

Electroweak Baryogenesis via Domain Walls in the N2HDM

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Partially based on [2407.14468](#) and [JHEP 04 \(2024\) 101](#)

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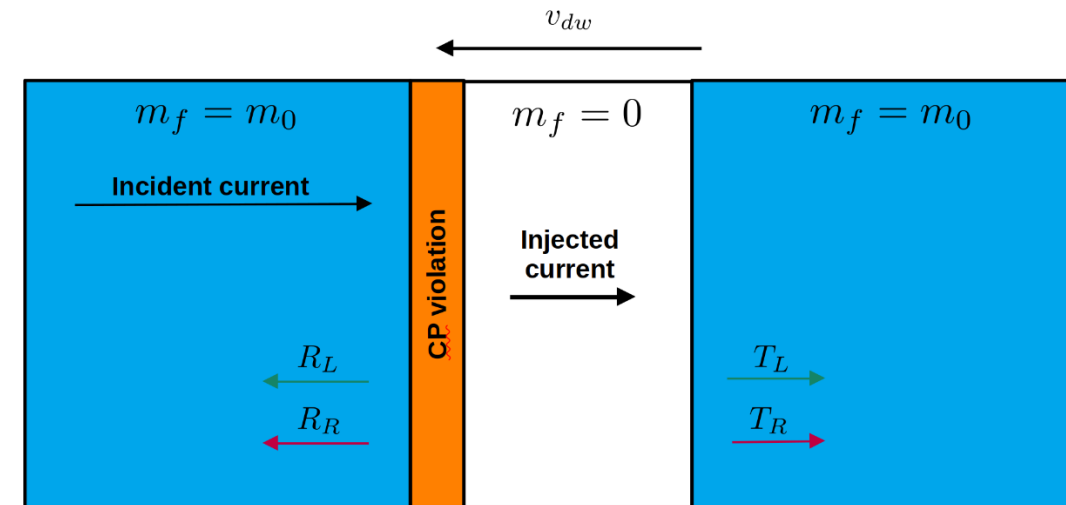
Motivation and main idea

Problem

- **Matter anti-matter asymmetry** cannot be solved using physics from the **standard model alone**.
- **Conventional electroweak baryogenesis relies on CP-violation** that is mainly constrained by EDM.

Proposed solution

- Several **BSM Higgs sectors** predict the formation of **topological defects** such as **domain walls** in the early universe (**without the need for a first order phase transition!**).
- The **scalar doublets** can have vanishing or very small VEVs **inside the domain wall**.
- **CP-violating vacuum condensates** generated **in the vicinity of the wall**.
- **Similar mechanism proposed in 2404.13035 (Brandenberger and Schröder)** in the context of **embedded domain walls**.



Introduction to Domain Walls

Simple definition

- **Domain walls** are a type of **topological defects** that arise after **spontaneous symmetry breaking** (SSB) of a **discrete symmetry** in the early universe.
- After SSB, **different regions** of the universe end up in different **degenerate vacua**. The universe is then divided into separate cells with the **boundary between** them called a **“domain wall”**.

Simplest example (real singlet scalar)

$$V(\phi) = \mu\phi^2 + \lambda\phi^4$$

$V(\Phi)$ is **invariant** under \mathbf{Z}_2 : $\phi \rightarrow -\phi$

- Universe gets **seperated into different cells** with **positive** and **negative minima** having the same probability to occur.

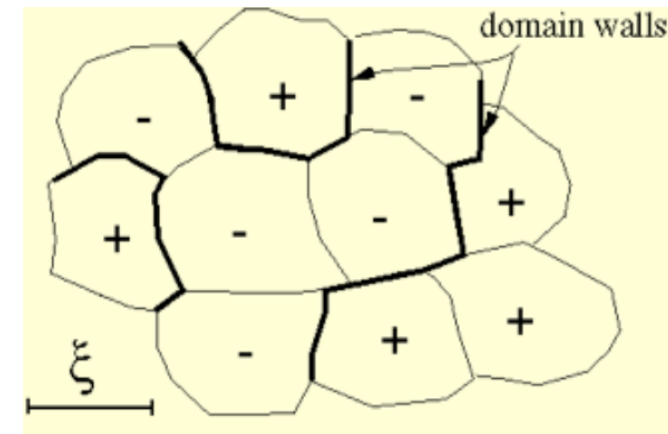


Fig from https://www.ctc.cam.ac.uk/outreach/origins/cosmic_structure_s_two.php

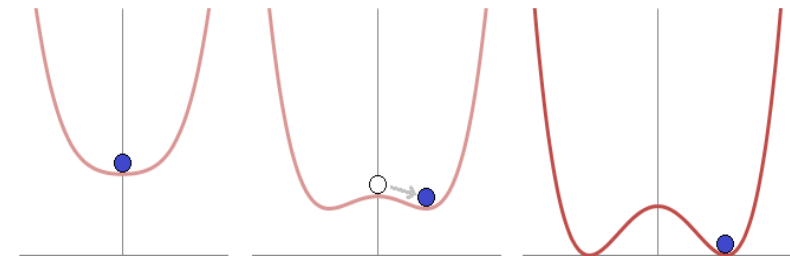


Fig from wikipedia

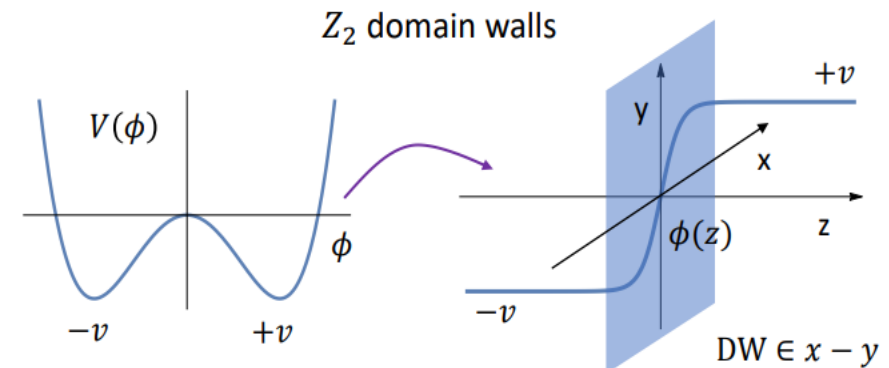


Fig from S. Blasi talk at DESY

The next-to-two-Higgs-doublet-model (N2HDM)

Add one extra doublet and one extra singlet to the Standard Model.

$$V_{N2HDM} = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 + m_{12}^2 (\Phi_1^\dagger \Phi_2 + h.c) + \frac{\lambda_1}{2} (\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2} (\Phi_2^\dagger \Phi_2)^2$$
$$+ \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) + \left[\frac{\lambda_5}{2} (\Phi_1^\dagger \Phi_2)^2 + h.c \right] \quad \text{Two Higgs doublets}$$
$$+ \frac{m_S^2}{2} \Phi_s^2 + \frac{\lambda_6}{8} \Phi_s^4 + \frac{\lambda_7}{2} \Phi_s^2 (\Phi_1^\dagger \Phi_1) + \frac{\lambda_8}{2} \Phi_s^2 (\Phi_2^\dagger \Phi_2). \quad \text{Singlet scalar component}$$

The N2HDM admits several discrete symmetries

- **Z₂ Symmetry:** $\Phi_1 \rightarrow \Phi_1$, $\Phi_2 \rightarrow -\Phi_2$, $\Phi_s \rightarrow \Phi_s$ (**softly broken by m_{12} term**). Used to forbid **Flavor-Changing-Neutral-Currents** at **tree level** when extended to the quarks in the Yukawa sector.
- **Z'₂ Symmetry:** $\Phi_1 \rightarrow \Phi_1$, $\Phi_2 \rightarrow \Phi_2$, $\Phi_s \rightarrow -\Phi_s$. **Unbroken** in the **standard N2HDM**. Leads to the formation of stable domain walls that are **cosmologically forbidden**. Problem solved by adding small soft breaking terms:

$$a\Phi_s, b\Phi_s^3, c_1\Phi_s\Phi_1^2, c_2\Phi_s\Phi_2^2, c_3\Phi_s\Phi_1\Phi_2, \dots$$

- We assume those terms are **very small** making them irrelevant for the **DW profiles (only relevant for determining the annihilation time of the DW network)**

The next-to-two-Higgs-doublet-model (N2HDM)

Possible types of vacua in the N2HDM:

$$\langle \Phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle \Phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_+ \\ \pm v_2 e^{i\xi} \end{pmatrix}, \quad \langle \Phi_s \rangle = \pm v_s.$$

The N2HDM admits several types of vacua after SSB:

- **Electrically charged vacuum:** $v_+ \neq 0$. Breaks $U(1)_{\text{em}}$ and leads to photons being massive \rightarrow unphysical.
- **CP-Violating vacuum:** $\xi \neq 0$. CP-violation due to phase between the doublets \rightarrow constrained by EDM
- **Neutral vacuum:** $v_+ = 0$, $\xi = 0$. Same behavior as the SM Higgs vacuum \rightarrow **used throughout this work**
- It was shown that it is possible to have **CP-violating** or **electric charge breaking vacua** localized **inside domain walls** of the **2HDM** (see **(Battye, Pilaftsis, Viatic) [2006.13273] JHEP**, **(Pilaftsis, Law) [2110.12550] PRD** and **(MYS, Moortgat-Pick) [2309.12398] JHEP**).
- Similar behavior in the **N2HDM** \rightarrow Opportunity for **electroweak baryogenesis via domain walls**.

Domain Wall solutions in the N2HDM

We focus on domain walls related to the Z'_2 symmetry breaking:

To get the domain wall solution:

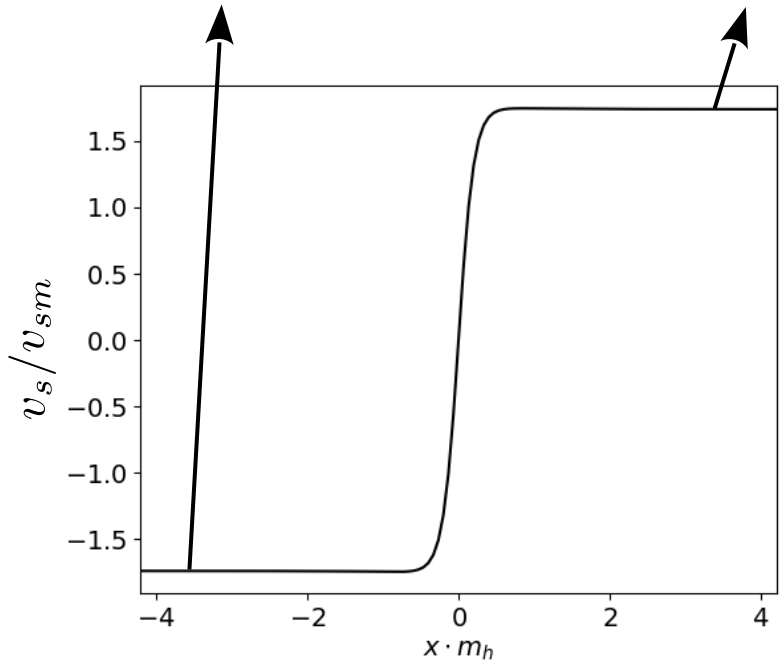
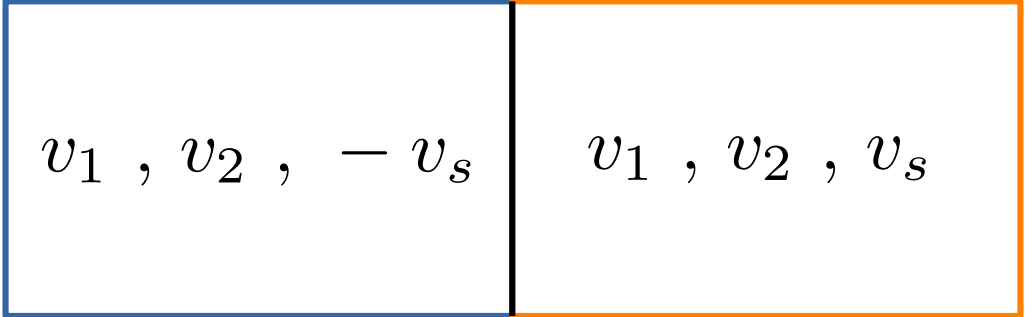
- Determine the boundary conditions
- Solve the equation of motion of the scalar fields:

$$\frac{d^2 v_s}{dx^2} - \frac{dV_{N2HDM}}{dv_s} = 0$$

$$\frac{d^2 v_1}{dx^2} - \frac{dV_{N2HDM}}{dv_1} = 0$$

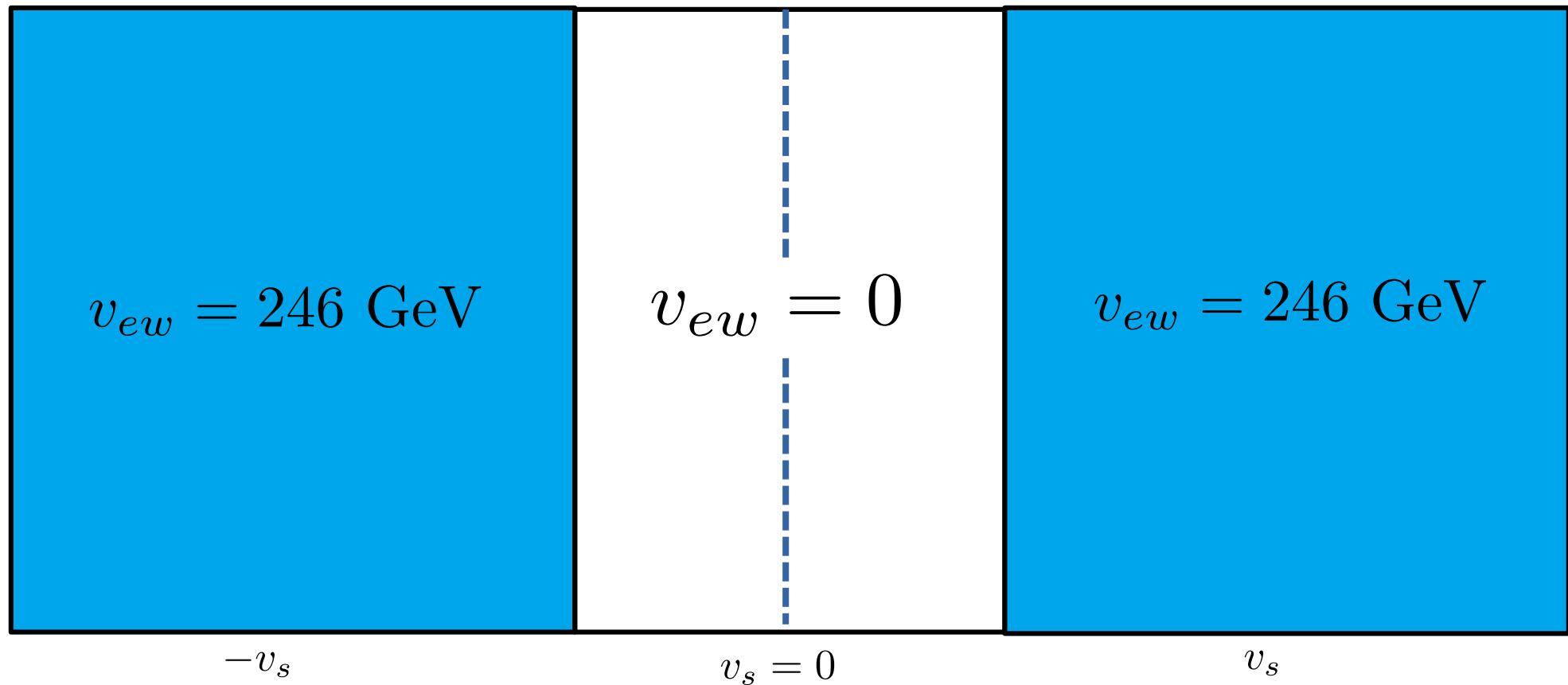
$$\frac{d^2 v_2}{dx^2} - \frac{dV_{N2HDM}}{dv_2} = 0$$

- This is done numerically using a gradient flow algorithm, see **Battye, Brawn, Pilaftsis 2011 (JHEP)**

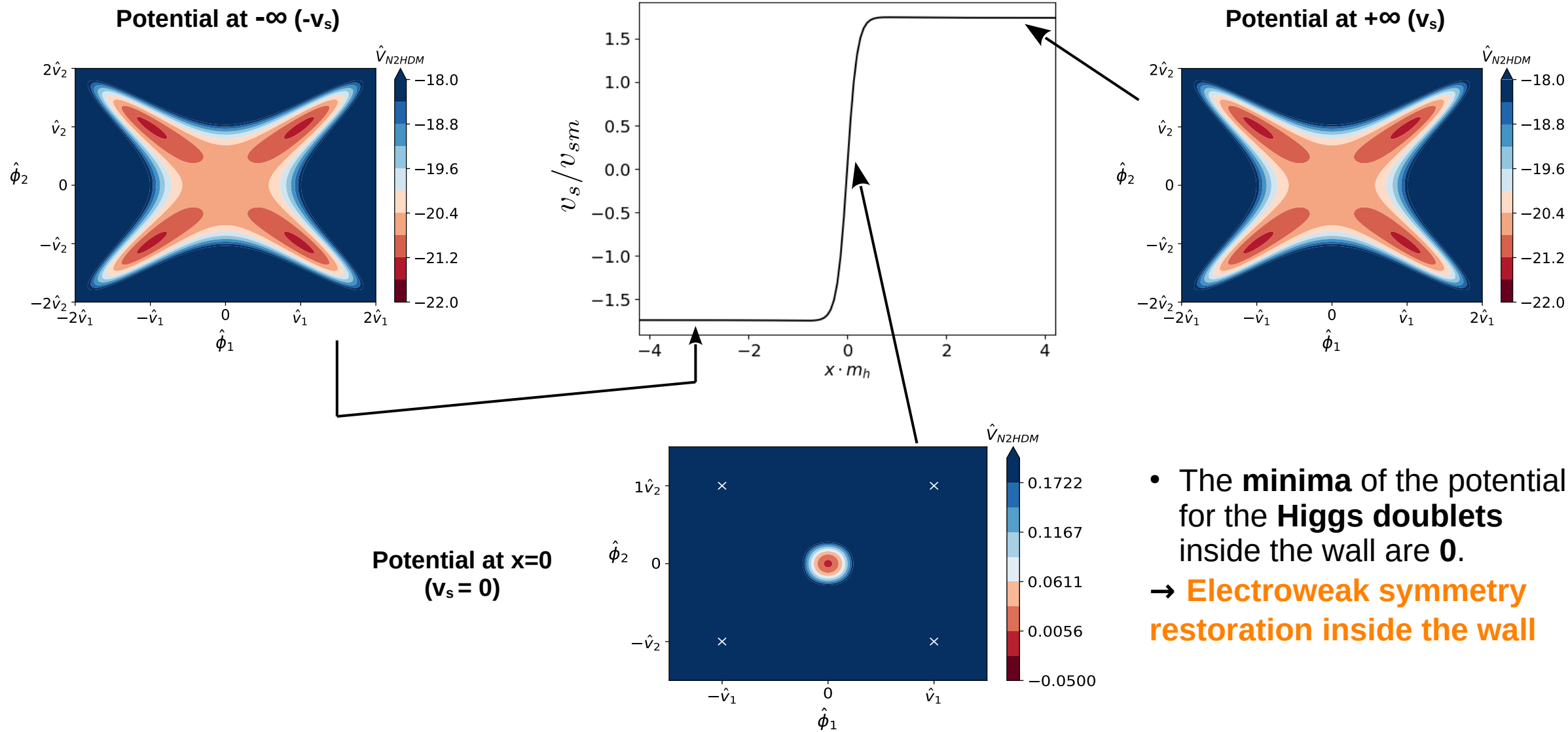


First ingredient for Baryogenesis via domain walls:

- The **electroweak symmetry** is **restored** inside and in the vicinity of the wall.
- Therefore **EW sphalerons** are **unsuppressed** inside the wall.



How to realise this ? Use the domain wall solution of the singlet in the N2HDM to make the 2HDM potential dependent on the space coordinate x :



- The **minima** of the potential for the **Higgs doublets** inside the wall are **0**.
- **Electroweak symmetry restoration inside the wall**

In the N2HDM the effective mass terms are:

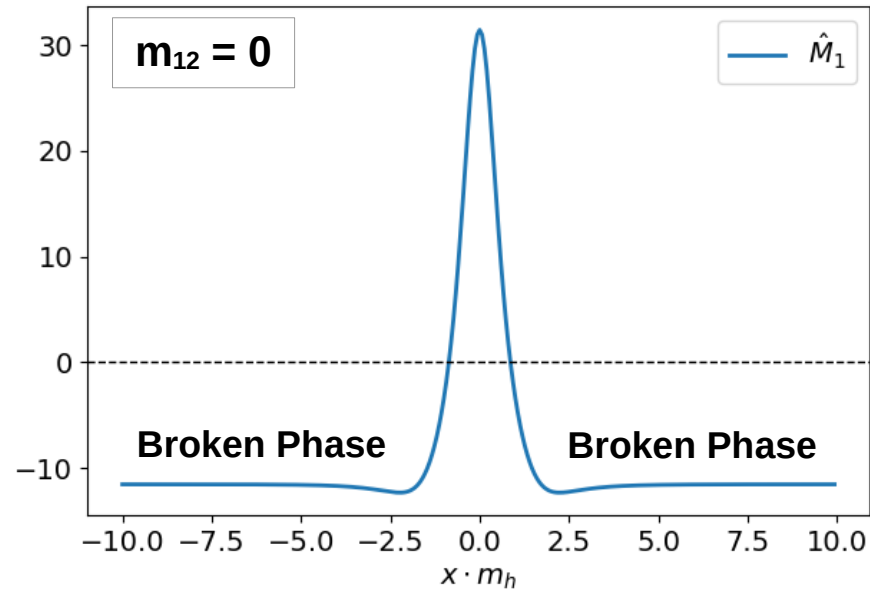
$$V_{N2HDM} = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 + m_{12}^2 (\Phi_1^\dagger \Phi_2 + h.c) + \frac{\lambda_1}{2} (\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2} (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) \\ + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) + \left[\frac{\lambda_5}{2} (\Phi_1^\dagger \Phi_2)^2 + h.c \right] + \frac{m_S^2}{2} \Phi_s^2 + \frac{\lambda_6}{8} \Phi_s^4 + \frac{\lambda_7}{2} \Phi_s^2 (\Phi_1^\dagger \Phi_1) + \frac{\lambda_8}{2} \Phi_s^2 (\Phi_2^\dagger \Phi_2).$$

- Extract the effective mass terms for the doublets:

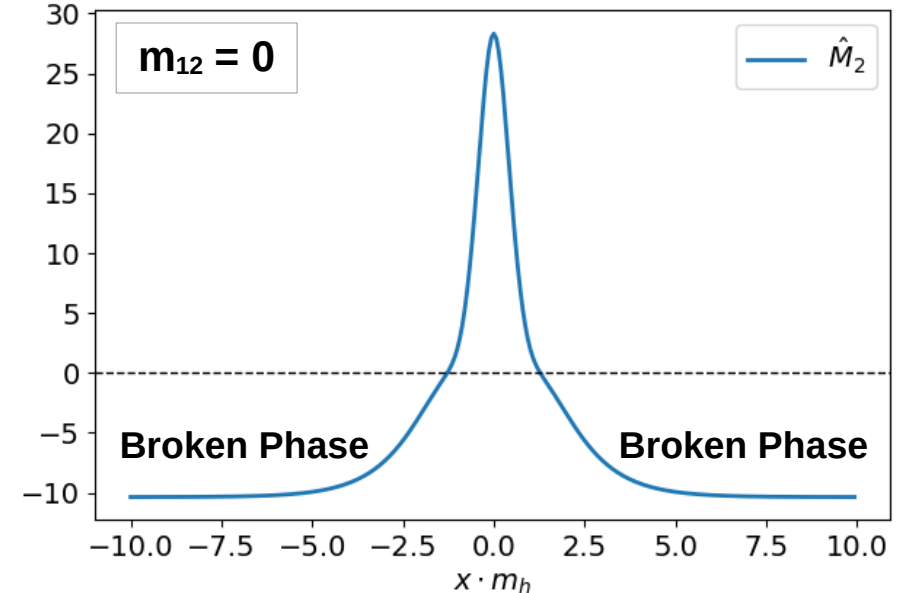
$$M_1 = \frac{m_{11}^2}{2} + \lambda_{345} v_2^2(x) + \frac{\lambda_7}{2} v_s^2(x)$$

$$M_2 = \frac{m_{22}^2}{2} + \lambda_{345} v_1^2(x) + \frac{\lambda_8}{2} v_s^2(x)$$

Symmetric Phase



Symmetric Phase



Verify the possibility of electroweak symmetry restoration by solving the EOMs of the scalar fields:

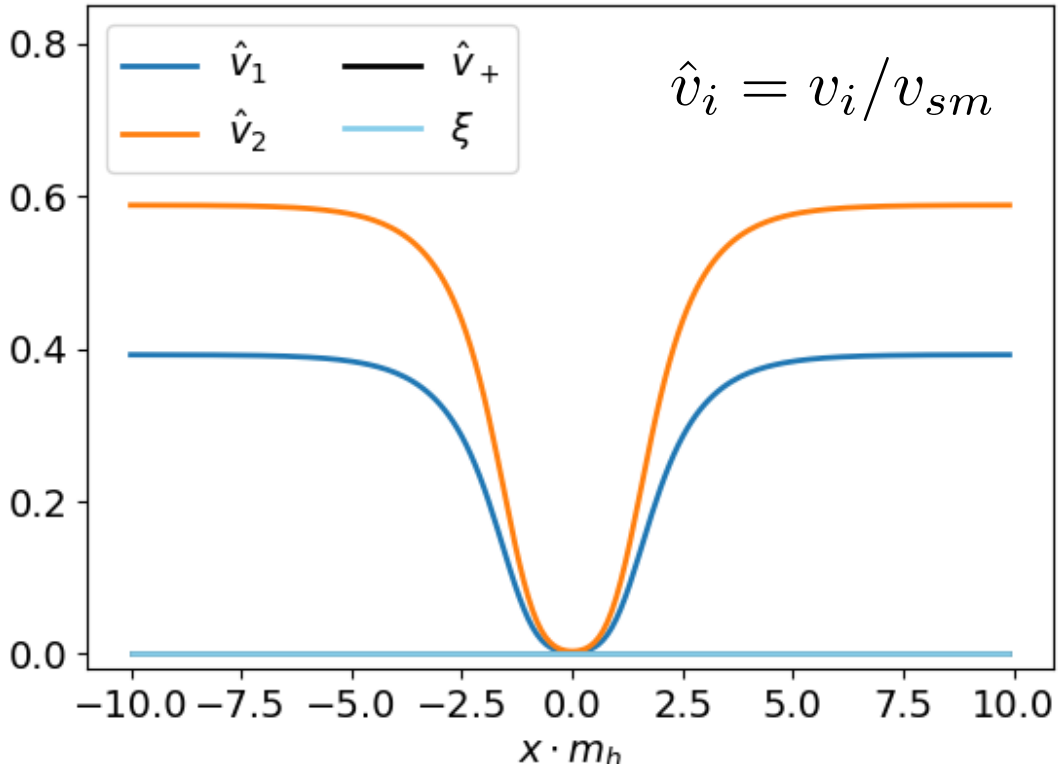
$$\frac{d^2 v_s}{dx^2} - \frac{dV_{N2HDM}}{dv_s} = 0 \quad \frac{d^2 v_2}{dx^2} - \frac{dV_{N2HDM}}{dv_2} = 0$$

$$\frac{d^2 v_1}{dx^2} - \frac{dV_{N2HDM}}{dv_1} = 0$$

• **Boundary conditions:**

$$\Phi_1(\pm\infty) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix} \quad \Phi_s(-\infty) = -v_s$$

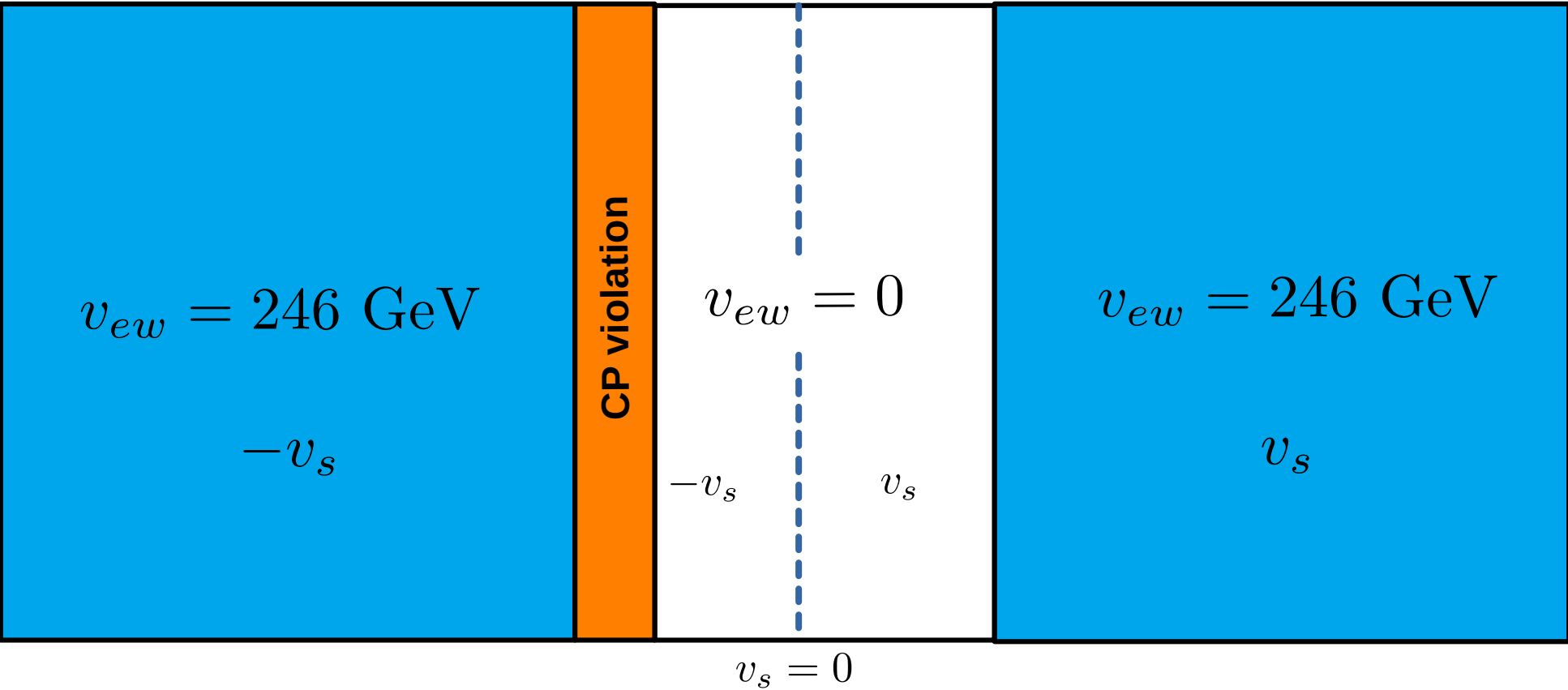
$$\Phi_2(\pm\infty) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix} \quad \Phi_s(+\infty) = v_s$$



- Indeed, the profiles of $v_1(x)$ and $v_2(x)$ **vanish inside** the singlet wall → **Electroweak symmetry restoration!**
- **Sphalerons are unsuppressed inside the wall.**

Second ingredient for Baryogenesis via domain walls:

- Need a **CP-violating phase** on the wall.
- **Left-handed particles** and **right-handed antiparticles** scatter off the wall with different rates.

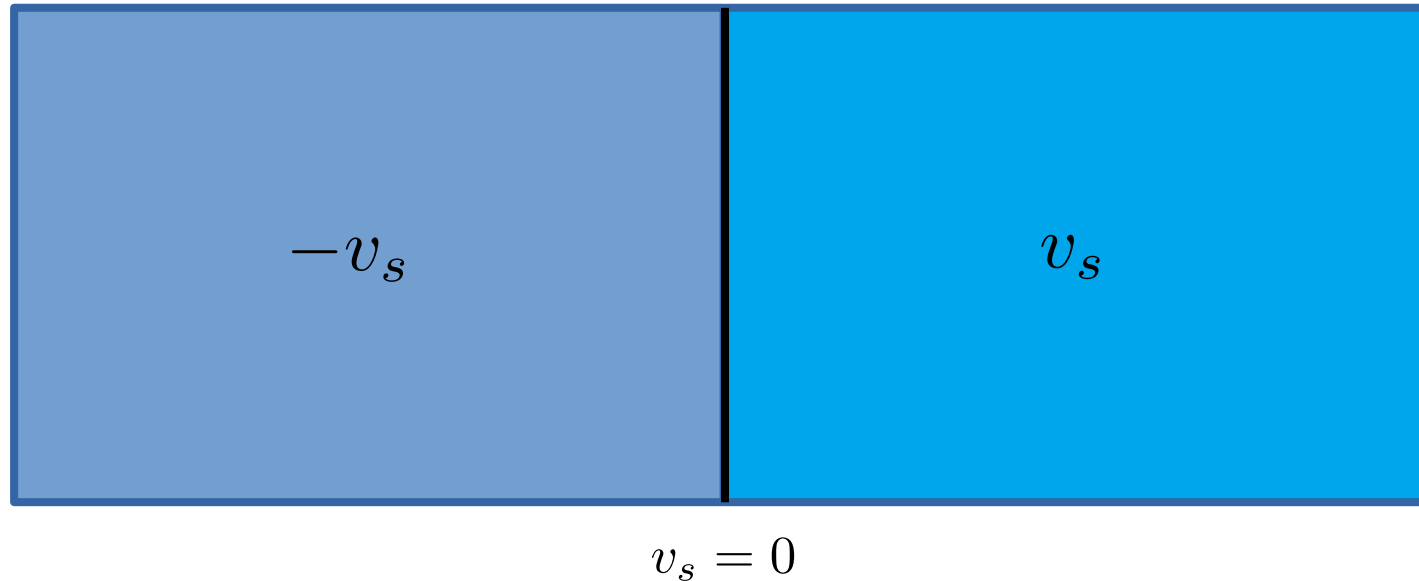


Different Goldstone modes on both domains

- θ and \mathbf{g}_i are the **Goldstone modes** related to $\mathbf{U}(1)_Y$ and $\mathbf{SU}(2)_L$
- In the early universe **different domains** can have **random values of the Goldstone modes**.

$$\langle \Phi_1 \rangle = U \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle \Phi_2 \rangle = U \frac{1}{\sqrt{2}} \begin{pmatrix} v_+ \\ \pm v_2 e^{i\xi} \end{pmatrix},$$
$$U = \exp(i\theta) \exp[(g_i \sigma_i)/(2v_{sm})].$$

Before EWSB

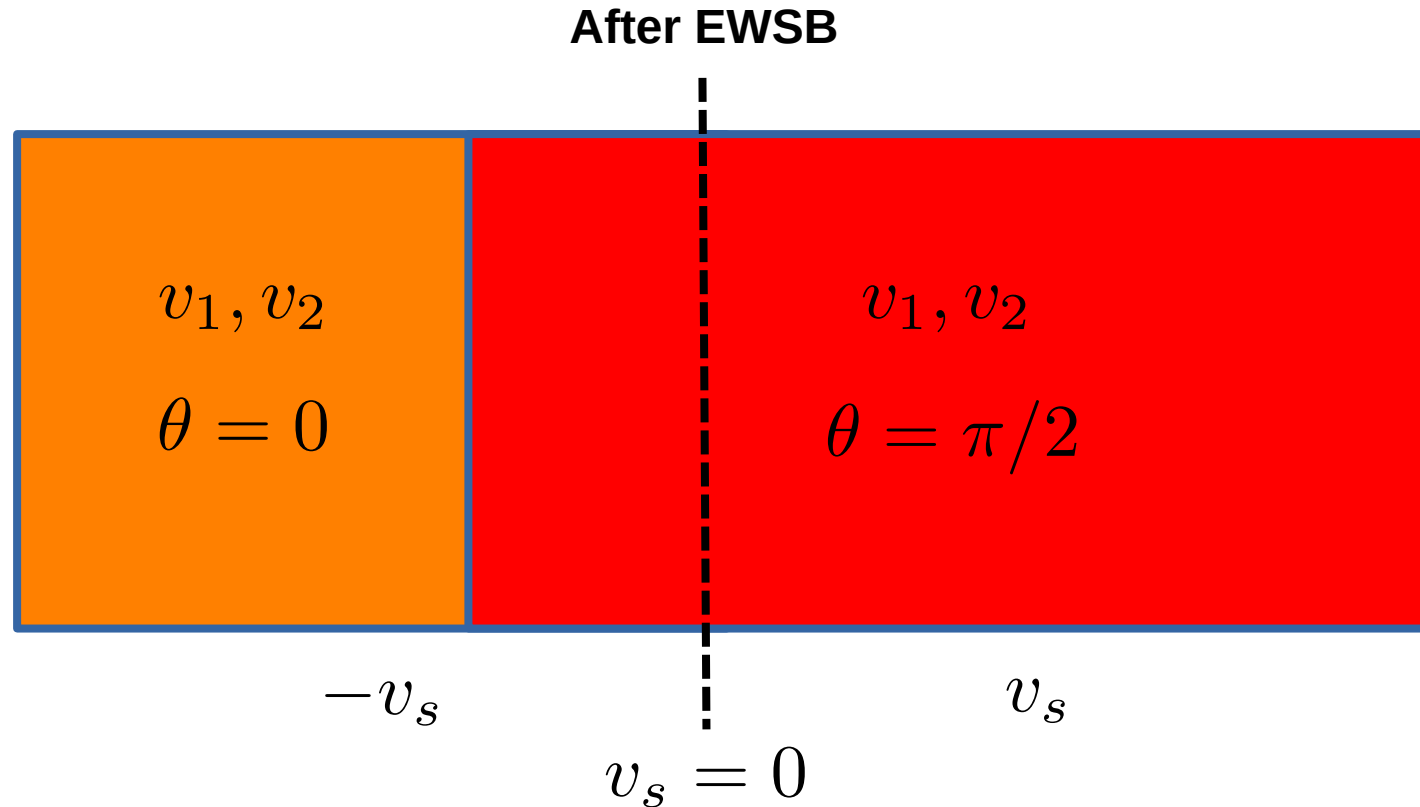


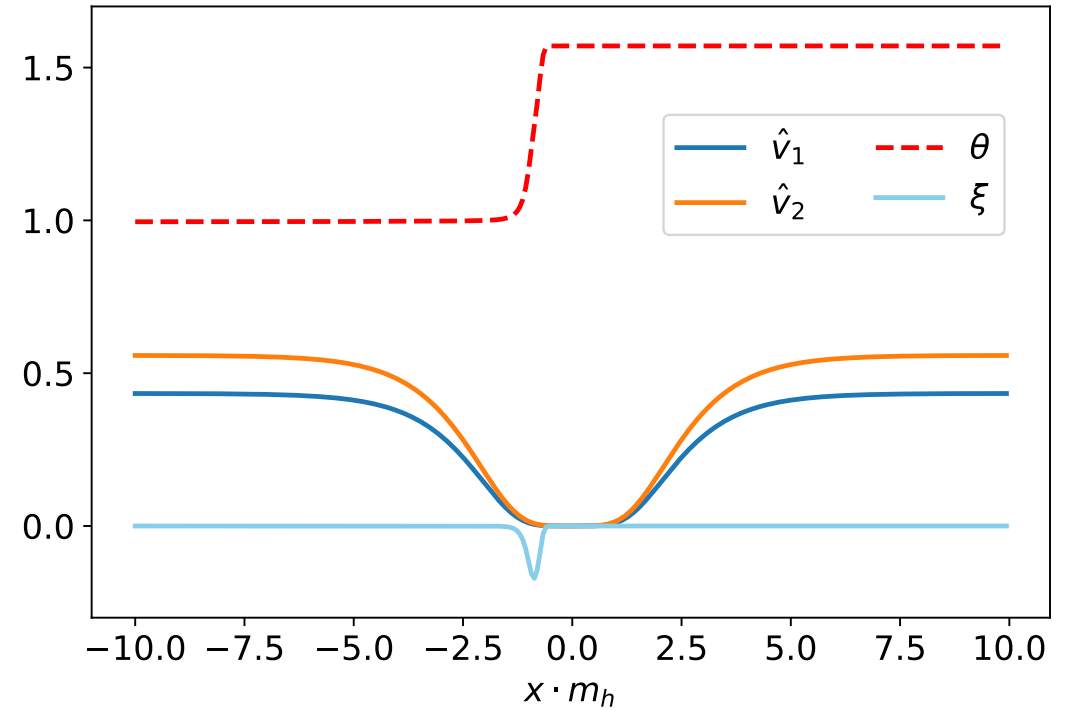
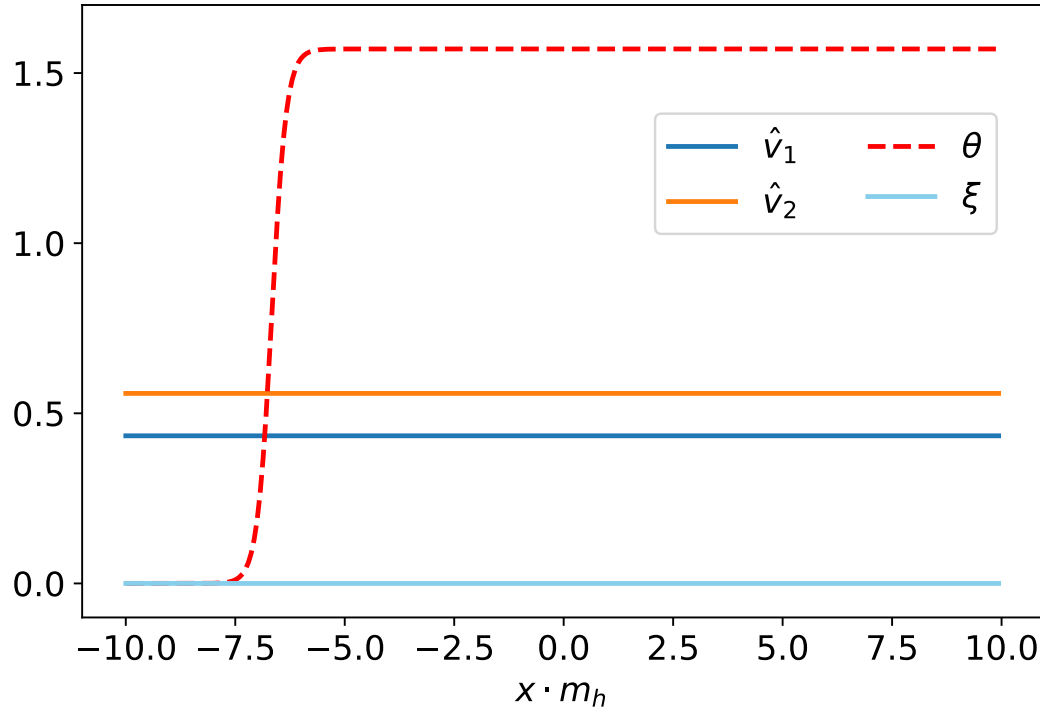
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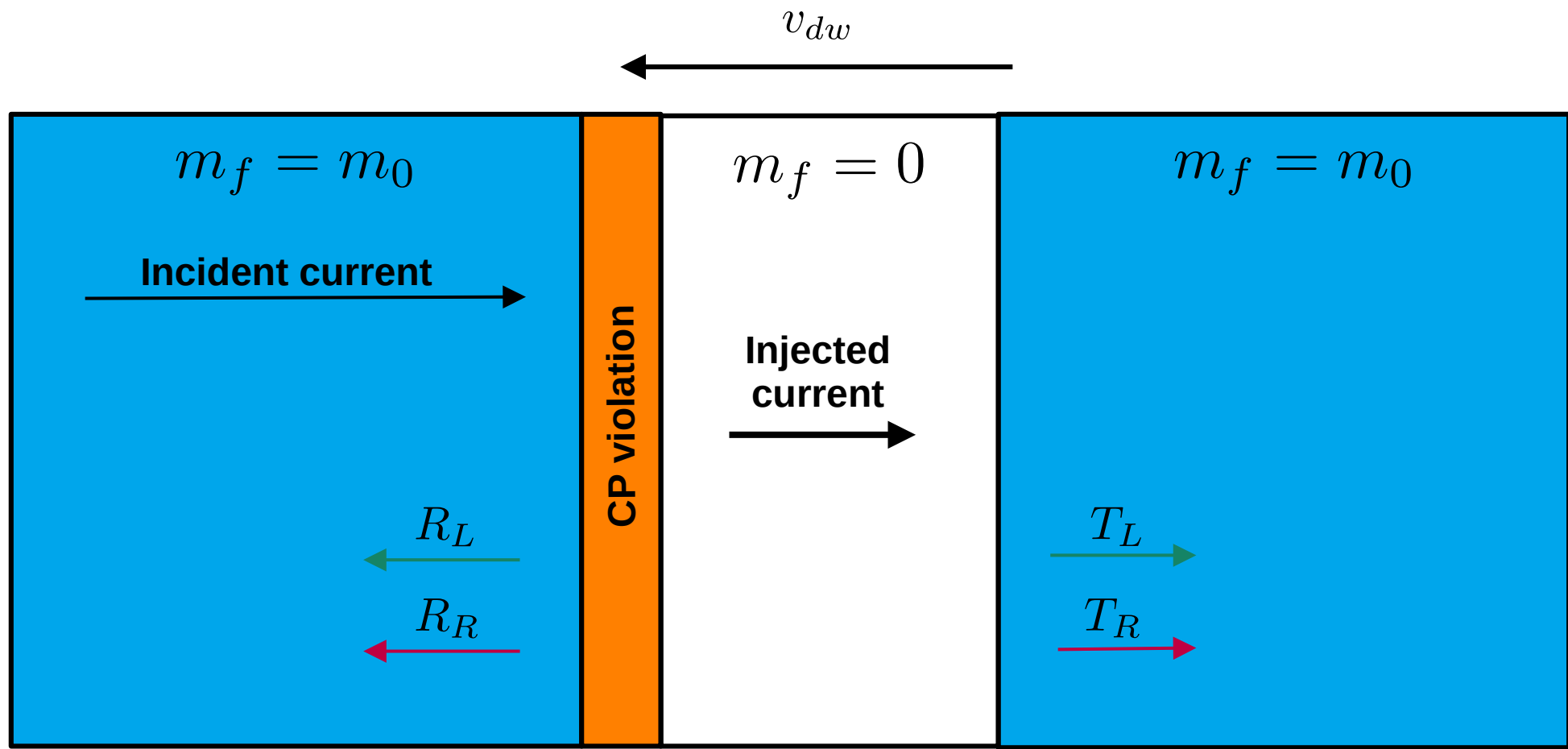


- Different Goldstone modes can induce **CP-violating and/or charge breaking vacua** located **inside** the wall.
- E.g. having **different θ** induces **CP-violating vacua localized in the vicinity of the wall.**
- For more details see **2110.12550 (Law, Pilaftsis)** and **2309.12398 (MYS, Moortgat-Pick).**

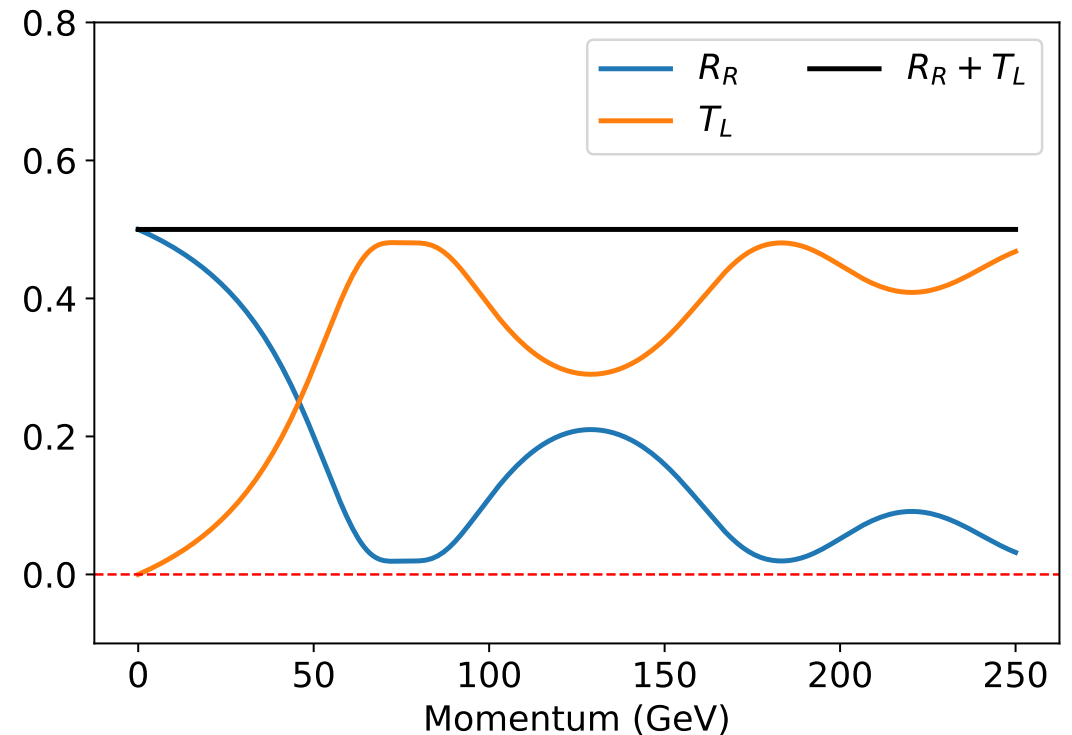
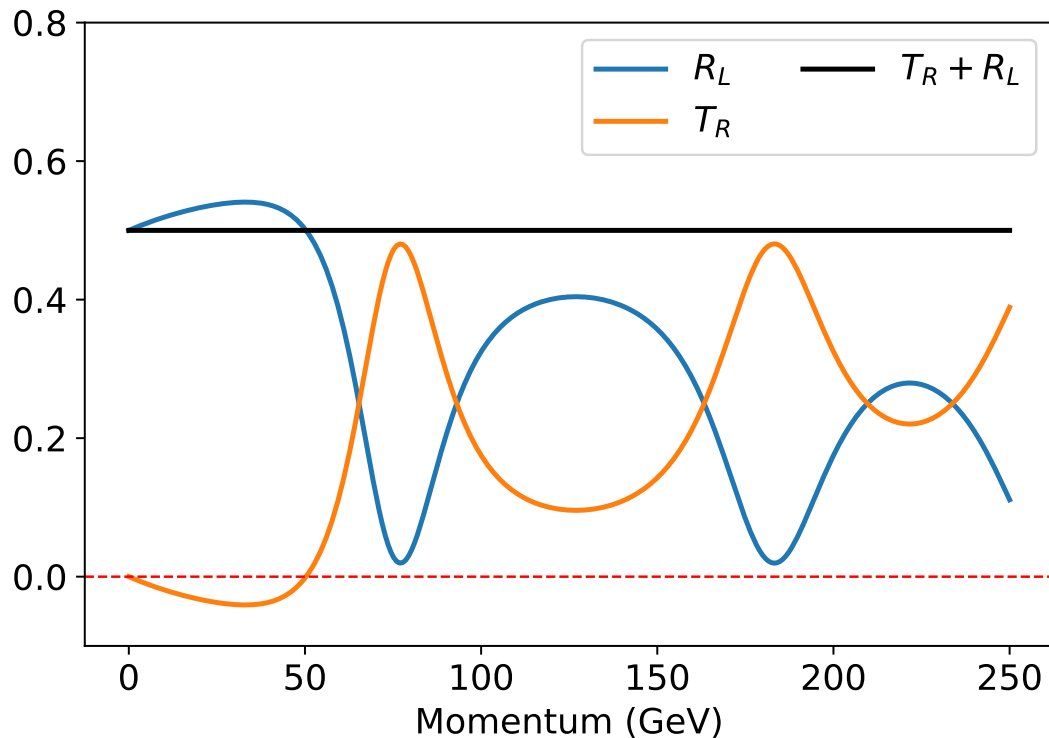
Fermion scattering off the wall

Solve the **Dirac equation** of a fermion in the **background of an x-dependent mass term**:

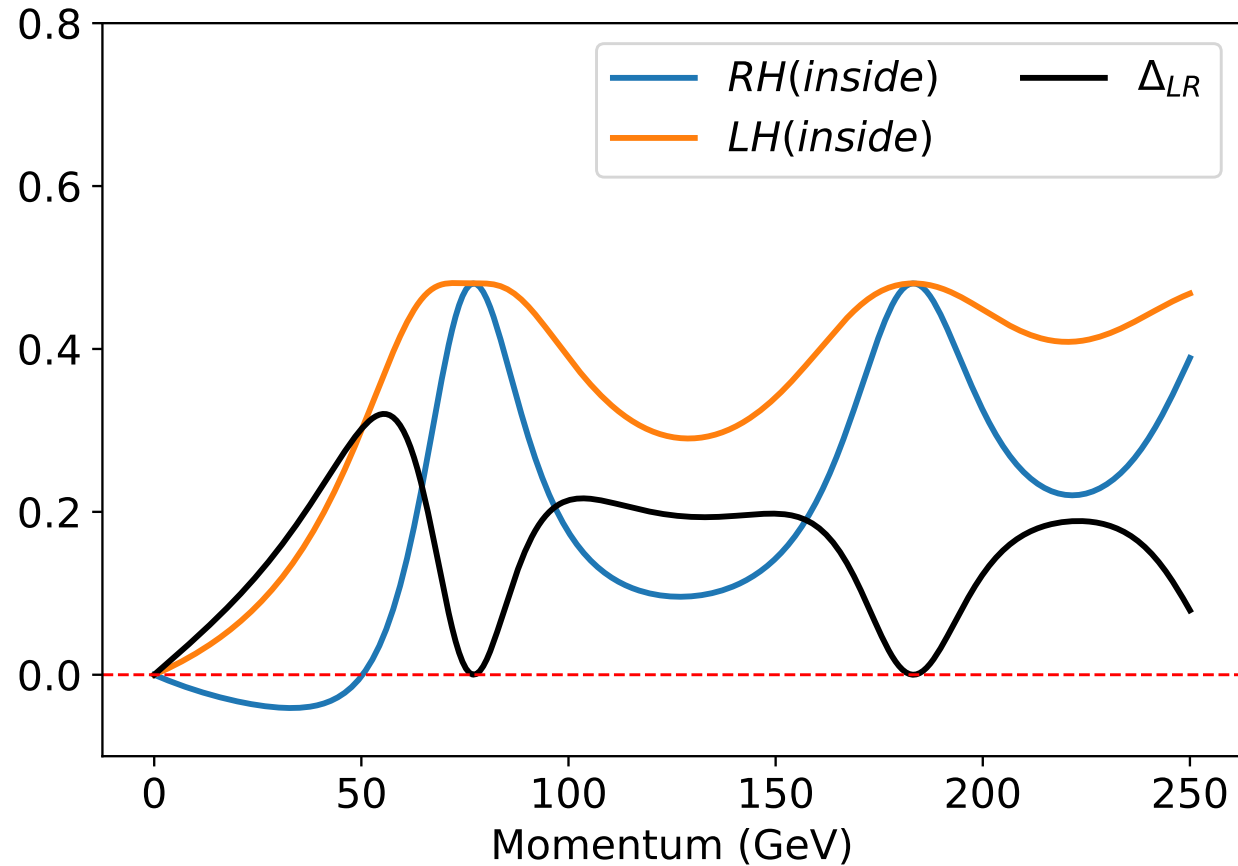
$$i\gamma^\mu D_\mu \psi + m(x)P_R\psi + m^*(x)P_L\psi = 0$$



Thin wall approximation (valid for small momenta). Results for top quarks.



- **CP violating mass term induces a difference in the reflection and transmission rates of left and right-handed particles.**
- **For small momenta, the transmission coefficients can become negative, while the reflection rate grows. Klein paradox for chiral particles ?**



- Difference between **left-handed particle** and **right-handed antiparticle** current inside the wall is non-zero due to **CP-violation on the outer wall**.

Excess flux of **left handed particles** over **right handed antiparticles** entering the wall:

$$J_L = \int \frac{d^3 p}{(2\pi)^3} f(p) \Delta_{LR}(p) \quad f(p) = \frac{|p_x|}{E} \frac{1}{1 + \exp\left(\frac{\gamma}{T} (E - v_d \sqrt{p_x^2 - m_t^2})\right)}$$

Ignoring diffusion effects (in the limit where the region of EWSR is big enough for a sphaleron), the baryon asymmetry normalized to entropy is:

$$\frac{n_b}{s} = \frac{6N_F}{T^3} \frac{\Gamma_s}{v_D} J_L L_{DW} \left(\frac{V_{BG}}{V} \right) \quad \Gamma_s = \kappa \alpha_W^4 T^4 \quad \text{Sphaleron rate inside the wall}$$

For top quarks with mass $m_t = 172$ GeV at $T=100$ GeV

$$v_D = 0.9$$

$$L_{DW} = 0.05 \text{ GeV}^{-1} \approx 5m_h$$

$$V_{BG}/V = 0.1$$

$$m_l = v_2 \xi = 0.15 \text{ GeV}$$

$$\Delta\theta = 0.1$$

$$\frac{n_b}{s} = 4.28 \times 10^{-11}$$

Experimental
observation
 $\approx 9 \times 10^{-11}$

Advantages of the mechanism

- Can **generate enough Baryogenesis** (at least in this simplified scenario).
- Does **not need a first order EW phase transition** (even a crossover would be fine).
- Can **evade EDM constraints** by using **CP-violating** effects generated by having **different Goldstone modes**.
- Works also in case of **EW symmetry non-restoration**.

Disadvantages of the mechanism

- Requires a **domain wall network** that needs to be **annihilated** (fine tuning of model parameters ?)
- The **amount of CP-violation** generated by having different Goldstone modes is **arbitrary** and **can have different signs for different regions**.
- Need for other more reliable ways to generate the CP-violation (e.g. a **CP-violating vacuum phase before the neutral one** ?)

Summary and conclusions

- 1) **Domain wall baryogenesis** ingredients can be achieved in the **N2HDM**.
- 2) Singlet Higgs field leads to **EW symmetry restoration** in the vicinity of the wall.
- 3) Regions with **different Goldstone modes** can generate **CP-violating VEVs in the vicinity of the wall** leading to a **chiral asymmetry for fermions inside the wall**.
- 4) **Preliminary results** following an approach similar to **2404.13035 (Brandenberger and Schröder)** show a baryogenesis comparable with the experimentally observed values.

Outlook

- A **precise calculation** of the baryogenesis rate taking into account **diffusion, finite temperature effects on the profile of the scalar fields** and a **bias term to annihilate the singlet domain walls**.

Thank you

Contact

Deutsches Elektronen-
Synchrotron DESY

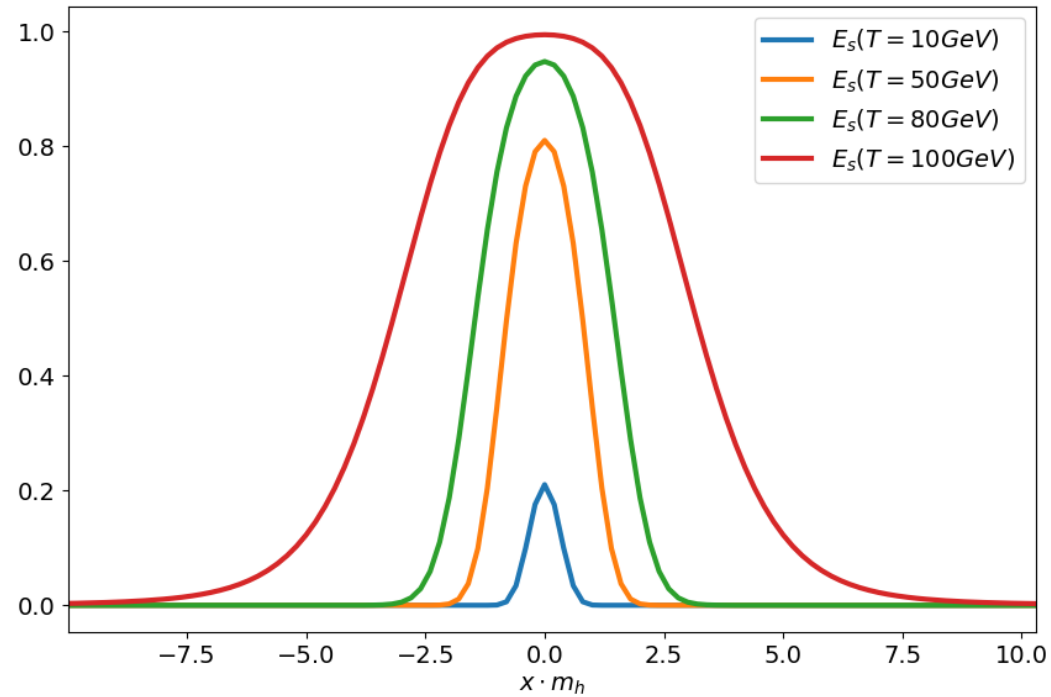
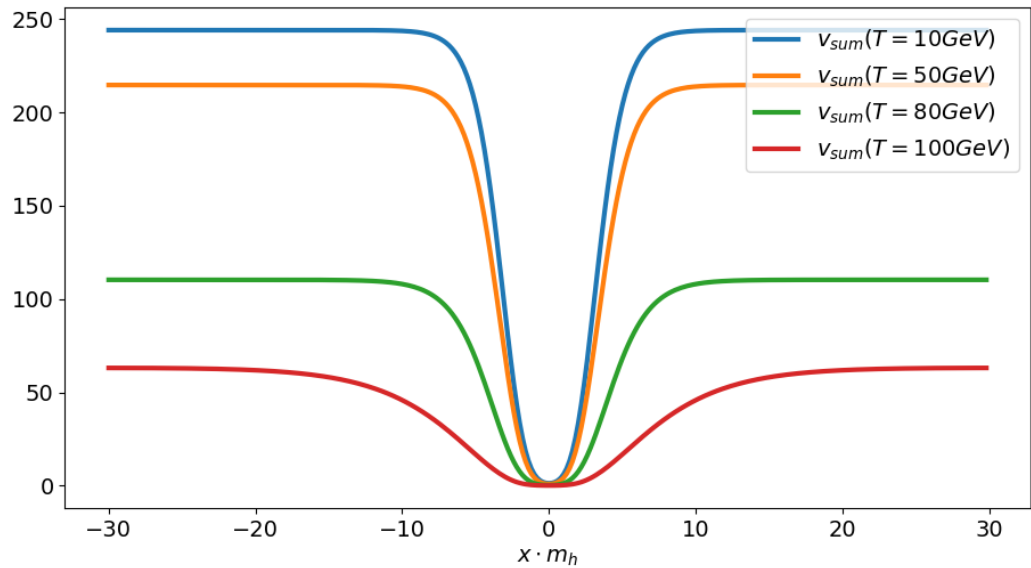
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Backup

Temperature effects



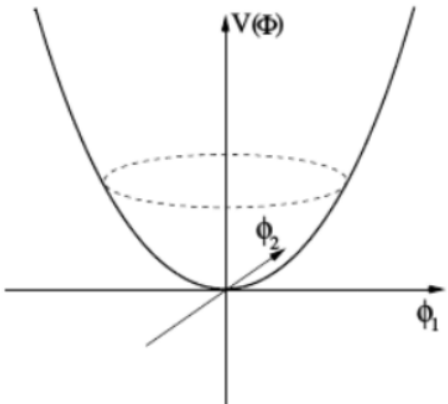
Sphaleron Suppression

Explanation

- For potentials of the form:

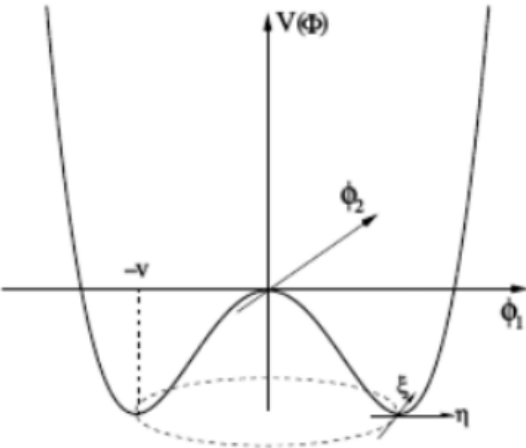
$$V = a_i \phi_i^2 + b_i \phi_i^4 + c_{ij} \phi_i \phi_j$$

- When **c_{ij}** terms vanish, the **phase** of the potential (**symmetric or broken**) is determined by the **sign** of the **mass term a_i** multiplying the **quadratic** field terms.
- For positive **a_i** the potential is in the **symmetric phase**.
- For negative **a_i** the potential is in the **broken phase**.



$a_i > 0$
 $b_i > 0$

Symmetric phase



$a_i < 0$
 $b_i > 0$

Broken phase

Conditions for electroweak symmetry restoration inside the wall

1. Need the **effective mass terms** to be **positive** inside the wall.

Define the change in the effective mass across the wall:

$$\Delta_1 = \lambda_{345}(v_2^2(0) - v_2^2(\pm\infty)) - \frac{\lambda_7}{2}v_s^2(\pm\infty) > 0$$

$$\Delta_2 = \lambda_{345}(v_1^2(0) - v_1^2(\pm\infty)) - \frac{\lambda_8}{2}v_s^2(\pm\infty) > 0$$

2. The **change in the effective mass** across the wall needs to happen in a **large enough space D** in order for the doublet fields to **converge** to a **very small value inside the wall**.

Relevant quantity influencing **D** is the **width of the singlet wall γ_s** :

$$\delta_s \propto (\sqrt{\lambda_6}v_s)^{-1}$$

- Neglecting contributions from terms proportional to λ_{345} , the dimensionless quantities $\mathbf{B}_{1,2} = \lambda_{7,8}/\lambda_6$ provide a good **parameter** for the **amount of symmetry restoration** inside the wall.

Verifying the different behaviors of the doublet fields inside the singlet wall

- Relevant **potential parameters** are: m_{11} , m_{22} , m_{12} , λ_{345} , λ_6 , λ_7 , λ_8 and v_s .
- Relevant **physical parameters** are then: m_{h1} , m_{h2} , m_{h3} , α_1 , α_2 , α_3 , v_s and m_{12} .

→ Perform a random parameter scan using **ScannerS** (20000 points) varying the **CP-even Higgs masses**, **mixing angles**, v_s and m_{12} .

- All points satisfy **theoretical constraints** of **boundedness from below**, **vacuum stability** and **perturbative unitarity**.
- All points satisfy the **experimental constraints** of **flavor physics**, **electroweak precision measurements S,T and U**.
- Also require **Z' symmetry restoration** in the early universe.
- The results are expressed in terms of:

$$r_{1,2} = \frac{v_{1,2}(0)}{v_{1,2}(\pm\infty)}$$

Ratio of the VEVs inside and outside the wall

Scan Parameters

$m_{h1} = 125.09 \text{ GeV}$
 $150 \text{ GeV} < m_{h2} < 400 \text{ GeV}$
 $500 \text{ GeV} < m_{h3} < 1100 \text{ GeV}$

$0.7 < \alpha_1 < 1.1$
 $-0.6 < \alpha_2 < -0.6$
 $0.5 < \alpha_3 < 1.57$

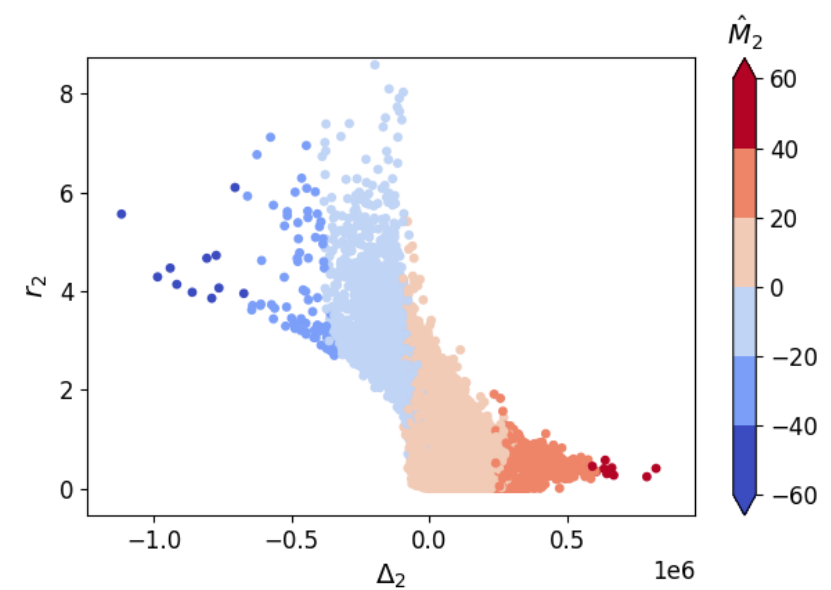
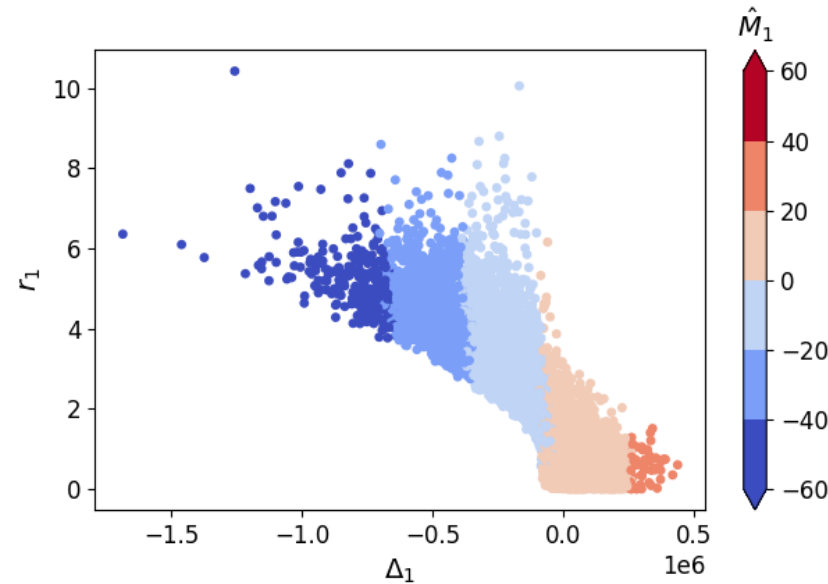
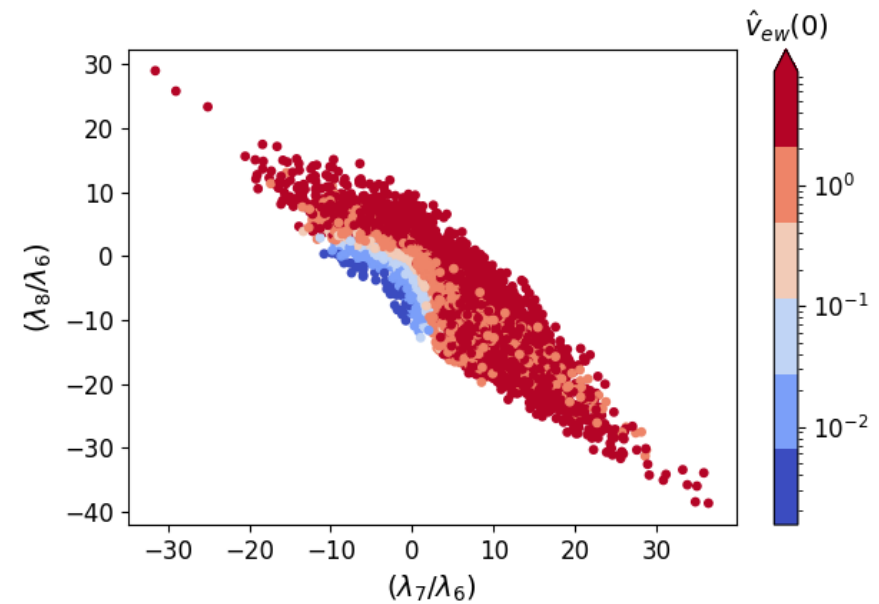
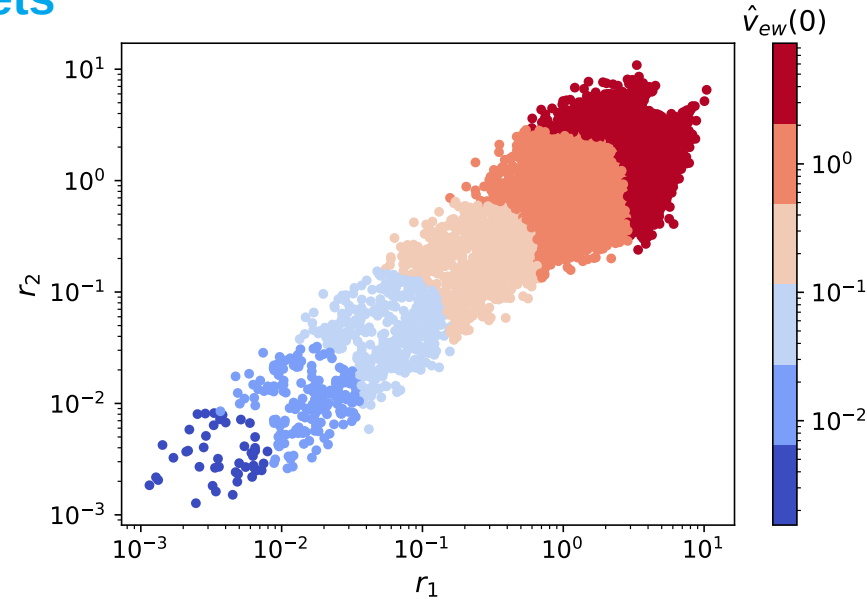
$200 \text{ GeV} < v_s < 3000 \text{ GeV}$
 $75000 \text{ GeV}^2 < m_{12} < 200000 \text{ GeV}^2$

$$\hat{v}_{ew}(0) = \frac{\sqrt{v_1^2(0) + v_2^2(0)}}{v_{sm}}$$

Measure of electroweak symmetry restoration

Different effects of the singlet domain wall on the VEVs of the Higgs doublets

- The results of the scan show that r_1 and r_2 can range from nearly **0.001** to **10**.
- Ratios **smaller than 1** possible mainly when λ_7 and λ_8 **negative**.
- **Negative $\Delta_{1,2}$** mainly lead to ratios **bigger** than 1.
- **Positive $\Delta_{1,2}$** mainly lead to ratios **smaller** than 1.
- Some **anomalous points** where the opposite behavior happens. Mainly due to $m_{12} \neq 0$.

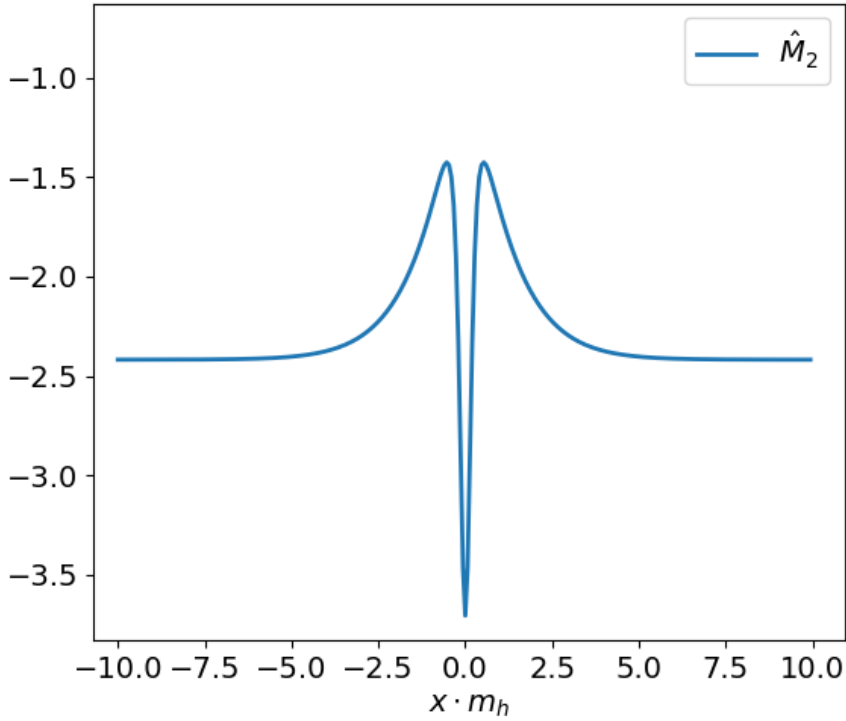
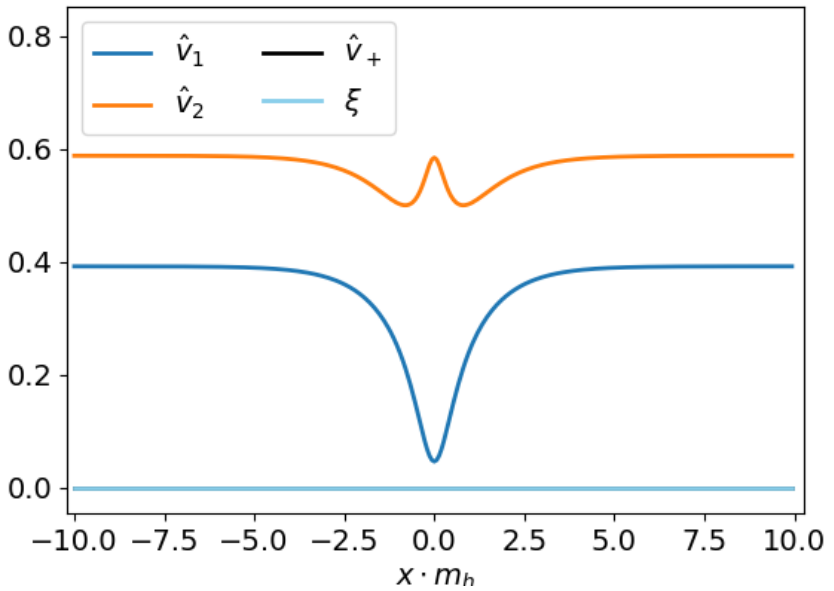


Some parameter points have $r_i < 1$ even for $\Delta_i < 0$ (and the opposite).

This is because the contribution of λ_{345} to the effective mass can be big for $x \approx 0$.

This behavior occurs for λ_8 positive and a thin domain wall, making the contribution from λ_8 to the effective mass localized at $x = 0$.

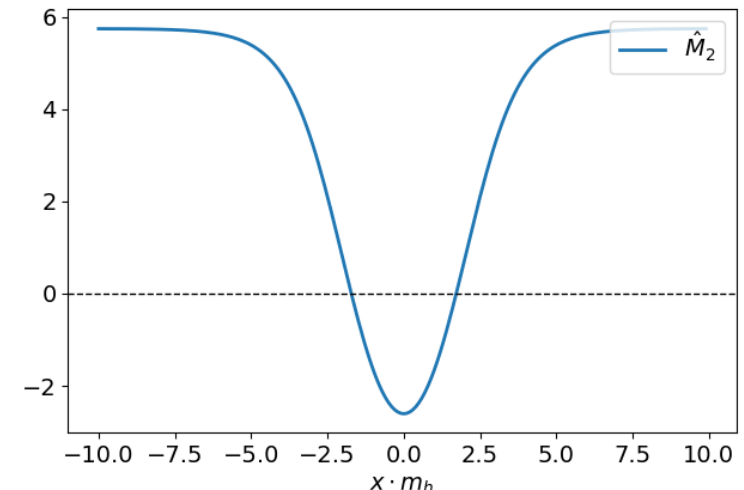
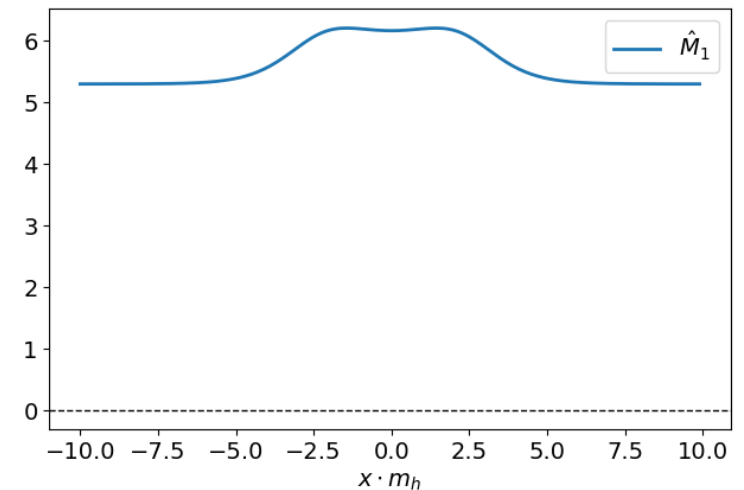
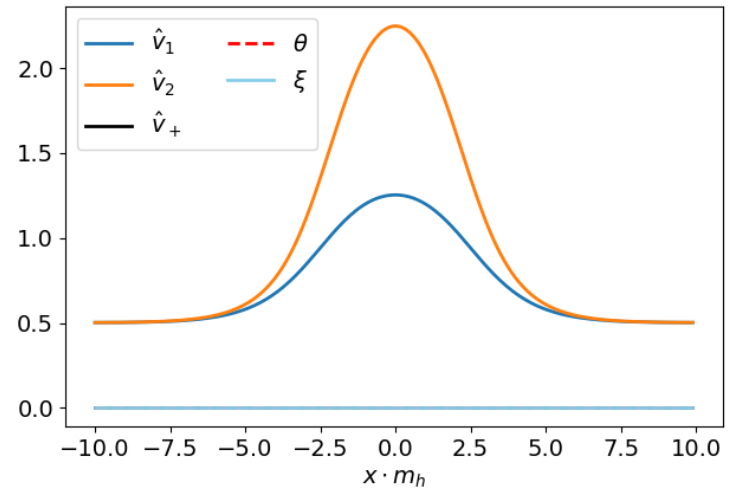
$$M_{eff,2} = \frac{m_{22}^2}{2} + \lambda_{345} v_1^2(x) + \frac{\lambda_8}{2} v_s^2(x)$$



$v_2(x=0)$ inside the wall is smaller than outside the wall. But Δ_2 is negative!

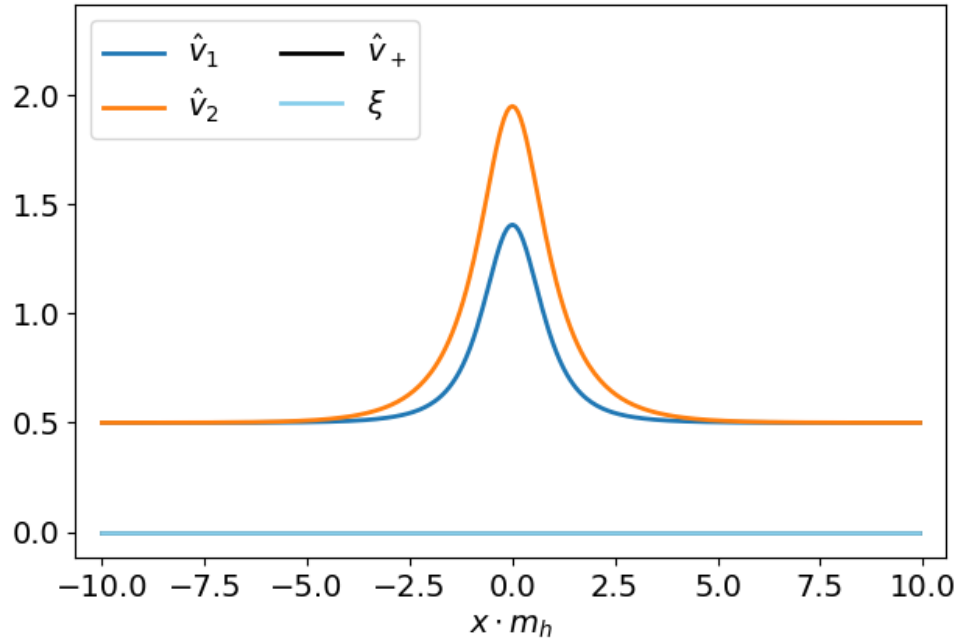
m_{12} anomalies

- Because $m_{12} \neq 0$, some parameter points will not have the minima of the 2HDM potential at $x=0$ ($v_s=0$) at the origin ($v_{1,2}=0$) even though the effective masses are **positive and higher inside the wall**.
- The minima of the Higgs doublets at $x=0$ will then converge to those **non-zero vevs**.



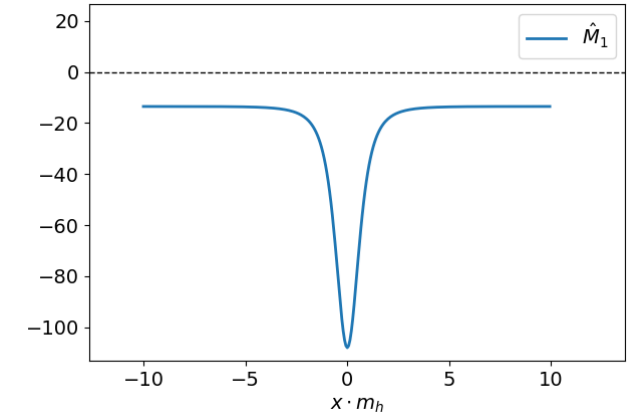
Same behavior for parameter points with $\Delta_2 > 0$ but $r_2 > 1$.

Also opposite behavior occurs: VEVs are bigger inside the wall:

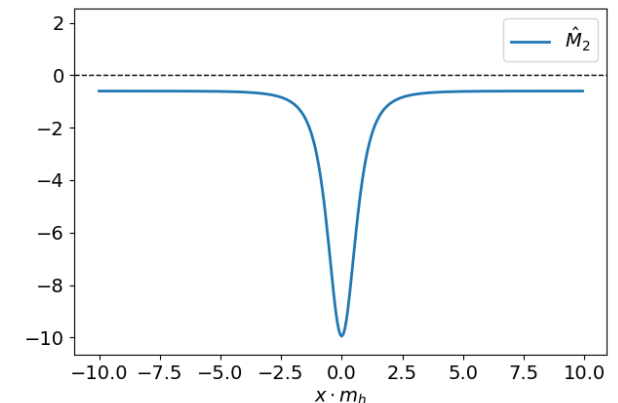


- This occurs when the **effective mass terms** become **more negative** inside the wall.
- Occurs in particular when λ_7 and λ_8 are **positive** (v_s vanishing inside the wall induces a **negative contribution**).
- Most particles get **reflected** off the wall.

$$M_1 = \frac{m_{11}^2}{2} + \lambda_{345} v_2^2(x) + \frac{\lambda_7}{2} v_s^2(x)$$

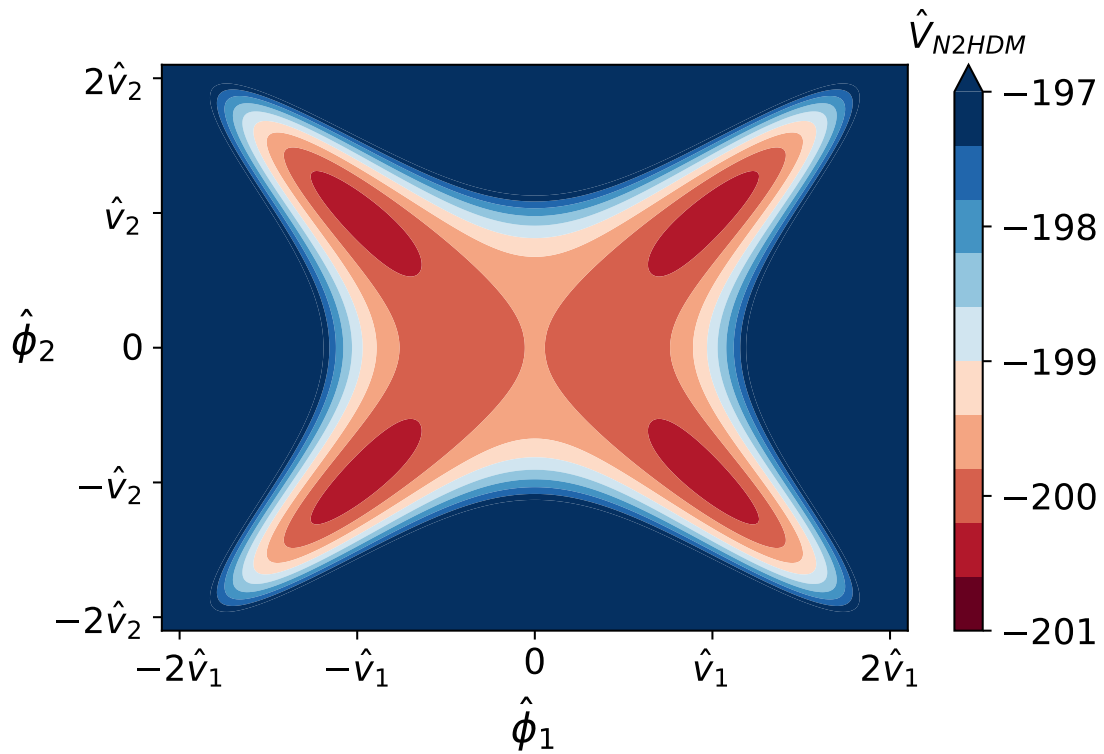


$$M_2 = \frac{m_{22}^2}{2} + \lambda_{345} v_1^2(x) + \frac{\lambda_8}{2} v_s^2(x)$$

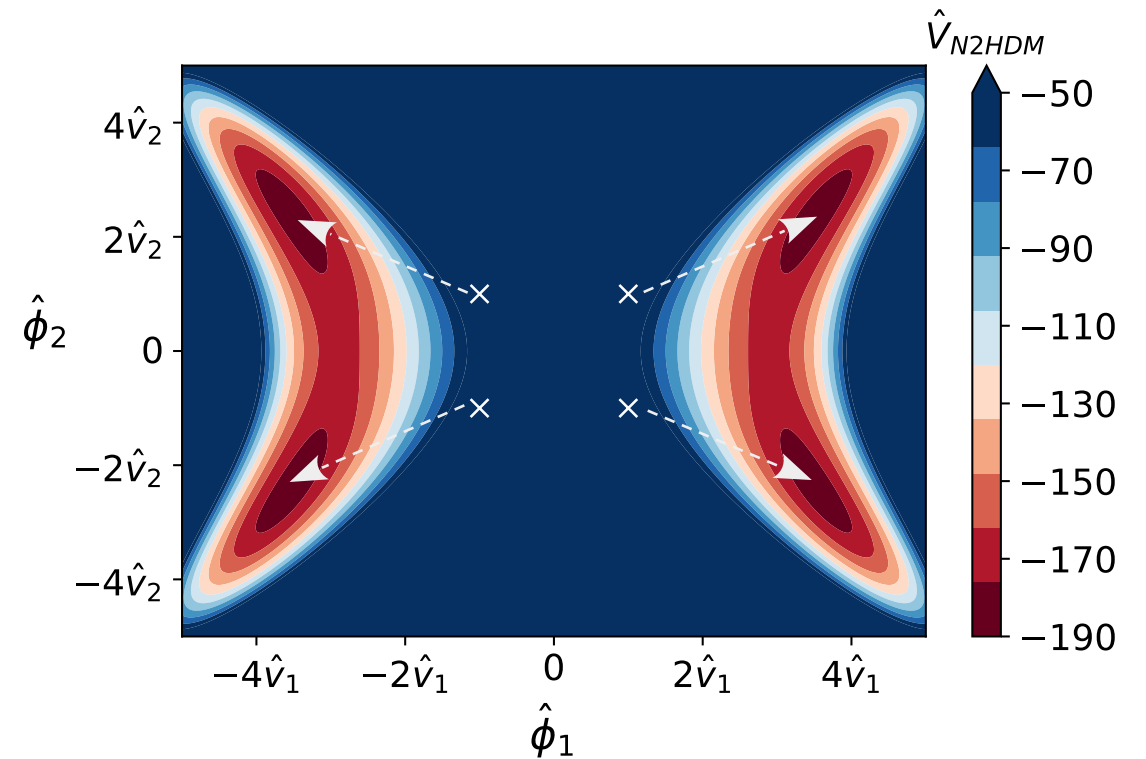


- The **effective mass terms get smaller inside the wall**, leading the **doublet minima** of the potential to “stretch”.

Outside the wall



Inside the wall

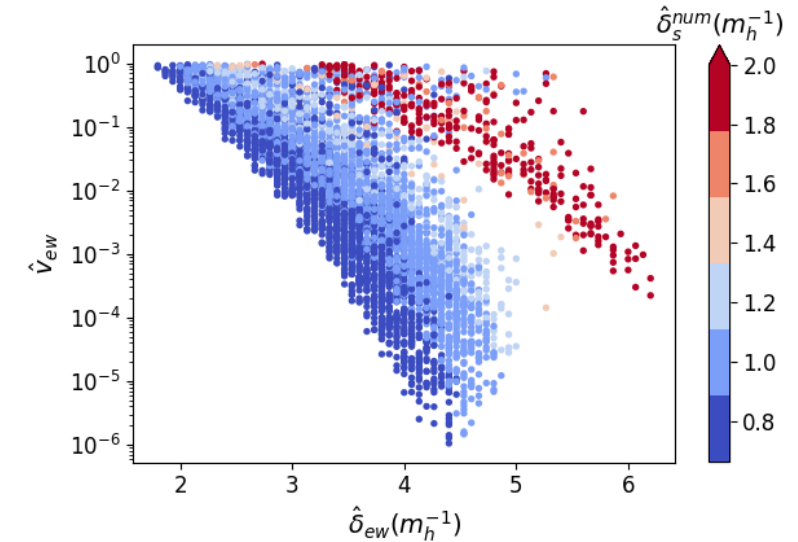
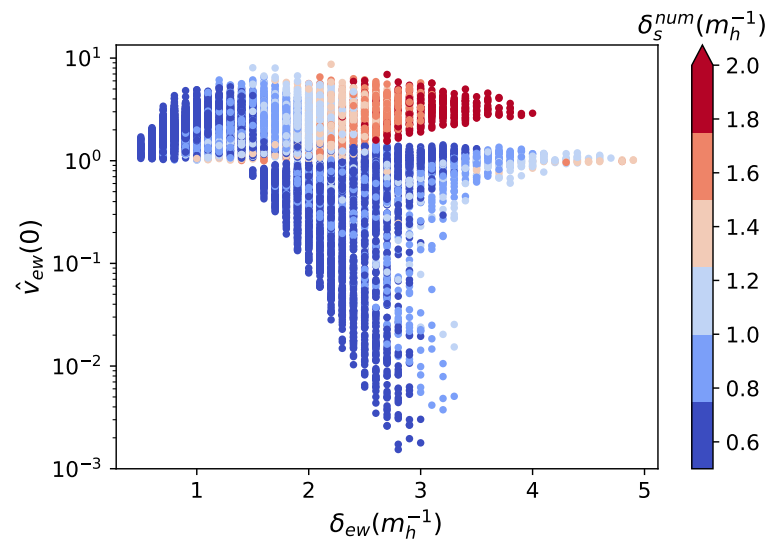


Width of the wall :

- For a model with only a real scalar singlet, the width of the wall is given by $\delta_s = \left(\frac{\sqrt{\lambda_6}}{2}v_s\right)^{-1}$
- In the case of the **N2HDM**, the **backreaction of the doublet fields** can substantially change the width of the singlet wall → **Need to evaluate the width numerically.**
- $\delta_s = \left(\frac{\sqrt{\lambda_6}}{2}v_s\right)^{-1}$ Is a good approximation in case of Higgs doublet decoupling or when $v_{1,2}(0) = 0$ inside the wall.

What about the width of doublet profiles in the vicinity of the wall ?

- Only possible to evaluate it numerically in a complex models such as the N2HDM.
- Proportional to the width of the singlet wall δ_s .
- Increases with **smaller $v_{ew}(0)$** . Electroweak symmetry restoring parameters usually have a **large width**.



Results from another scan with negative $\lambda_{7,8}$

Focus on scenarios that lead to electroweak symmetry breaking in a large region around the wall:

- **Smaller $v_{ew}(0)$** can be obtained for **large positive $\Delta_{1,2}$** and a **large region** where the effective mass term changes across the wall.
- When neglecting λ_{345} , $\Delta_{1,2} \times D$ **proportional to $\lambda_{7,8}/\lambda_6$**
- Large ratios $\lambda_{7,8}/\lambda_6$ lead to very small $v_{1,2}(0)$ in a large region around the wall.
- Using the mass basis for the couplings:

$$R = \begin{pmatrix} c_{\alpha_1} c_{\alpha_2} & s_{\alpha_1} c_{\alpha_2} & s_{\alpha_2} \\ - (c_{\alpha_1} s_{\alpha_2} s_{\alpha_3} + s_{\alpha_1} c_{\alpha_3}) & c_{\alpha_1} c_{\alpha_3} - s_{\alpha_1} s_{\alpha_2} s_{\alpha_3} & c_{\alpha_2} s_{\alpha_3} \\ -c_{\alpha_1} s_{\alpha_2} c_{\alpha_3} + s_{\alpha_1} s_{\alpha_3} & - (c_{\alpha_1} s_{\alpha_3} + s_{\alpha_1} s_{\alpha_2} c_{\alpha_3}) & c_{\alpha_2} c_{\alpha_3} \end{pmatrix}$$

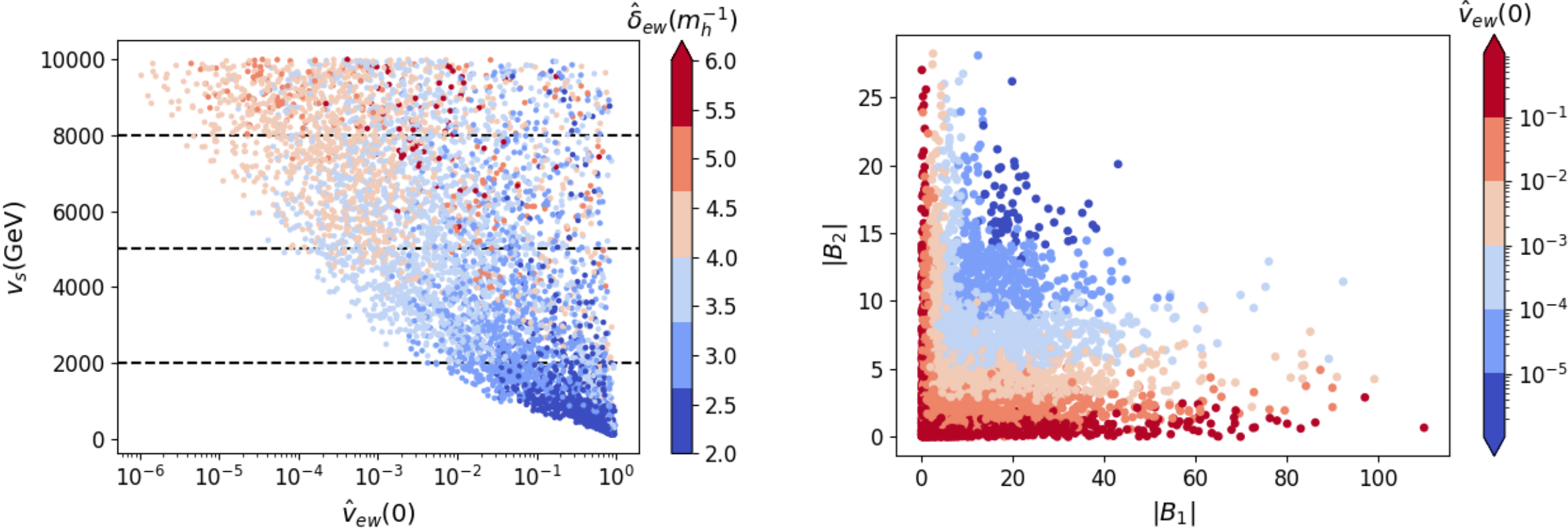
CP-even Higgs Mixing angles

$$\lambda_6 = \frac{m_{h_1}^2 R_{13}^2 + m_{h_2}^2 R_{23}^2 + m_{h_3}^2 R_{33}^2}{v_s^2} \quad \lambda_7 = \frac{R_{13} R_{11} m_{h_1}^2 + R_{23} R_{21} m_{h_2}^2 + R_{33} R_{31} m_{h_3}^2}{v_1 v_s} \quad \lambda_8 = \frac{R_{13} R_{12} m_{h_1}^2 + R_{23} R_{22} m_{h_2}^2 + R_{33} R_{32} m_{h_3}^2}{v_2 v_s}$$

$$\rightarrow \lambda_7/\lambda_6 = \left(\frac{v_s}{v_1} \right) \left(\frac{R_{13} R_{11} m_{h_1}^2 + R_{23} R_{21} m_{h_2}^2 + R_{33} R_{31} m_{h_3}^2}{m_{h_1}^2 R_{13}^2 + m_{h_2}^2 R_{23}^2 + m_{h_3}^2 R_{33}^2} \right)$$

- Look for **large v_s**
- Look for parameter points with **small λ_6** . For example **small masses**.

Parameter scan for small masses and large v_s



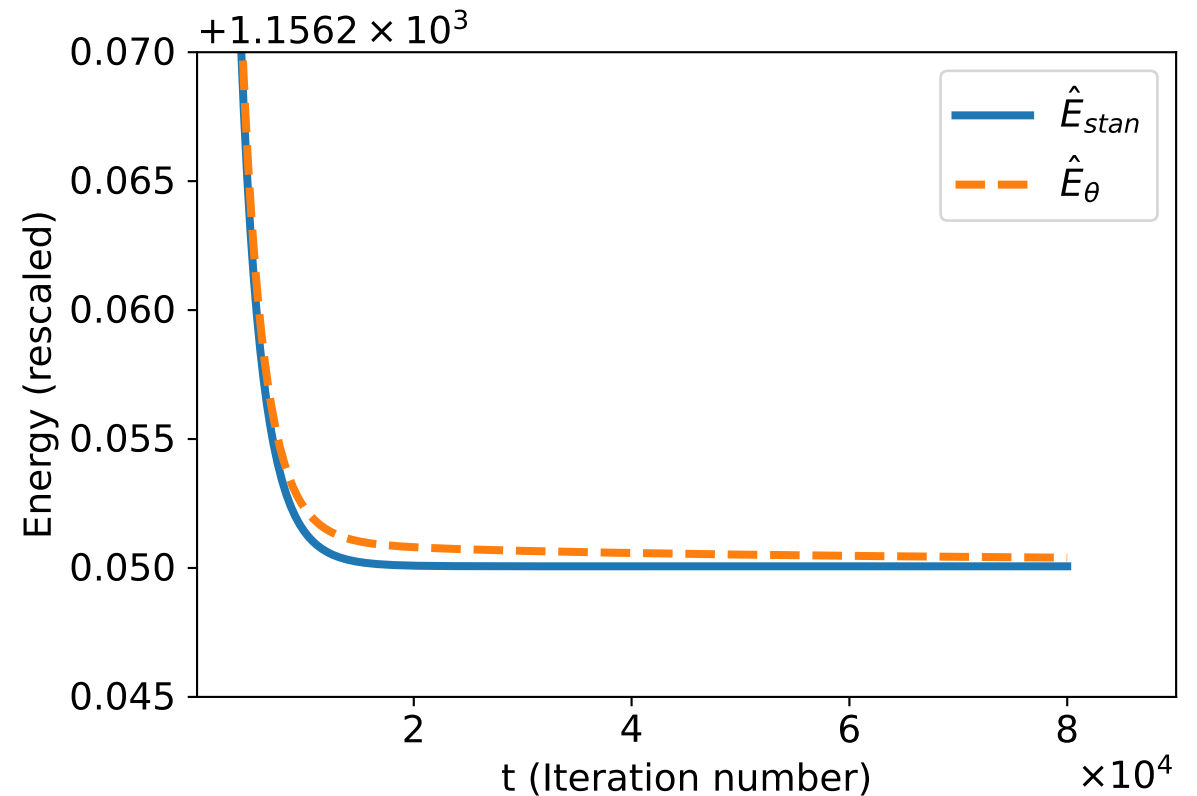
- Parameter points with **larger v_s** can lead to **electroweak symmetry restoration** in a **large region** around the wall

From EOM of the Goldstone mode $\theta(\mathbf{x})$

$$\rightarrow \frac{d\theta}{dx} = \frac{-v_2^2}{v_1^2 + v_2^2 + v_+^2} \frac{d\xi}{dx}$$

Pilaftsis, Law (2021)

- Solution with **CP-violation** has **higher energy** than the **standard solution**.
- CP-violating solution of the doublet fields will **decay** to the standard solution.



Energy functional of a CP-violating field configuration:

$$\mathcal{E}(x) = V_{N2HDM}(x) + \frac{1}{2}v_1^2(x) \left(\frac{d\theta}{dx} \right)^2 + \frac{1}{2}v_2^2(x) \left[\left(\frac{d\theta}{dx} \right)^2 + 2 \frac{d\theta}{dx} \frac{d\xi}{dx} \right] + \frac{1}{2}v_+^2(x) \left(\frac{d\theta}{dx} \right)^2.$$

Minimized for small v_1 and v_2 in the region where θ varies.

- The energy is smaller for parameter points where **v_1 and v_2 are minimal.**
- **In those parameter points, the CP-violating field configuration is longer lived.**