Scalar sector of CP4 3HDM

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Multi-Higgs-Doublet Model and General CP Symmetry

Multi-Higgs-doublet model:

- 2HDM [T.D.Lee, 1973], see review: [G.C.Branco et al, 2012; Bhattacharyya et al, 2015].
- 3HDM [Weinberg, 1976], see review: [Igor P. Ivanov, 2017].

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General CP (GCP) transformation[G. Ecker et al, 1987; Grimus, Rebelo, 1997]:

$$\phi_i \xrightarrow{GCP} X_{ij}\, \phi_j^*$$

Apply CP transformation twice:

$$\phi_i \xrightarrow{GCP} X_{ij}\, \phi_j^* \xrightarrow{GCP} (X X^*)_{ij}\, \phi_j$$

- Usual CP: $X = \mathbf{1}$.
- CP symmetry of order 2 (CP2): $X X^* = \mathbf{1}$.
- CP symmetry of order $2k$, if $(X X^*)^k = \mathbf{1}$.

CP4 3HDM

- Apply GCP symmetry to 2HDM [Ferreira et al, 2010] and 3HDM [Bree et al, 2024].
- 3HDM with CP symmetry of order 4 (CP4) was proposed in [Ivanov, Silva, 2015];
- The phenomenology of CP4 3HDM was discussed in [Ferreira et al, 2018; Zhao et al, 2023].

$$X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & i \\ 0 & -i & 0 \end{pmatrix}, \quad XX^* = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad (XX^*)^2 = 1.$$

$$\phi_1 \to \phi_1^*, \quad \phi_2 \to i\phi_3^*, \quad \phi_3 \to -i\phi_2^*$$

Higgs potential of CP4 3HDM

The Higgs potential with CP4 symmetry [Ivanov, Silva, 2015]:

$$\begin{aligned} V_0 &= -m_{11}^2(1^\dagger 1) - m_{22}^2(2^\dagger 2 + 3^\dagger 3) + \lambda_1(1^\dagger 1)^2 + \lambda_2[(2^\dagger 2)^2 + (3^\dagger 3)^2] \\ &+ \lambda_{34}(1^\dagger 1)(2^\dagger 2 + 3^\dagger 3) + \lambda_{34}'(2^\dagger 2)(3^\dagger 3) - \lambda_4'[(2^\dagger 2)(3^\dagger 3) - (2^\dagger 3)(3^\dagger 2)] \\ &- \lambda_4[(1^\dagger 1)(2^\dagger 2) - (1^\dagger 2)(2^\dagger 1) + (1^\dagger 1)(3^\dagger 3) - (1^\dagger 3)(3^\dagger 1)] \\ V_1 &= \lambda_5(3^\dagger 1)(2^\dagger 1) + \lambda_8(2^\dagger 3)^2 + \lambda_9(2^\dagger 3)(2^\dagger 2 - 3^\dagger 3) + h.c. \end{aligned}$$

with complex λ_8, λ_9, here $1, 2, 3 \equiv \phi_1, \phi_2, \phi_3$.

Vacuum expectation value(vev): $\langle\phi_1\rangle = v_1/\sqrt{2}$, $\langle\phi_2\rangle = v_2/\sqrt{2}$, $\langle\phi_3\rangle = v_3/\sqrt{2}$.

$\tan\beta = \sqrt{v_2^2 + v_3^2}/v_1\,, \quad \tan\psi = v_3/v_2$

Misalignment SM-like Higgs boson

Expand three doublets:

$$\phi_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\, h_1^+ \\ v_1 + \rho_1 + i a_1 \end{pmatrix}, \phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\, h_2^+ \\ v_2 + \rho_2 + i a_2 \end{pmatrix}, \phi_3 = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\, h_3^+ \\ v_3 + \rho_3 + i a_3 \end{pmatrix}$$

Misalignment SM-like Higgs boson

In a Higgs basis: $\langle\Phi_1\rangle = v/\sqrt{2}$, $\langle\Phi_2\rangle = 0$, $\langle\Phi_3\rangle = 0$.

$$\Phi_1 = \frac{1}{\sqrt{2}}\begin{pmatrix}\sqrt{2}\,G^+ \\ v + h_1 + iG^0\end{pmatrix}, \Phi_2 = \frac{1}{\sqrt{2}}\begin{pmatrix}\sqrt{2}w_2^+ \\ h_2 + i\eta_2\end{pmatrix}, \Phi_3 = \frac{1}{\sqrt{2}}\begin{pmatrix}\sqrt{2}w_3^+ \\ h_3 + i\eta_3\end{pmatrix}$$

- Two charged Higgs boson with a 2×2 mass matrix.
- Five neutral Higgs boson with a 5×5 mass matrix.

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- Two charged Higgs boson with a 2×2 mass matrix.
- Five neutral Higgs boson with a 5×5 mass matrix.

- Alignment: $h_{SM} = h_1$.
- Misalignment: $h_{SM} = c_\epsilon \cdot h_1 + s_\epsilon c_\alpha c_{\gamma_1} \cdot h_2 + s_\epsilon c_\alpha s_{\gamma_1} \cdot h_3 + s_\epsilon s_\alpha c_{\gamma_2} \cdot \eta_3 + s_\epsilon s_\alpha s_{\gamma_2} \cdot \eta_2$

(Abbreviation: $s_\alpha \equiv \sin\alpha$, $c_\alpha \equiv \cos\alpha$.)

Here c_ϵ plays the same role as $\sin(\beta - \alpha)$ in 2HDM, shows $h_{SM}VV$ coupling.

The mass matrix of neutral Higgs boson

The parameters in the matrix contain $m_{11}^2, m_{22}^2, \lambda_i$.

In Higgs basis $(h_1, h_2, h_3, \eta_3, \eta_2)$, the most general scalar mass matrix:

$$\tilde{\mathcal{M}} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{12} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{13} & a_{23} & a_{33} & a_{34} & a_{35} \\ a_{14} & a_{24} & a_{34} & a_{44} & a_{45} \\ a_{15} & a_{25} & a_{35} & a_{45} & a_{55} \end{pmatrix}.$$

The mass matrix of neutral Higgs boson

The parameters in the matrix contain $m_{11}^2, m_{22}^2, \lambda_i$.

In Higgs basis $(h_1, h_2, h_3, \eta_3, \eta_2)$, **tridiagonal** scalar mass matrix in CP4 3HDM:

$$\tilde{\mathcal{M}} = \begin{pmatrix} a_{11} & a_{12} & 0 & 0 & 0 \\ a_{12} & a_{22} & a_{23} & 0 & 0 \\ 0 & a_{23} & a_{33} & a_{34} & 0 \\ 0 & 0 & a_{34} & a_{44} & a_{23} \\ 0 & 0 & 0 & a_{23} & a_{55} \end{pmatrix}.$$

(a_{ij} are function of m_{11}^2, m_{22}^2 and λ_i, they're related)

Inversion procedure

A usual procedure: random scan in $\lambda_i \longrightarrow$ Physical Higgs [Ferreira et al, 2018].

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- $h_{SM} = c_\epsilon \cdot h_1 + s_\epsilon c_\alpha c_{\gamma_1} \cdot h_2 + s_\epsilon c_\alpha s_{\gamma_1} \cdot h_3 + s_\epsilon s_\alpha c_{\gamma_2} \cdot \eta_3 + s_\epsilon s_\alpha s_{\gamma_2} \cdot \eta_2$
- With eigenvalue: $m^2_{SM} = (125\,\text{GeV})^2$.

$$\begin{pmatrix} a_{11} & a_{12} & 0 & 0 & 0 \\ a_{12} & a_{22} & a_{23} & 0 & 0 \\ 0 & a_{23} & a_{33} & a_{34} & 0 \\ 0 & 0 & a_{34} & a_{44} & a_{23} \\ 0 & 0 & 0 & a_{23} & a_{55} \end{pmatrix} \begin{pmatrix} c_\epsilon \\ s_\epsilon c_\alpha c_{\gamma_1} \\ s_\epsilon c_\alpha s_{\gamma_1} \\ s_\epsilon s_\alpha c_{\gamma_2} \\ s_\epsilon s_\alpha s_{\gamma_2} \end{pmatrix} = m^2_{\text{SM}} \begin{pmatrix} c_\epsilon \\ s_\epsilon c_\alpha c_{\gamma_1} \\ s_\epsilon c_\alpha s_{\gamma_1} \\ s_\epsilon s_\alpha c_{\gamma_2} \\ s_\epsilon s_\alpha s_{\gamma_2} \end{pmatrix}$$

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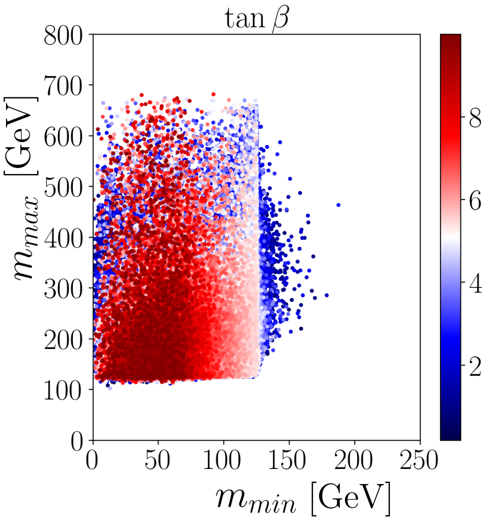
Use $m_{SM}^2, \epsilon, \alpha, \gamma_1, \gamma_2$ as input parameters.

Five coefficients in Higgs potential are not free anymore.

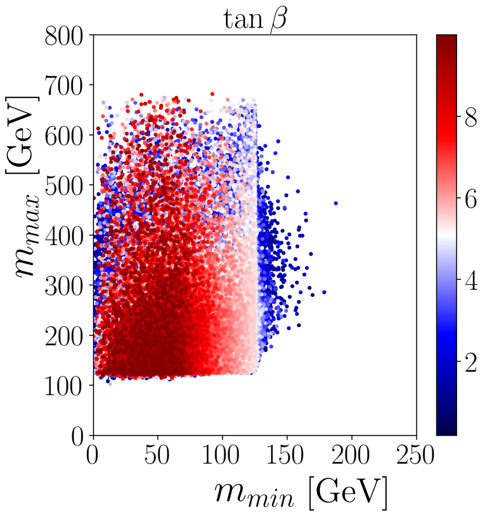
Angles $\epsilon, \alpha, \gamma_1, \gamma_2$ related to the Yukawa coupling of h_{SM} [Zhao et al, 2023].

The mass distribution of extra neutral Higgs (general scan)

- With constraints:
  - Bounded from below [Ferreira et al, 2018].
  - Unitarity and Perturbativity constraints [Bento et al, 2022].
  - STU constraints [Grimus et al, 2008].
- $m_{min}(m_{max})$ is the minimum(maximum) mass of the four extra Higgs boson.
- $\tan\beta = \sqrt{v_2^2 + v_3^2}/v_1$
- Plot show: No decoupling limit
- All points have $m_{min} < 200\,\text{GeV}$!!!

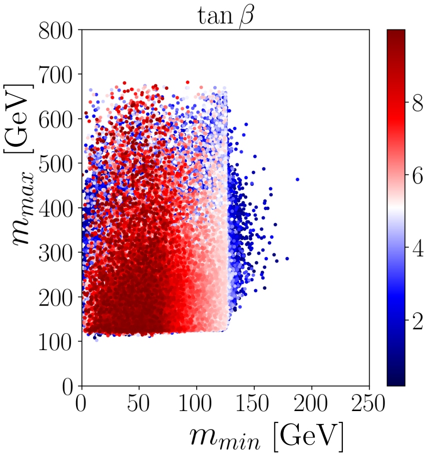


Few large m_{min} points

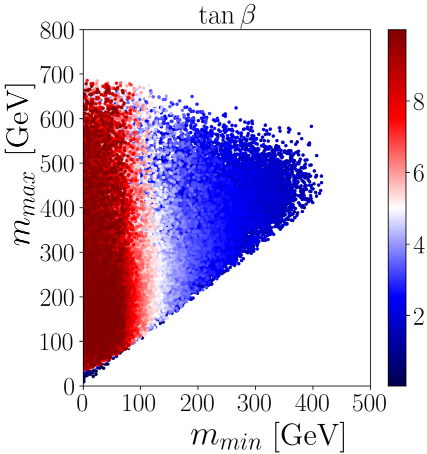


For $m_{min} < m_t$, the top quark $t \to H\,c$ [CMS, 2024] and $t \to H^{\pm}b$ [ATLAS, 2018] decay channels could easily exceed experimental constraints.

General Vs. Limit (Mismatch)



Misalignment



Alignment[Ferreira et al, 2018]

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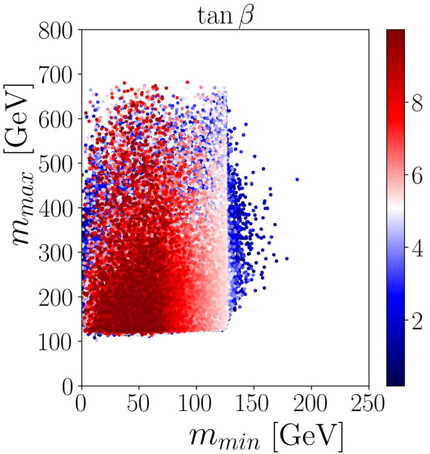
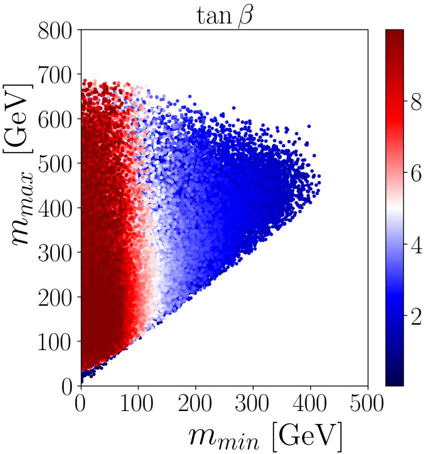


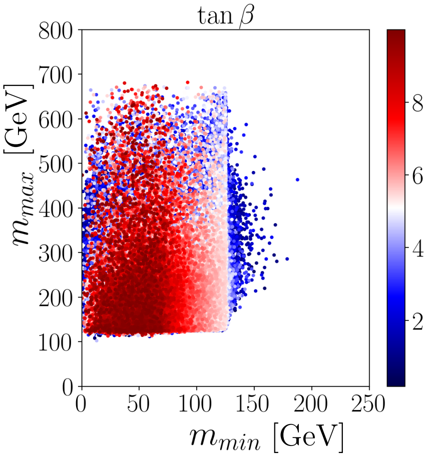
Misalignment

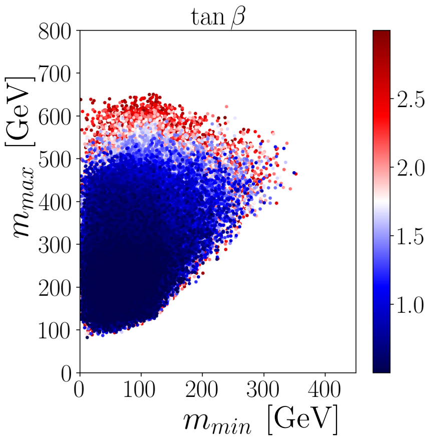


Alignment[Ferreira et al, 2018]

By analyzing the analytic form of mass matrix, we find that large m_{min} points ($m_{min} > 200\,\text{GeV}$) exist in a very narrow range in parameter space.

Focused scan in the high-mass region





$$\tan\beta \in [0.5, 2]\,, \quad \tan\psi \in [0.5, 3]\,, \quad |\tan\alpha| < 0.05\,, \quad |\tan\gamma_1|, \; |\tan\gamma_2| < 0.3$$

Outlook: Misalignment Higgs boson and FCNC

SM-like Higgs boson could induce FCNC in CP4 3HDM:

$$h_{SM} = c_\epsilon \cdot h_1 + s_\epsilon c_\alpha c_{\gamma_1} \cdot h_2 + s_\epsilon c_\alpha s_{\gamma_1} \cdot h_3 + s_\epsilon s_\alpha c_{\gamma_2} \cdot \eta_3 + s_\epsilon s_\alpha s_{\gamma_2} \cdot \eta_2$$

$$\mathcal{L} \supset \bar{d}_L \, \mathrm{N_d} \, d_R \, h_{SM} + h.c.$$

- Coupling matrix N_{d2} for h_2, η_2 [Zhao, 2023]:

$$N_{d2} = \begin{pmatrix} m_d \cot\beta & 0 & 0 \\ 0 & m_s \cot\beta & 0 \\ 0 & 0 & -m_b \tan\beta \end{pmatrix}.$$

- Coupling matrix N_{d3} for h_3, η_3:

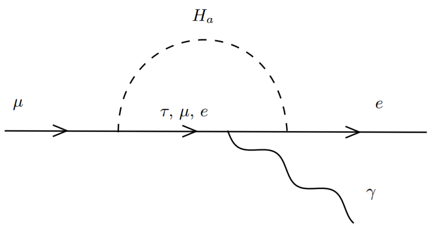
$$N_{d3} \propto \begin{pmatrix} -m_s c_{2\theta} & -m_s s_{2\theta} & 0 \\ -m_d s_{2\theta} & m_d c_{2\theta} & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

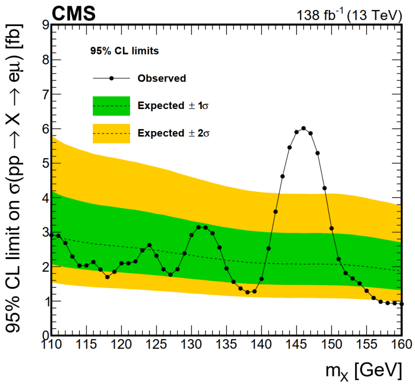
Control angles $\epsilon, \alpha, \gamma_1, \gamma_2 \implies$ control FCNC.

Outlook: LFV

Connect the scalar sector to the lepton sector:

- h_{SM} LFV decay.
- Explore $H \to \ell_i^+ \ell_j^-$ [CMS, 2023]:
  For instance: the coupling of $H \to e^+ e^-$ is proportional to m_μ.
- Explore $\mu \to e\,\gamma$:





Thanks!

Find high-mass points

Method: find a strategy to make all diagonal elements large enough at the same time.

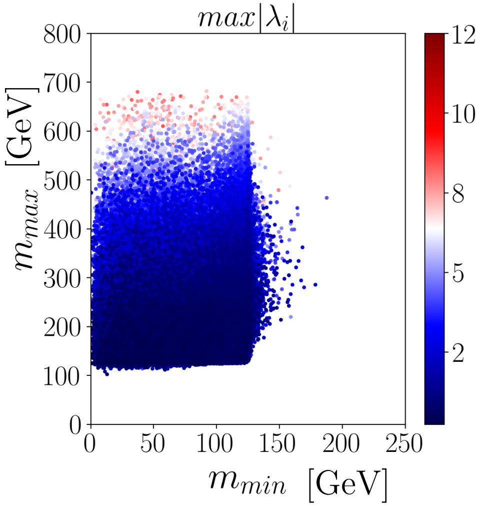
$$\begin{pmatrix} a_{11} & a_{12} & 0 & 0 & \cdots \\ a_{12} & a_{22} & a_{23} & 0 & \cdots \\ 0 & a_{23} & a_{33} & a_{34} & \cdots \\ 0 & 0 & a_{34} & a_{44} & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{pmatrix} = m_{\mathrm{SM}}^2 \cdot \mathbf{1} + a_{12} \begin{pmatrix} \tan x_2 & 1 & 0 & 0 & \cdots \\ 1 & \cot x_2 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 0 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{pmatrix}$$

$$+ \ a_{23} \begin{pmatrix} 0 & 0 & 0 & 0 & \cdots \\ 0 & \tan x_3 & 1 & 0 & \cdots \\ 0 & 1 & \cot x_3 & 0 & \cdots \\ 0 & 0 & 0 & 0 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{pmatrix} + \ a_{34} \begin{pmatrix} 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & \tan x_4 & 1 & \cdots \\ 0 & 0 & 1 & \cot x_4 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{pmatrix} + \ldots$$

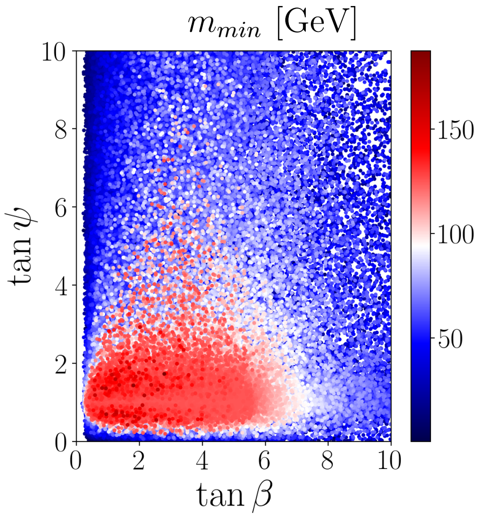
$$\tilde{\mathcal{M}} = m_{\rm SM}^2 \cdot \mathbf{1}_5 \;+\; \sin 2\beta\,(m_{11}^2 - m_{22}^2) \begin{pmatrix} c_\alpha c_{\gamma_1} \tan\epsilon & -1 & 0 & 0 & 0 \\ -1 & (c_\alpha c_{\gamma_1} \tan\epsilon)^{-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$- \frac{m_{\rm SM}^2\, c_\beta s_{\gamma_2}}{c_\beta c_{\gamma_2} + s_{\gamma_2} t_{2\psi}} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \tan\gamma_1 & -1 & 0 & 0 \\ 0 & -1 & \cot\gamma_1 & 0 & 0 \\ 0 & 0 & 0 & \tan\gamma_2 & -1 \\ 0 & 0 & 0 & -1 & \cot\gamma_2 \end{pmatrix}$$

$$+ \lambda_{89} v^2 s_\beta^2 \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & (c_{\gamma_2}/s_{\gamma_1}) \tan\alpha & -1 & 0 \\ 0 & 0 & -1 & (s_{\gamma_1}/c_{\gamma_2}) \cot\alpha & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$



$max|\lambda_i|$
m_{max} [GeV]
m_{min} [GeV]
0
100
200
300
400
500
600
700
800
0
50
100
150
200
250
2
5
8
10
12



m_{min} [GeV]
$\tan\psi$
$\tan\beta$
0
2
4
6
8
10
0
2
4
6
8
10
50
100
150