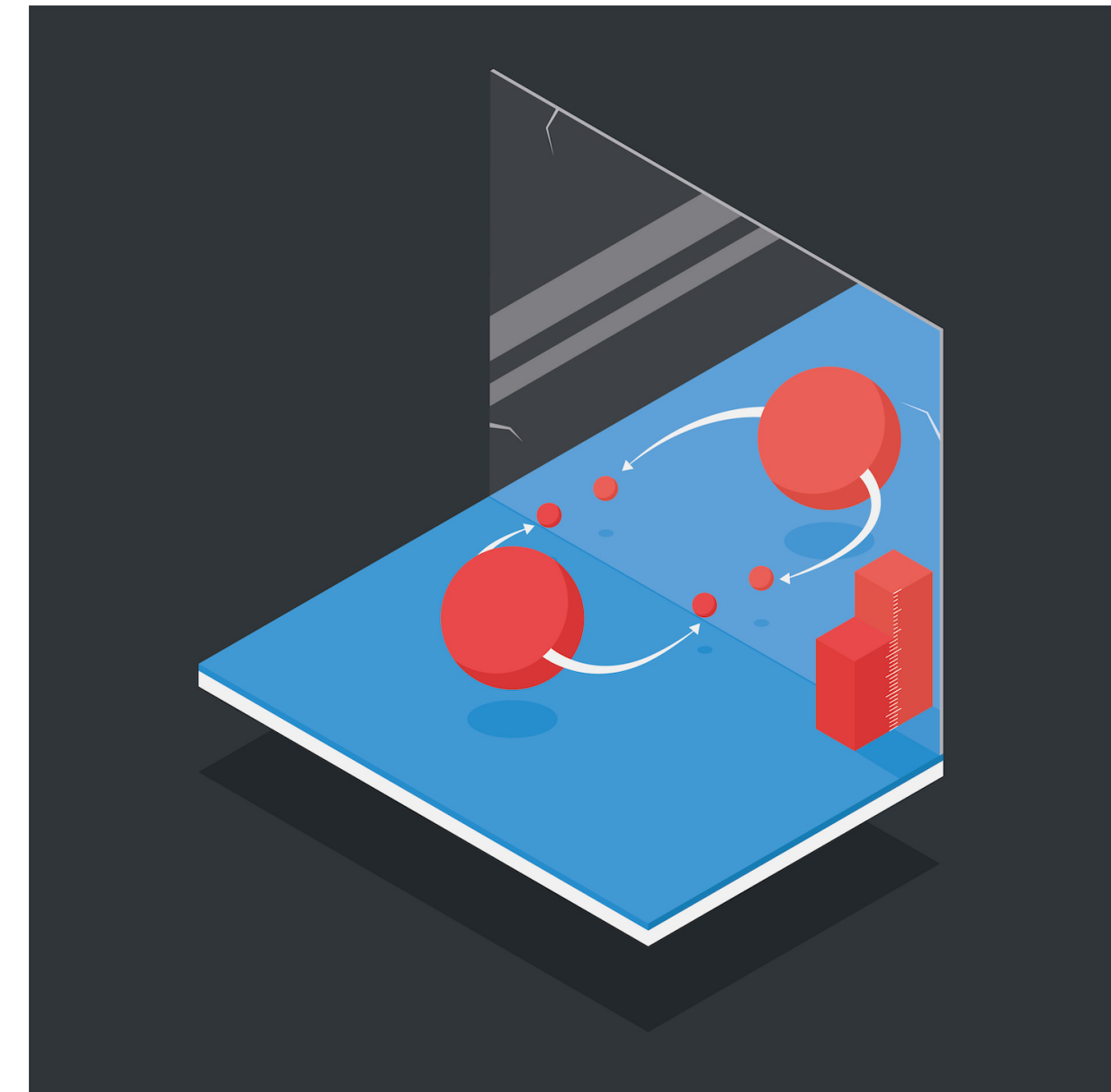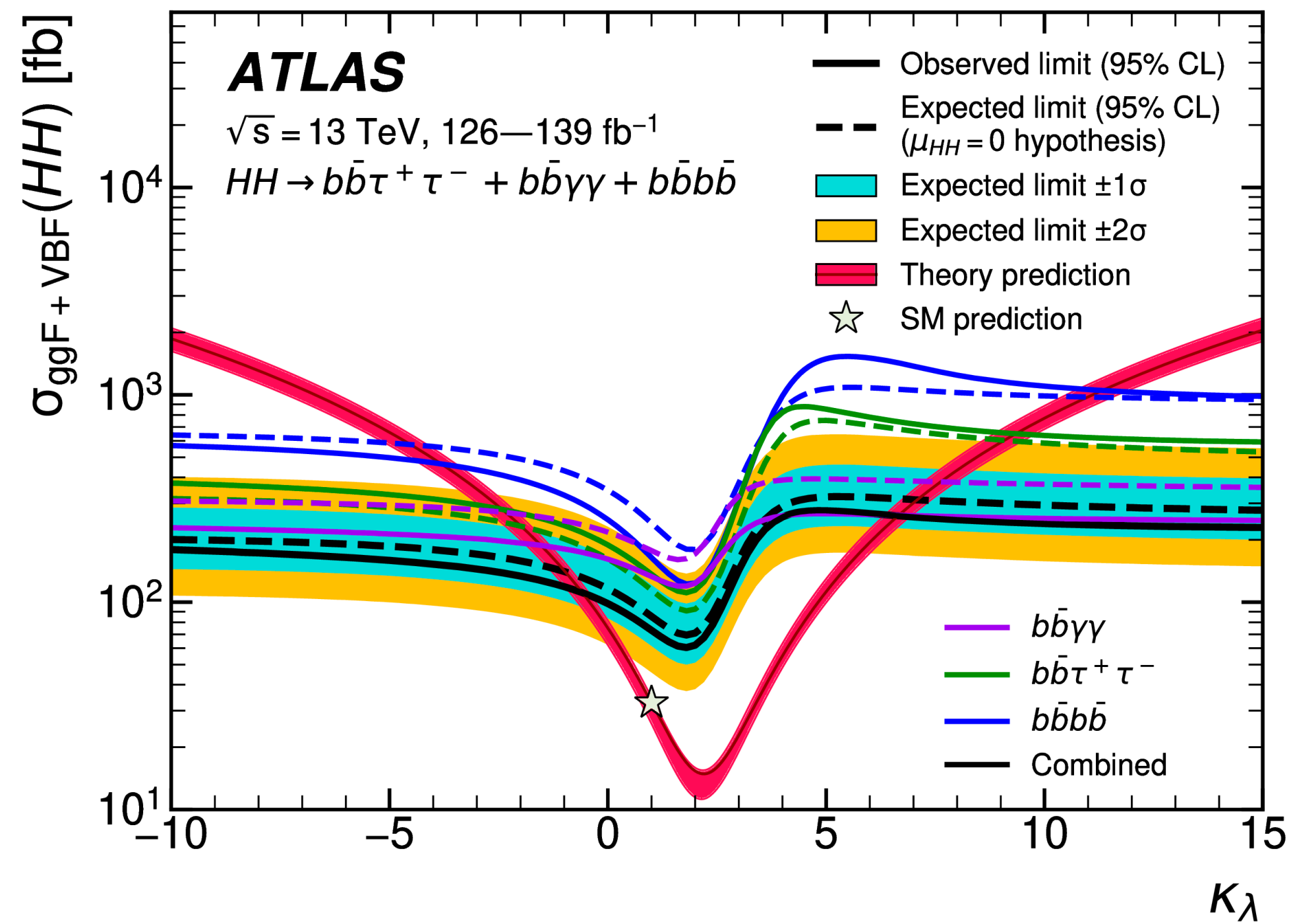# Multi-Higgs Production & Electroweak Phase Transition

Osama Karkout

Working with
Jorinde van de Vis, Marieke Postma, Andreas Papaefstathiou, Gilberto Tetlalmatzi, Tristan du Pree

Project: LHC Higgs results → matter-antimatter asymmetry

matter-antimatter asymmetry

Cosmic rays: $\bar{p}/p = 10^{-4}$

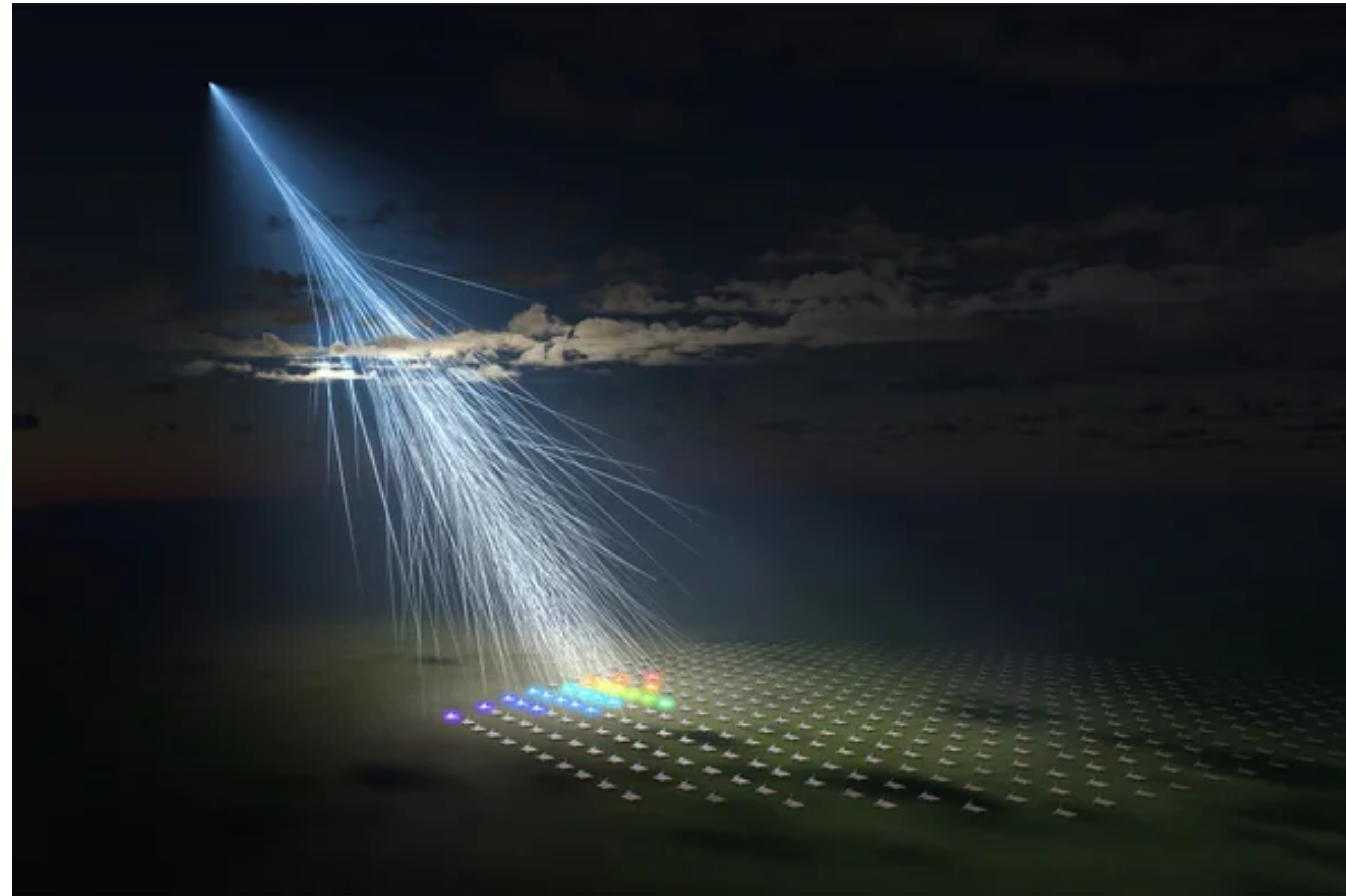= no ambient antiprotons ($\bar{p}$)

BIG DEAL!

Lorentz invariance
+
Hermitian Hamiltonian (physical observables are real)
=
matter-antimatter symmetry (CPT) is conserved!

True in SM and any BSM!!!!

Baryogenesis (matter-antimatter asymmetry)

Problem: we exist :(

(CPT) is conserved => need for **dynamical** mechanism to generate matter-antimatter asymmetry.

Sakharov conditions:

- Baryon number violation
- Loss of thermal equilibrium
- Break C and CP symmetries

In SM: all related to the Higgs field

https://arxiv.org/pdf/hep-ph/0609145.pdf

https://arxiv.org/pdf/2301.05197.pdf
http://www.laine.itp.unibe.ch/cosmology/lec09.pdf

BARYOGENESIS

James M. Cline

Baryogenesis (matter-antimatter asymmetry)

Sakharov is mostly known for his political activism for individual freedom, human rights, civil liberties

Baryogenesis (matter-antimatter asymmetry)

Problem: we exist :(

(CPT) is conserved => need for **dynamical** mechanism to generate matter-antimatter asymmetry.

Sakharov conditions:

- Baryon number violation
- **Loss of thermal equilibrium**
- Break C and CP symmetries

https://arxiv.org/pdf/hep-ph/0609145.pdf
https://arxiv.org/pdf/2301.05197.pdf
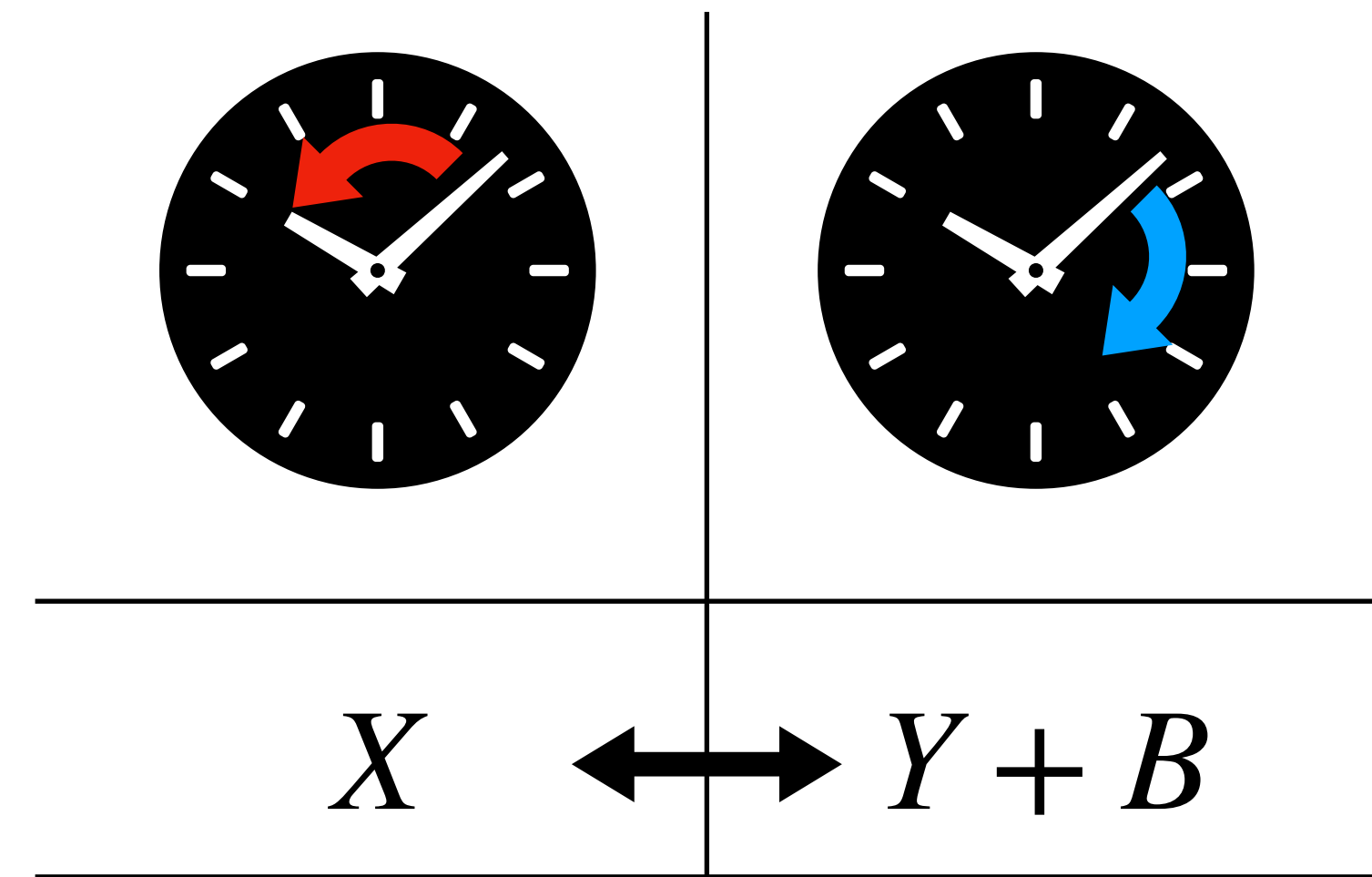http://www.laine.itp.unibe.ch/cosmology/lec09.pdf

BARYOGENESIS

James M. Cline

Electroweak Baryogenesis

Out of thermal equilibrium

$$X \longleftrightarrow Y + B$$
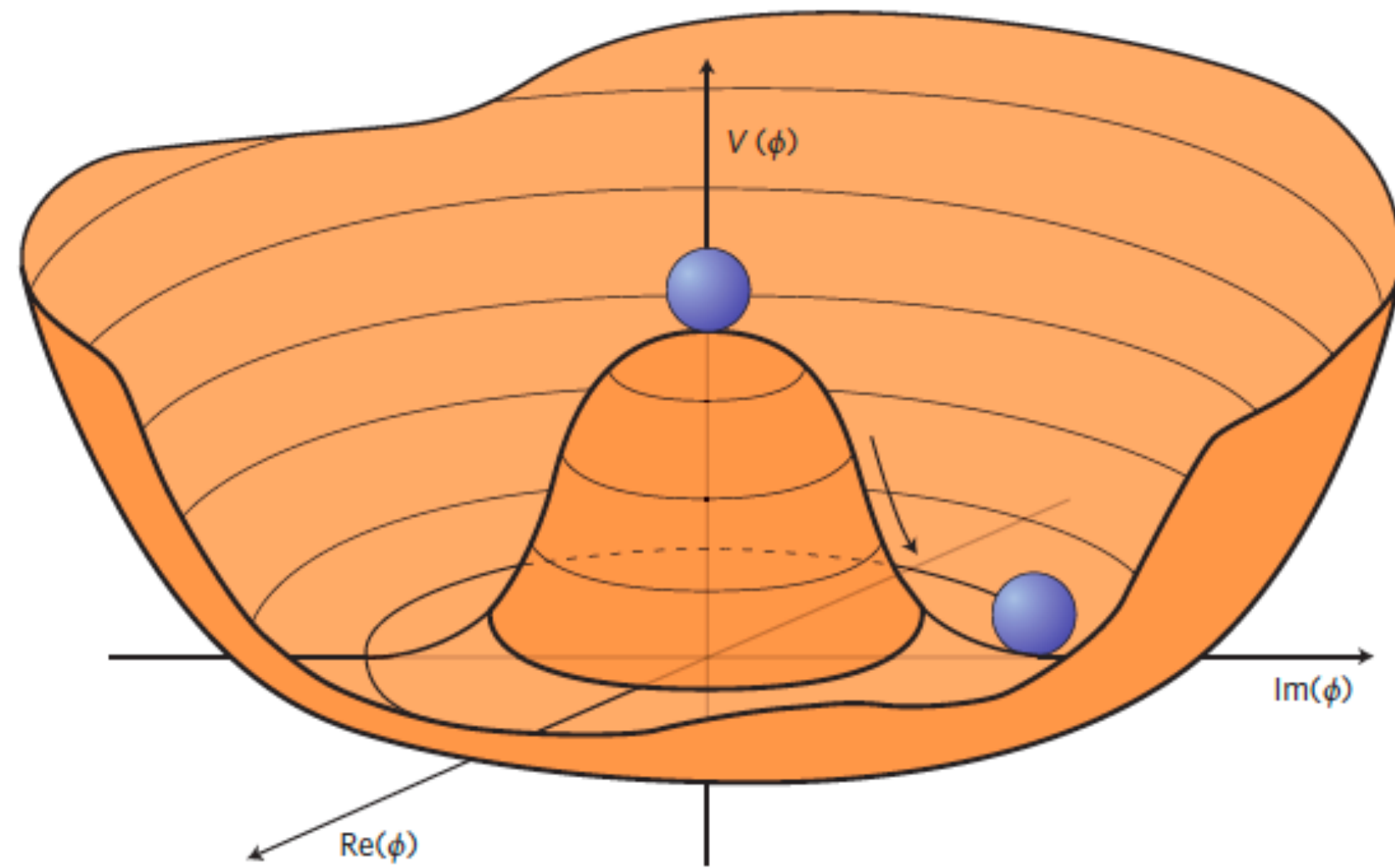
Electroweak symmetry breaking (EWSB) is a phase transition!

It can cause loss of thermal equilibrium if it is a First Order Phase Transition (FOPT)
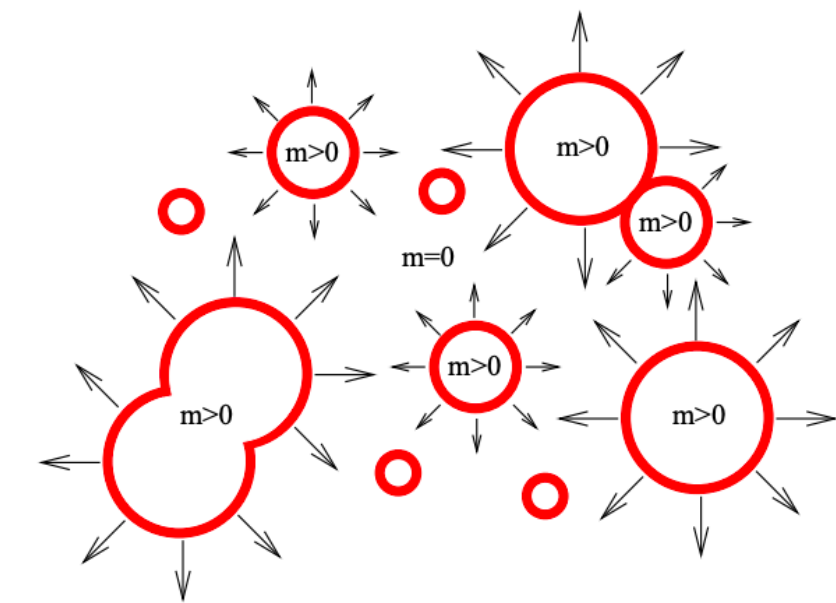
Electroweak Baryogenesis
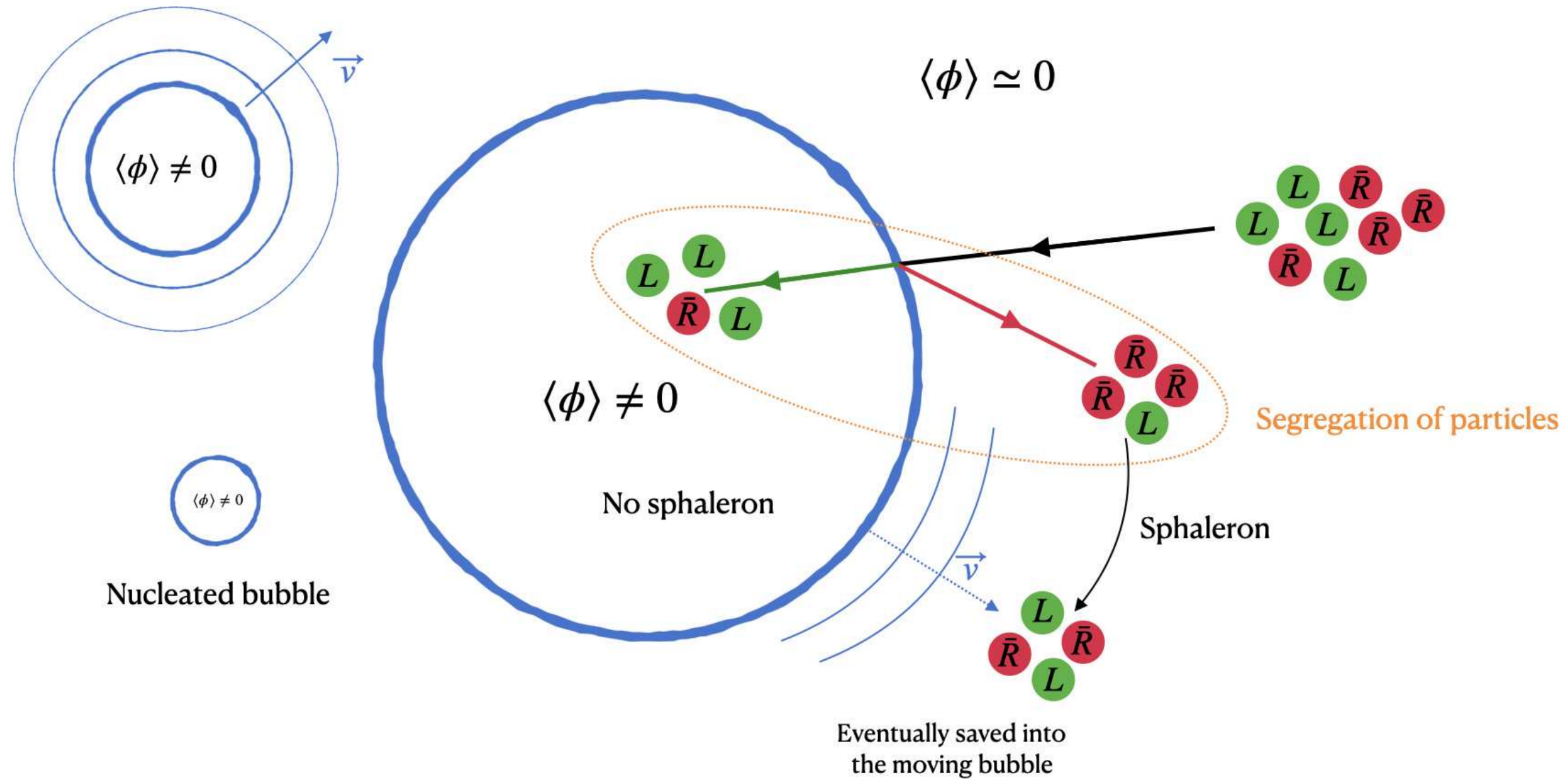
$\langle \phi \rangle \neq 0$

$\vec{v}$

$\langle \phi \rangle \simeq 0$

$\langle \phi \rangle \neq 0$

$\langle \phi \rangle \neq 0$

No sphaleron

$\vec{v}$

Nucleated bubble

Segregation of particles

Sphaleron

Eventually saved into
the moving bubble

Electroweak Phase Transition: Thermal QFT

Before symmetry breaking, Higgs potential is:

$$V(H) = -\frac{1}{2}\mu^2 H^2 + \frac{1}{4}\lambda H^4$$

Legend: $V(H) = -2H^2 + \frac{1}{4}H^4$

Electroweak Phase Transition: Thermal QFT

Before symmetry breaking, Higgs potential is:

$$V(H) = -\frac{1}{2}\mu^2 H^2 + \frac{1}{4}\lambda H^4$$

In Feynman diagrams:

Electroweak Phase Transition: Thermal QFT

Before symmetry breaking, Higgs potential is:

$$V(H) = -\frac{1}{2}\mu^2 H^2 + \frac{1}{4}\lambda H^4$$

In Feynman diagrams:

Higgs field is coupled to a thermal bath of fields.
at LO, this looks like:
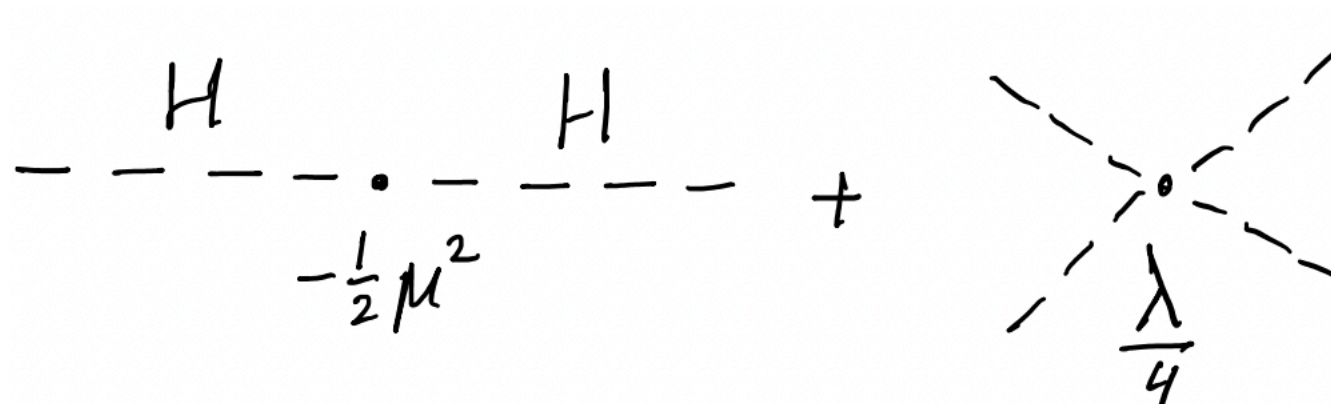
$$V_{eff}(H, T) = -\frac{1}{2}\mu^2 H^2 + \frac{1}{4}\lambda H^4 + \frac{\alpha}{2}T^2 H^2$$
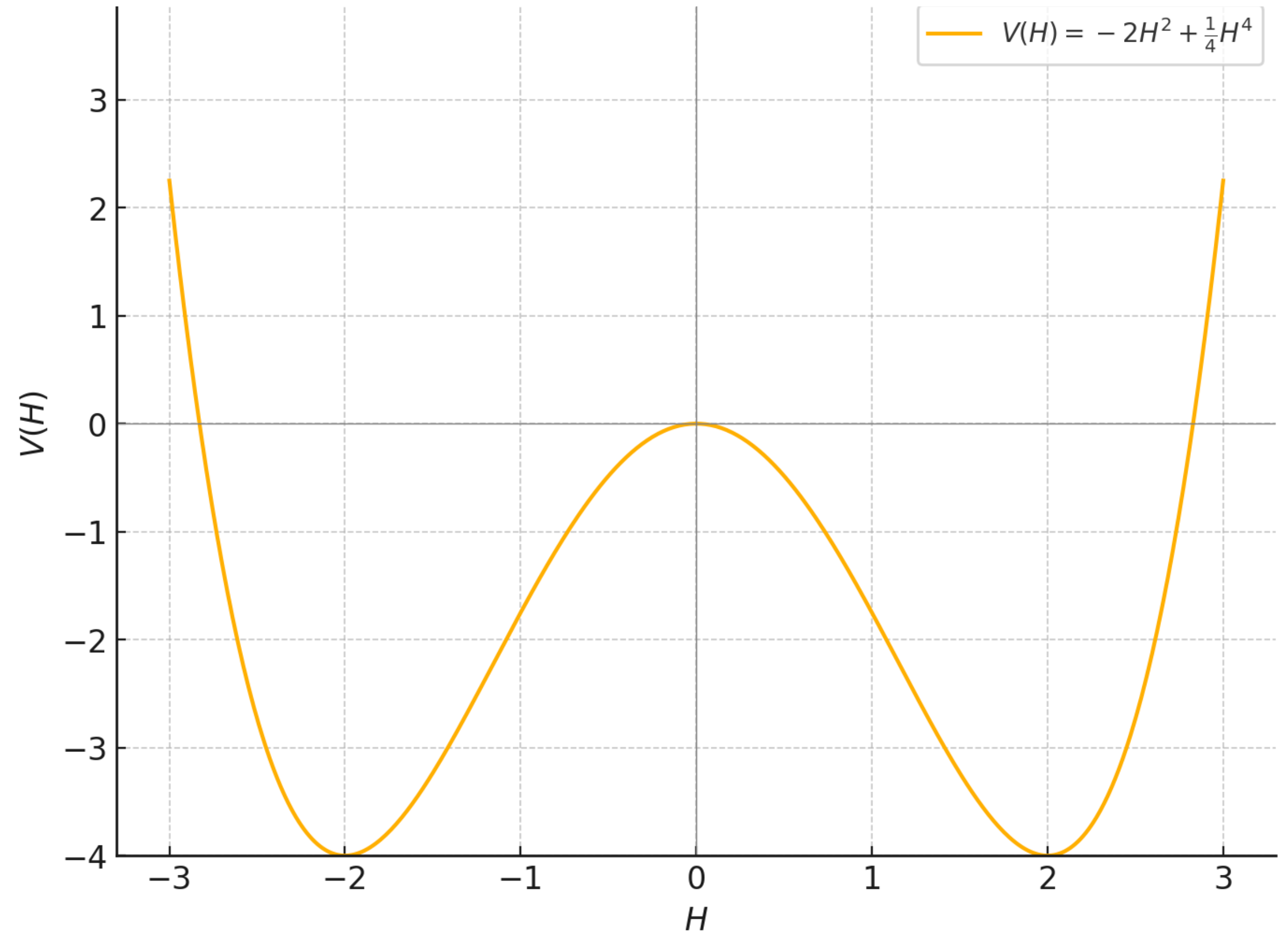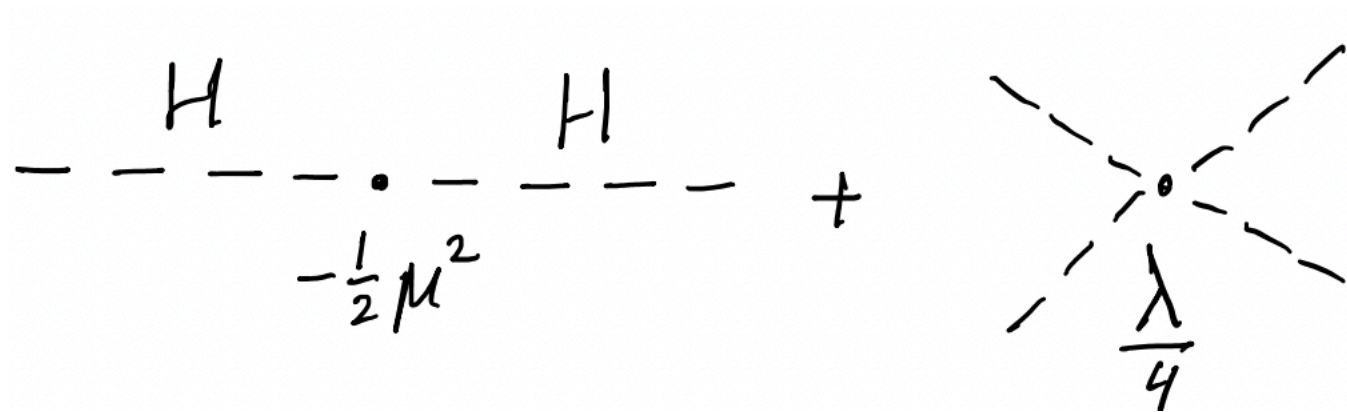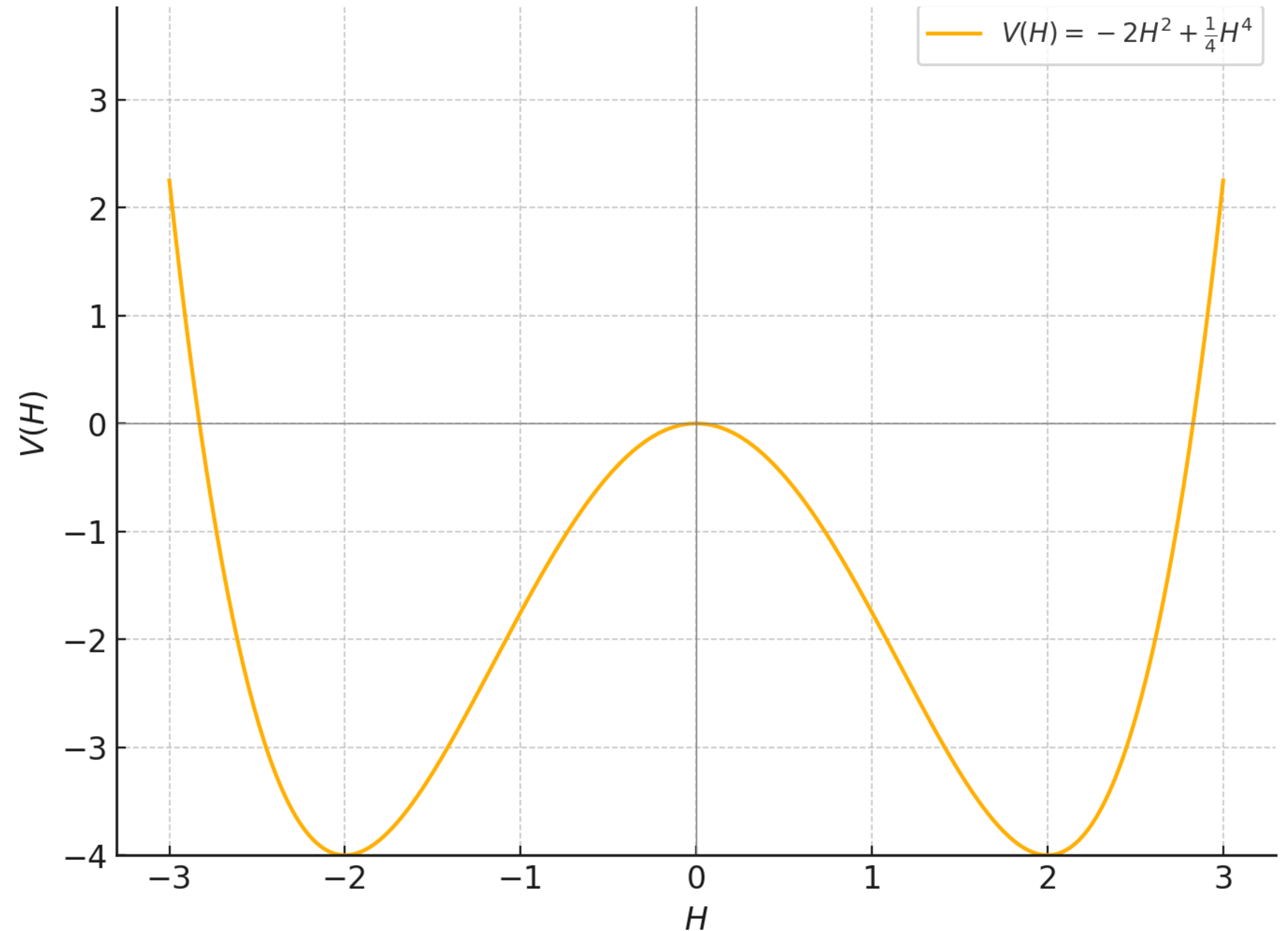
Electroweak Phase Transition: Thermal QFT

Before symmetry breaking, Higgs potential is:

$$V(H) = -\frac{1}{2}\mu^2 H^2 + \frac{1}{4}\lambda H^4$$

In Feynman diagrams:

$$V(H) = \quad \underset{-\frac{1}{2}\mu^2}{\underbrace{\quad \overset{H}{\vdots} \quad \cdot \quad \overset{H}{\vdots} \quad}} \quad + \quad \underset{\frac{\lambda}{4}}{\underbrace{\quad \times \quad}}$$

Higgs field is coupled to a thermal bath of fields.
at LO, this looks like:

$$\underset{\sim T^2}{\overset{H \quad\quad H}{\bigcirc}}$$

$$V_{eff}(H, T) = -\frac{1}{2}\mu^2 H^2 + \frac{1}{4}\lambda H^4 + \frac{\alpha}{2}T^2 H^2$$

$V(H)$

$T > T_c$

$T = T_c$

$T < T_c$

$T = 0$

H
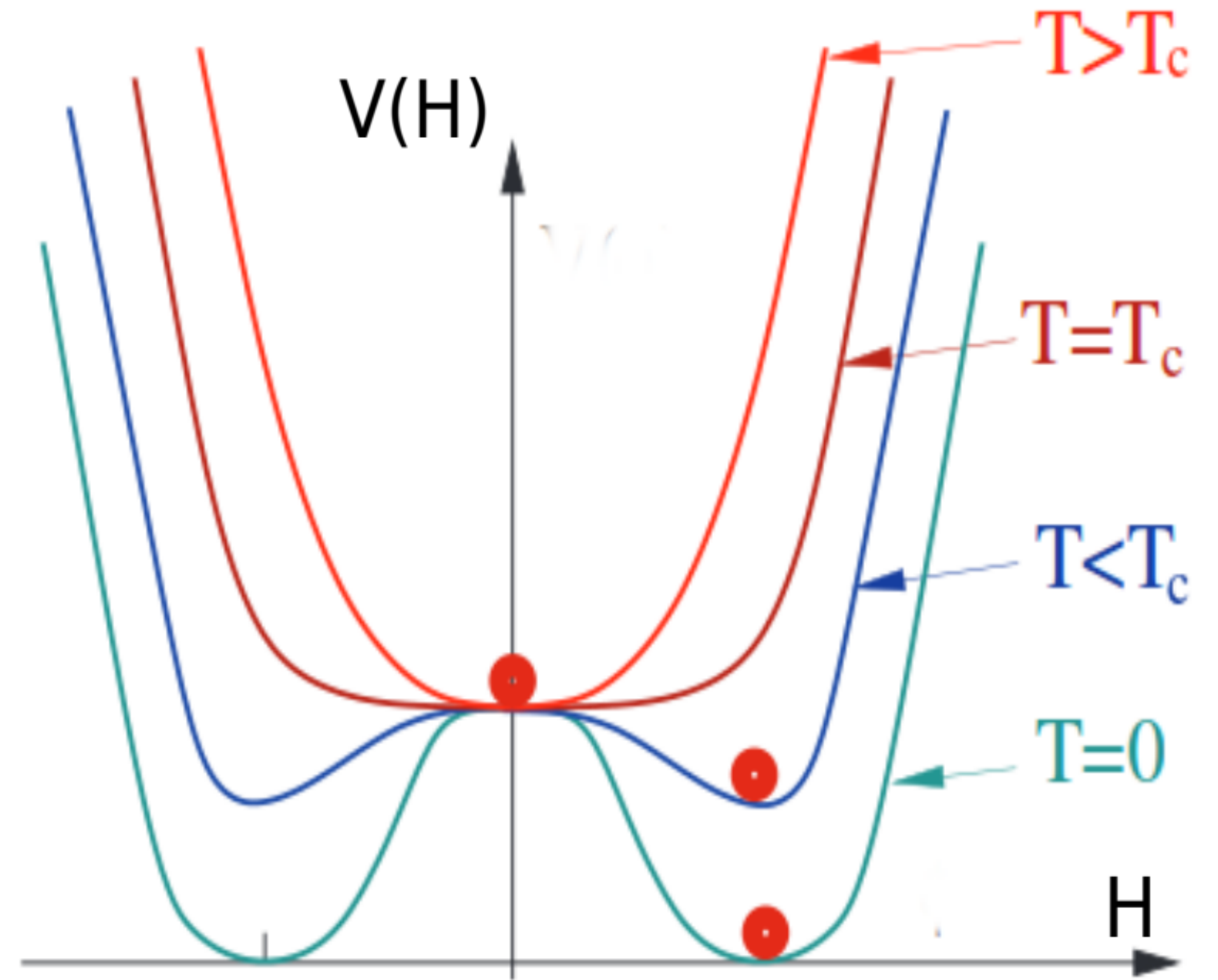
Electroweak Phase Transition: Thermal QFT

at NLO, the effective potential gets a cubic term

$$V_{eff}(H,T) = \frac{1}{2}(-\mu^2 + \alpha T^2)H^2 - \beta T(-\mu^2 + \gamma H^2)^{3/2} + \frac{1}{4}\lambda H^4$$

The values of (α, β, γ) depend on your theory

If you're lucky, you can get a barrier between two minima at some critical temperature T_c

No first order phase transition in SM
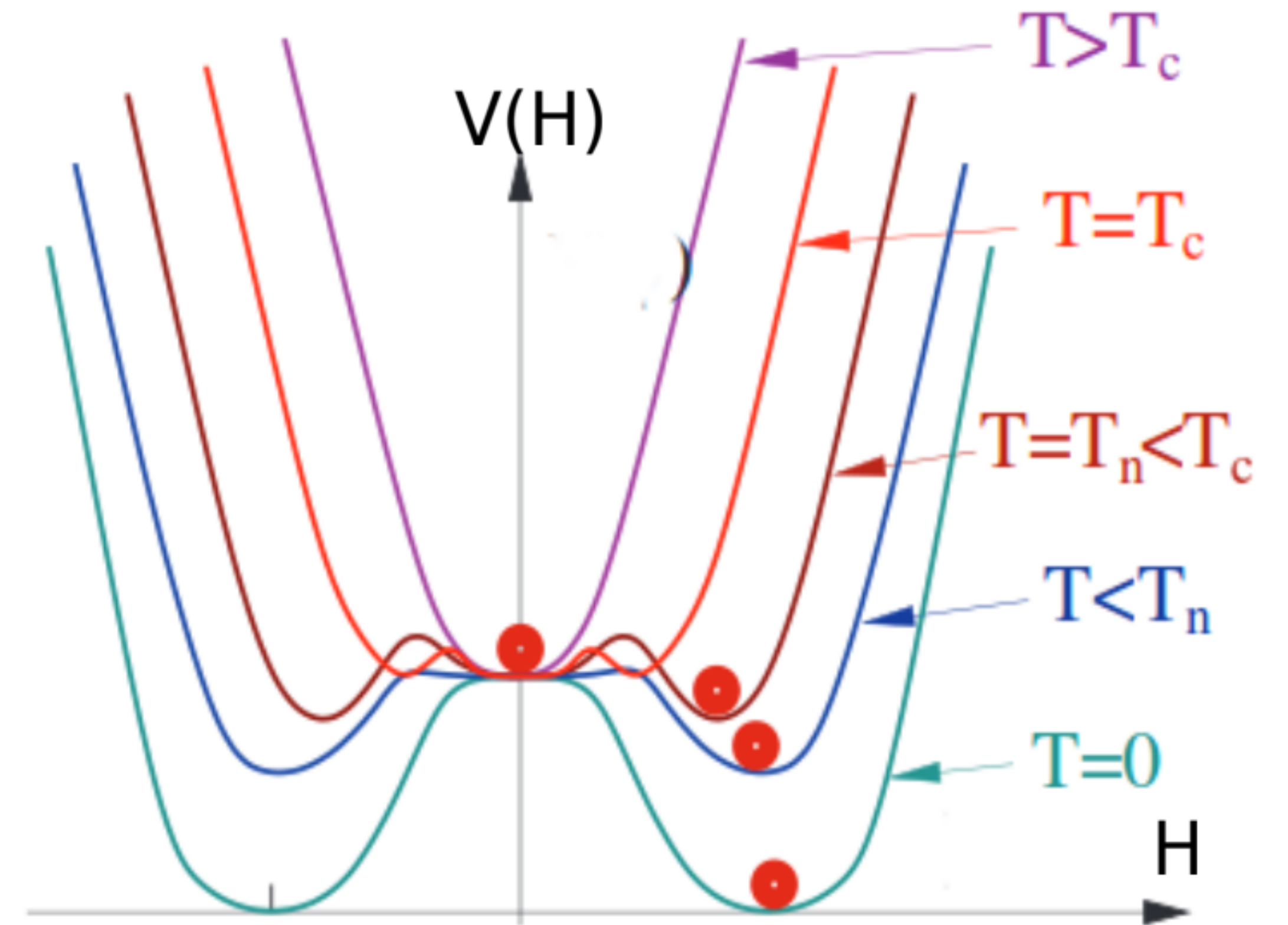
T_n = temperature of bubble nucleation

Electroweak Phase Transition: BSM

Idea:

1. add a scalar field S which couples to the Higgs.
2. This scalar field also has a phase transition! Going to a VEV for S

$$V(H) = -\frac{1}{2}\mu^2 H^2 + \frac{1}{4}\lambda H^4 + V(H,S)$$
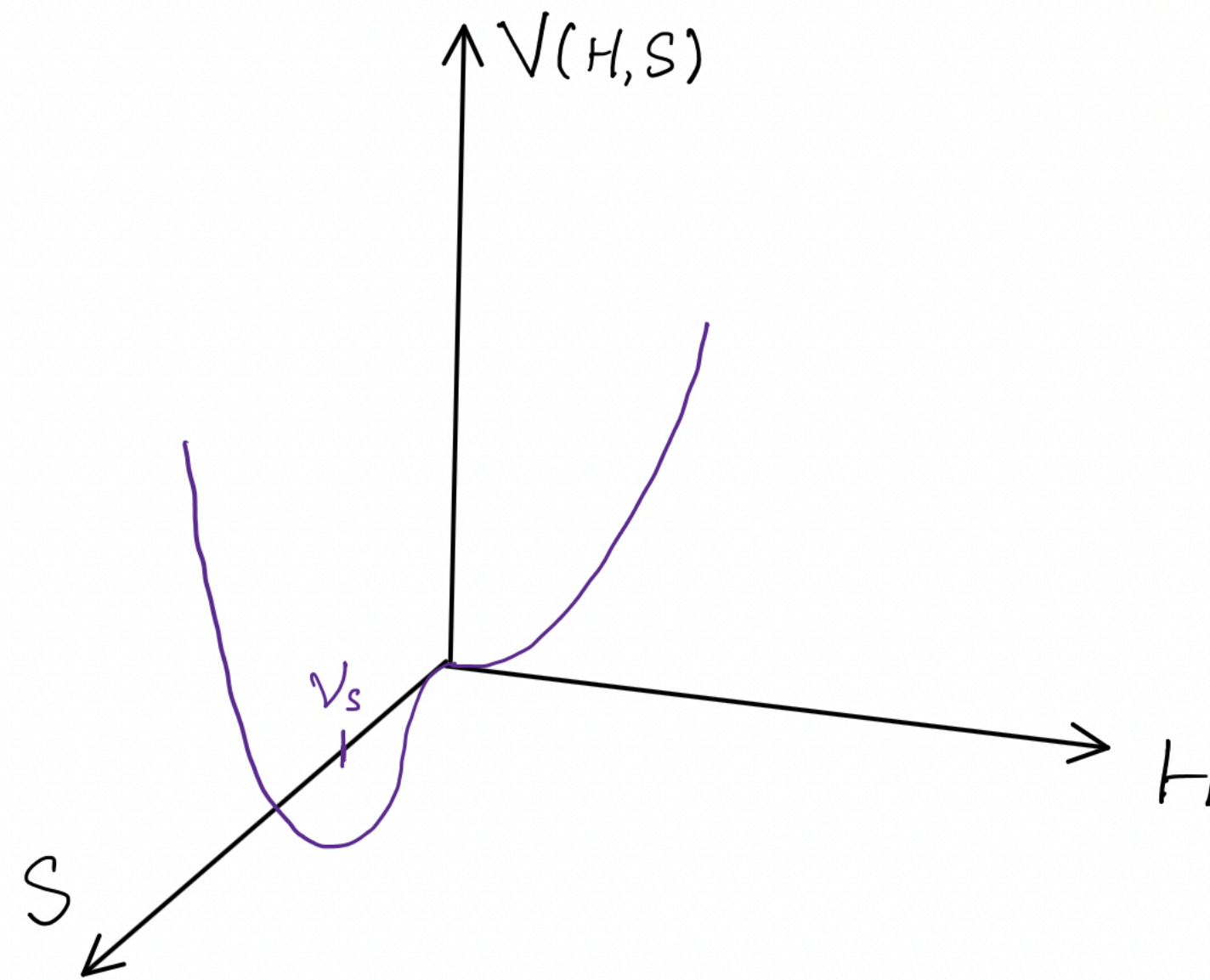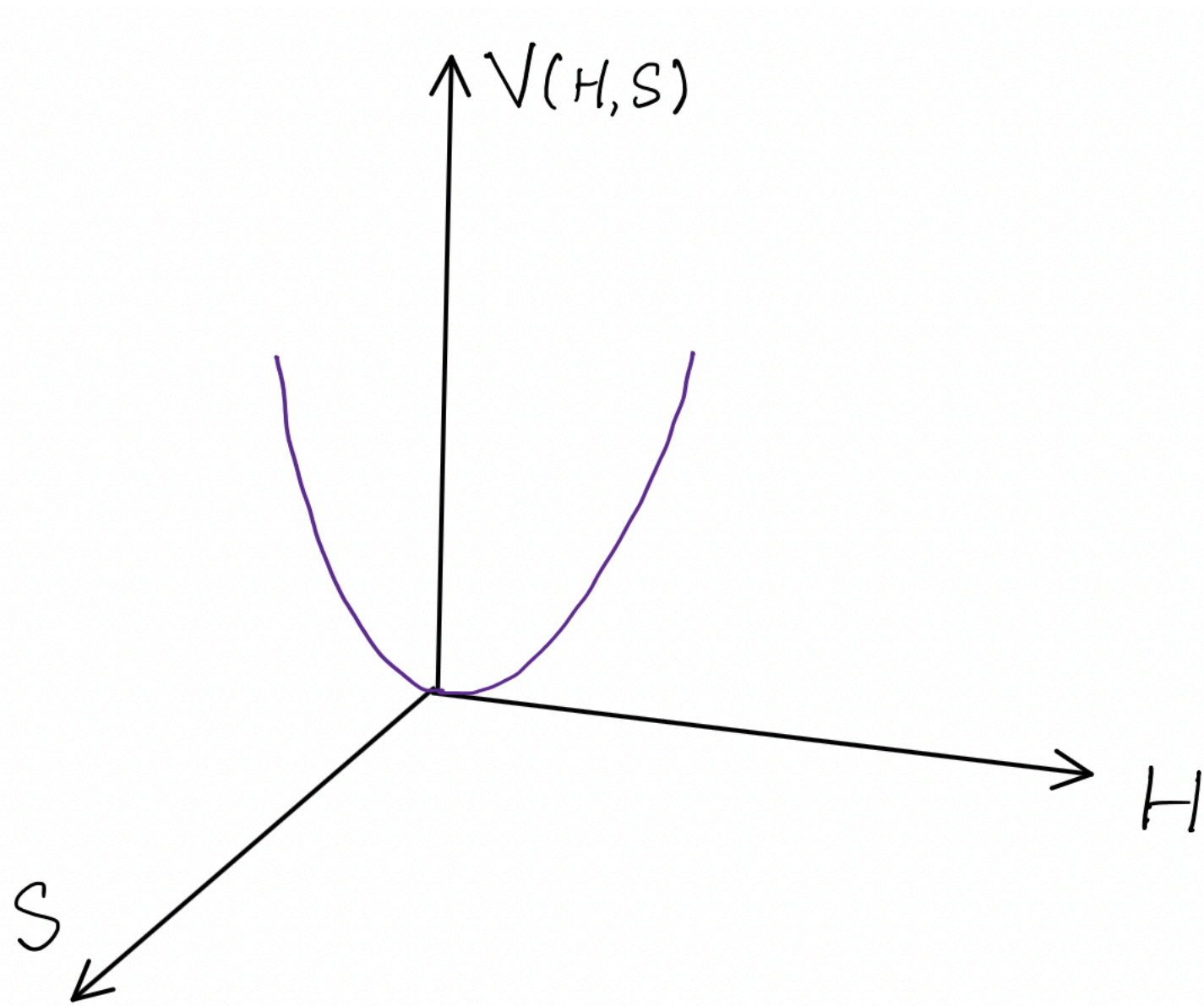
Electroweak Phase Transition: BSM

Idea:
1. add a scalar field S which couples to the Higgs.
2. This scalar field also has a phase transition! Going to a VEV for S
3. Form a potential barrier between the VEV of S and the VEV of H
4. Tunnel to the VEV of H: This is FOPT

$$V(H) = -\frac{1}{2}\mu^2 H^2 + \frac{1}{4}\lambda H^4 + V(H,S)$$
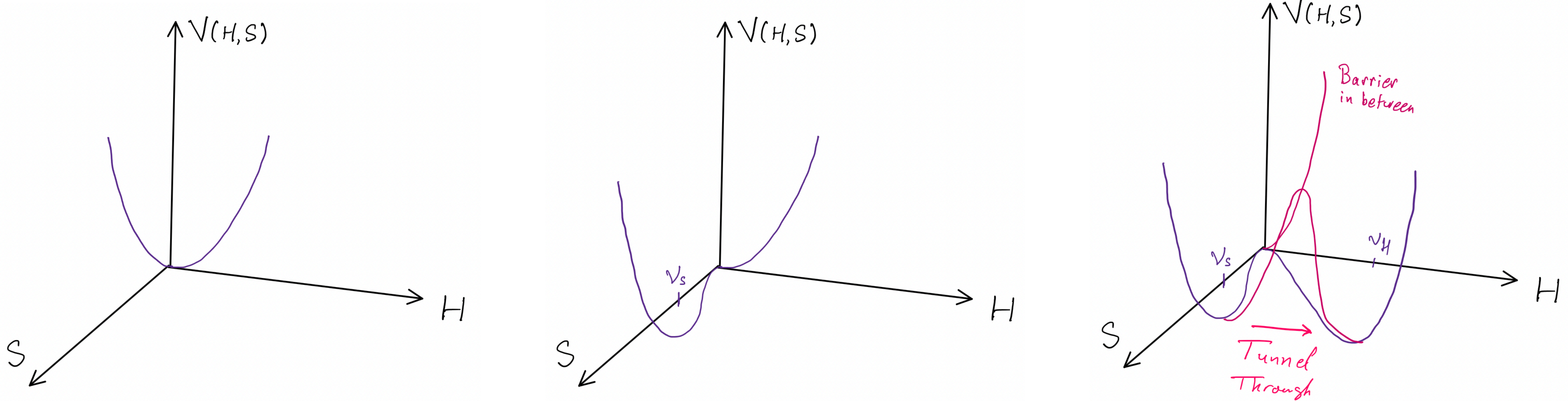
HHH searches are coming!
Maybe we will see enhancement of HHH production!
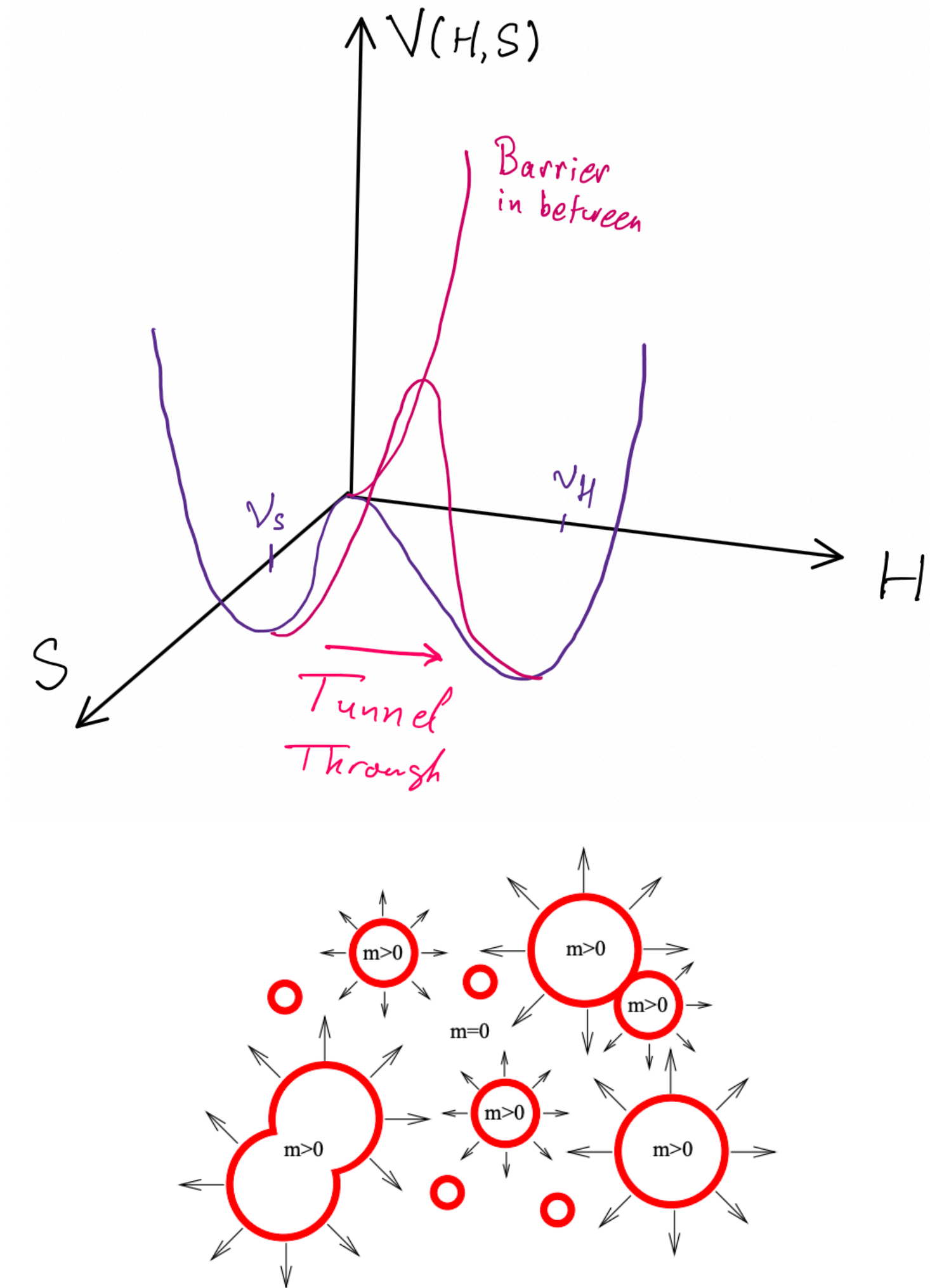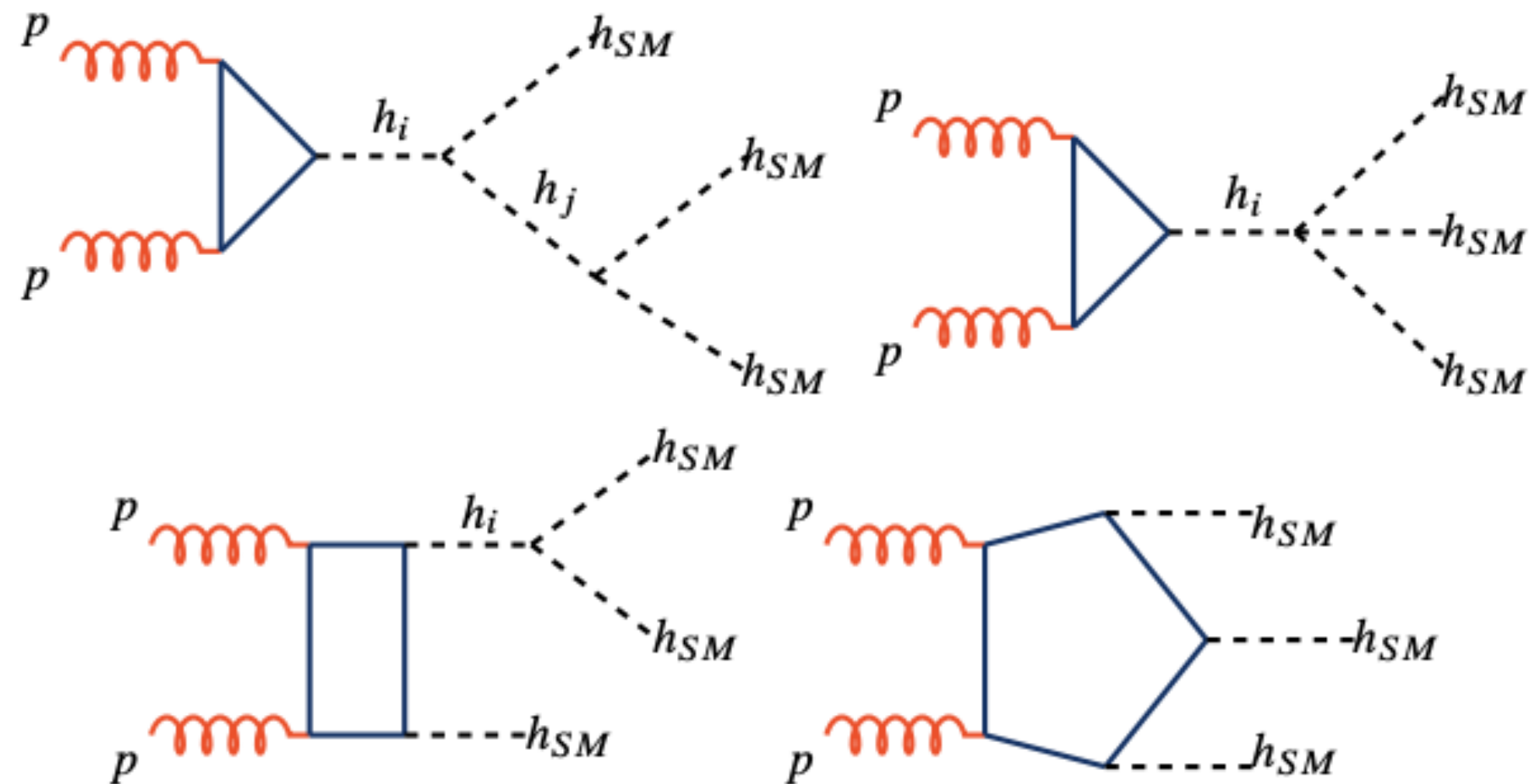
How to enhance HHH

Simplified BSM model predicting large HHH: **TRSM**.

SM + two singlets coupling to the Higgs.

$$V = \mu_\Phi^2 \Phi^\dagger \Phi + \lambda_\Phi (\Phi^\dagger \Phi)^2 + \mu_S^2 S^2 + \lambda_S S^4 + \mu_X^2 X^2 + \lambda_X X^4$$
$$+ \lambda_{\Phi S} \Phi^\dagger \Phi S^2 + \lambda_{\Phi X} \Phi^\dagger \Phi X^2 + \lambda_{SX} S^2 X^2 \, .$$

How to enhance HHH

Simplified BSM model predicting large HHH: **TRSM**.

SM + two singlets coupling to the Higgs.

$$V = \mu_\Phi^2 \Phi^\dagger \Phi + \lambda_\Phi (\Phi^\dagger \Phi)^2 + \mu_S^2 S^2 + \lambda_S S^4 + \mu_X^2 X^2 + \lambda_X X^4$$
$$+ \lambda_{\Phi S} \Phi^\dagger \Phi S^2 + \lambda_{\Phi X} \Phi^\dagger \Phi X^2 + \lambda_{SX} S^2 X^2 .$$

Scalars get VEVs! $\rightarrow$ Mixing:

$$\Phi = \begin{pmatrix} 0 \\ \frac{\phi_h + v}{\sqrt{2}} \end{pmatrix}, \quad S = \frac{\phi_S + v_S}{\sqrt{2}}, \quad X = \frac{\phi_X + v_X}{\sqrt{2}}$$

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = R \begin{pmatrix} \phi_h \\ \phi_S \\ \phi_X \end{pmatrix}$$

h1 can be our scalar particle of 125 GeV

Tania Robens,[1,*] Tim Stefaniak,[2,†] and Jonas Wittbrodt[2,‡]

How to enhance HHH

Simplified BSM model predicting large HHH: **TRSM**.

SM + two singlets coupling to the Higgs.

$$V = \mu_\Phi^2 \Phi^\dagger \Phi + \lambda_\Phi (\Phi^\dagger \Phi)^2 + \mu_S^2 S^2 + \lambda_S S^4 + \mu_X^2 X^2 + \lambda_X X^4$$
$$+ \lambda_{\Phi S} \Phi^\dagger \Phi S^2 + \lambda_{\Phi X} \Phi^\dagger \Phi X^2 + \lambda_{SX} S^2 X^2 .$$

Scalars get VEVs! → Mixing:

$$\Phi = \begin{pmatrix} 0 \\ \frac{\phi_h + v}{\sqrt{2}} \end{pmatrix}, \quad S = \frac{\phi_S + v_S}{\sqrt{2}}, \quad X = \frac{\phi_X + v_X}{\sqrt{2}}$$

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = R \begin{pmatrix} \phi_h \\ \phi_S \\ \phi_X \end{pmatrix}$$

$V(\phi, S, X)$

Remember:
Mixing requires nonzero VEV
For added scalars

h1 can be our scalar particle of 125 GeV

Tania Robens,[1,*] Tim Stefaniak,[2,†] and Jonas Wittbrodt[2,‡]

How to enhance HHH

Simplified BSM model predicting large HHH: **TRSM**.

SM + two singlets coupling to the Higgs.

$$V = \mu_\Phi^2 \Phi^\dagger \Phi + \lambda_\Phi (\Phi^\dagger \Phi)^2 + \mu_S^2 S^2 + \lambda_S S^4 + \mu_X^2 X^2 + \lambda_X X^4$$
$$+ \lambda_{\Phi S} \Phi^\dagger \Phi S^2 + \lambda_{\Phi X} \Phi^\dagger \Phi X^2 + \lambda_{SX} S^2 X^2 .$$

Scalars get VEVs! → Mixing:

$$\Phi = \begin{pmatrix} 0 \\ \frac{\phi_h + v}{\sqrt{2}} \end{pmatrix}, \quad S = \frac{\phi_S + v_S}{\sqrt{2}}, \quad X = \frac{\phi_X + v_X}{\sqrt{2}}$$

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = R \begin{pmatrix} \phi_h \\ \phi_S \\ \phi_X \end{pmatrix}$$

h1 can be our scalar particle of 125 GeV

Tania Robens,[1,*] Tim Stefaniak,[2,†] and Jonas Wittbrodt[2,‡]

HHH production is enhanced through **resonance**
xsec ~ 30 fb (~ HH production in SM)
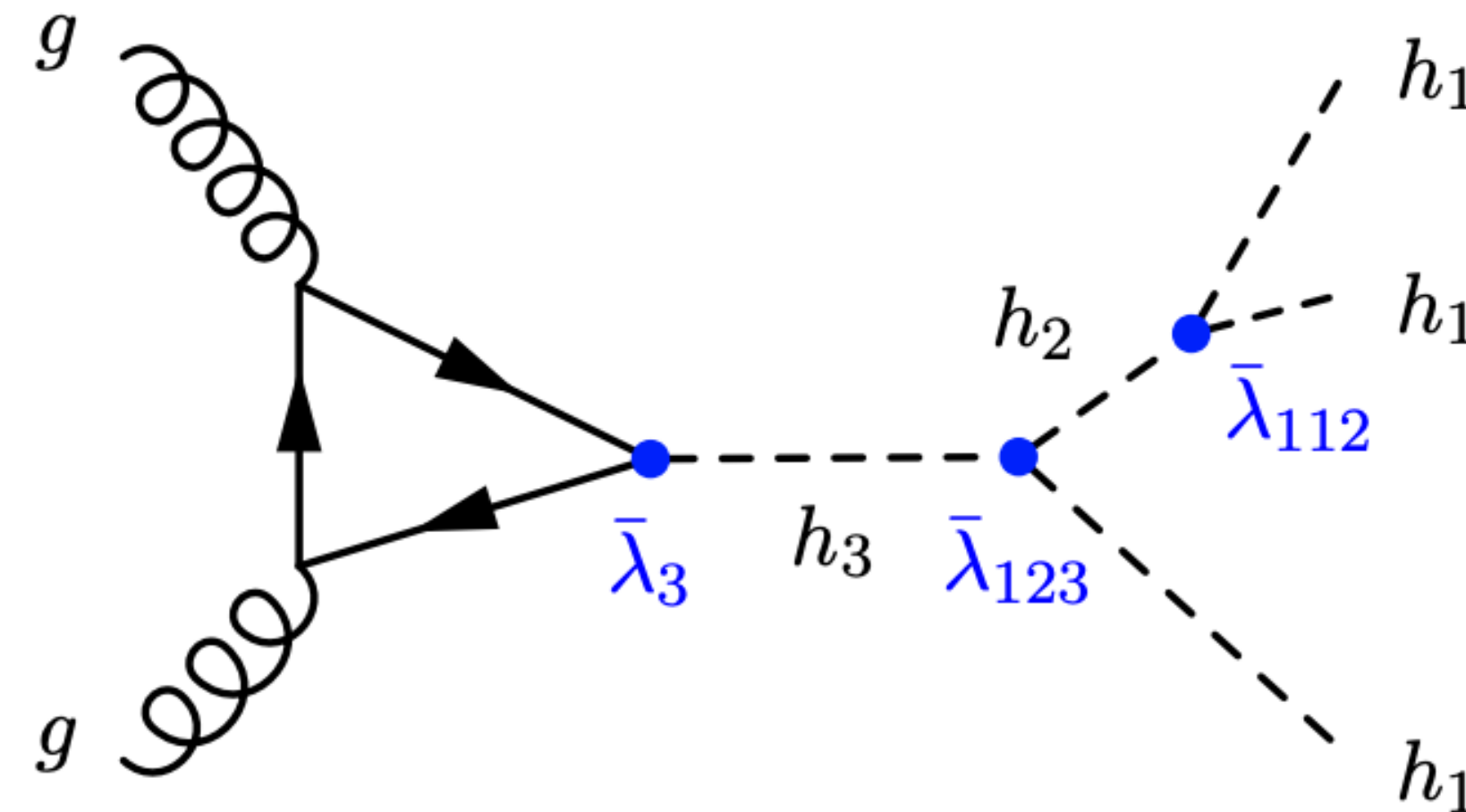
We updated this conclusion using
Stronger theoretical bounds (perturbativity)
and newest experimental bounds!

Osama Karkout,[1] **Andreas Papaefstathiou,**[2] **Marieke Postma,**[1,3] **Gilberto Tetlalmatzi-Xolocotzi,**[4,5] **Jorinde van de Vis,**[6] **Tristan du Pree**[1]

https://arxiv.org/pdf/2404.12425

Electroweak Phase Transition: TRSM

$$V = \mu_\Phi^2 \Phi^\dagger \Phi + \lambda_\Phi (\Phi^\dagger \Phi)^2 + \mu_S^2 S^2 + \lambda_S S^4 + \mu_X^2 X^2 + \lambda_X X^4$$
$$+ \lambda_{\Phi S} \Phi^\dagger \Phi S^2 + \lambda_{\Phi X} \Phi^\dagger \Phi X^2 + \lambda_{SX} S^2 X^2.$$
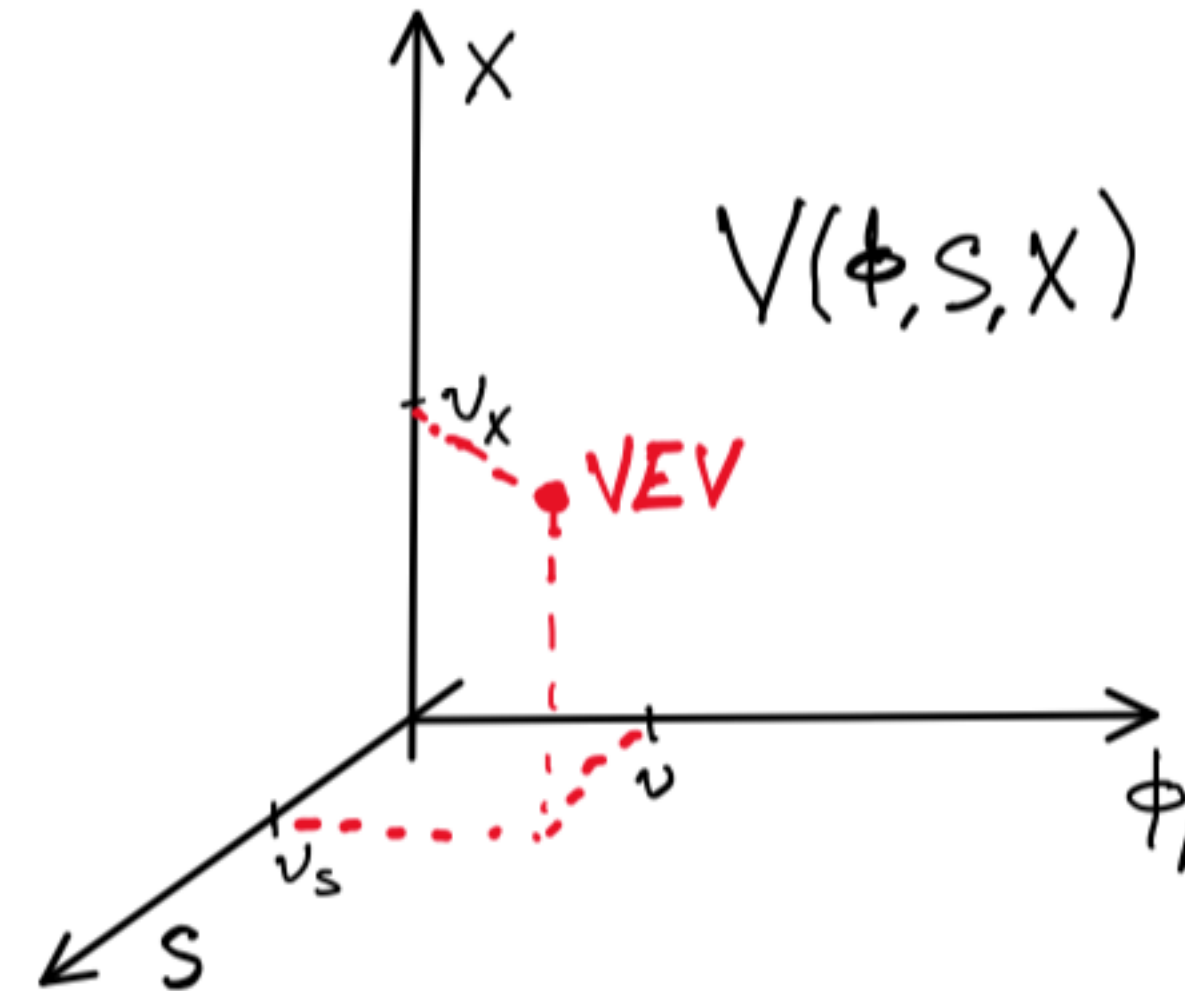
Mixing:

$$\Phi = \begin{pmatrix} 0 \\ \frac{\phi_h + v}{\sqrt{2}} \end{pmatrix}, \quad S = \frac{\phi_S + v_S}{\sqrt{2}}, \quad X = \frac{\phi_X + v_X}{\sqrt{2}} \qquad \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = R \begin{pmatrix} \phi_h \\ \phi_S \\ \phi_X \end{pmatrix}$$

Physical parameter space:

$$\{M_2, M_3, v_s, v_x, \theta_1, \theta_2, \theta_3\}$$

$M_1 = 125 \ GeV, \ v = 246 \ GeV$

Can we have First-Order Phase Transition (FOPT)?
For which parameters?
Does it come with HHH enhancement?

$$\{M_2, M_3, v_s, v_x, \theta_1, \theta_2, \theta_3\}$$
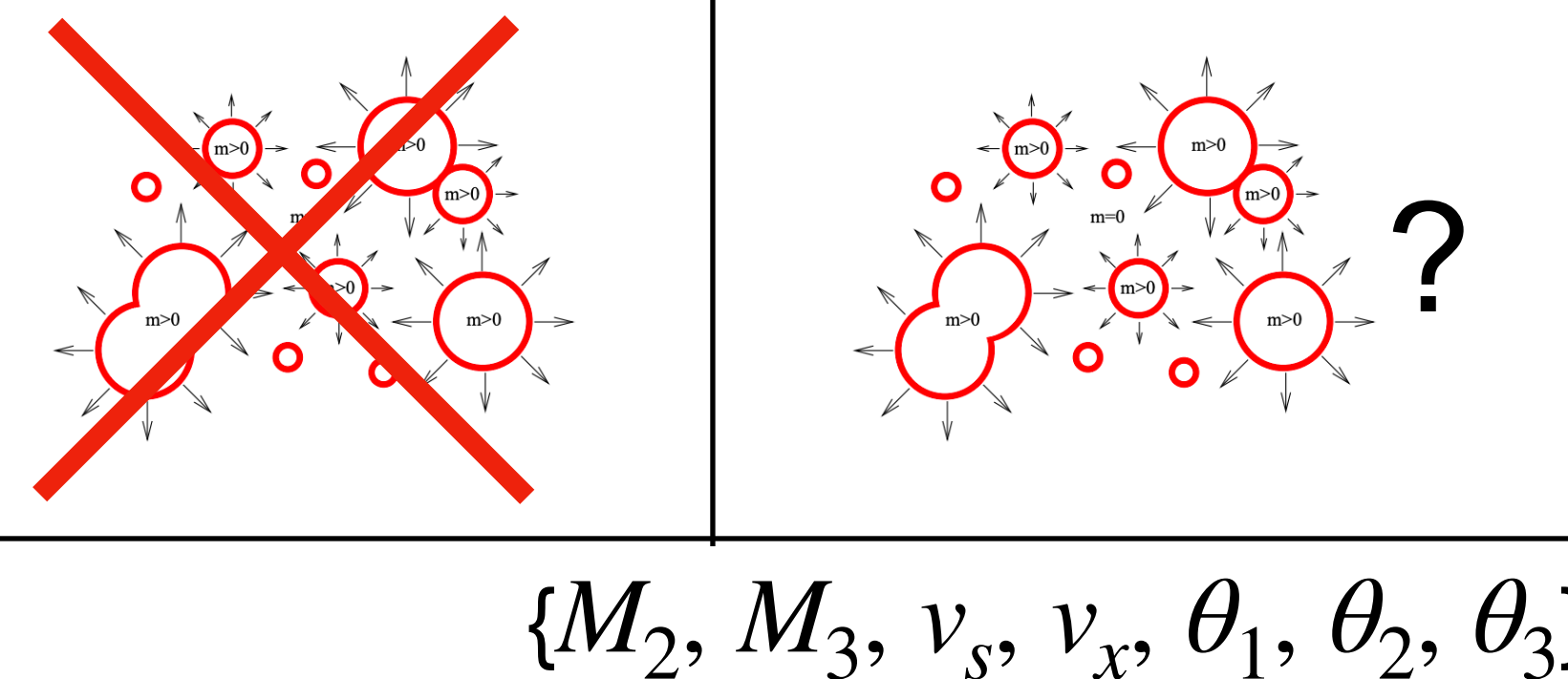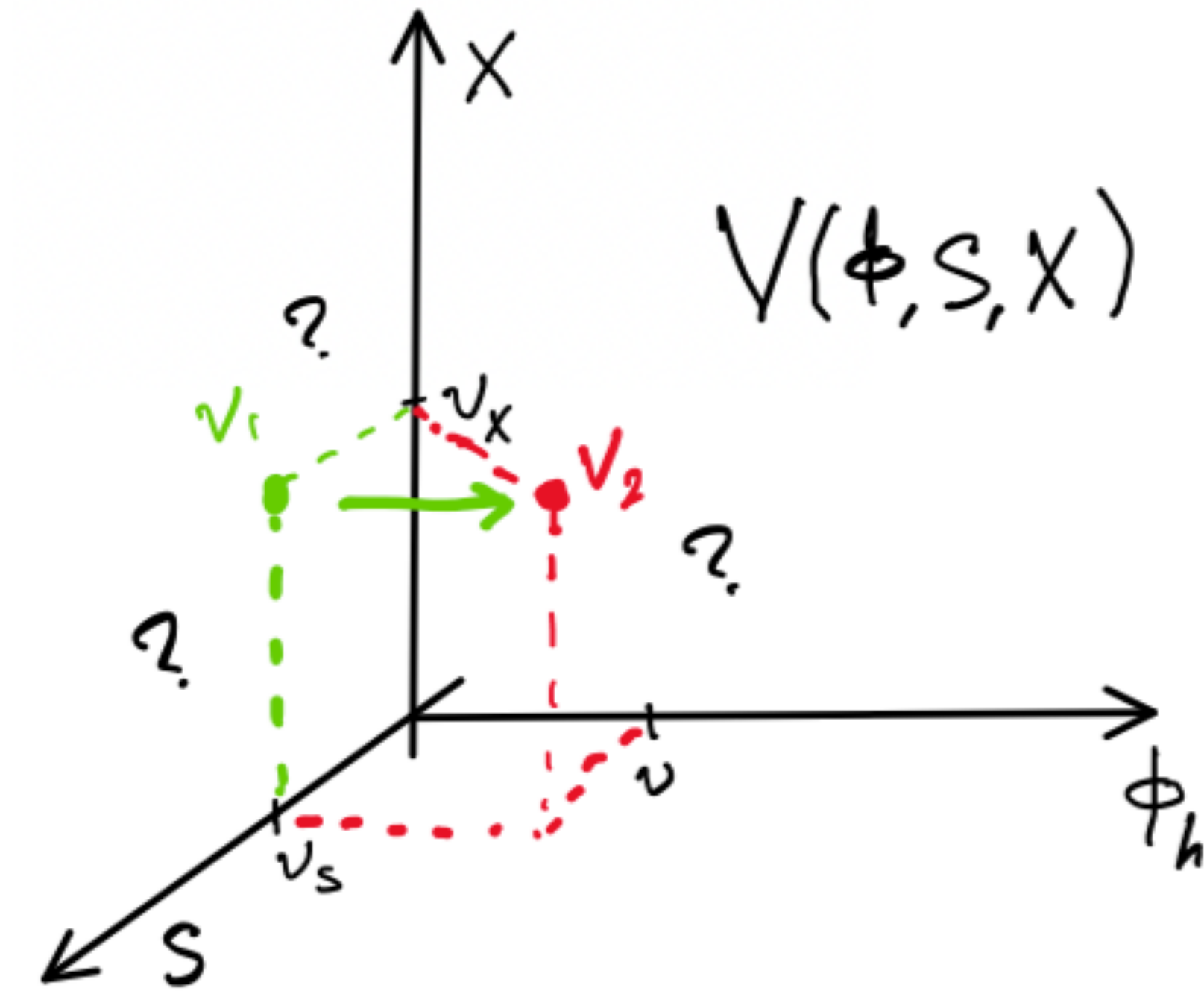
TRSM: HHH production and Higgs FOPT

g

g

h_1

h_1

h_1

h_1

h_2

h_3

$\bar{\lambda}_3$

$\bar{\lambda}_{123}$

$\bar{\lambda}_{112}$
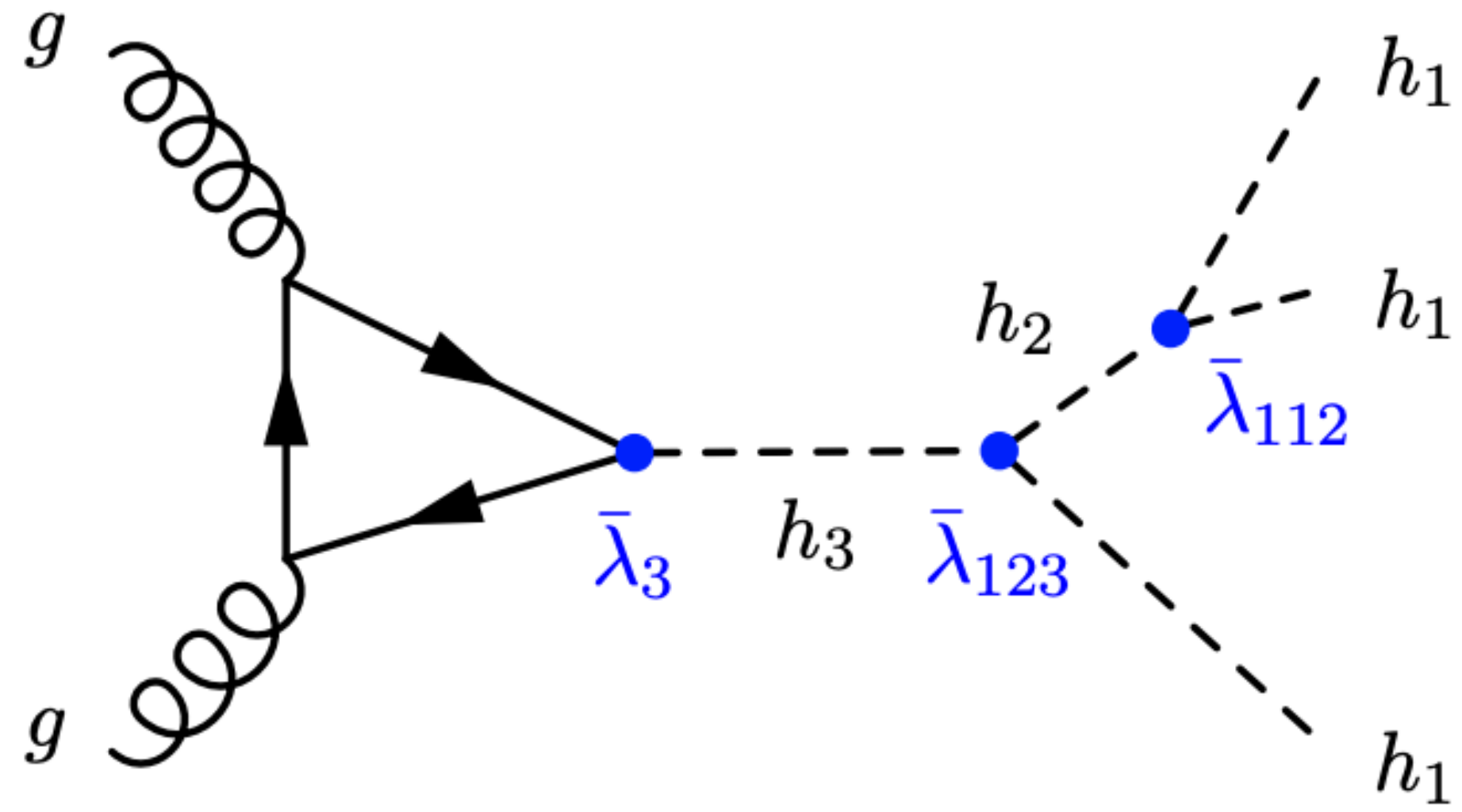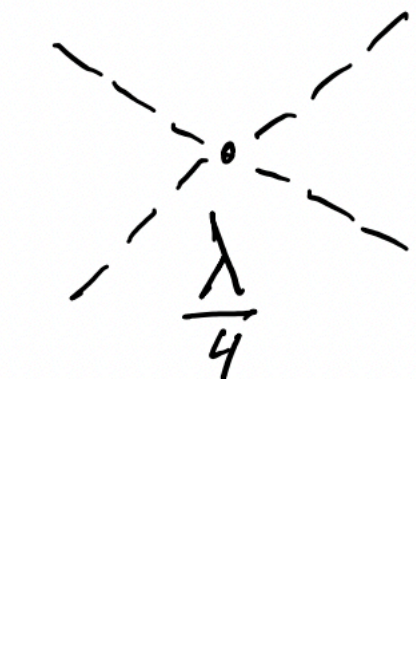
Need nonzero VEV
for two added scalars
for double resonance
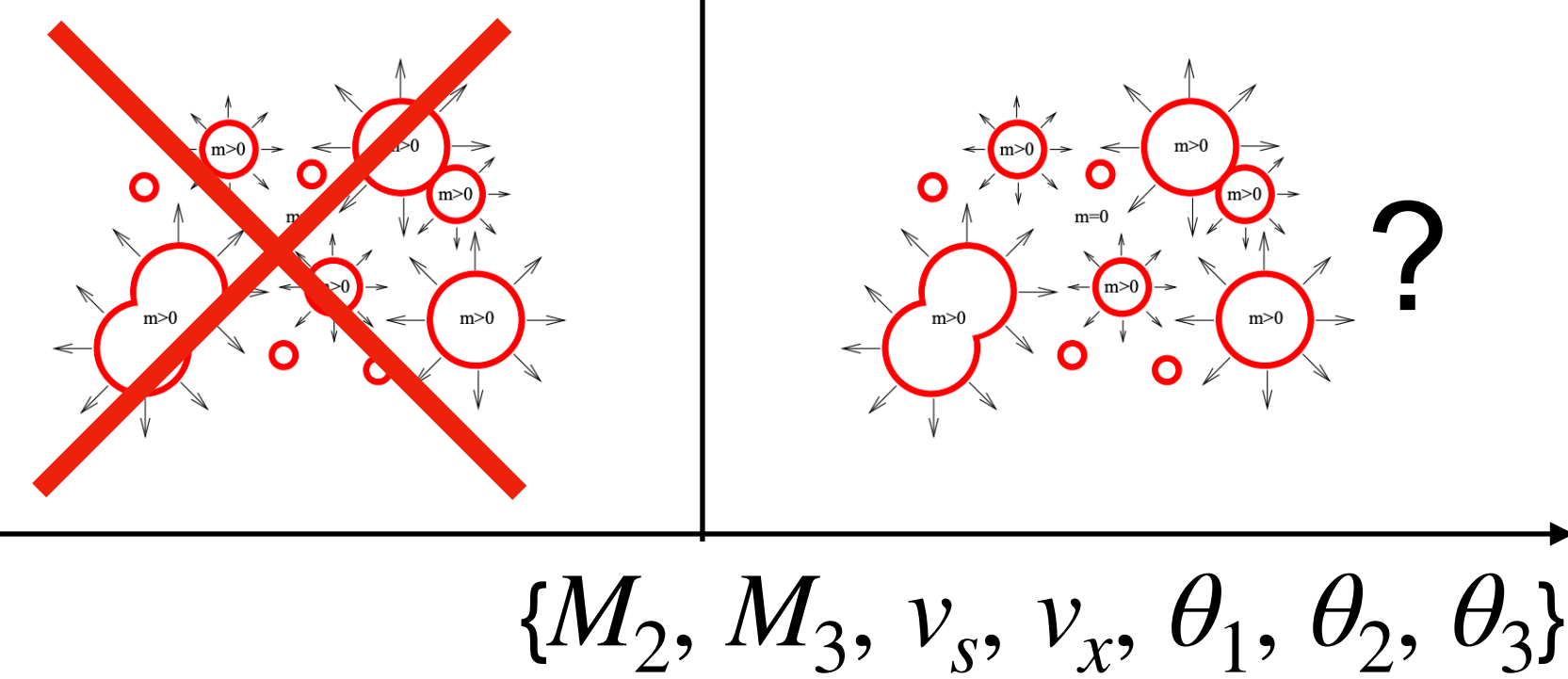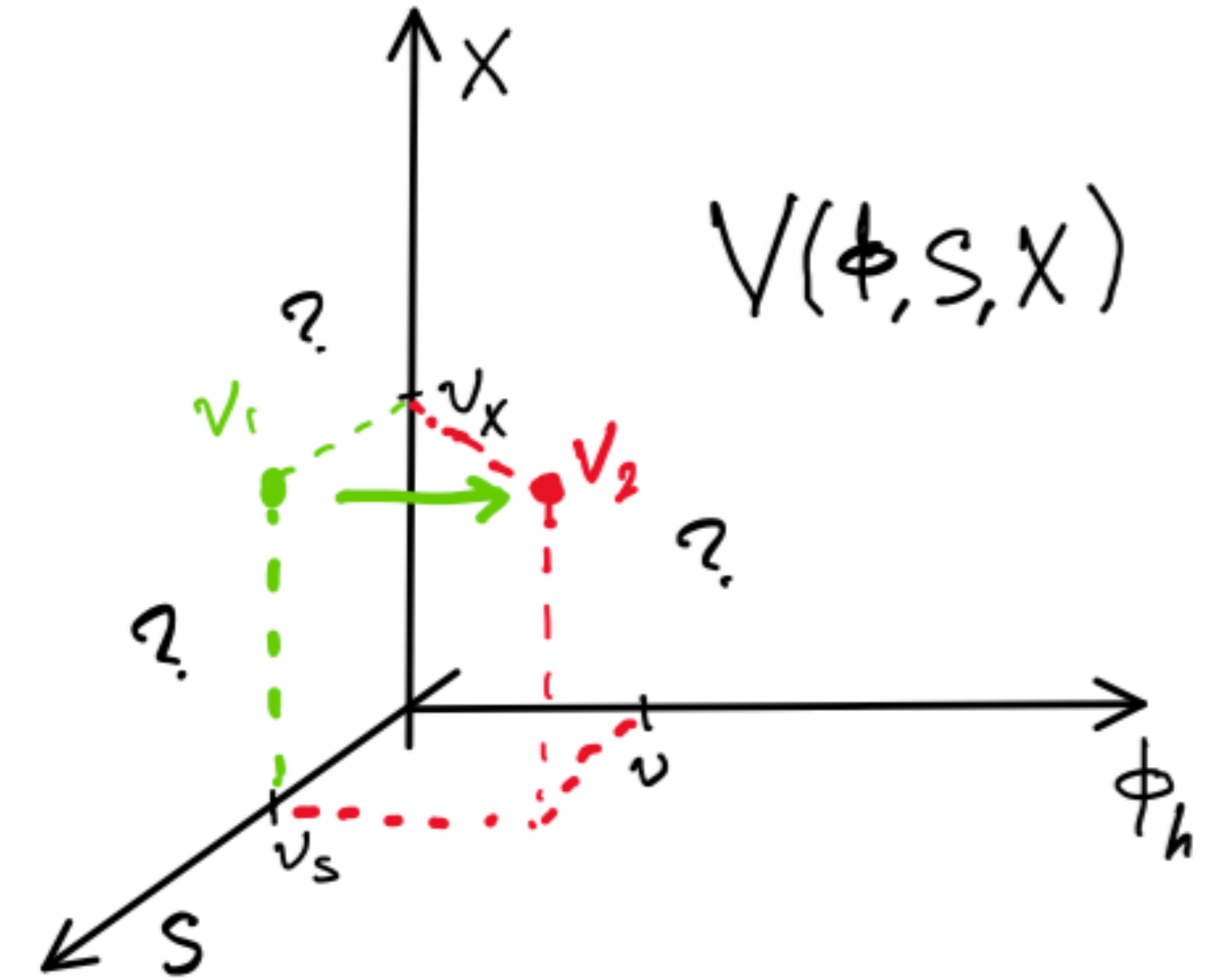
$V(\phi, S, X)$

v_r

v_x

V_2

v_s

v

$\{M_2, M_3, v_s, v_x, \theta_1, \theta_2, \theta_3\}$

PT in TRSM: start with thermal QFT

At LO: only masses get T contribution

$$m_1^2(T) = -\mu_1^2 + \frac{T^2}{48}\left(3g_1^2 + 9g_2^2 + 2(6y_t^2 + 12\lambda_1 + \lambda_{12} + \lambda_{13})\right),$$

$$m_2^2(T) = -\mu_2^2 + \frac{T^2}{24}\left(4\lambda_{12} + \lambda_{23} + 6\lambda_2\right),$$

$$m_3^2(T) = -\mu_3^2 + \frac{T^2}{24}\left(4\lambda_{13} + \lambda_{23} + 6\lambda_3\right),$$

resulting in an *effective* finite-temperature potential:

$$V_{\text{eff,LO}}(\phi_i, T) = \frac{1}{2}\sum_i m_i^2(T)\phi_i^2 + \frac{1}{4}\sum_{i \leq j}\lambda_{ij}\phi_i^2\phi_j^2.$$

PT in TRSM: start with thermal QFT

At LO: only masses get T contribution

$$V(H) = \underset{-\frac{1}{2}\mu^2}{\underbrace{\quad H \quad \cdot \quad H \quad}} + \underset{\frac{\lambda}{4}}{\underbrace{\qquad\qquad}}$$

$$+ \underset{\sim T^2}{\underbrace{\quad H \quad \bigcirc \quad H \quad}}$$

$$m_1^2(T) = -\mu_1^2 + \frac{T^2}{48}\left(3g_1^2 + 9g_2^2 + 2(6y_t^2 + 12\lambda_1 + \lambda_{12} + \lambda_{13})\right),$$

$$m_2^2(T) = -\mu_2^2 + \frac{T^2}{24}\left(4\lambda_{12} + \lambda_{23} + 6\lambda_2\right),$$

$$m_3^2(T) = -\mu_3^2 + \frac{T^2}{24}\left(4\lambda_{13} + \lambda_{23} + 6\lambda_3\right),$$

resulting in an *effective* finite-temperature potential:

$$V_{\text{eff,LO}}(\phi_i, T) = \frac{1}{2}\sum_i m_i^2(T)\phi_i^2 + \frac{1}{4}\sum_{i \leqslant j} \lambda_{ij}\phi_i^2\phi_j^2.$$

Started using Mathematica to numerically solve RGEs

We tried points with large HHH xsec: No FOPT!

What is going on?
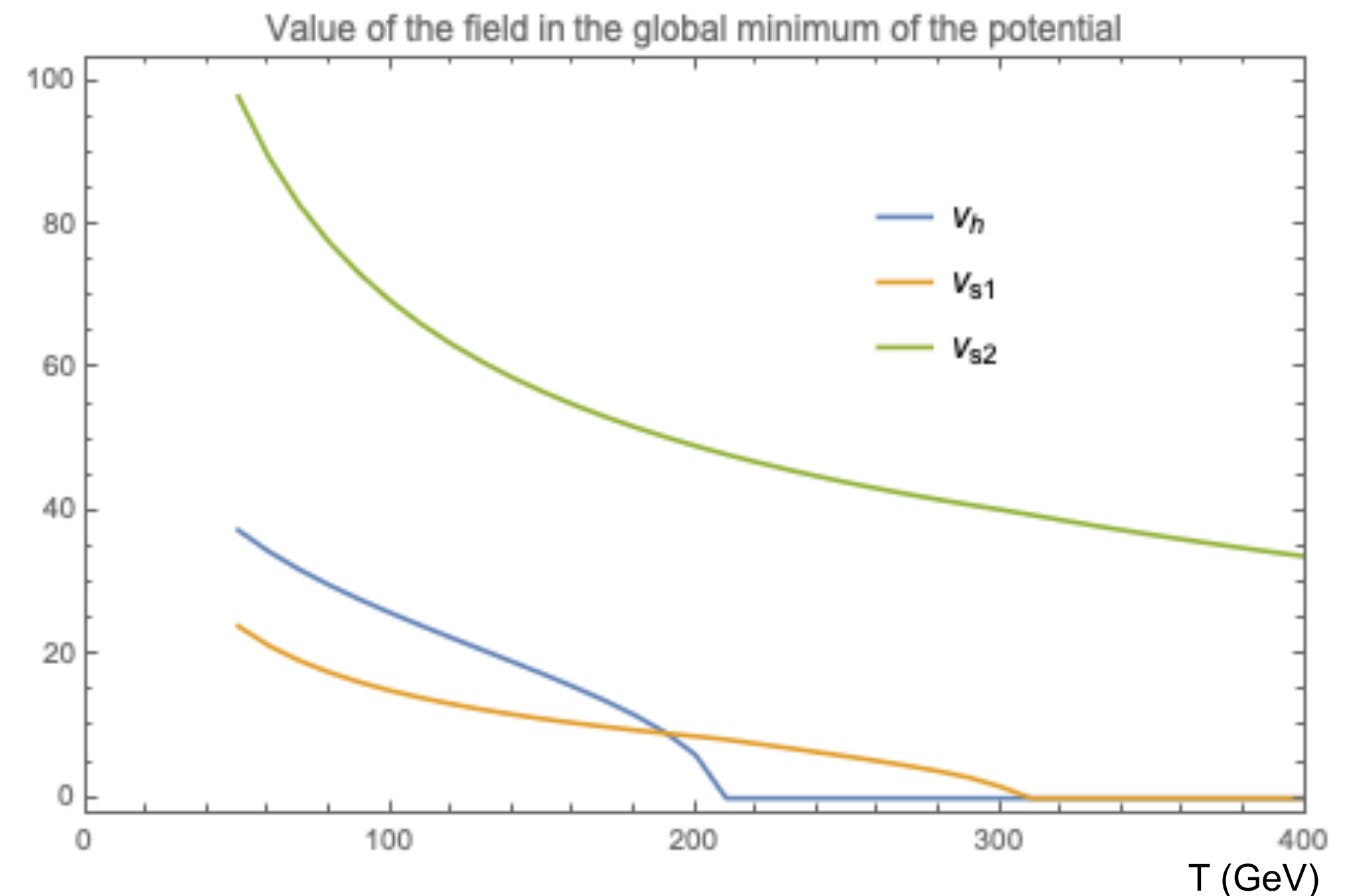
Value of the field in the global minimum of the potential

T (GeV)

PT in TRSM: start with only one scalar

Osama Karkout,[1] Andreas Papaefstathiou,[2] Marieke Postma,[1,3] Gilberto Tetlalmatzi-Xolocotzi,[4,5] Jorinde van de Vis,[6] Tristan du Pree[1]

$$V \in \frac{1}{2}m^2(T)H^2 + \frac{1}{4}\lambda H^4 + \frac{1}{2}m_S^2(T)S^2 + \frac{1}{4}\lambda_S S^4 + \frac{1}{2}\lambda_{HS}H_S^2 S^2$$

Extrema at: $\partial_H V = 0, \quad \partial_S V = 0$

Case 1: $\lambda\lambda_s - \lambda_{HS}^2 < 0$

There is a barrier between two minima:

$(0, v_S)$ and $(v_H, 0)$
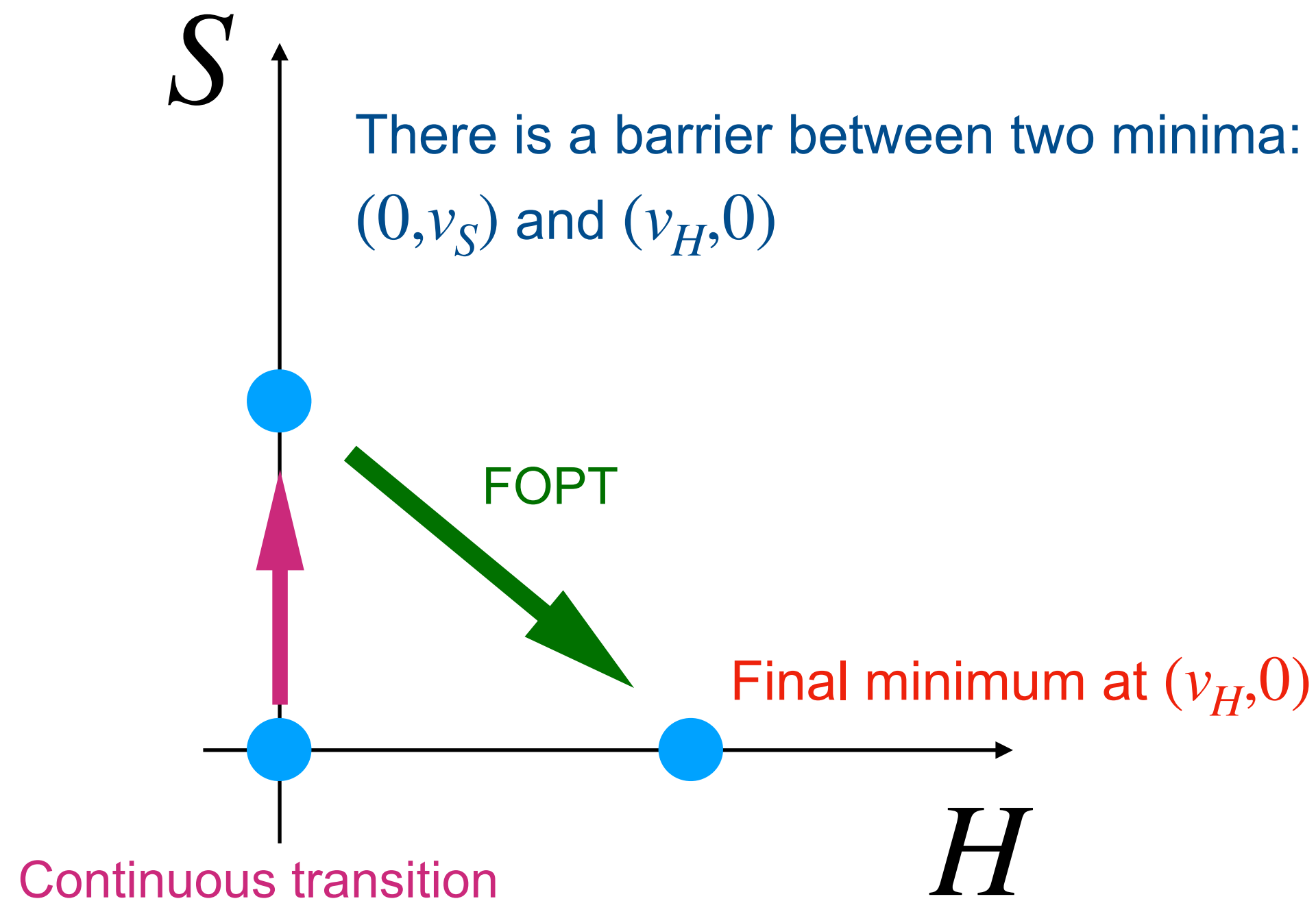
Takeaway: Since there is a barrier between the two axes (fields)
You cannot put a minimum there!
So the field S must end up with a zero VEV
Therefor: No Mixing! No resonant HH production!

S

FOPT

Final minimum at $(v_H, 0)$

Continuous transition

H

PT in TRSM: start with only one scalar

Osama Karkout,[1] Andreas Papaefstathiou,[2] Marieke Postma,[1,3] Gilberto Tetlalmatzi-Xolocotzi,[4,5] Jorinde van de Vis,[6] Tristan du Pree[1]

$$V \in \frac{1}{2}m^2(T)H^2 + \frac{1}{4}\lambda H^4 + \frac{1}{2}m_S^2(T)S^2 + \frac{1}{4}\lambda_S S^4 + \frac{1}{2}\lambda_{HS}H_S^2 S^2$$

Extrema at: $\partial_H V = 0, \quad \partial_S V = 0$

Case 1: $\lambda\lambda_s - \lambda_{HS}^2 < 0$

Case 2: $\lambda\lambda_s - \lambda_{HS}^2 > 0$

S

There is a barrier between two minima:

$(0, v_S)$ and $(v_H, 0)$

FOPT

Final minimum at $(v_H, 0)$

Continuous transition

H

Case 1: Yes FOPT! No resonant HH production!

S

Each extrema 'comes out' of a previous one: only one minimum at a time, no barriers

Final minimum at (v_H, v_S)

Continuous transitions

H
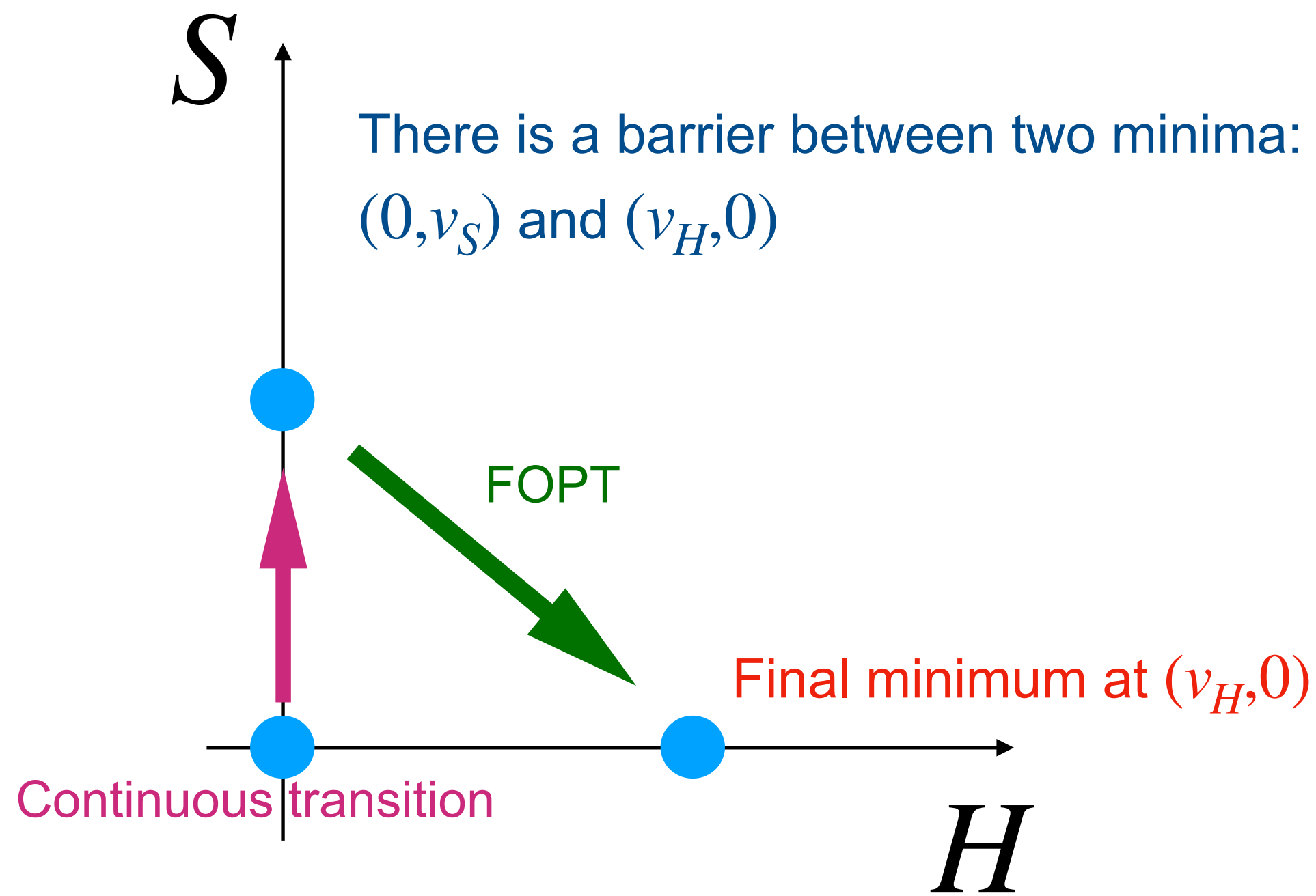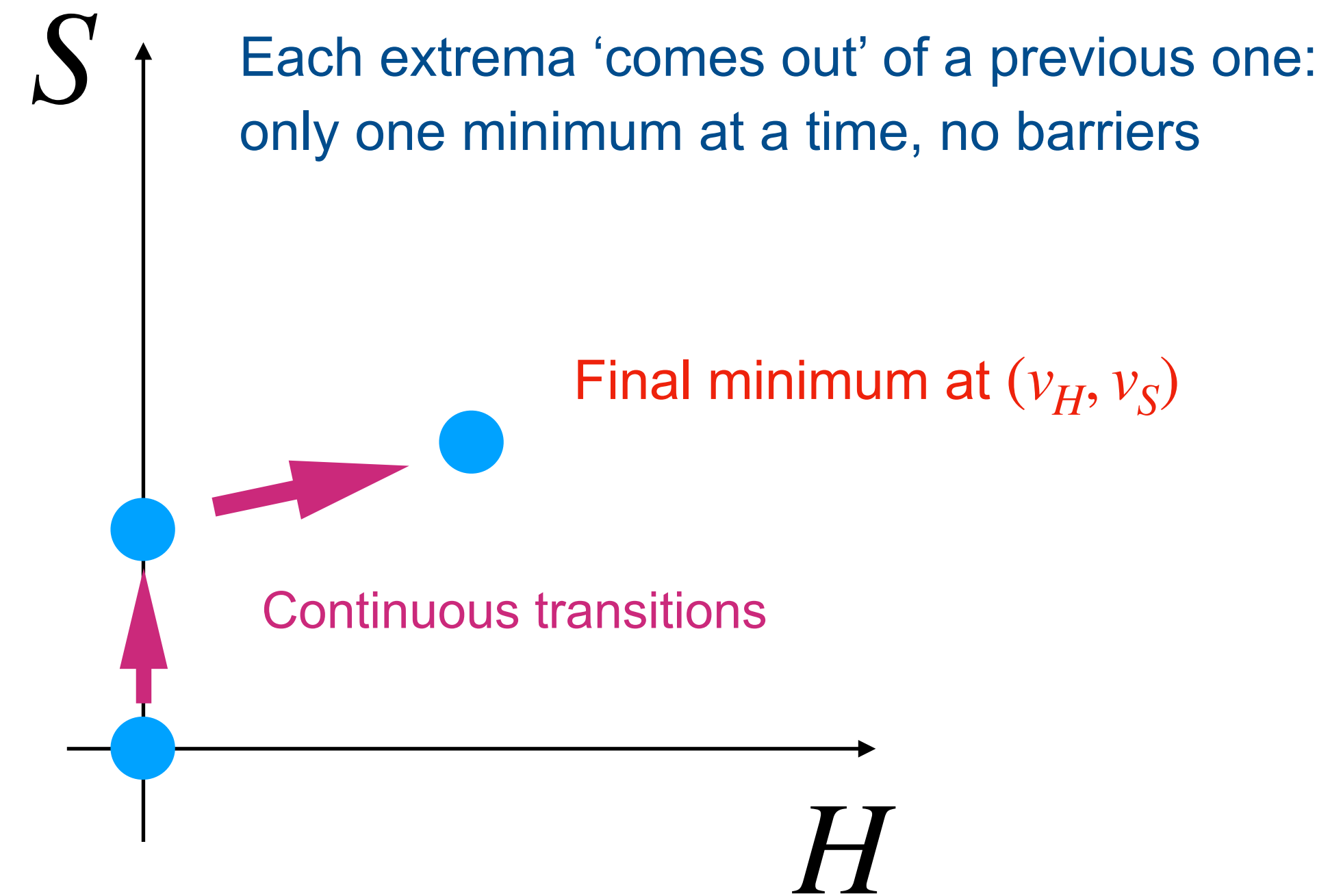
Case 2: NO FOPT! Yes resonant HH production!

PT in TRSM: with both scalars

Osama Karkout,[1] Andreas Papaefstathiou,[2] Marieke Postma,[1,3] Gilberto Tetlalmatzi-Xolocotzi,[4,5] Jorinde van de Vis,[6] Tristan du Pree[1]

Call the fields x_i

$$V(x_1, x_2, x_3) = \frac{1}{2}\sum_i m_i^2 x_i^2 + \frac{1}{4}\sum_{i,j} c_{ij} x_i^2 x_j^2,$$

Find all extrema by taking $\partial_i V = 0$

- Origin: $\mathbf{x}_0 \equiv (0,0,0)$.

- Axial extremum $\mathbf{x}_1 \equiv (x_1, 0, 0)$ with

$$x_1 = \sqrt{-m_1^2/c_{11}}.$$

- Planar extremum $\mathbf{x}_{12} \equiv (x_1, x_2, 0)$ with

$$x_1 = \sqrt{\frac{c_{12}m_2^2 - c_{22}m_1^2}{c_{11}c_{22} - c_{12}^2}}, \quad x_2 = \sqrt{\frac{c_{12}m_1^2 - c_{11}m_2^2}{c_{11}c_{22} - c_{12}^2}}.$$

- Bulk extremum $\mathbf{x}_{123} \equiv (x_1, x_2, x_3)$ with

$$x_1 = \frac{\sqrt{(c_{23}^2 - c_{22}c_{33})m_1^2 + (c_{12}c_{33} - c_{13}c_{23})m_2^2 + (c_{13}c_{22} - c_{12}c_{23})m_3^2}}{\sqrt{D}},$$

$$x_2 = \frac{\sqrt{(c_{12}c_{33} - c_{13}c_{23})m_1^2 + (c_{13}^2 - c_{11}c_{33})m_2^2 + (c_{11}c_{23} - c_{12}c_{13})m_3^2}}{\sqrt{D}},$$

$$x_3 = \frac{\sqrt{(c_{13}c_{22} - c_{12}c_{23})m_1^2 + (c_{11}c_{23} - c_{12}c_{13})m_2^2 + (c_{12}^2 - c_{11}c_{22})m_3^2}}{\sqrt{D}},$$

where

$$D = c_{11}c_{22}c_{33} + 2c_{12}c_{13}c_{23} - c_{13}^2 c_{22} - c_{11}c_{23}^2 - c_{12}^2 c_{33},$$

is the determinant of c_{ij}.

PT in TRSM: with both scalars

Osama Karkout,[1] Andreas Papaefstathiou,[2] Marieke Postma,[1,3] Gilberto Tetlalmatzi-Xolocotzi,[4,5] Jorinde van de Vis,[6] Tristan du Pree[1]
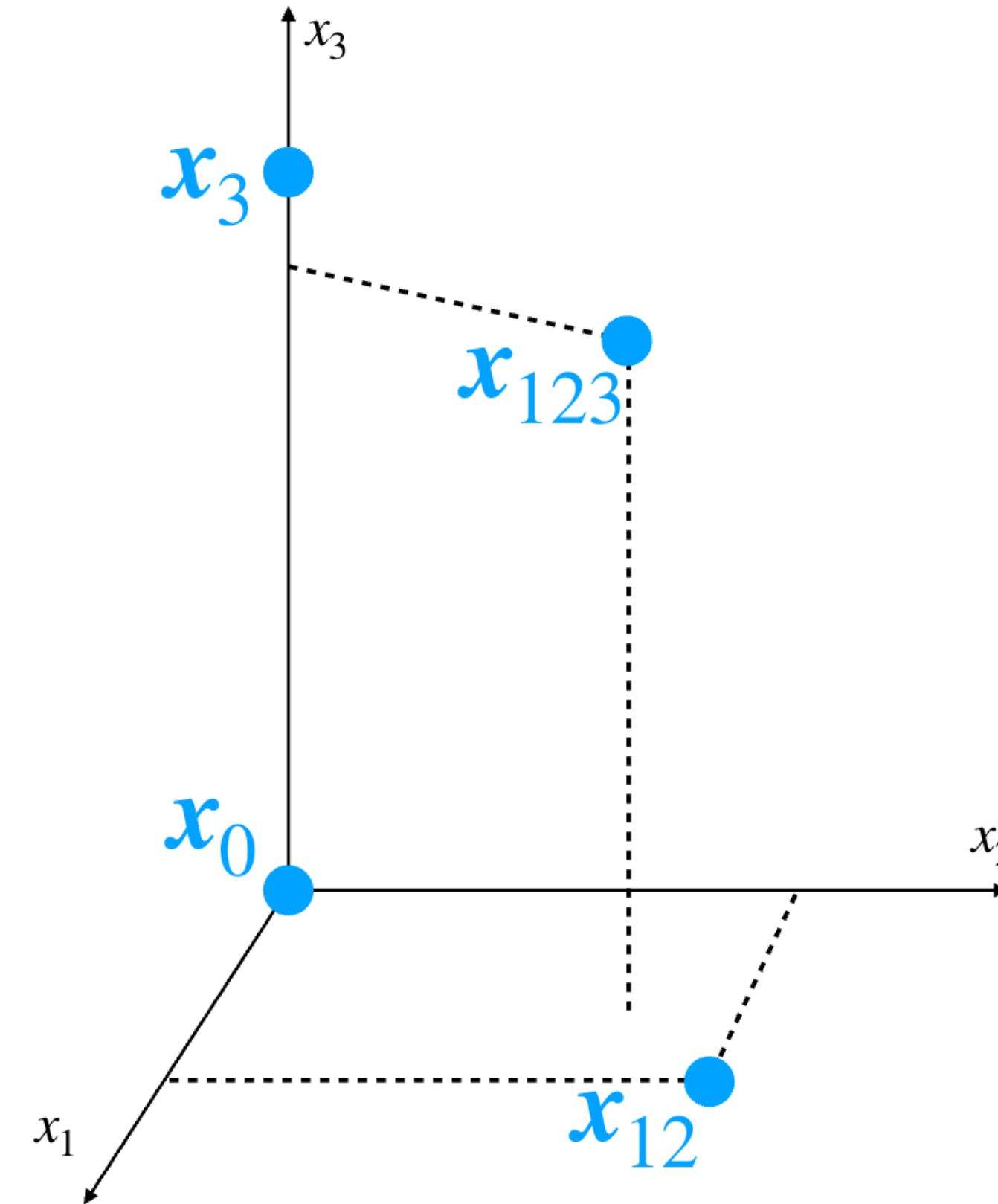
The extremum is a minimum if the eigenvalues of the Hessian of the potential h_{kl}, i.e. the mass matrix, evaluated at the extremum are all positive, with

$$h_{kl}(x_1, x_2, x_3) \equiv \partial_{x_k} \partial_{x_l} V(x_1, x_2, x_3) = (m_k^2 + \sum_i c_{ik} x_i^2)\delta_{kl} + 2c_{kl}x_k x_l. \qquad (4.16)$$

```
]:= curve = Simplify[Eigenvalues[Simplify[hessian /. Solutions[[16]]]]]
```


Full expression not available (original memory size: 5.8 MB)

Not even mathematica could help… insight needed.

PT in TRSM: with both scalars

Osama Karkout,[1] Andreas Papaefstathiou,[2] Marieke Postma,[1,3] Gilberto Tetlalmatzi-Xolocotzi,[4,5] Jorinde van de Vis,[6] Tristan du Pree[1]

$$V(x_1, x_2, x_3) = \frac{1}{2}\sum_i m_i^2 x_i^2 + \frac{1}{4}\sum_{i,j} c_{ij} x_i^2 x_j^2,$$

Insights:

- Z_2 symmetry: $(x \to -x)$ does not change the potential! I can focus on the positive x_i and generalise.

- The shape of the potential (whether an extremum is minimum) does not change if I scale the axes: $x^2 \to x$

Now the Hessian is simple: $h_{kl}(x_1, x_2, x_3) \equiv \partial_{x_k}\partial_{x_l} V(x_1, x_2, x_3) = \frac{1}{2}c_{kl}.$

PT in TRSM: with both scalars

Osama Karkout,[1] Andreas Papaefstathiou,[2] Marieke Postma,[1,3] Gilberto Tetlalmatzi-Xolocotzi,[4,5] Jorinde van de Vis,[6] Tristan du Pree[1]
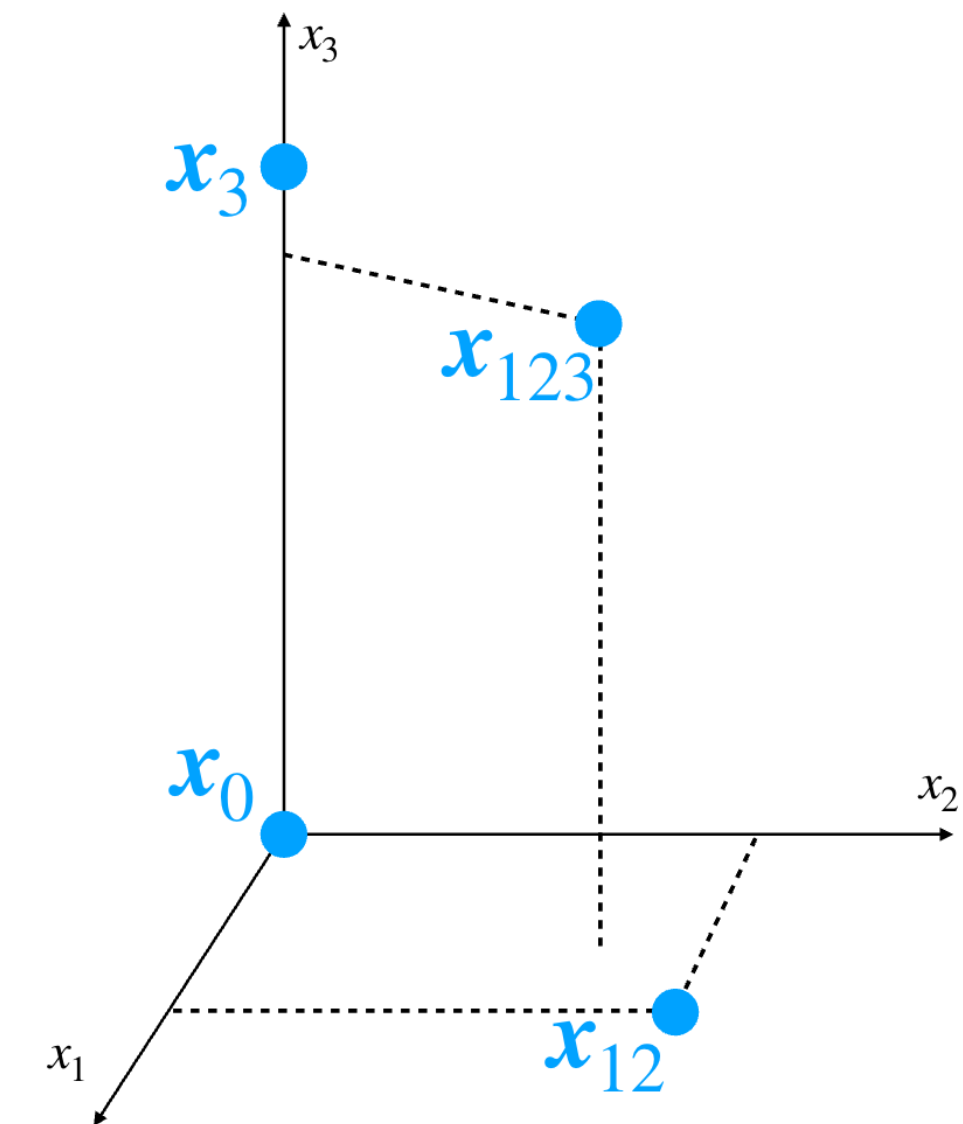
$$V(x_1, x_2, x_3) = \frac{1}{2}\sum_i m_i^2 x_i^2 + \frac{1}{4}\sum_{i,j} c_{ij} x_i^2 x_j^2,$$

Insights:

- Z_2 symmetry: $(x \to -x)$ does not change the potential! I can focus on the positive x_i and generalise.

- The shape of the potential (whether an extremum is minimum) does not change if I scale the axes: $x^2 \to x$

Now the Hessian is simple:

$$h_{kl}(x_1, x_2, x_3) \equiv \partial_{x_k}\partial_{x_l}V(x_1, x_2, x_3) = \frac{1}{2}c_{kl}.$$

For resonant HHH:

We demand that $\mathbf{x}_{123}$ is today's vacuum. The eigenvalues of the rescaled Hessian should then be positive. Sylvester's criterion, stating that a square Hermitian matrix is positive definite if *and only if* all the leading principal minors are positive, then gives

$$c_{ii} > 0, \quad \& \quad C_{ij} \equiv c_{ii}c_{jj} - c_{ij}^2 > 0, \quad \& \quad D > 0, \tag{4.18}$$

where

$$D = c_{11}c_{22}c_{33} + 2c_{12}c_{13}c_{23} - c_{13}^2 c_{22} - c_{11}c_{23}^2 - c_{12}^2 c_{33},$$

is the determinant of c_{ij}.

PT in TRSM: with both scalars

Osama Karkout,[1] Andreas Papaefstathiou,[2] Marieke Postma,[1,3] Gilberto Tetlalmatzi-Xolocotzi,[4,5] Jorinde van de Vis,[6] Tristan du Pree[1]
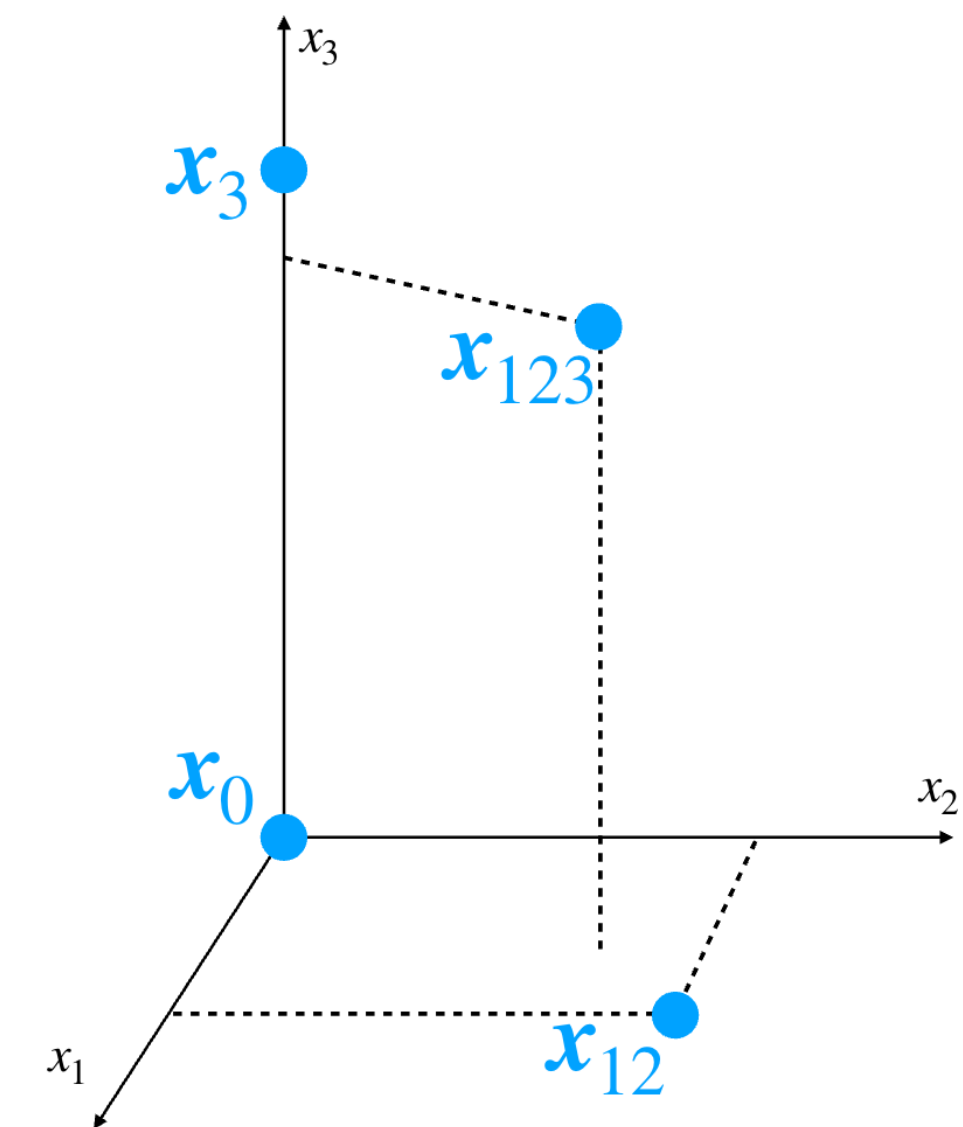
$$V(x_1, x_2, x_3) = \frac{1}{2}\sum_i m_i^2 x_i^2 + \frac{1}{4}\sum_{i,j} c_{ij} x_i^2 x_j^2,$$

$$c_{ii} > 0, \quad \& \quad \boxed{C_{ij} \equiv c_{ii}c_{jj} - c_{ij}^2 > 0, \quad \& \quad D > 0,}$$
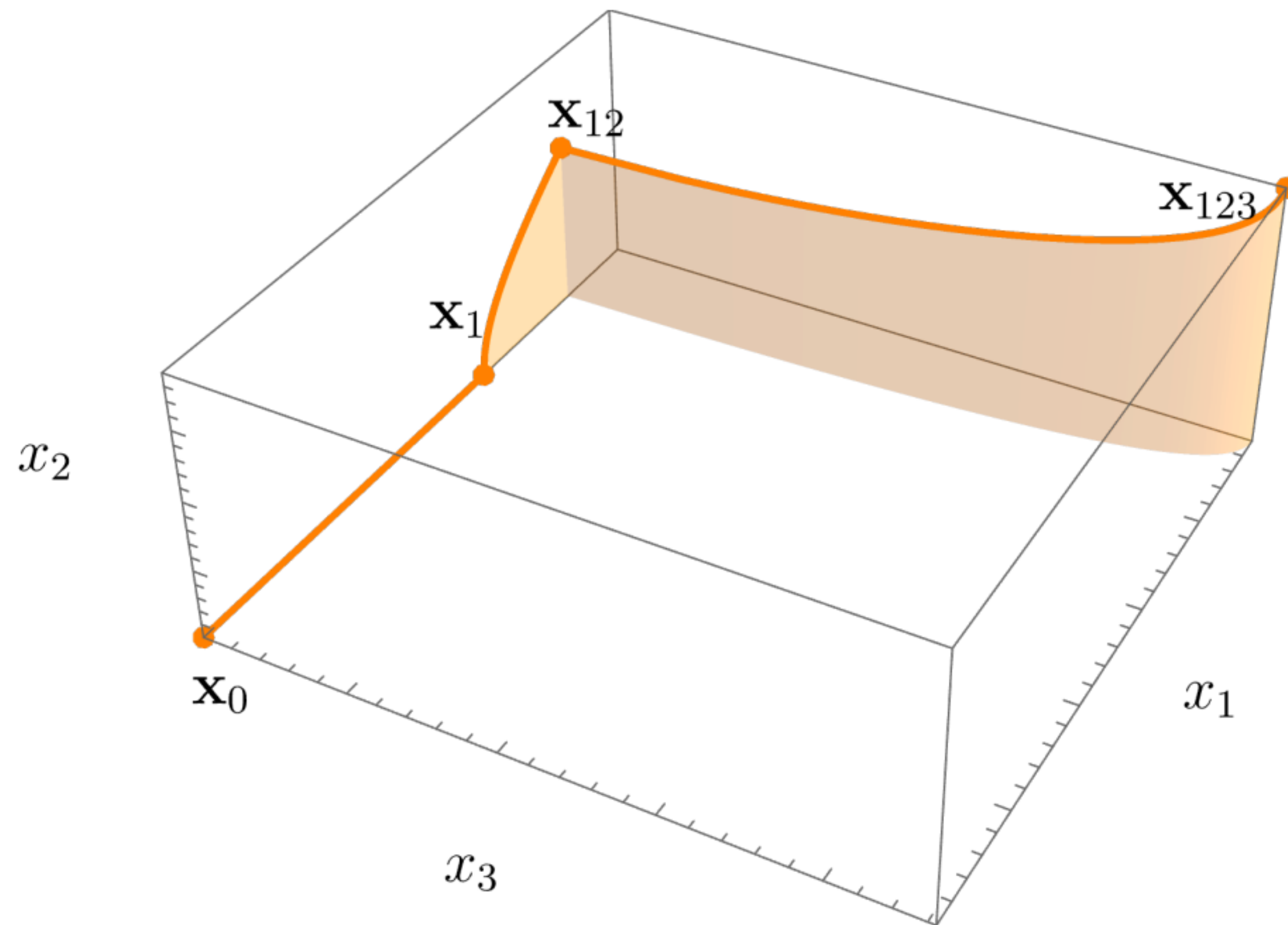
Each extrema 'comes out' of a previous one:
only one minimum at a time, no barriers

Final minimum at (v_H, v_S, v_x)

Continuous transitions

Case 2: NO FOPT! Yes resonant HHH production!

PT in TRSM: with both scalars

Osama Karkout,[1] Andreas Papaefstathiou,[2] Marieke Postma,[1,3] Gilberto Tetlalmatzi-Xolocotzi,[4,5] Jorinde van de Vis,[6] Tristan du Pree[1]

$$V(x_1, x_2, x_3) = \frac{1}{2} \sum_i m_i^2 x_i^2 + \frac{1}{4} \sum_{i,j} c_{ij} x_i^2 x_j^2,$$

$$c_{ii} > 0, \quad \& \quad \boxed{C_{ij} \equiv c_{ii} c_{jj} - c_{ij}^2 > 0, \quad \& \quad D > 0,}$$
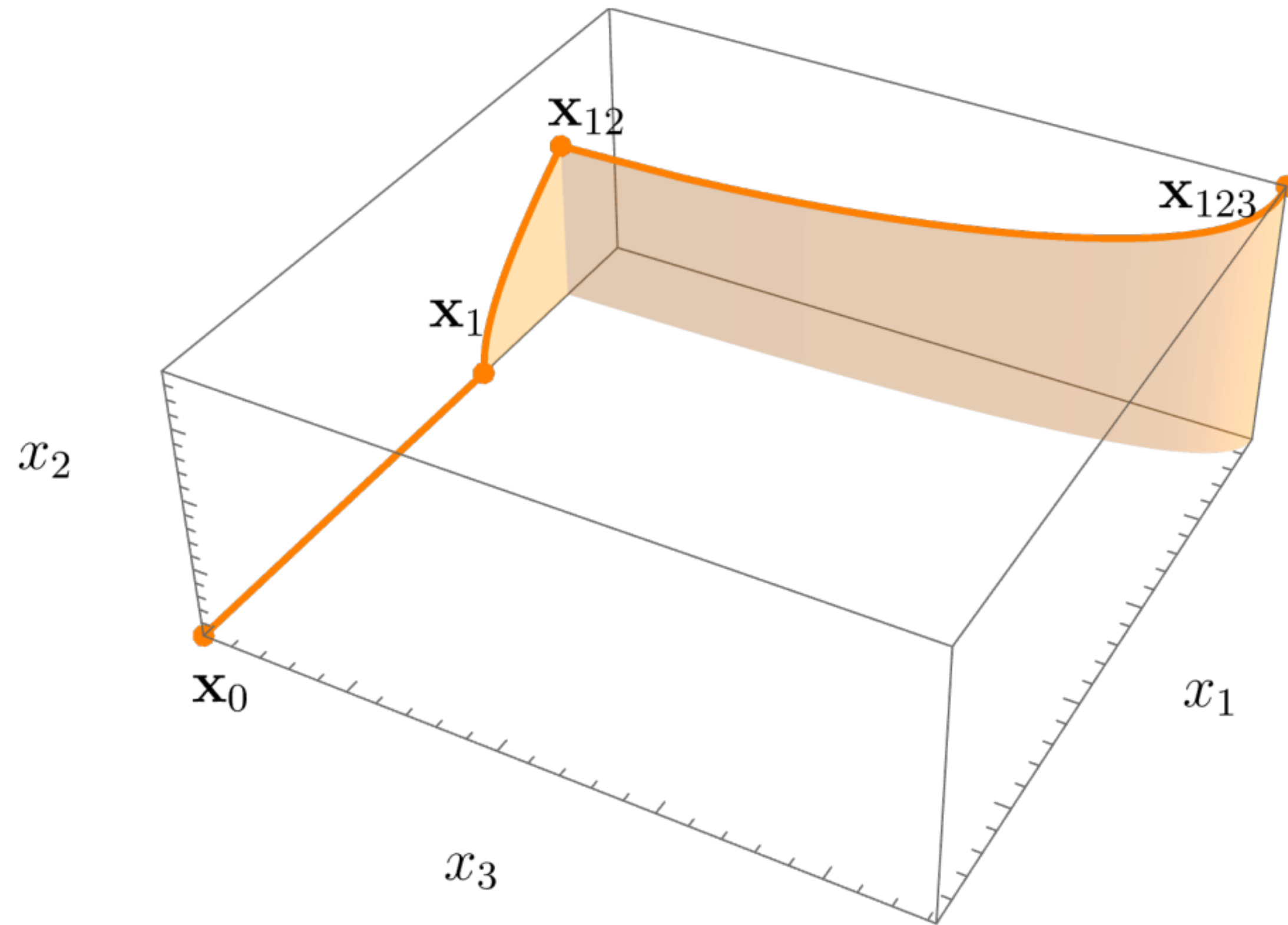
Going to NLO numerically solidifies the conclusion
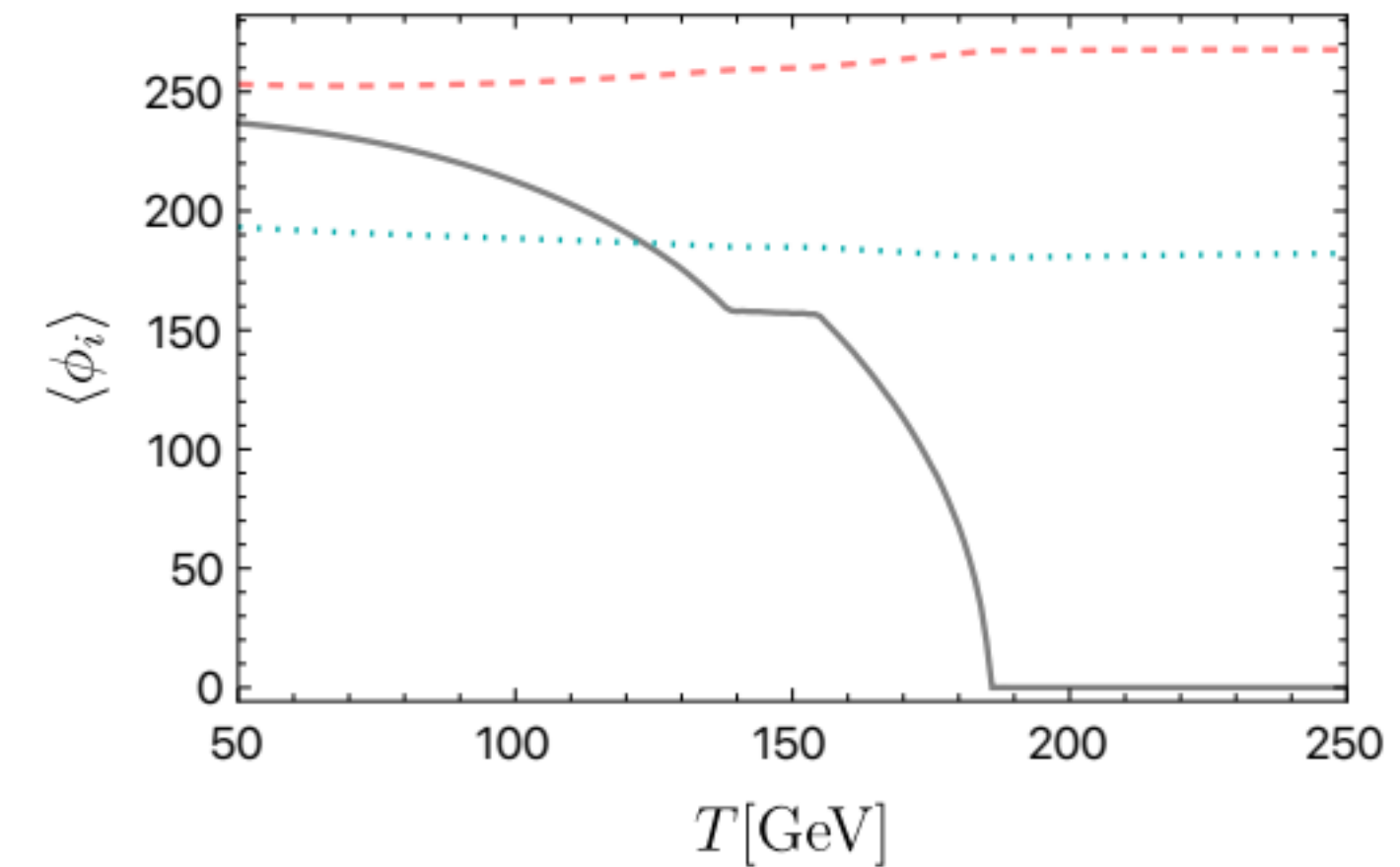
Figure 5: Evolution of the field expectation values in the minimum of the potential for the third BM point in Table 2. The Higgs field is represented by gray solid, ϕ_2 by dashed pink, and ϕ_3 by dotted cyan.

Case 2: NO FOPT! Yes resonant HHH production!

PT in TRSM: with both scalars

Osama Karkout,[1] Andreas Papaefstathiou,[2] Marieke Postma,[1,3] Gilberto Tetlalmatzi-Xolocotzi,[4,5] Jorinde van de Vis,[6] Tristan du Pree[1]

$$V(x_1, x_2, x_3) = \frac{1}{2}\sum_i m_i^2 x_i^2 + \frac{1}{4}\sum_{i,j} c_{ij} x_i^2 x_j^2,$$

$$c_{ii} > 0, \quad \& \quad C_{ij} \equiv c_{ii}c_{jj} - c_{ij}^2 > 0, \quad \& \quad D > 0,$$

$$\boxed{D < 0}$$

Final minimum at $(v_H, v_s, 0)$

$\mathbf{X}_{12}$

First-Order Phase Transition

x_2

$\mathbf{X}_0$

x_3

x_1

$\mathbf{X}_3$

Case 1: Yes FOPT! No resonant HHH production!

PT in TRSM: with both scalars

Osama Karkout,[1] Andreas Papaefstathiou,[2] Marieke Postma,[1,3] Gilberto Tetlalmatzi-Xolocotzi,[4,5] Jorinde van de Vis,[6] Tristan du Pree[1]

$$V(x_1, x_2, x_3) = \frac{1}{2}\sum_i m_i^2 x_i^2 + \frac{1}{4}\sum_{i,j} c_{ij} x_i^2 x_j^2,$$

$$c_{ii} > 0, \quad \& \quad C_{ij} \equiv c_{ii}c_{jj} - c_{ij}^2 > 0, \quad \& \quad D \; \cancel{>} \; 0,$$

$$\boxed{D < 0}$$
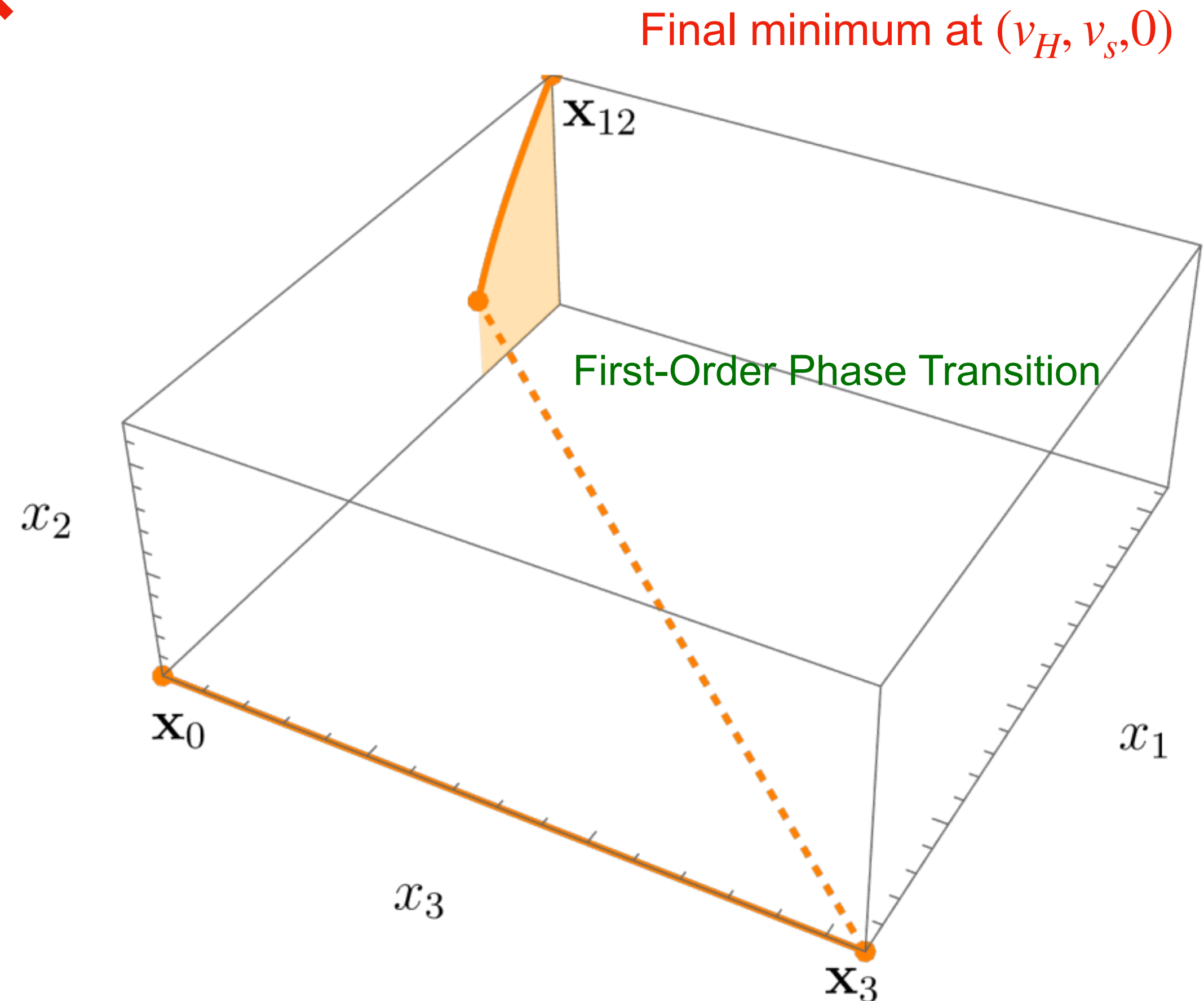
Final minimum at $(v_H, v_s, 0)$

Nightmare!
If we want FOPT,
we cannot detect it with HHH

First-Order Phase Transition

Case 1: Yes FOPT! No resonant HHH production!

PT in TRSM: with both scalars

Osama Karkout,[1] Andreas Papaefstathiou,[2] Marieke Postma,[1,3] Gilberto Tetlalmatzi-Xolocotzi,[4,5] Jorinde van de Vis,[6] Tristan du Pree[1]
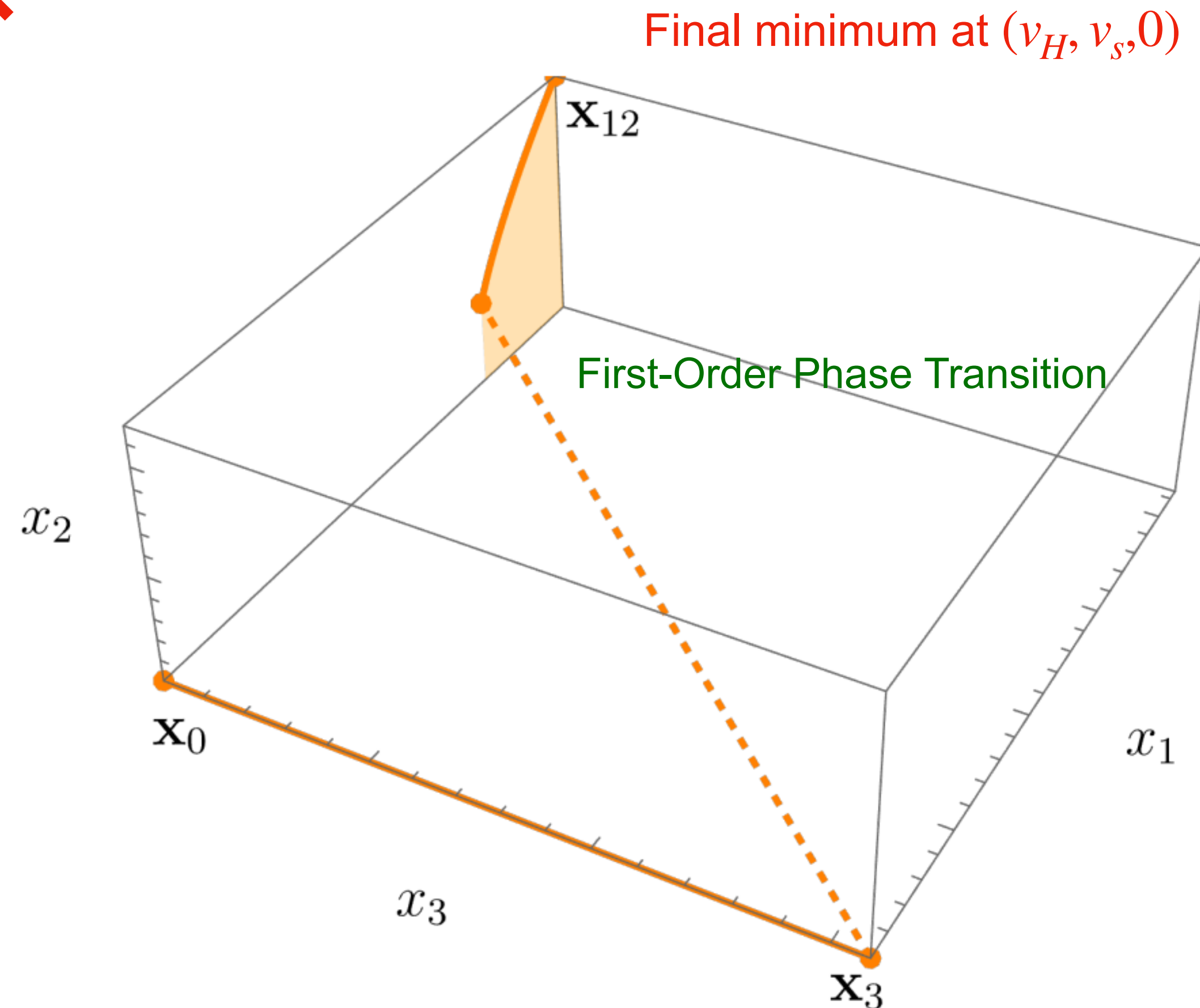
$$V(x_1, x_2, x_3) = \frac{1}{2}\sum_i m_i^2 x_i^2 + \frac{1}{4}\sum_{i,j} c_{ij} x_i^2 x_j^2,$$

$$c_{ii} > 0, \quad \& \quad C_{ij} \equiv c_{ii}c_{jj} - c_{ij}^2 > 0, \quad \& \quad D > 0,$$

$$\boxed{D < 0}$$

Silver lining:

Final minimum at $(v_H, v_s, 0) \rightarrow$ resonant HH production!!

Final minimum at $(v_H, v_s, 0)$

First-Order Phase Transition
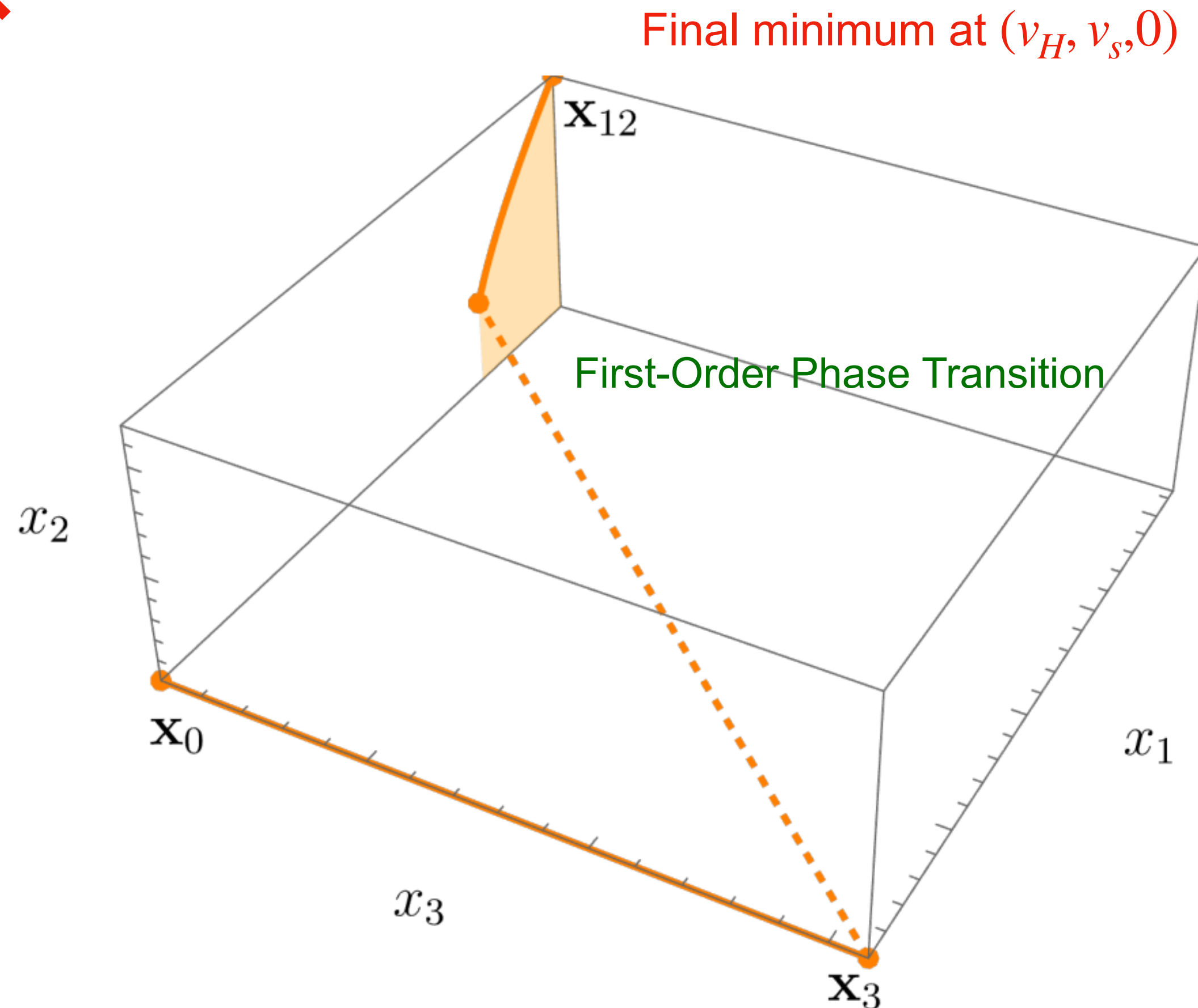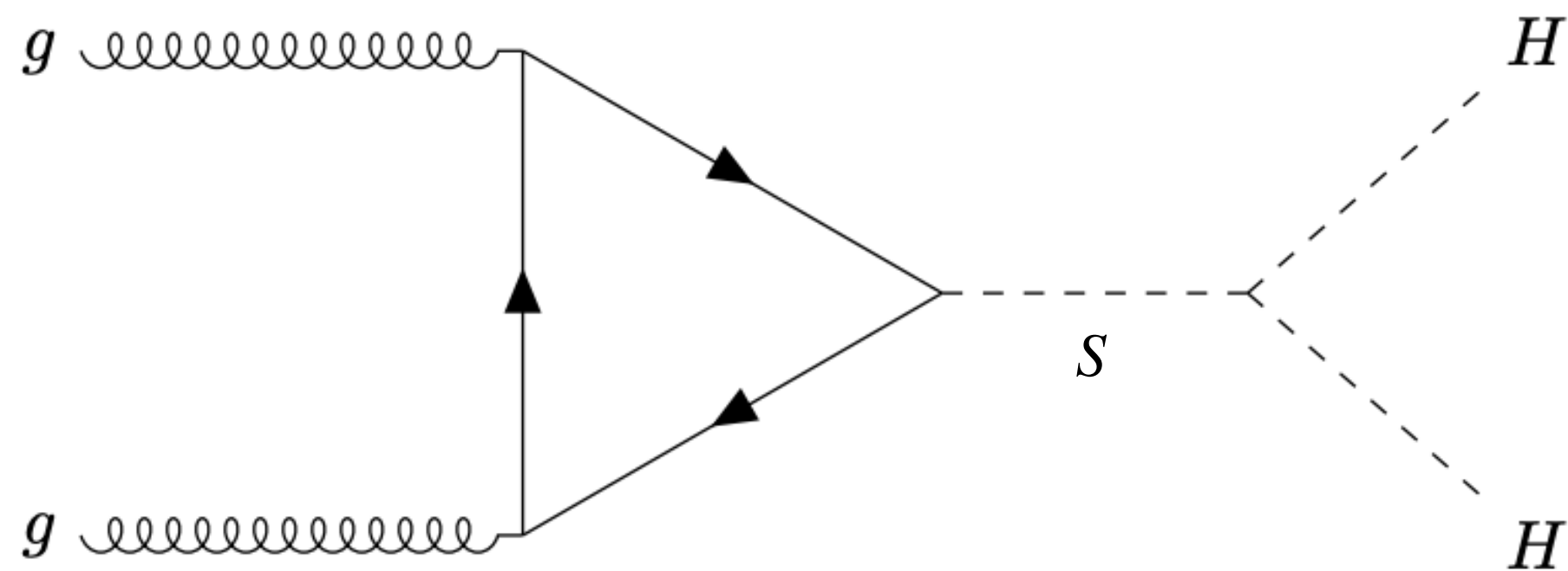
TRSM can accommodate both HH enhancement and FOPT!

Final minimum at $(v_H, v_s, 0)$

First-Order Phase Transition

x_2

x_1

x_3

$\mathbf{x}_{12}$

$\mathbf{x}_0$

$\mathbf{x}_3$

ATLAS

$\sqrt{s}$ = 13 TeV, 139 fb^{-1}

95% CL limits on σ (pp $\rightarrow$ X $\rightarrow$ HH) [fb]

m_X [GeV]

---- Comb. Exp.	---- $\tau_{lep}\tau_{had}$ Exp.
—•— Comb. Obs.	—•— $\tau_{lep}\tau_{had}$ Obs.
▇ Comb. Exp. ±1σ	---- $\tau_{had}\tau_{had}$ Exp.
▇ Comb. Exp. ±2σ	—•— $\tau_{had}\tau_{had}$ Obs.

?

Impossible for only one added scalar

PT in TRSM: with both scalars

Osama Karkout,[1] Andreas Papaefstathiou,[2] Marieke Postma,[1,3] Gilberto
Tetlalmatzi-Xolocotzi,[4,5] Jorinde van de Vis,[6] Tristan du Pree[1]

$$V(x_1, x_2, x_3) = \frac{1}{2}\sum_i m_i^2 x_i^2 + \frac{1}{4}\sum_{i,j} c_{ij} x_i^2 x_j^2,$$

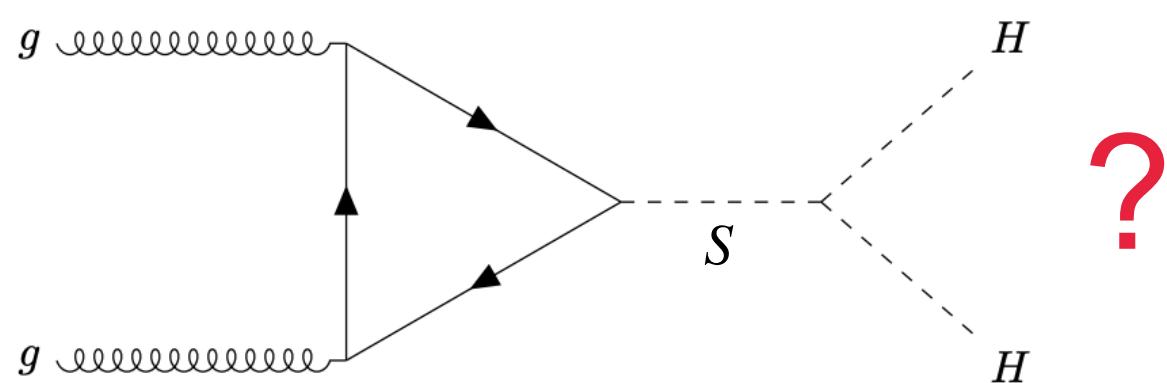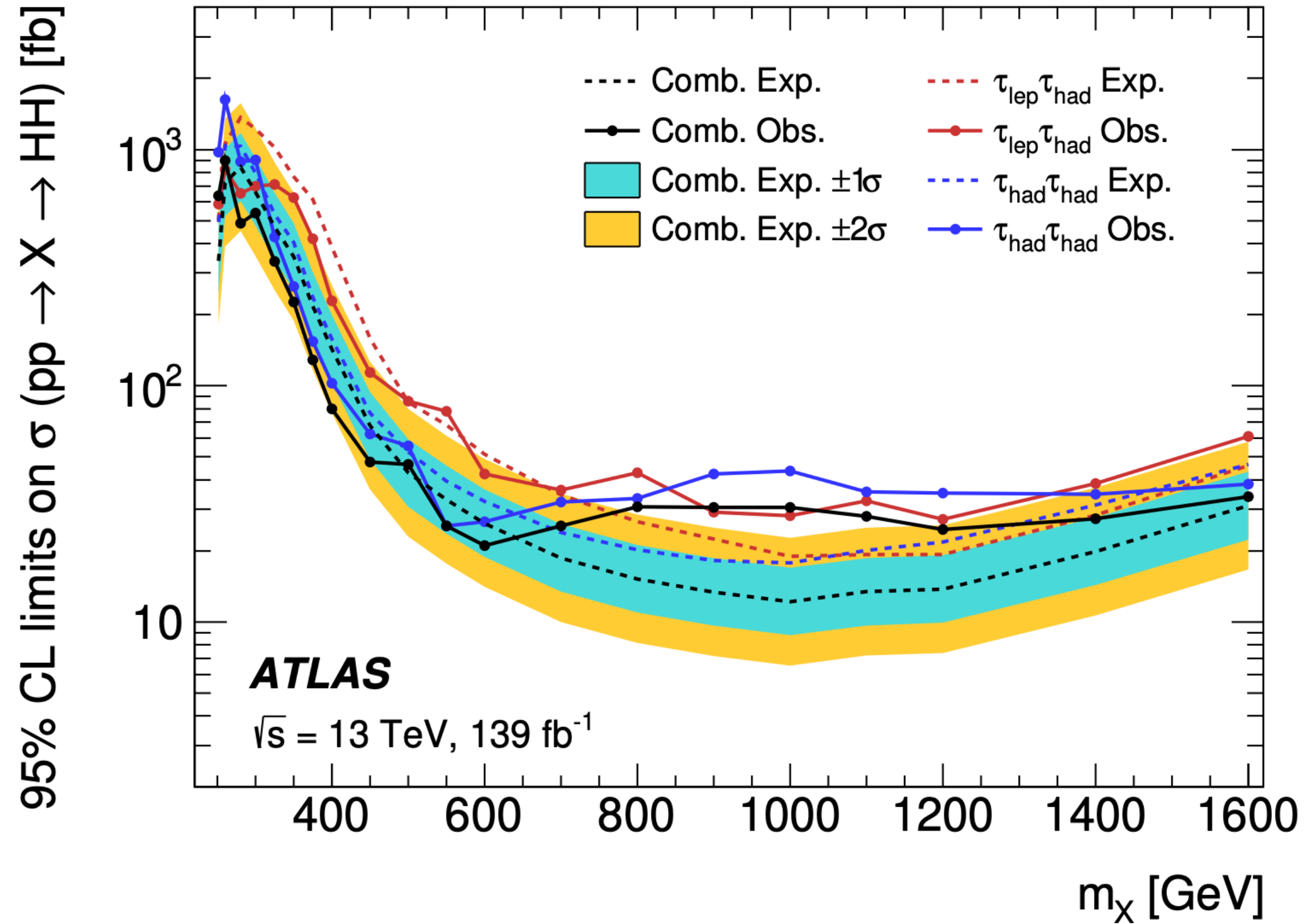$$c_{ii} > 0, \quad \& \quad C_{ij} \equiv c_{ii}c_{jj} - c_{ij}^2 > 0, \quad \& \quad D > 0,$$
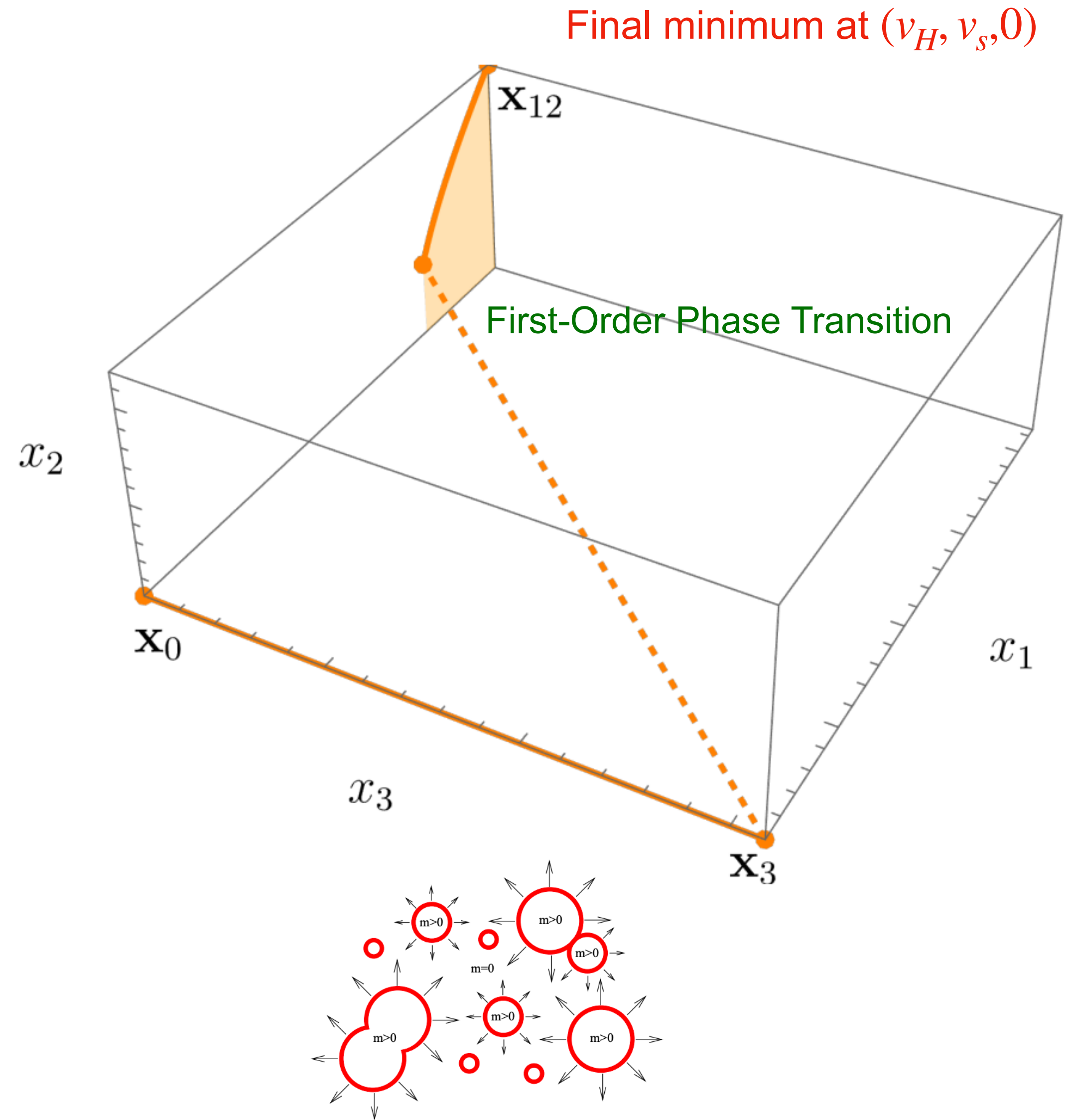
$$\boxed{D < 0}$$

Final minimum at $(v_H, v_s, 0)$

Another silver lining:

x_3 has 0 VEV -> no mixing + Z2 symmetry -> DM candidate!

First-Order Phase Transition

Relic density
Indirect detection

Direct detection

Collider
experiments
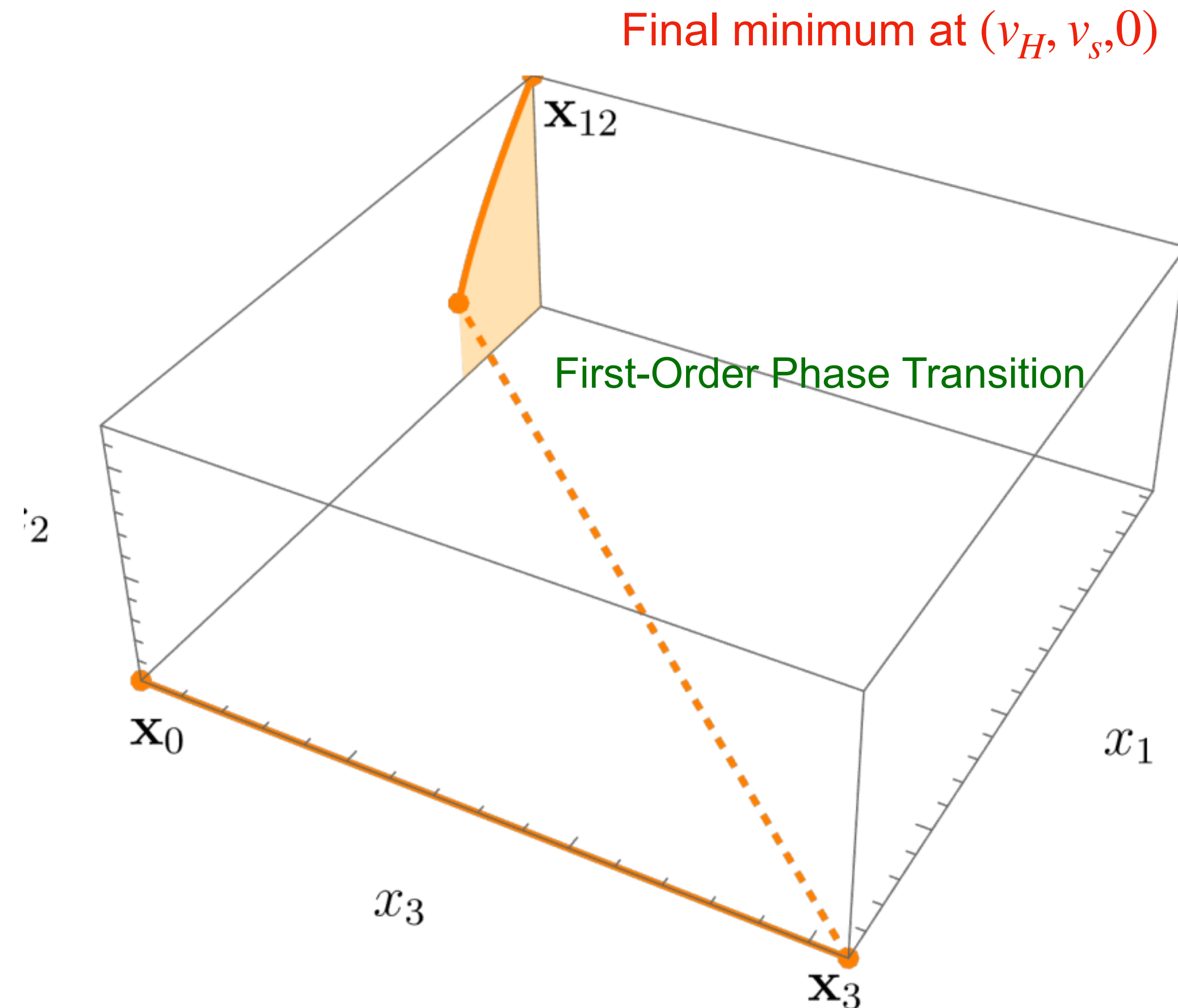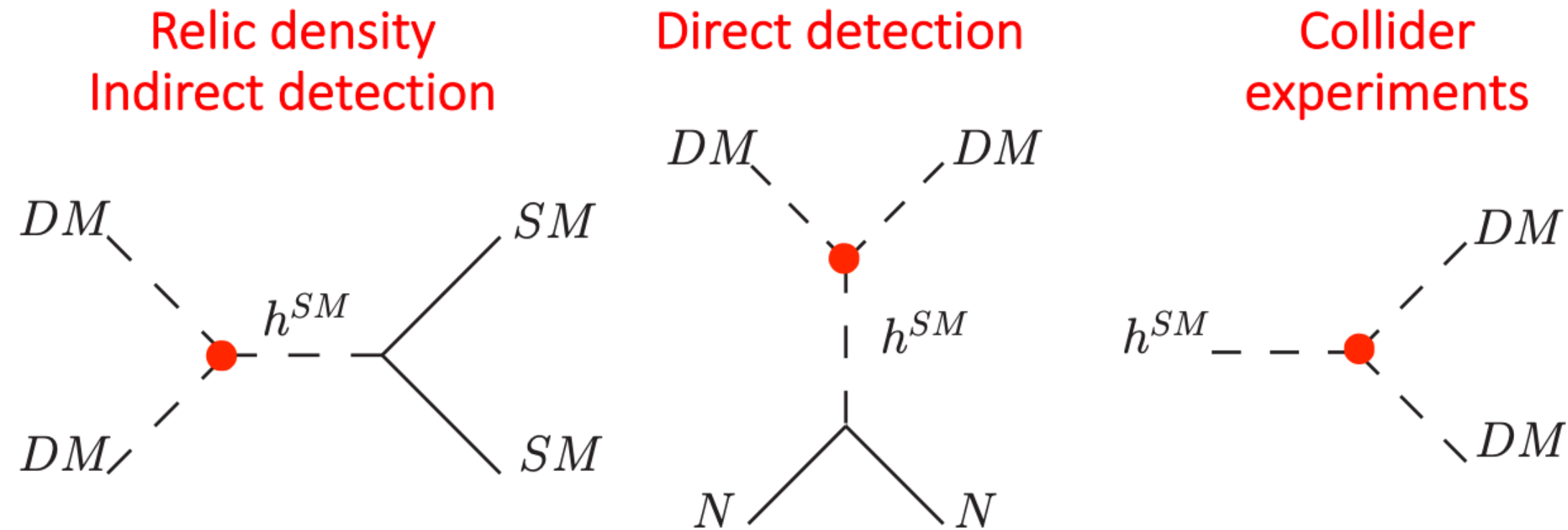
Future study: Model can be constraints from DM observables

Summary

Osama Karkout,[1] Andreas Papaefstathiou,[2] Marieke Postma,[1,3] Gilberto Tetlalmatzi-Xolocotzi,[4,5] Jorinde van de Vis,[6] Tristan du Pree[1]

- Z_2 symmetric TRSM can **enhance HHH** if both scalars have nonzero VEVs at zero temperature (today)

- Z_2 symmetric TRSM can accommodate **First Order Phase Transitions** (desired for matter-antimatter asymmetry)

- Z_2 symmetric TRSM **cannot accommodate both** at the same time! Zero scalar VEV required for FOPT

Ideas to achieve both FOPT and HHH enhancement:

- Add terms that break Z_2 symmetry
- Add yet another scalar ;)

$$V = \mu_\Phi^2 \Phi^\dagger \Phi + \lambda_\Phi (\Phi^\dagger \Phi)^2 + \mu_S^2 S^2 + \lambda_S S^4 + \mu_X^2 X^2 + \lambda_X X^4$$
$$+ \lambda_{\Phi S} \Phi^\dagger \Phi S^2 + \lambda_{\Phi X} \Phi^\dagger \Phi X^2 + \lambda_{SX} S^2 X^2.$$

- Z_2 symmetric TRSM can **enhance HH** and accommodate a **First Order Phase Transition (FOPT)**
- Further studies can include **gravitational waves** and **dark matter** constraints for TRSM

YOU CALLED FOR BACKUP?

Electroweak Baryogenesis

Baryon number violation

In SM: left handed B+L violated!

$$E_{sph} \sim \frac{8\pi v}{g_w}$$

Fig. 8. Energy of gauge field configurations as a function of Chern-Simons number.

v is the Higgs VEV

Electroweak Baryogenesis

Baryon number violation

In SM: left handed B+L violated!

$$\partial_\mu J^\mu_{B_L + L_L} = \frac{3g^2}{32\pi^2} \epsilon_{\alpha\beta\gamma\delta} W_a^{\alpha\beta} W_a^{\gamma\delta}$$

where $W_a^{\alpha\beta}$ is the SU(2) field strength.

$$\Delta B = \Delta L = \pm 3 \qquad\qquad (2.2)$$

Q_1

Q_2

Q_3

L_e

L_μ

L_τ

Fig. 4. The sphaleron.

$V(\phi)$

Sphaleron
ϕ_S

$n = \frac{1}{2}$

real space

$|1\rangle$

Map with $n = 1$

$\mathrm{Im}(\phi)$

$\mathrm{Re}(\phi)$

Map with $n = 0$ (winding number)

$|0\rangle$

E

E_{sph}

$$E_{sph} \sim \frac{8\pi v}{g_w}$$
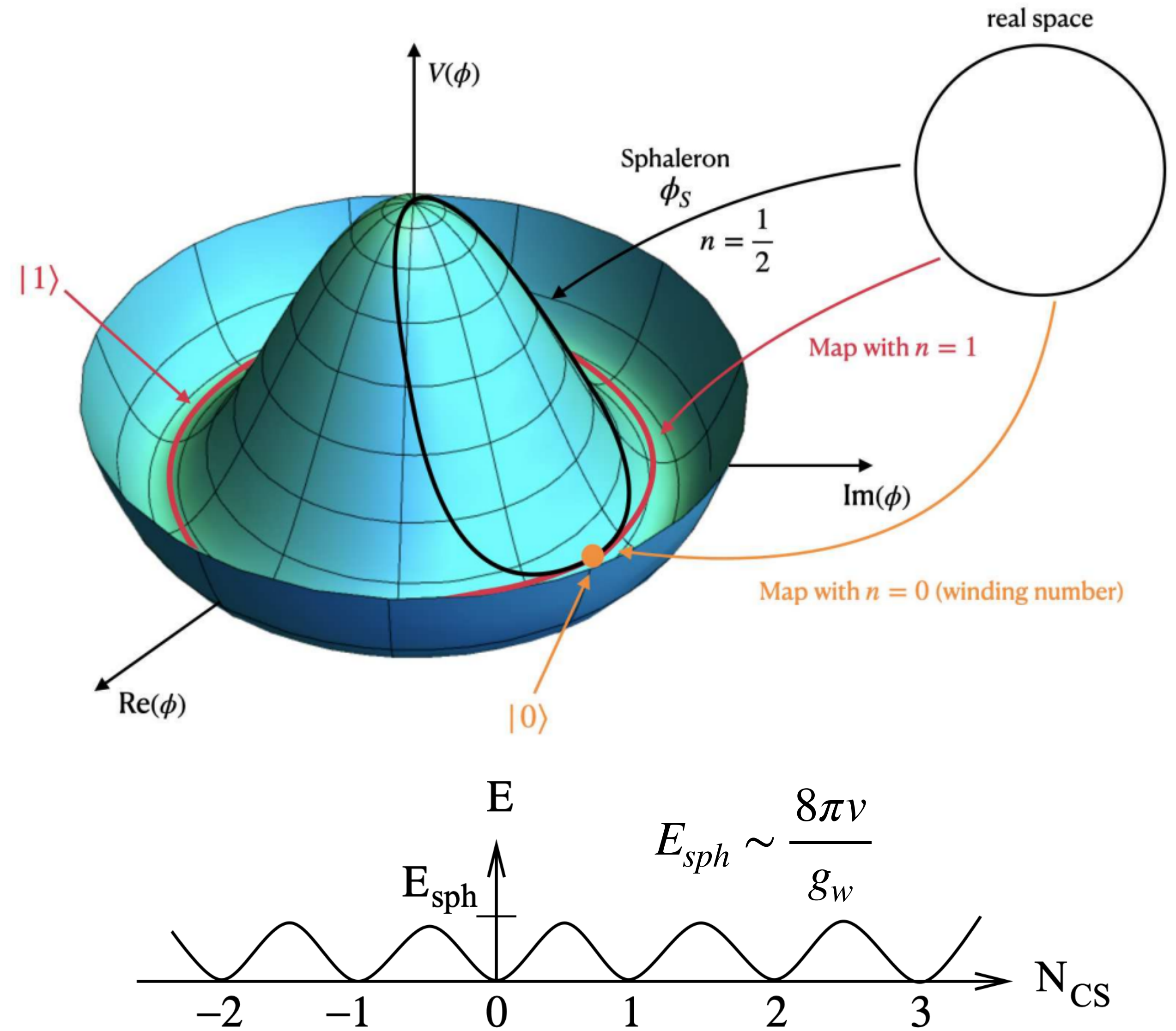
−2   −1   0   1   2   3 N_{CS}
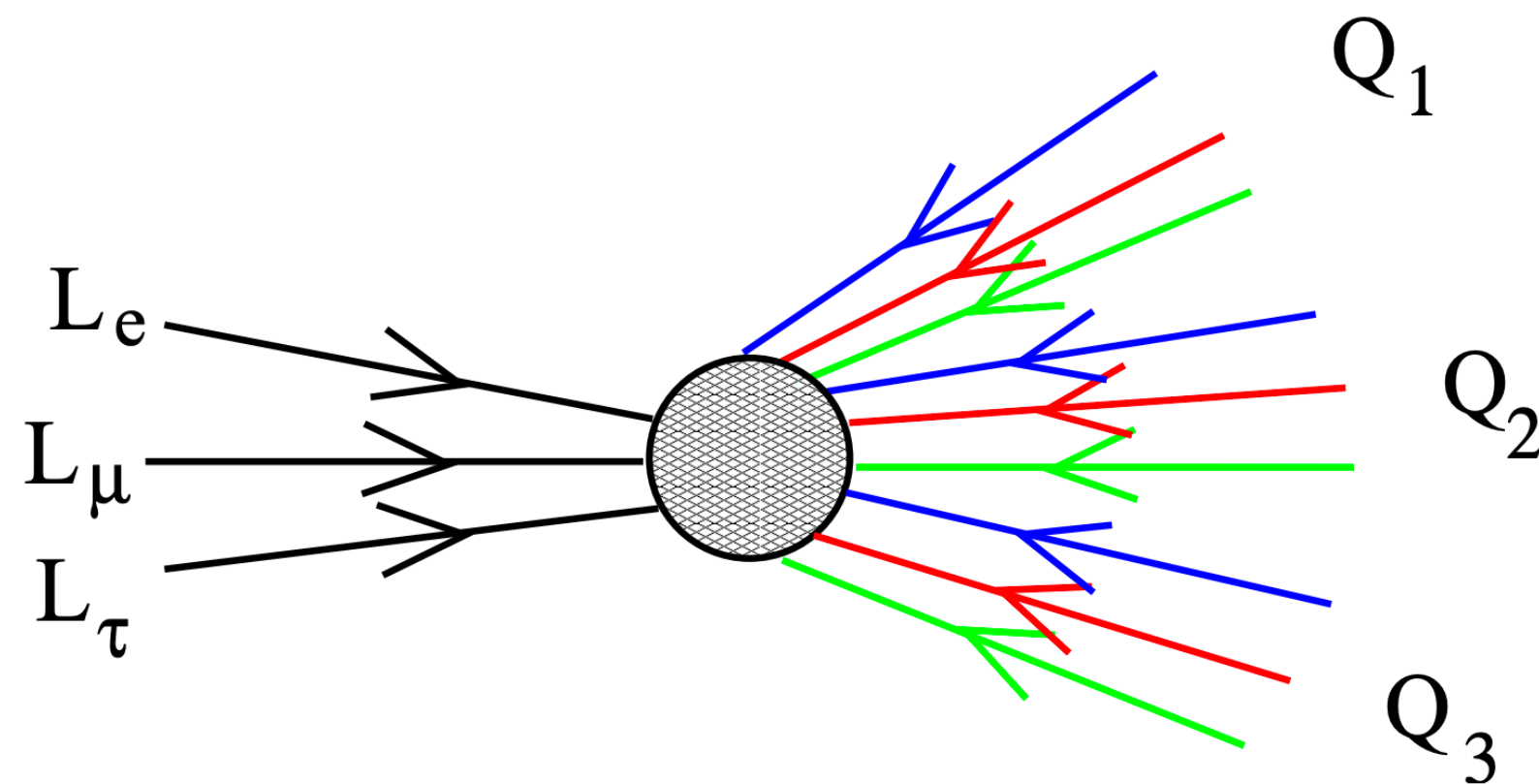
Fig. 8. Energy of gauge field configurations as a function of Chern-Simons number.

v is the Higgs VEV

Electroweak Baryogenesis

Baryon number violation

Rate of tunnelling to another vacuum:

$$\Gamma_{sph}(T) \sim e^{-E_{sph}/T} \sim e^{-v/T}$$

$$\Delta B = \Delta L = \pm 3 \qquad\qquad (2.2)$$

Fig. 4. The sphaleron.

Fig. 8. Energy of gauge field configurations as a function of Chern-Simons number.

$$E_{sph} \sim \frac{8\pi v}{g_w}$$

v is the Higgs VEV

Electroweak Baryogenesis

Baryon number violation

Rate of tunnelling to another vacuum:

$$\Gamma_{sph}(T) \sim e^{-E_{sph}/T} \sim e^{-\boxed{v/T}}$$

$$\Delta B = \Delta L = \pm 3 \qquad\qquad (2.2)$$
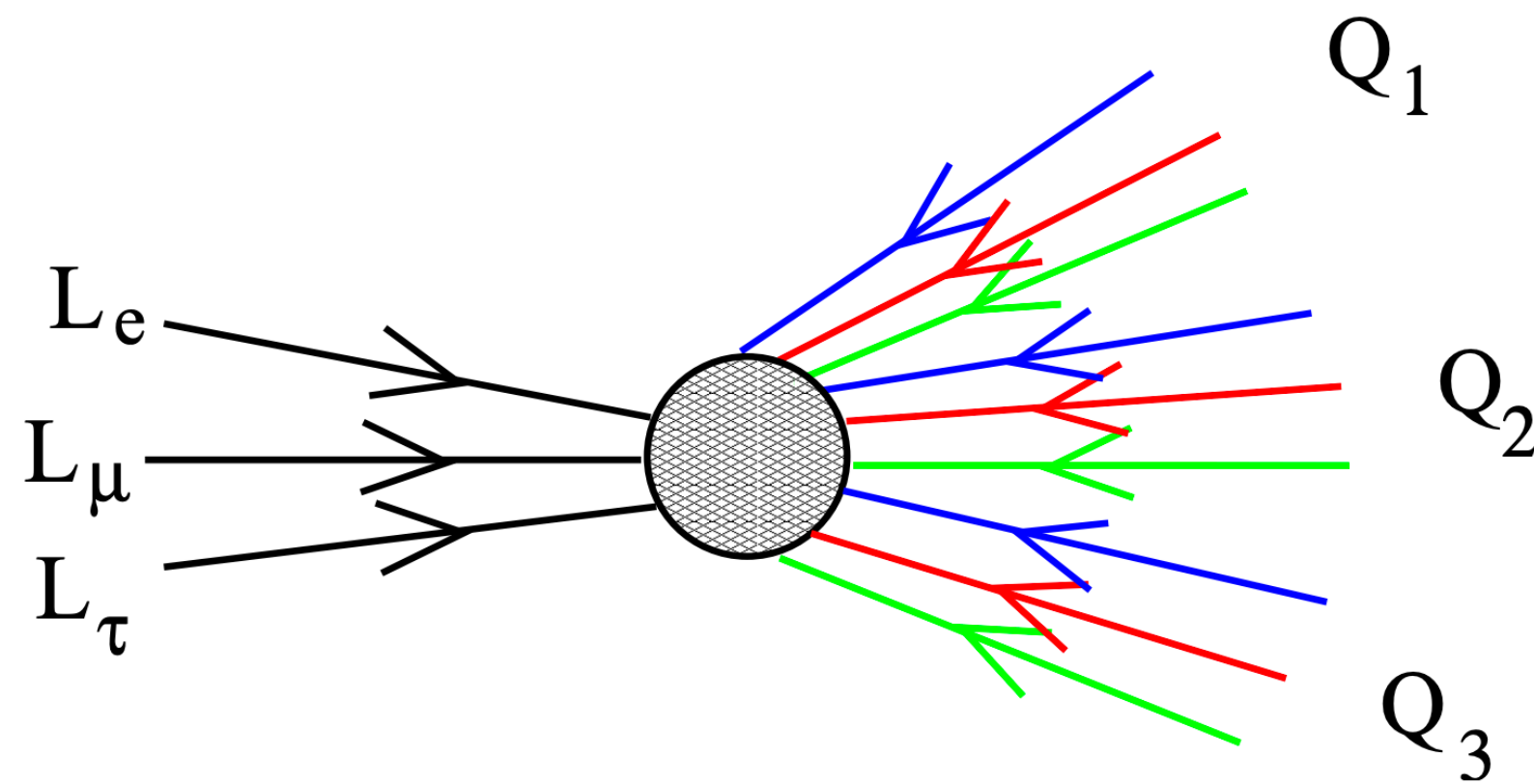
Fig. 4. The sphaleron.
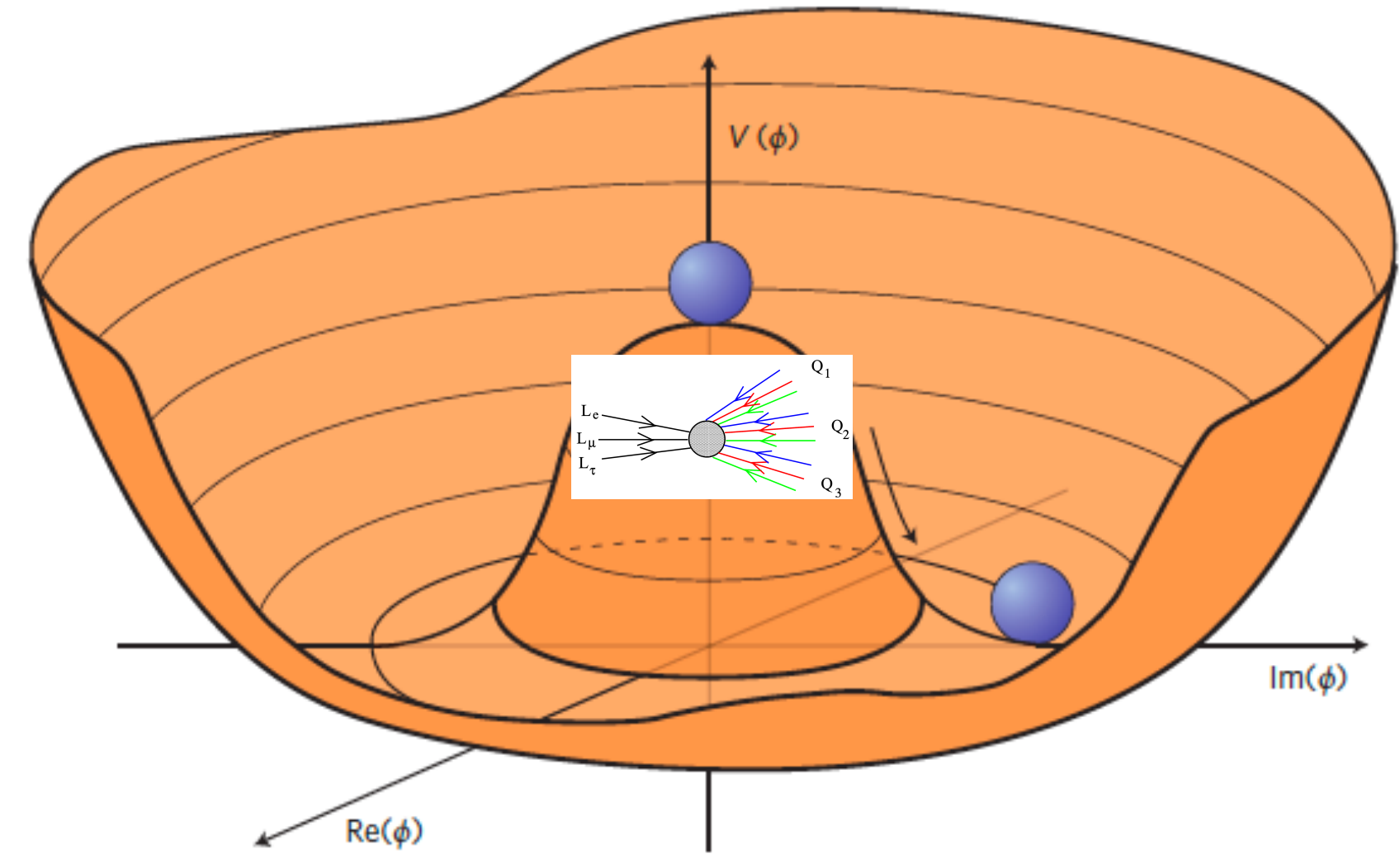
If EW symmetry is restored (VEV=0)
Sphalerons everywhere!
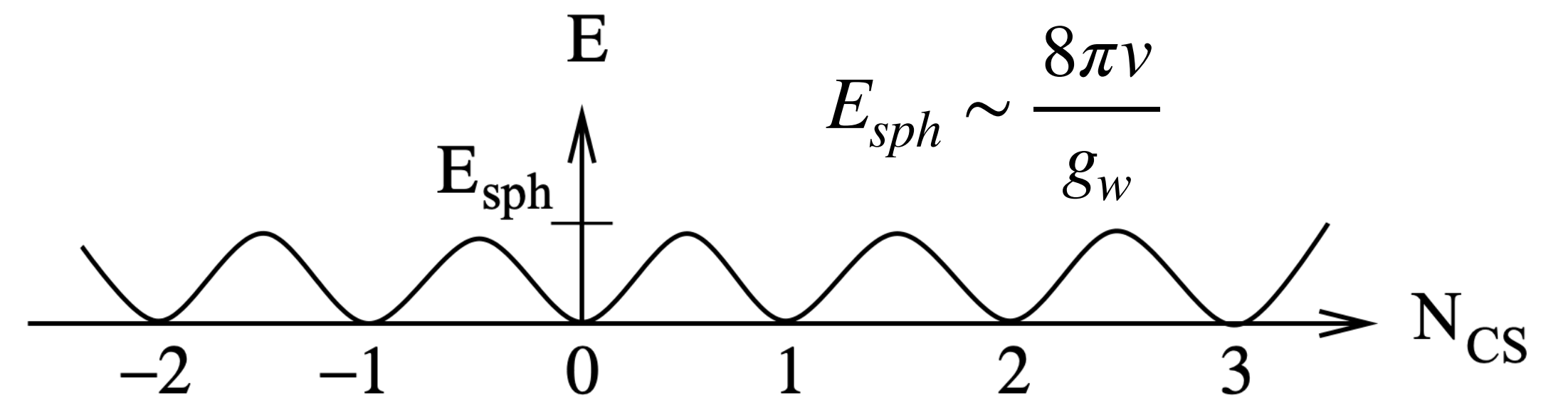
$$E_{sph} \sim \frac{8\pi v}{g_w}$$
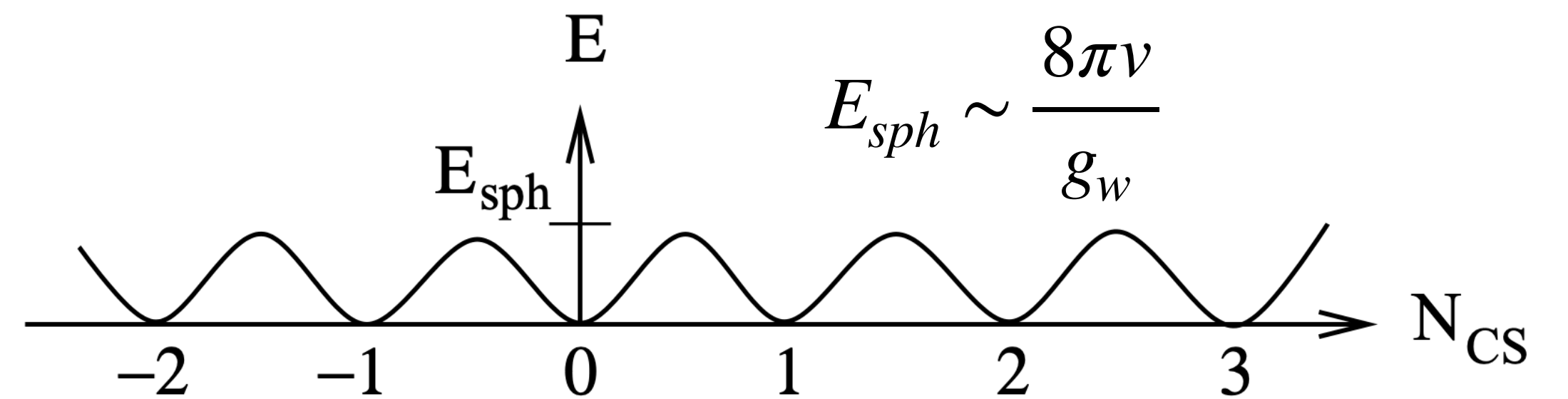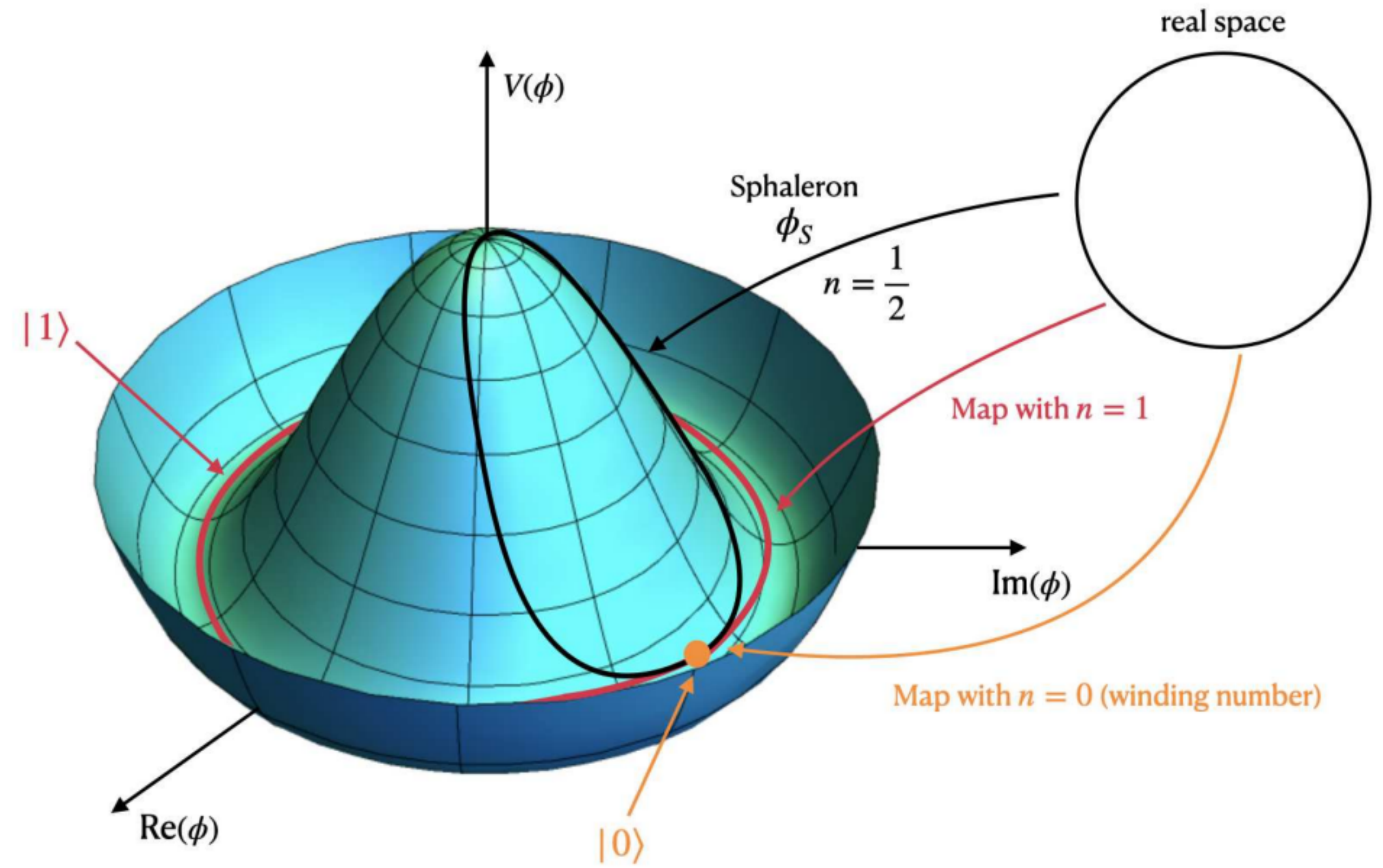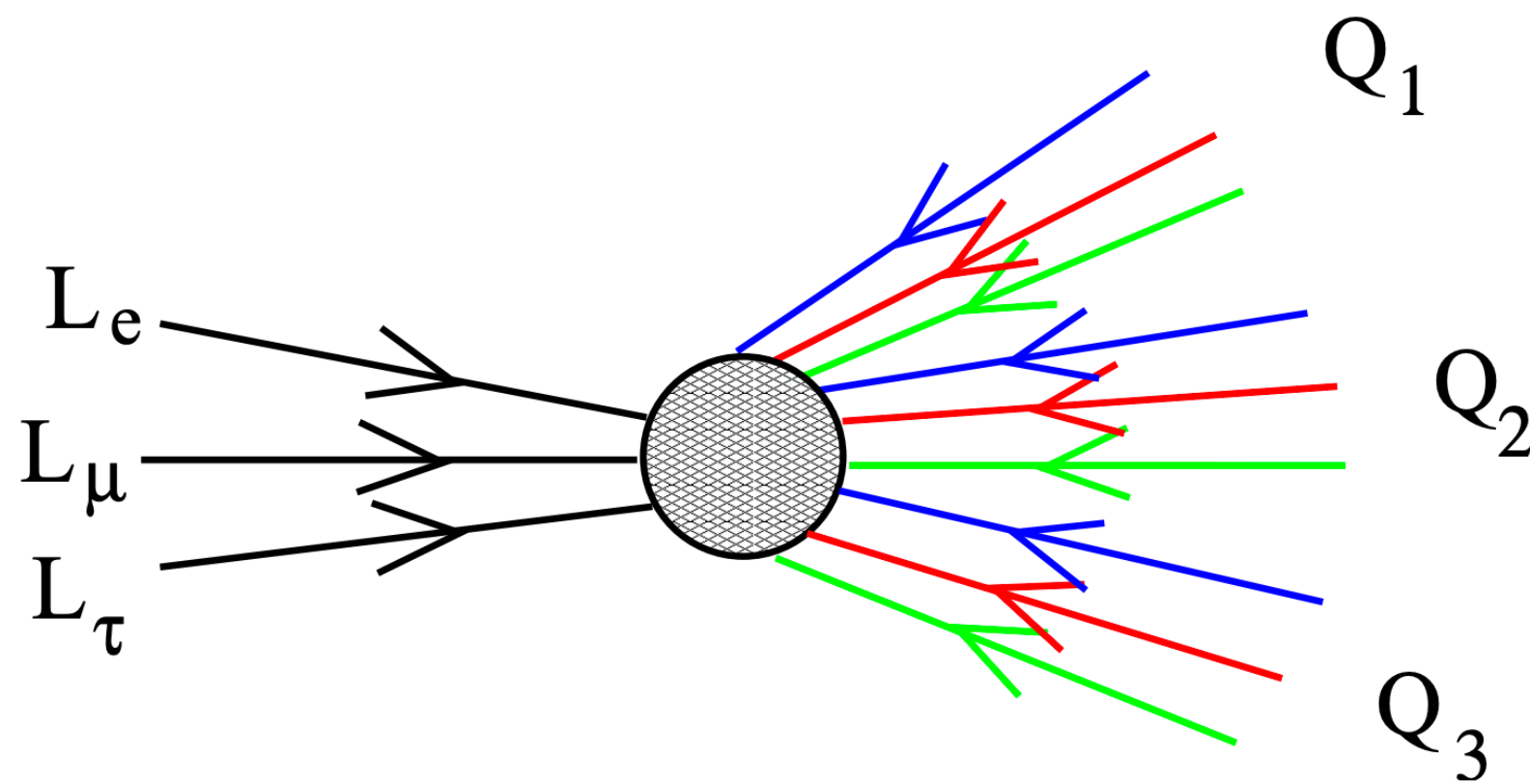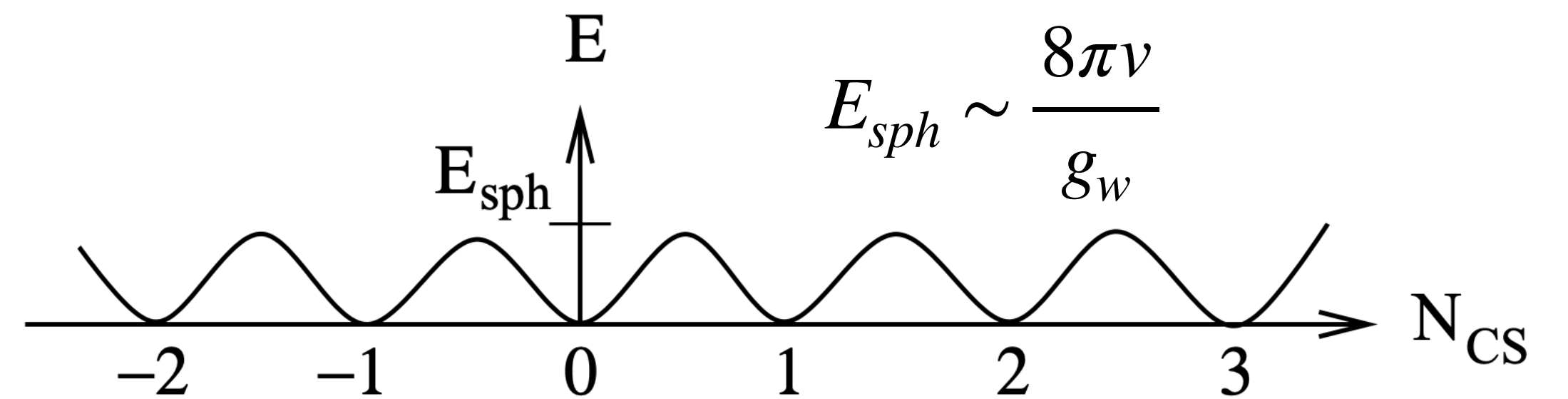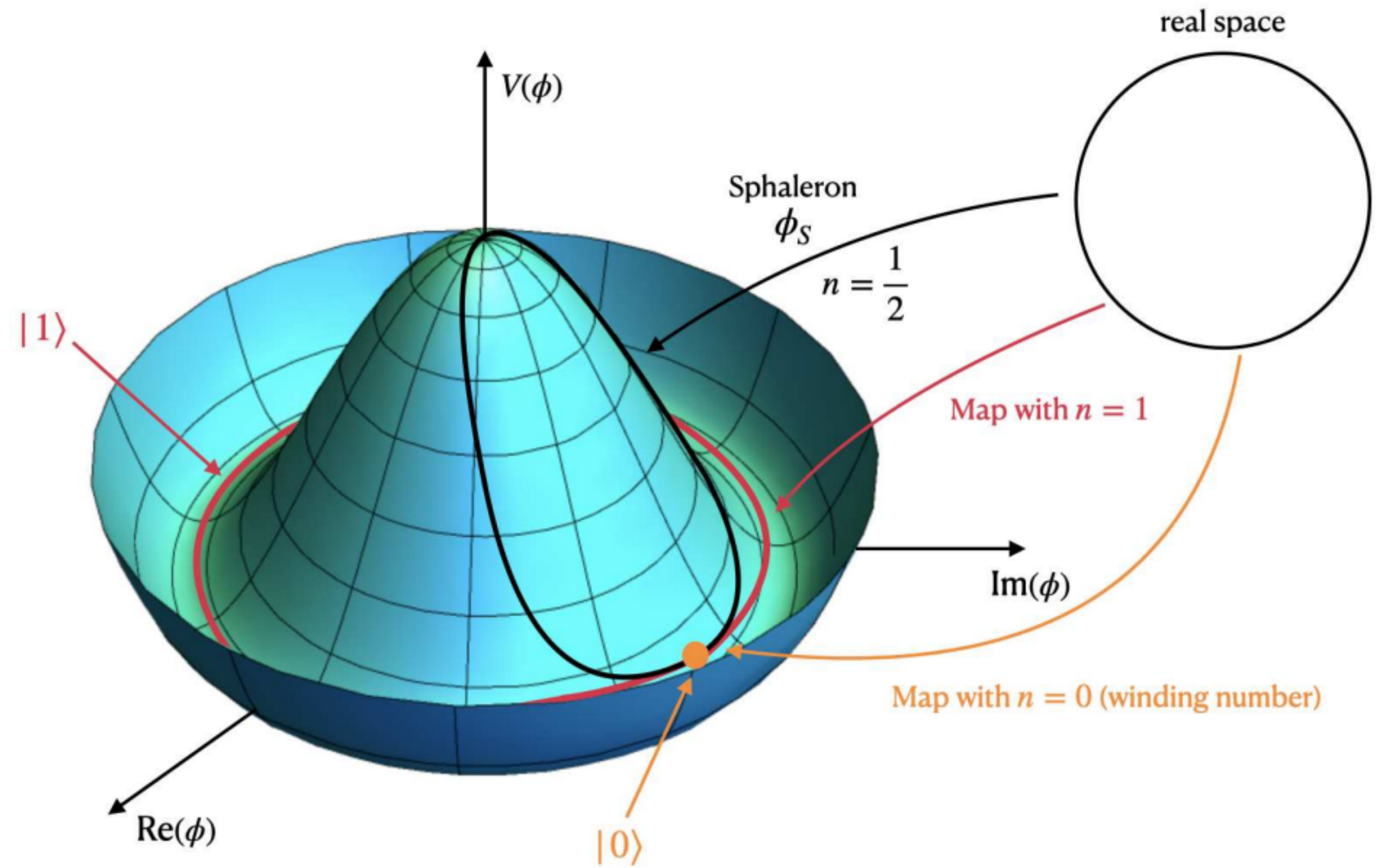

Fig. 8. Energy of gauge field configurations as a function of Chern-Simons number.

v is the Higgs VEV

Electroweak Baryogenesis

Charge and Charge+Parity symmetries (C and CP violation)

$$\begin{aligned} C : \quad & q_L \rightarrow \bar{q}_L \\ CP : \quad & q_L \rightarrow \bar{q}_R \end{aligned}$$

Under C conservation:

$X \rightarrow Y + B$ comes with $\bar{X} \rightarrow \bar{Y} + \bar{B}$

$$\Gamma(\bar{X} \rightarrow \bar{Y} + \bar{B}) = \Gamma(X \rightarrow Y + B)$$

The net rate of baryon production goes like the difference of these rates,

$$\frac{dB}{dt} \propto \Gamma(\bar{X} \rightarrow \bar{Y} + \bar{B}) - \Gamma(X \rightarrow Y + B)$$

CP violation is a longer story but also needed

Electroweak Baryogenesis

Charge and Charge+Parity symmetries (C and CP violation)

$$C: \quad q_L \rightarrow \bar{q}_L$$
$$CP: \quad q_L \rightarrow \bar{q}_R$$

In SM: CP violation in CKM matrix. Not enough though! BSM CP violation is more than welcomed.

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_2 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix}$$

Electroweak Baryogenesis

Charge and Charge+Parity symmetries (C and CP violation)

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & \begin{array}{c} c_1 c_2 c_3 \\ -s_2 s_3 e^{i\delta} \end{array} & \begin{array}{c} c_1 c_2 s_3 \\ +s_2 c_3 e^{i\delta} \end{array} \\ s_1 s_2 & \begin{array}{c} c_1 s_2 c_3 \\ +c_2 s_3 e^{i\delta} \end{array} & \begin{array}{c} c_2 s_2 s_3 \\ -c_2 c_3 e^{i\delta} \end{array} \end{pmatrix}$$

Backup

Osama Karkout,[1] Andreas Papaefstathiou,[2] Marieke Postma,[1,3] Gilberto Tetlalmatzi-Xolocotzi,[4,5] Jorinde van de Vis,[6] Tristan du Pree[1]

For the actual scan we have generated $530,000$ random points over the phase space defined by $M_2, M_3, v_2, v_3, \theta_{12}, \theta_{13}, \theta_{23}$. The ranges considered are as follows:

$$M_2 \in [255, 700]\ \text{GeV}, \qquad\qquad M_3 \in [350, 900]\ \text{GeV},$$
$$v_2 \in [0, 1000]\ \text{GeV}, \qquad\qquad v_3 \in [50, 1000]\ \text{GeV}. \qquad (3.1)$$

For the mixing angles $\theta_{12}, \theta_{13}, \theta_{23}$ we impose the following limits on the scaling factors [38, 68] of eq. (2.4):

$$0.95 \leqslant \kappa_1 \leqslant 1.00, \quad 0.0 \leqslant \kappa_2 \leqslant 0.25, \quad 0.0 \leqslant \kappa_3 \leqslant 0.25. \qquad (3.2)$$

Backup

Osama Karkout,[1] Andreas Papaefstathiou,[2] Marieke Postma,[1,3] Gilberto stan du Pree[1]

Viable points with $\sigma > 10 \times \sigma_{SM}(gg \rightarrow hhh)$@13.6 TeV

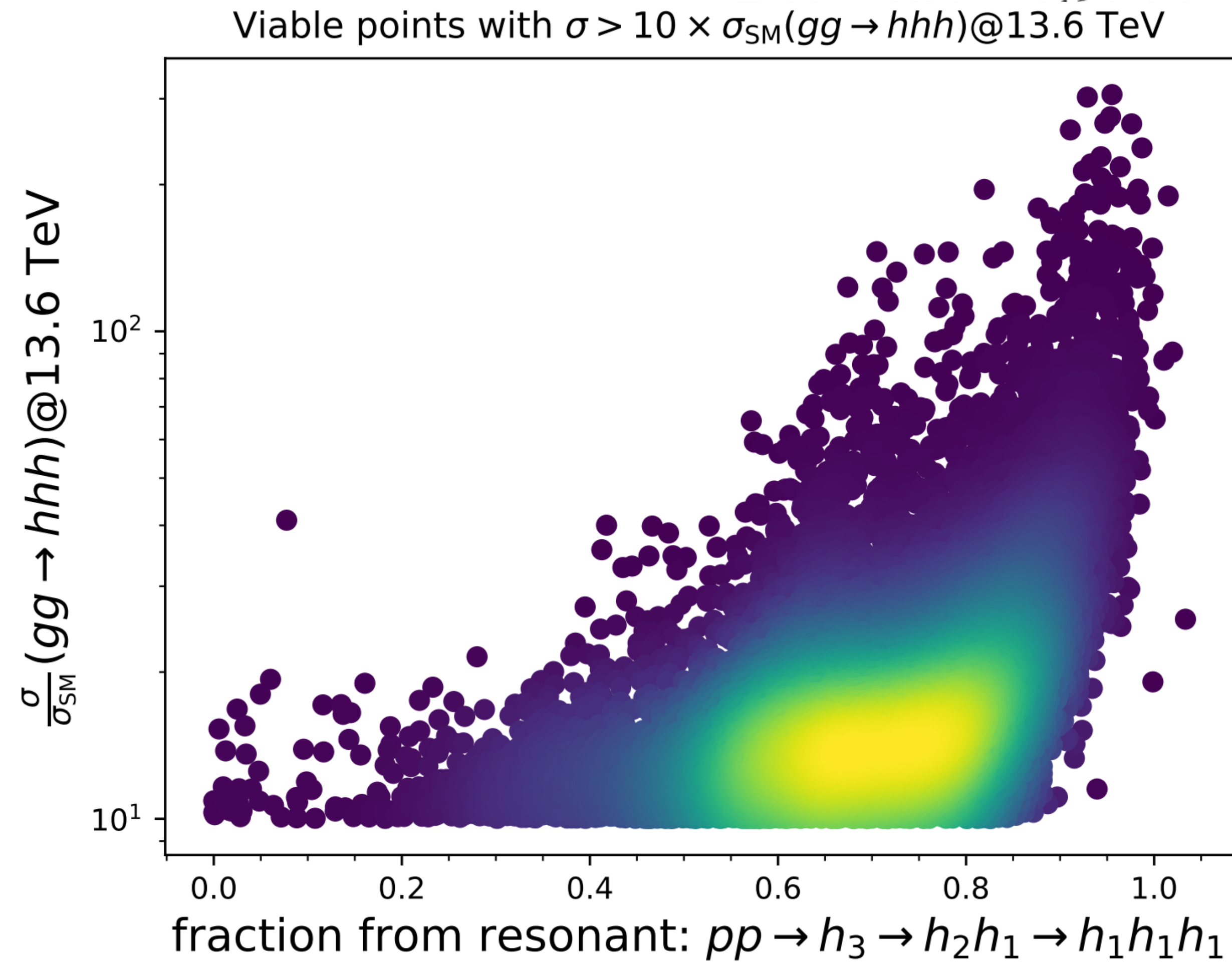**Figure 2**: Enhancement of the triple Higgs boson production cross section $\sigma(pp \rightarrow h_1 h_1 h_1)$ at 13.6 TeV, given in terms of multiples of the SM value, and the resonant fraction contribution from $pp \rightarrow h_3 \rightarrow h_2 h_1 \rightarrow h_1 h_1 h_1$. Only points with a factor 10 enhancement or greater are shown. The density of points increases from the dark blue to yellow shade.

Backup

Osama Karkout,[1] Andreas Papaefstathiou,[2] Marieke Postma,[1,3] Gilberto Tetlalmatzi-Xolocotzi,[4,5] Jorinde van de Vis,[6] Tristan du Pree[1]

https://arxiv.org/pdf/2404.12425

Benchmark points for enhanced triple Higgs production

M_2	M_3	v_2	v_3	θ_{12}	θ_{13}	θ_{23}	$\frac{\sigma}{\sigma_{SM}}$	Res. Frac.	μ_{pert}	$\frac{\mu_{\mathrm{pert}}}{\mu_{\mathrm{pole}}}$
259.0	495.0	215.8	180.8	6.191	0.163	5.691	306.025	0.955	2.7×10^2	7.3
270.6	444.7	122.4	847.2	0.268	0.030	0.522	302.361	0.929	1.8×10^2	7.3
268.6	452.7	137.8	784.8	0.263	0.023	0.645	275.616	0.954	2.4×10^2	7.3
272.6	480.7	928.3	143.7	3.098	2.9	2.375	267.245	0.948	1.4×10^2	7.2
269.0	409.8	138.0	599.4	0.244	0.004	0.773	266.439	0.976	2.4×10^2	7.2
269.1	486.9	227.5	307.9	0.074	6.149	2.631	157.583	0.956	4.3×10^2	8.0
259.2	577.0	289.0	275.6	0.137	6.148	2.324	145.470	0.781	1.2×10^4	7.2
283.7	575.0	259.4	330.4	0.137	6.152	2.299	122.546	0.779	3.0×10^3	7.2
264.3	469.3	207.3	359.5	0.285	6.277	0.692	119.121	0.999	5.4×10^3	7.3
266.5	461.9	653.1	229.0	2.889	3.046	1.015	112.794	0.863	5.3×10^4	8.0
259.2	399.7	444.5	217.0	2.917	3.046	1.047	103.717	0.973	1.2×10^5	8.0

Backup

Osama Karkout,[1] Andreas Papaefstathiou,[2] Marieke Postma,[1,3] Gilberto Tetlalmatzi-Xolocotzi,[4,5] Jorinde van de Vis,[6] Tristan du Pree[1]

The one-loop TRSM effective potential at finite temperature is:

$$V_T(\phi_i, T) = V(\phi_i) + V_{\mathrm{CW}}(\phi_i) + V_{\mathrm{c.t.}}(\phi_i) + V_{T,1-\mathrm{loop}}(\phi_i, T), \qquad (4.2)$$

with ϕ_i the field values defined in eq. (2.2) (with $\phi_i = v_i$ in the vacuum today). $V(\phi_i)$ is the tree-level potential of eq. (2.1), V_{CW} the standard zero-temperature one-loop 'Coleman-Weinberg' potential and $V_{\mathrm{c.t.}}$ the corresponding counterterms. The temperature-corrections are captured by $V_{T,1-\mathrm{loop}}$, which is given by

$$V_{T,1-\mathrm{loop}}(\phi, T) = \frac{T^4}{2\pi^2}\left[\sum_{\alpha=\Phi_i,W,Z} n_\alpha J_B[m_\alpha^2(\phi_i)/T^2] + n_t J_F[m_t^2(\phi_i)/T^2]\right]. \qquad (4.3)$$

Backup

Osama Karkout,[1] Andreas Papaefstathiou,[2] Marieke Postma,[1,3] Gilberto Tetlalmatzi-Xolocotzi,[4,5] Jorinde van de Vis,[6] Tristan du Pree[1]

https://arxiv.org/pdf/2404.12425

At temperatures large compared to the mass, the functions $J_{B,F}$ can be expanded in $m_\alpha^2(\phi_i)/T^2$ as

$$J_B(m_\alpha^2/T^2) = -\frac{\pi^4}{45} + \frac{\pi^2}{24}\frac{m_\alpha^2}{T^2} - \frac{\pi}{6}\frac{m_\alpha^3}{T^3} - \frac{1}{32}\frac{m_\alpha^4}{T^4}\left(\log\frac{m_\alpha^2}{16\pi^2 T^2} - \frac{3}{2} + 2\gamma_E\right)\cdots,$$

$$J_F(m_\alpha^2/T^2) = \frac{7\pi^4}{360} - \frac{\pi^2}{24}\frac{m_\alpha^2}{T^2} - \frac{1}{32}\frac{m_\alpha^4}{T^4}\left(\log\frac{m_\alpha^2}{\pi^2 T^2} - \frac{3}{2} + 2\gamma_E\right)\cdots, \tag{4.5}$$

Backup

Osama Karkout,[1] Andreas Papaefstathiou,[2] Marieke Postma,[1,3] Gilberto Tetlalmatzi-Xolocotzi,[4,5] Jorinde van de Vis,[6] Tristan du Pree[1]

https://arxiv.org/pdf/2404.12425

A.3 RGEs

The one-loop RGEs for the quartic couplings are

$$(4\pi)^2\beta_{\lambda_{11}} = 24\lambda_{11}^2 + \frac{\lambda_{22}^2}{2} + \frac{\lambda_{33}^2}{2} + \frac{3}{8}g_1^4 + \frac{9}{8}g_2^4 + \frac{3}{4}g_1^2g_2^2 - 6y_t^4 - 4\lambda_{11}\gamma_{\Phi_1},$$

$$(4\pi)^2\beta_{\lambda_{22}} = 18\lambda_{22}^2 + 2\lambda_{12}^2 + \frac{\lambda_{23}^2}{2},$$

$$(4\pi)^2\beta_{\lambda_{33}} = 18\lambda_{33}^2 + 2\lambda_{13}^2 + \frac{\lambda_{23}^2}{2},$$

$$(4\pi)^2\beta_{\lambda_{12}} = 4\lambda_{12}^2 + 12\lambda_{12}\lambda_{11} + 6\lambda_{12}\lambda_{22} + \lambda_{13}\lambda_{23} - 2\lambda_{12}\gamma_{\Phi_1},$$

$$(4\pi)^2\beta_{\lambda_{13}} = 4\lambda_{13}^2 + 12\lambda_{13}\lambda_{11} + 6\lambda_{13}\lambda_{33} + \lambda_{12}\lambda_{23} - 2\lambda_{13}\gamma_{\Phi_1},$$

$$(4\pi)^2\beta_{\lambda_{23}} = 4\lambda_{23}^2 + 6\lambda_{23}\lambda_{22} + 6\lambda_{23}\lambda_{33} + 4\lambda_{12}\lambda_{13}, \tag{A.5}$$

with $\beta_\lambda = \mu\partial\lambda/\partial\mu$ and $\gamma_{\Phi_1} = \left(\frac{3g_1^2}{4} + \frac{9g_2^2}{4} - 3y_t^2\right)$. The running of the gauge couplings and the top quark is as in the SM

$$(4\pi)^2\beta_{g_i} = b_i g_i^3,$$

$$(4\pi)^2\beta_{y_t} = \frac{9}{2}y_t^3 - y_t\left(\frac{2}{3}g_1^2 + 9g_3^2\right) - y_t\gamma_{\Phi_1}, \tag{A.6}$$

with $b_i = (41/6, -19/6, -7)$ for $i = 1, 2, 3$.

How to enhance HHH

$$\mathcal{L} = -\bar{\lambda}_{abc} h_a h_b h_c - \frac{1}{2} \bar{\lambda}_{aab} h_a^2 h_b - \frac{1}{3!} \bar{\lambda}_{aaab} h_a^3 h_b + \dots , \tag{2.5}$$

with

$$\bar{\lambda}_{abc} = (M_a^2 + M_b^2 + M_c^2) \sum_j \frac{R_{aj} R_{bj} R_{cj}}{v_j},$$

$$\bar{\lambda}_{aaab} = (3!) \sum_{ijk} \frac{M_k^2}{v_i v_j} R_{ki} R_{kj} (R_{ai}^2 R_{aj} R_{bj} + R_{ai} R_{bi} R_{aj}^2), \tag{2.6}$$

and R the mixing matrix of eq. (A.3). The tree-level amplitudes can then be written as (up to symmetry factors)

$$\mathcal{A}_1 \sim (\mathcal{A}_{pp \to h_1}^{\text{SM}} \kappa_3) \times \frac{\bar{\lambda}_{321} \bar{\lambda}_{211}}{D_3(p) D_2(p')}, \qquad \mathcal{A}_2^{(a)} \sim (\mathcal{A}_{pp \to h_1}^{\text{SM}} \kappa_a) \times \frac{\bar{\lambda}_{a111}}{D_a(p)}. \tag{2.7}$$
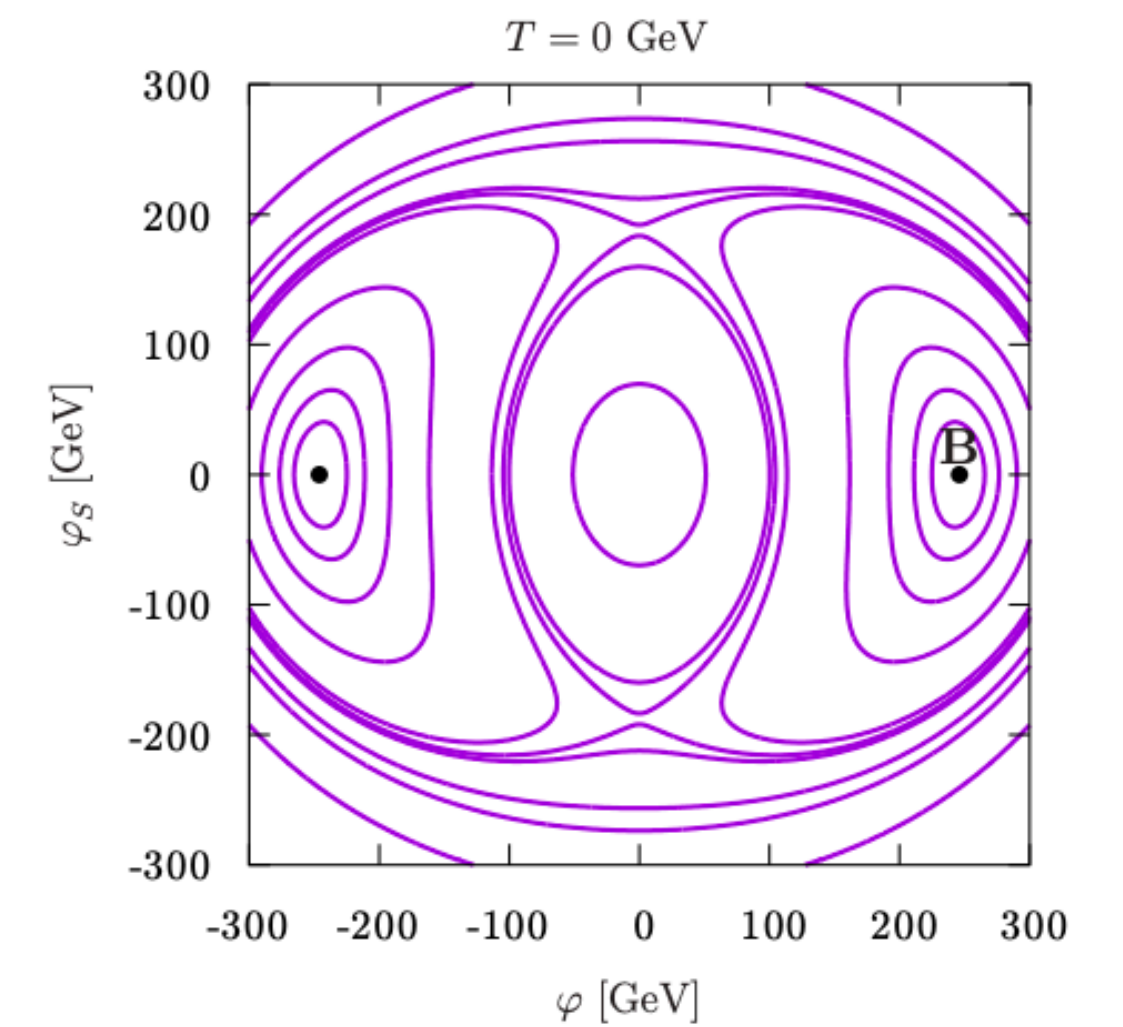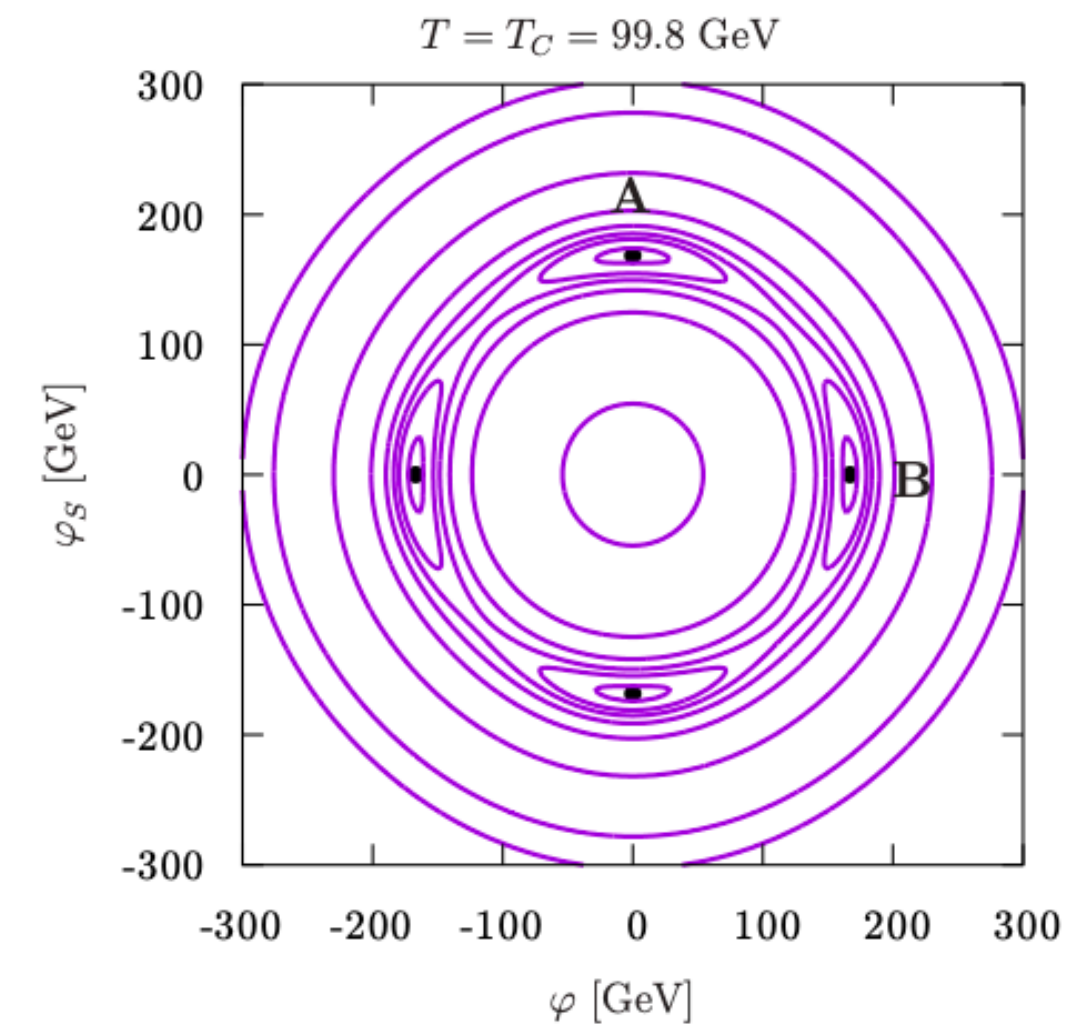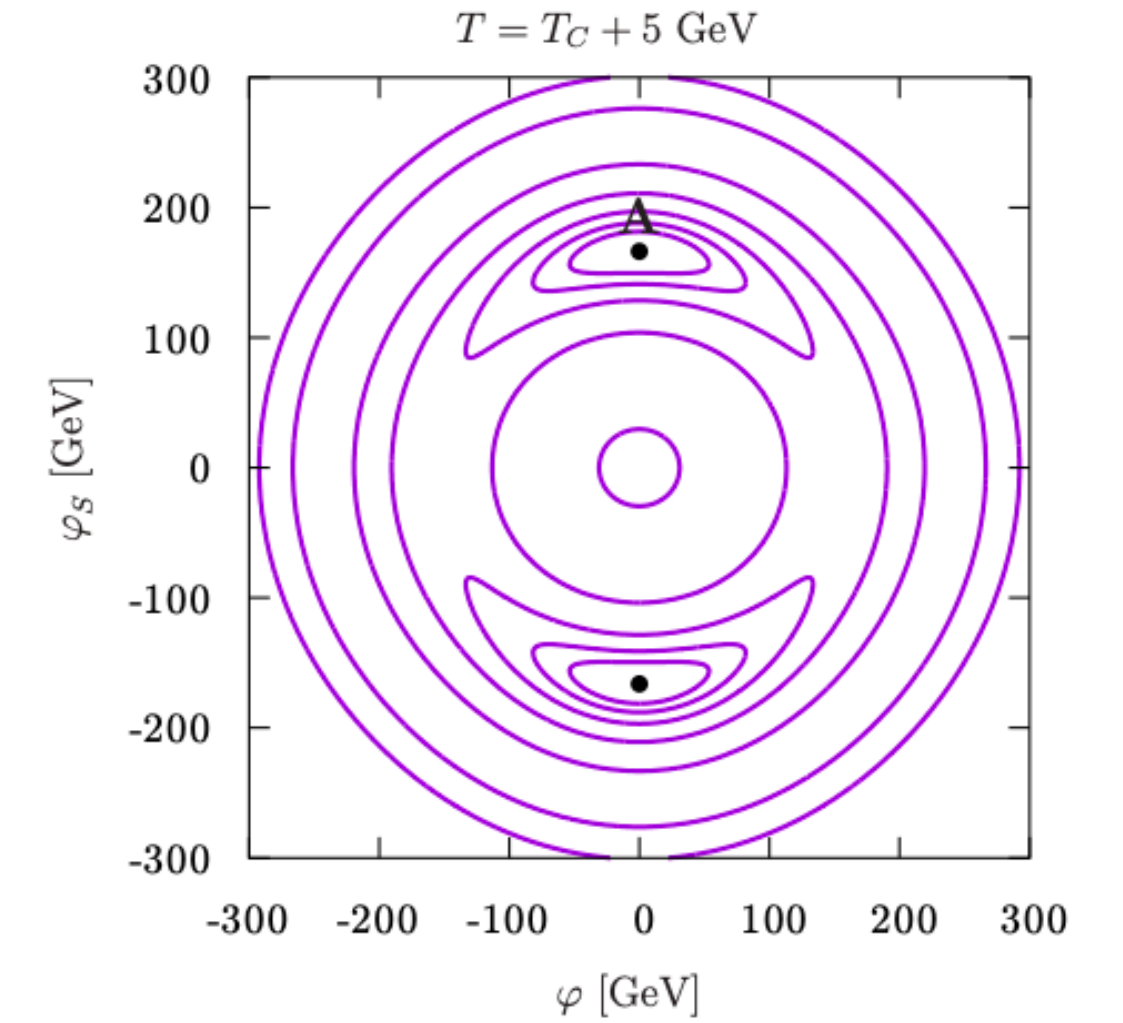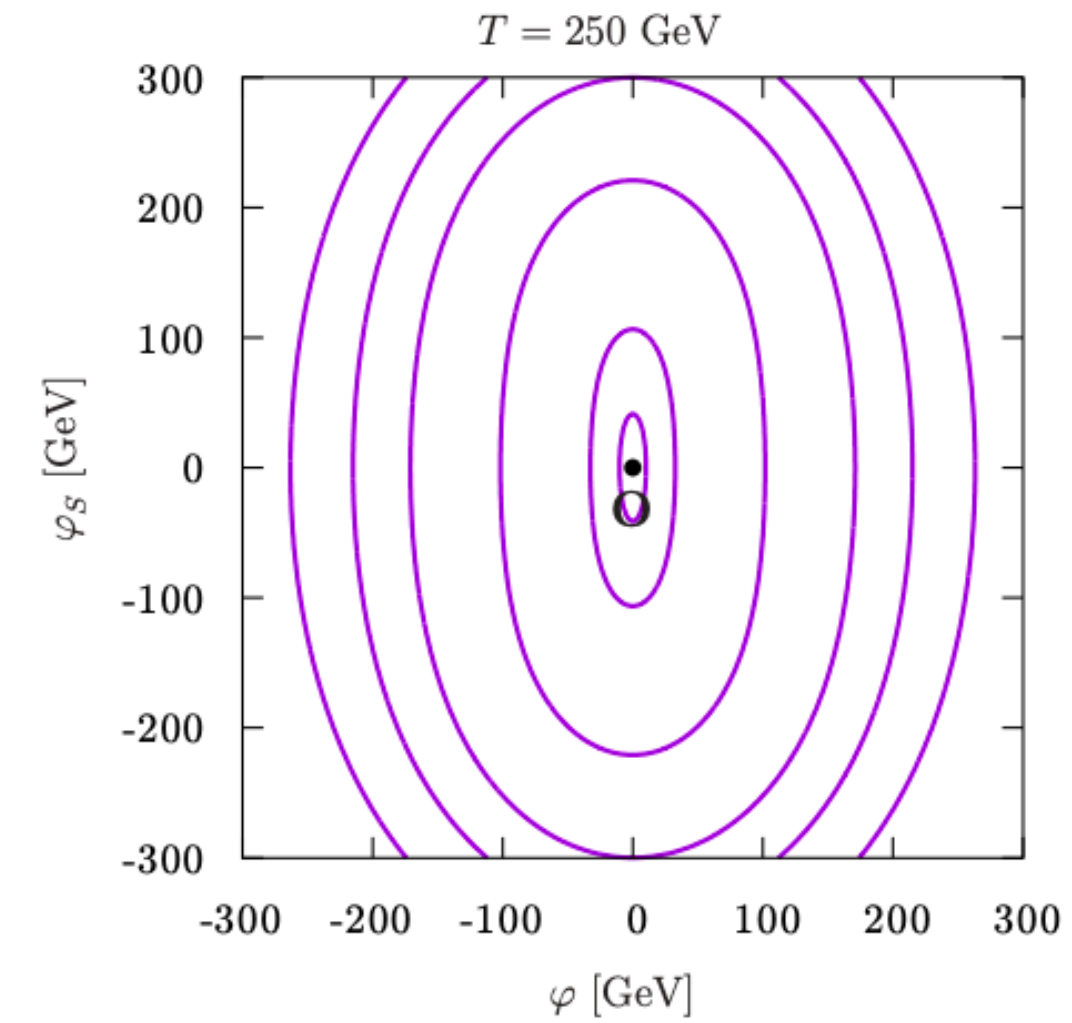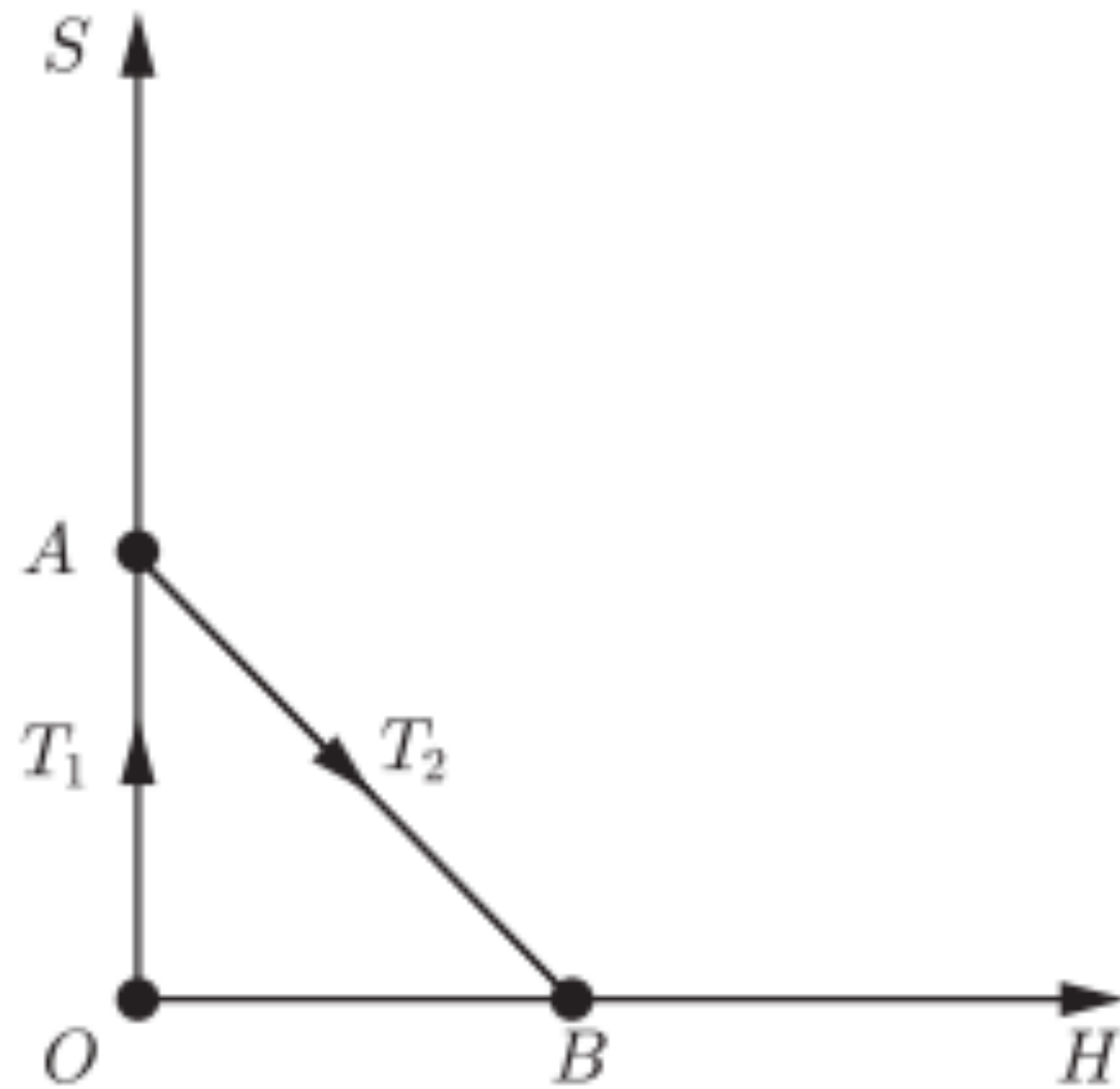
The inverse propagators are $D_a(p) = p^2 - M_a^2 + i M_a \Gamma_a$, with p the momentum flowing through the propagator, and Γ_a the decay width of h_a. On resonance, we have $|p^2 - M_a^2| \ll |M_a \Gamma_a|$.

Electroweak Phase Transition: BSM

Adding a scalar field can make a two-step FOPT!

$$V = -\frac{1}{2}\mu_h^2 h^2 + \frac{1}{4}\lambda_h h^4 + \frac{1}{2}\mu_s^2 s^2 + \frac{1}{4}\lambda_s s^4 + \frac{1}{4}\mu_m s h^2 + \frac{1}{4}\lambda_m s^2 h^2 + \mu_1^3 s + \frac{1}{3}\mu_3 s^3$$

$$V^{\text{high-}T}(\varphi, \varphi_S; T) = V_0(\varphi, \varphi_S) + \frac{1}{2}\left(\Sigma_H \varphi^2 + \frac{1}{2}\Sigma_S \varphi_S^2\right)T^2$$

Cheng-Wei Chiang,[1,2,3,4,*] Michael J. Ramsey-Musolf,[5,6,†] and Eibun Senaha[1,7,‡]

How to enhance HHH

Simplified BSM model predicting large HHH: **TRSM**.

SM + two singlets coupling to the Higgs.

$$V = \mu_\Phi^2 \Phi^\dagger \Phi + \lambda_\Phi \left(\Phi^\dagger \Phi\right)^2 + \mu_S^2 S^2 + \lambda_S S^4 + \mu_X^2 X^2 + \lambda_X X^4$$
$$+ \lambda_{\Phi S} \Phi^\dagger \Phi S^2 + \lambda_{\Phi X} \Phi^\dagger \Phi X^2 + \lambda_{SX} S^2 X^2 .$$
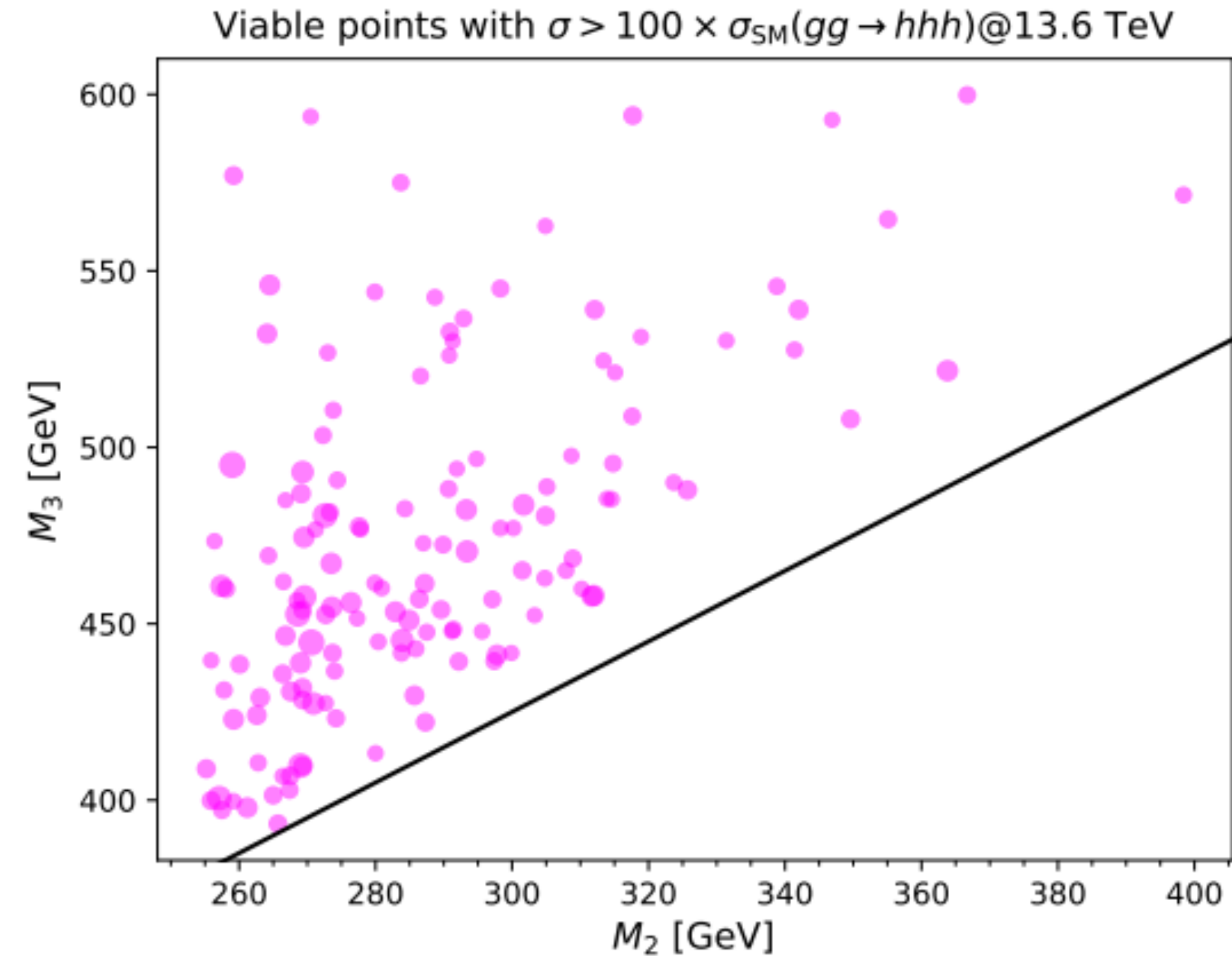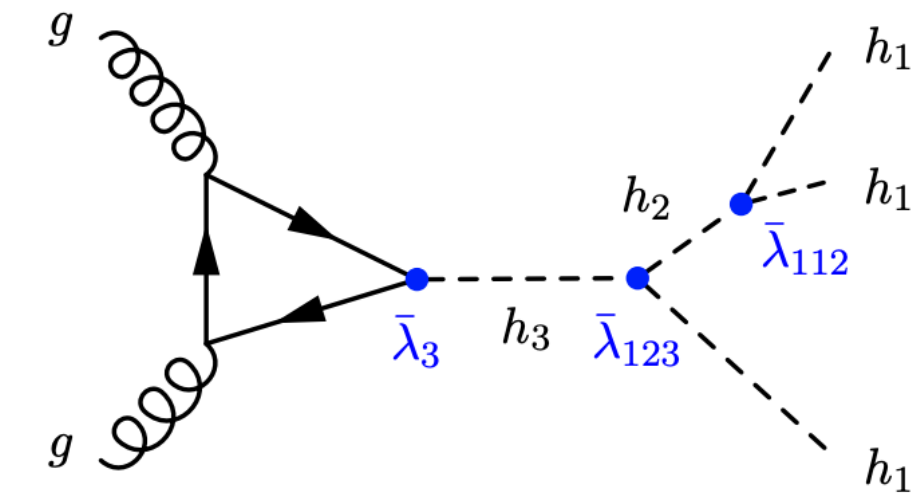
Scalars get VEVs! $\rightarrow$ Mixing:

$$\Phi = \begin{pmatrix} 0 \\ \frac{\phi_h + v}{\sqrt{2}} \end{pmatrix}, \quad S = \frac{\phi_S + v_S}{\sqrt{2}}, \quad X = \frac{\phi_X + v_X}{\sqrt{2}}$$

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = R \begin{pmatrix} \phi_h \\ \phi_S \\ \phi_X \end{pmatrix}$$

h1 can be our scalar particle of 125 GeV

Tania Robens,[1,*] Tim Stefaniak,[2,†] and Jonas Wittbrodt[2,‡]

Viable points with $\sigma > 100 \times \sigma_{SM}(gg \rightarrow hhh)$@13.6 TeV

Osama Karkout,[1] Andreas Papaefstathiou,[2] Marieke Postma,[1,3] Gilberto Tetlalmatzi-Xolocotzi,[4,5] Jorinde van de Vis,[6] Tristan du Pree[1]

https://arxiv.org/pdf/2404.12425