\times \exp\left(\int_{\boldsymbol{k}} J_\Lambda(\boldsymbol{k}) \left[\sum_O b_O^{\Lambda'} O[\delta^{(1)}_\Lambda + \delta^{(1)}_{\text{shell}}](-\boldsymbol{k}) \right] + \frac{1}{2} P_\epsilon^{\Lambda'} \int_{\boldsymbol{k}} J_\Lambda(\boldsymbol{k}) J_\Lambda(-\boldsymbol{k}) + \mathcal{O}[J_\Lambda^2 \delta^{(1)}_\Lambda,\ J_\Lambda^3] \right)$$

1) Expand the operators in terms of the number of shell fields and integrate those out!

$$O^{(n)}[\delta^{(1)}_\Lambda + \delta^{(1)}_{\text{shell}}] = O^{(n)}[\delta^{(1)}_\Lambda] + O^{(n),(1)_{\text{shell}}}[\delta^{(1)}_\Lambda, \delta^{(1)}_{\text{shell}}] + O^{(n),(2)_{\text{shell}}}[\delta^{(1)}_\Lambda, \delta^{(1)}_{\text{shell}}] \tag{2.8}$$
$$+ \ldots + O^{(n),(n-1)_{\text{shell}}}[\delta^{(1)}_\Lambda, \delta^{(1)}_{\text{shell}}] + O^{(n)}[\delta^{(1)}_{\text{shell}}],$$

2) Integrate the shells

$$\mathcal{S}^2_O[\delta^{(1)}_\Lambda] = \sum_{n\geq 2} \int \mathcal{D}\delta^{(1)}_{\text{shell}} \mathcal{P}[\delta^{(1)}_{\text{shell}}]\, O^{(n),(2)_{\text{shell}}}[\delta^{(1)}_\Lambda, \delta^{(1)}_{\text{shell}}](\boldsymbol{k})$$

$$\mathcal{S}^{11}_{OO'}[\delta^{(1)}_\Lambda](\boldsymbol{k},\boldsymbol{k}') = \sum_{n,n'\geq 1} \int \mathcal{D}\delta^{(1)}_{\text{shell}} \mathcal{P}[\delta^{(1)}_{\text{shell}}]\, O^{(n),(1)_{\text{shell}}}[\delta^{(1)}_\Lambda, \delta^{(1)}_{\text{shell}}](\boldsymbol{k})\, O'^{(n'),(1)_{\text{shell}}}[\delta^{(1)}_\Lambda, \delta^{(1)}_{\text{shell}}](\boldsymbol{k}')$$

Solutions

Henrique Rubira

1) Neglect fourth-order+ bias;

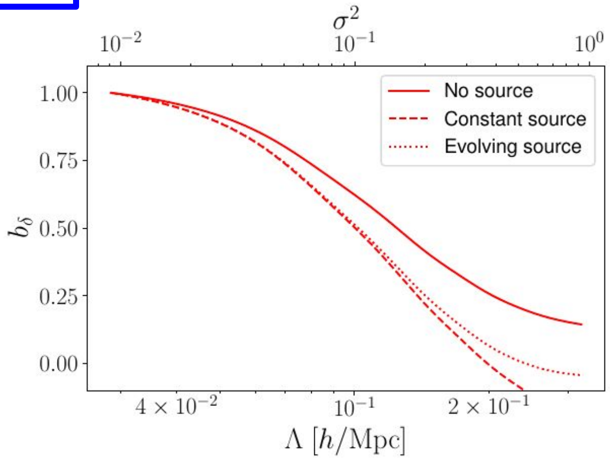
2) Neglect ***the running*** of fourth-order+ bias;

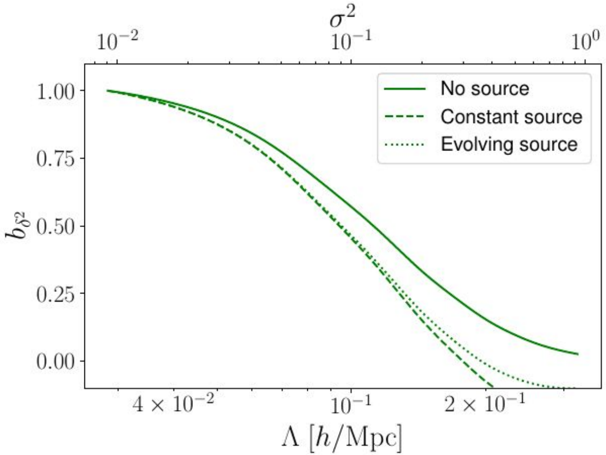
3) Ansatz for higher-order bias running.

$$b^{(O)}_{n=3+4}(\sigma^2) = b^{*(O)}_{n=3+4}\, e^{-c^{(O)}(\sigma^2-\sigma^2_*)}$$

Wilson-Polchinski RG-equations

$$\frac{db_\delta}{d\Lambda} = -\left[\frac{68}{21}b_{\delta^2} + 3b^*_{\delta^3} - \frac{4}{3}b^*_{\mathcal{G}_2\delta}\right]\frac{d\sigma^2_\Lambda}{d\Lambda}\,,$$
$$\frac{db_{\delta^2}}{d\Lambda} = -\left[\frac{8126}{2205}b_{\delta^2} + \frac{17}{7}b^*_{\delta^3} - \frac{376}{105}b^*_{\mathcal{G}_2\delta} + b^{*(\delta^2)}_{n=4}\right]\frac{d\sigma^2_\Lambda}{d\Lambda}\,,$$
$$\frac{db_{\mathcal{G}_2}}{d\Lambda} = -\left[\frac{254}{2205}b_{\delta^2} + \frac{116}{105}b^*_{\mathcal{G}_2\delta} + b^{*(\mathcal{G}_2)}_{n=4}\right]\frac{d\sigma^2_\Lambda}{d\Lambda}\,.$$





Conclusions:

1) **Neglecting source affects result;**
2) **Evolving source does not affect the result!**

EFTofLSS via Wilson Polchinski

(based on Carroll, Leichenauer, Pollack, 13)

$$Z[J] = \int \mathcal{D}\phi_{\rm in} \;\exp\left(S_0[\phi_{\rm in}] + J_i \phi^i[\phi_{\rm in}]\right)$$

with

$$S_0[\phi_{\rm in}] = -\frac{1}{2}\phi^i [P(\Lambda)^{-1}]_{ij}\phi^j$$

We use

$$\left.\frac{\partial \mathcal{Z}}{\partial J_\Lambda \ldots \partial J_\Lambda}\right|_{J_\Lambda = 0}$$

Since

$$\langle \phi^{i_1} \cdots \phi^{i_n}\rangle = \int \mathcal{D}\phi_{\rm in}\; \phi^{i_1}[\phi_{\rm in}] \cdots \phi^{i_n}[\phi_{\rm in}] e^{S_0[\phi_{\rm in}]}$$

$$\phi^i_{\rm SPT} \equiv K^i_{{\rm SPT}\,j}\,\phi^j_{\rm in} + \frac{1}{2} K^i_{{\rm SPT}\,jk}\,\phi^j_{\rm in}\phi^k_{\rm in} + \cdots$$

Advantages:
- Path integral formulation
- Systematic generation EFT structure (coefficients are closed under RG flow)
- Keeps small (yet-perturbative) modes in the theory

$$\frac{d}{d\Lambda} K^{j_1\cdots j_n}_{i_1\cdots i_m} = -\frac{1}{2}\left(\frac{dP^{ij}}{d\Lambda} K^{j_1\cdots j_n}_{ij i_1\cdots i_m} + \frac{dP^{ij}}{d\Lambda}\sum_{k=0}^{m}\sum_{l=0}^{n}\binom{m}{k}\binom{n}{l} K^{j_1\cdots j_l}_{i i_1\cdots i_k} K^{j_{l+1}\cdots j_n}_{j i_{k+1}\cdots i_m}\right)$$

Historical overview and frameworks

Henrique Rubira

- **Dim Reg, scale transformations and applications to QED**: Stueckelberg, Petermann, Gell-Mann, Low ~1953
- **RG in condensed matter**: Kadanoff, 1966
- **RG in the continuum, derivation of RG equations and critical phenomena**: Callan and Symanzik 1970, Kenneth Wilson, 1970/71 (Nobel Prize 1982)
- **RG via path integrals**: Polchinski, 1984

Framework 1 (a la Wilson/Polchinski):

$$\Lambda \frac{d}{d\Lambda} Z[J] = 0$$

Sliding cutoff,
integrate out modes between cutoffs

$$\Lambda \to \Lambda'$$

Framework 2:
Sliding renormalization conditions (e.g. Dim Reg), no UV regulator

$$\frac{\partial g}{\partial \ln \mu} = \beta(g)$$

- More practical for computations