Renormalization of the general Two-Higgs-Doublet Model

Seminar Talk

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Structure of the talk

1. Renormalization schemes and their properties
2. The general THDM
   - Motivation
   - Lagrangian
   - Renormalization
3. Running of λ_{hhh} in different schemes

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$$\mathcal{O}_1 = \mathcal{P}_1(\lambda_1, \ldots, \lambda_n)$$
$$\ldots$$
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  $$\mathcal{O}_1 = \mathcal{P}_1(\lambda_1, \ldots, \lambda_n) \quad \ldots \quad \mathcal{O}_n = \mathcal{P}_n(\lambda_1, \ldots, \lambda_n)$$
- perturbative stability
  ↪ heuristically measured, e.g., via "reduction of scale dependence"

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The general THDM: Motivation

The SM describes most of Nature extremely well!
$\hookrightarrow$ **why study SM extensions at all?**

Some phenomena not explained by the SM:

- gravitation
- matter-antimatter asymmetry of the universe
- dark matter
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Why study the general THDM? [Lee, Branco, Lavoura, Silva, Haber, O'Neil]

- one of the simplest SM extensions
- doublet extensions automatically preserve custodial symmetry
- why the "general" one?
  - contains different "types" as limits in parameter space
  - sources for CP violation
  - can be viewed as "effective" theory for the MSSM scalar sector

The general THDM: Lagrangian

Field content of the SM, except for the scalar sector:

$$\Phi_1(x) = \begin{pmatrix} \phi_1^+(x) \\ \frac{1}{\sqrt{2}}(v_1 + h_1(x) + i\chi_1(x)) \end{pmatrix}, \qquad \Phi_2(x) = e^{i\delta} \begin{pmatrix} \phi_2^+(x) \\ \frac{1}{\sqrt{2}}(v_2 + h_2(x) + i\chi_2(x)) \end{pmatrix}.$$

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This yields the classical Lagrangian

$$\mathcal{L}_{\mathrm{class}}^{\mathrm{THDM}} = \mathcal{L}_{\mathrm{Fermion}}^{\mathrm{SM}} + \mathcal{L}_{\mathrm{Gauge}}^{\mathrm{SM}} + \mathcal{L}_{\mathrm{Higgs}}^{\mathrm{THDM}} + \mathcal{L}_{\mathrm{Yukawa}}^{\mathrm{THDM}}.$$

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Higgs sector

$$\mathcal{L}_{\mathrm{Higgs}}^{\mathrm{THDM}} = (D_\mu \Phi_1)^\dagger (D^\mu \Phi_1) + (D_\mu \Phi_2)^\dagger (D^\mu \Phi_2) - V_{\mathrm{THDM}},$$

with Higgs potential

$$\begin{aligned} V_{\mathrm{THDM}} = {} & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - (m_{12}^2 \Phi_1^\dagger \Phi_2 + \mathrm{h.c.}) \\ & + \frac{1}{2}\lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2}\lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) \\ & + \left[\frac{1}{2}\lambda_5 (\Phi_1^\dagger \Phi_2)^2 + (\lambda_6 \Phi_1^\dagger \Phi_1 + \lambda_7 \Phi_2^\dagger \Phi_2) \Phi_1^\dagger \Phi_2 + \mathrm{h.c.} \right] \end{aligned}$$

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The general THDM: transition to the Higgs basis

Changing the basis in field space via [Branco, Lavoura, Silva, Gunion, Haber]

$$H_a = \sum_{b=1}^{2} U_{ab}\Phi_b, \qquad\qquad U = \frac{1}{\sqrt{v_1^2 + v_2^2}} \begin{pmatrix} v_1 & v_2 e^{-i\delta} \\ -v_2 & v_1 e^{-i\delta} \end{pmatrix},$$

yields the new doublets

$$H_1(x) = \tfrac{1}{\sqrt{v_1^2+v_2^2}} \begin{pmatrix} \overbrace{v_1\phi_1^+(x) + v_2\phi_2^+(x)}^{=G^+(x)} \\ \tfrac{1}{\sqrt{2}}\big(\underbrace{v_1^2 + v_2^2}_{=v^2} + \underbrace{v_1 h_1(x) + v_2 h_2(x)}_{=N_1(x)} + i(\underbrace{v_1\chi_1(x) + v_2\chi_2(x)}_{=G_0(x)})\big) \end{pmatrix},$$

$$H_2(x) = \tfrac{1}{\sqrt{v_1^2+v_2^2}} \begin{pmatrix} \overbrace{v_1\phi_2^+(x) - v_2\phi_1^+(x)}^{=H^+(x)} \\ \tfrac{1}{\sqrt{2}}\big(\underbrace{v_1 h_2(x) - v_2 h_1(x)}_{=N_2(x)} + i(\underbrace{v_1\chi_2(x) - v_2\chi_1(x))}_{=N_3(x)}\big) \end{pmatrix}.$$

The general THDM: transition to the Higgs basis

Redefining the parameters

$$\widetilde{m}_{11}^2 = m_{11}^2 c_\beta^2 + m_{22}^2 s_\beta^2 - \mathrm{Re}\{m_{12}^2 e^{i\delta}\} s_{2\beta}, \qquad \widetilde{m}_{22}^2 = m_{11}^2 s_\beta^2 + m_{22}^2 c_\beta^2 + \mathrm{Re}\{m_{12}^2 e^{i\delta}\} s_{2\beta},$$
$$\widetilde{m}_{12}^2 = \tfrac{1}{2}(m_{11}^2 - m_{22}^2) s_{2\beta} + \mathrm{Re}\{m_{12}^2 e^{i\delta}\} c_{2\beta} + i\mathrm{Im}\{m_{12}^2 e^{i\delta}\},$$
$$\widetilde{\lambda}_1 = \lambda_1 c_\beta^4 + \lambda_2 s_\beta^4 + \tfrac{1}{2}\lambda_{345} s_{2\beta}^2 + 2 s_{2\beta}[c_\beta^2 \mathrm{Re}\{\lambda_6 e^{i\delta}\} + s_\beta^2 \mathrm{Re}\{\lambda_7 e^{i\delta}\}],$$
$$\widetilde{\lambda}_2 = \lambda_1 s_\beta^4 + \lambda_2 c_\beta^4 + \tfrac{1}{2}\lambda_{345} s_{2\beta}^2 - 2 s_{2\beta}[s_\beta^2 \mathrm{Re}\{\lambda_6 e^{i\delta}\} + c_\beta^2 \mathrm{Re}\{\lambda_7 e^{i\delta}\}],$$
$$\widetilde{\lambda}_3 = \lambda_3 + \tfrac{1}{4} s_{2\beta}^2(\lambda_1 + \lambda_2 - 2\lambda_{345}) - s_{2\beta} c_{2\beta} \mathrm{Re}\{(\lambda_6 - \lambda_7) e^{i\delta}\},$$
$$\widetilde{\lambda}_4 = \lambda_4 + \tfrac{1}{4} s_{2\beta}^2(\lambda_1 + \lambda_2 - 2\lambda_{345}) - s_{2\beta} c_{2\beta} \mathrm{Re}\{(\lambda_6 - \lambda_7) e^{i\delta}\},$$
$$\widetilde{\lambda}_5 = \tfrac{1}{4} s_{2\beta}^2(\lambda_1 + \lambda_2 - 2\lambda_{345}) + \mathrm{Re}\{\lambda_5 e^{2i\delta}\} + i c_{2\beta} \mathrm{Im}\{\lambda_5 e^{2i\delta}\} - s_{2\beta} c_{2\beta} \mathrm{Re}\{(\lambda_6 - \lambda_7) e^{i\delta}\} - i s_{2\beta} \mathrm{Im}\{(\lambda_6 - \lambda_7) e^{i\delta}\},$$
$$\widetilde{\lambda}_6 = -\tfrac{1}{2} s_{2\beta}[\lambda_1 c_\beta^2 - \lambda_2 s_\beta^2 - \lambda_{345} c_{2\beta} - i\mathrm{Im}\{\lambda_5 e^{2i\delta}\}] + c_\beta c_{3\beta} \mathrm{Re}\{\lambda_6 e^{i\delta}\} + s_\beta s_{3\beta} \mathrm{Re}\{\lambda_7 e^{i\delta}\} + i c_\beta^2 \mathrm{Im}\{\lambda_6 e^{i\delta}\} + i s_\beta^2 \mathrm{Im}\{\lambda_7 e^{i\delta}\},$$
$$\widetilde{\lambda}_7 = -\tfrac{1}{2} s_{2\beta}[\lambda_1 s_\beta^2 - \lambda_2 c_\beta^2 + \lambda_{345} c_{2\beta} + i\mathrm{Im}\{\lambda_5 e^{2i\delta}\}] + s_\beta s_{3\beta} \mathrm{Re}\{\lambda_6 e^{i\delta}\} + c_\beta c_{3\beta} \mathrm{Re}\{\lambda_7 e^{i\delta}\} + i s_\beta^2 \mathrm{Im}\{\lambda_6 e^{i\delta}\} + i c_\beta^2 \mathrm{Im}\{\lambda_7 e^{i\delta}\},$$

with $\lambda_{345} := \lambda_3 + \lambda_4 + \mathrm{Re}\{\lambda_5 e^{2i\delta}\}, \qquad \tan\beta = \dfrac{v_2}{v_1},$

yields Higgs potential of the form

$$\begin{aligned} V_{\mathrm{THDM}} = {} & \widetilde{m}_{11}^2 H_1^\dagger H_1 + \widetilde{m}_{22}^2 H_2^\dagger H_2 - (\widetilde{m}_{12}^2 H_1^\dagger H_2 + \mathrm{h.c.}) \\ & + \frac{1}{2}\widetilde{\lambda}_1 (H_1^\dagger H_1)^2 + \frac{1}{2}\widetilde{\lambda}_2 (H_2^\dagger H_2)^2 + \widetilde{\lambda}_3 (H_1^\dagger H_1)(H_2^\dagger H_2) + \widetilde{\lambda}_4 (H_1^\dagger H_2)(H_2^\dagger H_1) \\ & + \left[\frac{1}{2}\widetilde{\lambda}_5 (H_1^\dagger H_2)^2 + (\widetilde{\lambda}_6 H_1^\dagger H_1 + \widetilde{\lambda}_7 H_2^\dagger H_2) H_1^\dagger H_2 + \mathrm{h.c.}\right]. \end{aligned}$$

The general THDM: physical parametrization

Expanding to second order in the component fields yields

$$V^{(2)}_{\mathrm{THDM}} = -\overbrace{\left(-\widetilde{m}^2_{11} v - \tfrac{1}{2}\widetilde{\lambda}_1 v^3\right)}^{=t_{N_1}} N_1 - \overbrace{\left(\mathrm{Re}\{\widetilde{m}^2_{12}\} v - \tfrac{1}{2}\mathrm{Re}\{\widetilde{\lambda}_6\} v^3\right)}^{=t_{N_2}} N_2 - \overbrace{\left(-\mathrm{Im}\{\widetilde{m}^2_{12}\} v + \tfrac{1}{2}\mathrm{Im}\{\widetilde{\lambda}_6\} v^3\right)}^{=t_{N_3}} N_3$$
$$+ \left(H^+, \quad G^+\right) \mathbf{M}_{\mathrm{charged}} \begin{pmatrix} H^- \\ G^- \end{pmatrix} + \frac{1}{2}\left(N_1, \quad N_2 \quad N_3 \quad G_0\right) \mathbf{M}_{\mathrm{neutral}} \begin{pmatrix} N_1 \\ N_2 \\ N_3 \\ G_0 \end{pmatrix},$$

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with

$$\mathbf{M}_{\mathrm{charged}} = \begin{pmatrix} \overbrace{\widetilde{m}_{22}^2 + \tfrac{1}{2}\widetilde{\lambda}_3 v^2}^{=M^2_{\mathrm{H}^+}} & -\tfrac{1}{v}(t_{N_2} + i t_{N_3}) \\ -\tfrac{1}{v}(t_{N_2} - i t_{N_3}) & -\tfrac{t_{N_2}}{v} \end{pmatrix},$$

$$\mathbf{M}_{\mathrm{neutral}} = \begin{pmatrix} \tfrac{1}{v}(\widetilde{\lambda}_1 v^3 - t_{N_1}) & \tfrac{1}{v}(\mathrm{Re}\{\widetilde{\lambda}_6\} v^3 - t_{N_2}) & -\tfrac{1}{v}(\mathrm{Im}\{\widetilde{\lambda}_6\} v^3 + t_{N_3}) & 0 \\ \tfrac{1}{v}(\mathrm{Re}\{\widetilde{\lambda}_6\} v^3 - t_{N_2}) & M^2_{\mathrm{H}^+} + \tfrac{1}{2}(\widetilde{\lambda}_4 + \mathrm{Re}\{\widetilde{\lambda}_5\}) v^2 & -\tfrac{1}{2}\mathrm{Im}\{\widetilde{\lambda}_5\} v^2 & \tfrac{t_{N_3}}{v} \\ -\tfrac{1}{v}(\mathrm{Im}\{\widetilde{\lambda}_6\} v^3 + t_{N_3}) & -\tfrac{1}{2}\mathrm{Im}\{\widetilde{\lambda}_5\} v^2 & M^2_{\mathrm{H}^+} + \tfrac{1}{2}(\widetilde{\lambda}_4 - \mathrm{Re}\{\widetilde{\lambda}_5\}) v^2 & -\tfrac{t_{N_2}}{v} \\ 0 & \tfrac{t_{N_3}}{v} & -\tfrac{t_{N_2}}{v} & -\tfrac{t_{N_1}}{v} \end{pmatrix}.$$

The general THDM: physical parametrization

Perform rotation in field space

$$\begin{pmatrix} N_1 \\ N_2 \\ N_3 \\ G_0 \end{pmatrix} =: \mathbf{R}_{4\times 4} \begin{pmatrix} H \\ h \\ A_0 \\ G_0 \end{pmatrix} \equiv \begin{pmatrix} c_1 c_2 & s_1 c_2 & -s_2 & 0 \\ c_1 s_2 s_3 - s_1 c_3 & s_1 s_2 s_3 + c_1 c_3 & c_2 s_3 & 0 \\ c_1 s_2 c_3 + s_1 s_3 & s_1 s_2 c_3 - c_1 s_3 & c_2 c_3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} H \\ h \\ A_0 \\ G_0 \end{pmatrix}, \quad c_j \equiv \cos(\theta_j), s_j \equiv \sin(\theta_j).$$

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Demanding diagonal mass matrix (up to tadpole contributions)

$$(\mathbf{R}_{4\times 4}^{\mathrm{T}} \mathbf{M}_{\mathrm{neutral}} \mathbf{R}_{4\times 4})^{(3\times 3)} \stackrel{!}{=} \mathrm{diag}\{M_{\mathrm{H}}^2, M_{\mathrm{h}}^2, M_{\mathrm{A_0}}^2\},$$

implies 6 independent conditions.

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$\hookrightarrow$ Use them to trade old parameters [Degrande, 2014]

$$\widetilde{\lambda}_1, \widetilde{\lambda}_4, \mathrm{Re}\{\widetilde{\lambda}_5\}, \mathrm{Im}\{\widetilde{\lambda}_5\}, \mathrm{Re}\{\widetilde{\lambda}_6\}, \mathrm{Im}\{\widetilde{\lambda}_6\},$$

for masses and mixing angles

$$M_{\mathrm{H}}, M_{\mathrm{h}}, M_{\mathrm{A_0}}, \theta_1, \theta_2, \theta_3.$$

Renormalization transformation

Parameters:

$$
\begin{aligned}
t_{N_1,0} &= t_{N_1} + \delta t_{N_1}, & t_{N_2,0} &= t_{N_2} + \delta t_{N_2}, & t_{N_3,0} &= t_{N_3} + \delta t_{N_3},\\
\theta_{1,0} &= \theta_1 + \delta\theta_1, & \theta_{2,0} &= \theta_2 + \delta\theta_2, & \theta_{3,0} &= \theta_3 + \delta\theta_3,\\
M^2_{\mathrm{h},0} &= M^2_{\mathrm{h}} + \delta M^2_{\mathrm{h}}, & M^2_{\mathrm{H},0} &= M^2_{\mathrm{H}} + \delta M^2_{\mathrm{H}}, & M^2_{\mathrm{A}_0,0} &= M^2_{\mathrm{A}_0} + \delta M^2_{\mathrm{A}_0},\\
M^2_{\mathrm{H}^+,0} &= M^2_{\mathrm{H}^+} + \delta M^2_{\mathrm{H}^+}, & \widetilde{\lambda}_{2,0} &= \widetilde{\lambda}_2 + \delta\widetilde{\lambda}_2, & \widetilde{\lambda}_{3,0} &= \widetilde{\lambda}_3 + \delta\widetilde{\lambda}_3,\\
& & \widetilde{\lambda}_{7,0} &= \widetilde{\lambda}_7 + \delta\widetilde{\lambda}_7, & &
\end{aligned}
$$

Fields:

$$
\begin{pmatrix} G^\pm_{\mathrm{B}} \\ H^\pm_{\mathrm{B}} \end{pmatrix} = \begin{pmatrix} 1 + \frac{1}{2}\delta Z_{\mathrm{G}^+} & \frac{1}{2}\delta Z_{\mathrm{GH}^+} \\ \frac{1}{2}\delta Z_{\mathrm{HG}^+} & 1 + \frac{1}{2}\delta Z_{\mathrm{H}^+} \end{pmatrix} \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix},
$$

$$
\begin{pmatrix} G_{0,\mathrm{B}} \\ h_{\mathrm{B}} \\ H_{\mathrm{B}} \\ A_{0,\mathrm{B}} \end{pmatrix} = \begin{pmatrix} 1 + \frac{1}{2}\delta Z_{\mathrm{G}} & \frac{1}{2}\delta Z_{\mathrm{Gh}} & \frac{1}{2}\delta Z_{\mathrm{GH}} & \frac{1}{2}\delta Z_{\mathrm{GA}} \\ \frac{1}{2}\delta Z_{\mathrm{hG}} & 1 + \frac{1}{2}\delta Z_{\mathrm{h}} & \frac{1}{2}\delta Z_{\mathrm{hH}} & \frac{1}{2}\delta Z_{\mathrm{hA}} \\ \frac{1}{2}\delta Z_{\mathrm{HG}} & \frac{1}{2}\delta Z_{\mathrm{Hh}} & 1 + \frac{1}{2}\delta Z_{\mathrm{H}} & \frac{1}{2}\delta Z_{\mathrm{HA}} \\ \frac{1}{2}\delta Z_{\mathrm{AG}} & \frac{1}{2}\delta Z_{\mathrm{Ah}} & \frac{1}{2}\delta Z_{\mathrm{AH}} & 1 + \frac{1}{2}\delta Z_{\mathrm{A}_0} \end{pmatrix} \begin{pmatrix} G_0 \\ h \\ H \\ A_0 \end{pmatrix},
$$

Renormalization conditions

Tadpoles:

Choose $t_{N_j} = 0$ at tree-level $\quad$ ($\leftarrow$ minimizes classical potential at $H = h = A_0 = 0$)

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$$\langle 0|h|0\rangle = \underbrace{\bullet\overset{h}{\text{- - - -}}\times}_{=i\delta t_h} + \bullet\overset{h}{\text{- - -}}\text{(1L)} + \ldots \overset{!}{=} 0$$

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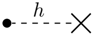
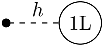
$\langle 0|h|0\rangle =$  $+$  $+ \quad \dots \quad \stackrel{!}{=} 0$

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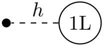
$\Rightarrow \qquad \delta t_h = -i$  $+ \quad \dots$

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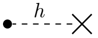
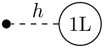
$\langle 0|h|0\rangle =$ 

$+$ 

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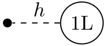
$\Rightarrow \qquad \delta t_h = -i$ 

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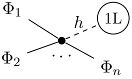
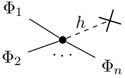
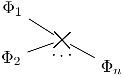
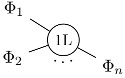
$\Gamma_{1\mathrm{L}}^{\Phi_1 \ldots \Phi_n} =$ 

$+$ 

$+$ 

$+$ 

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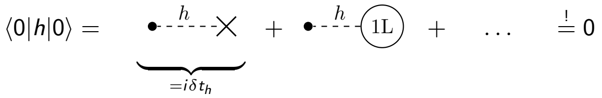
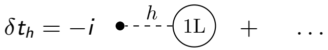


$\Rightarrow$



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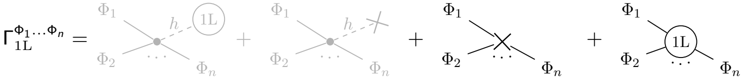

Renormalization conditions

Different tadpole schemes:

$$V^{(2)}_{\mathrm{THDM}} \supset -\overbrace{\left(-\widetilde{m}^2_{11}\, v - \frac{1}{2}\widetilde{\lambda}_1 v^3\right)}^{=t_{N_1}} N_1$$

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Trade tadpole parameter for

- potential parameter: Parameter-renormalized tadpole scheme (PRTS) [Denner,1993]
  $\hookrightarrow$ perturbatively stable, but gauge dependent
- field shift parameter: Fleischer-Jegerlehner tadpole scheme (FJTS) [Fleischer, Jegerlehner, 2001]
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Conditions for remaining RCs:

Masses and fields $\rightarrow$ on-shell conditions:

$$\begin{aligned} \widetilde{\mathrm{Re}}\Gamma^{h_j h_k}_{\mathrm{R}}(p)\Big|_{p^2=m^2_{h_j}} &= 0, \\ \lim_{p^2\to m^2_{h_j}} \frac{1}{p^2-m^2_{h_j}} \widetilde{\mathrm{Re}}\Gamma^{h_j h_j}_{\mathrm{R}}(p) &= 1. \end{aligned}$$

Rest $\rightarrow$ (modified) minimal subtraction.

Structure of the talk

1. Renormalization schemes and their properties
2. The general THDM
   - Motivation
   - Lagrangian
   - Renormalization
3. Running of λ_{hhh} in different schemes

Trilinear Higgs self-coupling: computation

Goal:

Compute and compare running of λ_{hhh} at 1L in three different tadpole schemes!

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Goal:

Compute and compare running of λ_{hhh} at 1L in three different tadpole schemes!

Steps of the computation:

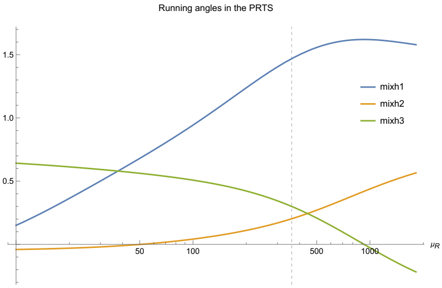
- implementation of different tadpole schemes
  - including two different (but equivalent) instances of the FJTS
- parameter input-value conversion between different schemes
  - different RSs furnish different relations between observables and parameters $\rightarrow$ extracted input values must be adjusted
  - this also yields a naive estimate for perturbative stability
- compute running of the relevant input parameters in different schemes
- insert properly converted running parameters in results for λ_{hhh}

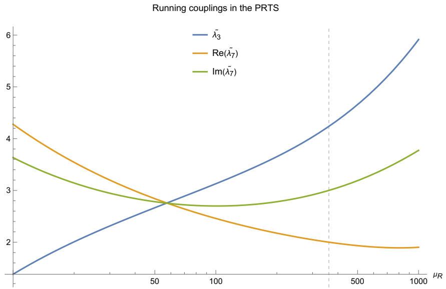
Trilinear Higgs self-coupling: preliminary results

Parameter region:

For all parameters except the running ones:
take type 1 symmetric limit and go into "low mass" benchmark scenario

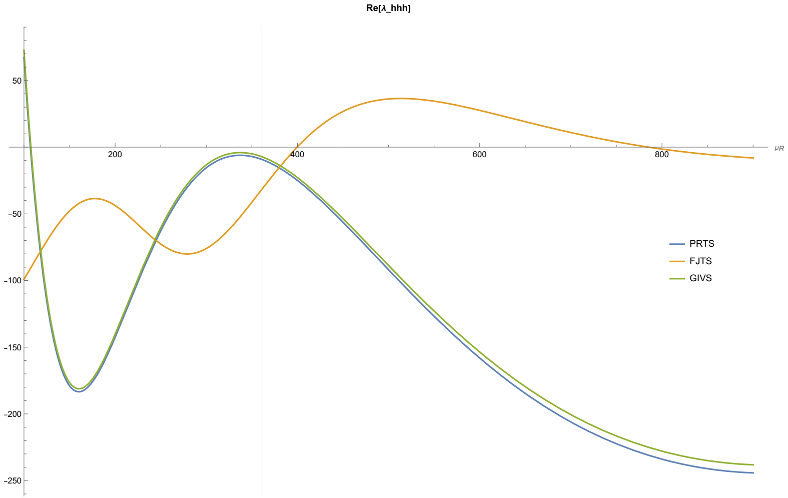
Running of mixing angles and couplings:





Trilinear Higgs self-coupling: preliminary results

Running of NLO contributions to λ_{hhh}:



Summary and Outlook

Summary:

- implementation of the general THDM
- renormalization in different schemes
- NLO calculation of λ_{hhh} and study of its running

Work in progress:

- finishing up the λ_{hhh} analysis
- performing similar computation for decay $H \to hh$

Thanks!

Backup slides

Perturbation theory in a nutshell

Starting point: Lagrangian $\mathcal{L}(\psi, \partial\psi)$ of some QFT model

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$$\underset{\substack{\uparrow\\ \text{Gell-Mann-}\\ \text{Low}}}{G(x_1, \dots, x_n) =} \frac{\langle 0 | T\left\{ \psi_{\mathrm{I}}(x_1) \dots \psi_{\mathrm{I}}(x_n) \exp\left[i \int d^4 x \mathcal{L}_{\mathrm{I}} \right] \right\} | 0 \rangle}{\langle 0 | T\left\{ \exp\left[i \int d^4 x \mathcal{L}_{\mathrm{I}} \right] \right\} | 0 \rangle}$$

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order-by-order expansion! $\longrightarrow$

$$= \left(1 + i \int d^4 x \mathcal{L}_\mathrm{I} + \dots \right)$$

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$\uparrow$

Gell-Mann-Low

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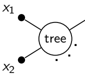
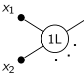
$$G(x_1, \dots, x_n) = \frac{\langle 0 | T \left\{ \psi_{\mathrm{I}}(x_1) \dots \psi_{\mathrm{I}}(x_n) \overbrace{\exp\left[i \int d^4 x \mathcal{L}_{\mathrm{I}} \right]}^{= \left(1 + i \int d^4 x \mathcal{L}_{\mathrm{I}} + \dots \right)} \right\} | 0 \rangle}{\langle 0 | T \left\{ \exp\left[i \int d^4 x \mathcal{L}_{\mathrm{I}} \right] \right\} | 0 \rangle} =$$  $+$  $+ \dots$

$\uparrow$ Gell-Mann-Low

$\uparrow$ Wick, Feynman

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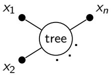
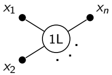


$+$



$+ \ldots$

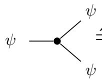
Gell-Mann-Low

Wick, Feynman

$\hookrightarrow$ rules can be read off $\mathcal{L}$

Renormalization

Example: $\mathcal{L} \supset \frac{\lambda_0}{3!}\psi^3 \quad \Rightarrow$



$\hat{=} \ i\lambda_0$

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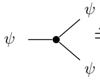
Example: $\mathcal{L} \supset \frac{\lambda_0}{3!}\psi^3 \quad \Rightarrow$ 

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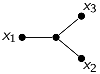
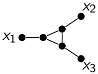
$\hookrightarrow$ Green function:

$G(x_1, x_2, x_3) =$ 

$+$ 

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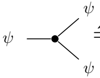

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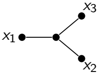

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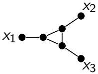

$+ \quad \dots \quad \sim \left(\lambda_0 + c_1\Delta_{\mathrm{UV}} + c_2 + \dots\right)$

Common bookkeeping:

$$\lambda_0 = \lambda + \delta\lambda$$

"renormalized coupling"
(finite expansion parameter)

"renormalization constant"
(contains UV-divergencies)

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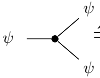
Example: $\mathcal{L} \supset \frac{\lambda_0}{3!}\psi^3 \quad \Rightarrow$ 

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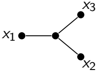
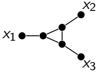
$G(x_1, x_2, x_3) =$ 

$+$ 

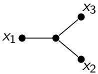
$+ \quad \dots \quad \sim \left(\lambda_0 + c_1\Delta_{\mathrm{UV}} + c_2 + \dots\right)$

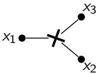
Common bookkeeping:

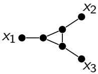
$$\lambda_0 = \lambda + \delta\lambda$$

"renormalized coupling"
(finite expansion parameter)

"renormalization constant"
(contains UV-divergencies)

$=$ 

$+$ 

$+$ 

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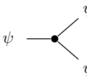
Renormalization

Example: $\mathcal{L} \supset \frac{\lambda_0}{3!}\psi^3 \quad \Rightarrow$ 

$\hat{=}\; i\lambda_0$

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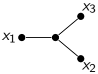
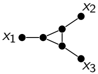
$G(x_1, x_2, x_3) =$ 

$+$ 

$+ \quad \ldots \quad \sim \Big(\lambda_0 + c_1\Delta_{\mathrm{UV}} + c_2 + \ldots\Big)$

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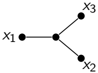
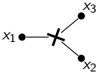
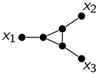
$$\lambda_0 = \underbrace{\lambda}_{\substack{\text{“renormalized coupling”}\\ \text{(finite expansion parameter)}}} + \underbrace{\delta\lambda}_{\substack{\text{“renormalization constant”}\\ \text{(contains UV-divergencies)}}} \;=\; (\lambda_{\overline{\mathrm{MS}}} - c_{\mathrm{fin}}) + (c_{\mathrm{fin}} + \underbrace{\delta\lambda_{\overline{\mathrm{MS}}}}_{=-c_1\Delta_{\mathrm{UV}}})$$

$=$ 

$+$ 

$+$ 

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Note: choice of RS is in principal unphysical!
<u>However:</u> in practice it influences properties of perturbative calculations!

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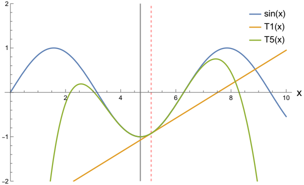

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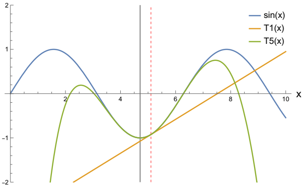

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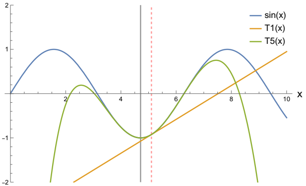

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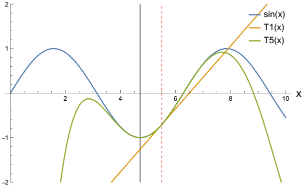

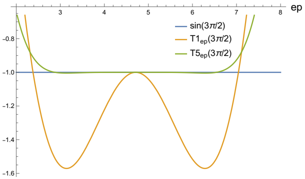

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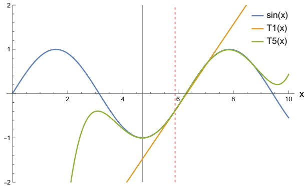

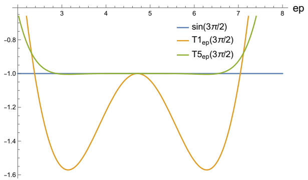

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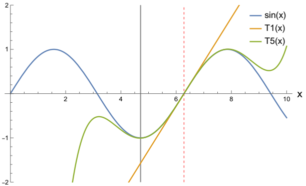

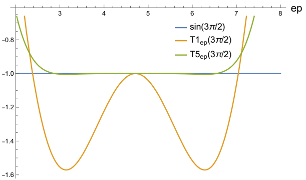

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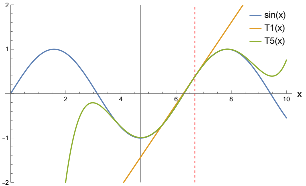

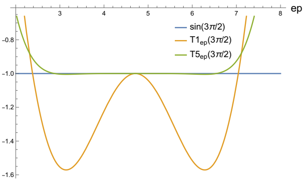

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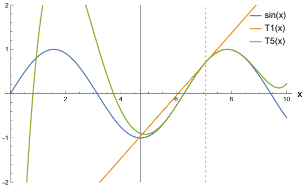

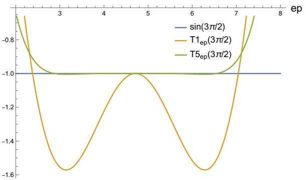

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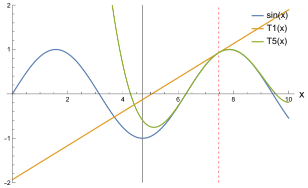

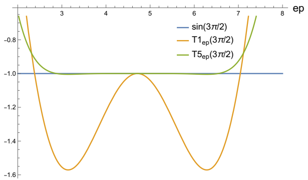

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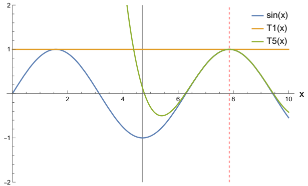

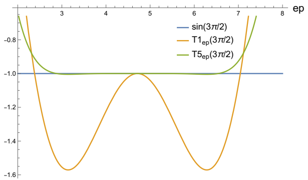

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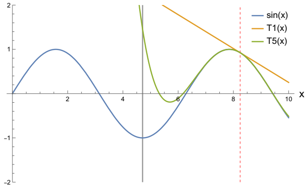

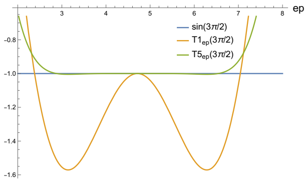

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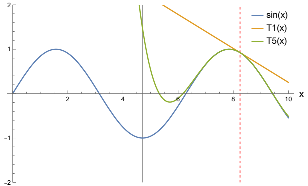

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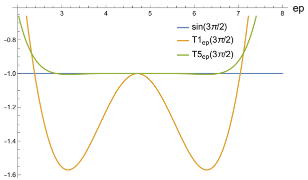

$\Rightarrow$ **desirable property:** perturbative stability
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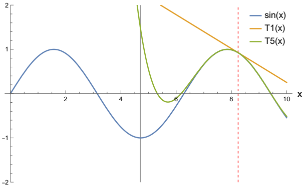

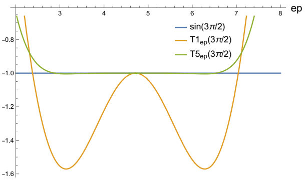

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Other desirable properties of RSs:

- gauge independence
- simplification of calculation

Tools used in the calculation

`Mathematica` packages for

- implementing the model:
  `FeynRules`
- performing the renormalization transformation:
  `MoGRe`
- generating the diagrams and computing, simplifying, manipulating the corresponding expressions:
  `FeynArts`, `FormCalc`
- dealing with appearing loop integrals:
  `FeynCalc`, `PackageX`

Input parameters

$$
\begin{aligned}
m_{\mathrm{e}} &= 0.510\,998\,950\,00\,\mathrm{MeV}, m_{\mu} = 105.658\,375\,5\,\mathrm{MeV}, m_{\tau} = 1776.86\,\mathrm{MeV}, \\
m_{\mathrm{u}} &= 0.1\,\mathrm{GeV}, \qquad m_{\mathrm{c}} = 0.986\,\mathrm{GeV}, \qquad m_{\mathrm{t}} = 172.5\,\mathrm{GeV}, \\
m_{\mathrm{d}} &= 0.1\,\mathrm{GeV}, \qquad m_{\mathrm{s}} = 0.1\,\mathrm{GeV}, \qquad m_{\mathrm{b}} = 4.18\,\mathrm{GeV}, \\
M_{\mathrm{W}} &= 80.377\,\mathrm{GeV}, \qquad M_{\mathrm{Z}} = 91.1876\,\mathrm{GeV}, M_{\mathrm{h}} = 125\,\mathrm{GeV}, \\
\alpha(0) &= \frac{e^2}{4\pi} = \frac{1}{137.035999139}, \alpha_{\mathrm{s}}(M_{\mathrm{Z}}) = 0.118, \\
\sin\theta^1_{\mathrm{CKM}} &= 0.22650, \sin\theta^2_{\mathrm{CKM}} = 0.00361, \sin\theta^3_{\mathrm{CKM}} = 0.04053, \\
\delta_{\mathrm{CKM}} &= 1.196.
\end{aligned}
$$

$$
\mu_R = \frac{M_{\mathrm{H}} + M_{\mathrm{h}} + M_{\mathrm{A_0}} + 2M_{\mathrm{H^+}}}{5}
$$