Two-loop form factors for Dark Matter production from colored Standard Model particles

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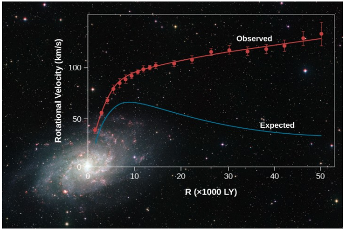
Overview

- Why study Dark Matter
- Framework
- EFT vs Simplified model
- DM annihilation - gluon and quark channels
- Method of Differential Equations

Why study Dark Matter ?

- Studies such as the orbital velocity curve of spiral galaxies and the motion of galaxies within galaxy clusters suggest the presence of 27% Dark Matter(DM) among the total mass-energy content of the observable universe.



- Explanation- Modified Gravity or Dark Matter(DM)?
  Irregularities in Uranus' orbit led to Neptune's discovery but precession of the perihelion of Mercury could only be explained by General relativity.
- Indirect searches look for excess of events over the background, such as galactic nuclei or relic density. DM annihilation processes are relevant.
- Direct searches such as in particle colliders or underground experiments are based on scattering of DM with SM particles, DM SM $\rightarrow$ DM SM. Mono-jet processes are relevant.

Dark Matter as BSM particle

UV complete approach:
In the **Gluphilic Scalar Dark Matter Model** (GSDM)[1], the interaction Lagrangian is given by,

$$\boldsymbol{\mathcal{L}_{int}} = \partial_\mu \chi^* \partial^\mu \chi - m_\chi^2 |\chi|^2 + (D_\mu \phi)^\dagger D^\mu \phi - m_\phi^2 |\phi|^2 + \lambda_d \chi^* \chi \phi^\dagger \phi$$

where χ is complex scalar DM which is gauge singlet, ϕ is the colored complex scalar mediator and D_μ is the covariant derivative, $D_\mu \phi = \partial_\mu \phi - i g_s \frac{\lambda^a}{2} G_\mu^a \phi$.

For the process $\boldsymbol{pp \to \chi\bar{\chi}}$,

The parton level contributions come from channels-

$$gg \to \chi\chi^*$$
$$q\bar{q} \to \chi\chi^*$$

where g is gluon and q is quark.
The relic density is can be used to constrain the parameters as,

$$\sigma v_\chi(gg) = \frac{\lambda_d^2}{64\pi^3 m_\chi^2} |M|^2$$

[1] Rohini M. Godbole et al. arXiv:1506.01408.

EFT vs Exact

For the process $pp \to \chi\chi^* j$

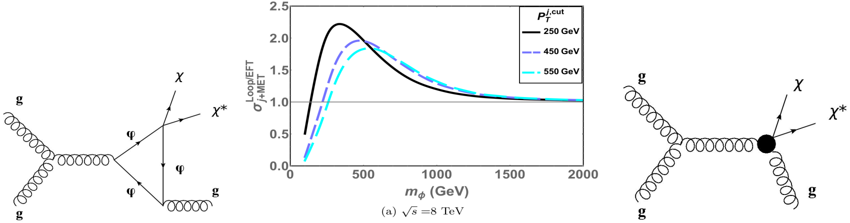


Figure: Ratio of the full calculation to the EFT approximation, as a function of m_ϕ

In **EFT** approach,

$$\mathcal{L}_{eff} = \frac{g_s^2}{m_\phi^2} |\chi|^2 \mathcal{G}^a_{\mu\nu} \mathcal{G}_a^{\mu\nu}$$

- EFT is valid for mediator masses > 1.5 TeV.
- Since we consider less massive mediators, exact model of GSDM would be appropriate.

Gluon channel

We consider the DM pair production process,

$$g(p_1)g(p_2) \to \chi\chi*$$

The amplitude can be expanded as-

$$\mathcal{M}_{gg\to\chi\chi*} = \delta^{ab}\left(\epsilon_1.\epsilon_2 - \frac{(\epsilon_1.p_2)(\epsilon_2.p_1)}{(s/2)}\right)\frac{\alpha_s}{4\pi}\left(\mathcal{M}_{LO} + \frac{\alpha_s}{4\pi}\mathcal{M}_{NLO} + \mathcal{O}(\alpha_s^2)\right)$$

Where $p_1^2 = p_2^2 = 0$ and $p_1.p_2 = s/2$.
The amplitude in terms of form factors is-

$$\mathcal{M}^{\mu\nu} = \delta^{ab}\left(F_1\ (s/2)\ g^{\mu\nu} + F_2\ p_1^{\nu}p_2^{\mu}\ \right)$$

The form factors are related at total amplitude level due to current conservation,

$$p_{1,\mu}\mathcal{M}^{\mu\nu} = p_{2,\nu}\mathcal{M}^{\mu\nu} = 0$$

which implies $F_2 = -F_1$.
Hence, the amplitude in terms of form factor F is-

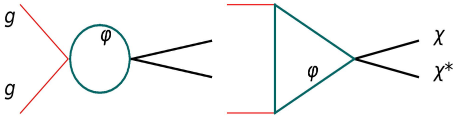
$$\mathcal{M}^{\mu\nu} = F * \delta^{ab}\left((s/2)\ g^{\mu\nu} - p_1^{\nu}p_2^{\mu}\ \right)$$

Projector -

$$\mathcal{P}_1^{\mu\nu} = \frac{1}{(d-2)}(g^{\mu\nu} - \frac{p_1^{\nu}p_2^{\mu}}{(s/2)})$$

1-Loop contributions

The Feynman diagrams at Leading order are,



Leading order 1 loop analytical result for form factor is-

$$F = \frac{g_s^2 \lambda}{s}(s + m_\phi^2 \ \log[\frac{2m_\phi^2 - s + \sqrt{s(s - 4m_\phi^2)}}{2m_\phi^2}]^2) + \mathcal{O}(\epsilon)$$

There will be UV divergences individually, but overall amplitude is UV finite. No Infrared Divergences due to massive propagators.

Parametrization of dimensionless ratio, $\tau = \frac{4m_\phi^2}{s}$, $x = -\frac{\sqrt{1-\tau}-1}{\sqrt{1-\tau}+1} + i\epsilon$

Why NLO ?
The Leading Order(One-Loop) Corrections suffer from large scale uncertainties $\sim 30\%$, Next-to-Leading Order (NLO) corrections are necessary to reduce the error.

The Feynman diagrams at 2-Loop order with maximum number of propagators are,

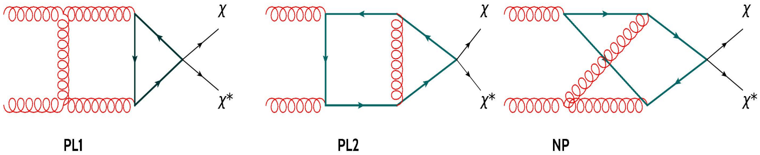


Figure: Topmost topologies in $gg \to \chi\chi*$, red lines are massless gluons, green lines are massive mediator and thin black lines are DM particles

- Since k_1 and k_2 are variables, they can be exchanged. Using translation invariance properties of Feynman integrals, we can shift loop momenta $k_j \to p_i + k_j$. Also exchanging and replacing p_i and p_j is possible. Using these, scalar integrals can be mapped to minimum of 3 integral families.
- IBP reduction of scalar integrals leads to 18 Master Integrals.
- The Integration by Parts (IBP) identities are based on fact that within dimensional regularisation the integral of a total derivative vanishes as there are no boundary terms,

$$\int\int \left(\frac{\partial}{\partial k_j}.l_i\right) \frac{d^d k_1 \ d^d k_2}{((k_1+p_1)^2 - m_\phi^2)^{a_1}((k_1+p_2)^2 - m_\phi^2)^{a_2}..((k_2+p_n)^2 - m_\phi^2)^{a_n}} = 0$$

where l_i can be loop momenta or external momenta.

- This process is similar to $gg \to H$, Master Integrals available in arXiv:2001.06295 [Charalampos Anastasiou et al.]

Renormalization

The pole structure at NLO two-loop can be ,

$$\mathcal{M} = \frac{1}{\epsilon_{IR}^2}\{...\} + \frac{1}{\epsilon_{UV/IR}^1}\{...\} + \mathcal{O}(\epsilon^0)$$

The Infrared divergences will have a Universal IR structure at NLO as[2]

$$M_{IR} = -g_s^2\left(\frac{-s}{4\pi\mu^2}\right)^{-\epsilon} e^{-\gamma_E \epsilon}\left(\frac{2N_c}{\epsilon^2} + \frac{2\beta}{\epsilon}\right)\mathcal{M}_{LO}$$

$$M_{UV} = -\left(\frac{-s}{\mu^2}\right)^{-\epsilon}\left(\delta Z_{\alpha_s}\ \mathcal{M}_{LO} + \delta Z_{m_\phi}\frac{\partial}{\partial m_\phi}\mathcal{M}_{LO} + \ \delta Z_\lambda\ \mathcal{M}_{LO}\right)$$

[2] Catani & Seymour.,A General Algorithm for Calculating Jet Cross Sections in NLO QCD arXiv:9605323.

Result: The finite part of virtual contributions

$$
\begin{aligned}
\mathcal{F}_{gg} = \frac{1}{2880N(x-1)^4(x+1)} \Big(& 3(-1+x)\left(3375 - 6810x + 15(454-225x)x^3 + 192\pi^4(x+x^3)\right) \\
& + 20\pi^2(3 + (-2+x)x(-11+28x+6x^2)) + 138240(1-x)\left(3(x+x^3) - 2N^2(x+2x^3)\right)\mathrm{HPL}(\{-4\},x) \\
& - 23040N^2(-1+x)^2x(1+x)\mathrm{HPL}(\{-2\},x)^2 \\
& + 11520(1-x)\left(N^2x(23+31x^2) - 27(x+x^3)\right)\mathrm{HPL}(\{4\},x) \\
& - 184320N^2(-1+x)^2x(1+x)\mathrm{HPL}(\{-3,1\},x) \\
& - 92160N^2(-1+x)^2x(1+x)\mathrm{HPL}(\{2,-2\},x) \\
& - 184320N^2(-1+x)^2x(1+x)\mathrm{HPL}(\{3,-1\},x) \\
& + 46080N^2(-1+x)^2x(1+x)\mathrm{HPL}(\{-2,-1\},x)\log(x) + 92160N^2(-1+x)^2x(1+x)\mathrm{HPL}(\{-2,1\},x)\log(x) \\
& + 92160N^2(-1+x)^2x(1+x)\mathrm{HPL}(\{2,-1\},x)\log(x) \\
& - 1440x\mathrm{HPL}(\{3\},x)\left(-((1+x)(-32+88N^2(-1+x)^2+59x-32x^2)\right) \\
& - 5760(1-x)x\mathrm{HPL}(\{-3\},x)\left(-((-27+20N^2)(-1+x^2)\right) \\
& - 11520N^2(-1+x)^2(1+x)\log(1-x)^2((-1+x)^2 - x\log(x)^2) + \\
& 240(1-x)\mathrm{HPL}(\{-2\},x)\left(-((-1+x)(-9(-1-9x+9x^2+x^3) + 8N^2(-6+(-15+2\pi^2)x\right. \\
& - 12x(-1+x^2)(-27+20N^2+48N^2\log(1-x))\log(x) + 48x(N^2(-5+x^2)+2(1+x^2))\log(x)^2) \\
& - 60\mathrm{HPL}(\{2\},x)\left(3((-1+x)(1+x)^2(3+x(-16+3x))\right. \\
& + 32N^2(1-x)x(4\pi^2(-1+x^2) - 3\log(x)(20(-1+x^2)+(3+11x^2)\log(x)))) \\
& - 16N^2(1-x)\left(630(-1+x)^3(1+x) - 2\pi^4x(-19+55x^2) - 30\pi^2(-1+x)^2(2+x(7+2x))\right. \\
& - 15\log(x)\left(-2(24+x(57+\pi^2(6-14x^2)+3x(-18+x(-13+4x))))\log(x)\right. \\
& + x(21-19x^2)\log(x)^3 + 12(-1+x)(9+x(4-\pi^2(1+x)+x(-16+3x)) \\
& - 360x(7(-1+x^2)+2(-7+15x^2)\log(x))\zeta(3)) \\
& + 180(1-x)(1+x)\log(1-x)\left(\log(x)\left((1+x)(3+x(-16+3x)) - 40(-1+x)x\log(x)\right)\right. \\
& + 32N^2(1-x)\left(6(-1+x)^2 + \log(x)\left(4+2(-2+\pi^2)x - x\log(x)(11+4\log(x))\right) + 36x\zeta(3)\right) + \\
& 30\left((36+x(930+64\pi^2(-1+x)(1+x^2)+3x(-435+x(-139+3x(71+x)))))\log(x)^2\right. \\
& + 2x(1+x)(86+x(-215+134x))\log(x)^3 + 8(-1+x)x(1+x^2)\log(x)^4 \\
& - 48x(1+x)(49+x(-103+49x))\zeta(3) \\
& + 2\log(x)\left(3+36(-1+x)^3(1+x(10+x))\log(1+x)\right. \\
& + x(336 - 2\pi^2(1+x)(32+x(-69+32x)) - 3x(337+x(-337+x(112+x))) \\
& + 768(-1+x)(1+x^2)\zeta(3)))) \Big)
\end{aligned}
$$

For comparison with EFT amplitudes, the small and large mass expansions are carried out through PolyLogTools [C. Duhr et al arXiv:1904.07279]

Quark Channel

We consider the DM pair production process,

$$q(p_1)\bar{q}(p_2) \to \chi\chi*$$

The amplitude can be expanded as-

$$\mathcal{M}_{qq\to\chi\chi*} = \bar{v}(p_2)\ \Gamma\ u(p_1)$$

$$\Gamma = \frac{\alpha_s^2}{(4\pi)^2}\left(\mathcal{M}_{LO} + \frac{\alpha_s}{4\pi}M_{NLO} + \mathcal{O}(\alpha_s^2)\right)$$

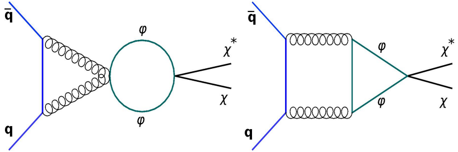
Where $p_1^2 = p_2^2 = m_q^2$ and $p_1.p_2 = s/2 - m_q^2$.
Choosing an ansatz for projector such that it results in trace and satisfy the projector condition, we get,

$$P = \frac{1}{(\text{Trace}[\not{p}_1\not{p}_2 - m_q^2 I])}\bar{u}(p_1)\quad v(p_2)$$

The form factor is given by,

$$\mathcal{F} = \frac{1}{2(s-4m_q^2)}\ \text{Trace}\left[(\not{p}_1\not{p}_2 - m_q^2 I)\ \Gamma\right]$$

The Feynman diagrams at 2-Loop order with maximum number of propagators are,



- Since $\not{p}_1 u(p_1) = \bar{v}(p_2)\not{p}_2 \;= 0$, it can be shown that the amplitude vanishes for massless quarks.
- Also, the quark leg with two vertices and a quark propagator lead to trace over odd number of γ matrices in form factor. Hence, a massless quark leg would give zero contribution.
- Only 1 Integral Family is sufficient and there are 20 Master Integrals after IBP reduction.
- From literature the Master Integrals are obtained from "Top-induced contributions to $H \to b\bar{b}$ and $H \to c\bar{c}$ at (α_s^3)" [Roberto Mondini et al arXiv:2006.03563]

Results - Quark Channel

Since this is a leading order amplitude, it must be finite and devoid of any divergences. Our result is finite. Parametrization of scaleless ratios,

$$-\frac{s}{m_\phi^2} = \frac{(1-w^2)^2}{w^2}, \; -\frac{m_q^2}{m_\phi^2} = \frac{(1-w^2)^2 z^2}{(1-z^2)^2 w^2}$$

$$\mathcal{M}_{LO} = \Bigg\{ ... + \frac{32w\left(w^2+1\right)z\left(w^4\left(z^4-4z^2+1\right)-2w^2\left(3z^4-8z^2+3\right)+z^4-4z^2+1\right)}{9m_\phi\left(w^2-1\right)^4\left(z^2+1\right)^3}$$

$$G(0,w)\left(-6G(0,-1,z)+6G(0,0,z)-6G(0,1,z)+\pi^2\right)+...\Bigg\}$$

Where $G(a1, a2, .., x)$ are Multiple Polylogarithms.

Method of Differential Equations

- The master integrals are solved using Canonical form of Differential Equations[3].
- Every Master Integral I_i is differentiated w.r.t. scaleless ratios.
- Since the MIs form a complete basis, using IBP identities $\partial_x \mathcal{I}_i$ are expressed in terms of MIs.
- The differential equations take the following form,

$$\partial_x \mathcal{I}_i(\kappa, y, z, \epsilon) = A^x_{ij}(\kappa, y, z, \epsilon) \mathcal{I}_j(\kappa, y, z, \epsilon)$$

  where x is $\kappa = \frac{s}{m_\phi^2}$, $y = \frac{t}{m_\phi^2}$ and $z = \frac{u}{m_\phi^2}$

- This gives us coupled partial differential eqns which are solved to obtain analytical results in terms of special functions such as Multiple Polylogs.
- Particularly after converting the Differential Equations to canonical form, we can iteratively integrate and obtain $I(x)$ for different orders of ϵ,

$$(d + \epsilon \tilde{A}(x_n)(\sum_{j=0}^{\infty} \overrightarrow{I}^j(x_n).\epsilon^j) = 0$$

[3] Johannes M. Henn arXiv:1304.1806.

Summary

- Methodology of Loop calculations
- 2-Loop Virtual corrections for gluon channel.
- Leading order 2-Loop contribution for quark channel.

What Next ?

- Computing Real corrections and NLO cross section.
- Constraints on DM parameters from relic density data.
- Compute NLO for Mono-jet case and constrain the parameters from Collider Data.

Thank You!

Backup slides I

At NLO DM+jet cross section is given by,

$$\sigma^{NLO} = \int_{\tau_0}^{1} dx_1 \int_{\tau_0/x_1}^{1} dx_2\, f_g(x_1) \int dx_2\, f_g(x_2)[\hat{\sigma}_B^{(0)}(gg \to \chi\chi^* g) + \hat{\sigma}_V^{(1)}(gg \to \chi\chi^* g)]$$
$$+ \int_{\tau_0}^{1} dx_1 \int_{\tau_0/x_1}^{1} dx_2\, f_g(x_1) \int dx_2\, f_g(x_2)[\hat{\sigma}_R^{(1)}(gg \to \chi\chi^* gg)]$$

Since

$$|M|^2 = |(\alpha_s^{3/2} M_B + \alpha_s^{5/2} M_v + \alpha_s^2 M_R)|^2$$

Collinear divergence, $\theta \to 0$

$\int \frac{dk}{(k+p_2)^2} = \int \frac{dk}{(k^2+p_2^2+2k.p_2)} = \int \frac{dk}{k^0.p_2^0 - |\bar{k}||\bar{p_2}|\cos\theta}$

Backup slides II

- There will be IR divergences which are cancelled by divergences due to phase space integrals.

$$\sigma = \frac{1}{4s}\int \frac{d^3p_1}{(2\pi)^3 2E_1}\frac{d^3p_2}{(2\pi)^3 2E_2}(2\pi)^4\delta^4(p_1+p_2-\sum P_f)|M_{fi}|^2$$

- It can be soft where $p_i \to 0$, collinear where $\theta \to 0$ or both.
- They are necessary for finite predictions as KLN theorem guarantees infrared divergences cancel at the same order for partonic cross sections.

Gluon propagator counter term-

$$\delta_g = \frac{g_s^2 \delta^{ab}}{12} \{2(22 + 4nf)p_1^\mu p_1^\nu + (12m_\phi^2 - (53 + 8 * nf)p.p)g^{\mu\nu}\}$$

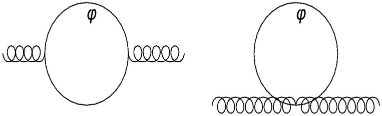
Phi propagator counter term-

$$\delta_{m_\phi} = -\frac{g_s^2 \delta^{ij}}{3}(8m_\phi^2) \quad \text{,Mass renormalisation}$$

$$\delta_\phi = \frac{g_s^2 \delta^{ij}}{3}(4p.p) \quad \text{,Wavefunction renormalisation}$$

Modifications in strong coupling constant α_s

For the case of gluon propagator, the additional diagrams are-



Z_g is the coupling renormalization constant. For DM+QCD it is given by,

$$Z_{g,DM}^{MS} = 1 - \frac{g_R^2}{(4\pi)^2} \frac{(11C_a - 4T_R N_f - 2T_R N_\phi)}{6} \frac{2}{4-d} + \mathcal{O}(g_R^4)$$

This corresponds to β functions at 1-Loop as,

$$\beta_{DM} = \frac{1}{(4\pi)^2} \frac{(11C_a - 4T_R N_f - T_R N_\phi)}{3}$$

The running in α is given by,

$$\alpha(\mu_R) = \frac{\alpha(\tilde{\mu}_R)}{1 + (4\pi)\alpha(\tilde{\mu}_R)\beta \log\left(\frac{\mu_R^2}{\tilde{\mu}_R^2}\right)}$$

Results I

The pole structure at two loop can be

$$\mathcal{M} = \frac{1}{\epsilon_{IR}^2}\{...\} + \frac{1}{\epsilon_{UV/IR}^1}\{...\} + \mathcal{O}(\epsilon^0)$$

Since this is a Next-to-leading order amplitude, we expect divergences.

- The Infrared divergences will have Universal IR structure at NLO as[3]

$$M_{IR} = -g_s^2(\frac{-s}{4\pi\mu^2})^{-\epsilon}e^{-\gamma_E\epsilon}(\frac{2\beta}{\epsilon} + \frac{2N_c}{\epsilon^2})\mathcal{M}_{LO}$$

$$M_{UV} = -(\frac{-s}{\mu^2})^{-\epsilon}(2\ \delta Z_g\ \mathcal{M}_{LO} + \delta Z_m \frac{\partial}{\partial m_\phi}\mathcal{M}_{LO})$$

  where

$$\delta Z_g = -g_s^2\frac{\beta}{\epsilon}$$

- A combination of UV renormalization and IR subtraction should make the amplitude finite.

Results II

$$\mathrm{CoEpsInv} = -\frac{2}{(x-1)^2(x+1)}\left(-24(x+1)x\mathrm{HPL}(\{-3\},x) + 12(x+1)x\log(x)\mathrm{HPL}(\{-2\},x)\right.$$
$$+ 18x^2\zeta(3) + 7x^3 - 7x^2 - 4x^2\log^3(x) + 6x^2\log(1-x)\log^2(x) + x^2\log^2(x)$$
$$- 6x^3\log(1-x) + 6x^2\log(1-x) + \pi^2x^2\log(x) - 10x^2\log(x) + 18x\zeta(3)$$
$$- 7x - 4x\log^3(x) + 6x\log(1-x)\log^2(x) + x\log^2(x) + 6x\log(1-x)$$
$$\left. + \pi^2x\log(x) + 4x\log(x) - 6\log(1-x) + 6\log(x) + 7\right)$$

$$\mathrm{UV}_{CT} = \frac{2\beta\left(\frac{x\log^2(x)}{(x-1)^2} - 1\right)}{\mathrm{eps}} - \frac{4x\left(\log(x)\left(\left(x^2-1\right)\log(x) + 6x\right) - 3\left(x^2-1\right)\right)}{\mathrm{eps}(x-1)^3(x+1)}$$

$$\alpha_R(\mu^2) = \frac{\alpha_R(\mu_0^2)}{1 + \alpha_R(\mu_0^2) b_0 log \frac{\mu^2}{\mu_0^2}}$$

Plot of α_S vs μ-