

# Minimal U(1) two-Higgs-doublet models for quark and lepton flavour

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In collaboration with: J. R. Rocha, F.R. Joaquim, R. G. Felipe

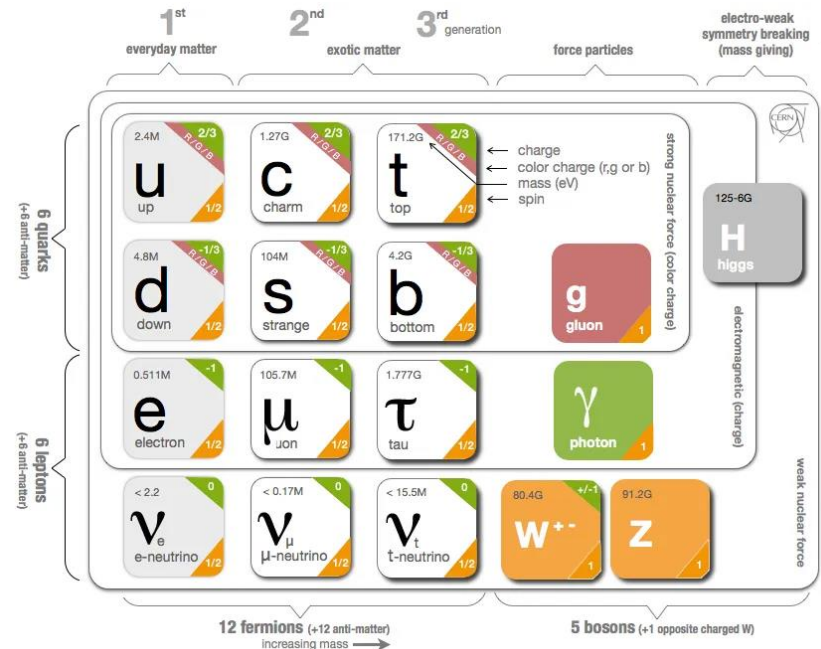
arXiv: **2406.03331** [hep-ph]

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# Introduction

The **Standard Model** of Particle Physics:

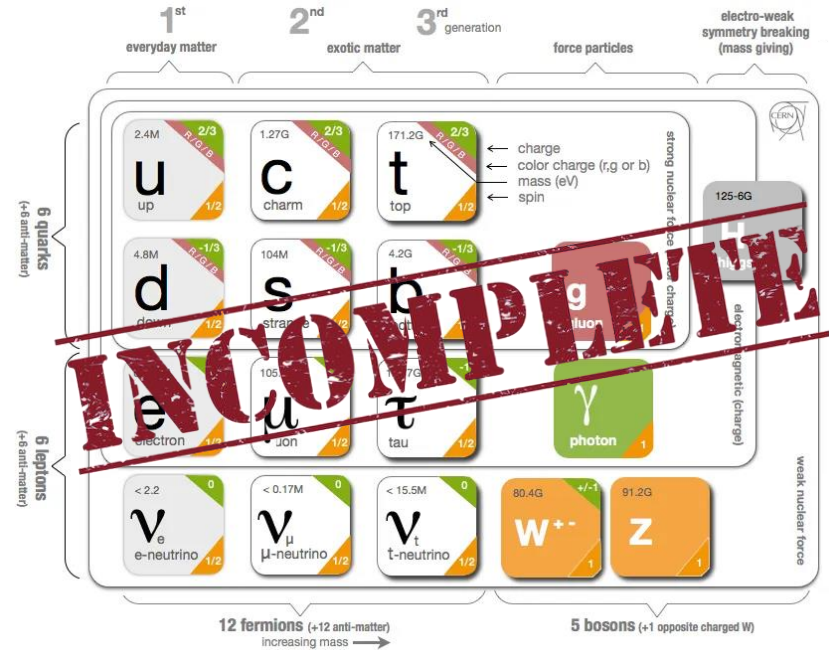
- ✓ Quark mixing is encoded in the CKM matrix;
- ✓ This flavour structure is the only known source of CP violation;
- ✓ The CKM parameters have been determined with extreme precision.



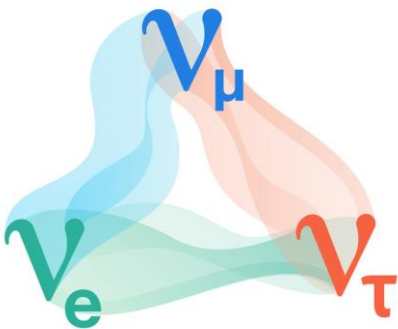
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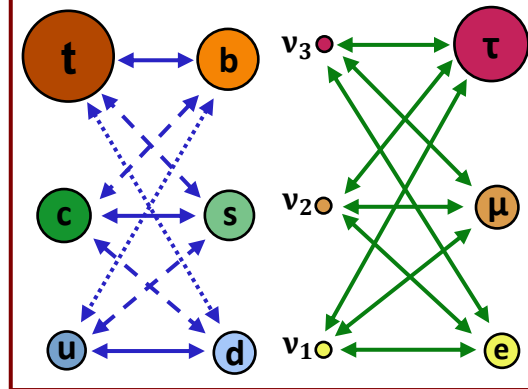
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## Neutrino Oscillations



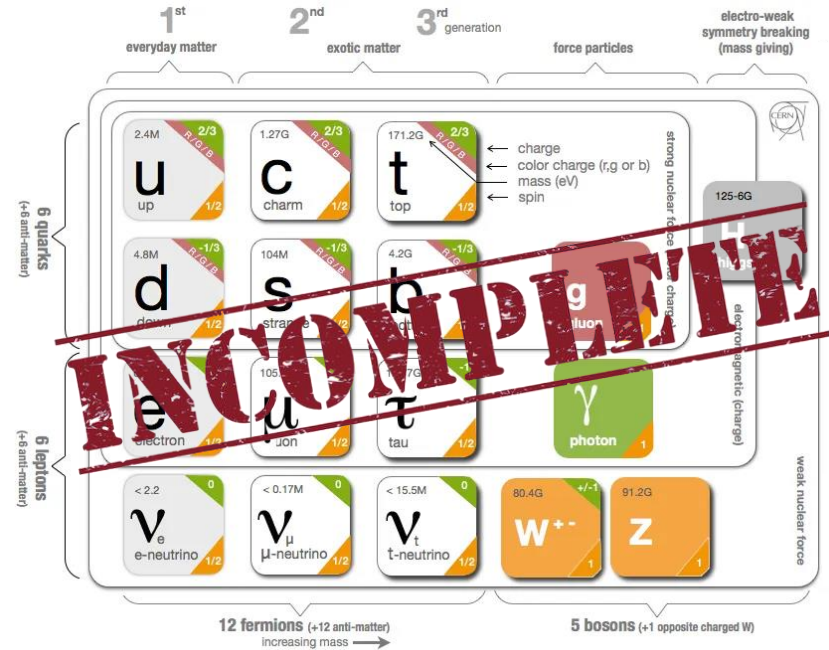
## Flavour Puzzle



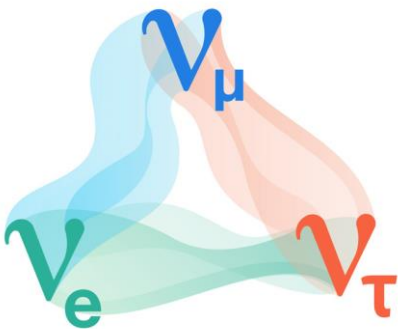
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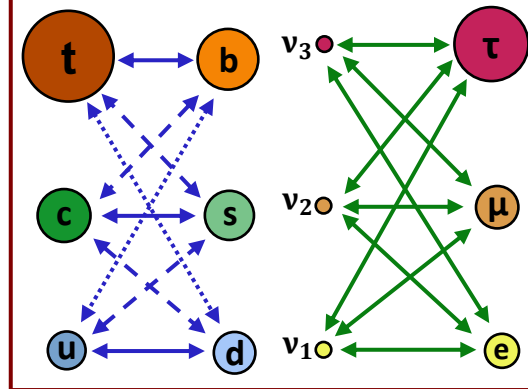
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## Neutrino Oscillations



## Flavour Puzzle



**The SM must be extended!**

# Neutrino masses and mixing

## EFFECTIVE THEORY with SM fields

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \delta\mathcal{L}^{d=5} + \delta\mathcal{L}^{d=6} + \dots, \quad \delta\mathcal{L}^{D=d} \equiv \sum_k \frac{\mathcal{O}_k^{(d)}}{\Lambda^{d-4}}$$

# Neutrino masses and mixing

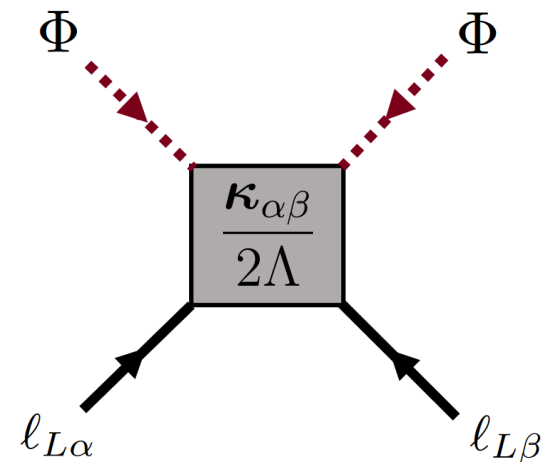
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The lowest  $d > 4$  operator is **unique (Weinberg Operator)**

(Weinberg, 1979)

$$\delta\mathcal{L}^{d=5} = \frac{1}{2\Lambda} \kappa_{\alpha\beta} \left( \overline{\ell_{\alpha L}^C} \tilde{\Phi}^* \right) \left( \tilde{\Phi}^\dagger \ell_{\beta L} \right) + \text{H.c.}$$



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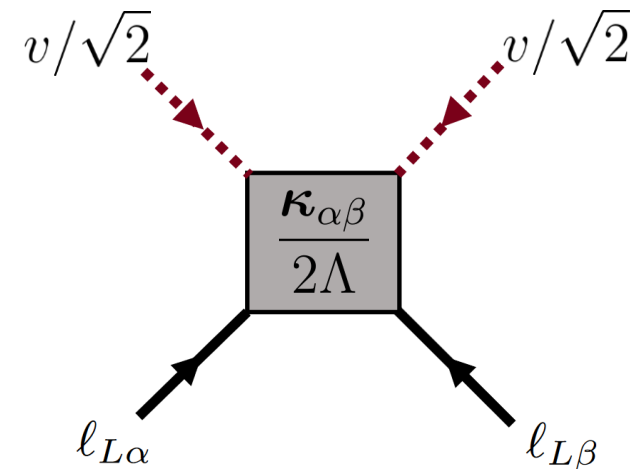
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**EWSB**



$$\mathcal{L}_m^{\text{Majorana}} = -\frac{1}{2} \mathbf{M}_{\nu\alpha\beta} \overline{\nu_{\alpha L}^C} \nu_{\beta L} + \text{H.c.}$$



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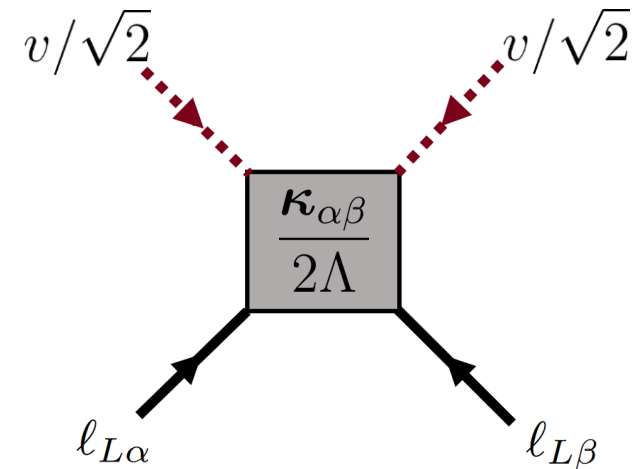
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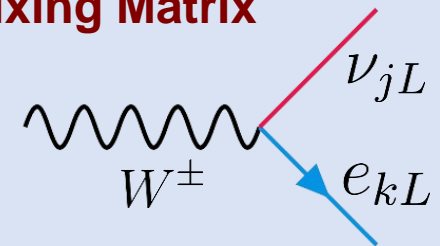
### Majorana Mass Eigenstates

$$\nu_{\alpha L} \rightarrow (\mathbf{U}_L^\nu)_{\alpha j} \nu_{jL}$$

$$\mathbf{U}_L^{\nu T} \mathbf{M}_\nu \mathbf{U}_L^\nu = \text{diag}(m_1, m_2, m_3)$$

### Lepton Mixing Matrix

$$\mathbf{U}_\ell = \mathbf{U}_L^{e\dagger} \mathbf{U}_L^\nu$$





# Softly-broken U(1)-symmetric 2HDM

The **SM** does not allow for the implementation of **Abelian flavour symmetries**



(Branco, et al., 2012)

**2HDM**

$$\Phi_{1,2} = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\phi_{1,2}^+ \\ v_{1,2} + \rho_{1,2} + i\eta_{1,2} \end{pmatrix}$$

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$$\begin{aligned} V = & \mu_{11}^2 \left( \Phi_1^\dagger \Phi_1 \right) + \mu_{22}^2 \left( \Phi_2^\dagger \Phi_2 \right) + \mu_{12}^2 \left( \Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1 \right) \\ & + \frac{\lambda_1}{2} \left( \Phi_1^\dagger \Phi_1 \right)^2 + \frac{\lambda_2}{2} \left( \Phi_2^\dagger \Phi_2 \right)^2 \\ & + \lambda_3 \left( \Phi_1^\dagger \Phi_1 \right) \left( \Phi_2^\dagger \Phi_2 \right) + \lambda_4 \left( \Phi_1^\dagger \Phi_2 \right) \left( \Phi_2^\dagger \Phi_1 \right) \end{aligned}$$

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**Mass Eigenstates**

$$\begin{matrix} m_h & m_I \\ m_H & m_{H^\pm} \end{matrix}$$

**Alignment Limit**

$$\beta - \alpha = \pi/2$$

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**Mass Eigenstates**

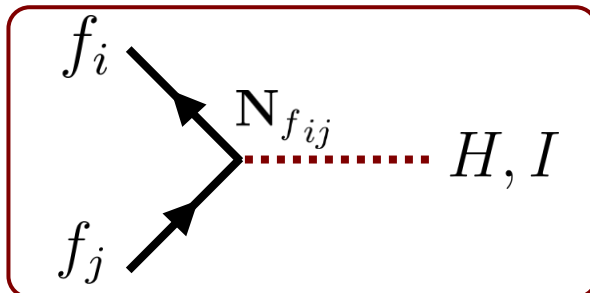
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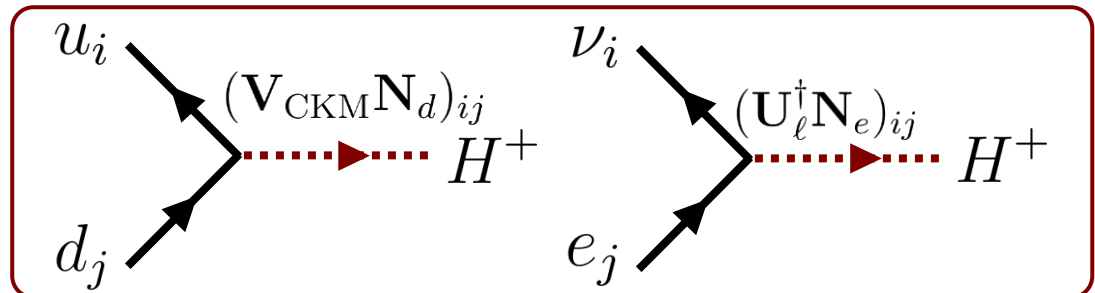
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Expanding the **Yukawa Lagrangian** in the **mass eigenstates**:

**FCNC**



**FCCC**

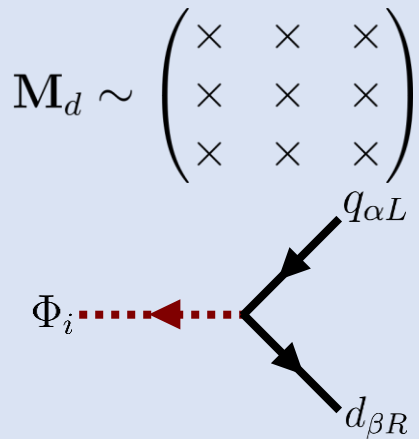


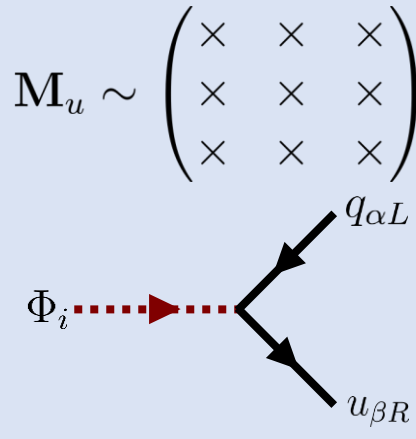
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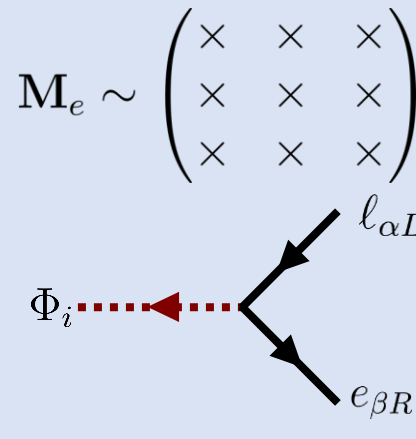
## GOAL

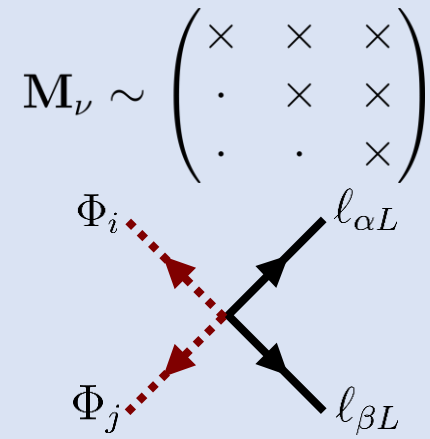
Reduce the number of free parameters in the mass matrices and make the theory more predictive

↓ Introduce flavour charges

$$\mathbf{M}_d \sim \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix}$$


$$\mathbf{M}_u \sim \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix}$$


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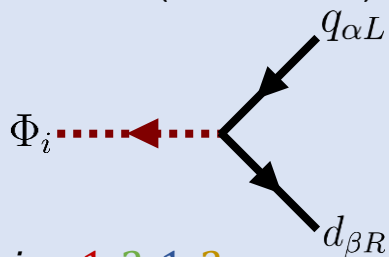
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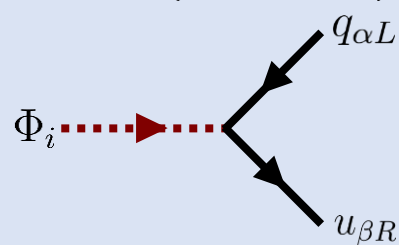
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$$M_d \sim \begin{pmatrix} 0 & 0 & \otimes \\ 0 & \otimes & 0 \\ \otimes & 0 & \otimes \end{pmatrix}$$



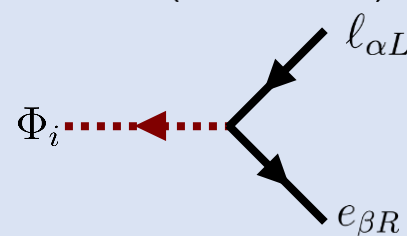
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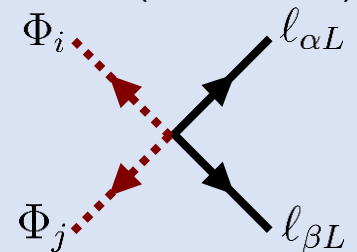
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$$M_e \sim \begin{pmatrix} 0 & 0 & \otimes \\ 0 & \otimes & 0 \\ \otimes & 0 & \otimes \end{pmatrix}$$



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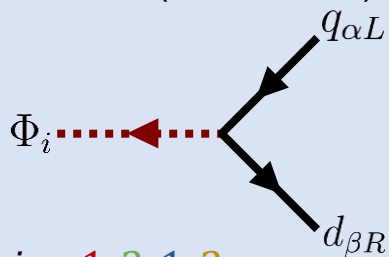
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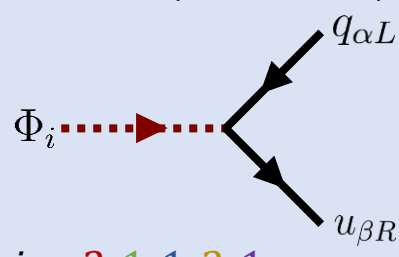
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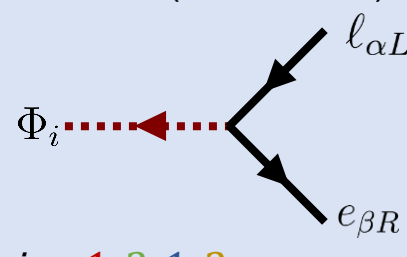
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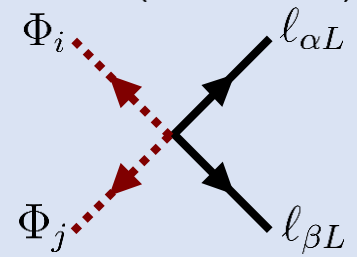
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**Example:**

$$\Phi_{1,2} \rightarrow q_{1L} + d_{2R}$$

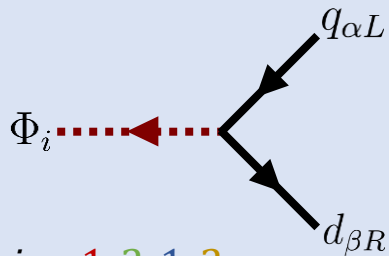
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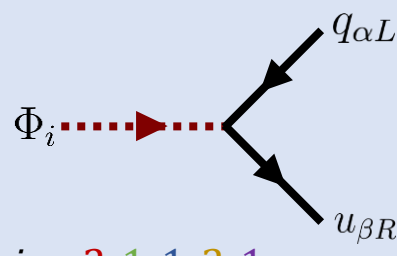
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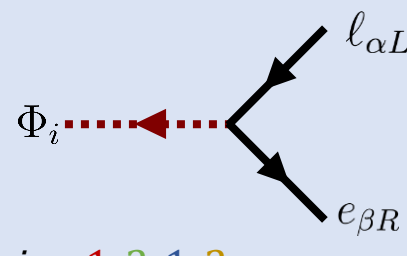
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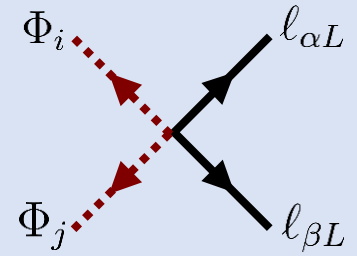
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Example:

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Flavour charge is not conserved

$$Q_{\Phi_{1,2}} - Q_{q_{1L}} + Q_{d_{2R}} \neq 0$$



# Maximally-restrictive textures from $U(1)$ symmetries

**Procedure**

**Equivalence classes** with  
the maximum number of  
zeros

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Solve system of equations  
for the field charges

# Maximally-restrictive textures from U(1) symmetries

## Procedure

Equivalence classes with the maximum number of zeros



Solve system of equations for the field charges



Test compatibility at the  $1\sigma$  CL for all observables

## Experimental Data

Parameter	Best fit $\pm 1\sigma$
$m_d (\times \text{MeV})$	$4.67^{+0.48}_{-0.17}$
$m_s (\times \text{MeV})$	$93.4^{+8.6}_{-3.4}$
$m_b (\times \text{GeV})$	$4.18^{+0.03}_{-0.02}$
$m_u (\times \text{MeV})$	$2.16^{+0.49}_{-0.26}$
$m_c (\times \text{GeV})$	$1.27 \pm 0.02$
$m_t (\times \text{GeV})$	$172.69 \pm 0.30$
$\theta_{12}^q (^\circ)$	$13.04 \pm 0.05$
$\theta_{23}^q (^\circ)$	$2.38 \pm 0.06$
$\theta_{13}^q (^\circ)$	$0.201 \pm 0.011$
$\delta^q (^\circ)$	$68.75 \pm 4.5$

Quarks

Parameter	Best Fit $\pm 1\sigma$
$m_e (\times \text{keV})$	$510.99895000 \pm 0.00000015$
$m_\mu (\times \text{MeV})$	$105.6583755 \pm 0.0000023$
$m_\tau (\times \text{GeV})$	$1.77686 \pm 0.00012$
$\Delta m_{21}^2 (\times 10^{-5} \text{ eV}^2)$	$7.50^{+0.22}_{-0.20}$
$ \Delta m_{31}^2  (\times 10^{-3} \text{ eV}^2) [\text{NO}]$	$2.55^{+0.02}_{-0.03}$
$ \Delta m_{31}^2  (\times 10^{-3} \text{ eV}^2) [\text{IO}]$	$2.45^{+0.02}_{-0.03}$
$\theta_{12}^\ell (^\circ)$	$34.3 \pm 1.0$
$\theta_{23}^\ell (^\circ) [\text{NO}]$	$49.26 \pm 0.79$
$\theta_{23}^\ell (^\circ) [\text{IO}]$	$49.46^{+0.60}_{-0.97}$
$\theta_{13}^\ell (^\circ) [\text{NO}]$	$8.53^{+0.13}_{-0.12}$
$\theta_{13}^\ell (^\circ) [\text{IO}]$	$8.58^{+0.12}_{-0.14}$
$\delta^\ell (^\circ) [\text{NO}]$	$194^{+24}_{-22}$
$\delta^\ell (^\circ) [\text{IO}]$	$284^{+26}_{-28}$

Leptons

# Maximally-restrictive textures from U(1) symmetries

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Equivalence classes with the maximum number of zeros



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Add nonzero entry

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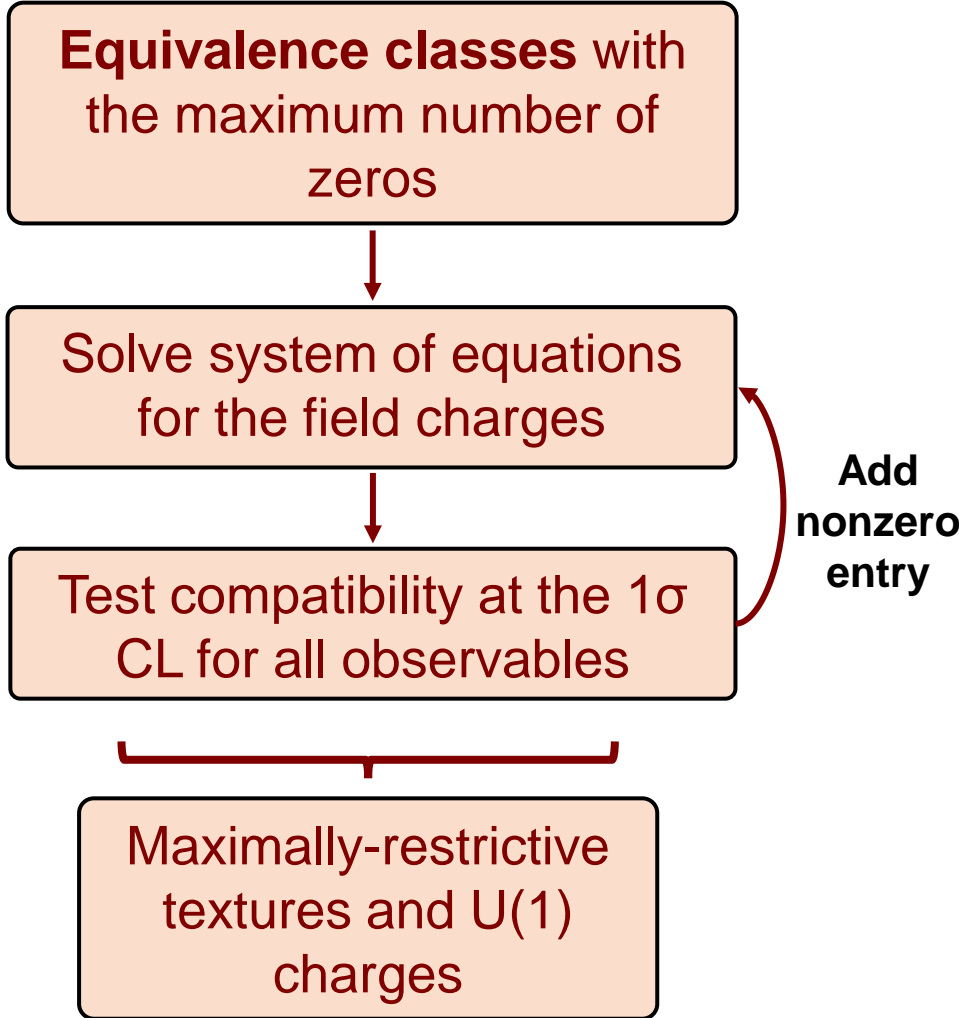
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$ \Delta m_{31}^2  (\times 10^{-3} \text{ eV}^2) [\text{NO}]$	$2.55^{+0.02}_{-0.03}$
$ \Delta m_{31}^2  (\times 10^{-3} \text{ eV}^2) [\text{IO}]$	$2.45^{+0.02}_{-0.03}$
$\theta_{12}^\ell (^\circ)$	$34.3 \pm 1.0$
$\theta_{23}^\ell (^\circ) [\text{NO}]$	$49.26 \pm 0.79$
$\theta_{23}^\ell (^\circ) [\text{IO}]$	$49.46^{+0.60}_{-0.97}$
$\theta_{13}^\ell (^\circ) [\text{NO}]$	$8.53^{+0.13}_{-0.12}$
$\theta_{13}^\ell (^\circ) [\text{IO}]$	$8.58^{+0.12}_{-0.14}$
$\delta^\ell (^\circ) [\text{NO}]$	$194^{+24}_{-22}$
$\delta^\ell (^\circ) [\text{IO}]$	$284^{+26}_{-28}$

Leptons

# Maximally-restrictive textures from U(1) symmetries

## Procedure



## Experimental Data

Parameter	Best fit $\pm 1\sigma$
$m_d (\times \text{MeV})$	$4.67^{+0.48}_{-0.17}$
$m_s (\times \text{MeV})$	$93.4^{+8.6}_{-3.4}$
$m_b (\times \text{GeV})$	$4.18^{+0.03}_{-0.02}$
$m_u (\times \text{MeV})$	$2.16^{+0.49}_{-0.26}$
$m_c (\times \text{GeV})$	$1.27 \pm 0.02$
$m_t (\times \text{GeV})$	$172.69 \pm 0.30$
$\theta_{12}^q (^\circ)$	$13.04 \pm 0.05$
$\theta_{23}^q (^\circ)$	$2.38 \pm 0.06$
$\theta_{13}^q (^\circ)$	$0.201 \pm 0.011$
$\delta^q (^\circ)$	$68.75 \pm 4.5$

Quarks

Parameter	Best Fit $\pm 1\sigma$
$m_e (\times \text{keV})$	$510.99895000 \pm 0.00000015$
$m_\mu (\times \text{MeV})$	$105.6583755 \pm 0.0000023$
$m_\tau (\times \text{GeV})$	$1.77686 \pm 0.00012$
$\Delta m_{21}^2 (\times 10^{-5} \text{ eV}^2)$	$7.50^{+0.22}_{-0.20}$
$ \Delta m_{31}^2  (\times 10^{-3} \text{ eV}^2) [\text{NO}]$	$2.55^{+0.02}_{-0.03}$
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Leptons

# Maximally-restrictive textures from U(1) symmetries

## U(1) charges

$\mathbb{Z}_5$			
$(\mathbf{M}_e, \mathbf{M}_\nu)$	$(\delta_1, \delta_2, \delta_3)$	$(\epsilon_1, \epsilon_2, \epsilon_3)$	
$(5_1^e, 2_3^\nu)$	$(-1, -3, 1)$	$(1, -5, -1)$	
$(5_1^e, 2_7^\nu)$	$(-1, -2, 0)$	$(0, -3, -1)$	
$(5_1^e, 2_{10}^\nu)$	$(0, -1, 1)$	$(1, -2, 0)$	

$\mathbb{Z}_4$			
$(\mathbf{M}_d, \mathbf{M}_u)$	$(\alpha_1, \alpha_2, \alpha_3)$	$(\beta_1, \beta_2, \beta_3)$	$(\gamma_1, \gamma_2, \gamma_3)$
$(4_3^d, \mathbf{P}_{12} 5_1^u \mathbf{P}_{23})$	$(0, 1, 2)$	$(2, 1, 0)$	$(3, 2, 0)$
$(4_3^d, \mathbf{P}_{123} 5_1^u \mathbf{P}_{12})$	$(0, 1, 2)$	$(2, 1, 0)$	$(3, 0, 1)$
$(5_1^d, \mathbf{P}_{12} 4_3^u)$	$(0, -1, 1)$	$(1, -2, 0)$	$(2, 1, 0)$
$(5_1^d, \mathbf{P}_{321} 4_3^u \mathbf{P}_{23})$	$(0, -1, 1)$	$(1, -2, 0)$	$(-1, 1, 0)$

## Maximally restrictive mass matrices

Quarks	Leptons
$4_3^d \sim \begin{pmatrix} 0 & 0 & \times \\ 0 & \times & \times \\ \times & \times & 0 \end{pmatrix}$	$5_1^e \sim \begin{pmatrix} 0 & 0 & \times \\ 0 & \times & 0 \\ \times & 0 & \times \end{pmatrix}$
$5_1^d \sim \begin{pmatrix} 0 & 0 & \times \\ 0 & \times & 0 \\ \times & 0 & \times \end{pmatrix}$	$2_3^\nu \sim \begin{pmatrix} \times & \times & \bullet \\ \cdot & 0 & \bullet \\ \cdot & \cdot & 0 \end{pmatrix}$
$\mathbf{P}_{12} 5_1^u \mathbf{P}_{23} \sim \begin{pmatrix} 0 & 0 & \times \\ 0 & \bullet & 0 \\ \times & \times & 0 \end{pmatrix}$	$2_7^\nu \sim \begin{pmatrix} \times & 0 & \bullet \\ \cdot & 0 & \times \\ \cdot & \cdot & \bullet \end{pmatrix}$
$\mathbf{P}_{123} 5_1^u \mathbf{P}_{12} \sim \begin{pmatrix} 0 & \times & \bullet \\ 0 & 0 & \times \\ \times & 0 & 0 \end{pmatrix}$	$2_{10}^\nu \sim \begin{pmatrix} \times & \bullet & 0 \\ \cdot & \times & \bullet \\ \cdot & \cdot & 0 \end{pmatrix}$
$\mathbf{P}_{12} 4_3^u \sim \begin{pmatrix} 0 & \bullet & \times \\ 0 & 0 & \times \\ \times & \times & 0 \end{pmatrix}$	
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$(4_3^d, \mathbf{P}_{12} 5_1^u \mathbf{P}_{23})$	$(0, 1, 2)$	$(2, 1, 0)$	$(3, 2, 0)$
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$(5_1^d, \mathbf{P}_{321} 4_3^u \mathbf{P}_{23})$	$(0, -1, 1)$	$(1, -2, 0)$	$(-1, 1, 0)$

**“Decoupled” entry in the matrices of type “5” lead to zeros in the  $N_k$  matrices**

## Maximally restrictive mass matrices

Quarks	Leptons
$4_3^d \sim \begin{pmatrix} 0 & 0 & \times \\ 0 & \times & \times \\ \times & \times & 0 \end{pmatrix}$	$5_1^e \sim \begin{pmatrix} 0 & 0 & \times \\ 0 & \times & 0 \\ \times & 0 & \times \end{pmatrix}$
$5_1^d \sim \begin{pmatrix} 0 & 0 & \times \\ 0 & \times & 0 \\ \times & 0 & \times \end{pmatrix}$	$2_3^\nu \sim \begin{pmatrix} \times & \times & \bullet \\ \cdot & 0 & \bullet \\ \cdot & \cdot & 0 \end{pmatrix}$
$\mathbf{P}_{12} 5_1^u \mathbf{P}_{23} \sim \begin{pmatrix} 0 & 0 & \times \\ 0 & \bullet & 0 \\ \times & \times & 0 \end{pmatrix}$	$2_7^\nu \sim \begin{pmatrix} \times & 0 & \bullet \\ \cdot & 0 & \times \\ \cdot & \cdot & \bullet \end{pmatrix}$
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# Maximally-restrictive textures from $U(1)$ symmetries

Minimal flavour patterns for **quarks**:

- ✓ Four different models;
- ✓ There is a total of ten independent parameters, **matching** the number of observables;



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**Predictions**



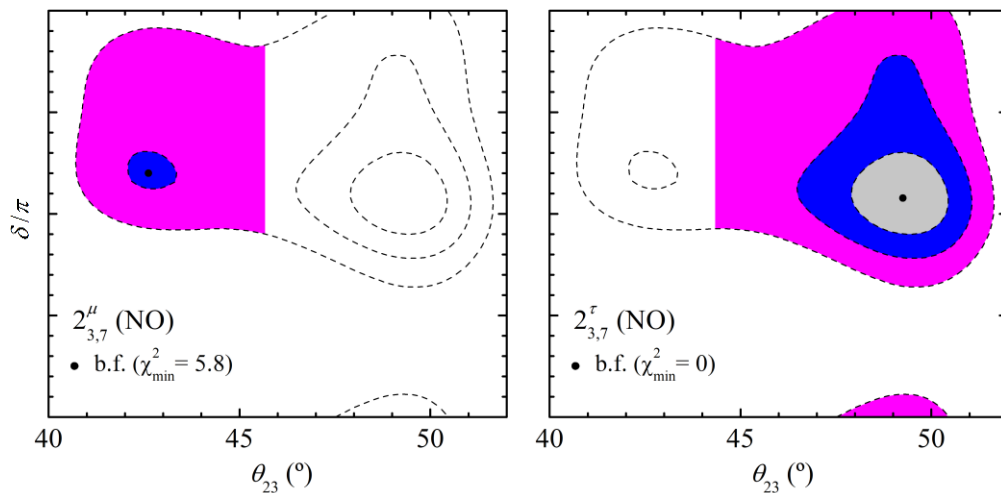
$$\mathbf{NO:} \quad m_2 = \sqrt{m_1^2 + \Delta m_{21}^2}, \quad m_3 = \sqrt{m_1^2 + \Delta m_{31}^2}$$

$$\mathbf{IO:} \quad m_1 = \sqrt{m_3^2 + |\Delta m_{31}^2|}, \quad m_2 = \sqrt{m_3^2 + \Delta m_{21}^2 + |\Delta m_{31}^2|}$$

$$m_{\beta\beta} = \left| c_{12}^2 c_{13}^2 m_1 + s_{12}^2 c_{13}^2 m_2 e^{-i\alpha_{21}} + s_{13}^2 m_3 e^{-i\alpha_{31}} \right|$$

# Lepton sector predictions - NO

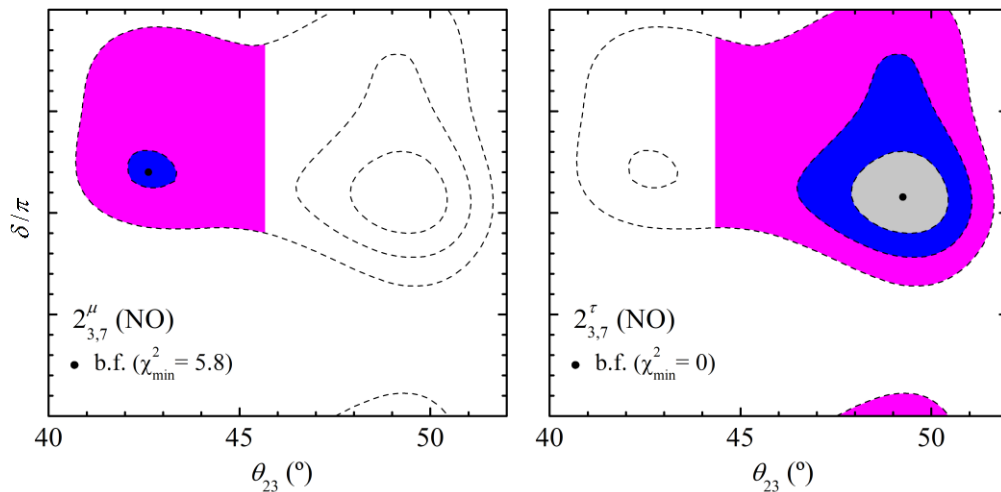
The symmetry-constrained lepton models provide **predictions** for the **neutrino sector**, for example:



For NO,  $2_{3,7}^{\mu}$  and  $2_{3,7}^{\tau}$  select the **first** and **second octant** for the atmospheric mixing angle  $\theta_{23}$ , respectively

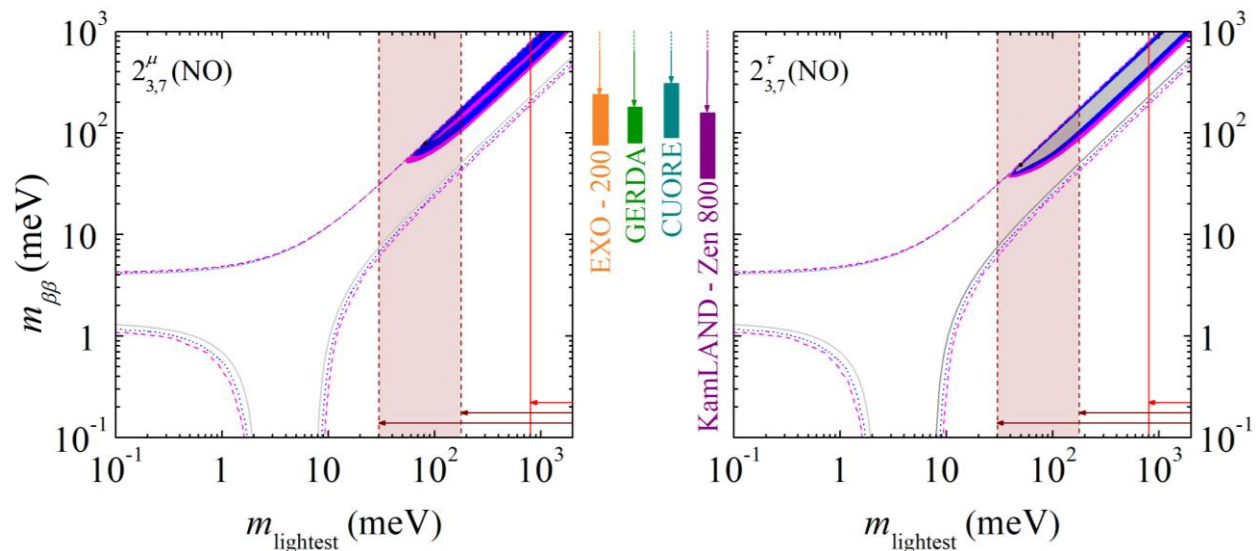
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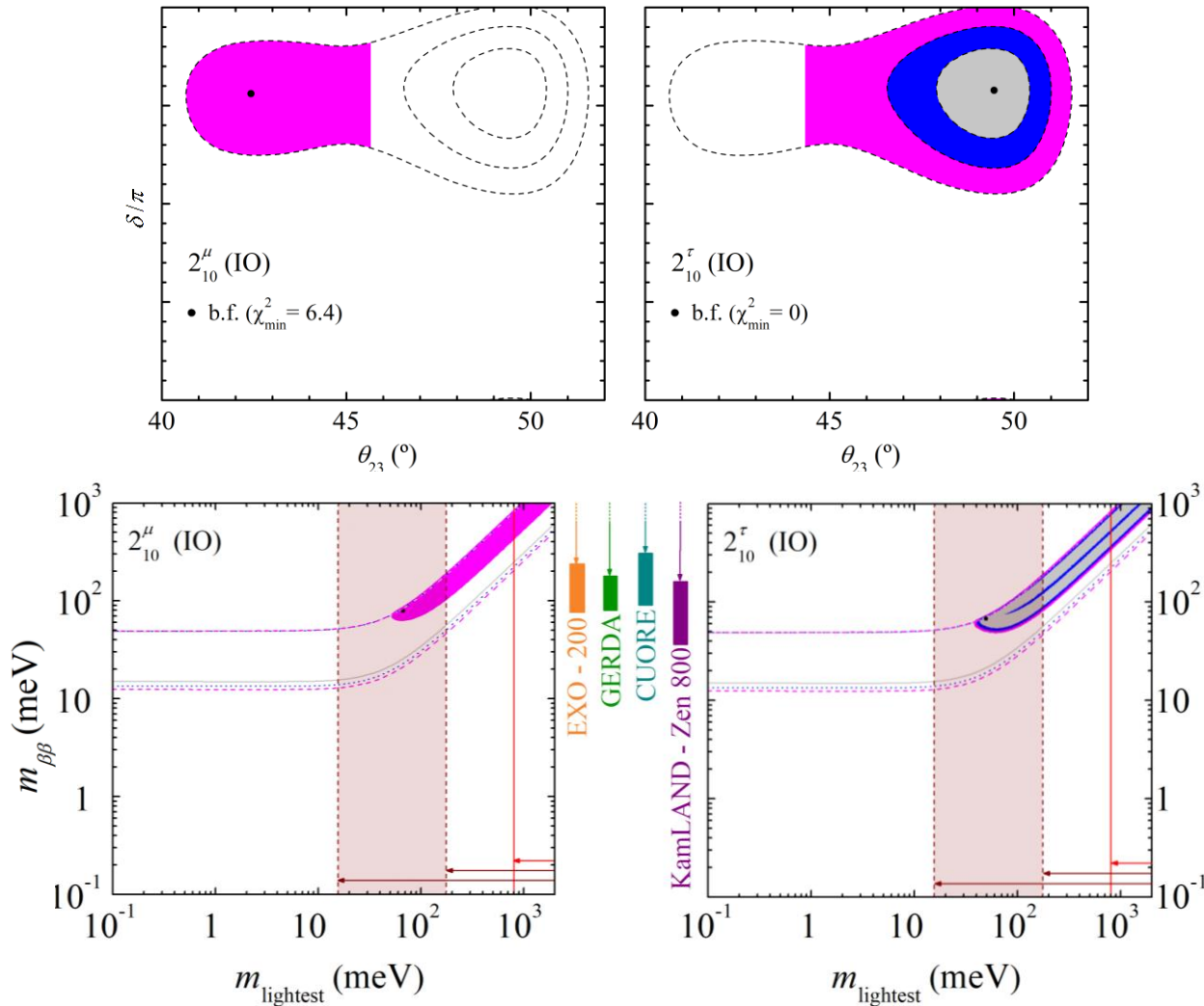
For NO,  $2_{3,7}^{\mu}$  and  $2_{3,7}^{\tau}$  select the **first** and **second octant** for the atmospheric mixing angle  $\theta_{23}$ , respectively

The lower bounds on  $m_{\beta\beta}$  are **within the sensitivity** of  $0\nu\beta\beta$  decay experiments, while being simultaneously in **tension with cosmological** constraints on  $m_{lightest}$



# Lepton sector predictions - IO

There are models that behave similarly for **inverted ordering** (IO), namely  $2_{10}^{\mu}$  and  $2_{10}^{\tau}$



# Numerical procedure and phenomenological analysis

For the numerical analysis of the phenomenology of maximally-restrictive matrices, a private *Python* code was developed, which works as follows:

Random values for  $\tan \beta$ ,  $m_I$ ,  $m_H$ ,  $m_{H^\pm}$

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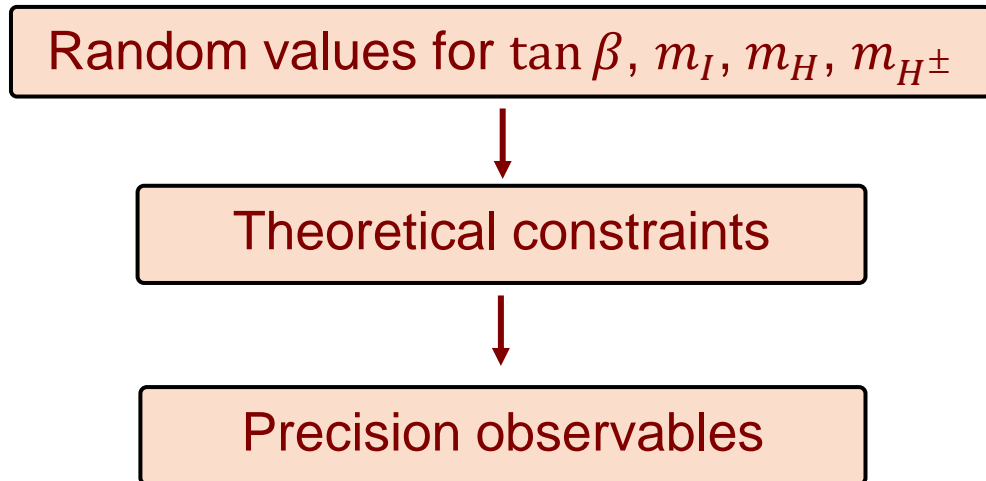
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Theoretical constraints

# Numerical procedure and phenomenological analysis

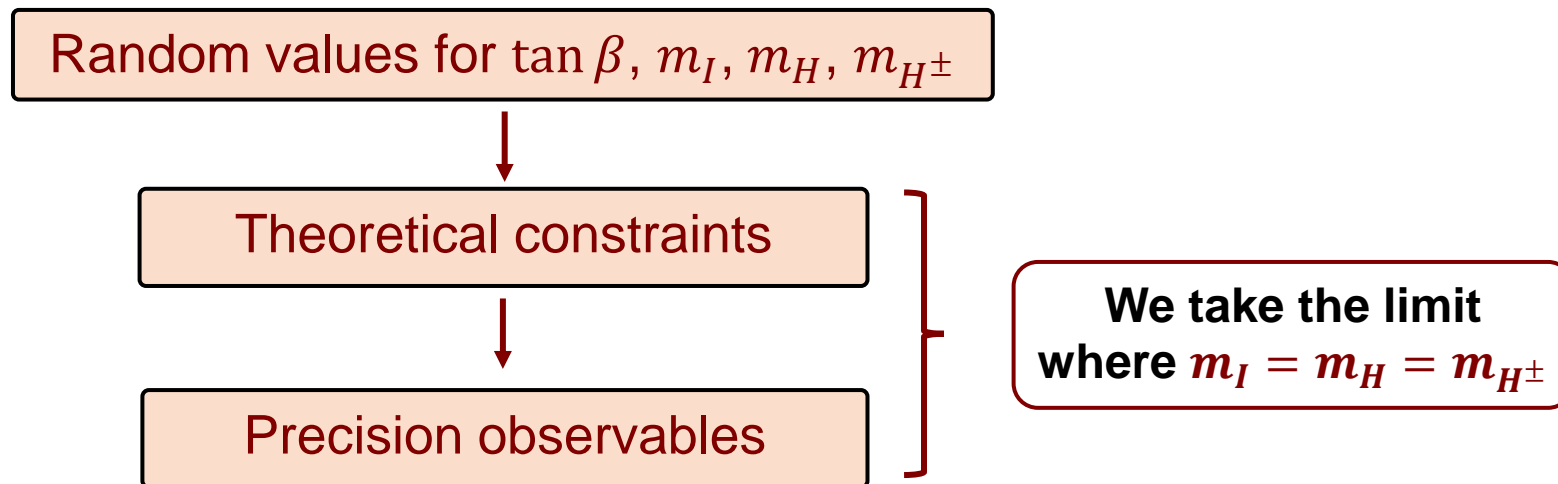
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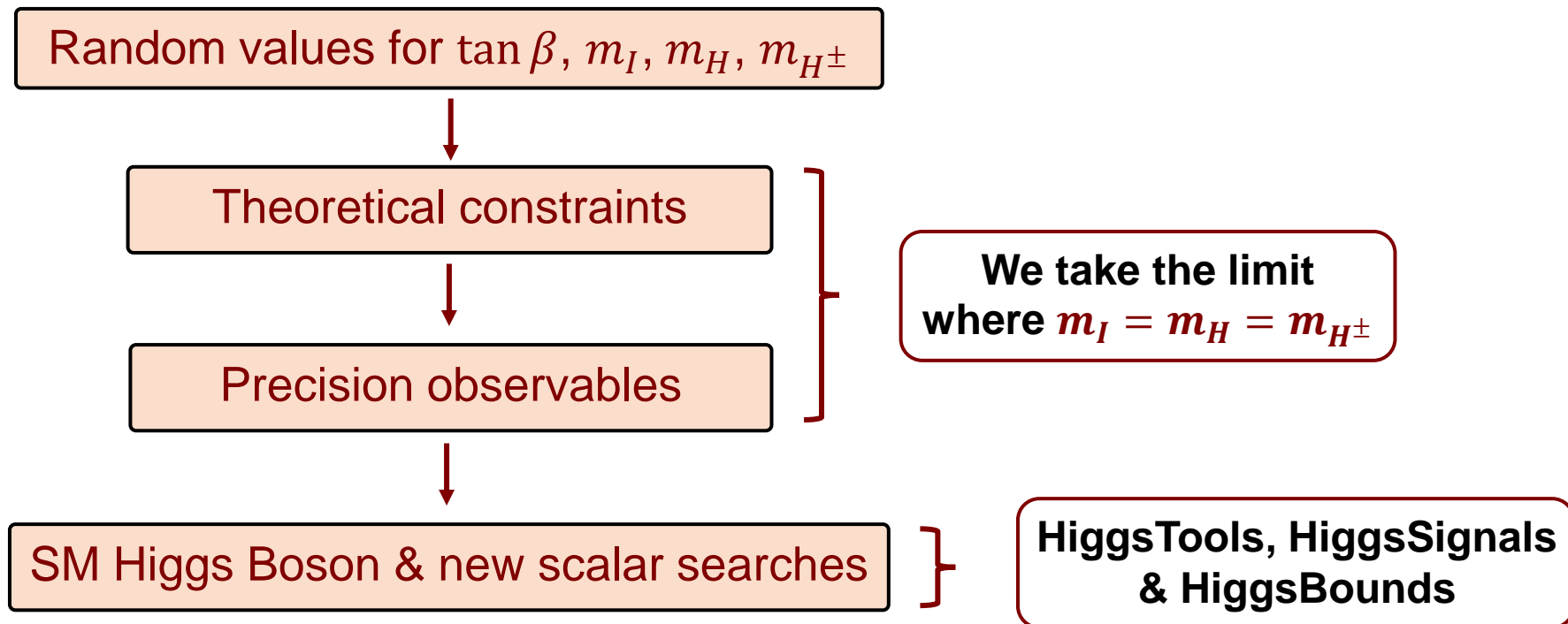
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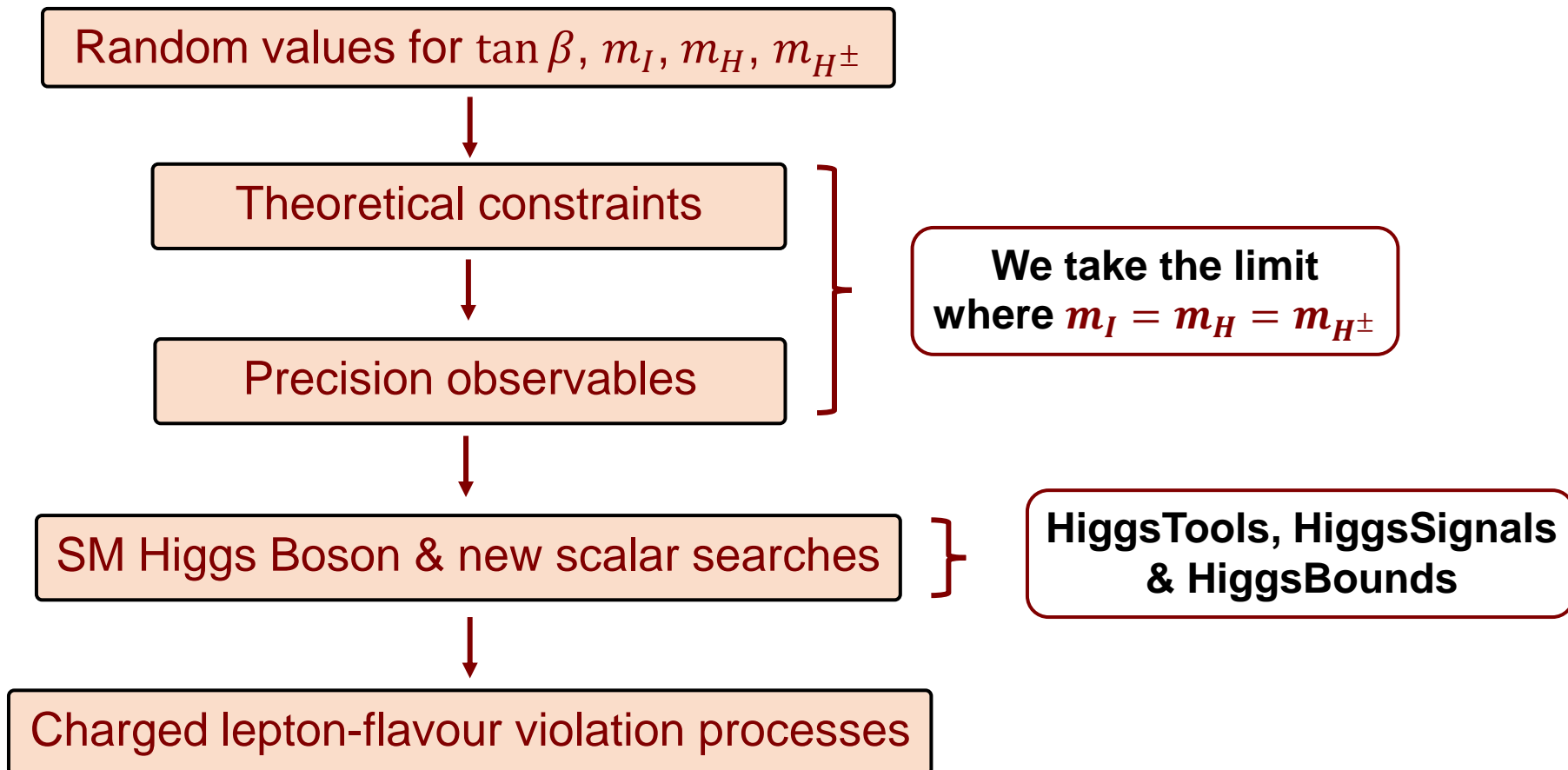
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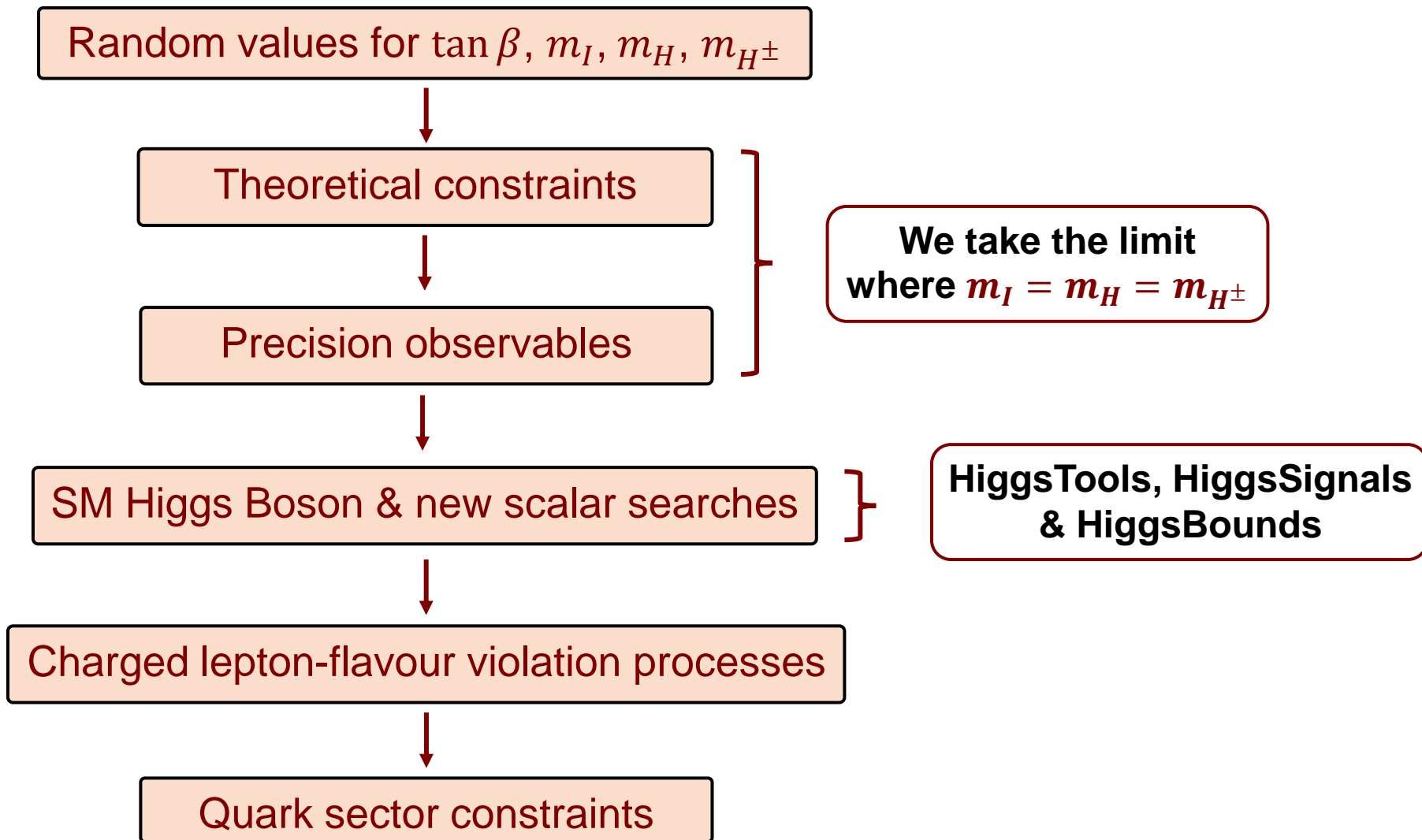
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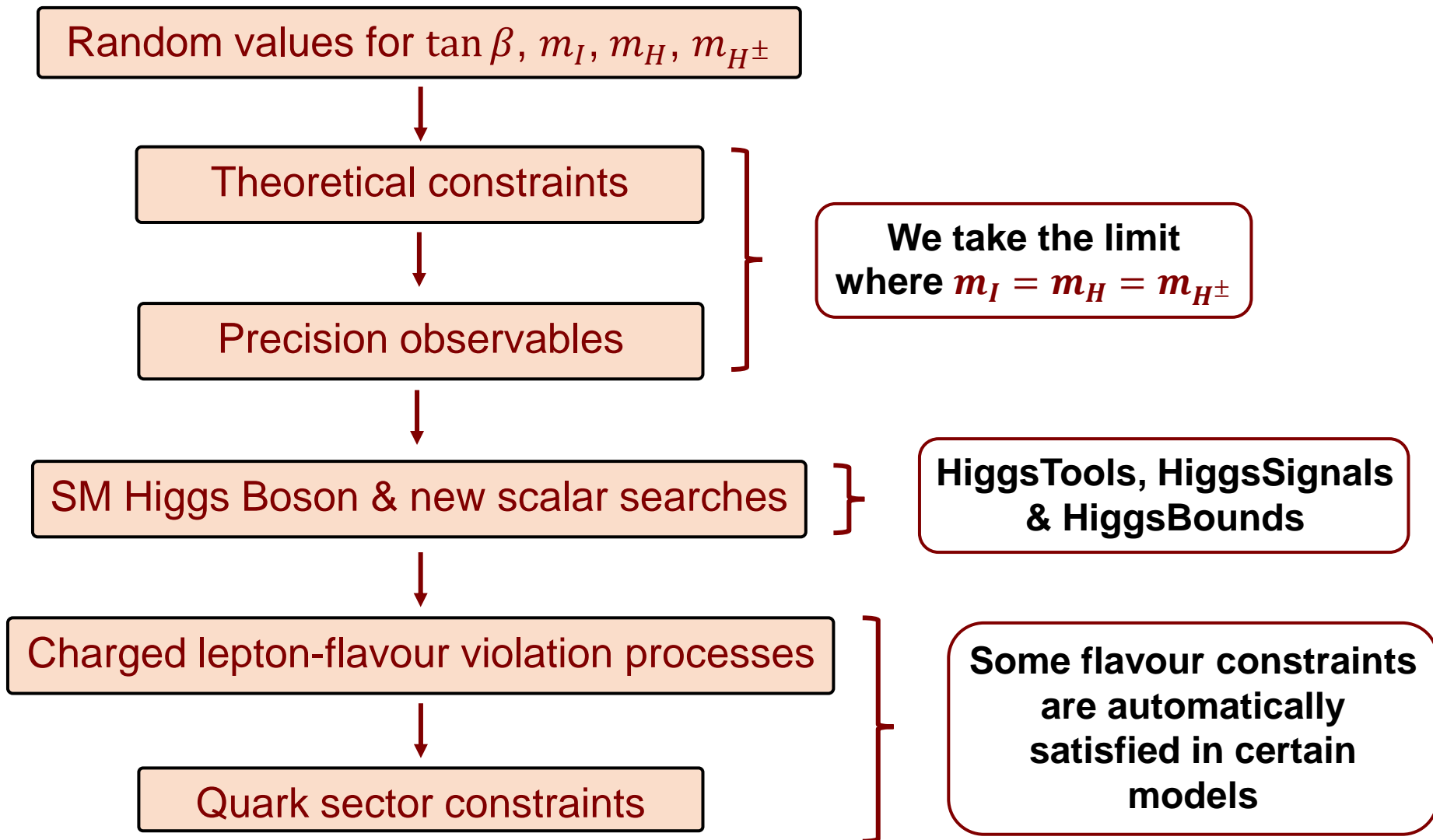
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The mass matrices labelled "5" exhibit an isolated non-zero entry in a given row and column, which coincides with the mass of a fermion translating into:

$$5^{d,u,e} : \mathbf{N}_{d,u,e} \sim \begin{pmatrix} \times & 0 & 0 \\ 0 & \times & \times \\ 0 & \times & \times \end{pmatrix}, \quad 5^{s,c,\mu} : \mathbf{N}_{s,c,\mu} \sim \begin{pmatrix} \times & 0 & \times \\ 0 & \times & 0 \\ \times & 0 & \times \end{pmatrix}, \quad 5^{b,t,\tau} : \mathbf{N}_{b,t,\tau} \sim \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & \times \end{pmatrix}$$

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To directly observe the effect of flavour symmetries, consider the NP contribution to the  $\bar{K}_0 \rightarrow K_0$  transition:

$$M_{21}^{\text{NP}} = \frac{f_k^2 m_K}{96v^2} \left\{ [(\mathbf{N}_d^*)_{ds}^2 + (\mathbf{N}_d)_{sd}^2] \frac{10m_k^2}{(m_s + m_d)^2} \left( \frac{1}{m_I^2} - \frac{c_{\beta-\alpha}^2}{m_h^2} - \frac{s_{\beta-\alpha}^2}{m_H^2} \right) + 4(\mathbf{N}_d^*)_{ds}(\mathbf{N}_d)_{sd} \left[ 1 + \frac{6m_K^2}{(m_s + m_d)^2} \left( \frac{1}{m_I^2} + \frac{c_{\beta-\alpha}^2}{m_h^2} + \frac{s_{\beta-\alpha}^2}{m_H^2} \right) \right] \right\}$$

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$$\Delta m_K^{\text{NP}} = 2|M_{21}^{\text{NP}}| = 0 \quad \varepsilon_K = \varepsilon_K^{\text{SM}} - \frac{\text{Im}(M_{21}^{\text{NP}} \lambda_u^{*2})}{\sqrt{2} \Delta m_K |\lambda_u|^2}$$

The two constraints associated with  $K^0$  are **inherently satisfied** for  $d$  or  $s$  decoupled

## Yukawa perturbativity bounds

$$\tan^2 \beta \leq \frac{2\pi v^2}{|(\mathbf{M}_1^x)_{ij}|^2} - 1, \quad \tan^2 \beta \geq 1 / \left( \frac{2\pi v^2}{|(\mathbf{M}_2^x)_{ij}|^2} - 1 \right)$$

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## Lepton sector constraints

We only consider the lepton model  $(5_1^e, 2_3^v)_{\text{NO}}$ , as the conclusions do not differ with a more detailed analysis.

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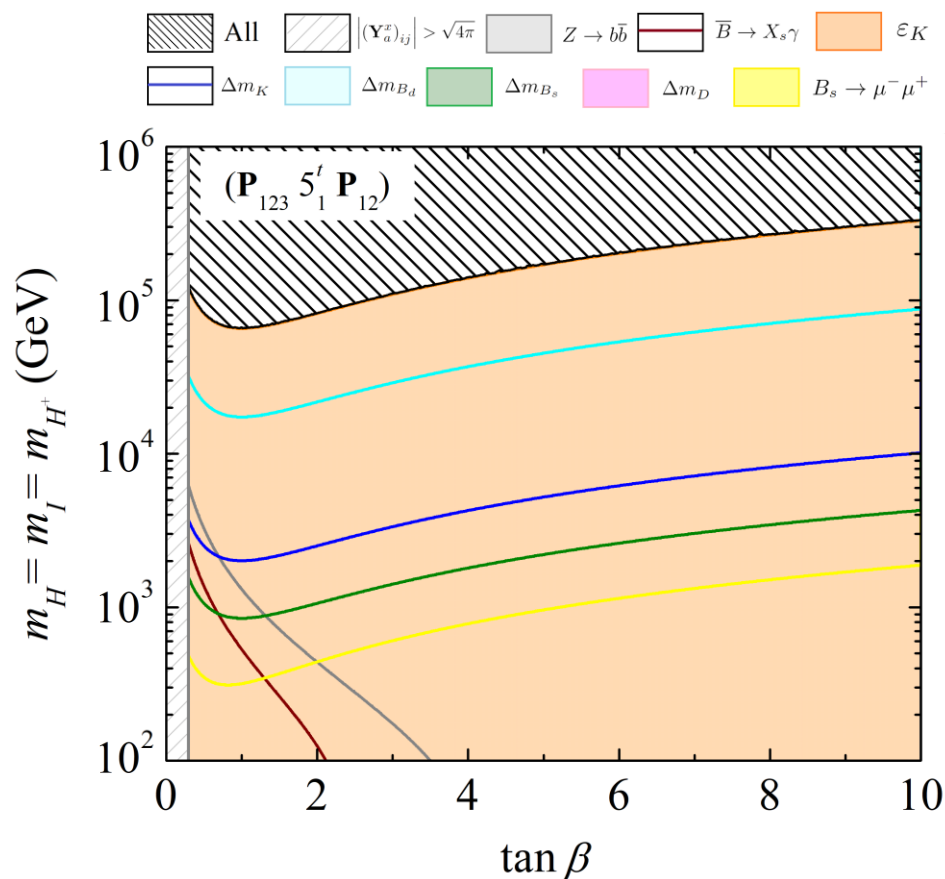
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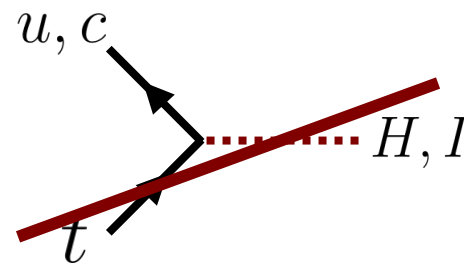
## Most restrictive constraints

Only some constraints shape the allowed region  $(\tan \beta, \{m_H = m_I = m_{H^\pm}\})$ , which we refer to as the **most restrictive constraints**.

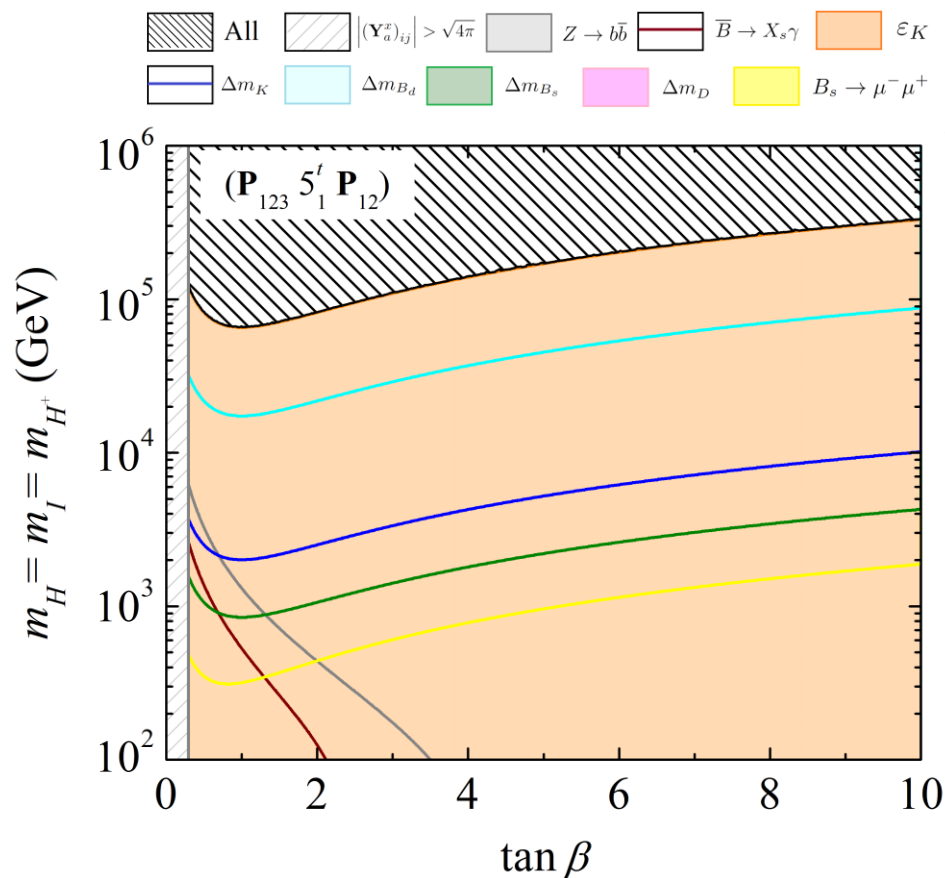
# Numerical procedure and phenomenological analysis



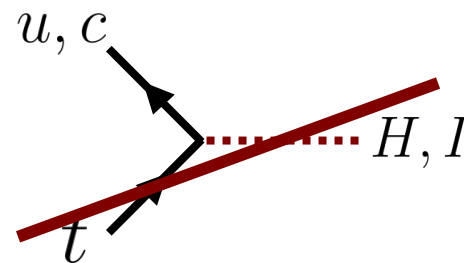
$$\mathbf{N}_t \sim \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & \times \end{pmatrix}$$



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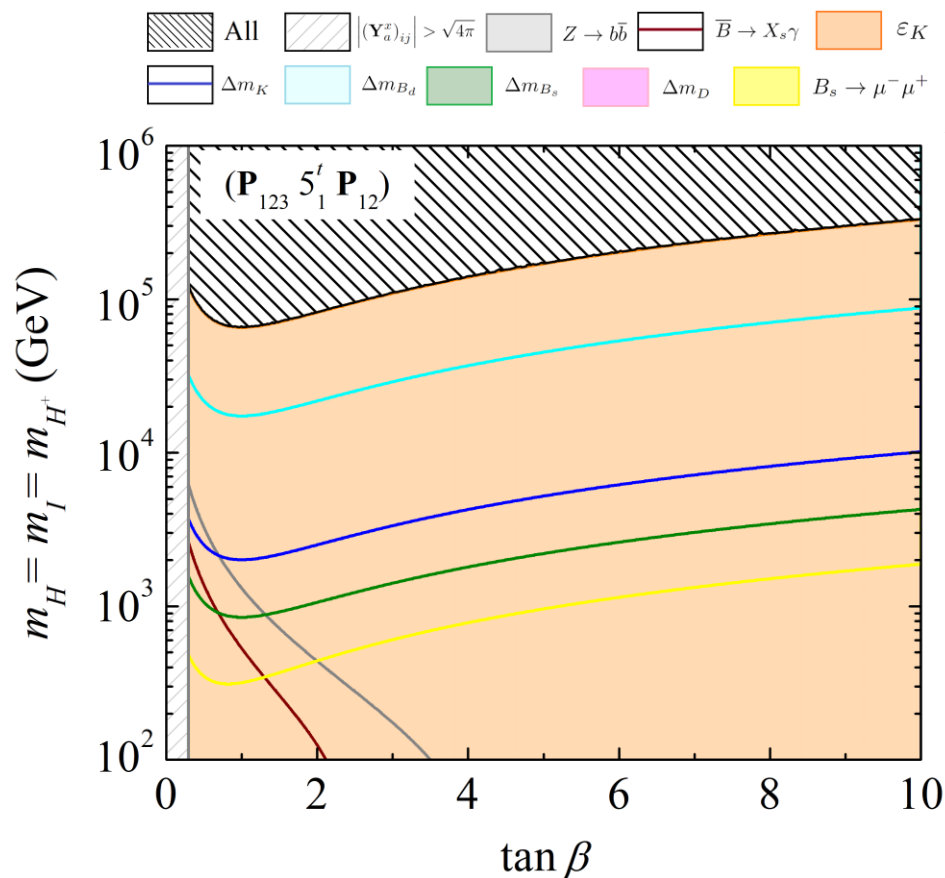


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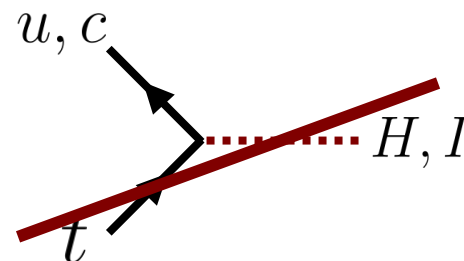


↓  
**None of the most restrictive constraints are automatically satisfied.**

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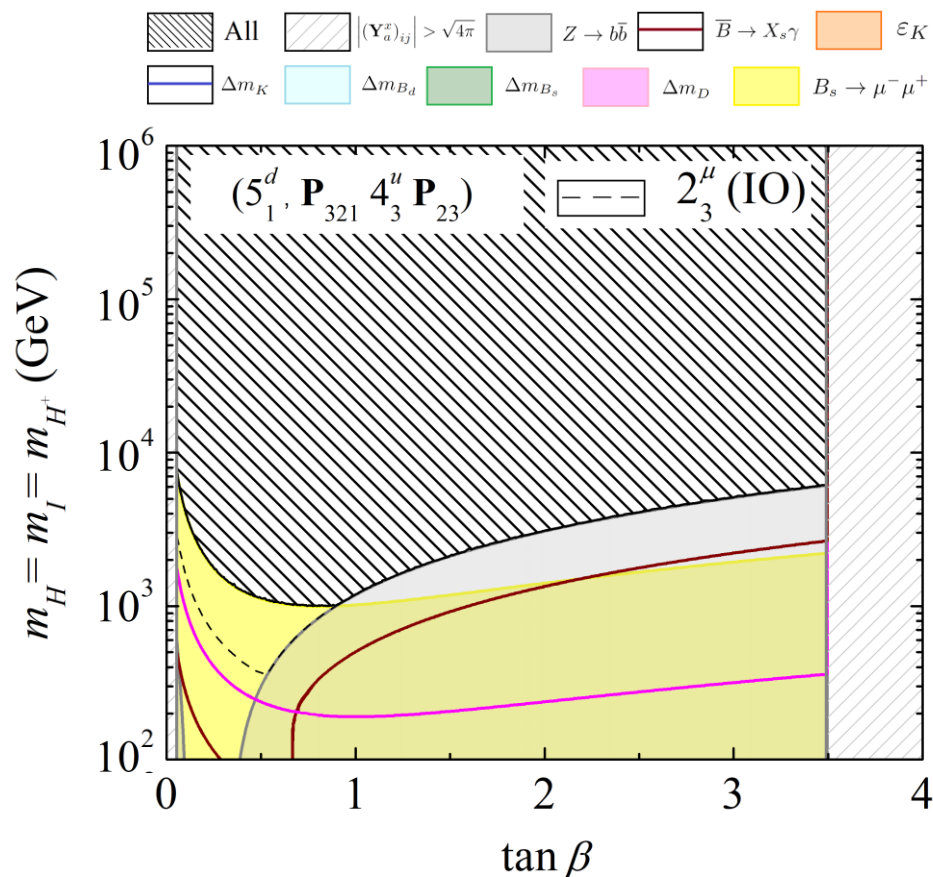
None of the most restrictive constraints are automatically satisfied.

The decoupled state could be picked to satisfy some constraints, for example  $d$

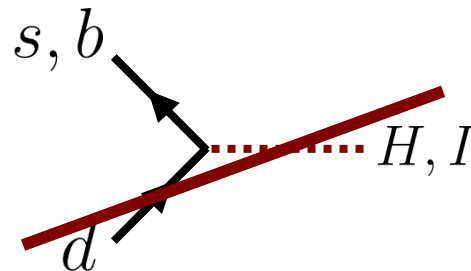
Observable	Constraint	Decoupled state
$ \varepsilon_K $	$(2.228 \pm 0.011) \times 10^{-3}$	$(u, d, s)$
$\Delta m_K^{\text{NP}}$	$< 3.484 \times 10^{-15} \text{ GeV}$	$(d, s)$
$\Delta m_{B_d}$	$(3.334 \pm 0.013) \times 10^{-13} \text{ GeV}$	$(d, b)$
$\Delta m_{B_s}$	$(1.1693 \pm 0.0004) \times 10^{-11} \text{ GeV}$	$(s, b)$
$\Delta m_D^{\text{NP}}$	$< 6.56 \times 10^{-15} \text{ GeV}$	$(u, c)$



# Numerical procedure and phenomenological analysis

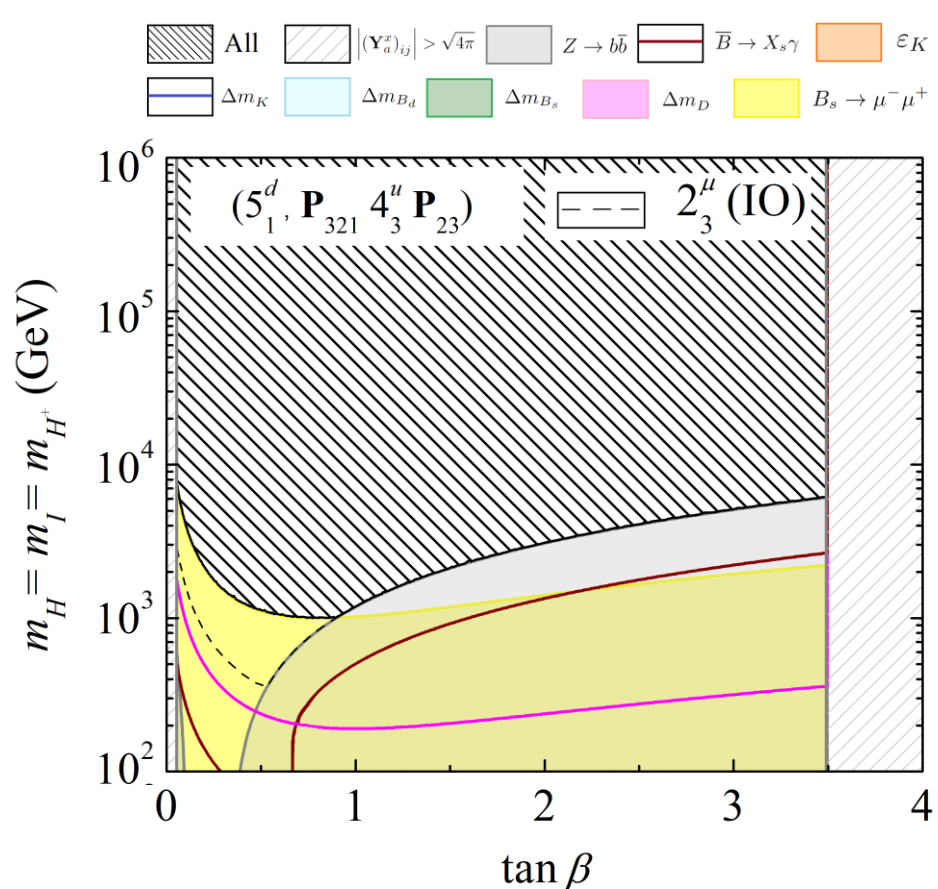


$$\mathbf{N}_d \sim \begin{pmatrix} \times & 0 & 0 \\ 0 & \times & \times \\ 0 & \times & \times \end{pmatrix}$$

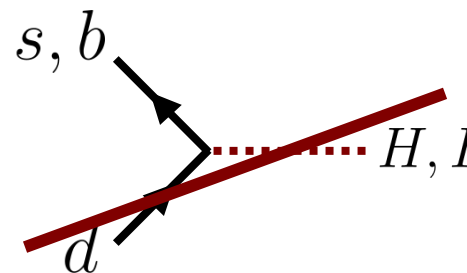


- ✓ K mesons:  $K^0(d\bar{s})$ :  $\Delta m_K, \varepsilon_K$
- ✓ B mesons:  $B_d^0(d\bar{b})$ :  $\Delta m_{B_d}, \text{Br}(B_d^0 \rightarrow \mu^+ \mu^-)$

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**This model highlights the effectiveness of Abelian flavour symmetries in aligning theoretical frameworks with highly constrained experimental observations.**

# Summary and outlook

## Work done:

- ✓ Study of the theoretical framework of the **minimal U(1) 2HDM for flavour**;
- ✓ Identification of the **maximally-restrictive pairs of quark and lepton mass matrices** compatible with current masses, mixing and CP violation data;
- ✓ Lepton sector **predictions**;
- ✓ **Phenomenological study** (analytical and numerical) of the quark and charged lepton sectors.

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# Thank you !