Electroweak hierarchy from conformal and custodial symmetry:

“Custodial Naturalness”

Andreas Trautner

based on:

arXiv:2407.15920 w/ **Thede de Boer** and Manfred Lindner

Extended Scalar Sectors From All Angles

Workshop, CERN

21.10.24

MAX-PLANCK-GESELLSCHAFT

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Outline

- Hierarchy problem
- General idea of “Custodial Naturalness”
- Minimal model
- Numerical analysis, experimental constraints and predictions
- Extensions and embeddings
- Conclusions

Disclaimer: For this talk in 4D, scale invariance $\sim$ conformal invariance.

Electroweak scale hierarchy problem

Not a problem *in* the Standard Model (SM). [Bardeen '95]

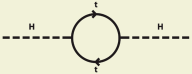
However, in presence of heavy scales Λ_{high}, it remains puzzling that

(see, however, [Mooij, Shaposhnikov '21], [K.-S. Choi '24])

$$m_h^2 \propto \Lambda_{\text{high}}^2 ,$$

which, in case e.g. $\Lambda_{\text{high}} \sim M_{\text{Pl}}$, is not supported by observation.

Symmetry based solutions:



- Supersymmetry.
- Composite Higgs ($h =$ pNGB of some new strongly coupled sector).

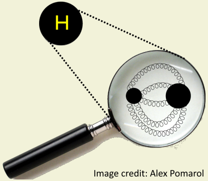


Image credit: Alex Pomarol

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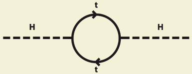

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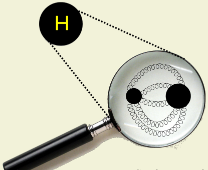

Image credit: Alex Pomarol

However, neither is Nature close-to supersymmetric,
nor do the Higgs measurements hint at compositeness.
Also: No top-partners observed.

But: SM *is* close to **scale invariant**, *explicitly* broken only by $\mu_H \, (\sim m_h \sim v_{\text{EW}})_{\text{SM}}$.

Conformal "solution"

- The SM exhibits classical scale symmetry, only explicitly broken by $\mu_H^2 \, |H|^2$.
- Quantum corrections *can* spontaneously generate $\mu_H^2 \sim \Lambda_{\mathrm{CW}}^2 \sim \mathrm{e}^{-\frac{\lambda}{g^4}} \Lambda_{\mathrm{high}}^2$, [Coleman, Weinberg '73]
- …But in SM this parametrically only works for $m_h \sim m_t \sim \mathcal{O}(10\,\mathrm{GeV})$. [Weinberg '76]

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New here: Higgs as pNGB of spontaneosuly broken **custodial symmetry** avoids this problem.

✓ Technically natural suppression of EW scale.

✓ Only elementary fields, no compositeness, all perturbative.

✓ No top partners, marginal top Yukawa like in SM.

"Custodial Naturalness" – General Idea

Assumptions:

1. Classical scale invariance.
2. New complex scalar Φ + new $\mathrm{U}(1)_\mathrm{X}$ gauge symmetry. $\mathrm{SU}(3)_\mathrm{c}\times\mathrm{SU}(2)_\mathrm{L}\times\mathrm{U}(1)_\mathrm{Y}\times\mathrm{U}(1)_\mathrm{X}$
3. High-scale $\mathrm{SO}(6)$ **custodial** symmetry of scalar potential:

$$\Rightarrow \qquad V(H,\Phi) \;=\; \lambda\left(|H|^2+|\Phi|^2\right)^2 \qquad \text{at} \;\; \mu = \Lambda_\mathrm{high} \equiv M_\mathrm{Pl}\,.$$

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Both, scale invariance + $\mathrm{SO}(6)$ are broken by quantum effects.

- ***If*** $\mathrm{SO}(6)$ *were* classically exact $\rightarrow$ [Coleman, Weinberg '73] $\rightarrow$ VEVs $\langle\Phi\rangle\&\langle H\rangle$.

$\Rightarrow$ $\mathrm{SO}(6)\xrightarrow{\langle\mathbf{6}\rangle}\mathrm{SO}(5)$: massive dilaton + 4 *would-be* NGBs + massless NGB "h".

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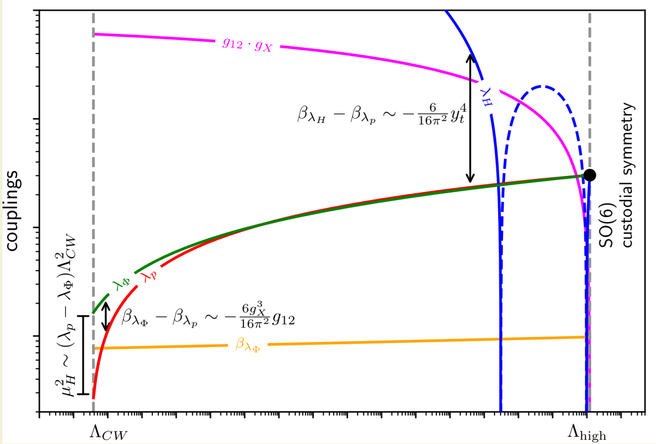
- Realistically: $\mathrm{SO}(6)$ explicitly broken by: y_t, g_{Y} & g_{X}, g_{12}, ..., e.g. y_{new}

$\Rightarrow$ $\mathrm{SO}(6) \xrightarrow{\langle\mathbf{6}\rangle} \mathrm{SO}(5)$: massive dilaton + 4 *would-be* NGBs + massive pNGB “h”.

General Idea – RGE evolution is key

below M_{Pl} : $\quad V_{\text{tree}}(H,\Phi) \;=\; \lambda_H |H|^4 + 2\,\lambda_p |\Phi|^2 |H|^2 + \lambda_\Phi |\Phi|^4$.



Actual running for a benchmark point. Dashed=negative.
β_i: Beta function coefficients.

Custodial sym. (C.S.) breaking:

- dominant breaking: y_t

  $\Rightarrow \quad \langle H \rangle \ll \langle \Phi \rangle$

- splitting $\lambda_\Phi - \lambda_p$ **requires** a new breaking of C.S.

Minimal C.S. breaking:

$\mathrm{U}(1)_{\mathrm{X}} - \mathrm{U}(1)_{\mathrm{Y}}$
gauge kinetic mixing g_{12}.

This generates "$\lambda_\Phi - \lambda_p$."

General Idea – Masses and EW scale

Masses of physical real scalars $h_\Phi \subset \Phi$ and $h \subset H$: $\langle\Phi\rangle = \frac{v_\Phi}{\sqrt{2}}, \langle H\rangle = \frac{v_h}{\sqrt{2}}$

Dilaton: $$m_{h_\Phi}^2 \approx \frac{3\, g_X^4}{8\pi^2}\, v_\Phi^2$$

pNGB Higgs: $$m_h^2 \approx 2\left[\lambda_\Phi\left(1+\frac{g_{12}}{2\, g_X}\right)^2 - \lambda_p\right] v_\Phi^2\ .$$

- This corresponds to $m_{h_\Phi}^2 \approx \beta_{\lambda_\Phi} v_\Phi^2$ and $m_h^2 \approx 2\left(\lambda_\Phi \beta_{\lambda_p}/\beta_{\lambda_\Phi} - \lambda_p\right) v_\Phi^2$.
- λ_H can stay at its SM value.
- EW scale VEV gets to keep the SM relation

$$v_H^2 \approx \frac{m_h^2}{2\lambda_H}\ .$$

⇒ The **EW scale is custodially suppressed** compared to the intermediate scale v_Φ of spontaneous scale and custodial symmetry violation.

Minimal Model

Field	#Gens.	$\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$	$\mathrm{U}(1)_X$	$\mathrm{U}(1)_{B-L}$
Q	3	$(\mathbf{3}, \mathbf{2}, +\frac{1}{6})$	$-\frac{2}{3}$	$+\frac{1}{3}$
u_R	3	$(\mathbf{3}, \mathbf{1}, +\frac{2}{3})$	$+\frac{1}{3}$	$+\frac{1}{3}$
d_R	3	$(\mathbf{3}, \mathbf{1}, -\frac{1}{3})$	$-\frac{5}{3}$	$+\frac{1}{3}$
L	3	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$+2$	-1
e_R	3	$(\mathbf{1}, \mathbf{1}, -1)$	$+1$	-1
ν_R	3	$(\mathbf{1}, \mathbf{1}, 0)$	$+3$	-1
H	1	$(\mathbf{1}, \mathbf{2}, +\frac{1}{2})$	$+1$	0
Φ	1	$(\mathbf{1}, \mathbf{1}, 0)$	$+1$	$q_\Phi^{\mathrm{B-L}} = -\frac{1}{3}$

$$Q^{(\mathrm{X})} \equiv 2\,Q^{(\mathrm{Y})} + \frac{1}{q_\Phi^{\mathrm{B-L}}}\,Q^{(\mathrm{B-L})}$$

- The only free parameter of the charge assignment is $q_\Phi^{\mathrm{B-L}}$.
- Constrained to $\frac{1}{3} \lesssim |q_\Phi^{\mathrm{B-L}}| \lesssim \frac{5}{11}$; special value: $q_\Phi^{\mathrm{B-L}} = -\frac{16}{41}$. Let us fix $q_\Phi^{\mathrm{B-L}} = -\frac{1}{3}$.

Note: Our model is very similar to “classical conformal extension of minimal $B-L$ model”, but $q_\Phi^{\mathrm{B-L}} \neq -2$.
[Iso, Okada, Orikasa ’09]

Numerical analysis

- SM parameters $G_{\rm F}$, m_h, $m_t \longleftrightarrow$ parameters λ, g_X and y_t (@$\Lambda_{\rm high} \sim M_{\rm Pl}$).
- Remaining free parameter: g_{12}. Can fix $g_{12}|_{M_{\rm Pl}} = 0 \quad \Leftrightarrow \quad$ C.S. fixes all d.o.f.'s.

Same number of parameters as the SM!

$\rightarrow$ Properties of Z' and h_Φ are predictions of the model.

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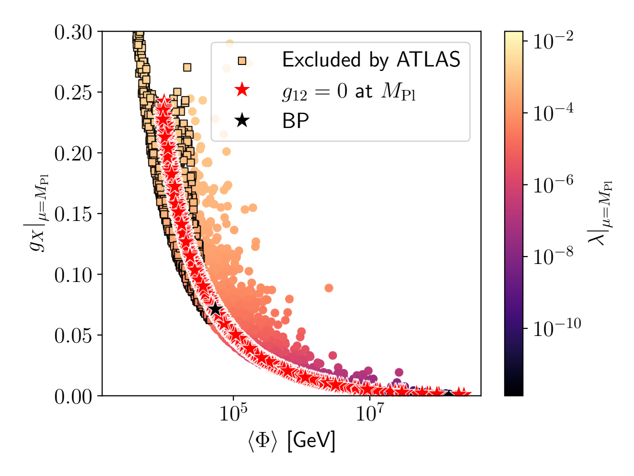
Parameter scan

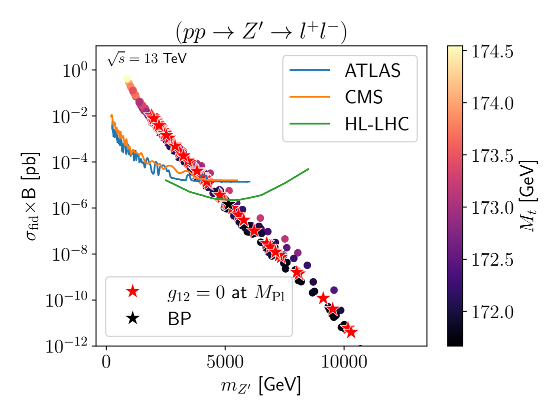
- Impose $\mathrm{SO}(6)$ symmetric BC's @M_Pl: $\lambda_{H,\Phi,p}|_{M_\mathrm{Pl}} = \lambda|_{M_\mathrm{Pl}}$ and $g_{12}|_{M_\mathrm{Pl}} = 0$.
- 2-loop running with PyR@TE. [Sartore, Schienbein '21]
- Iteratively determine intermediate scale Φ_0, match to SM at $\mu_0 \sim \mathcal{O}(g_X \Phi_0)$.
- Numerically minimize 1-loop V_eff (at μ_0), compute v_Φ and v_H, m_{h_Φ}, m_h, $\lambda_{H,\Phi,p}$, match to 1-loop $V_\mathrm{eff}^\mathrm{SM}$ (+dilaton hidden scalar, corrections negligible).
- From μ_0 down to m_t 2-loop running.
- Require $v_H^\mathrm{exp} = 246.2 \pm 0.1\,\mathrm{GeV}$, as well as g_L, g_Y, g_3 and y_t within SM errors.
- Low scale new couplings g_X, g_{12} and masses $m_{Z'}$, m_{h_Φ} are predictions.

Parameter space



Parameters at $\mu = M_{\rm Pl}$. All points shown reproduce the correct EW scale.
New scale $\langle\Phi\rangle = v_\Phi/\sqrt{2}$ is prediction. (m_h, M_t not imposed as constraint).

Phenomenological constraints



- $Z' \to l^+l^-$ resonance searches require $m_{Z'} \gtrsim 4\,\mathrm{TeV}$. (di-jets are weaker)
- EW precision: Additional custodial breaking shifts m_Z,

$$\Delta m_Z \propto -m_Z \langle H\rangle^2/(2\langle\Phi\rangle^2)\ .$$

Constraint: $\langle\Phi\rangle \gtrsim 18\,\mathrm{TeV}$, weaker than direct Z' searches.

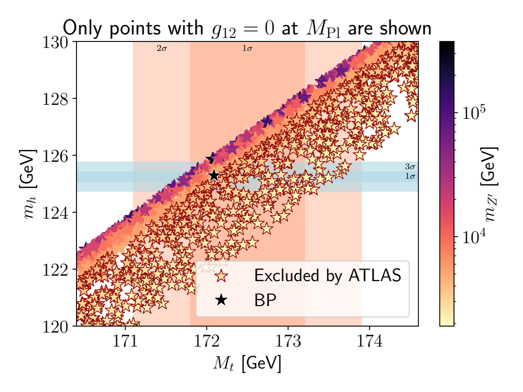
- Dilaton-higgs mixing:

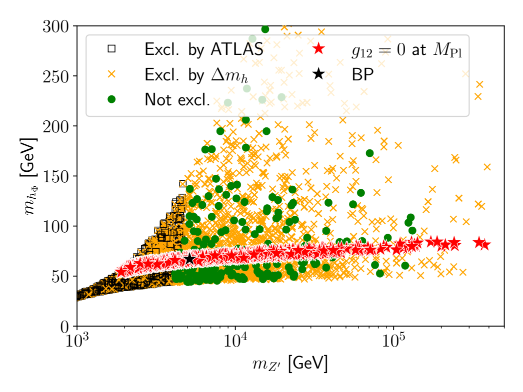
$$\mathcal{O}_{h_\Phi} \approx \sin\theta \times \mathcal{O}^{\rm SM}_{h\to h_\Phi}\ .$$

For $m_{h_\Phi} \sim 75\,\mathrm{GeV}$, $\sin\theta \lesssim 10^{-1}$ is a-OK.
(typical values for us are BP: $\sin\theta \sim 10^{-2.5}$)

- Neglect dilaton-gauge-gauge coupling from trace anomaly, suppressed by $\frac{v_h}{v_\Phi}$.

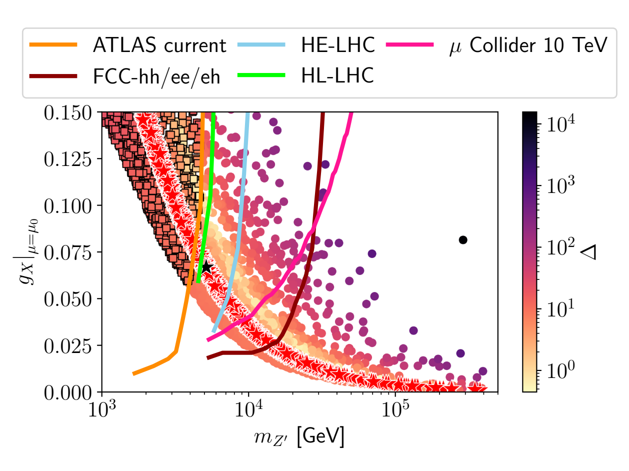
Reproductions and predictions





All points shown reproduce the correct EW scale. M_t: top pole mass.

Fine tuning and Future collider projections



Projections are for hypercharge universal Z' from [R.K. Ellis et al. '20]

Fine tuning:

$$\Delta \;:=\; \max_{g_i} \left| \frac{\partial \ln \frac{\langle H\rangle}{\langle \Phi\rangle}}{\partial \ln g_i} \right| .$$

Barbieri-Giudice measure.
[Barbieri, Giudice '88]

The choice of $\langle H\rangle/\langle\Phi\rangle$ automatically subtracts the shared sensitivity of VEVs to variation of g_i. [Anderson, Castano '95]

Red stars: $g_{12}|_{M_{\rm Pl}} = 0$.

Black star: benchmark point.

Prime target: Z' at FC, Dilaton production(+displaced dec.) at Higgs factories.

Extensions and embeddings

"Custodial Naturalness" is reasonably stable under variation of boundary conditions, charge assignments, addition of extra particles.

[de Boer, Lindner, AT 'XX]

Minimal model portals: $|\Phi|^2|H|^2$ and $X^{\mu\nu}Y_{\mu\nu}$, in extensions also $\overline{L}\widetilde{H}\Psi_{\text{new}}$.

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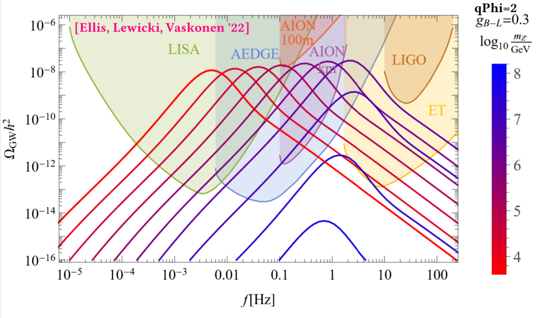
Additional fermions can:

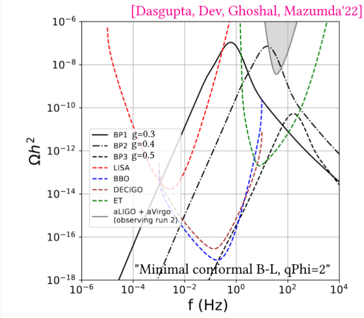
- Provide ingredients for neutrino mass generation, [Iso, Okada, Orikasa '09] [Foot, Kobakhidze, McDonald, Volkas '07]
- Be part of the dark matter, [S. Okada '18]
- "Cure" SM vacuum instability. [(Das), Oda, Okada, Takahashi '15('16)]

- Custodial symmetry could originate from UV fixed point $\leftrightarrow$ quantum criticality. e.g. [Litim, Sannino '14]
- GUT embeddings $G_{\text{cust.}} \subset G_{\text{GUT}}$ allow to constrain $q^{\Phi}_{\text{B}-\text{L}}$ and compute the size of gauge-kinetic mixing g_{12}.
- Note: We have ignored finite-T effects here, this is yet to be done!

Gravitational wave signals?

- We have ignored finite-T effects so far. This is yet to be done.
- CW transition is known to be first order $\rightarrow$ Gravitational wave signals.
  see e.g. [Litim, Wetterich, Tetradis '97], [Dasgupta, Dev, Ghoshal, Mazumdar '22],[Huang, Xie '22]
- In fact, the "minimal conformal $B-L$ model" is prototype for **strong supercooling** $\rightarrow$ strong GW signal from bubble collisions. see e.g. [Ellis,Lewicki,Vaskonen'20]





Quantitative predictions for our specific case have yet to be worked out!

Conclusions

- Classical scale invariance + extended custodial symmetry (here $\mathrm{SO}(6)$)

$\Rightarrow$ New mechanism to explain large scale separation and little hierarchy problem.

- Minimal model: Φ + $\mathrm{U}(1)_{\mathrm{X}}$ gauge: same number of parameters as the SM.
- Predicts light scalar dilaton $m_\Phi \sim 75\,\mathrm{GeV}$ + Z' at $4-100\,\mathrm{TeV}$.
- Top mass at lower end of currently allowed 1σ region.
- Perfect model to motivate new colliders + Higgs factory + GR waves.
- Many extensions and details to explore.

Thank You!

Image credit: CERN

Backup slides

Details of the potential and matching

Effective potential for background fields H_b and Φ_b @1-loop $\overline{\text{MS}}$:

$(-1)^{2s_i} \equiv {}_{(-)}^{+}1$ for bosons(fermions), $n_i \equiv \#\text{d.o.f}$
$C_i = \frac{5}{6}\left(\frac{3}{2}\right)$ for vector bosons(scalars/fermions).

$$V_{\text{eff}} = V_{\text{tree}} + \sum_i \frac{n_i(-1)^{2s_i}}{64\pi^2} m_{i,\text{eff}}^4 \left[\ln\left(\frac{m_{i,\text{eff}}^2}{\mu^2}\right) - C_i\right] .$$

Two different *analytical* expansions: First

$$V_{\text{EFT}}(H_b) := V_{\text{eff}}\left(H_b, \tilde{\Phi}(H_b)\right) , \qquad \text{with} \qquad \left.\frac{\partial V_{\text{eff}}}{\partial \Phi_b}\right|_{\Phi_b = \tilde{\Phi}(H_b)} = 0 .$$

Using $\Phi_0 := \Phi(H_b/\Phi_b = 0)$, we expand V_{EFT} in $H_b \ll \Phi_0$, $\curvearrowright$ RG-scale independent expression

$$V_{\text{EFT}} \approx 2\left[\lambda_p - \left(1 + \frac{g_{12}}{2\, g_X}\right)^2 \lambda_\Phi\right] \Phi_0^2 H_b^2 + \frac{\lambda_p \lambda_H}{16\pi^2}[...] .$$

This expression illustrates the origin of the Higgs mass and EW scale suppression.

Alternatively, take $\mu = \mu_0 := \sqrt{2} g_X \Phi_0 \mathrm{e}^{-1/6} \sim \langle\Phi\rangle$ and "t Hooft-like" expansion $\frac{\lambda_p}{\lambda_H} \sim \frac{H_b^2}{\Phi_0^2} \sim \epsilon^2 \to 0$,

$$V_{\text{EFT}} = -\frac{6\, g_X^4}{64\pi^2}\Phi_0^4 + 2\,\lambda_p \Phi_0^2 H_b^2 + \lambda_H H_b^4 + \sum_{i=\text{SM}} \frac{n_i(-1)^{2s_i}}{64\pi^2} m_{i,\text{eff}}^4 \left[\ln\left(\frac{m_{i,\text{eff}}^2}{\mu_0^2}\right) - C_i\right] .$$

This expression facilitates matching to the SM at scale μ_0.

Details of the potential and matching II

For all practical purpose the usual CW relation holds:

$$\Phi_0^2 \approx \exp\left\{-\frac{16\pi^2\lambda_\Phi}{3g_X^4} - \ln(2g_X^2) + \frac{1}{3} + \ldots\right\}\mu^2 \ . \tag{1}$$

Analytically we can use $H_b \ll \tilde{\Phi}(0) := \Phi_0$ and the leading order expression for Φ_0 reads

$$\frac{1}{16\pi^2}\ln\left(\frac{\Phi_0^2}{\mu^2}\right) \ = \ -\frac{\lambda_\Phi + \frac{1}{16\pi^2}\left\{q_\Phi^4 g_X^4\left[3\ln\left(2q_\Phi^2 g_X^2\right) - 1\right] + 4\,\lambda_p^2\left(\ln 2\lambda_p - 1\right)\right\}}{3\,q_\Phi^4 g_X^4 + 4\,\lambda_p^2} \ . \tag{2}$$

Alternatively, we can use the ϵ expansion, and Φ_0 at $\mathcal{O}(\epsilon^0)$ reads

$$\frac{1}{16\pi^2}\ln\left(\frac{\Phi_0^2}{\mu^2}\right) \ = \ -\frac{\lambda_\Phi + \frac{1}{16\pi^2}\left\{q_\Phi^4 g_X^4\left[3\ln\left(2q_\Phi^2 g_X^2\right) - 1\right]\right\}}{3\,q_\Phi^4 g_X^4} \ . \tag{3}$$

This is an example for the difference between the two expansion schemes. Note that our quantitative analysis is not based on any of these expansions but uses a fully numerical minimization of the effective potential to compute $\langle\Phi\rangle$ and $\langle H\rangle$.

Integrating out scalar in non-conformal model

Consider a simple two complex scalar system with a potential given by

$$V = -m_H^2|H|^2 - m_\Phi^2|\Phi|^2 + \frac{\lambda_H}{2}|H|^4 + \lambda_p|H|^2|\Phi|^2 + \frac{\lambda_\Phi}{2}|\Phi|^4.$$

For $m_\Phi^2 > 0$ and $-m_H^2 + m_\Phi^2\frac{\lambda_p}{\lambda_\Phi} > 0$, this potential has a minimum at $\langle\Phi\rangle := \frac{v_\Phi}{\sqrt{2}} = \sqrt{\frac{m_\Phi^2}{\lambda_\Phi}}, \langle H\rangle = 0$.

Integrating out the heavy field Φ at tree level, we find the low energy potential

$$\begin{aligned} V_{\text{EFT}} &= \left(-m_H^2 + \lambda_p\frac{v_\Phi^2}{2}\right)|H|^2 + \frac{1}{2}\left(\lambda_H + \frac{\lambda_p^2}{\lambda_\Phi}\right)|H|^4 \\ &= \left(-m_H^2 + \lambda_p\frac{m_\Phi^2}{\lambda_\Phi}\right)|H|^2 + \frac{1}{2}\left(\lambda_H + \frac{\lambda_p^2}{\lambda_\Phi}\right)|H|^4. \end{aligned}$$

The light field is massless at tree level if $\lambda_\Phi\, m_H^2 = \lambda_p\, m_\Phi^2$.
A special point fulfilling this condition is $m_H^2 = m_\Phi^2 := m^2$ and $\lambda_p = \lambda_\Phi := \lambda$. At this point the original potential is given by

$$V = -m^2\left(|H|^2 + |\Phi|^2\right) + \frac{\lambda}{2}\left(|H|^2 + |\Phi|^2\right)^2 + \frac{\lambda_H - \lambda}{2}|H|^4$$

This potential is symmetric up to the quartic term of H which can violate the symmetry badly without affecting the light mass term at tree level.

Benchmark point 1 (BP)

μ [GeV]	g_X	g_{12}	λ_H	λ_p	λ_Φ	y_t	m_{h_Φ} [GeV]	$m_{Z'}$ [GeV]	m_h [GeV]	v_H [GeV]
$1.2 \cdot 10^{19}$	0.0713	0.	$\lambda_H = \lambda_p = \lambda_\Phi = 3.3030 \cdot 10^{-5}$			0.377	-	-	-	-
4353	0.0668	0.0093	**0.084**	$-1.6 \cdot 10^{-6}$	$-2.5 \cdot 10^{-11}$	**0.795**	67.0	5143	**132.0**	**263.0**
172	-	-	0.13	-	-	0.930	-	-	125.3	246.1

Table: Input parameters of an example benchmark point (BP) at the high scale (top) and corresponding predictions at the matching scale μ_0 (middle) and M_t (bottom). At μ_0 the bold parameters also correspond to the parameters of the one-loop SM effective potential. The numerical result for the VEV of Φ is $\langle\Phi\rangle = v_\Phi/\sqrt{2} = 54407\,\mathrm{GeV}$.

One-loop RGE's

Neglect all Yukawas besides y_t and take general $\mathrm{U}(1)_\mathrm{X}$ charges $q_{H,\Phi}$.

$$\beta_{\lambda_H} = \frac{1}{16\pi^2}\left[+\frac{3}{2}\left(\left(\frac{g_Y^2}{2}+\frac{g_L^2}{2}\right)+2\left(q_H g_X+\frac{g_{12}}{2}\right)^2\right)^2+\frac{6}{8}g_L^4-6y_t^4 + 24\lambda_H^2+4\lambda_p^2+\lambda_H\left(12y_t^2-3g_Y^2-12\left(q_H g_X+\frac{g_{12}}{2}\right)^2-9g_L^2\right)\right],$$

$$\beta_{\lambda_\Phi} = \frac{1}{16\pi^2}\left(+6q_\Phi^4 g_X^4+20\lambda_\Phi^2+8\lambda_p^2-12\lambda_\Phi q_\Phi^2 g_X^2\right),$$

$$\beta_{\lambda_p} = \frac{1}{16\pi^2}\left[+6q_\Phi^2 g_X^2\left(q_H g_X+\frac{g_{12}}{2}\right)^2+8\lambda_p^2 + \lambda_p\left(8\lambda_\Phi+12\lambda_H-\frac{3}{2}g_Y^2-6q_\Phi^2 g_X^2-6\left(q_H g_X+\frac{g_{12}}{2}\right)^2-\frac{9}{2}g_L^2+6y_t^2\right)\right],$$

$$\beta_{g_{12}} = \frac{1}{16\pi^2}\left[-\frac{14}{3}g_X g_Y^2-\frac{14}{3}g_X g_{12}^2+\frac{41}{3}g_Y^2 g_{12}+\frac{179}{3}g_X^2 g_{12}+\frac{41}{6}g_{12}^3\right].$$

The dominant splitting of $\lambda_\Phi-\lambda_p$ via running (for benchmark charges) is given by

$$\beta_{\lambda_\Phi}-\beta_{\lambda_p} = -\frac{6\,g_{12}\,g_X^2}{16\pi^2}\left(g_X+\frac{g_{12}}{4}\right)-\frac{\lambda_p}{16\pi^2}\left[6y_t^2-\frac{9}{2}g_L^2-\frac{3}{2}g_Y^2+12(\lambda_H-\lambda_p)\right]+\ldots,$$

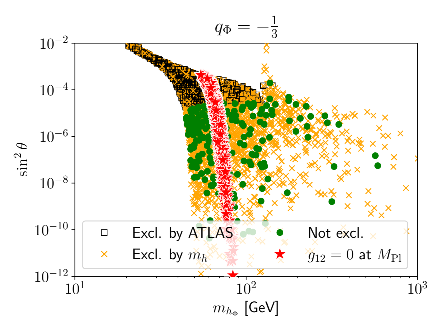
We do the numerical running with the full two-loop beta functions computed with `PyR@TE`.

Higgs-dilaton mixing

A crude analytic expression for the Higgs-dilaton mixing angle is

$$\tan\theta \approx \frac{2\left[\lambda_p - \left(1+\frac{g_{12}}{2g_X}\right)^2\left(\lambda_\Phi - \frac{3g_X^4}{16\pi^2}\right)\right] v_H v_\Phi}{m_h^2 - m_{h_\Phi}^2}\,.$$

Note: We use a fully numerical evaluation of all masses and mixings for our analysis which also confirms the analytic approximations.



Gravitational wave signals?

Quantitative predictions for our specific case have yet to be worked out! But see: