[made with unstability.ai]

Dissecting Multi-Higgs Boson Production in New Physics Models

Andreas Papaefstathiou

Kennesaw State University, Georgia, USA

KENNESAW STATE UNIVERSITY

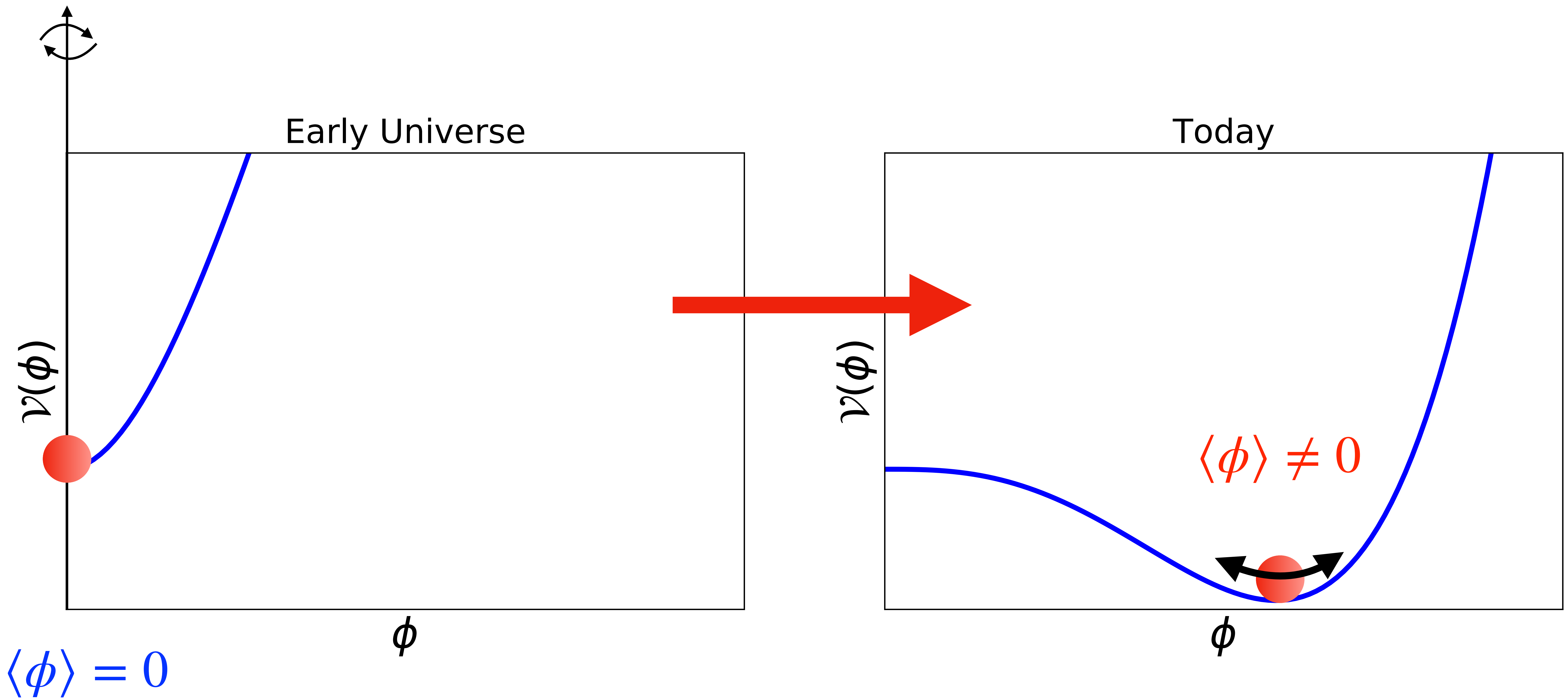
Extended Scalar Sectors From All Angles
@ CERN, October 21-26, 2024

Symmetry Breaking

Breaking the Symmetry in the Standard Model

$$\mathcal{V}(\phi) = \bullet\,|\phi|^2 + \blacksquare\,|\phi|^4$$

Higgs field (ϕ) potential

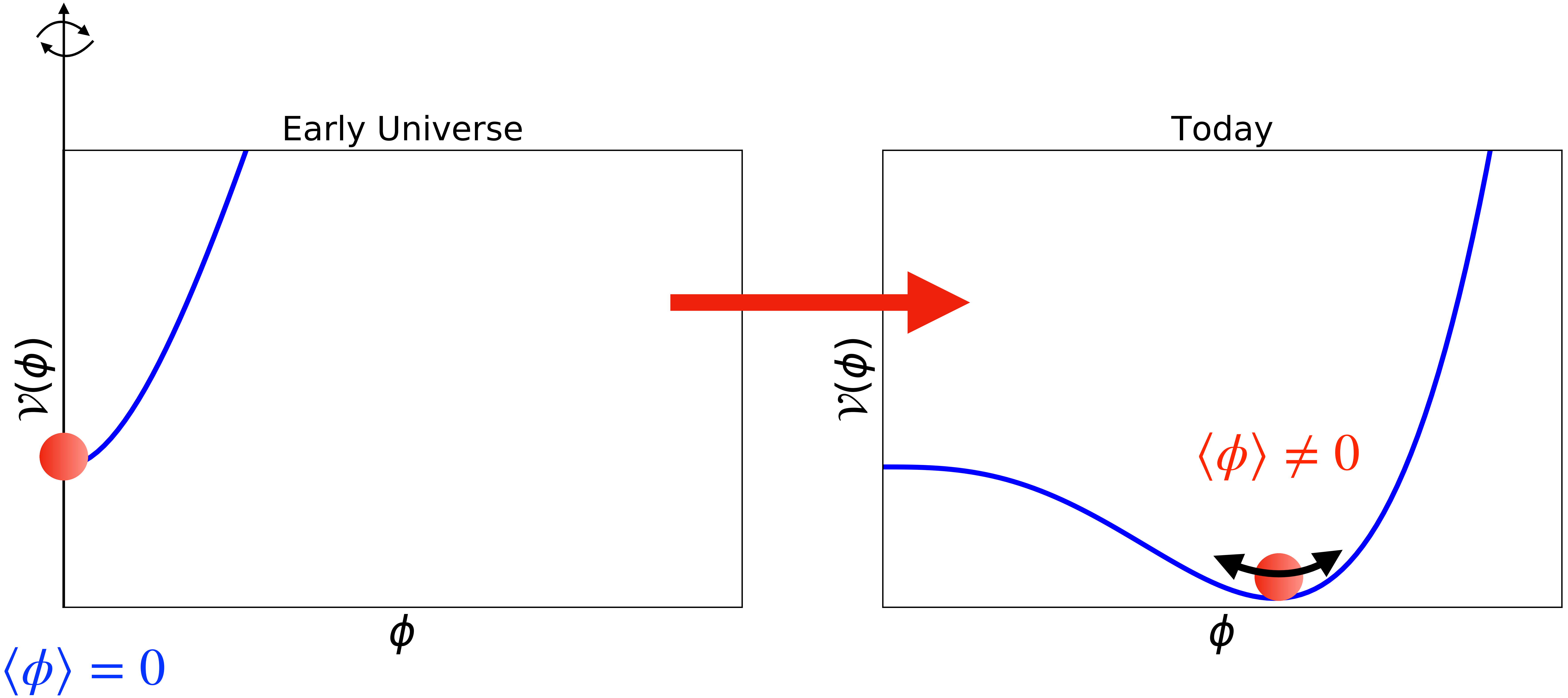


Breaking the Symmetry in the Standard Model

$$\mathcal{V}(\langle\phi\rangle + h) = \bullet\, h^2 + \blacktriangle\, h^3 + \blacksquare\, h^4$$

→ h **is the Higgs boson! (LHC, 2012)**

Higgs field (ϕ) potential



Motivation: The Higgs Boson's Potential

$\mathcal{V}(\langle\phi\rangle + h) = \bullet h^2 + \blacktriangle h^3 + \blacksquare h^4$ → **the Higgs boson's self-interactions.**

→ **Determine shape of this potential by measuring:**

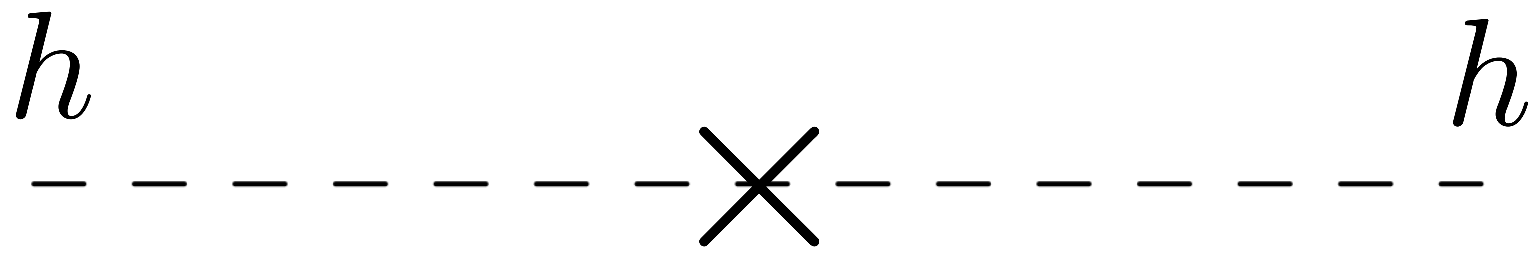
$$\{\bullet, \blacktriangle, \blacksquare\}$$

Motivation: The Higgs Boson’s Potential

$\mathcal{V}(\langle\phi\rangle + h) = \bullet h^2 + \blacktriangle h^3 + \blacksquare h^4$ → **the Higgs boson’s self-interactions.**

→ **Determine shape of this potential by measuring:**

$$\{\bullet, \blacktriangle, \blacksquare\}$$



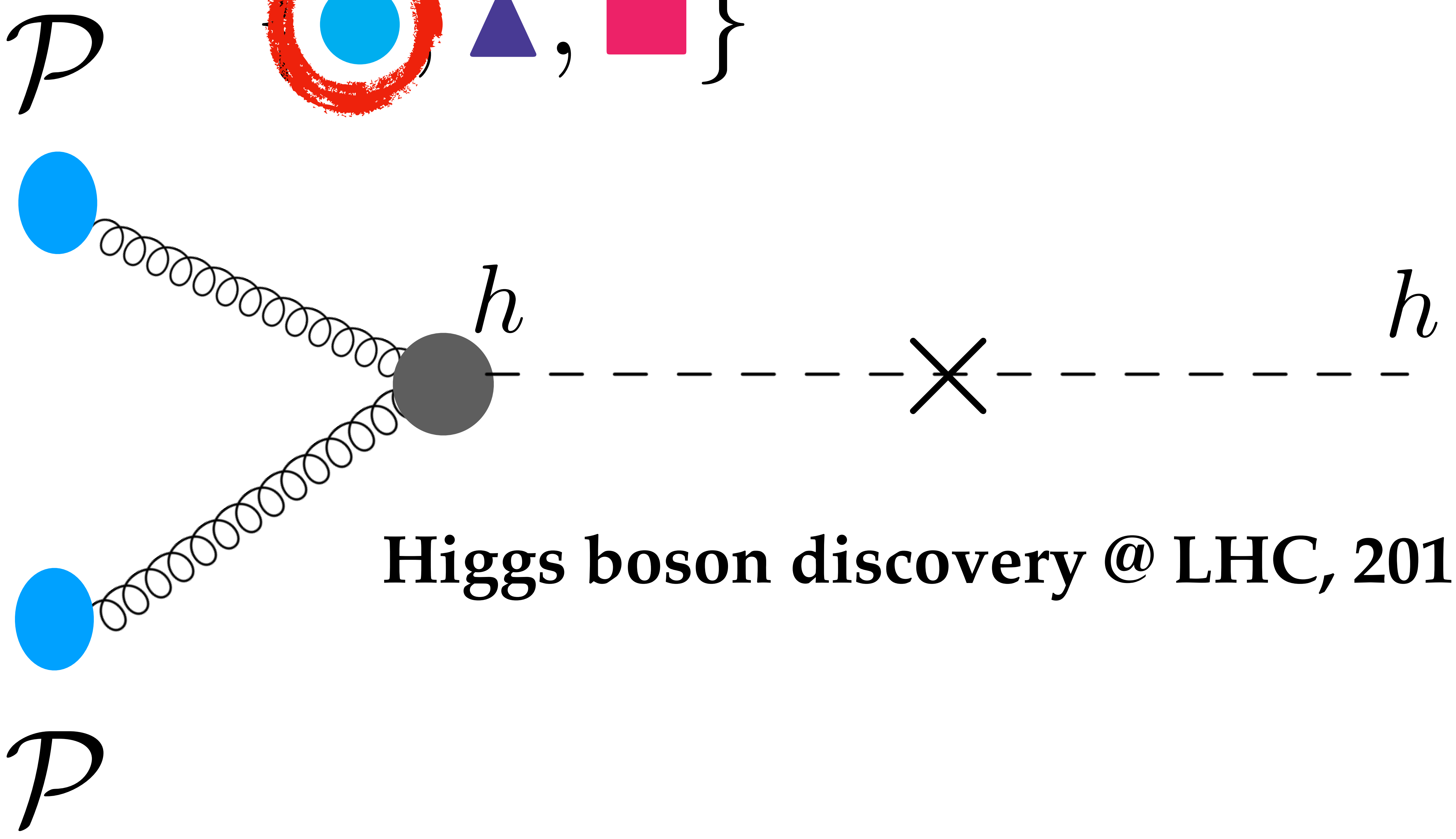
Higgs boson discovery @ LHC, 2012 ✅

Motivation: The Higgs Boson's Potential

$\mathcal{V}(\langle\phi\rangle + h) = \bullet h^2 + \blacktriangle h^3 + \blacksquare h^4$ → **the Higgs boson's self-interactions.**

→ **Determine shape of this potential by measuring:**

$\{\bullet, \blacktriangle, \blacksquare\}$

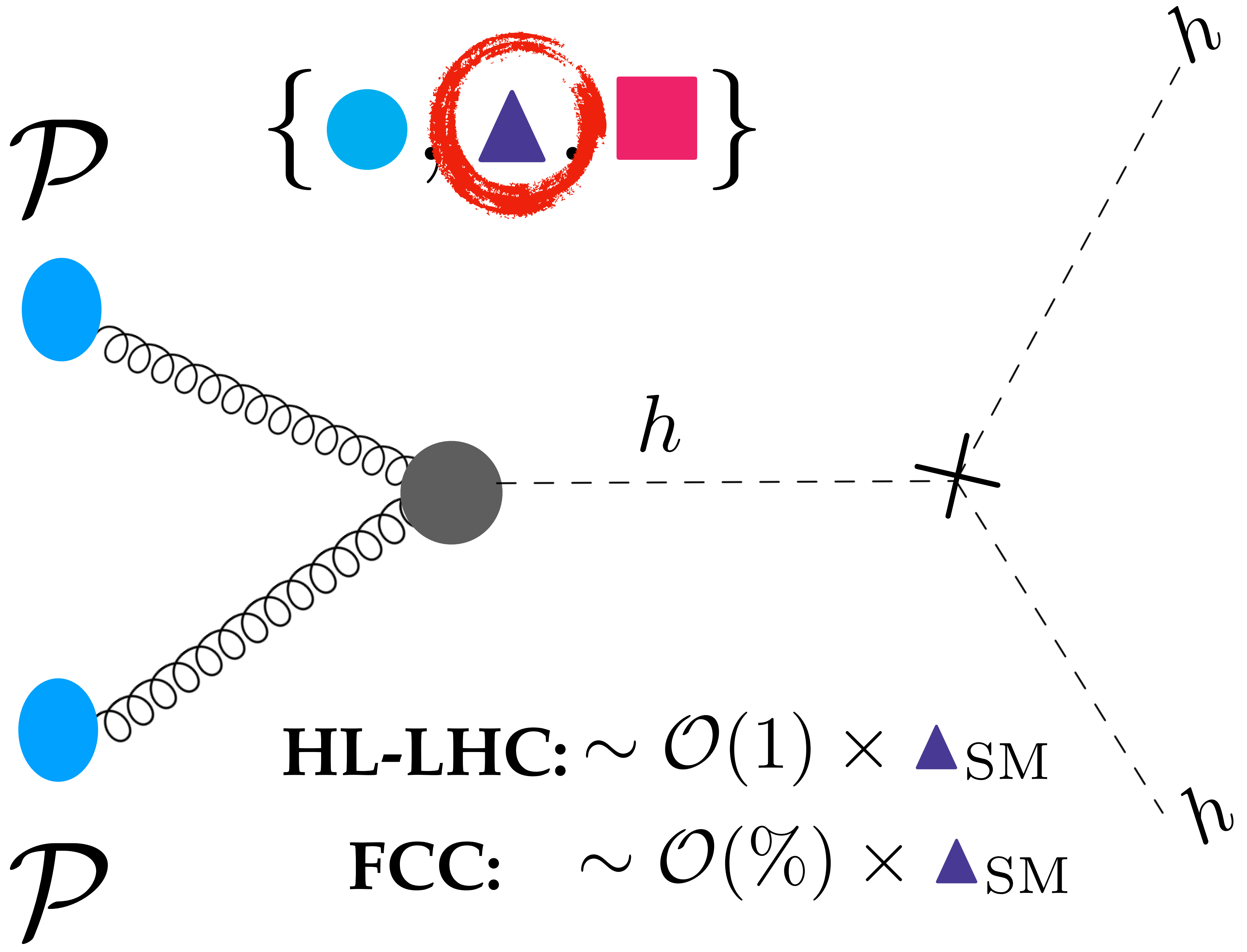


Higgs boson discovery @ LHC, 2012 ✅

Motivation: The Higgs Boson's Potential

$\mathcal{V}(\langle\phi\rangle + h) = \bullet h^2 + \blacktriangle h^3 + \blacksquare h^4$ → **the Higgs boson's self-interactions.**

→ **Determine shape of this potential by measuring:**



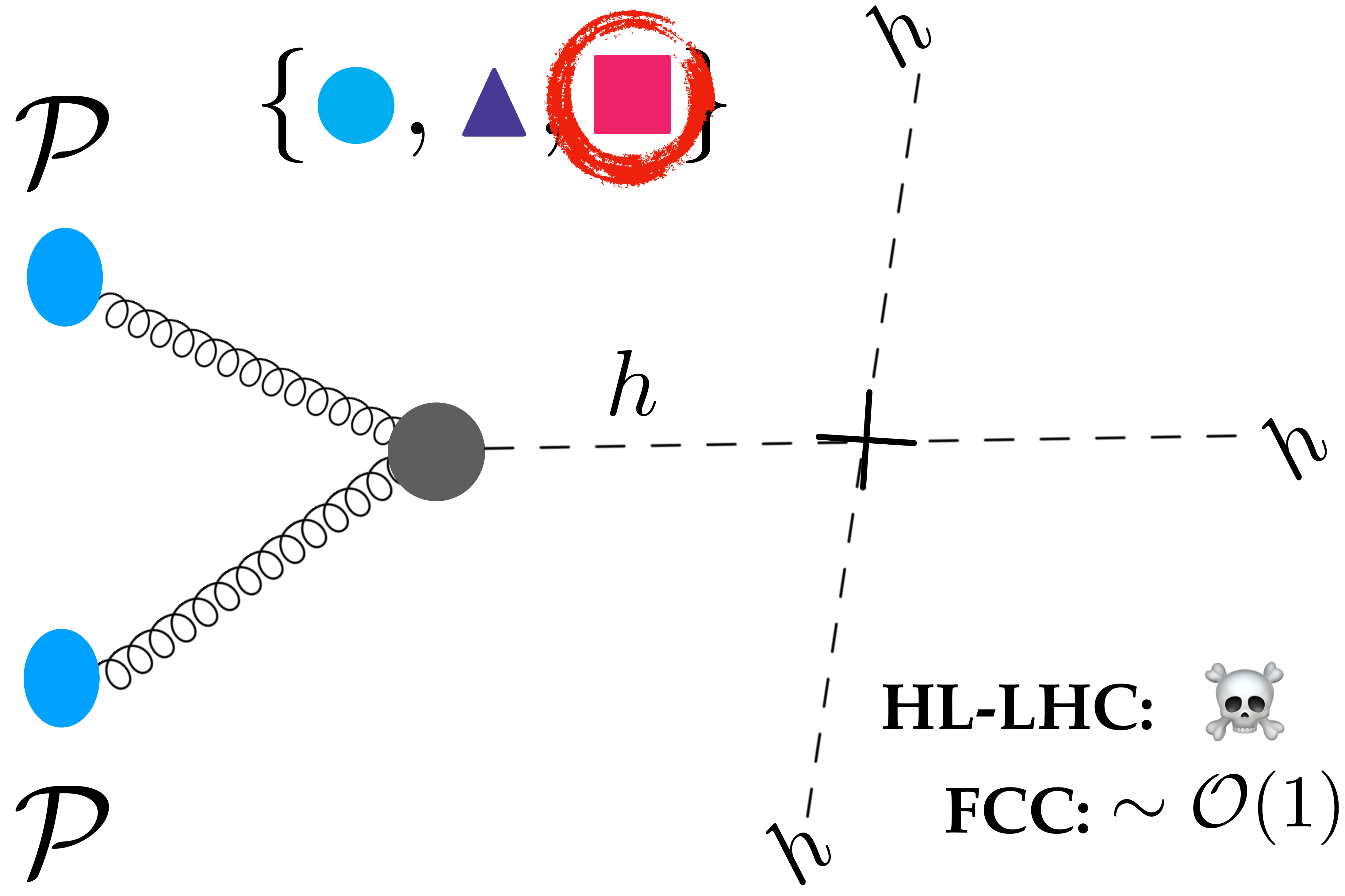
HL-LHC: $\sim \mathcal{O}(1) \times \blacktriangle_{\mathrm{SM}}$

FCC: $\sim \mathcal{O}(\%) \times \blacktriangle_{\mathrm{SM}}$

Motivation: The Higgs Boson's Potential

$\mathcal{V}(\langle\phi\rangle + h) = \bullet h^2 + \blacktriangle h^3 + \blacksquare h^4$ → **the Higgs boson's self-interactions.**

→ **Determine shape of this potential by measuring:**



HL-LHC: ☠️

FCC: $\sim \mathcal{O}(1) \times \blacksquare_{\mathrm{SM}}$

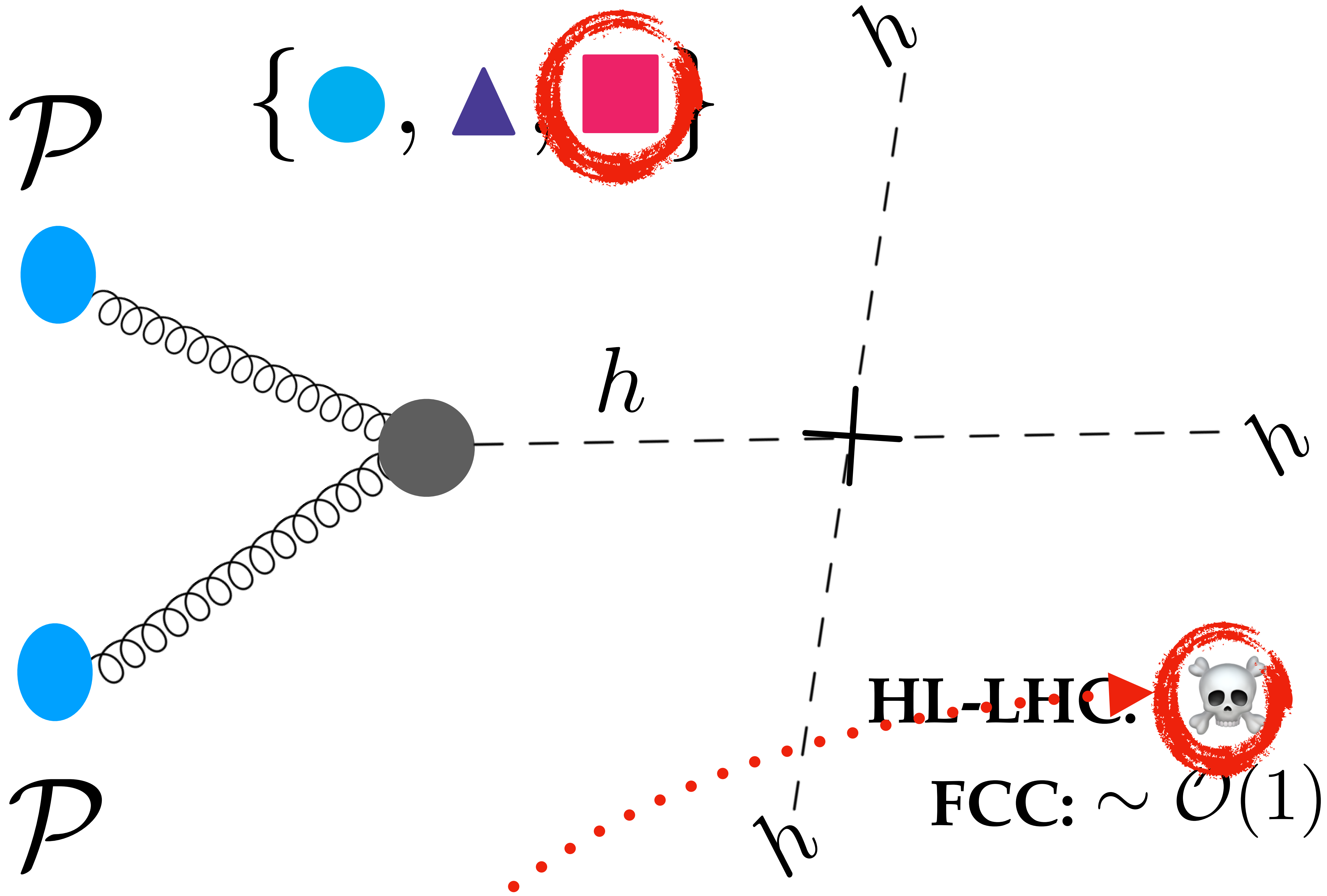
[e.g. AP, Sakurai, **arXiv:1508.06524**,
Fuks, Kim, Lee, **arXiv:1510.07697** & **arXiv:1704.04298**,
AP, Tetlalmatzi-Xolocotzi, Zaro, **arXiv:1909.09166**, …]

Motivation: The Higgs Boson's Potential

$$\mathcal{V}(\langle\phi\rangle + h) = \bullet h^2 + \blacktriangle h^3 + \blacksquare h^4$$ → **the Higgs boson's self-interactions.**

→ **Determine shape of this potential by measuring:**

$\{\bullet, \blacktriangle, \blacksquare\}$



HL-LHC: ☠

FCC: $\sim \mathcal{O}(1) \times \blacksquare_{\mathrm{SM}}$

[AP, Robens, Tetlalmatzi-Xolocotzi, **arXiv:2101.00037** +
Karkout, AP, Postma, Tetlalmatzi-Xolocotzi, van de Vis, du Pree, **arXiv:2404.12425**,
AP, Tetlalmatzi-Xolocotzi, **aXiv:2312.13562**]
SEE LATER!

[e.g. AP, Sakurai, **arXiv:1508.06524**,
Fuks, Kim, Lee, **arXiv:1510.07697** & **arXiv:1704.04298**,
AP, Tetlalmatzi-Xolocotzi, Zaro, **arXiv:1909.09166**, …]

SM Multi-Higgs Boson Production "Fun" Facts

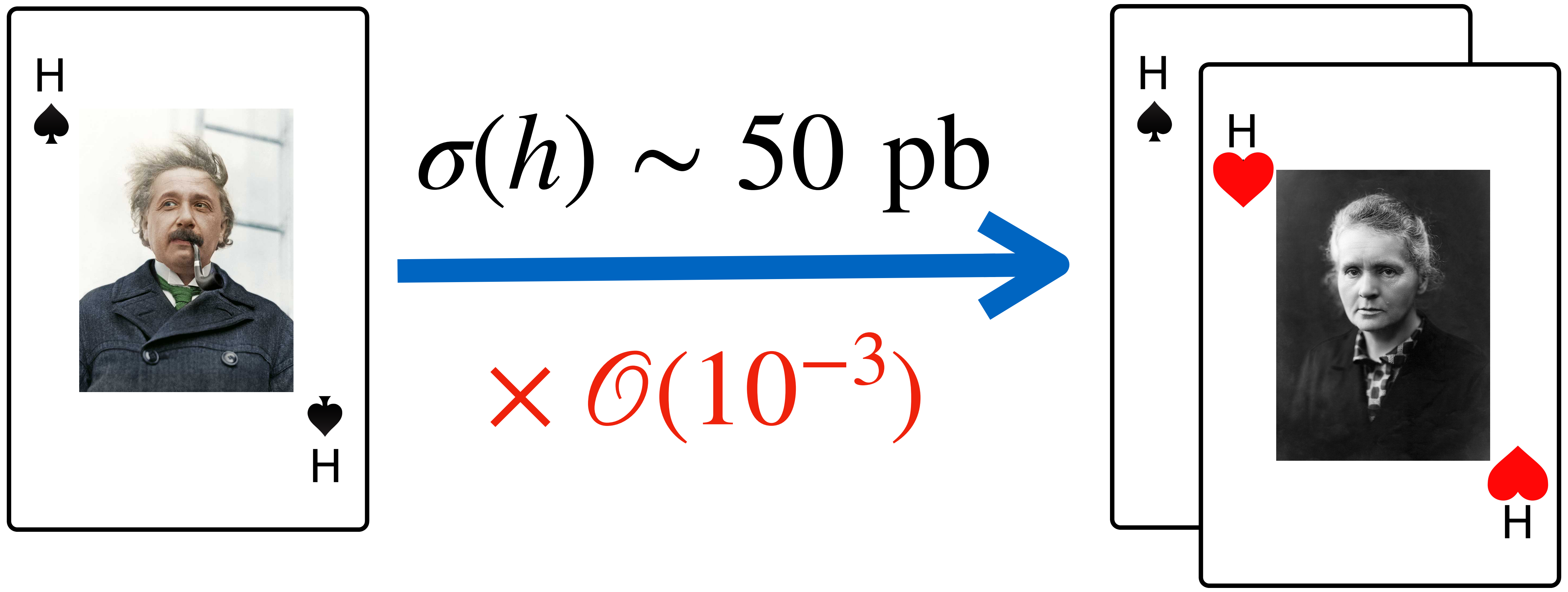
- ∃ factor of $\mathcal{O}(10^{-3})$ each time you "draw" an extra Higgs boson @ pp colliders.

$\sigma(h) \sim 50$ pb

SM, 14 TeV

SM Multi-Higgs Boson Production “Fun” Facts

- ∃ factor of $\mathcal{O}(10^{-3})$ each time you “draw” an extra Higgs boson @ pp colliders.



$\sigma(h) \sim 50$ pb

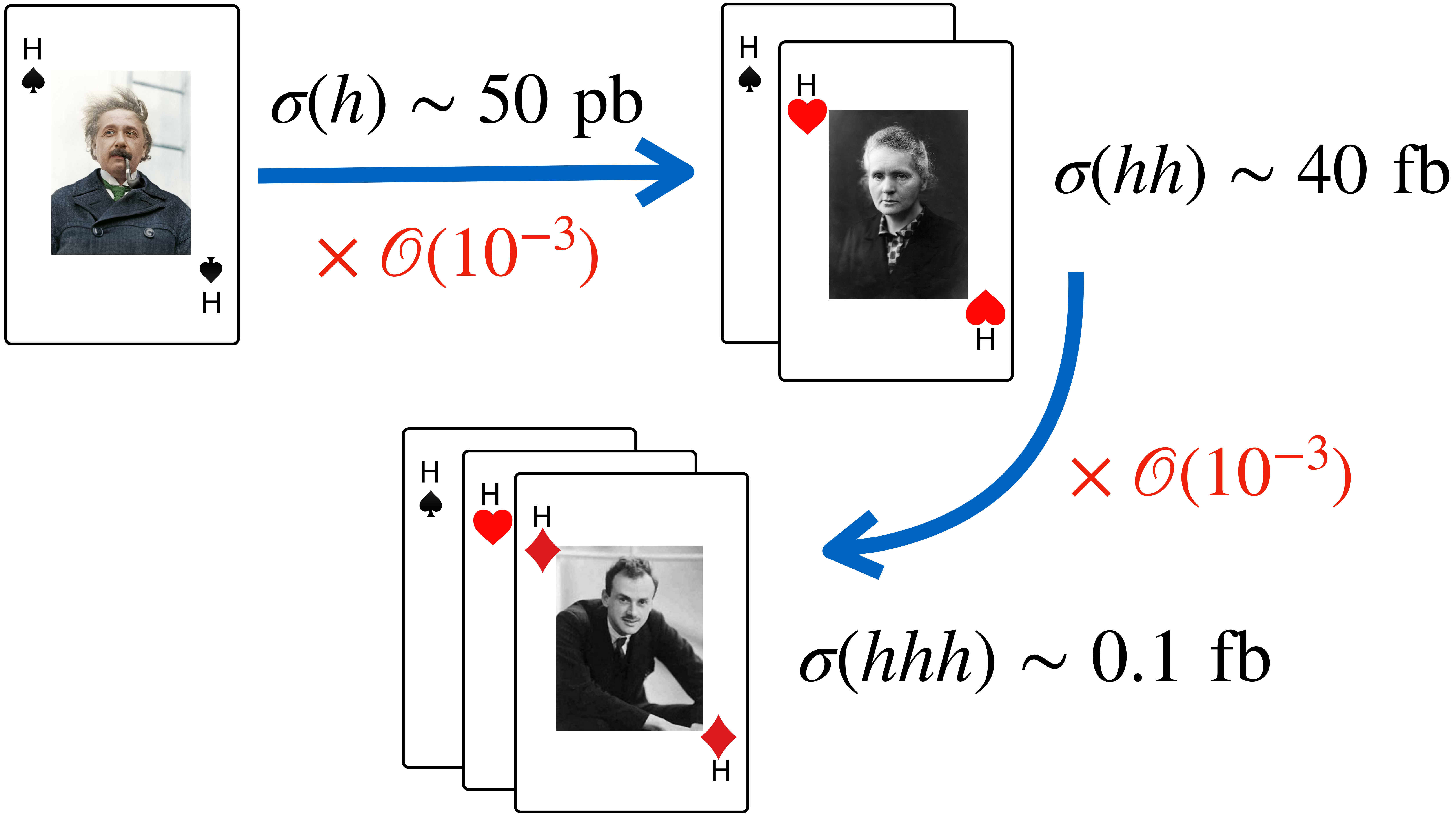
$\times\, \mathcal{O}(10^{-3})$

$\sigma(hh) \sim 40$ fb

SM, 14 TeV

SM Multi-Higgs Boson Production “Fun” Facts

- $\exists$ factor of $\mathcal{O}(10^{-3})$ each time you “draw” an extra Higgs boson @ pp colliders.



$\sigma(h) \sim 50$ pb

$\times\, \mathcal{O}(10^{-3})$

$\sigma(hh) \sim 40$ fb

$\times\, \mathcal{O}(10^{-3})$

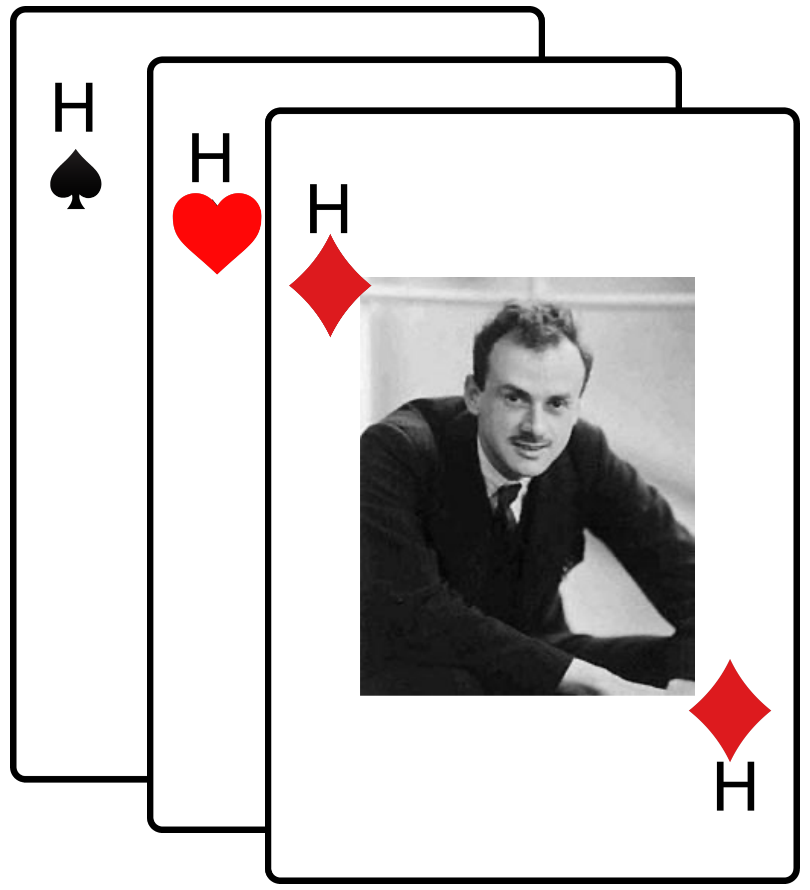
$\sigma(hhh) \sim 0.1$ fb

SM, 14 TeV

SM Multi-Higgs Boson Production "Fun" Facts

- ∃ factor of $\mathcal{O}(10^{-3})$ each time you "draw" an extra Higgs boson @ pp colliders.

SM, 14 TeV

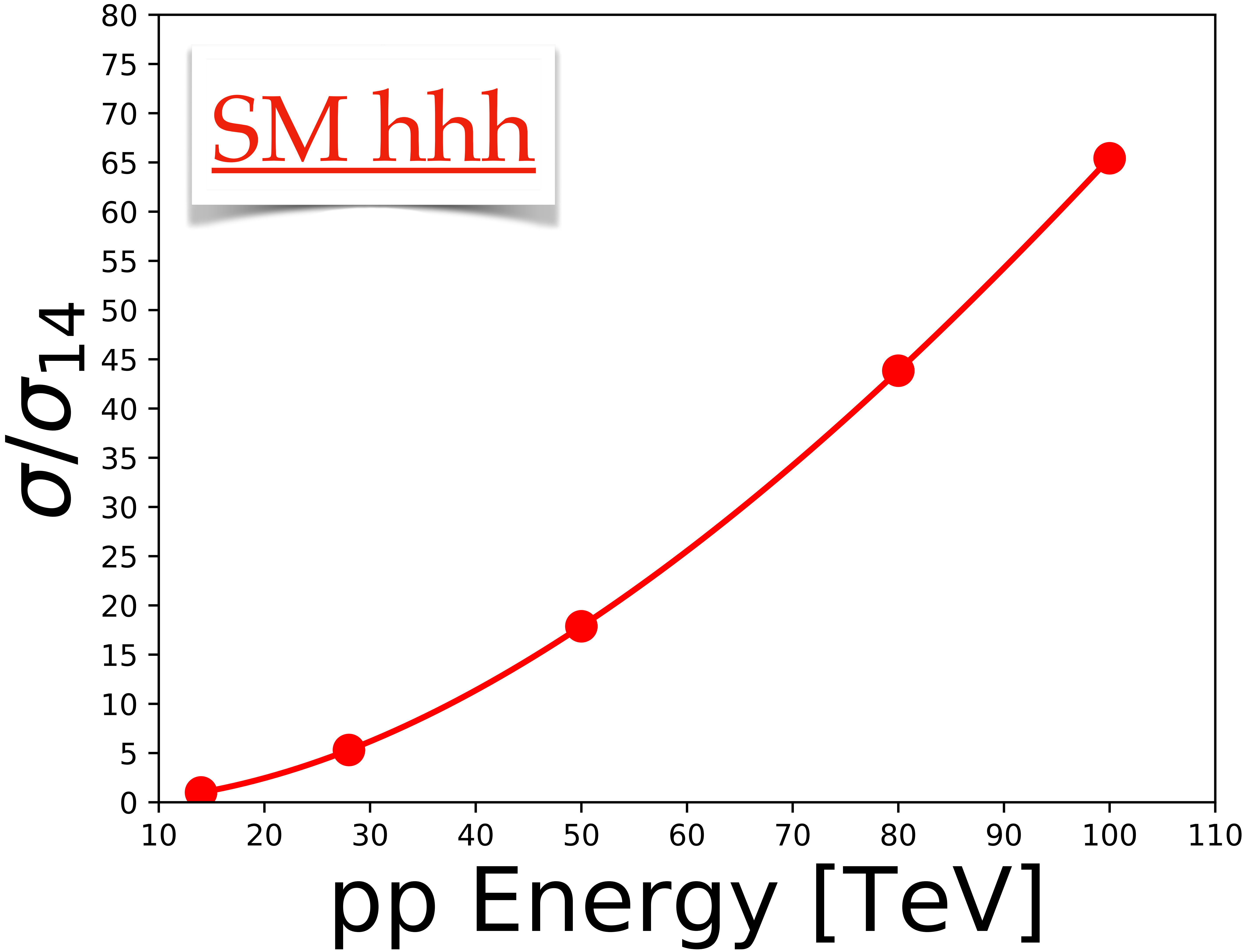


$\times\,\mathcal{O}(10^{-3})$

$\sigma(hhh) \sim 0.1$ fb

SM hhh Boson Production “Fun” Facts

- Cranking up the **pp COM energy** could help!

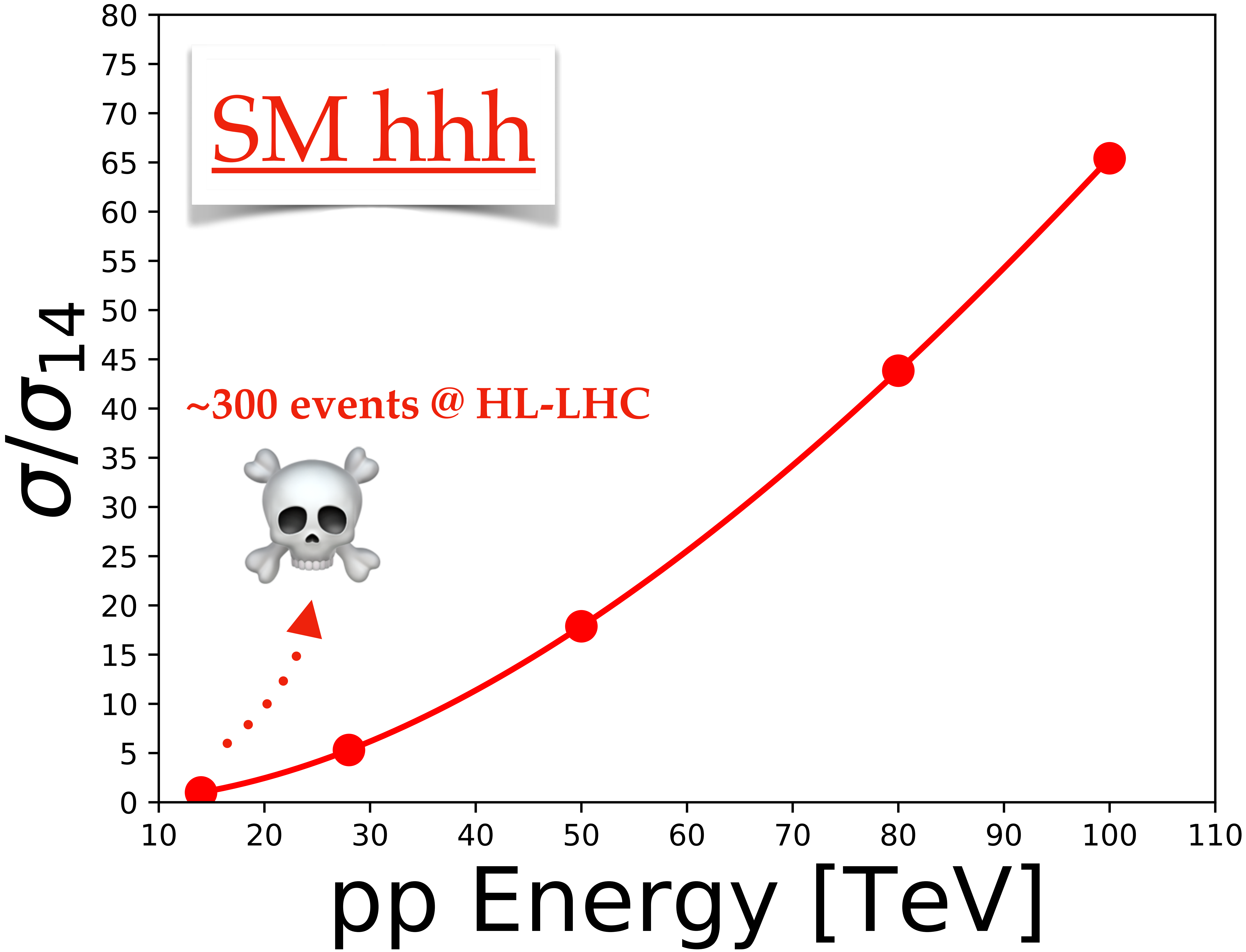


~ ×60 increase in cross section as pp COM 14 TeV → 100 TeV.

Andreas Papaefstathiou

SM hhh Boson Production "Fun" Facts

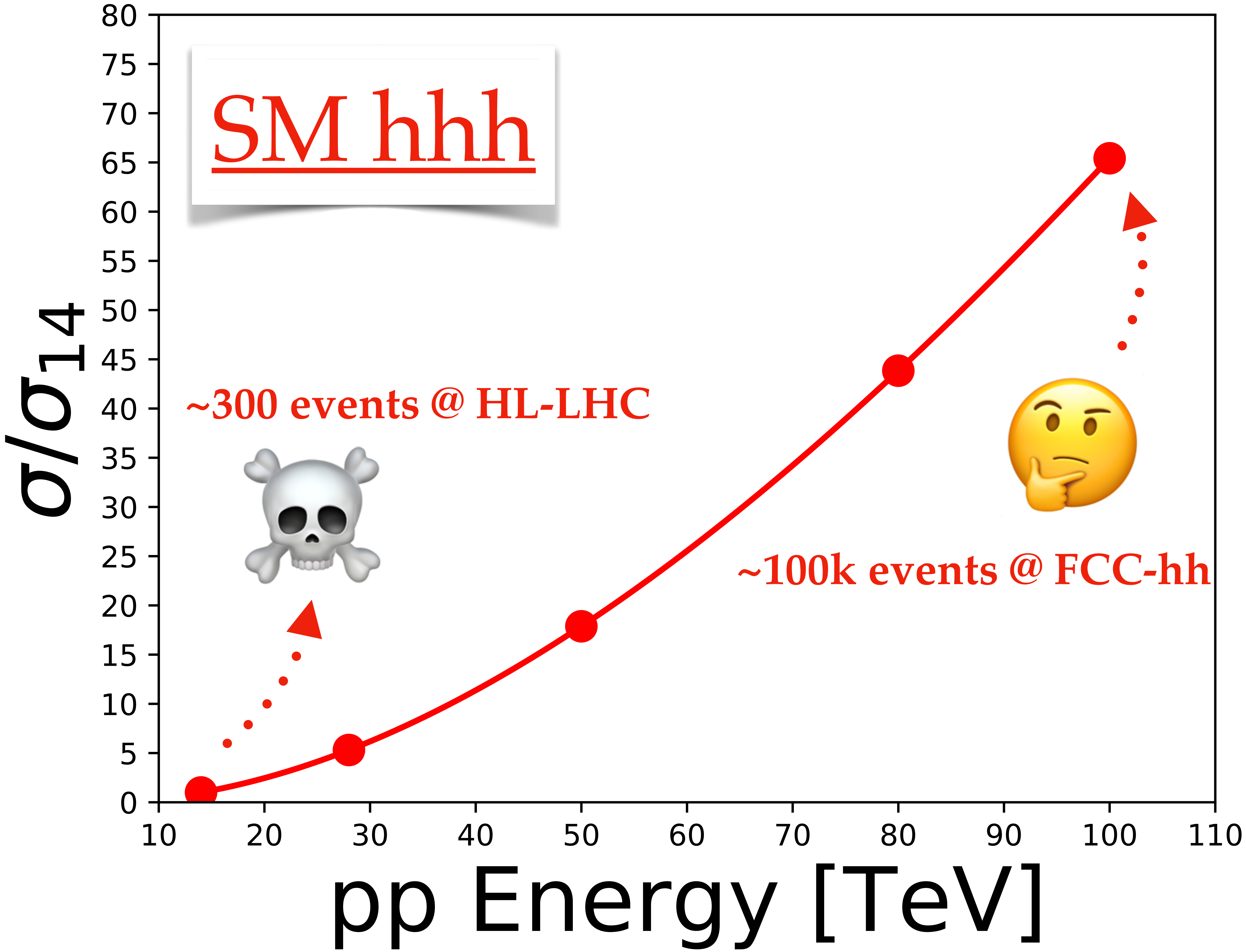
- Cranking up the **pp COM energy** could help!



~ ×60 increase in cross section as pp COM 14 TeV → 100 TeV.

SM hhh Boson Production “Fun” Facts

- Cranking up the **pp COM energy** could help!



~300 events @ HL-LHC

~100k events @ FCC-hh

~ ×60 increase in cross section as pp COM 14 TeV → 100 TeV.

THE SECRET INGREDIENT IS ALWAYS LOVE

THE SECRET
INGREDIENT
IS ALWAYS
~~LOVE~~ NEW PHYSICS

THE SECRET INGREDIENT IS ALWAYS ~~LOVE~~ NEW PHYSICS

Here:

A. **hhh** in SM+2 singlet scalar fields,

B. **hhh** with anomalous couplings.

A. **hhh** in SM+2 singlet scalar fields,

SM + Two Real Singlet Scalars [= TRSM]

- Add **two real singlet scalar fields** S, X**.**
- & impose **discrete** $\mathcal{Z}_2$ **symmetries:** $\mathcal{Z}_2^S : S \to -S,\ X \to X$

  $\mathcal{Z}_2^X : X \to -X,\ S \to S$

⇒ TRSM scalar potential:

$$\begin{aligned}\mathcal{V}(\phi, S, X) = {} & \bullet\, |\phi|^2 + \blacksquare |\phi|^4 + \bullet S^2 + \blacksquare S^4 + \bullet X^2 + \blacksquare X^4 \\ & + \blacksquare S^2 X^2 \\ & + \blacksquare |\phi|^2 S^2 + \blacksquare |\phi|^2 X^2\end{aligned}$$

SM + Two Real Singlet Scalars [= TRSM]

- Go through electroweak symmetry breaking…

⇒ Get **three** scalar bosons: $h_1, h_2, h_3 \to h_1 \approx$ SM-like Higgs boson.

⇒ **hhh** detectable at the LHC! [**AP**, Robens, Tetlalmatzi-Xolocotzi, **arXiv:2101.00037**]

✨**see Gilberto Tetlalmatzi-Xolocotzi's talk!**✨

SM + Two Real Singlet Scalars [= TRSM]

- Go through electroweak symmetry breaking…

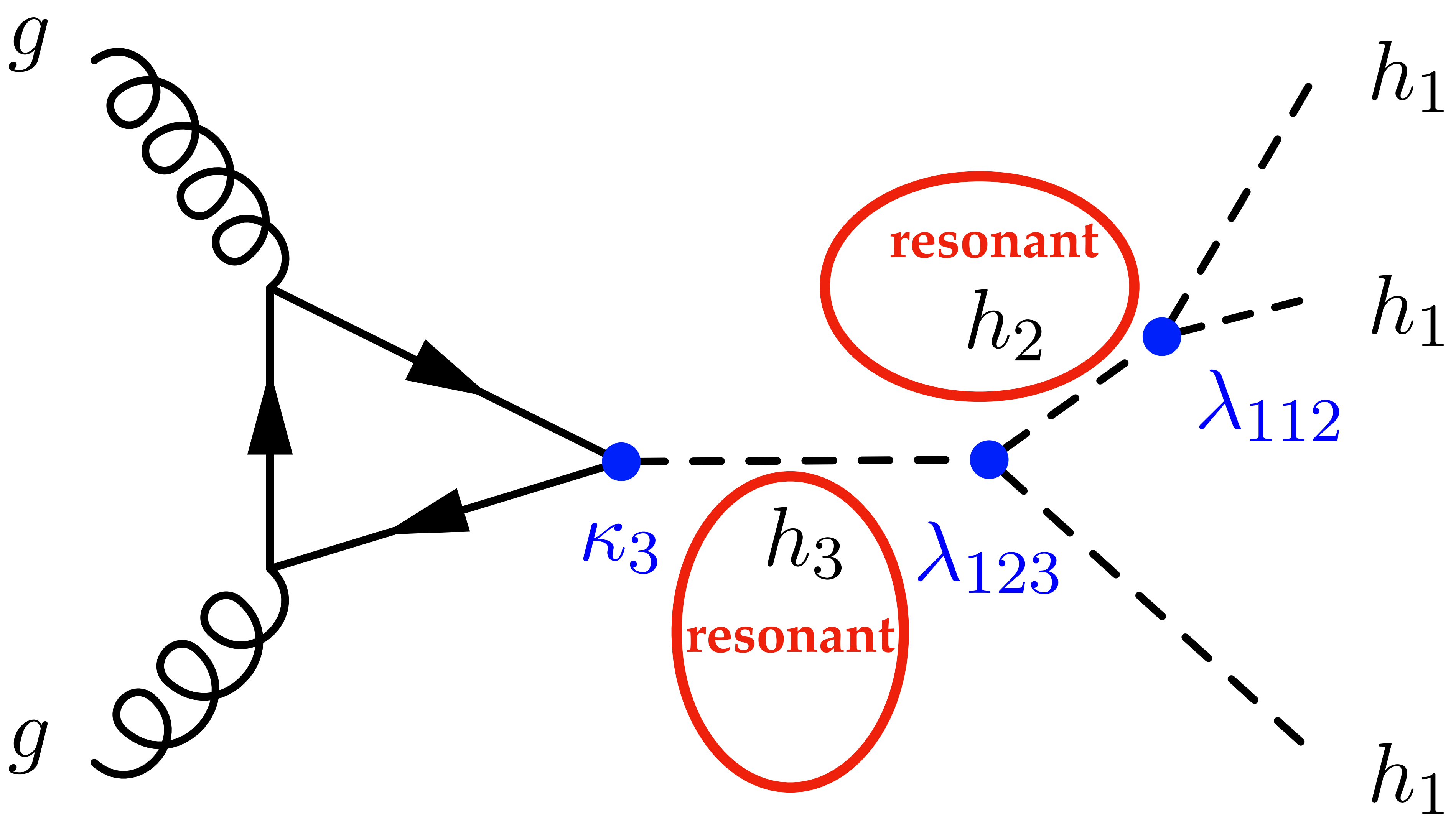
⇒ Get **three** scalar bosons: $h_1, h_2, h_3 \to h_1 \approx$ SM-like Higgs boson.

⇒ **hhh** detectable at the LHC! [**AP**, Robens, Tetlalmatzi-Xolocotzi, **arXiv:2101.00037**]

✨see Gilberto Tetlalmatzi-Xolocotzi's talk!✨

$$\text{through:} \quad pp \to h_3 \to h_2 h_1 \to h_1 h_1 h_1$$

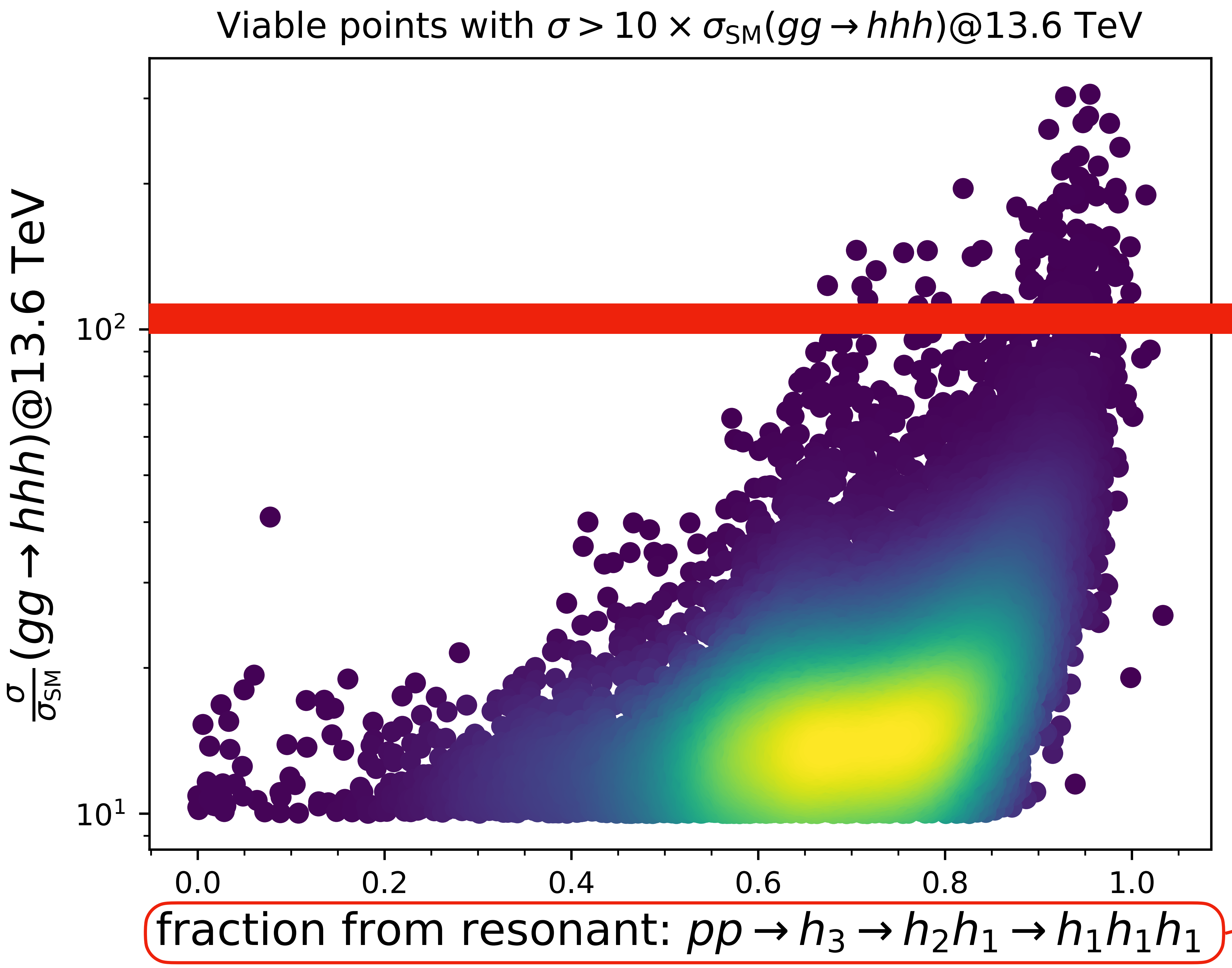
→ **"Double-Resonant Enhancement"!**



hhh in the TRSM

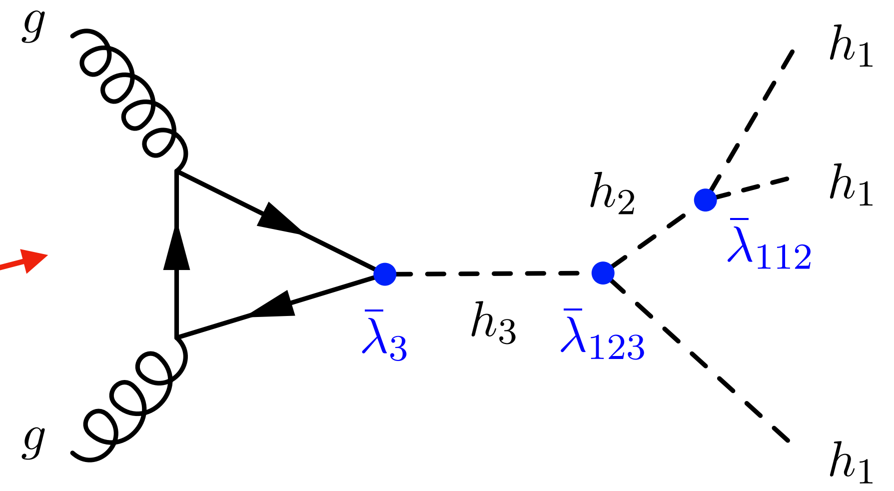
[Karkout, **AP**, Postma, Tetlalmatzi-Xolocotzi,
van de Vis, du Pree, **arXiv:2404.12425**]

Enhancement over SM



Enhancements $\mathcal{O}(100) \times \mathrm{SM}$!

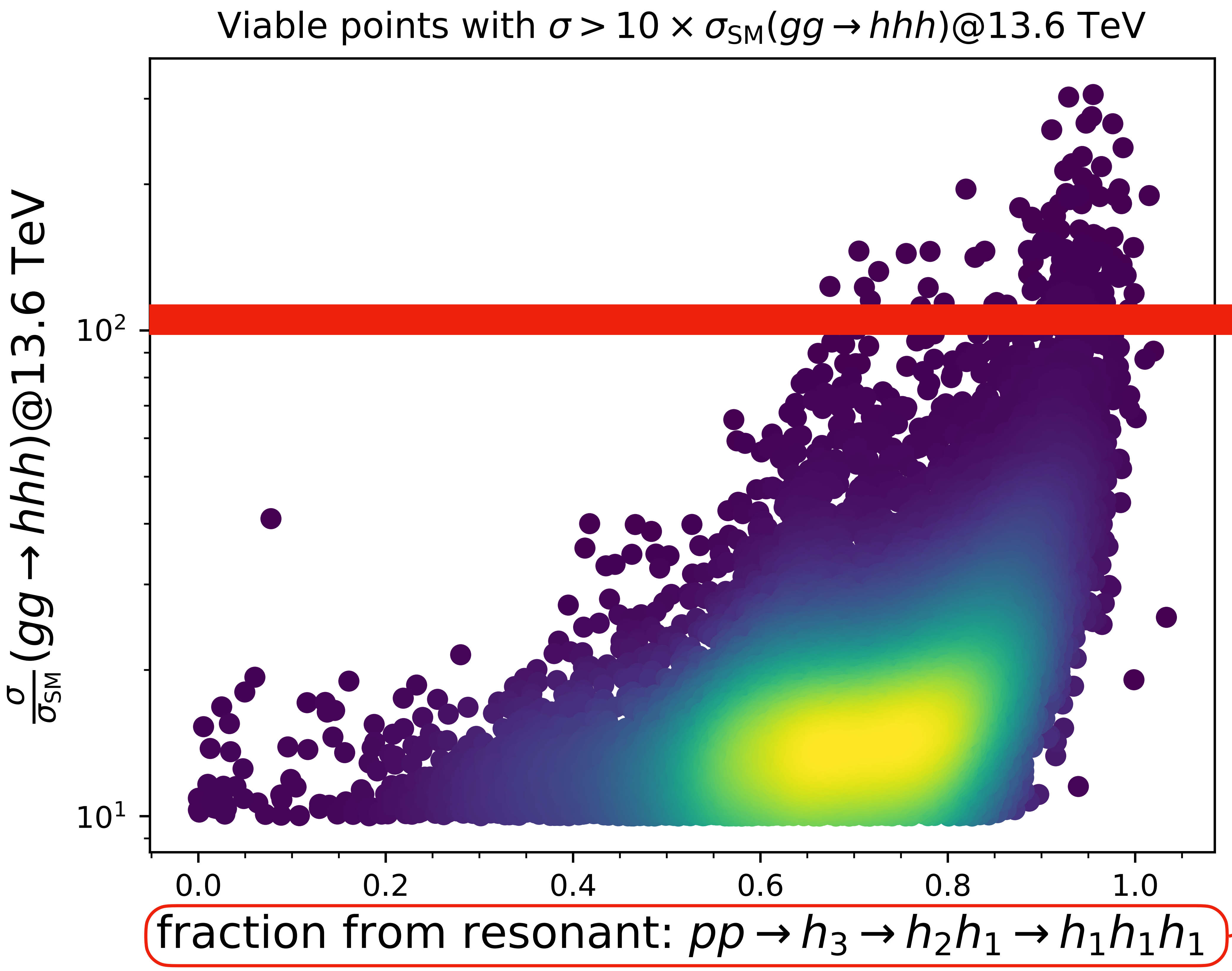
= How much of the total cross section comes from… ?



hhh in the TRSM

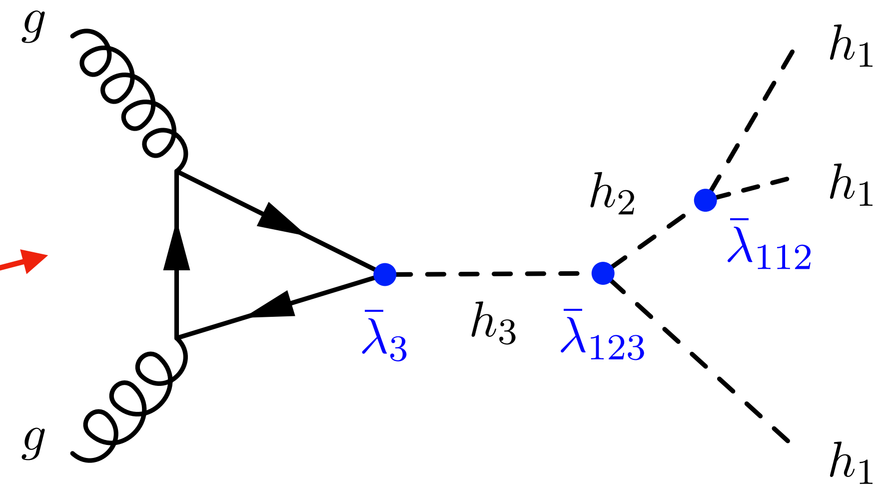
[Karkout, **AP**, Postma, Tetlalmatzi-Xolocotzi, van de Vis, du Pree, **arXiv:2404.12425**]

Enhancement over SM



Enhancements $\mathcal{O}(100) \times \mathrm{SM}$!

= How much of the total cross section comes from… ?



- Narrow-width ($\Gamma_2 \ll M_2$ & $\Gamma_3 \ll M_3$) double-resonant production **suggests a "simplified" factorized approach**.
- Coming soon! [**AP**, Tetlalmatzi-Xolocotzi, **arXiv:24⁉.⁉⁉**]

FO-EWPT and hhh in the TRSM

- *Q*: Can there be a **First-Order Electroweak Phase Transition** in the **TRSM**, related to **electro-weak baryogenesis**?

- and if so, will this lead to **enhanced hhh at the LHC?**

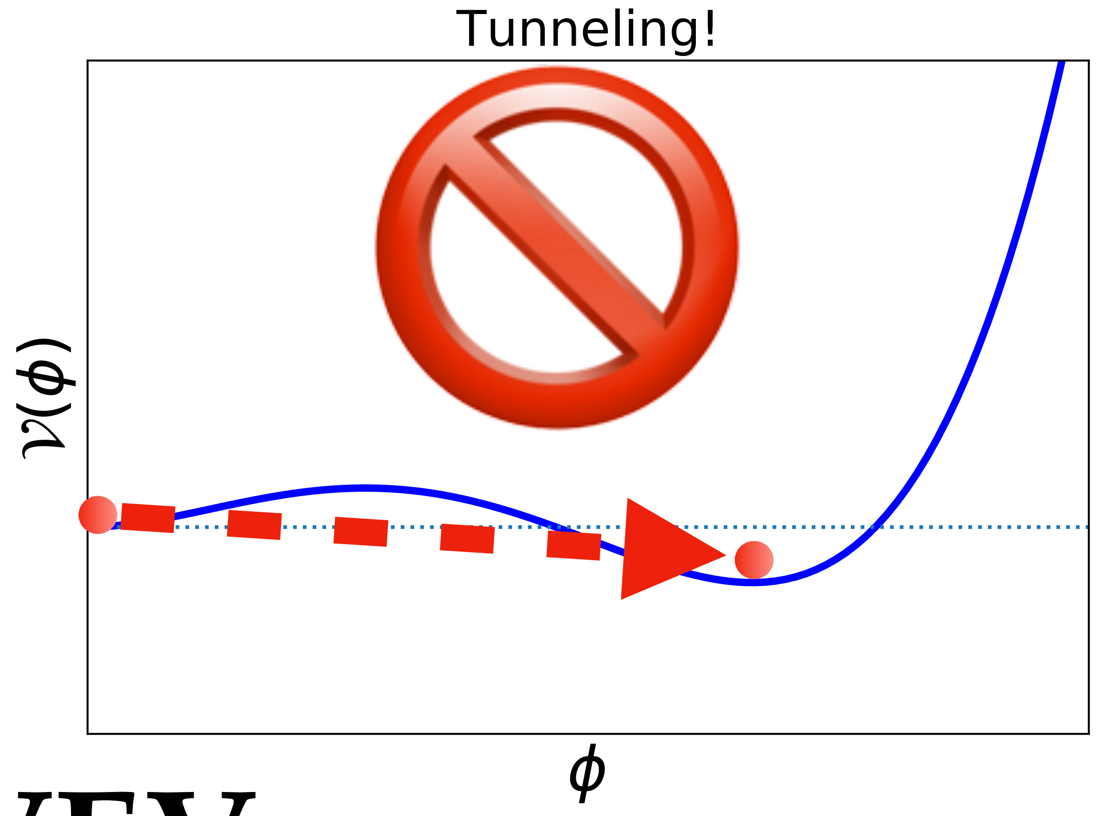
[Karkout, **AP**, Postma, Tetlalmatzi-Xolocotzi, van de Vis, du Pree, **arXiv:2404.12425**]

FO-EWPT and hhh in the TRSM

- *Q*: Can there be a **First-Order Electroweak Phase Transition** in the **TRSM**, related to **electro-weak baryogenesis**?

- and if so, will this lead to **enhanced hhh at the LHC?**

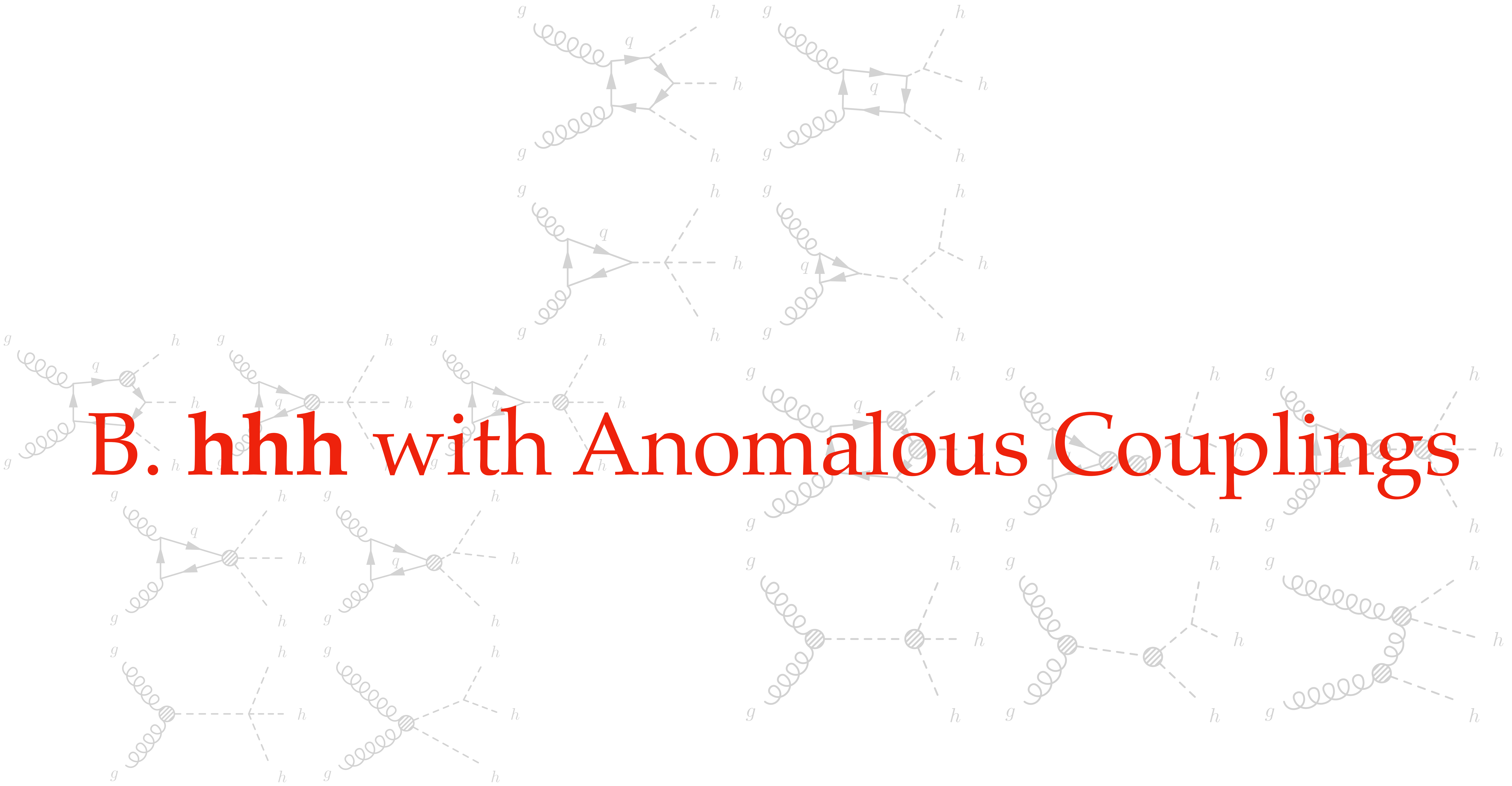
[Karkout, **AP**, Postma, Tetlalmatzi-Xolocotzi, van de Vis, du Pree, **arXiv:2404.12425**]



- **NO!** ✨**see Osama Karkout's talk!**✨
  - **FO-EWPT & enhanced hhh** are **mutually exclusive**!
  - "barrier" **not** generated **if both new scalars attain a VEV,**
  - and **non-zero VEVs are necessary** for sufficient mixing!

➡ **Removing the $\mathcal{Z}_2$ restrictions might help!**

B. **hhh** with Anomalous Couplings



D=6-Inspired Anomalous Couplings

- Add **higher-dimensional operators** to the SM Lagrangian!

→ Capture **effects of new particles at Scales** ≫ **Collision Energies**.

- e.g. Add D=6 operators **relevant to multi-Higgs boson production**, of the form $\frac{\mathcal{O}_6}{\Lambda^2}$:

SM →

$$\mathcal{L}_{h^n} \supset -\mu^2|\phi|^2 - \lambda|\phi|^4 - \left(y_t\bar{Q}_L\phi^c t_R + y_b\bar{Q}_L\phi b_R + \mathrm{h.c.}\right)$$

D=6 →

$$+\frac{c_H}{2\Lambda^2}(\partial^\mu|\phi|^2)^2 - \frac{c_6}{\Lambda^2}\lambda_{\mathrm{SM}}|\phi|^6 + \frac{\alpha_s c_g}{4\pi\Lambda^2}|\phi|^2 G^a_{\mu\nu}G_a^{\mu\nu}$$

$$-\left(\frac{c_t}{\Lambda^2}y_t|\phi|^2\bar{Q}_L\phi^c t_R + \frac{c_b}{\Lambda^2}y_b|\phi|^2\bar{Q}_L\phi b_R + \mathrm{h.c.}\right)$$

[see e.g. Goertz, **AP**, Yang, Zurita, **arXiv:1410.3471** for $pp \to hh$ study in this framework]

For 1-loop computations see: **smeft@nlo**: [Degrande, Durieux, Maltoni, Mimasu, Vryonidou, Zhang, **arXiv:2008.11743**]

D=6-Inspired Anomalous Couplings

[**AP**, Tetlalmatzi-Xolocotzi, **arXiv:2312.13562**]

- Go through EWSB…

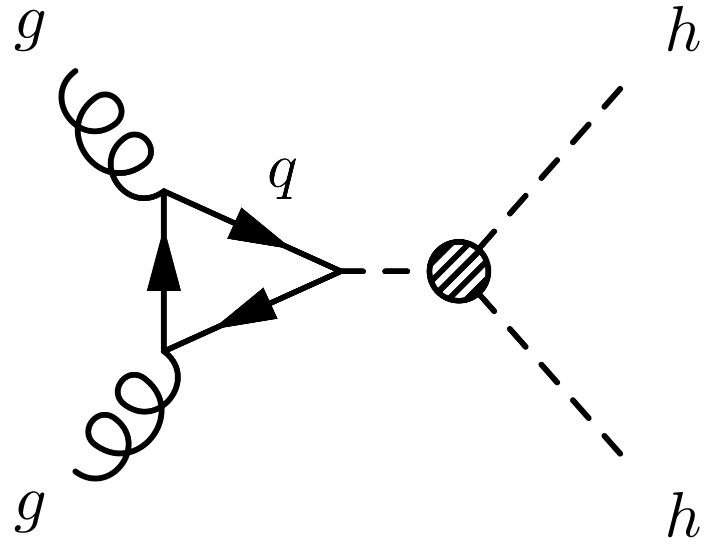
⇒ in terms of the physical scalar Higgs boson h:

$$
\begin{aligned}
\mathscr{L}_{\mathrm{D=6}} \supset & -\frac{m_h^2}{2v}\left(1+c_6\right)h^3 - \frac{m_h^2}{8v^2}\left(1+6c_6\right)h^4 \\
& + \frac{\alpha_s c_g}{4\pi}\left(\frac{h}{v}+\frac{h^2}{2v^2}\right)G^a_{\mu\nu}G^{\mu\nu}_a \\
& - \left[\frac{m_t}{v}\left(1+c_t\right)\bar{t}_L t_R h + \frac{m_b}{v}\left(1+c_b\right)\bar{b}_L b_R h + \mathrm{h.c.}\right] \\
& - \left[\frac{m_t}{v^2}\left(\frac{3c_t}{2}\right)\bar{t}_L t_R h^2 + \frac{m_b}{v^2}\left(\frac{3c_b}{2}\right)\bar{b}_L b_R h^2 + \mathrm{h.c.}\right] \\
& - \left[\frac{m_t}{v^3}\left(\frac{c_t}{2}\right)\bar{t}_L t_R h^3 + \frac{m_b}{v^3}\left(\frac{c_b}{2}\right)\bar{b}_L b_R h^3 + \mathrm{h.c.}\right],
\end{aligned}
$$

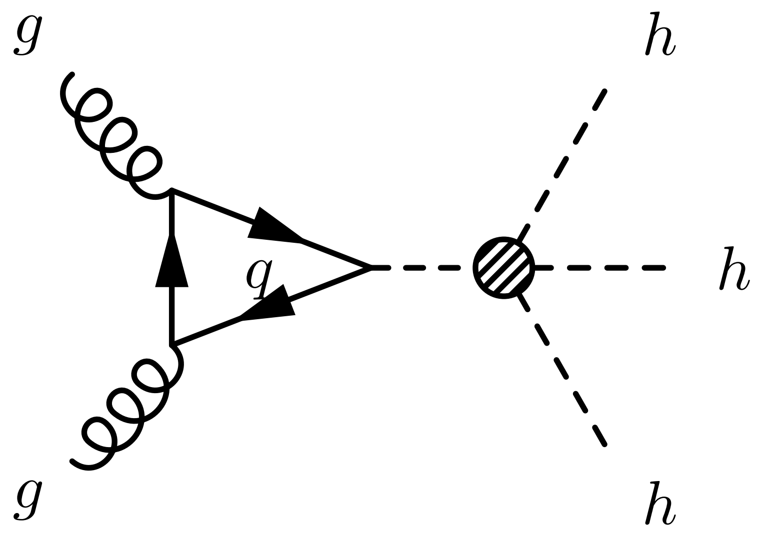
D=6-Inspired Anomalous Couplings

[**AP**, Tetlalmatzi-Xolocotzi, **arXiv:2312.13562**]

- Go through EWSB…  ⇒ in terms of the physical scalar Higgs boson h:



$$\mathscr{L}_{\text{D=6}} \supset \boxed{-\frac{m_h^2}{2v}\left(1+c_6\right)h^3 - \frac{m_h^2}{8v^2}\left(1+6c_6\right)h^4}$$
$$+\frac{\alpha_s c_g}{4\pi}\left(\frac{h}{v}+\frac{h^2}{2v^2}\right)G^a_{\mu\nu}G^{\mu\nu}_a$$
$$-\left[\frac{m_t}{v}\left(1+c_t\right)\bar{t}_L t_R h + \frac{m_b}{v}\left(1+c_b\right)\bar{b}_L b_R h + \text{h.c.}\right]$$
$$-\left[\frac{m_t}{v^2}\left(\frac{3c_t}{2}\right)\bar{t}_L t_R h^2 + \frac{m_b}{v^2}\left(\frac{3c_b}{2}\right)\bar{b}_L b_R h^2 + \text{h.c.}\right]$$
$$-\left[\frac{m_t}{v^3}\left(\frac{c_t}{2}\right)\bar{t}_L t_R h^3 + \frac{m_b}{v^3}\left(\frac{c_b}{2}\right)\bar{b}_L b_R h^3 + \text{h.c.}\right],$$

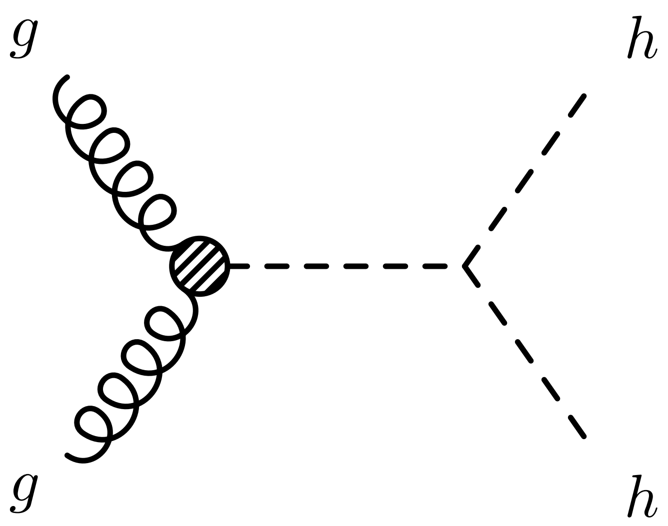


D=6-Inspired Anomalous Couplings

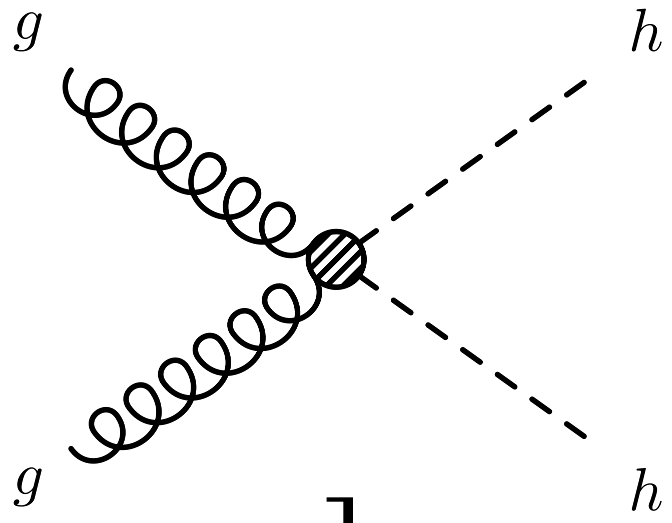
[**AP**, Tetlalmatzi-Xolocotzi, **arXiv:2312.13562**]

- Go through EWSB…

⇒ in terms of the physical scalar Higgs boson *h*:



$$
\begin{aligned}
\mathscr{L}_{\mathrm{D=6}} \supset & -\frac{m_h^2}{2v}\left(1+c_6\right)h^3 - \frac{m_h^2}{8v^2}\left(1+6c_6\right)h^4 \\
& \boxed{+\frac{\alpha_s c_g}{4\pi}\left(\frac{h}{v}+\frac{h^2}{2v^2}\right)G^a_{\mu\nu}G^{\mu\nu}_a} \\
& -\left[\frac{m_t}{v}\left(1+c_t\right)\bar{t}_L t_R h + \frac{m_b}{v}\left(1+c_b\right)\bar{b}_L b_R h + \text{h.c.}\right] \\
& -\left[\frac{m_t}{v^2}\left(\frac{3c_t}{2}\right)\bar{t}_L t_R h^2 + \frac{m_b}{v^2}\left(\frac{3c_b}{2}\right)\bar{b}_L b_R h^2 + \text{h.c.}\right] \\
& -\left[\frac{m_t}{v^3}\left(\frac{c_t}{2}\right)\bar{t}_L t_R h^3 + \frac{m_b}{v^3}\left(\frac{c_b}{2}\right)\bar{b}_L b_R h^3 + \text{h.c.}\right],
\end{aligned}
$$



D=6-Inspired Anomalous Couplings

[**AP**, Tetlalmatzi-Xolocotzi, **arXiv:2312.13562**]

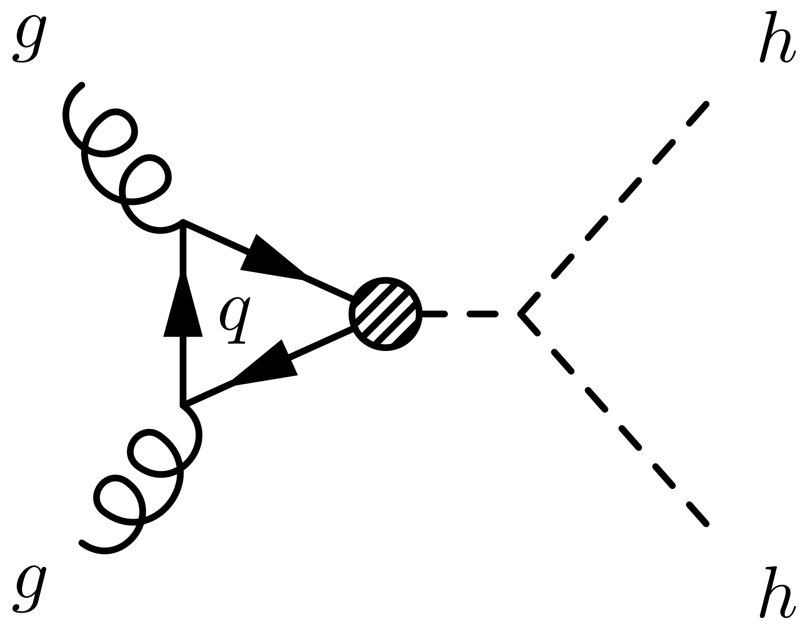
- Go through EWSB…

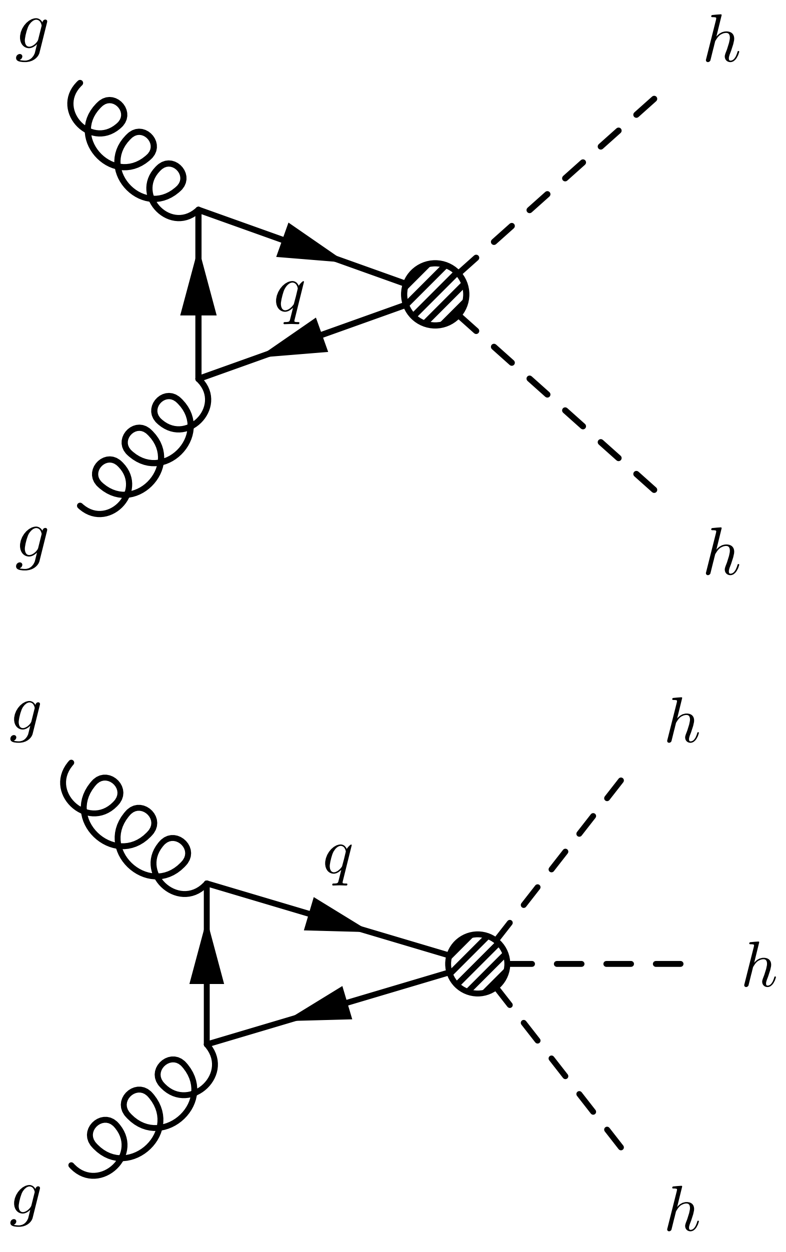
⇒ in terms of the physical scalar Higgs boson h:

$$\mathscr{L}_{\mathrm{D=6}} \supset -\frac{m_h^2}{2v}\left(1+c_6\right)h^3 - \frac{m_h^2}{8v^2}\left(1+6c_6\right)h^4$$

$$+\frac{\alpha_s c_g}{4\pi}\left(\frac{h}{v}+\frac{h^2}{2v^2}\right)G^a_{\mu\nu}G^{\mu\nu}_a$$

$$\boxed{\begin{aligned}&-\left[\frac{m_t}{v}\left(1+c_t\right)\bar{t}_L t_R h + \frac{m_b}{v}\left(1+c_b\right)\bar{b}_L b_R h + \mathrm{h.c.}\right]\\&-\left[\frac{m_t}{v^2}\left(\frac{3c_t}{2}\right)\bar{t}_L t_R h^2 + \frac{m_b}{v^2}\left(\frac{3c_b}{2}\right)\bar{b}_L b_R h^2 + \mathrm{h.c.}\right]\\&-\left[\frac{m_t}{v^3}\left(\frac{c_t}{2}\right)\bar{t}_L t_R h^3 + \frac{m_b}{v^3}\left(\frac{c_b}{2}\right)\bar{b}_L b_R h^3 + \mathrm{h.c.}\right],\end{aligned}}$$





D=6-Inspired Anomalous Couplings

[**AP**, Tetlalmatzi-Xolocotzi, **arXiv:2312.13562**]

- A slightly more "general" picture is obtained by "**dissociating**" the operators as:

$$
\begin{aligned}
\mathscr{L}_{\text{Pheno}} \supset & -\frac{m_h^2}{2v}\left(1+d_3\right)h^3-\frac{m_h^2}{8v^2}\left(1+d_4\right)h^4 \\
& +\frac{\alpha_s}{4\pi}\left(c_{g1}\frac{h}{v}+c_{g2}\frac{h^2}{2v^2}\right)G^a_{\mu\nu}G^{\mu\nu}_a \\
& -\left[\frac{m_t}{v}\left(1+c_{t1}\right)\bar{t}_L t_R h+\frac{m_b}{v}\left(1+c_{b1}\right)\bar{b}_L b_R h+\text{h.c.}\right] \\
& -\left[\frac{m_t}{v^2}\left(\frac{3c_{t2}}{2}\right)\bar{t}_L t_R h^2+\frac{m_b}{v^2}\left(\frac{3c_{b2}}{2}\right)\bar{b}_L b_R h^2+\text{h.c.}\right] \\
& -\left[\frac{m_t}{v^3}\left(\frac{c_{t3}}{2}\right)\bar{t}_L t_R h^3+\frac{m_b}{v^3}\left(\frac{c_{b3}}{2}\right)\bar{b}_L b_R h^3+\text{h.c.}\right],
\end{aligned}
$$

Recover D=6 by setting:

$$
\begin{aligned}
d_3 &= c_6, \\
d_4 &= 6c_6, \\
c_{g1} = c_{g2} &= c_g, \\
c_{f1} = c_{f2} = c_{f3} &= c_f\,.
\end{aligned}
$$

Note: This can be also be motivated via the Electro-weak Chiral Lagrangian,

[see e.g. Buchalla, Catá, Krause arXiv:1307.5017]

D=6-Inspired Anomalous Couplings

[**AP**, Tetlalmatzi-Xolocotzi, **arXiv:2312.13562**]

- A slightly more "general" picture is obtained by "**dissociating**" the operators as:

$$
\begin{aligned}
\mathscr{L}_{\text{Pheno}} \supset & -\frac{m_h^2}{2v}\,(1+d_3)\,h^3 - \frac{m_h^2}{8v^2}\,(1+d_4)\,h^4 \\
& + \frac{\alpha_s}{4\pi}\left(c_{g1}\frac{h}{v} + c_{g2}\frac{h^2}{2v^2}\right) G^a_{\mu\nu} G^{\mu\nu}_a \\
& - \left[\frac{m_t}{v}\,(1+c_{t1})\,\bar{t}_L t_R h + \frac{m_b}{v}\,(1+c_{b1})\,\bar{b}_L b_R h + \text{h.c.}\right] \\
& - \left[\frac{m_t}{v^2}\left(\frac{3c_{t2}}{2}\right)\bar{t}_L t_R h^2 + \frac{m_b}{v^2}\left(\frac{3c_{b2}}{2}\right)\bar{b}_L b_R h^2 + \text{h.c.}\right] \\
& - \left[\frac{m_t}{v^3}\left(\frac{c_{t3}}{2}\right)\bar{t}_L t_R h^3 + \frac{m_b}{v^3}\left(\frac{c_{b3}}{2}\right)\bar{b}_L b_R h^3 + \text{h.c.}\right],
\end{aligned}
$$

instead of c_g

Recover D=6 by setting:

$$
\begin{aligned}
d_3 &= c_6, \\
d_4 &= 6c_6, \\
c_{g1} = c_{g2} &= c_g, \\
c_{f1} = c_{f2} = c_{f3} &= c_f\,.
\end{aligned}
$$

***Note*: This can be also be motivated via the Electro-weak Chiral Lagrangian,**

[see e.g. Buchalla, Catá, Krause arXiv:1307.5017]

D=6-Inspired Anomalous Couplings

[**AP**, Tetlalmatzi-Xolocotzi, **arXiv:2312.13562**]

- A slightly more "general" picture is obtained by "**dissociating**" the operators as:

$$
\begin{aligned}
\mathscr{L}_{\text{Pheno}} \supset & -\frac{m_h^2}{2v}\left(1+d_3\right)h^3 - \frac{m_h^2}{8v^2}\left(1+d_4\right)h^4 \\
& + \frac{\alpha_s}{4\pi}\left(c_{g1}\frac{h}{v}+c_{g2}\frac{h^2}{2v^2}\right)G^a_{\mu\nu}G^{\mu\nu}_a \\
& - \left[\frac{m_t}{v}\left(1+c_{t1}\right)\bar{t}_L t_R h + \frac{m_b}{v}\left(1+c_{b1}\right)\bar{b}_L b_R h + \text{h.c.}\right] \\
& - \left[\frac{m_t}{v^2}\left(\frac{3c_{t2}}{2}\right)\bar{t}_L t_R h^2 + \frac{m_b}{v^2}\left(\frac{3c_{b2}}{2}\right)\bar{b}_L b_R h^2 + \text{h.c.}\right] \\
& - \left[\frac{m_t}{v^3}\left(\frac{c_{t3}}{2}\right)\bar{t}_L t_R h^3 + \frac{m_b}{v^3}\left(\frac{c_{b3}}{2}\right)\bar{b}_L b_R h^3 + \text{h.c.}\right],
\end{aligned}
$$

instead of c_t

Recover D=6 by setting:

$$
\begin{aligned}
d_3 &= c_6, \\
d_4 &= 6c_6, \\
c_{g1} = c_{g2} &= c_g, \\
c_{f1} = c_{f2} = c_{f3} &= c_f\,.
\end{aligned}
$$

Note: This can be also be motivated via the Electro-weak Chiral Lagrangian,

[see e.g. Buchalla, Catá, Krause arXiv:1307.5017]

D=6-Inspired Anomalous Couplings

[**AP**, Tetlalmatzi-Xolocotzi, **arXiv:2312.13562**]

- **Further modify** to match more closely **LHC experiments' definitions**:

$$
\begin{aligned}
\mathscr{L}_{\text{PhenoExp}} \supset & -\lambda_{\text{SM}} v \left(1+d_3\right) h^3 - \frac{\lambda_{\text{SM}}}{4}\left(1+d_4\right) h^4 \\
& + \frac{\alpha_s}{12\pi}\left(c_{g1}\frac{h}{v} - c_{g2}\frac{h^2}{2v^2}\right) G^a_{\mu\nu} G^{\mu\nu}_a \\
& - \left[\frac{m_t}{v}\left(1+c_{t1}\right)\bar{t}_L t_R h + \frac{m_b}{v}\left(1+c_{b1}\right)\bar{b}_L b_R h + \text{h.c.}\right] \\
& - \left[\frac{m_t}{v^2} c_{t2} \bar{t}_L t_R h^2 + \frac{m_b}{v^2} c_{b2} \bar{b}_L b_R h^2 + \text{h.c.}\right] \\
& - \left[\frac{m_t}{v^3}\left(\frac{c_{t3}}{2}\right)\bar{t}_L t_R h^3 + \frac{m_b}{v^3}\left(\frac{c_{b3}}{2}\right)\bar{b}_L b_R h^3 + \text{h.c.}\right],
\end{aligned}
$$

Defined: $\lambda_{\text{SM}} = m_h^2/2v^2$.

Obtain **CMS-like** parametrization by:

$$
\begin{aligned}
\kappa_\lambda &= (1+d_3), \\
k_t &= c_{t1}, \\
c_2 &= c_{t2}, \\
c_g &= c_{g1}, \\
c_{gg} &= c_{2g}\,.
\end{aligned}
$$

And **ATLAS-like** parametrization by:

$$
\begin{aligned}
c_{hhh} &= (1+d_3), \\
c_{ggh} &= 2c_{g1}/3, \\
c_{gghh} &= -c_{g2}/3.
\end{aligned}
$$

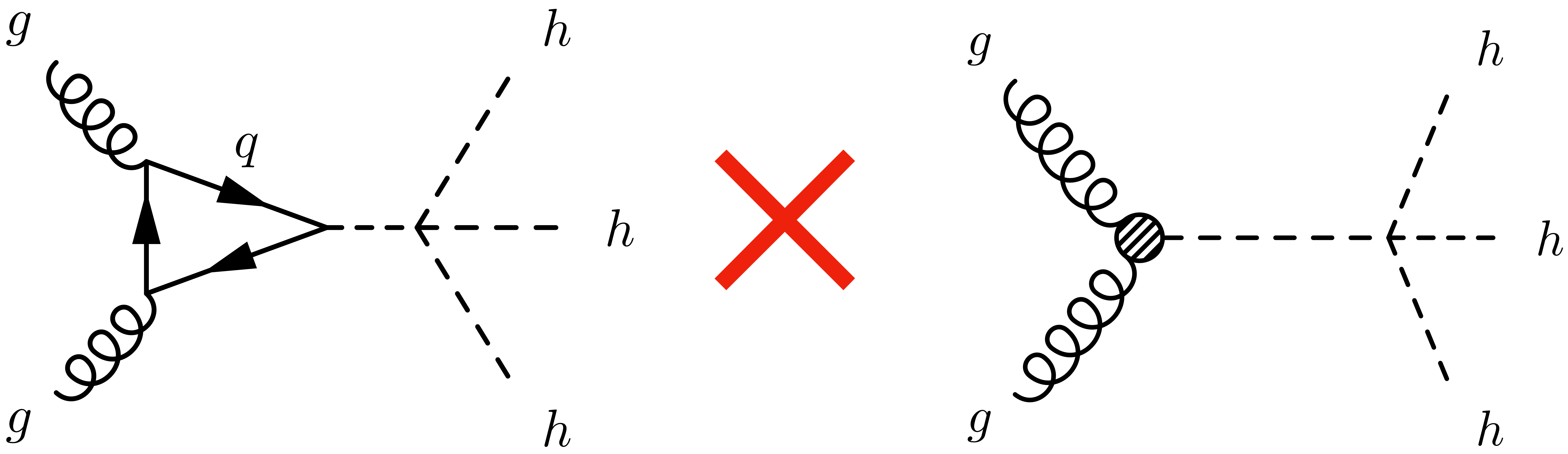
Monte Carlo Implementation of Anomalous Couplings

- We have implemented a **MadGraph5_aMC@NLO "loop" model** for $\mathscr{L}_{\text{PhenoExp}}$.

- **Includes Loop × Tree Level interference between diagrams.**

  [see: Hirschi, https://cp3.irmp.ucl.ac.be/projects/madgraph/wiki/LoopInducedTimesTree]

- e.g.:

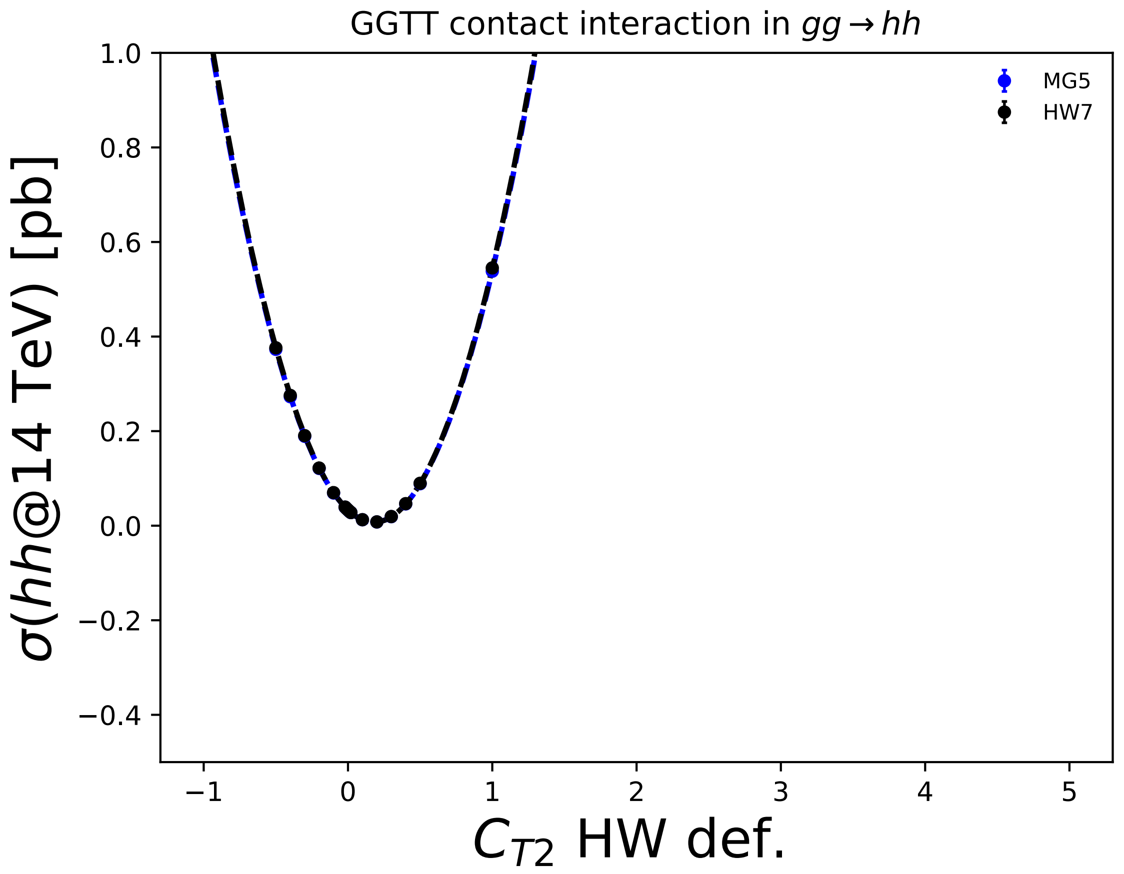


- **Model available at https://gitlab.com/apapaefs/multihiggs_loop_sm.**
  [includes necessary patch for MG5_aMC].

Model Validation

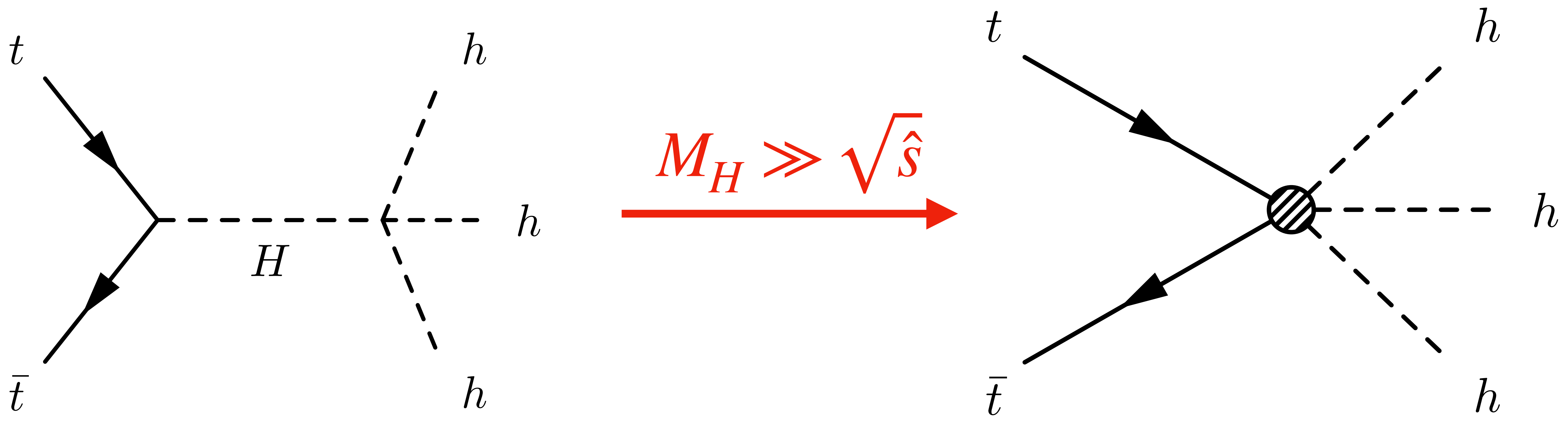
[**AP**, Tetlalmatzi-Xolocotzi, **arXiv:2312.13562**]

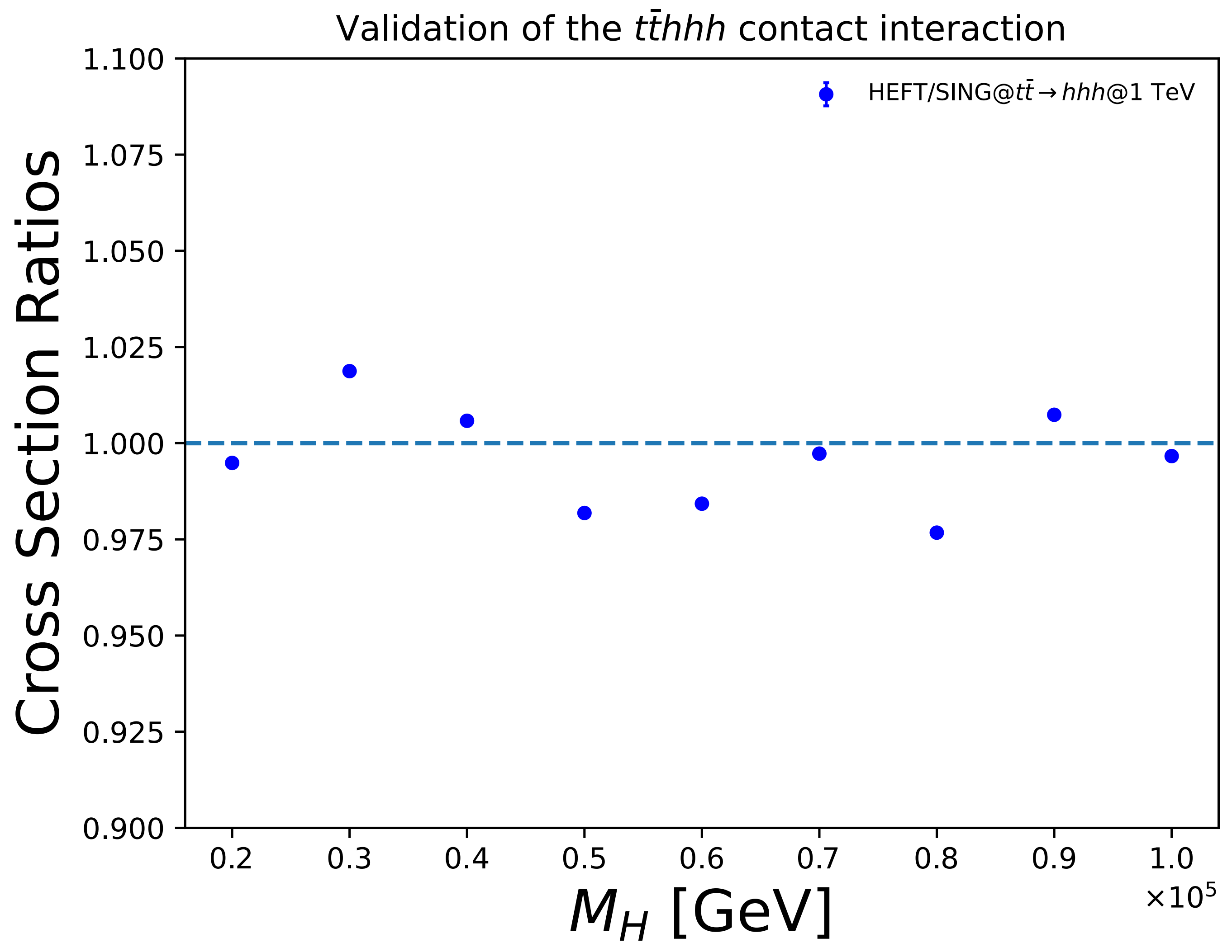
- Most couplings **validated** vs. a **HERWIG 7** $pp \to hh$ implementation, e.g.:



✓

- "New" non-trivial coupling that appears, $\propto c_{t3} t\bar{t}h^3$ has been **validated** via an **"EFT" limit**, in the $t\bar{t} \to hhh$ process:



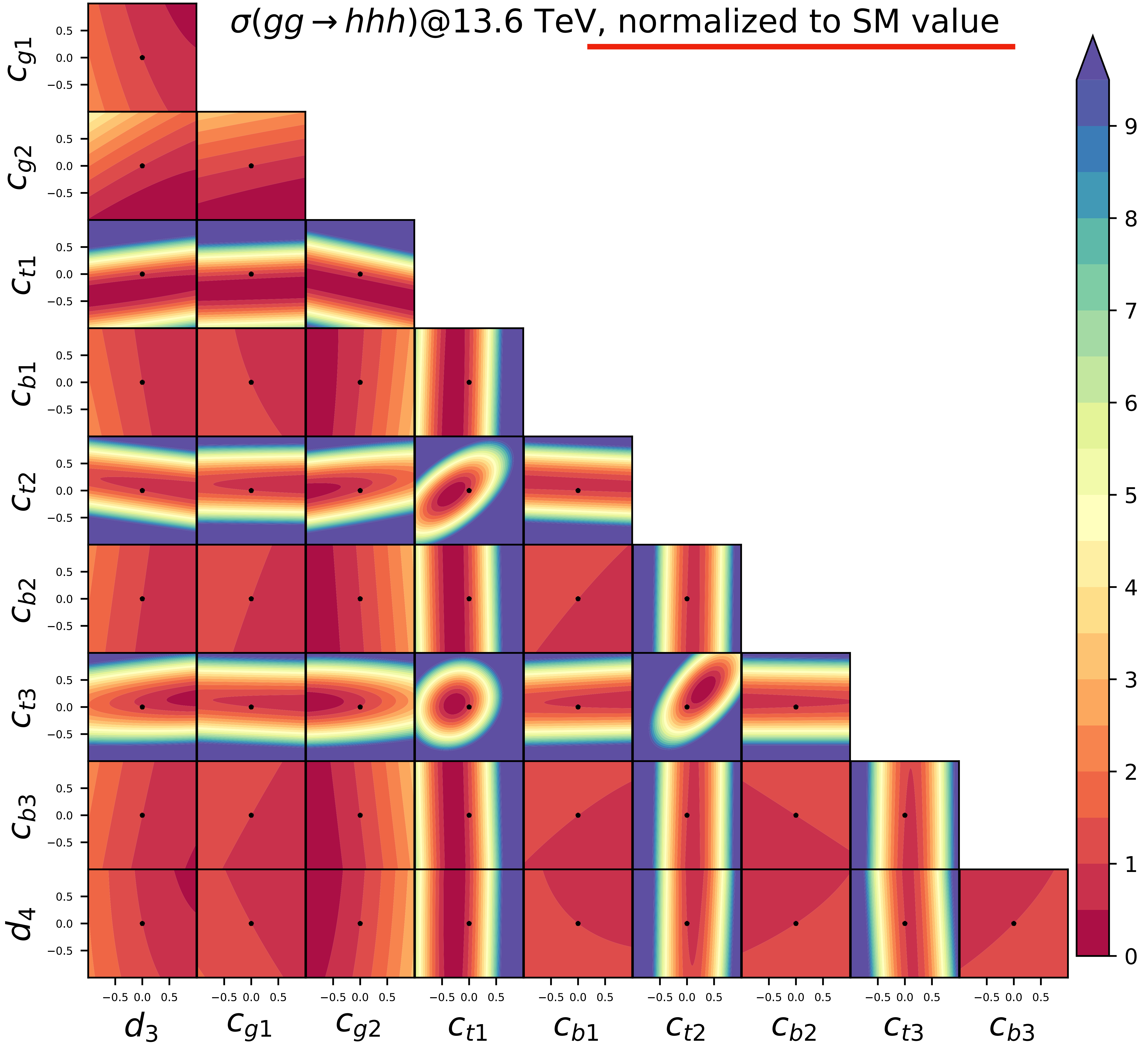


✓

hhh Cross Sections @ 13.6 TeV

[**AP**, Tetlalmatzi-Xolocotzi, **arXiv:2312.13562**]

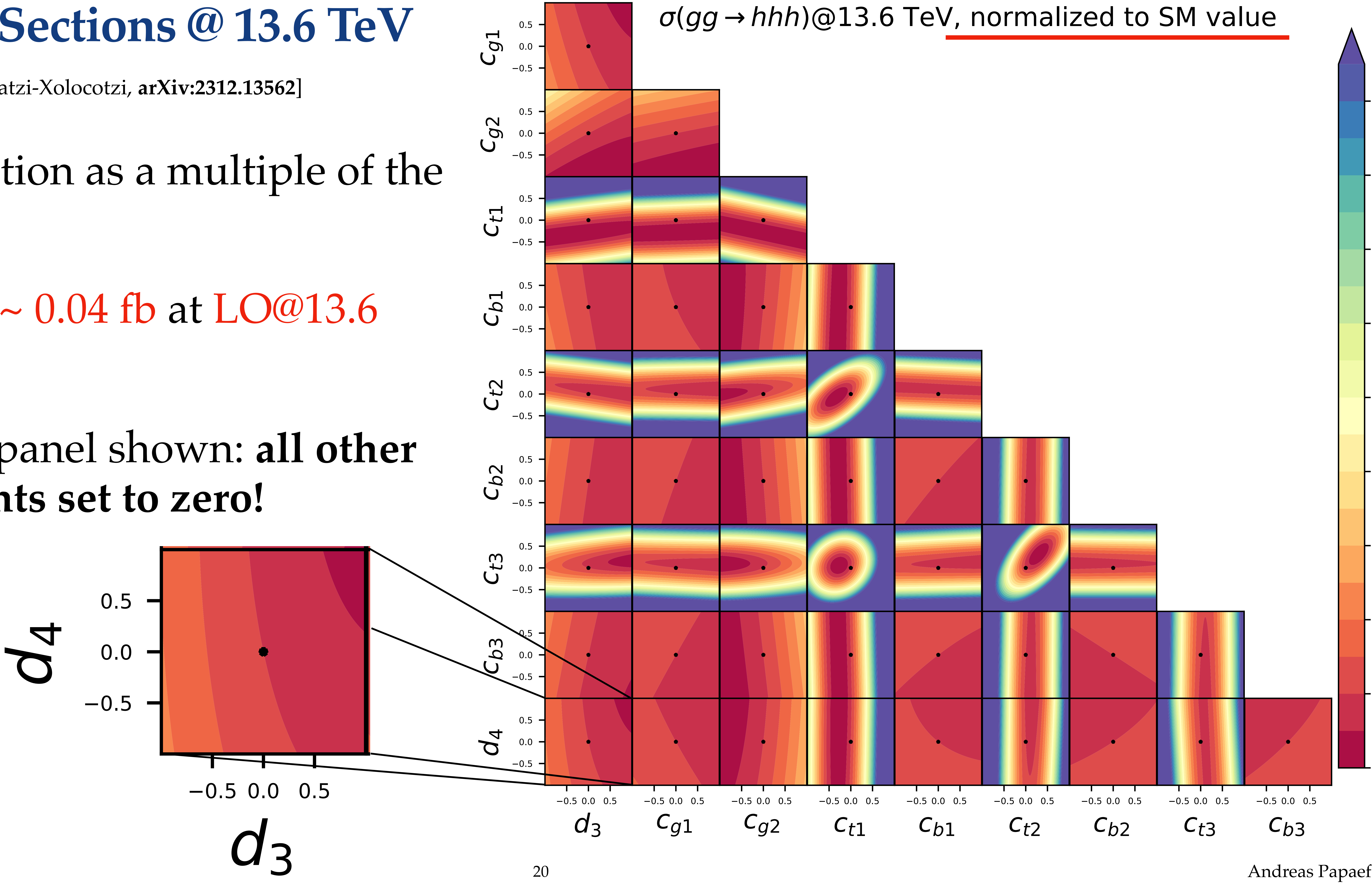
- Cross section as a multiple of the SM.
- n.b.: $\sigma_{\mathrm{SM}} \sim 0.04$ fb at LO@13.6 TeV.
- Each 2D panel shown: **all other coefficients set to zero!**



hhh Cross Sections @ 13.6 TeV

[**AP**, Tetlalmatzi-Xolocotzi, **arXiv:2312.13562**]

- Cross section as a multiple of the SM.
- n.b.: $\sigma_{\rm SM}$ ~ 0.04 fb at LO@13.6 TeV.
- Each 2D panel shown: **all other coefficients set to zero!**



Anomalous Couplings Constraints

- Other processes constrain (at LO) all coefficients **except** $\{c_{t3}, d_4\}$ **(→ only in hhh)**.
- **Projected constraints for other coefficients:**

Percentage uncertainties			
	HL-LHC	FCC-hh	Ref.
$\delta(d_3)$	50	5	[145] (table 12)
$\delta(c_{g1})$	2.3	0.49	[145] (table 3)
$\delta(c_{g2})$	5	1	[140] (Figure 12, right)
$\delta(c_{t1})$	3.3	1.0	[145] (table 3)
$\delta(c_{t2})$	30	10	[140] (Figure 12, right)
$\delta(c_{b1})$	3.6	0.43	[145] (table 3)
$\delta(c_{b2})$	30	10	assumed same as c_{t2}

✨ For details, see: [**AP**, Tetlalmatzi-Xolocotzi, **arXiv:2312.13562**]✨!

Anomalous Couplings Constraints

- Other processes constrain (<u>at LO</u>) all coefficients **<u>except</u>** $\{c_{t3}, d_4\}$ **(→ only in hhh).**
- **Projected constraints for other coefficients:**

Percentage uncertainties			
	HL-LHC	FCC-hh	Ref.
$\delta(d_3)$	50	5	[145] (table 12)
$\delta(c_{g1})$	2.3	0.49	[145] (table 3)
$\delta(c_{g2})$	5	1	[140] (Figure 12, right)
$\delta(c_{t1})$	3.3	1.0	[145] (table 3)
$\delta(c_{t2})$	30	10	[140] (Figure 12, right)
$\delta(c_{b1})$	3.6	0.43	[145] (table 3)
$\delta(c_{b2})$	30	10	assumed same as c_{t2}

✨ For details, see:
[**AP**, Tetlalmatzi-Xolocotzi,
arXiv:2312.13562]✨!

Anomalous Couplings Constraints

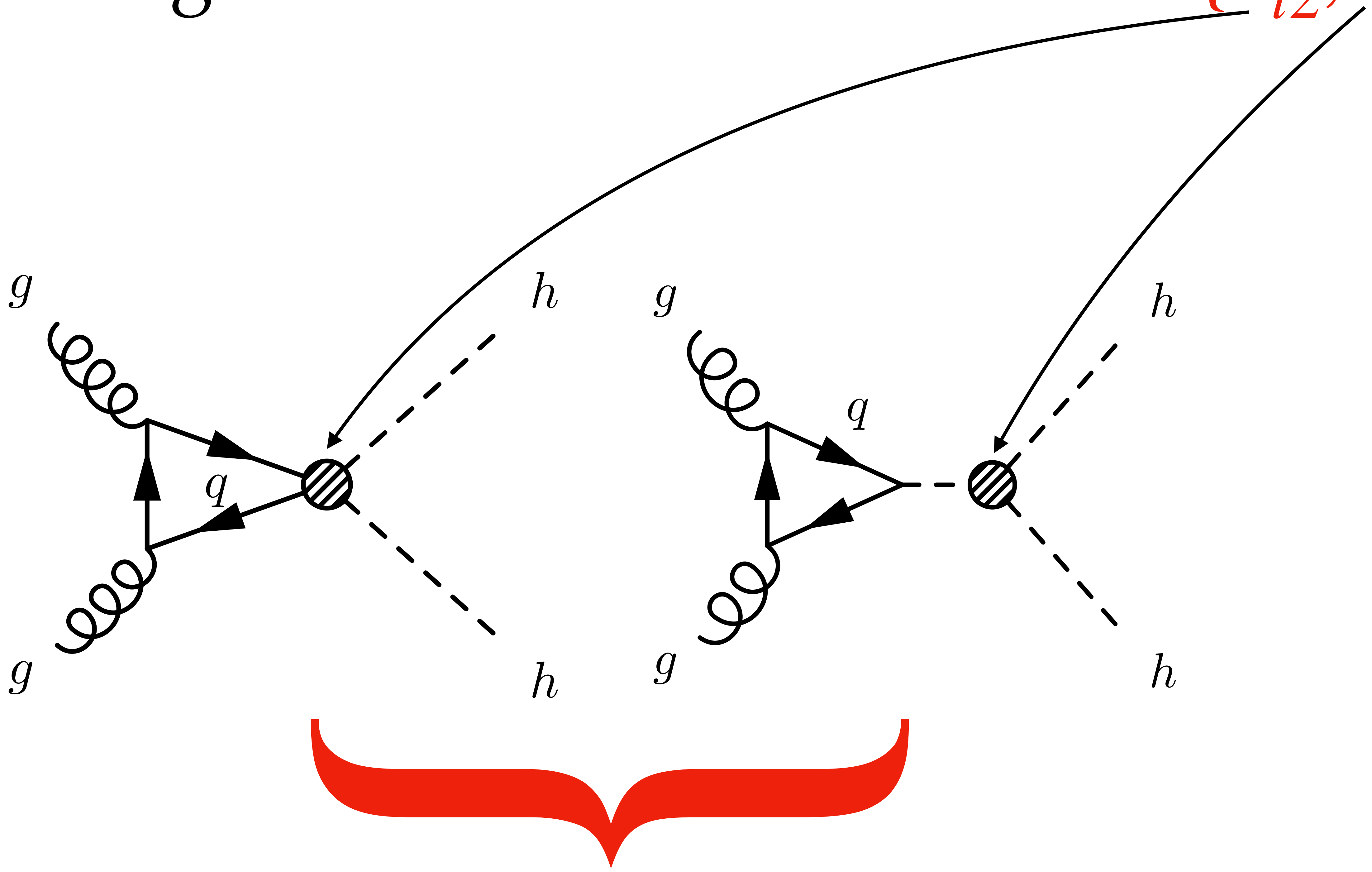
[**AP**, Tetlalmatzi-Xolocotzi, **arXiv:2312.13562**]

- Focusing on a model with **non-zero**: $\{c_{t2}, d_3, c_{t3}, d_4\}$:

Anomalous Couplings Constraints

[**AP**, Tetlalmatzi-Xolocotzi, **arXiv:2312.13562**]

- Focusing on a model with **non-zero**: $\{c_{t2}, d_3, c_{t3}, d_4\}$:

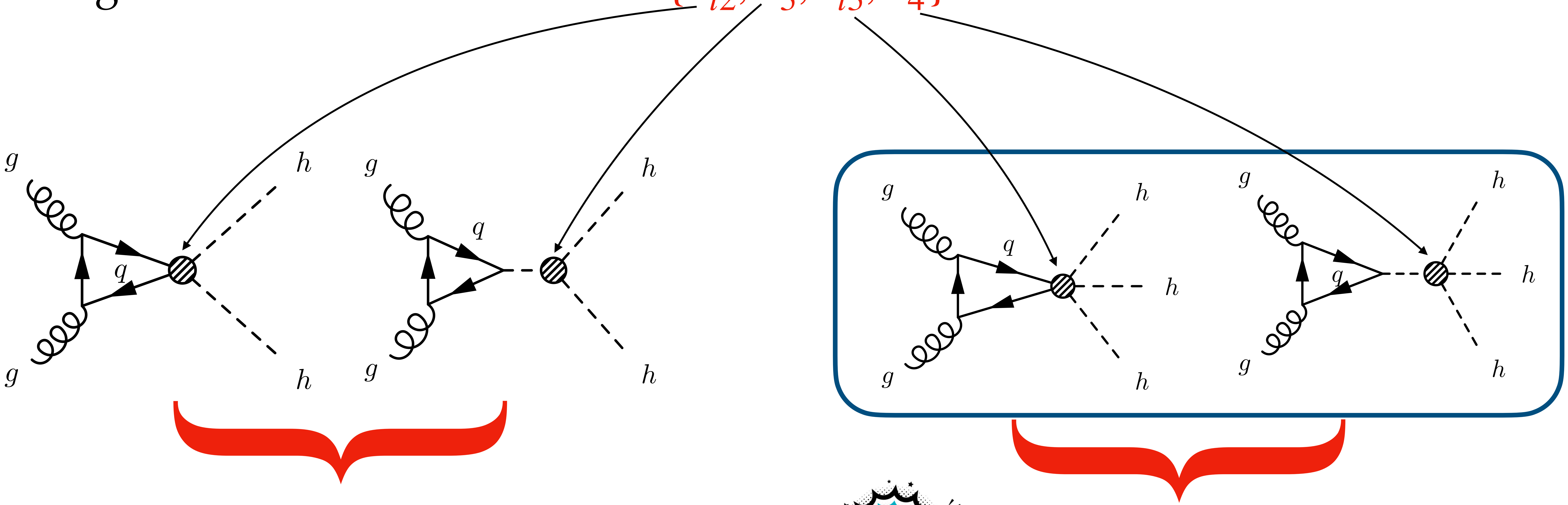


constrained by $pp \to hh$

Anomalous Couplings Constraints

[**AP**, Tetlalmatzi-Xolocotzi, **arXiv:2312.13562**]

- Focusing on a model with **non-zero**: $\{c_{t2}, d_3, c_{t3}, d_4\}$:



constrained by $pp \to hh$

constrained by $pp \to hhh$

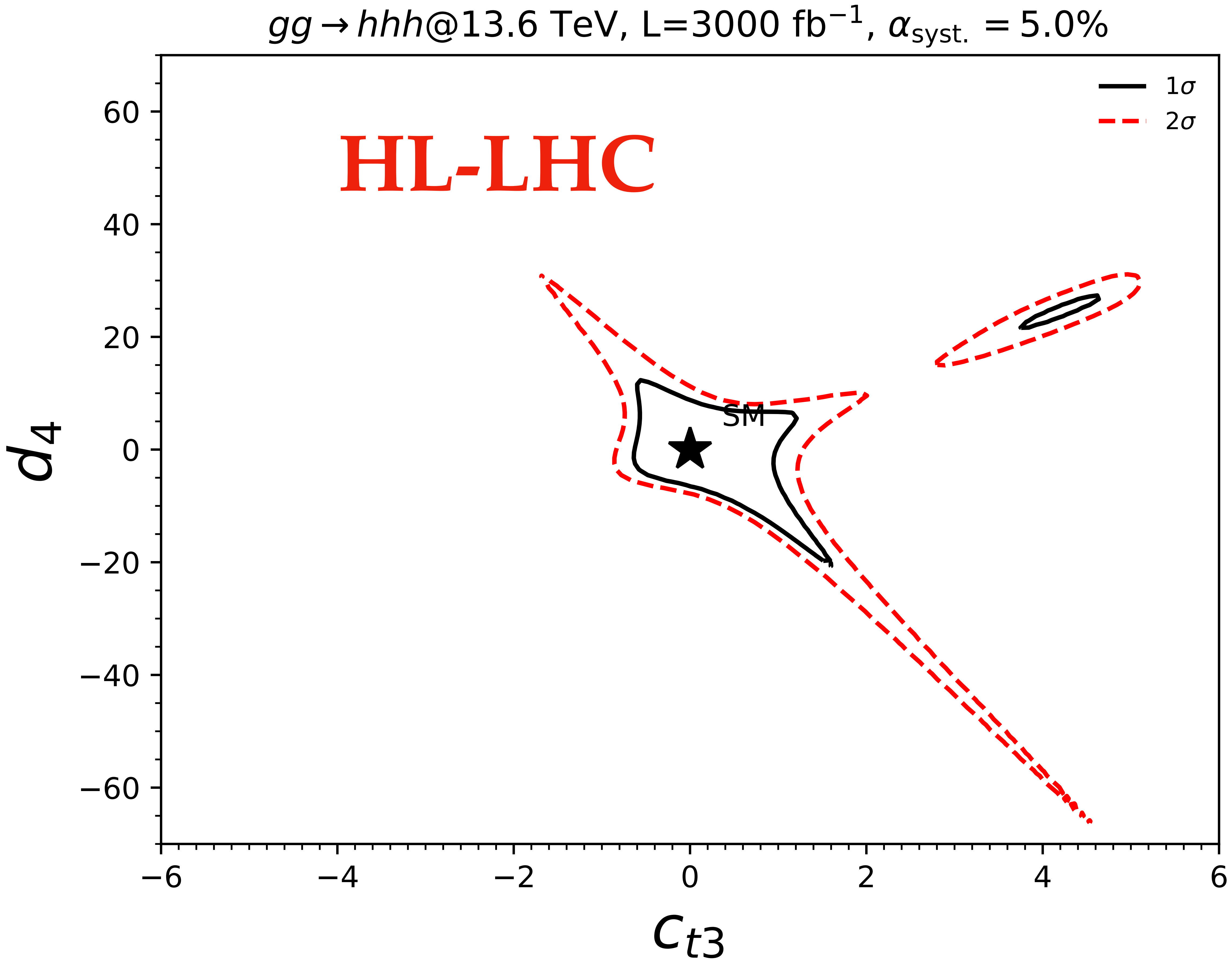
Anomalous Couplings Constraints

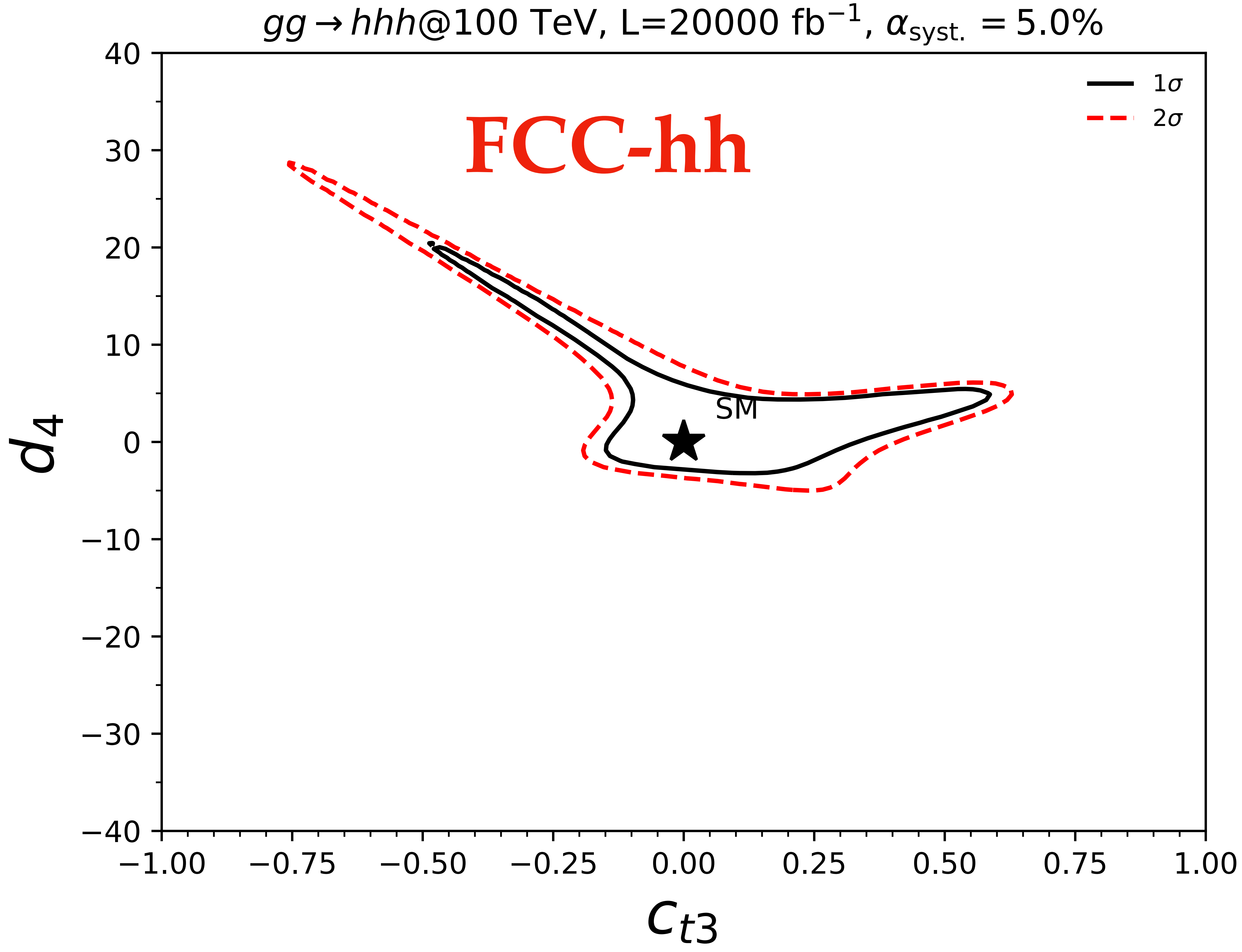
[**AP**, Tetlalmatzi-Xolocotzi, **arXiv:2312.13562**]

- Focusing on a model with only $\{c_{t2}, d_3, c_{t3}, d_4\}$,

✨see Gilberto Tetlalmatzi-Xolocotzi's talk!✨

- Using the **hhh → 6 b-jet final state,** and marginalizing over $\{c_{t2}, d_3\}$ within projected constraints:





Anomalous Couplings Constraints

[**AP**, Tetlalmatzi-Xolocotzi, **arXiv:2312.13562**]

- Focusing on a model with **only** $\{c_{t2}, d_3, c_{t3}, d_4\}$,
- Using the **hhh → 6 b-jet final state,** and **marginalizing** over $\{c_{t2}, d_3\}$ within projected constraints:

	HL-LHC 68%	HL-LHC 95%	FCC-hh 68%	FCC-hh 95%
d_4	$[-6.6, 12.4]$	$[-10.0, 21.3]$	$[-3.9, 10.5]$	$[-10.6, 18.8]$
c_{t3}	$[-0.6, 1.1]$	$[-0.9, 3.6]$	$[-0.1, 0.3]$	$[-0.4, 0.6]$

$$\Rightarrow \quad \begin{array}{l} c_{t3} \sim \mathcal{O}(0.1 - 1) \\ d_4 \sim \mathcal{O}(10) \end{array}$$

KENNESAW STATE UNIVERSITY

Summary & Outlook

- **Multi-Higgs production processes** → crucial rôle in understanding EWSB.
  - e.g. **hhh** production → **Higgs quartic self-coupling**.
- **SM hhh** → hopeless at the LHC,
- BUT: **Enhanced** in models with **extended scalar sectors** or **anomalous interactions**. **Could we see hints of hhh at the LHC?**
  - TRSM (**two singlets**) → through double-resonant process. **Related to baryogenesis?**
  - **Anomalous coupling picture** → an agnostic framework for **h/hh/hhh**.

    MG5_aMC model: https://gitlab.com/apapaefs/multihiggs_loop_sm
- **Questions merit investigation @ LHC & future colliders!**

Summary & Outlook

- **Multi-Higgs production processes** → crucial rôle in understanding EWSB.
  - e.g. **hhh** production → **Higgs quartic self-coupling**.
- **SM hhh** → hopeless at the LHC,
- BUT: **Enhanced** in models with **extended scalar sectors** or **anomalous interactions**. **Could we see hints of hhh at the LHC?**
  - TRSM (**two singlets**) → through double-resonant process. **Related to baryogenesis?**
  - **Anomalous coupling picture** → an agnostic framework for **h/hh/hhh**.

    MG5_aMC model: https://gitlab.com/apapaefs/multihiggs_loop_sm
- **Questions merit investigation @ LHC & future colliders!**

✨*Thanks!*

Questions?✨

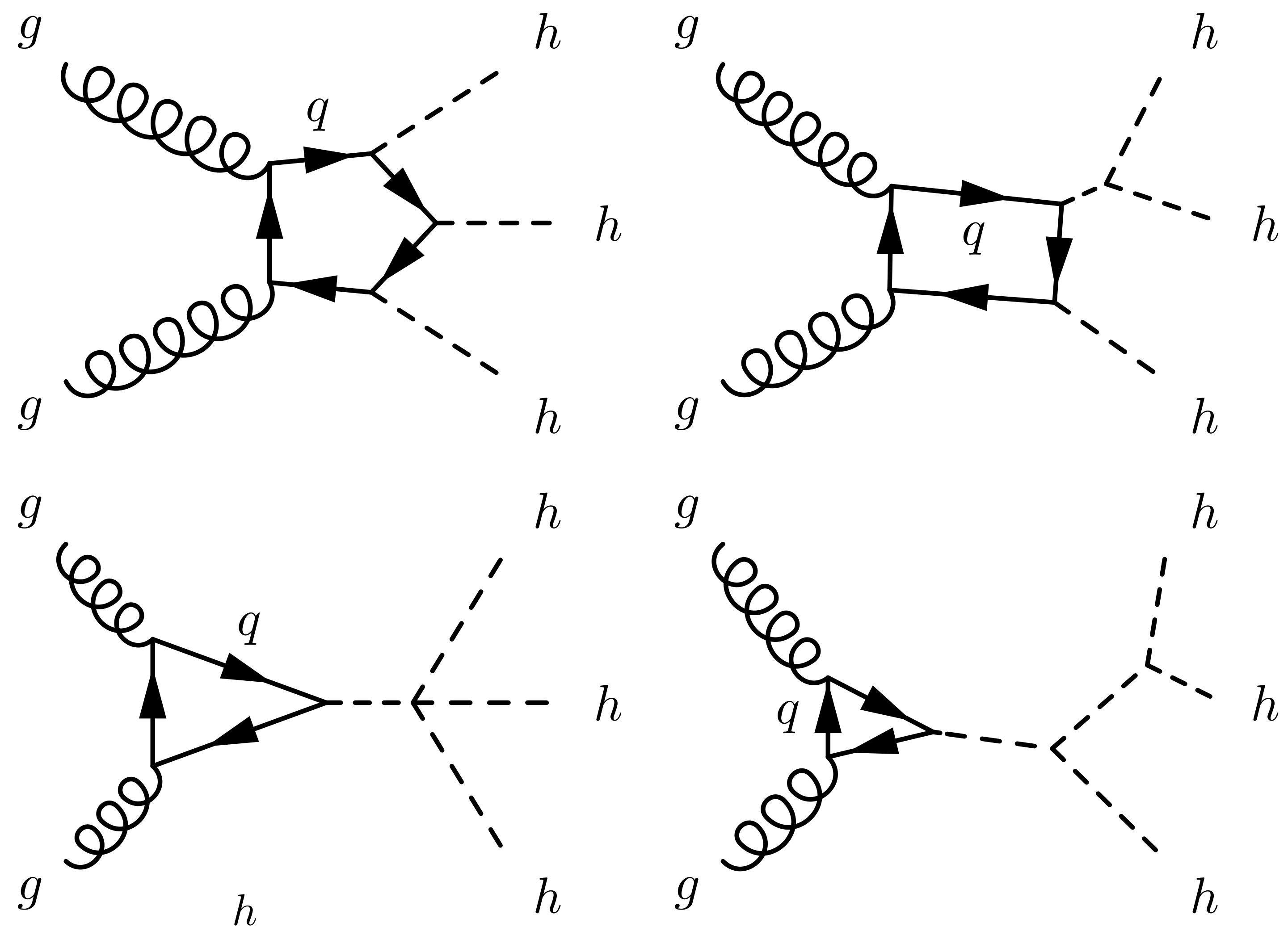
Appendices

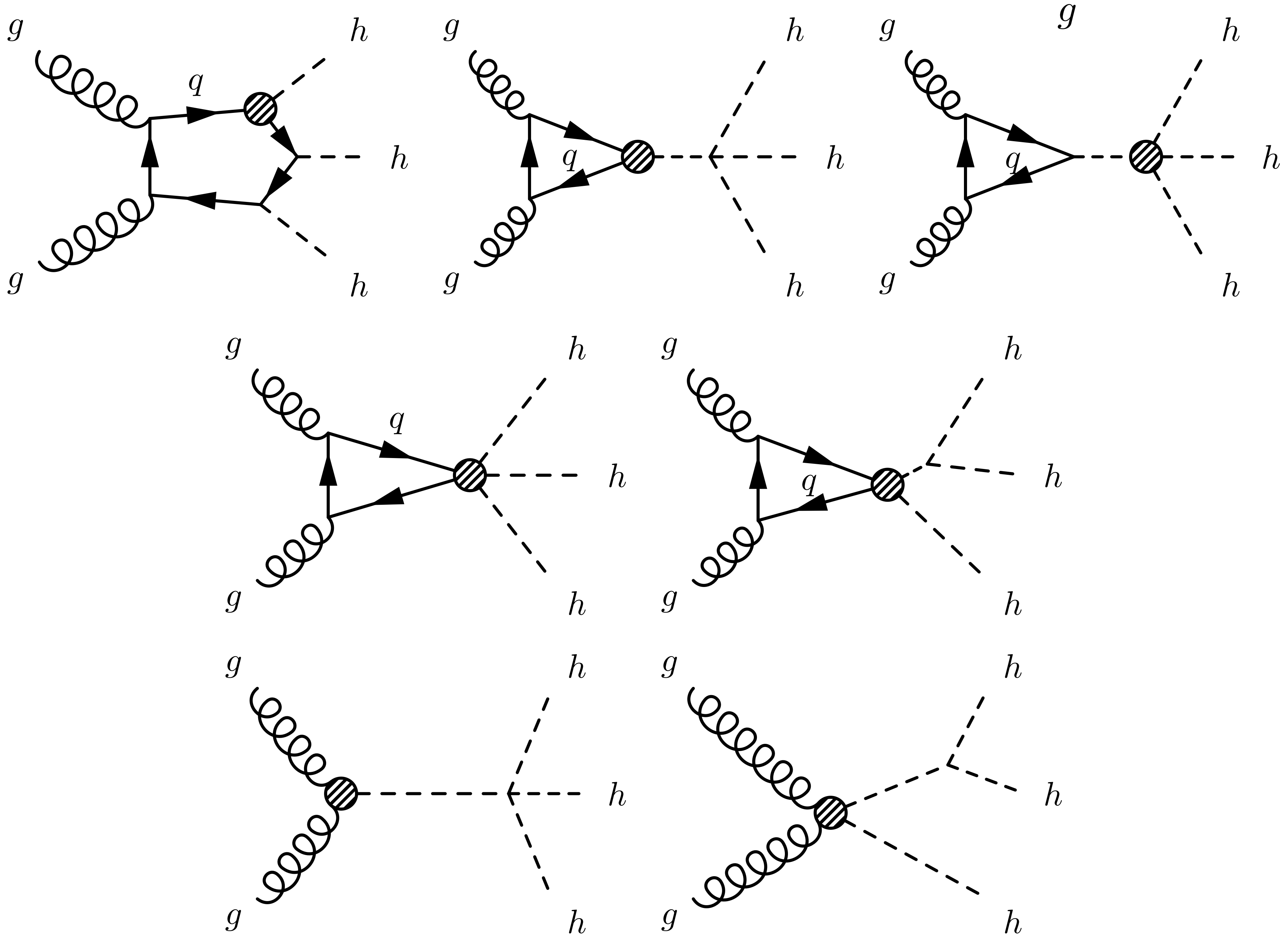
TRSM Monte Carlo Event Generation

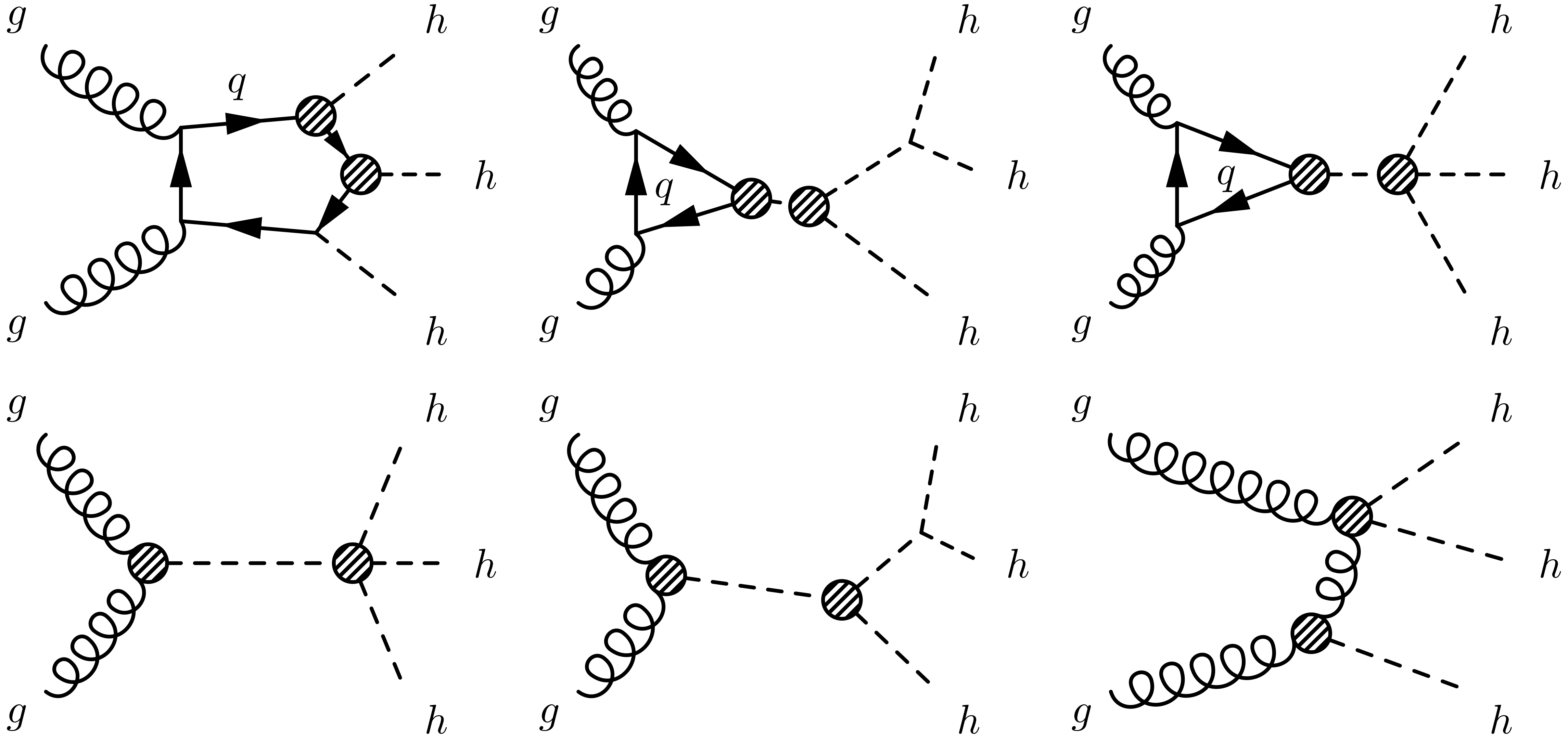
- We have implemented a **MadGraph5_aMC@NLO (MG5_aMC) "loop" model for the TRSM:**
  - **MG5_aMC input parameters**: the three mixing angles, two masses/widths and **all** the scalar couplings (only 7 are independent in **TRSM**).
  - Comes with a **Python script** that:
    - allows conversion of M_2, M_3 + three mixing angles + two VEVs to the **MG5_aMC** model input,
    - calculates several single-production cross sections, branching ratios, widths,
    - and writes associated **MG5_aMC** parameter card (**param_card.dat**) automatically.
  - **Get it at:** https://gitlab.com/apapaefs/twosinglet.

  [AP, Tania Robens, Gilberto Tetlalmatzi-Xolocotzi, arXiv:2101.00037]

hhh with Anomalous Couplings







hhh: Final states

Assume: K-factor = 2.

[Maltoni, Vryonidou, Zaro, 1408.6542]

$hhh \to$ final state	BR (%)	$N_{20\mathrm{ab}^{-1}}$
$(b\bar{b})(b\bar{b})(b\bar{b})$	19.21	22207
$(b\bar{b})(b\bar{b})(WW_{1\ell})$	7.20	8328
$(b\bar{b})(b\bar{b})(\tau\bar{\tau})$	6.31	7297
$(b\bar{b})(\tau\bar{\tau})(WW_{1\ell})$	1.58	1824
$(b\bar{b})(b\bar{b})(WW_{2\ell})$	0.98	1128
$(b\bar{b})(WW_{1\ell})(WW_{1\ell})$	0.90	1041
$(b\bar{b})(\tau\bar{\tau})(\tau\bar{\tau})$	0.69	799
$(b\bar{b})(b\bar{b})(\gamma\gamma)$	0.23	263

→ Fuks, Kim, Lee, 1510.07697, Fuks, Kim, Lee, 1704.04298.

→Kilian, Sun, Yan, Zhao, Zhao, 1702.03554.

→ AP, Sakurai, 1508.06524, Chen, Yan, Zhao, Zhao, Zhong, 1510.04013, Fuks, Kim, Lee, 1510.07697.

[AP, Sakurai, 1508.06524]

The 6b final state, analysis [AP, Gilberto Tetlalmatzi-Xolocotzi, Marco Zaro, arXiv:1909.09166]

- What can we learn about the anomalous couplings via **hhh** at 13.6 TeV?
- Begin by using the **6 b-jet final state!**

1. Require 6 tagged b-jets.

1 ____________________
2 ____________________
3 ____________________
4 ____________________
5 ____________________
6 ____________________

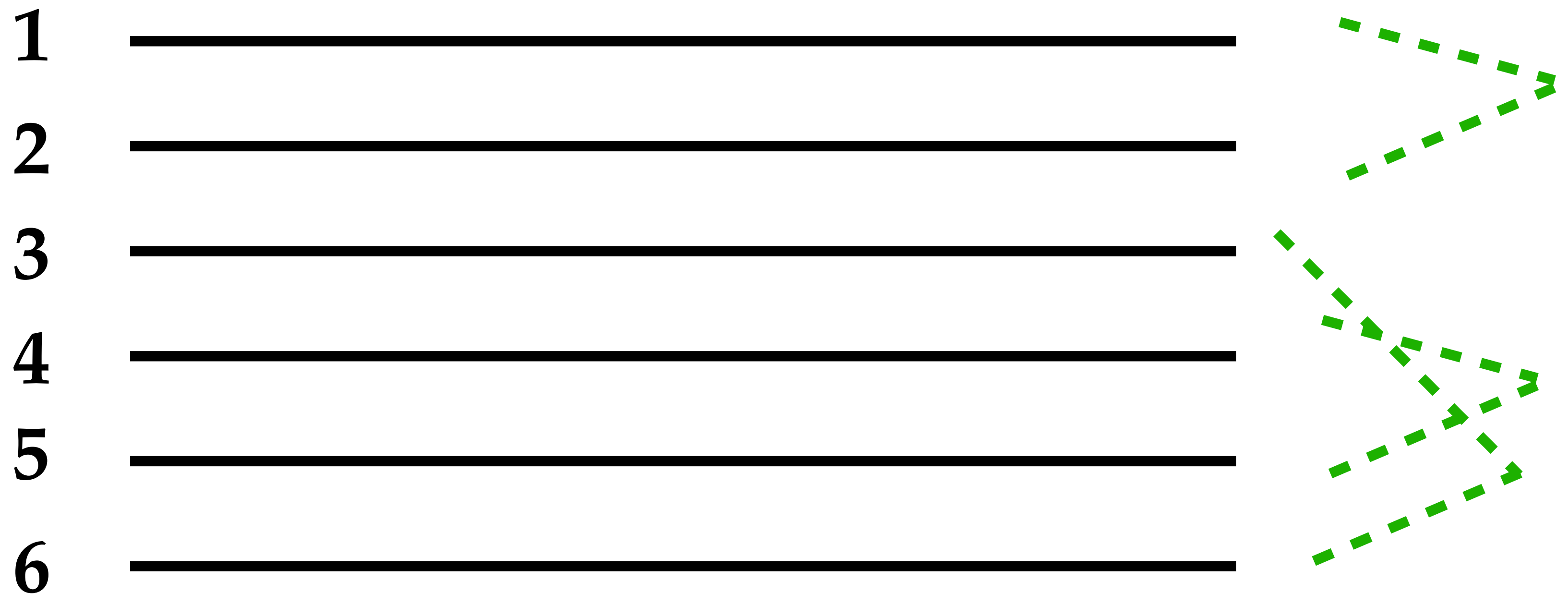
The 6b final state, analysis [AP, Gilberto Tetlalmatzi-Xolocotzi, Marco Zaro, **arXiv:1909.09166]**

- What can we learn about the anomalous couplings via **hhh** at 13.6 TeV?

- Begin by using the **6 b-jet final state!**

1. Require 6 tagged b-jets.

2. Consider pairings of the b-jets.

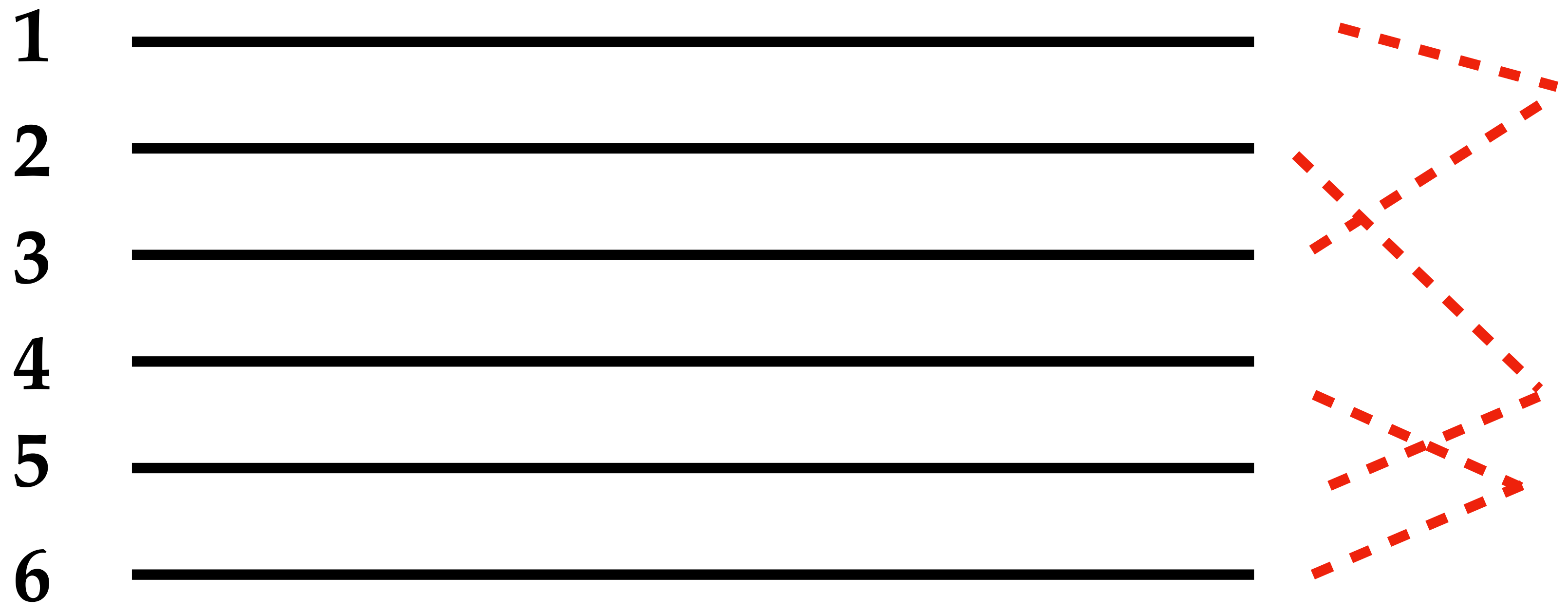


The 6b final state, analysis [AP, Gilberto Tetlalmatzi-Xolocotzi, Marco Zaro, arXiv:1909.09166]

- What can we learn about the anomalous couplings via **hhh** at 13.6 TeV?
- Begin by using the **6 b-jet final state!**

1. Require 6 tagged b-jets.

2. Consider pairings of the b-jets.



The 6b final state, analysis [AP, Gilberto Tetlalmatzi-Xolocotzi, Marco Zaro, arXiv:1909.09166]

- What can we learn about the anomalous couplings via **hhh** at 13.6 TeV?
- Begin by using the **6 b-jet final state!**

1. Require 6 tagged b-jets.

2. Consider pairings of the b-jets.

3. For each pairing construct:

$$\chi^2 = \sum_{qr\in\text{pairings } I} (M_{qr} - m_h^2)^2$$

≡ sum of squared differences from Higgs mass (~125 GeV)

The 6b final state, analysis [AP, Gilberto Tetlalmatzi-Xolocotzi, Marco Zaro, arXiv:1909.09166]

- What can we learn about the anomalous couplings via **hhh** at 13.6 TeV?

- Begin by using the **6 b-jet final state!**

1. Require 6 tagged b-jets.

2. Consider pairings of the b-jets.

3. For each pairing construct:

$$\chi^2 = \sum_{qr\in\text{pairings } I} (M_{qr} - m_h^2)^2$$

≡ sum of squared differences from Higgs mass (~125 GeV)

⇒ 4. Pairing that gives $\underbrace{\text{minimum } \chi^2}_{\chi^2_{\text{min}}}$ determines “reconstructed Higgs boson”.

The 6b final state, analysis

h_r^i →**Higgs boson candidates**

observable	cut
$p_{T,b}$	> 45 GeV
$\|\eta_b\|$	< 3.2
$\Delta R_{b,b}$	> 0.3
$p_T(h_r^i)$	$> [170, 120, 0]$ GeV, $i = 1, 2, 3$
χ^2_{min}	< 17 GeV
$\Delta m_{\text{min, mid, max}}$	$< 8, 8, 11$ GeV
$\Delta R(h_r^i, h_r^j)$	$< [3.5, 3.5, 3.5]$, $(i, j) = [(1,2), (1,3), (2,3)]$
$\Delta R_{bb}(h_r^i)$	$< [3.5, 3.5, 3.5]$, $i = 1, 2, 3$

← **the three terms in χ^2_{min}.**

signal/backgrounds after analysis

Process	σ_{GEN} (pb)	$\sigma_{\text{NLO}} \times \text{BR}$ (pb)	$\varepsilon_{\text{analysis}}$	$N^{\text{cuts}}_{20\ \text{ab}^{-1}}$
hhh (SM)	2.88×10^{-3}	1.06×10^{-3}	0.0131	278
QCD $(b\bar{b})(b\bar{b})(b\bar{b})$	26.15	52.30	2.6×10^{-5}	27116
$q\bar{q} \to hZZ \to h(b\bar{b})(b\bar{b})$	8.77×10^{-4}	4.99×10^{-4}	1.8×10^{-4}	~ 2
$q\bar{q} \to ZZZ \to (b\bar{b})(b\bar{b})$	7.95×10^{-4}	7.95×10^{-4}	1.2×10^{-5}	< 1
ggF $hZZ \to h(b\bar{b})(b\bar{b})$	1.08×10^{-4}	1.23×10^{-4}	$\mathcal{O}(10^{-3})$	~ 2
ggF $ZZZ \to (b\bar{b})(b\bar{b})$	1.36×10^{-5}	2.73×10^{-5}	2×10^{-5}	$\ll 1$
$h(b\bar{b})(b\bar{b})$	1.46×10^{-2}	1.66×10^{-2}	5.4×10^{-4}	179
$hh(b\bar{b})$	1.40×10^{-4}	9.11×10^{-5}	2.8×10^{-4}	~ 1
$hhZ \to hh(b\bar{b})$	4.99×10^{-3}	1.61×10^{-3}	7.2×10^{-4}	23
$hZ(b\bar{b}) \to h(b\bar{b})(b\bar{b})$	9.08×10^{-3}	1.03×10^{-2}	1.4×10^{-4}	29
$ZZ(b\bar{b}) \to (b\bar{b})(b\bar{b})(b\bar{b})$	2.87×10^{-2}	5.74×10^{-2}	1×10^{-5}	11
$Z(b\bar{b})(b\bar{b}) \to (b\bar{b})(b\bar{b})(b\bar{b})$	0.93	1.87	3×10^{-5}	1121
$\sum$ backgrounds				2.8×10^{4}

Reducible backgrounds

process	σ_{GEN} (pb)	$\sigma_{\mathrm{GEN}} \times \mathscr{P}(6\,b-\text{jets})$ (pb)
$(b\bar{b})(b\bar{b})(c\bar{c})$	76.8	0.768
$(b\bar{b})(c\bar{c})(c\bar{c})$	75.6	0.00756
$(c\bar{c})(c\bar{c})(c\bar{c})$	22.5	22.5×10^{-5}
$(b\bar{b})(b\bar{b})(jj)$	1.32×10^{4}	1.32
$(b\bar{b})(jj)(jj)$	9.79×19^{5}	0.00979
$(jj)(jj)(jj)$	1.37×10^{6}	1.37×10^{-6}

c.f. σ_{GEN}(6b)= 26.15 pb

$\uparrow$

applied:

$$\mathcal{P}_{c\to b} = 0.1$$
$$\mathcal{P}_{j\to b} = 0.01$$

⇒ **Assuming perfect b-tagging + identical analysis efficiency to QCD 6b:**

→~10% contribution from reducible backgrounds.

for P(b-tagging) = 0.8:

→~30% contribution.

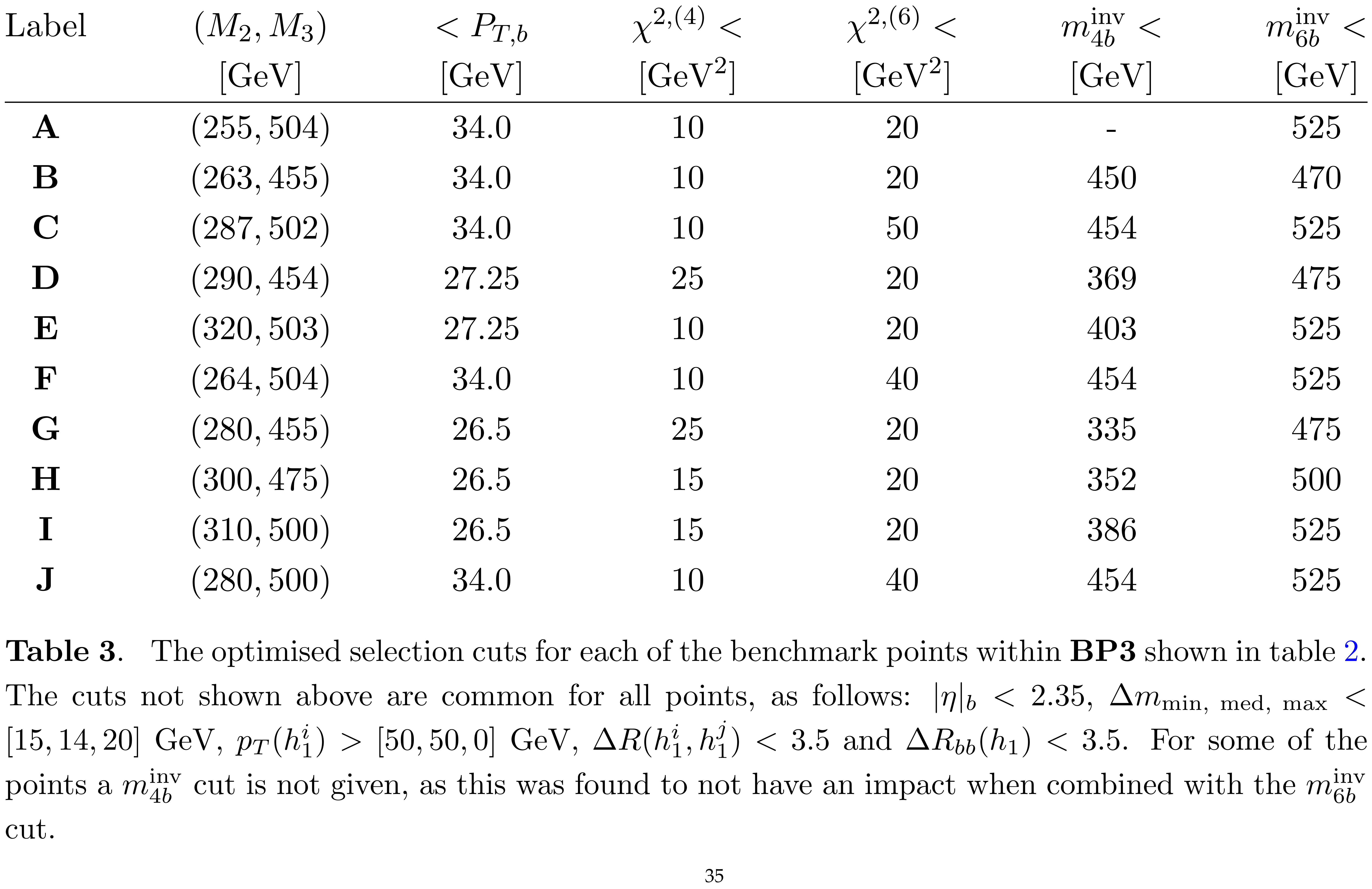
TRSM hhh → 6b analysis details

Introduce two observables: $\chi^{2,(4)} = \sum_{qr \in I} \left(M_{qr} - M_1 \right)^2$

$$\chi^{2,(6)} = \sum_{qr \in J} \left(M_{qr} - M_1 \right)^2$$

→ constructed from different pairings of 4 and 6 b-tagged jets, M_{qr} is the invariant mass of the pairing qr.

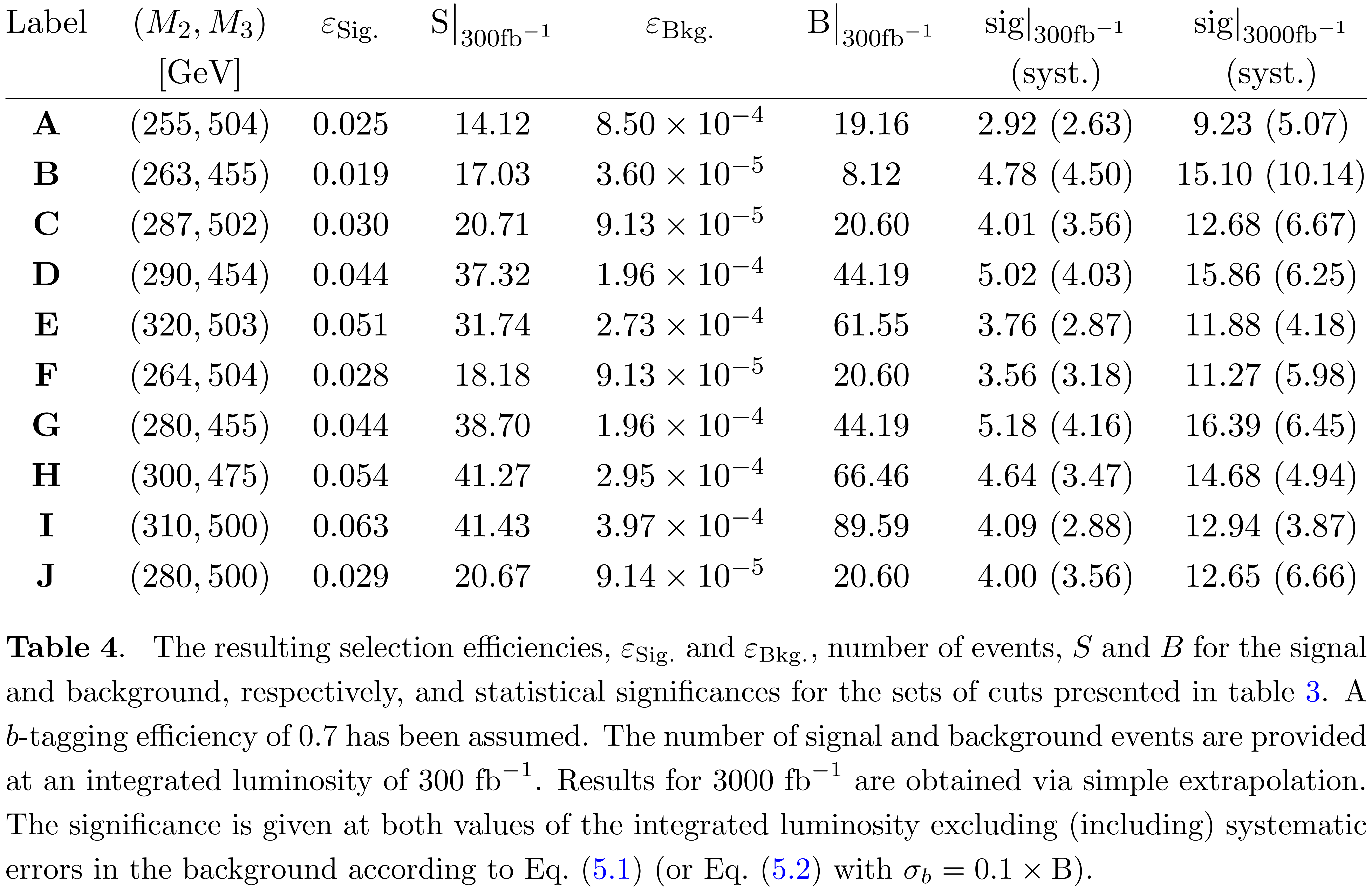
TRSM hhh → 6b analysis details



Label	(M_2, M_3) [GeV]	$< P_{T,b}$ [GeV]	$\chi^{2,(4)} <$ [GeV^2]	$\chi^{2,(6)} <$ [GeV^2]	$m_{4b}^{\text{inv}} <$ [GeV]	$m_{6b}^{\text{inv}} <$ [GeV]
A	$(255, 504)$	34.0	10	20	-	525
B	$(263, 455)$	34.0	10	20	450	470
C	$(287, 502)$	34.0	10	50	454	525
D	$(290, 454)$	27.25	25	20	369	475
E	$(320, 503)$	27.25	10	20	403	525
F	$(264, 504)$	34.0	10	40	454	525
G	$(280, 455)$	26.5	25	20	335	475
H	$(300, 475)$	26.5	15	20	352	500
I	$(310, 500)$	26.5	15	20	386	525
J	$(280, 500)$	34.0	10	40	454	525

Table 3. The optimised selection cuts for each of the benchmark points within **BP3** shown in table 2. The cuts not shown above are common for all points, as follows: $|\eta|_b < 2.35$, $\Delta m_{\text{min, med, max}} < [15, 14, 20]$ GeV, $p_T(h_1^i) > [50, 50, 0]$ GeV, $\Delta R(h_1^i, h_1^j) < 3.5$ and $\Delta R_{bb}(h_1) < 3.5$. For some of the points a m_{4b}^{inv} cut is not given, as this was found to not have an impact when combined with the m_{6b}^{inv} cut.

TRSM hhh → 6b analysis details (Signal vs Bkg)



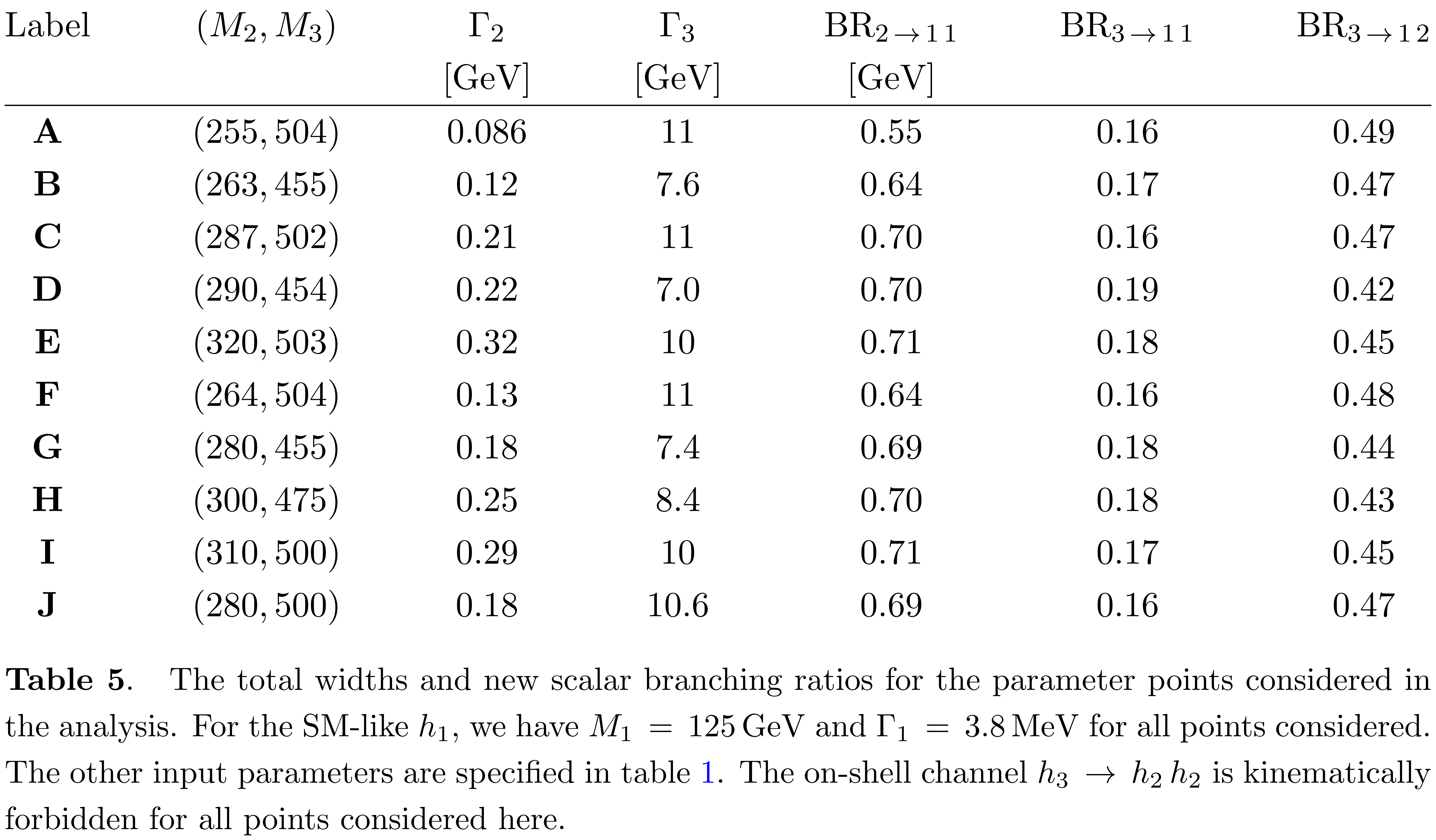
Label	(M_2, M_3) [GeV]	$\varepsilon_{\text{Sig.}}$	$\text{S}\big\|_{300\text{fb}^{-1}}$	$\varepsilon_{\text{Bkg.}}$	$\text{B}\big\|_{300\text{fb}^{-1}}$	$\text{sig}\big\|_{300\text{fb}^{-1}}$ (syst.)	$\text{sig}\big\|_{3000\text{fb}^{-1}}$ (syst.)
A	$(255, 504)$	0.025	14.12	8.50×10^{-4}	19.16	2.92 (2.63)	9.23 (5.07)
B	$(263, 455)$	0.019	17.03	3.60×10^{-5}	8.12	4.78 (4.50)	15.10 (10.14)
C	$(287, 502)$	0.030	20.71	9.13×10^{-5}	20.60	4.01 (3.56)	12.68 (6.67)
D	$(290, 454)$	0.044	37.32	1.96×10^{-4}	44.19	5.02 (4.03)	15.86 (6.25)
E	$(320, 503)$	0.051	31.74	2.73×10^{-4}	61.55	3.76 (2.87)	11.88 (4.18)
F	$(264, 504)$	0.028	18.18	9.13×10^{-5}	20.60	3.56 (3.18)	11.27 (5.98)
G	$(280, 455)$	0.044	38.70	1.96×10^{-4}	44.19	5.18 (4.16)	16.39 (6.45)
H	$(300, 475)$	0.054	41.27	2.95×10^{-4}	66.46	4.64 (3.47)	14.68 (4.94)
I	$(310, 500)$	0.063	41.43	3.97×10^{-4}	89.59	4.09 (2.88)	12.94 (3.87)
J	$(280, 500)$	0.029	20.67	9.14×10^{-5}	20.60	4.00 (3.56)	12.65 (6.66)

Table 4. The resulting selection efficiencies, $\varepsilon_{\text{Sig.}}$ and $\varepsilon_{\text{Bkg.}}$, number of events, S and B for the signal and background, respectively, and statistical significances for the sets of cuts presented in table 3. A b-tagging efficiency of 0.7 has been assumed. The number of signal and background events are provided at an integrated luminosity of 300 fb^{-1}. Results for 3000 fb^{-1} are obtained via simple extrapolation. The significance is given at both values of the integrated luminosity excluding (including) systematic errors in the background according to Eq. (5.1) (or Eq. (5.2) with $\sigma_b = 0.1 \times \text{B}$).

TRSM BP3 Definition

Parameter	Value
M_1	125.09 GeV
M_2	$[125,\ 500]$ GeV
M_3	$[255,\ 650]$ GeV
θ_{hS}	-0.129
θ_{hX}	0.226
θ_{SX}	-0.899
v_S	140 GeV
v_X	100 GeV
κ_1	0.966
κ_2	0.094
κ_3	0.239

TRSM BP3 Benchmark Point Info



Label	(M_2, M_3)	Γ_2 [GeV]	Γ_3 [GeV]	$\text{BR}_{2\to 1\,1}$ [GeV]	$\text{BR}_{3\to 1\,1}$	$\text{BR}_{3\to 1\,2}$
A	$(255, 504)$	0.086	11	0.55	0.16	0.49
B	$(263, 455)$	0.12	7.6	0.64	0.17	0.47
C	$(287, 502)$	0.21	11	0.70	0.16	0.47
D	$(290, 454)$	0.22	7.0	0.70	0.19	0.42
E	$(320, 503)$	0.32	10	0.71	0.18	0.45
F	$(264, 504)$	0.13	11	0.64	0.16	0.48
G	$(280, 455)$	0.18	7.4	0.69	0.18	0.44
H	$(300, 475)$	0.25	8.4	0.70	0.18	0.43
I	$(310, 500)$	0.29	10	0.71	0.17	0.45
J	$(280, 500)$	0.18	10.6	0.69	0.16	0.47

Table 5. The total widths and new scalar branching ratios for the parameter points considered in the analysis. For the SM-like h_1, we have $M_1 = 125\,\text{GeV}$ and $\Gamma_1 = 3.8\,\text{MeV}$ for all points considered. The other input parameters are specified in table 1. The on-shell channel $h_3 \to h_2\,h_2$ is kinematically forbidden for all points considered here.

hhh in the TRSM [14 TeV]

- Focus on a particular family of benchmark points: **“Benchmark Plane 3”** = **“BP3”** in [Robens, Stefaniak, Wittbrodt, arXiv:1908.08554].

Label	(M_2, M_3) [GeV]	$\sigma(pp \to h_1 h_1 h_1)$ [fb]
A	$(255, 504)$	32.40
B	$(263, 455)$	50.36
C	$(287, 502)$	39.61
D	$(290, 454)$	49.00
E	$(320, 503)$	35.88
F	$(264, 504)$	37.67
G	$(280, 455)$	51.00
H	$(300, 475)$	43.92
I	$(310, 500)$	37.90
J	$(280, 500)$	40.26

- In BP3: All params fixed except M_2, M_3!

Cross section can be much higher than in the SM! 😲

→ c.f. SM: σ ~ 0.1 fb @ 14 TeV.

[**AP**, Robens, Tetlalmatzi-Xolocotzi, arXiv:2101.00037]

hhh in the TRSM “BP3” [14 TeV]

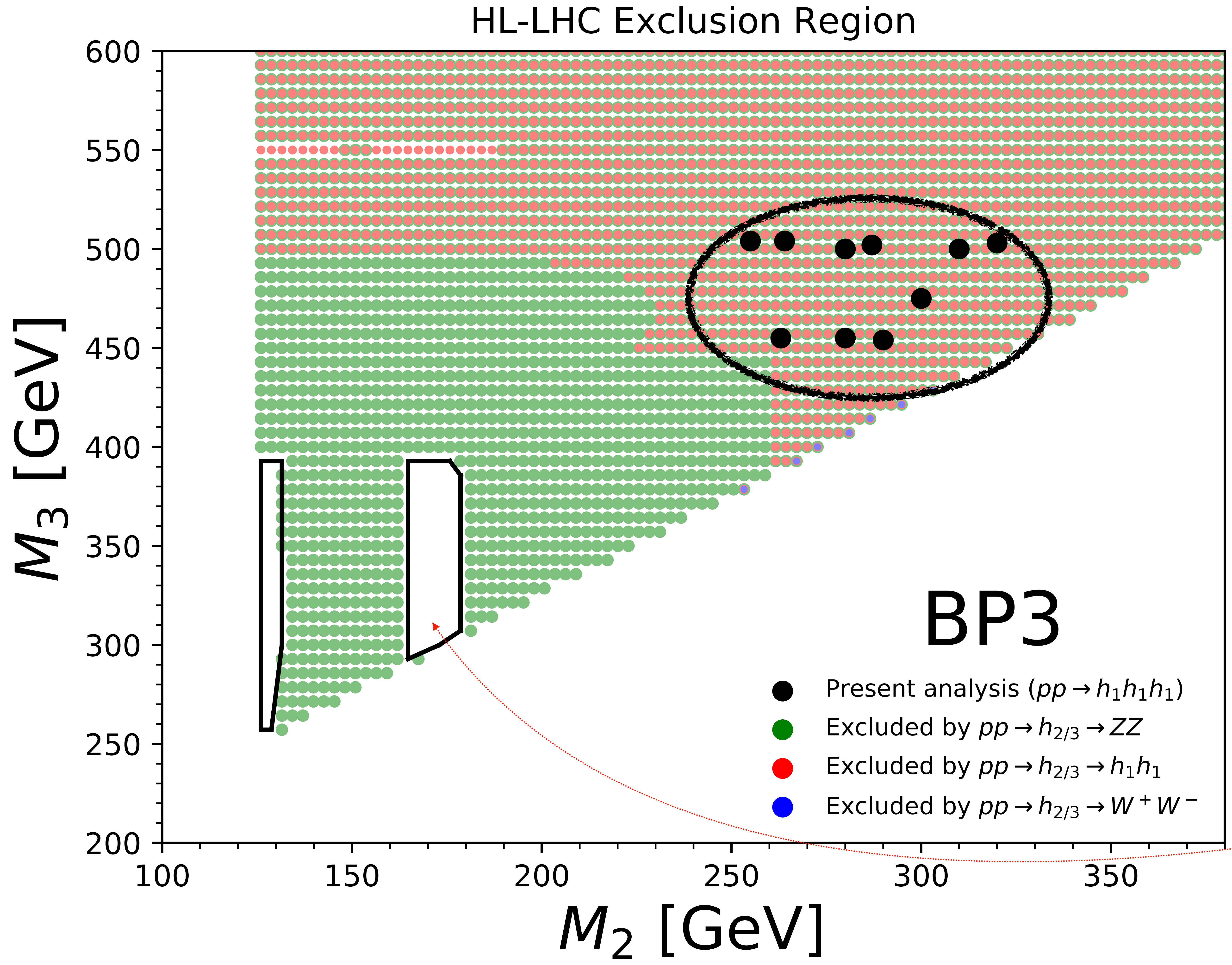
- Search for **hhh** via: $pp \to (b\bar{b})(b\bar{b})(b\bar{b}) \to$ **6 b-jets**.
- About **20%** of the **hhh** final state!
- Significances **large**, even when including systematic uncert.:

[**AP**, Robens, Tetlalmatzi-Xolocotzi, arXiv:2101.00037]

Label	$\text{sig}\vert_{300\text{fb}^{-1}}$ (syst.)	$\text{sig}\vert_{3000\text{fb}^{-1}}$ (syst.)
A	2.92 (2.63)	9.23 (5.07)
B	4.78 (4.50)	15.10 (10.14)
C	4.01 (3.56)	12.68 (6.67)
D	5.02 (4.03)	15.86 (6.25)
E	3.76 (2.87)	11.88 (4.18)
F	3.56 (3.18)	11.27 (5.98)
G	5.18 (4.16)	16.39 (6.45)
H	4.64 (3.47)	14.68 (4.94)
I	4.09 (2.88)	12.94 (3.87)
J	4.00 (3.56)	12.65 (6.66)

hhh in the TRSM “BP3” [14 TeV]

- **hhh** will (**probably?**) **not be a discovery channel,**
- but could be **important in determining the parameters of the model**, if scalars are discovered!



Note: regions near $M_2 \sim 130$ GeV and $M_2 \sim 170$ GeV will remain viable at the end of the HL-LHC.

Could help solve the “**inverse problem**” in the **TRSM**?

[**AP**, Robens, Tetlalmatzi-Xolocotzi, arXiv:2101.00037]

Monte Carlo Implementation of Anomalous Couplings

- Get the **MG5_aMC model** at: https://gitlab.com/apapaefs/multihiggs_loop_sm.

- [A patch to **MG5_aMC** to enable **Loop** × **Tree** is included].

- Can generate events either at:

  - **SM**^2 + interference of [**SM** × **One**-Insertion diagrams], i.e.:
    $|\mathscr{M}|^2 = |\mathscr{M}_{\mathrm{SM}}|^2 + 2\mathrm{Re}\{\mathscr{M}^*_{\mathrm{SM}}\mathscr{M}_{1-\mathrm{ins.}}\} \propto 1 + c_i$

    or

  - **SM**^2 + interference of [**SM** × **One** or **Two** insertion diagrams] + [**One** Insertion]^2, i.e.:
    $|\mathscr{M}|^2 = |\mathscr{M}_{\mathrm{SM}}|^2 + 2\mathrm{Re}\{\mathscr{M}^*_{\mathrm{SM}}\mathscr{M}_{1-\mathrm{ins.}}\} + 2\mathrm{Re}\{\mathscr{M}^*_{\mathrm{SM}}\mathscr{M}_{2-\mathrm{ins.}}\} + |\mathscr{M}_{1-\mathrm{ins.}}|^2$
    $\propto 1 + c_i + c_j c_k + c_\ell^2$